Measurements of Nuclear Effects and the $\bar{\nu}_\mu + H \to \mu^+ + n$

Cross Section in MINER$\nu$A with Neutron Tagging

by

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Biographical Sketch

The author attended the University of Chicago and graduated with a Bachelor of Arts in Physics with honors in 2013. During this time, he joined the CP-violation group working on the KOTO experiment. He was a post-graduate research assistant at the University of Chicago until 2014.

The author began his graduate career at the University of Rochester in 2014. He joined the MINERνA collaboration under the supervision of Professors Kevin McFarland and Arie Bodek. The author pursued his studies in absentia between 2016 and 2020 at the Fermi National Accelerator Laboratory in Batavia, Illinois. He played an active role in detector maintenance, phenomenological studies of neutrino-nucleus interactions, software development, and data analysis, culminating with the physics results described in this thesis and publications that the author is a main contributor of:


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Abstract

MINERνA, or Main INjector ExpeRiment for ν-A, at Fermilab, is an experiment dedicated to the study of neutrino-nucleus interactions in the GeV regime. Its goal is to illustrate the interplay between hadronic and nuclear physics and measure intranuclear dynamics crucial for the present and future neutrino oscillation measurements. We first measure a set of variables sensitive to how Monte Carlo (MC) simulations of neutrino-nucleus interactions implement binding energy and then move on to measure the antineutrino CCQE cross section on the hydrogen targets in MINERνA’s CH detector. We have developed a method to preferentially select events on the hydrogen by comparing outgoing neutrons’ directions to theoretical neutron directions assuming two-body interactions. We measured the cross section, extracted the axial form factor, and performed a Z-expansion fit. We observe larger values in the axial form factor at high $Q^2$ than current best fits. Finally, we show a preliminary selection of events with both protons and neutrons to investigate nuclear processes responsible for producing these final states.
Contributors and Funding Sources

This work was supervised by a committee consisting of the Advisor Kevin McFarland (Department of Physics and Astronomy), Co-advisor Arie Bodek (Department of Physics and Astronomy), Professor Alice Quillen (Department of Physics and Astronomy), Professor Andrew Berger (The Institute of Optics), and chaired by Professor Kara Bren (Department of Chemistry).

This thesis contains work performed by the author as a member of the MINERνA collaboration and descriptions made possible by other collaborators’ work. Chapter 2 contain descriptions of a work jointly performed by the author with Professor Arie Bodek. Chapter 3 describes the MINERνA detector and physics simulation and includes descriptions of work done by other collaborators. Chapter 4 contains a publication[1] which the author is the primary contributor. The analysis used a framework to which the entire MINERvA collaboration contributed, with special thanks to Tammy Walton, Xianguo Lu, and Daniel Ruterbories. Chapters 5 and 7 are the work of the author. Chapter 6 has benefited from the the collaboration between the author and his advisor, Kevin McFarland.

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Chapter 1

Introduction

1.1 Imagine the Cosmos through a Different Light

Looking up a typical moonless summer night sky at the Cherry Springs state park in Pennsylvania, USA, in the Northern Hemisphere, I am immediately awed by the splendor of a cosmos untainted by man-made light pollution. The Milky Way splits the night sky into two halves and casts faint shadows from objects on the ground below. Altair, Vega, and Deneb, members of the prominent Summer Triangle, are virtually unrecognizable, drowned out by the countless fainter stars visible to my naked eyes.

The human eye has evolved and became a sophisticated photo-sensor. A photon passing through the cornea is focused by the lens to arrive at the retina.[4] Here, it unloads its energy and excites a flurry of electrical signals that pass through the nerves to the brain, where it is finally recognized and imaged. The complex reconstruction algorithm inside our brain allows us to detect both the direction and the energy (color) of a photon, which gives us the ability to see the starry skies
1.1 Imagine the Cosmos through a Different Light

and the impulse to understand it. As we shall discover later in this thesis, the eye-neuron-brain system is also the basis for discovering and understanding other fundamental particles.

