Conductor Current Density and Magnet Design Variations

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The magnetic field strength of superconducting magnets is limited by the critical current density of the superconductor. Therefore, the current density of the conductor has direct influence on the performance of the magnet. This is the reason why we have been pushing for the current density improvement of the Nb3Sn conductors to promote the construction of the VLHC high field magnet.

However, there are several causes to deviate the benefit of high current density from the linear increase of the magnetic field. A careful consideration has to be made to find the appropriate emphasis on the development of the high current density conductors. Following analysis looks at the design variations of the magnet associated with the conductor current densities.

1.Factors Affecting the Field Strength

Current density curve

The current density of superconductor is a function of the magnetic field. A linear approximation such as used in ROXIE is not enough for the design consideration in wide range. A simple function of the Ic characteristics of Nb3Sn conductor is expressed as:

$$Ic \propto ((\frac{B_c - B}{300} + 1)^3 - 1)^{1/3}$$

Unlike other conventional functions, this formula fits to the measurement data well even for the low field region. 28 T is a good number for Bc of the titanium added Nb3Sn conductors. Since we increase the magnetic field to take advantage of higher current density, the actual usable current density increase is less than the increase of the current density at the same field.

Load line effect

Magnets are operated not on the B-constant line but on a load line. The load line of the magnet usually has 4 to 7% increase at the conductor. Difference increases with the field if the enhancement at the coil is the same percentage. For the simplicity, fixed 6% is used in the following analysis. If the center field **B** of the magnet is calculated,

$$Jc = Jc(1.06 \times B)$$

is the actual current density usable for the design of the magnet.

Copper ratio

It is necessary to have certain amount of copper to survive through quenches. If the field increases, the necessary amount of copper increases according to the increase of the stored energy. The copper ratio g has to be increased with the current density of superconductor. Thus:

$$g \propto B^2 j_c$$

Change of copper ratio effectively dilutes the advantage of high critical current density. Moreover, if the current density becomes too high for the cross section of the conductor, it becomes impossible to operate the magnet even if the entire cross section of the conductor is copper. There is an absolute limitation for the size of the conductor and if the conductor size is increased, certainly, there is no need for the high current density. Since stored energy of a magnet is also proportional to the length of the magnet, copper ratio has to be increased with the length of the magnet. However, since this is highly dependent on the protection heater and quench detection system, it is not always linear to the length of the magnet.

Spacer volume and iron saturation

To generate higher filed with two-shell cosine theta magnet; we need to use wider cable. The spacer in the coil becomes large when the cable becomes wider compared to the aperture. Since the key stone angle of the conductor is usually not large enough, this space efficiency, h is a function of aperture, **r** and cable width, **w**:

$$h \propto \frac{2r}{2r + w}$$

The thickness of the insulation also affects to the magnetic field strength. If we use thick insulation, it effectively reduce the current density. The space efficiency, h**i** to include the insulation thickness **di** is:

$$h_i = \frac{0.7w}{0.7w + 2di}$$

where, 0.7 is the assumed aspect ratio of the cable.

The iron saturation effect becomes larger when the field is increased. The wider cable makes the location of current far from the aperture center and the field generation efficiency decreases accordingly, The mirror image produced by the iron becomes smaller if the radius of the iron hole becomes large due to thick winding. This factor appears in the field calculation formula.

Field Calculation Formula

The calculation of the field could be done using ROXIE or ANSYS but the result should be similar to the approximated analytical formula. The field generated by an arc-shell shaped coil is:

