# Focussing Antiprotons inside the Production Target

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Assume a uniform current density J and a cylindrical conductor of radius $R_0$. At the radial position $r \leq R_0$ the magnetic field is given by

$$B_\theta(r) = \mu_0 \frac{r}{2} J \qquad \text{(MKSA units)}$$

Focussing is related to the field gradient, $\frac{dB_\theta}{dr} = \frac{B_\theta(r)}{r} = \frac{\mu_0}{2} J$. To calculate the focussing parameters, we can simply relate to classic formulae (given, for instance, in the paper by M. Sands, SLAC 121, page 18 and 77).

The transfer matrices are functions of the parameter $k = \frac{ec}{\varepsilon} \frac{\partial B}{\partial r}$, where $\varepsilon$ is the particle energy. Expressing the energy in eV, $\varepsilon = eE$ and using the expression for the field gradient applicable to our case we get

$$K = \frac{\mu_0 c}{2} \frac{J}{E(eV)}$$

Depending on whether the value of K is positive, zero, or negative in a particular segment <u>s</u> of the trajectory, the motion in <u>r</u> will have one of the forms:

$$K > 0 \qquad r = a \cos (\sqrt{K}\, s + b)$$

$$K = 0 \qquad r = as + b$$

$$K < 0 \qquad r = \cosh (\sqrt{-K}\, s + b)$$

where a, b are initial constants. The transfer matrix is:

$$\begin{pmatrix} \cos \sqrt{K}\,\ell & \frac{1}{\sqrt{K}} \sin \sqrt{K}\,\ell \\ -\sqrt{K} \sin \sqrt{K}\,\ell & \cos \sqrt{K}\,\ell \end{pmatrix}$$

where $\ell = s_2 - s_1$ and $K \geq 0$. (For $K < 0$, the usual replacements to hyperbolic functions apply.)

The motion can be related to the betatron function $\beta$, correlated to the motion by the well known expression:

$x(s) = a\sqrt{\beta} \cos(\phi - \delta)$ with $\phi = \int_0^s \frac{ds}{\beta}$. Comparing expressions, we obtain $\beta = \text{const} = \frac{1}{\sqrt{K}}$. Therefore, we can write

$$\beta^2 = \frac{2\,E}{\mu_0 cJ}$$

The current density which is needed to achieve a specified $\beta$ is given by

$$J = \frac{2}{\mu_0 C} \frac{E}{\beta^2} = 5.3 \times 10^{-3} \frac{E}{\beta^2}$$

where E is expressed in eV. Setting for instance $E = 6 \times 10^9$ eV ($P = 6$ GeV/c) and $\beta = 0.01$ m, we find $J = 3.2 \times 10^5$ A/mm$^2$, which has apparently been achieved (U. Sejdal et al., Z. Naturforsch, 30a, 1168 (1975)).

The object of the front end of the focussing channel is the one of concentrating the largest number of $\bar{p}$'s within the acceptance $E_\epsilon$ of the accumulator. The maximum production angle $\theta$ is related to $E_\epsilon$ through the well known formula

$\Omega \doteq \pi\theta^2 = E_0/\beta$, where $\beta$ is the value of the $\beta$-function at the production point. We denote by $d^2N = Y\, d\Omega\, dZ$ the number of $\bar{p}$'s which are produced by the proton beam in the slab dZ of target, within the valid angle $d\Omega$ and the momentum acceptance of the accumulator. The yield Y is normalized to the incoming proton flux. The total number of antiprotons collected from a target of length $\ell$ is given by:

$$N_{\bar{p}} = E_0\, Y e^{-\ell/\ell_0} \int_0^{\ell} \frac{ds}{\beta(s)} = E_0\, Y\, e^{-\ell/\ell_0}\, \Delta\phi$$

where $\ell_0$ is the attenuation length for protons and antiprotons in the target, and $\Delta\phi = \int_0^{\ell} \frac{ds}{\beta(s)}$ is the betatron phase advance over the target length. We have neglected the angular dependence of the production cross section and antiproton production by secondaries inside the target. Both approximations are generally justified in our application.

We now distinguish two cases:

(i) target centered around a minimum of the beta function $\beta = \beta_0$ and no internal focussing. There the betatron phase advance is given by the well known expression

$$\Delta\phi = 2 \arctan (\ell/2\beta_0)$$

For $\ell >> 2\beta_0$, $\Delta\phi \cong \pi$.

The target length which gives the maximum yield of antiprotons collected in the accumulator acceptance $E_0$ for a given incident beam is obtained by simple differentiation:

$$\left[1 + \left(\frac{\ell_m}{2\beta_0}\right)\right]^2 \quad \arctan\left(\frac{\ell_m}{2\beta_0}\right) = \frac{\ell_0}{\beta_0}$$

In the approximation $\ell_m >> 2\beta_0$ we find $\ell_m = \sqrt{\frac{8}{\pi}\,\ell_0\,\beta}$. Since also $\ell_m << \ell_0$, the attenuation can be neglected and the antiprotons collected are given by

$$N_{\bar{p}}(\max) \cong \pi\, E_0\, Y \qquad (\ell_m >> 2\beta_0)$$

independent of the actual value of $\beta_0$.

(ii) Target traversed by a constant current density to focus antiprotons. Inside the target, $\beta = \beta_0$ and therefore

$$\Delta\phi = \ell/\beta_0$$

The optimal target length $\ell_m$ is then $\ell = \ell_m$, for which we find

$$N_{\bar{p}}(\max) = e^{-1}\, E_0\, Y\, \ell_0/\beta_0 \simeq 0.368\, E_0\, Y\, \ell_0/\beta_0 .$$

We can now compare cases (i) and (ii). For optimized conditions we find that target focussing gives us a net gain factor G given by:

$$G \cong \frac{1}{e\pi}\,\frac{\ell_0}{\beta_0} = 0.117\,\frac{\ell_0}{\beta_0} \qquad (\beta_0 \rightarrow 0)$$

Remembering that $\beta_0^2 = \frac{2E}{\mu_0 c J}$, we finally arrive to the formula:

$$G \cong \sqrt{\frac{\mu_0\, c}{2\, e^2 \pi^2}}\;\ell_0\sqrt{\frac{J}{E}} = 1.61\,\ell_0\sqrt{\frac{J}{E}}$$

where units are MKSA and the energy E is given in eV. Note that the result is independent of the acceptance of the ring (within the approximation that the yield remains proportional to solid angle), and it is obtained for the limit $\beta_0 \to 0$. This limit is reached rather slowly and for intermediate cases one has to use numerical estimates. For instance for $\ell_0 = 10$ cm (Tungsten) and $\beta_0 = 1$ cm, we find $G = 2.49$.