Today, we have acquired a pretty comprehensive picture of the Universe and, to a large extent, the physical laws governing both the largest inter-stellar objects and the smallest building blocks of matter. On the macroscopic scale, we have the theory of general relativity that successfully describes the motion of objects in a warped space-time[5], the bending of light[6] by large gravitational bodies, as well as predicted the existence of the monstrous black holes[7] and the ripples in space-time that is the gravitational waves[8]. On the microscopic scale, we have combined quantum mechanics with special relativity to form the quantum field theory that eventually gave rise to the Standard Model(SM) of particle physics. The SM has been a terrific framework in understanding all the fundamental forces other than gravity – the electromagnetic (EM) force, the weak force, and the strong force – and how they interact with the collection of 17 fundamental particles and their antiparticle counterparts, making up the observable universe[9]. Figure 1.1 illustrates these particles.

A particle is either a boson or a fermion depending on its spin. A boson has integer spin (0,1,2, etc), while a fermion has half-integer spin (1/2,3/2,etc).

There are four bosons in the Standard Model that act as the carrier of the fundamental interactions – photon (the EM force), gluon (the strong force), W and Z bosons (the weak force). The Higgs boson was discovered at CERN[11, 12] in 2012. It was theorized that the Higgs boson is responsible for giving most particles their mass.[13, 14]

The fermions, on the other hand, are the fundamental building blocks of the
1.1 Imagine the Cosmos through a Different Light

Figure 1.1: The known Fermion elementary particles and the Boson force carriers[10].

The world around us. We know that the everyday world around us is formed by elements composed of protons, neutrons, and electrons. A proton has a single positive charge, an electron has a single negative charge, while the neutrons are electrically neutral. The protons and neutrons are bound together inside the nucleus by the strong nuclear force. The strong force is powerful enough to overcome the electrostatic repulsion between protons. Electrons move around the nucleus in their respective orbitals lending each element its distinctive chemical properties.

Among these three particles, only the electron is fundamental (Fig 1.1). The electron is classified as a lepton because of its small mass. The electron is a charged lepton. Both the proton and the neutron can be further decomposed into more
fundamental building blocks called the quarks. Two up quarks and one down quarks (uud) make up a proton. Two down quarks and one up quark (udd) make up a neutron. Gluons, which mediate the strong force, hold the quarks together. Both the quark and the gluon have a property called the “color charge”. A color charge can be either red, green, or blue. Each color has an anti-color (anti-red, anti-green, or anti-blue). It is a complicated system compared to an electron with only one type of charge, the electric charge. The interactions of these three color charges cause the quarks to be asymptotically free\[15\], a phenomenon that, when quarks are closer together they experience a smaller force and have a lower potential energy. The force between quarks increases with distance and therefore it is impossible observe a free quark because the energy required to separate two quarks in a nucleon increases with distance till it is energetically more favorable to create a quark-antiquark pair out of the vacuum which results in a pion plus nucleon final state.

Each particle (matter) in the SM has an antiparticle (antimatter) counterpart. As far as we know, an antiparticle is the same as the corresponding particle except for its electrical charges. For example, a positron is the antiparticle of an electron, it is identical to an electron in every way except the opposite charge. Antiprotons and antineutrons are made of the corresponding antiquarks, so while an antiproton is postively charged, an antineutron is still electrically neutral. When a particle and its antiparticle meets, they annihilate into pure energy through Einstein’s famous equation

\[ E = mc^2. \]  

(1.1)

The reverse process is also true—matter-antimatter pairs can be created when there is enough energy present.
Every element in the Universe today is formed by the two quarks and the electron. As we probe the Universe with increasing energy, we start to see heavier versions of the trio (families). Each of the quarks and the electron have two heavier cousins. The up quark has the charm (c) and the heavier top (t) quark, while the down quark has the strange (s) and the heavier bottom (b) quark. As for the electron, it has the muon (µ) and the heavier tau (τ) lepton.

