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Quench Protection Requirement of VLHC And Other Magnets

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Since VLHC magnets will have high field and long length, quench protection is not an obvious issue. VLHC magnet has to be designed not only with magnetic field and mechanical strength. Temperature estimation after a quench has to be carefully considered.

1.Temperature rise after quench

The energy balance between heat generation and the temperature of every portion of the conductor under adiabatic condition is:

$$RI^2 = C\frac{dT}{dt}$$

Where, R, I, C, T are resistance, current, volume heat capacity and temperature, respectively. Heat transfer to helium has to be considered to be precise but quench phenomena is usually fast enough to neglect the heat transfer.
Considering resistivity of superconductor is much higher than that of copper, current is carried only by the stabiliser. If the copper to superconductor ratio is r,

$$\frac{r_{Cu}}{S}(jS)^2 = \left(C_{Cu}m_{Cu}\frac{r}{r+1} + C_{Sc}m_{Sc}\frac{1}{r+1}\right)S\frac{dT}{dt}$$

where S is the cross section ρ is the resistivity. C is given par mass and m is the specific gravity of the material. From this equation, we get:

$$j^2dt = \frac{C_{Cu}m_{Cu}r + C_{Sc}m_{Sc}}{(r+1)r_{Cu}}\,dT$$

Integrating both side,

$$\int_0^\infty j^2dt = \int_{4.2}^{T}\left(\frac{C_{Cu}m_{Cu}}{r_{Cu}}\frac{r}{(r+1)} + \frac{C_{Sc}m_{Sc}}{r_{Cu}}\frac{1}{(r+1)}\right)dT$$

Or,

$$\int_0^\infty I^2dt = S^2\int_{4.2}^{T}\left(\frac{C_{Cu}m_{Cu}}{r_{Cu}}\frac{r}{(r+1)} + \frac{C_{Sc}m_{Sc}}{r_{Cu}}\frac{1}{(r+1)}\right)dT$$

Left-hand side of the equation is determined by the protection system of the magnet representing the current decay after the quench. Taking 10^6 as a unit, Mega I× I Time-integral is called MIITS number. Right hand side is a function of the temperature determined by the material and the size of the conductor. Every MIITS number corresponds to the maximum temperature of the conductor after a quench. Historically, MIITS were introduced in Tevatron project to experimentally determine the allowed maximum energy deposition on the conductor. Even if there is some ambiguity in the material characteristics, allowed MIITS for the conductor can be experimentally found and magnets are safe if it is operated within the MIITS limit.

The correspondence between MIITS and maximum temperature of the conductor can be calculated using known material characteristics. The resistivity of copper of certain RRR is often expressed by the formula

$$R = R(T) + \frac{R(300)}{RRR}$$

And the function R(T) is:

$$\frac{\log R(T)}{R(300)} = -9.600976 - 12.52445(\mathrm{logT}) + 8.309361(\mathrm{logT})^2 - 1.583458(\mathrm{logT})^3$$
$$+0.0993132(\mathrm{logT})^4$$

according to M.C.Jones et al. (Cryogenics 18(1978)337). The heat capacity of metals are given by Debye function as:

$$C_v = 9R\left(\frac{T}{T_D}\right)^3 \int_{\frac{T_D}{T}}^{\infty} \frac{x^4 \exp(x)}{\exp(x) - 1} dx / Z$$

where R is the gas constant and Z is the mass number. T_D is the Debye Temperature of the material. T_D=343K and 275K are used for copper and superconductor.
A numerical calculation for MIITS corresponding to 300K for 1mm^2 conductor with various copper ratio and RRR is summarised in the following table.

	RRR				
Copper Ratio	70	100	130	160	190
0.852	0.0547	0.0582	0.0629	0.0675	0.0716
1.000	0.0598	0.0636	0.0687	0.0736	0.0781
1.200	0.0657	0.0699	0.0754	0.0807	0.0856
1.600	0.0750	0.0796	0.0858	0.0917	0.0971
2.000	0.0819	0.0869	0.0935	0.0999	0.1057
4.000	0.1004	0.1063	0.1141	0.1216	0.1284

Previous projects such as SSC and LHC seem to be designed with MIITS numbers corresponding to 300K. Epoxy impregnated coil might need special attention on the maximum allowed temperature. If the magnet system can keep MIITS in the magnet within these numbers by mean of energy extraction or accelerated quench propagation using heaters, the magnet is considered to be safe through quenches. To get values for conductors with different cross sections other than 1mm^2, multiplication with square of cross section is necessary. The numbers for our 1mm diameter, 28-strand cables are:

