$\bar{p}$ Note #36

Accumulation Rate of $\bar{p}$ in the Scheme Main Ring - Precooler - Storage Ring

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In this note we have made estimates of accumulating speed of antiprotons in the scheme $p\bar{p}$ colliding beams at Fermilab. This is based upon experimental data on electron cooling (Novosibirsk, 1978).[1] It is possible to describe the results by two formulas which define the dependence of the cooling rate upon different parameters:

1. Damping rate of transverse oscillation

$$\lambda_{\perp} = \frac{21\, r_e\, c\, \Delta Q}{\beta^2 \gamma^2 R_0 \beta^* \left[\alpha^2 + \theta_{\perp}^2 + 11 \frac{\theta_{\parallel}^2}{\gamma^2}\right] \sqrt{\theta_{\alpha}^2 + \theta_{\perp}^2 + \frac{\theta_{\parallel}^2}{\gamma^2}}} \tag{1}$$

2. Longitudinal drag force

$$F'_{\parallel} = \frac{d \Delta P_{\parallel}}{dt} = \frac{12\, r_e\, mc^2\, \Delta Q}{\beta^2 \gamma^2 R_0 \beta^* \left[\left(\frac{\alpha}{2}\right)^2 + \theta_{\perp}^2 + \frac{\theta_{\parallel}^2}{\gamma^2}\right] \sqrt{\left(\frac{\theta_{\alpha}}{2}\right)^2 + \theta_{\perp}^2 + \frac{\theta_{\parallel}^2}{\gamma^2}}} \tag{2}$$

Where: r^c - is the classic of electron radius

C - velocity of light; 3.10^{10} cm/sec

M - proton mass $MC^2 = 938$ MeV

ΔQ - betatron tune shift of antiprotons due to electron beam

$\beta = v/c$ is the average velocity of electrons and protons

Ro - average radius of storage ring

β^* - is the β function in the cooling straight section

α - is coefficient which takes into account the distortion of the magnetic force lines in the electron device

$\theta_\perp = \frac{P_\perp}{P_\parallel}$ is the angular spread in the antiproton beam

$\theta_\parallel = \frac{\Delta P_\parallel}{P_\parallel}$ the longitudinal spread momentum in the antiproton beam

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}$$ relativistic factor

For instance in the cooling of antiproton $\alpha^2 \ll \theta_\perp \ll \theta_\perp$ due to that in the future we shall neglect terms $\frac{\alpha^2}{4}$ and $\theta_\perp^2$

For a strong focusing storage ring Q>>1 and the longitudinal spread is much greater than the transverse spread. Then the fraction of antiprotons with $\frac{\Delta\theta_\parallel}{\gamma} \sim \theta_\perp$ will damp in a time $\tau_\perp \sim \frac{1}{\lambda_\perp}$:

$$\tau_\perp^{-1} = \frac{2 r_e c \Delta Q}{\beta \gamma^2 \beta^* R_o \theta_\perp^2} \qquad (3)$$

For accumulation of the total momentum spread of the antiprotons, the electron beams energy must be scanned across the total momentum spread of the antiprotons. Naturally, the scanning process increases the total damping time.

$$\tau_{\parallel} = \int_{-\Delta p_{\parallel}}^{\Delta p_{\parallel}} \frac{dp}{\Gamma_{\parallel}} \tag{4}$$

Where 2 $\Delta p_{\parallel}$ is the momentum spread in the antiproton beam. As a result of integration in formula (4) we have

$$\tau_{\parallel}^{-1} = \frac{12\, r_e\, c\, \Delta Q}{\gamma R_0 \beta^* \theta_{\perp}^2 \theta_{\parallel}} \tag{5}$$

The time $\tau_{\parallel}$ is given by the acceleration cycle of protons in the Main Ring ($\tau_{\parallel}$ =T). Expression (5) defines the phase space of antiprotons as that which is possible to cool between cycles in the Main Ring.

$$\theta_{\perp}^2 \theta_{\parallel} = \frac{12\, r_e\, c\, \Delta Q\, T}{\gamma R_0 \beta^*} \tag{6}$$

Injection of antiprotons will be at an energy 4 GeV $\gamma_0 = 4$ and electron cooling will be at an energy of 200 MeV, $\beta=0.566$, $\gamma=1.2$ Then it is necessary to decelerate the antiprotons. Deceleration will lead to increased angular and momentum spread of antiprotons.

$$\theta_\parallel^0 = \frac{\beta\gamma}{\beta_0\gamma_0}\theta_\parallel$$

$$\theta_\perp^0 = \sqrt{\frac{\beta\gamma}{\beta_0\gamma_0}}\,\theta_\perp \qquad (7)$$

Here the value which has an asterisk is related to the production energy of 4 GeV. Using formulas 6 and 7, it is possible to get angle spreads of particles at the production energy which can be cooled after deceleration.

$$\theta_\perp^{0\,2}\theta_\parallel^0 = \frac{\beta^2\gamma}{\beta_0^2\gamma^2}\left(\frac{12\, r_e\, c\, \Delta Q\, T}{R_0\, \beta^*}\right) \qquad (8)$$

The number of antiprotons which will be injected in this phase space is equal to:^{2}

$$N_{\bar p} = N_p \cdot n\, \theta_\parallel' \,\frac{\theta_\perp^{0\,2}}{\theta_{\bar p}^2}\left(\frac{\beta^*}{\beta_t}\right) \qquad (9)$$

Where β_t is of the value of β function at the target

$$n = \frac{8\pi}{3} \frac{M_\pi M_p c^2 P}{\sigma_{in}} \frac{d^3\sigma}{dp^3}$$

M_π - is rest mass of Π meson

M_p - is rest mass of proton

σ_{in} - is absorption cross section of antiprotons

σ - is production cross section of antiprotons

For conversion from the energy protons 70 GeV to antiprotons with energies of 4 GeV the value n = 0.01

σ'_{n} is the momentum spread of injected antiprotons.