Now, we have a basic picture of the subatomic world around us – only the up, down quarks and the electron are commonplace. These three particles account for all the elements in the Universe. Nevertheless, as is discussed below, none of the stable elements heavier than hydrogen would exist without the neutral leptons called neutrinos.

Unlike the photon, the neutrino rarely interacts with ordinary matter. Since the neutrino couples exclusively to the weak nuclear force, its existence was only postulated in the 1930s by Pauli and Fermi to explain how β-decay, a process where an unstable nucleus emits an electron or positron, could conserve energy, momentum, and spin[16]. The neutrino was discovered much later, in 1955, in the Cowan and Rein neutrino experiment[17]. This experiment became possible when intense antineutrino beams generated by nuclear reactors became available.

Beyond the Earth, trillions of nuclear reactors are burning. Each star in the sky is fusing light elements into heavier elements in its core while releasing radiative energies according to Eq. (1.1). Because of their large interaction cross sections, the photons generated in the core of stars scatter from the material inside the sun and take many years to reach the stellar surface. Neutrinos, on the other hand, take only a few seconds because they only interact weakly.

As such, neutrinos are also one of the most abundant particles in the Universe
besides photons. Besides the stars’ interior, neutrinos are believed to have formed in the primordial matter soup of the big bang. They quickly decoupled from the rest of the matter just a second after the big bang, at a temperature of \( \sim 1 \) MeV. It is a staggering \( 11.6 \times 10^9 \) K in SI unit! The Universe is essentially transparent to the big bang neutrinos, and their momenta decrease as the Universe expands. These relic neutrinos form a cosmic neutrino background (CvB) and their number density is estimated to be \( n^0_\nu = 339.5 \text{ cm}^{-3} \), compared with \( n_\gamma = 411 \text{ cm}^{-3} \) for the Cosmic Microwave Background (CMB) photons[9]. The CMB photons also decoupled much later, after the electrons and protons recombined to become neutral atoms when the universe was 370,000 years old[9]. The CMB radiation is a blackbody radiation with mean temperature 2.73 K. The CMB is isotropic, meaning that the CMB is the same regardless of where we measure it in the sky, with small local temperature and polarization disturbances in the \( 10^{-5} \) and \( 10^{-6} \) levels respectively[9]. The anisotropies in the CMB could teach us a great deal about the conditions of the early Universe. The protons and electrons before recombination are in a constant state of oscillation (baryon acoustic oscillation) related to the density of matter. When they recombine, this information become imprinted on the CMB.

Our current cosmological model postulates that the Universe began at the Big Bang, when matter and antimatter are created from a state of pure energy. If matter and antimatter are really identical, we would expect them to completely annihilate each other to leave our Universe in a radiative desert. The fact that we live in a matter-dominated Universe means there is more to the SM and the Big Bang Cosmology than we know. Three conditions proposed by Sakharov are necessary to dynamically generate the matter-antimatter asymmetry required to produce a matter-dominated Universe. First, there must be processes that change the total
number of baryons (particles composed of quarks). Second, the interaction of matter and antimatter is out of equilibrium. Finally, there must be physical processes that proceed differently between matter and antimatter. In fact, the SM has recipes for all the conditions[18]. The first condition is satisfied with the sphaleron process that turns baryons into leptons. This process is suppressed at low energy but is not during the initial stage of the Big Bang. The Universe was out of equilibrium when the strong force decoupled from the electroweak force some $10^{-32}$ second after the Big Bang. Finally, a phenomenon called the charge parity violation (CPV) already exists in the SM. Conserving the CP symmetry requires the physical laws governing the particle interactions to be invariant by replacing a particle with its antiparticle and inverting its spatial coordinates. While CPV exists in the hadronic sector in the Kaon[19] and the B-meson[20], their size is too small to drive the matter-antimatter asymmetry. CPV in the leptonic sector, on the other hand, could indirectly cause the matter-antimatter asymmetry we observe today. While there is no definitive evidence that leptonic CPV exists, measurements of neutrino oscillation, to be discussed in more detail in Sec 1.2.2, may provide the definitive answer.