$$B = 0.8jwSin(q)\left(1 + \frac{3r^2 + w^2 + 3rw}{3R^2}\frac{m-1}{m+1}\right)$$

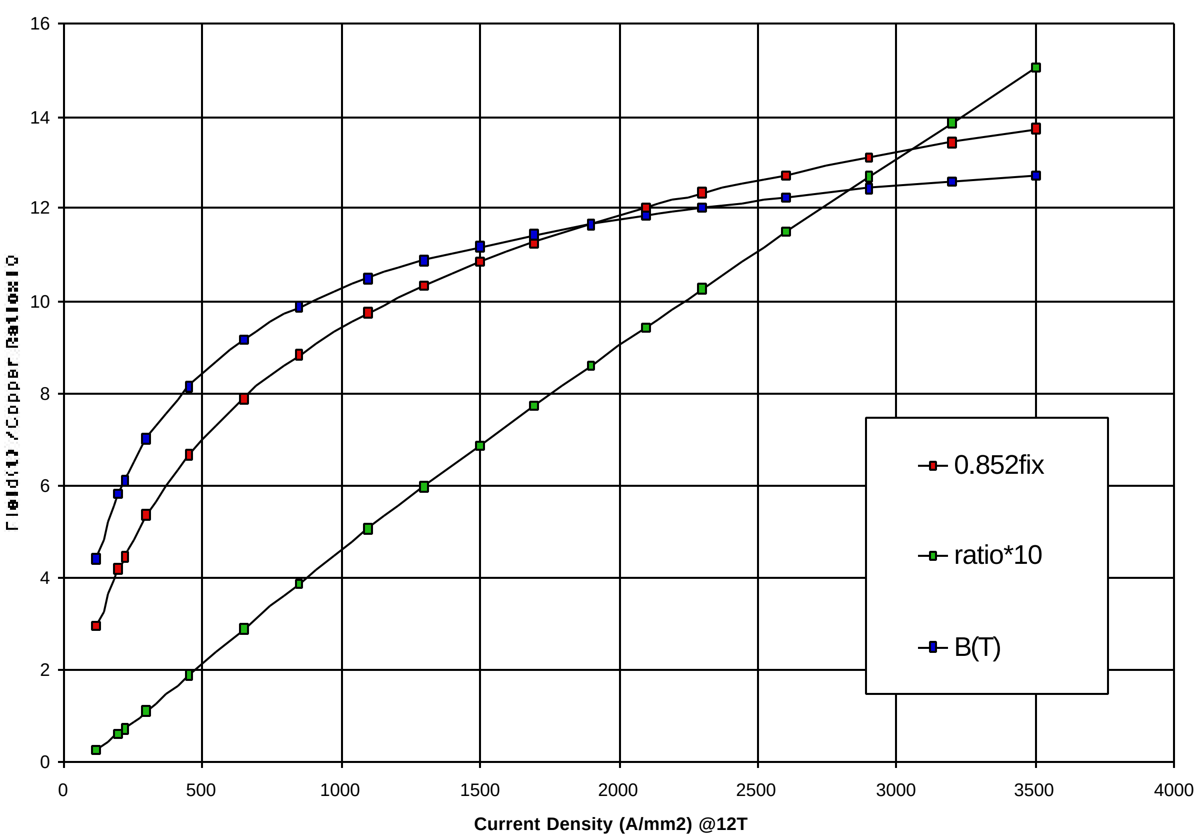
where, **R** is the radius of the iron hole and m is the permeability of the iron. The arc angle θ is nominally 60 degree but it can be adjusted to a reference design calculated by ROXIE. The field given by this equation and the one used to determine the critical current density should be consistent. Numerical adjustment to find the solution can be done using Excel spread sheet very conveniently. A macro routine in Excel can do the solution finding automatically. Comparison among various designs was done using these relationships.

2. Design Analysis

Current Density Improvement for the VLHC Dipole

Using the equations explained in the previous section, analysis over a large variety of magnet design can be made in a spread sheet. The improvement of the magnetic field strength due to the increase of the conductor current density for the 45mm aperture VLHC model magnet is plotted in the Fig.1. The analysis using ROXIE for the fixed copper ratio 0.825 appears in the MT16 paper. It is shown that the effect of copper ratio change due to the current density increase further reduces the gain in the magnetic field. As a matter of fact, the gain in the magnetic field is very little even if we get a 3000A/mm2 conductor. We will never reach 13T and, if we build a long magnet for the real accelerator, we have to use more copper and the obtained field will be reduced. However, if we stay in the range of 10 to 11 T, it is possible to construct magnets.