$\theta_{\bar{p}}^2 = \frac{2M_\pi M_p}{p^2} \simeq \frac{0.3}{\gamma_0^2\beta_0^2}$ is the rms angular spread of the antiprotons. The momentum spread of antiprotons must be decreased by using rf gymnastics or stochastic cooling in such a way that it will be in the range that is possible to cool by electron cooling.

$$\theta' = \frac{\Pi}{\ell}\, g\, \theta^0 \qquad (10)$$

Where $\frac{\Pi}{\ell}$ is a ratio of circumference of antiproton storage bunching of lengths of antiprotons.

g is a shrinking factor of momentum spread due to stochastic cooling. Using formulas 8, 9 and 10, it is possible to have an expression for the accumulation rate of antiprotons.

(11)

If we inject antiproton beams with bunch lengths $\ell = 400$ meters (equal to the circumference of the precooler - it means we need not regard the possibility of rf gymnastics).[3]

$$\Delta Q - 1 \,, \quad g - 10 \,, \quad \beta_\perp = 3\,cm \,, \quad N_{ppp} = 3\,10^{13}$$

In this case we have an accumulation rate of antiprotons of $2.4 \cdot 10^7 \frac{\bar{p}}{sec}$. With such speed we can reach $N\bar{p}$ - 10^{11} in 1.5 hours. It's remarkable (see formula 9) that the accumulation rate of antiprotons does not depend upon the circumference of the antiproton storage ring. This result comes from the dependencies $\theta_\perp^{o2}\beta^* \sim \frac{1}{R_0}$, $N \sim N_p \frac{R_0}{\ell}$ However, decreasing the circumference of the antiproton storage ring will lead to an increase of the density of the electron current $j \sim \frac{1}{R_0^2}$ increasing ΔQ:

$$j\eta = \frac{e\,\Delta Q\,(\gamma\beta)^3 c}{\pi\, r_p\, R_0\, \beta^*} \qquad (12)$$

Here η is the ratio of the cooling length to the circumference of the storaging ring. The increase in the cross section of the electron beam will be

$$S = \pi\left(\beta^* \theta_\perp\right)^2 \sim \frac{1}{R_0}$$

In the example as given above, $\eta = 0.5$, $\theta_\parallel = \pm 2 \cdot 10^{-2}$, $\theta_\perp = 1.5 \cdot 10^{-3}$

The value of electron current density will be 0.5 $\frac{A}{cm^2}$, the total current will be 14 A, and the diameter of the cathode will be 6 cm. Let us take for example an accumulation of antiprotons in the test ring. In this scheme restrictions will be connected to the phase space of the booster

$$\mathcal{E}_\perp = 20\pi\, 10^{6}\ m\,rad, \qquad \frac{\Delta P_\parallel}{P_\parallel} = \pm\, 2.5\, 10^{-3}$$

After deceleration the angle divergence of antiprotons in the test ring will be $\theta_\parallel = 4.10^{-4}$, $\theta_\parallel = \pm 2.5\cdot10^{-3}$

For such angular divergence the damping time will be only 1 sec for an electron current of 30 A. For this scheme where antiprotons will be injected into the booster, the useful momentum spread of an energy of 4 GeV will be $\theta_\parallel^0 = 0.8\cdot10^{-3}$ and transverse angular spread of $\theta_\perp^0 = 3.7\cdot10^{-4}$.

Using these numbers and formula (9) it is possible to get

$$N_{\bar{p}} = N_p \cdot 5.3\cdot10^{-8}$$

If we redistribute the proton beam in the Main Ring into three bunches of 150 m lengths, then eject them from the Main Ring at intervals of 1 sec to hit the target, then inject $\bar{p}$ into the booster, then decelerate $\bar{p}$ and inject them into the storage ring, we will use all accelerated protons. Accumulation speed for this scheme will be $1.6\cdot10^6$ $\bar{p}$ per pulse or $2.3\cdot10^5$ antiprotons per second.

References

1. N. Dikansky, V. Kononov, V. Kudelainen, I. Meshkov, V. Parkhomchuk, D. Pestrikov, A. Skrinsky, B. Sukhina, *"Investigation of Fast Electron Cooling"* National Accelerator Conference, Dubna, USSR (*1978*).
2. G. Budker, A. Skrinsky, *"Electron Cooling and New Possibilities in Physics of Elementary Particles"* USPEKHI Physicheskikh Naur; Vol. 124, Issue 4 (*April, 1978*).
3. *"A Conceptual Design for a High Luminosity Proton-Antiproton Source at Fermilab"* (*July, 1979*).