If we could detect neutrinos as readily as we able to do detect photons, we would gain immediate knowledge about the interior of stars. We would also be able to observe the universe when it it was only 1-second old. We could even directly communicate with people on the other side of the Earth [21] with neutrinos!

To achieve this, we must understand neutrino properties and how the neutrino interacts with matter. We must also advance the detection and analytical technologies available to us.
1.2 The Neutrinos We Know

1.2.1 Neutrinos in the Standard Model

Neutrinos are leptons. In the standard model (Fig 1.1), there are 3 flavor of neutrinos – electron neutrino ($\nu_e$), muon neutrino ($\nu_\mu$), and tau neutrino ($\nu_\tau$). Each of the neutrinos is electrically neutral and interact only through the weak force. Another interesting property is that each neutrino flavor is related to a charged lepton through the lepton number. An electron has an electron number of +1, its antiparticle, the positron, has an electron number of -1. The electron neutrino (antineutrino), on the other hand, also has an electron number of +1(-1). The same goes for the muon and the tau lepton – each of them has its own version of lepton number. The total lepton number during an interaction must always be conserved.

Taking beta decay as an example, a neutron decays into a proton, an electron ($L_e = 1$) and an antielectron neutrino ($L_e = -1$):

$$n \rightarrow p + e^- + \bar{\nu}_e \tag{1.2}$$

Another example can be found in the charged current (CC) quasielastic (QE) reaction of an antimuon neutrino with a proton:

$$\bar{\nu}_\mu + p \rightarrow n + \mu^+ \tag{1.3}$$

where we have $L_{\bar{\mu}} = -1$ on both sides of the equation. When a muon decays into an electron, it must simultaneously emit a $\bar{\nu}_\mu$ and a $\bar{\nu}_e$ so that the total lepton number after the decay is still $L_\mu = 1$.

Therefore, when a neutrino interacts with matter (which is rare), we can infer the
neutrino’s flavor by summing up the total lepton number, which is conserved during an interaction. Conservation of the lepton number governs how neutrinos should interact in the Standard Model, without further mystery. However, a mystery does exist. Just like we would not expect a muon to spontaneously turn into an electron without the neutrinos to conserve lepton numbers, we would have no reason to suspect that a neutrino violates the conservation of lepton numbers.

Nevertheless, this is what we observe. The first indication of something amiss was in 1976[22] when scientists tried to measure the solar neutrino flux by counting the number of chlorine atoms turned into argon by the electron neutrino capture process

\[ \nu_e + ^{37}\text{Cl} \rightarrow ^{37}\text{Ar} + e^- . \]  

(1.4)

They found only 1/3 of the expected neutrinos.

The sun should produce only electron neutrinos, primarily through the proton-proton (pp) chain,

\[ p + p \rightarrow d + e^+ + \nu_e , \]  

(1.5)

where \( d \) is a deuteron consisting of a proton and a neutron. Additional neutrinos are produced by the production and decay of heavier elements directly produced in the pp chain[23]. Neutrinos are also produced in the carbon-oxygen-nitrogen(CNO) cycle in the sun to a lesser extent[24].

Twenty-five years later, in 2001, the Sudbury Neutrino Observatory (SNO) in Canada confirmed that the missing 2/3 of the neutrinos do exist through the neutral current (NC) scattering of the remaining neutrinos. These neutrinos begin with the electron flavor deep inside the sun’s furnace but changed flavor on their way towards Earth. Capture of \( \nu_\mu \) and \( \nu_\tau \) on chlorine is energetically prohibited because the rest
mass of a muon is 105 MeV/c², and a tau lepton is at a whopping 1.776 GeV/c². The most energetic solar neutrino has an energy of only 18 MeV\[23\]. This is not enough to trigger the equivalent reactions in Eq.(1.4). The NC process does not change the lepton species, therefore is not inhibited in the muon and tau neutrinos. For an NC interaction on a nucleon, the neutrino deposits energy on the hit nucleon and continues unchanged:

\[ \nu + p(n) \rightarrow \nu + p(n). \] (1.6)

The SNO experiment is sensitive to both the charged current (CC) and neutral current (NC) processes, and is able to observe the interactions from the neutrinos that changed species during flight.