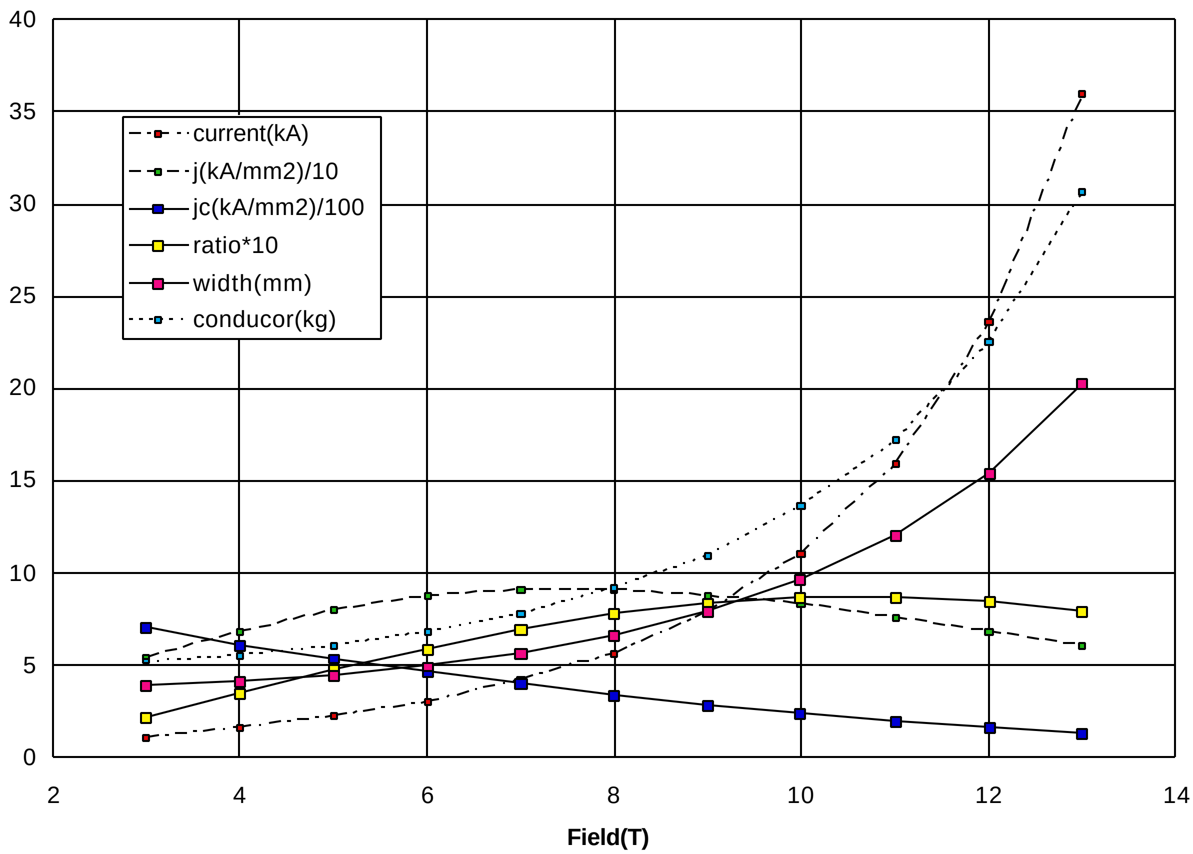
Fig1. VLHC Magn



VLHC design variations

If the strand diameter of the superconductor is changed, we can design magnets at various magnetic field. Figure 2 shows design parameters of the magnet with respect to the design field. The copper ratio has to be changed with the change of design field. In general, if the design field is high, the copper ratio of the conductor becomes large. However, in the very high field range, the copper ratio becomes small again. This is because the current density at high field is less. The problem with high field design is the operation current and the winding of the coil. High field design requires a thick conductor which is mechanically difficult to wind. For example, a 13T magnet with our 1860A/mm2 superconductor would be operated at 35KA and the width of the cable will be 20mm, which is almost impossible to wind on a 45mm mandrel. Very large operation current introduces a problem of current lead and consequently the large heat load of the cryogenic system. Again, going beyond 11 T is not a good choice of design.

Fig.2 VLHC Magnet Variatio



TEVATRON Upgrade Magnet

It is interesting to consider of the TEVATRON upgrade magnet using these formulae. The figure is the plot of the design variations of the 6m-dipole magnet with copper ratio 1.2 for 10 T design. The aperture of the magnet is 60 mm. It is possible to design various field using commercially easily available conductor of 1580 A/mm2 current density at 12T. Since the effective current density at the coil decreases with the field, the required amount of conductor rapidly increases with the field. The difference between 10 T design and 12T design is almost 100%. The required amount of conductor has to be doubled just to increase 2T of magnetic field. Considering the reasonable operation current of 16kA and short time for the development as well as the economy of the conductor, I would like to suggest that it is appropriate to remain at 10T for the TEVATRON up-grade magnet.

Fig3. Tevatron Up-grade Magne