	RRR				
Copper Ratio	70	100	130	160	190
0.852	26.4	28.1	30.4	32.6	34.6
1.000	28.9	30.7	33.2	35.6	37.7
1.200	31.7	33.8	36.4	39.0	41.3
1.600	36.2	38.5	41.4	44.3	46.9
2.000	39.6	42.0	45.2	48.3	51.1
4.000	48.5	51.4	55.1	58.7	62.0

2.Estimation of MIITS

The MIITS number of a magnet is dependent on the protection system but a rule of thumb estimation is possible. The stored energy of a magnet is calculated through a

$$U = \frac{B^2}{2\mu_0} D_{eff}{}^2 \frac{\pi}{4} L$$

ROXIE or ANSYS run but more quickly given by;
where U, B, Deff and L are stored energy, magnetic field, effective diameter and length of the magnet, respectively. D$_{eff}$ seems to be 70% more than the physical aperture of the magnet judging from the previously built magnets.
Since the inductance of the magnet Lm , with operation current I, is given by:

$$Lm = \sqrt{\frac{2U}{I^2}}$$

The time constant for the extraction of energy by a damping resistor is

$$\tau = \frac{Lm}{V_{init}}$$

if the maximum voltage across the magnet is limited by Vmax. Then the integration of the current square through the exponential decay of the current is:

$$\int_0^\infty I^2 dt = \frac{1}{2} \tau I_q{}^2$$

However, this is the most pessimistic estimation because the current decays faster than the exponential decay given by the external resistance. Internal resistance due to propagation of quench usually has large effect. If the current decay is linear,

$$\int_0^\infty I^2 dt = \frac{1}{3} \tau I_q{}^2$$

By use of quench heaters, MIITS can be further reduced. The numbers used for SSC and LHC are close to

$$\int_0^\infty I^2 dt = \frac{1}{4} \tau I_q{}^2$$

Of course, if we can install large enough heaters on every block of the coil, we could reduce the MIITS further more. However, there is always delay for the detection of quench and thermal time constant to heat up the coil. Such delay time should be added to the MIIT number.

$$MIIT = I_q{}^2 t_{delay} + \int_0^\infty I^2 dt$$

Delay time of 80 m sec was used in LHC design but there is a room for the shortening. A numerical calculation for various designs are summarised as:

		VLHC Model	TevDoubler	5TeV	Tevatron	SSC	LHC
field	[T]	11	10	11.25	4.5	6.6	8.4
aperture	[m]	0.045	0.06	0.06	0.075	0.05	0.056
length	[m]	12	6	6	6	16	14
stored energy	[MJ]	2.687	1.974	2.498	0.625	1.592	2.831
operation current	[kA]	18	15.4	20	4.5	6.5	11.5
inductance	[mH]	16.6	16.6	12.5	61.7	75.4	42.8
extraction voltage	[V]	500	500	500	500	500	500
time constant	[sec]	0.597	0.513	0.500	0.555	0.980	0.985
delay time	[sec]	0.05	0.05	0.05	0.05	0.08	0.08
delay MIIT	[MIIT]	16.2	11.858	20	1.0125	3.38	10.58
exponential decay MIIT	[MIIT]	96.7	60.8	99.9	5.6	20.7	130.2
linear decay MIIT	[MIIT]	64.5	40.5	66.6	3.7	13.8	65.1
propagation MIIT	[MIIT]	48.4	30.4	50.0	2.8	10.3	32.6
total MIIT (Maximum)	[MIIT]	112.9	72.7	119.9	6.6	24.1	140.8
total MIIT (guessed)	[MIIT]	64.6	42.3	70.0	3.8	13.7	43.1

The MIIT number for the Tevatron, SSC, and LHC conductors are:

	LHC	SSC	Tevatron
Copper RRR	70	70	70
c/s ratio	1.6	1.5	1.8
MIITS /(mm^2)^2	0.075	7.29E-02	7.87E-02
strand diameter	1.065	0.8	0.681

number of strands	28	30	23
300K MIIT	46.6	17.24	4.39

Comparing these numbers with the MIITS number estimated in the previous table is very interesting. Although MIITS estimations are based on rough assumptions the MIITS numbers turn out to be slightly below the allowed conductor MIITS. Therefore, this estimation can be a useful guideline for the design of magnet under consideration. The Tevatron up-grade magnets are very tight for the MIIT margin. The VLHC magnet is now under the 1m model stage. Obviously, it requires much more MIITS if we extend the magnet length in the same way as SSC or LHC. It requires either a drastic improvement in the protection system or a drastic increase of the copper ratio. The increase of the conductor size to accommodate enough amount of copper is even necessary.