1.2.2 Neutrino Mysteries

Neutrino Mass

The fact that the neutrino flavors oscillate is startling but perhaps not unexpected. In order for neutrinos to change flavor, the neutrinos must have a mass. However, this is in contradiction with the Standard Model in which the neutrinos are massless.

The mass of particles is generated in the standard Model through the process of spontaneous symmetry breaking involving the Higgs boson[15]. A mass term appears from the Higgs field when a left-handed particle couples to a right-handed version of the particle (see Eq.(1.7)).

Due to its vector-axial vector (V-A) structure, the weak interaction does not couple to right-handed neutrinos. In addition, there is no observational evidence of their existence either[15]. If right-handed neutrinos exist, they do not participate in
1.2 The Neutrinos We Know

the SM processes. They are therefore regarded as “sterile”.

\[ L^D = -m_D \bar{\psi} \psi = -m_D (\bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R), \]

\[ L^M = -\frac{1}{2} m_M (\bar{\psi}^c_L \psi_L + \bar{\psi}_L \psi^c_L). \]

(1.7)

(1.8)

Because neutrinos are electrically neutral, they could still generate mass without
the sterile neutrinos[25]. If neutrinos are their own antiparticles, they could couple
to themselves and still generate mass (see Eq.(1.8), where the superscript c indicate
a chiral transformation). A fermion being its own antiparticle is called a Majorana
fermion. Searches for the Majorana nature of the neutrinos are underway by looking
for a process called the neutrinoless double beta decay (0νββ). When a nucleus un-
derwent two beta decays simultaneously, the two neutrinos produced may annihilate
if they are Majorana.

As of today experiments[26, 27] have not found any conclusive evidence that
neutrinos are Majorana particles. There are also no direct evidence that sterile
neutrinos exist.

Neutrino Oscillation

As previously established, neutrinos interact as one of the flavor state \( l \), where \( l \)
corresponds to either electron, muon, or the tau lepton. If neutrinos have mass,
then their flavor oscillation can be readily explained.

Suppose there are mass states different from the flavor states. We can rewrite
each neutrino flavor state as a linear combination of the neutrino mass states:

\[ |\nu_\alpha\rangle = \sum_{i=1}^{n} U_{\alpha i}^* |\nu_i\rangle, \]

(1.9)
where $\alpha \in \{e, \mu, \tau\}$ and $i \in \{1, 2, 3\}$ corresponding to the 3 flavor eigenstates and the 3 mass eigenstate, which is also the minimum number of mass eigenstates required to describe our observations of neutrino oscillation. More mass eigenstates could exist, but their interactions are not governed by the weak interactions in the Standard Model. Neutrinos outside the Standard Models are “sterile”.

The matrix $U$ is a $3 \times 3$ unitary matrix describing how to combine neutrino mass eigenstates to form flavor states. The unitary matrix has four components:

$$U = U_{12} \cdot U_{23} \cdot U_{31} \cdot U_4,$$  \hfill (1.10)

where

$$U_{12} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$ \hfill (1.11)

$$U_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix},$$ \hfill (1.12)

$$U_{13} = \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{CP}} & 0 & c_{13} \end{pmatrix},$$ \hfill (1.13)

and

$$U_4 = \begin{pmatrix} e^{i\eta_1} & 0 & 0 \\ 0 & e^{i\eta_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \hfill (1.14)$$
For Eq. (1.11) – (1.14), $c_{ij}$ and $s_{ij}$ denote $\cos \theta_{ij}$ and $\sin \theta_{ij}$ respectively. The $\theta_{ij}$ are a set of three mixing angles that affect the probability of finding a neutrino in a certain flavor state after it starts propagating. $\delta_{\text{CP}}$ is a CP-violating phase, and $\eta_k$ are 2 phases related to the Majorana mass. For convenience, the Majorana phases can be absorbed into the neutrino’s state so that only three angles and 1 phase remain.

The matrix $U$ is often called the PMNS (Pontecorvo-Maki-Nakagawa-Sakata) matrix[28, 29] after the fashion of CKM matrix describing quark mixing.

To see how the existence of the neutrino mass eigenstates gives rise to neutrino oscillation, let’s consider a neutrino of flavor $\nu_\alpha$ after propagating for a time $t$:

$$|\nu_\alpha(t)\rangle = \sum_{i=1}^{n} U^*_{\alpha i} |\nu_i(t)\rangle,$$  \hspace{1cm} (1.15)

The probability that this neutrino can undergo a charged current (CC) interaction producing a charged lepton of flavor $\beta$ is

$$P_{\alpha \beta} = |\langle \nu_\beta | \nu_\alpha(t) \rangle|^2 = \left| \sum_{i=1}^{n} \sum_{j=1}^{n} U^*_{\alpha i} U_{\beta j} \langle \nu_j | \nu_i(t) \rangle \right|^2.$$  \hspace{1cm} (1.16)

Assuming the neutrino propagates as a plane wave, we have, for each mass state

$$|\nu_i(t)\rangle = e^{-iE_i t} |\nu_i(0)\rangle.$$  \hspace{1cm} (1.17)

$E_i = \sqrt{p_i^2 + m_i^2}$ is the total energy of the neutrino mass eigenstate $\nu_i$, $p_i$ and $m_i$ are the mass eigenstate’s momentum and mass respectively. While we lack any direct knowledge of neutrino masses, analyses of cosmological observations and particle experiments have placed an upper bound on the sum of neutrino masses at $\sum m_i \leq
0.15eV/c^2. The energy of neutrinos produced in nuclear reactions typically is on the order of MeV. Therefore we can safely assume that most of the neutrinos we can detect are relativistic enough that the mass term is negligible. Therefore

\[ E_i = \sqrt{p_i^2 + m_i^2} \approx p + \frac{m_i^2}{2E}, \] (1.18)

and

\[ \langle \nu_j | \nu_i(t) \rangle = \delta_{ij} \exp \left[ -i \left( p + \frac{m_i^2}{2E} \right) t \right], \] (1.19)

by taking the orthogonal properties of the mass eigenstates. Then

\[
\begin{align*}
\langle \nu_j | \nu_i(t) \rangle \langle \nu_k(t) | \nu_l \rangle^* &= \delta_{ij} \delta_{kl} \exp \left[ -i \left( \frac{m_i^2 - m_k^2}{2E} \right) t \right] \\
&= \delta_{ij} \delta_{kl} \exp \left[ -i \left( \frac{\Delta m_{ik}^2}{2E} \right) t \right] \\
&= \delta_{ij} \delta_{kl} \exp \left[ -i \left( \frac{\Delta m_{ik}^2}{2E} \right) x \right].
\end{align*}
\] (1.20)

In the last line of Eq.(1.20), we have used the relation \( x \approx ct \) for a relativistic neutrino to denote its phase as a function of the distance it traveled. Since \( c = 1 \) in the natural unit, we drop \( c \) entirely from the equation. \( \Delta(m_{ik})^2 = m_i^2 - m_k^2 \) is the difference in the mass squared between the \( i \)th and \( k \)th neutrino mass eigenstate. The sign of \( \Delta(m_{ik})^2 \) could be either positive or negative.
Expanding Eq.(1.16), we have

\[ P_{\alpha\beta} = \sum_i^n |U_{\alpha i}|^2 |U_{\beta i}|^2 \]

\[ + 2 \sum_{i<j}^n \text{Re} \left[ U_{\alpha i} U^*_{\beta i} U_{\alpha j} U^*_{\beta j} \right] \cos \left( \frac{\Delta m^2_{ij} L}{2E} \right) \]  \hspace{1cm} (1.21)

\[ + 2 \sum_{i<j}^n \text{Im} \left[ U_{\alpha i} U^*_{\beta i} U_{\alpha j} U^*_{\beta j} \right] \sin \left( \frac{\Delta m^2_{ij} L}{2E} \right). \]

We can further express the real part of Eq.(1.21) in term of sine only using double angle identities to get

\[ P_{\alpha\beta} = \delta_{\alpha\beta} - 4 \sum_{i<j}^n \text{Re} \left[ U_{\alpha i} U^*_{\beta i} U_{\alpha j} U^*_{\beta j} \right] \sin^2 \left( \frac{\Delta m^2_{ij} L}{4E} \right) \]  \hspace{1cm} (1.22)

\[ + 2 \sum_{i<j}^n \text{Im} \left[ U_{\alpha i} U^*_{\beta i} U_{\alpha j} U^*_{\beta j} \right] \sin \left( \frac{\Delta m^2_{ij} L}{2E} \right), \]

and

\[ \frac{\Delta m^2_{ij} L}{4E} = 1.267 \frac{\Delta m^2_{ij}}{eV^2} \frac{L}{m/MeV}. \]  \hspace{1cm} (1.23)

In a simplified 2 neutrino oscillation paradigm, the probability for \( \nu_\alpha \) to oscillate into the distinct \( \nu_\beta \) flavor is therefore

\[ P_{\alpha\beta} = \sin^2(2\theta) \sin^2 \left( \frac{\Delta m^2 L}{4E} \right), \]  \hspace{1cm} (1.24)

This probability equation contains 2 unknowns and 2 variables we can (sort of) control. The maximum amplitude of the oscillation probability depends on the value of the mixing angle \( \theta \). The squared mass difference between the 2 neutrino eigenstates can be determined from the shape of the oscillation probability as a
function of the detection distance for any given energy. For maximal mixing the oscillation term is close to 1, $|\Delta m^2_{ij}|L/4E = \pi$. Therefore, we define the oscillation length as

$$L_{ij}^{osc} / m = 2.480 \frac{E/\text{MeV}}{|\Delta m^2_{ij}|/\text{eV}^2}.$$ (1.25)

The existence of the 3 neutrino flavor and 3 neutrino mass eigenstate complicates the situation. Currently, we have measured 2 mass splitting terms $\Delta m^2_{21} \sim +7.39 \times 10^{-5} \text{ eV}^2$ and $\Delta m^2_{32} \sim 2.525(-2.512) \times 10^{-3} \text{ eV}^2[30]$.

There is no ambiguity in the sign of $\Delta m^2_{21}$ as $m_1$ has been defined to contain the largest fraction of $\nu_e$ by convention. There is, however, uncertainty in whether $m_3$ is more or less massive than $m_2$. The value in the bracket indicates the size of $\Delta m^2_{32}$ if $m_3 < m_2$. The neutrinos are considered normal order (NO) if $m_3$ is more massive, and inverse order (IO) if $m_3$ is less massive than $m_2$. 
1.2 The Neutrinos We Know

Figure 1.2 shows the two mass splitting scenarios. The mass ordering is a central unsolved problem in neutrino physics. With the exception of $\theta_{12} = +33.48^\circ$ and $\Delta m_{21}^2 = +7.39 \times 10^{-5} \text{ eV}^2$, which are unambiguously determined in solar neutrino measurements[30], the values of both the $\theta_{23}$ and the $\theta_{13}$ mixing angles are dependent on the mass ordering. $\delta_{\text{CP}}$ is the size of CP violation in the leptonic sector awaiting precise measurement. If neutrinos violate CP symmetry, the neutrino will oscillate differently from the antineutrino, and the $\delta_{\text{CP}}$ will be non-zero.

The fact that both the solar neutrinos and terrestrial neutrinos must travel through matter to reach us needs to be taken into account. Electron neutrino (antineutrino) may coherently scatter on the dense electron medium in matter during propagation, which effectively causes the neutrino (antineutrino) to experience a potential energy $V = \pm \sqrt{2} G_F n_e [9]$, where $n_e$ is the electron density and the $+(−)$ sign is for neutrino (antineutrino). This potential energy alters the effective masses of the mass eigenstates and also causes the neutrino oscillation probabilities to depend on the neutrino energy and the electron densities. For example, the survival probability of electron neutrinos from the Sun with energy over 10 MeV is $P_{ee} = \sin^2 \theta_{12}$ and below 1 MeV is $P_{ee} = 1 - 1/2 \sin^2 2\theta_{12} [9]$. The matter effects will also cause neutrinos and antineutrinos to oscillate differently, resulting in apparent CP violation that needs to be corrected.

1.2.3 Neutrino Oscillation Experiments

The determination of the mass ordering is a primary goal of the next-generation neutrino oscillation experiments. Currently, three next-generation neutrino oscillation experiments are under construction. They are the Deep Underground Neutrino Experiment (DUNE)[32] in the USA, the Hyper-K[33] experiment in Japan, and the
1.2 The Neutrinos We Know

Jiangmen Underground Neutrino Observatory (JUNO) [34] in China. Their physics potentials build on the success of the current and the previous experiments.

Both DUNE and Hyper-K are accelerator-based neutrino experiments. To produce neutrinos, a proton beam in an accelerator facility collides with a graphite target [35, 36] to produce pions which then decay to muons and neutrinos:

\[
\pi^+ \rightarrow \mu^+ + \nu_\mu, \tag{1.26}
\]

\[
\pi^- \rightarrow \mu^- + \bar{\nu}_\mu.
\]

When a charged pion decays at rest, the muon and the neutrino are isotropically produced back-to-back to conserve energy and momentum. Taking the rest mass of the charged pion to be 139.6 MeV/c\(^2\), the muon to be 105.7 MeV/c\(^2\), and the neutrino to be essentially massless, we can solve for the exact muon and neutrino momenta to be 29.78 MeV/c in opposite directions. The forward momentum of the pions will depend on the energy of the proton beam. A set of “horns” (see Fig 1.3 and Ref. [35] for more information) can change the directions of pions and kaons by a magnetic field. For forward current negative particles will be “selected out”, leaving behind mostly positive pions and kaons that produce neutrinos. By reversing the electric current (reverse current) in the horns, we select negative pions and kaons which produce antineutrinos. The decays pions and kaons decay results in a beam which is a mixture of muons and neutrinos. The muons’ spatial and momentum distributions can be monitored to estimate the energy distribution of the neutrino beam. A dense absorber and a thick stretch of Earth are used to stop the muons, allowing only neutrinos to go forward.

The neutrinos produced in this fashion are energetic. We describe the neutrino
beam energy profile through a quantity called the “neutrino flux” \((\Phi)\). It is defined as the number of neutrinos per unit area \((\text{cm}^2)\) per energy bin \((\text{GeV})\) for each proton that hits the target \((\text{proton on target, POT})\) and has the unit of \(\text{cm}^{-2}\text{GeV}^{-1}\text{POT}^{-1}\).

The neutrino energy distributions in the flux can be adjusted to suit the experiment’s need. Neutrino oscillation experiments aim to measure neutrinos at the \(L^{\text{osc}}\) to maximize the physics potential. Therefore the distance from the neutrino source \((\text{baseline})\) to the neutrino detector dictates the energy distributions of the various neutrino flavors.

The same applies to reactor neutrino experiments. Since the antineutrino energies from reactors have energies of only a few MeV, the detectors must be placed at a much closer distance to the neutrino source in order to observe neutrino oscillation.

The T2K[36] experiment situated in Japan has a limited baseline of 295 km. The T2K experiment measured \(\sin^2(2\theta_{13})\) by measuring a \(\nu_e\) appearance in the \(\nu_\mu\) beam[37]. It also made precision measurements of \(\sin^2(2\theta_{23})\) and \(\Delta m^2_{32}\)[38]. The
T2K neutrino flux peaks at about 0.6 GeV[39]. The NOνA[40] baseline, on the other hand, is located 810 km from the neutrino source and investigates the same oscillation parameter range as T2K. Therefore its flux peaks at a higher energy at 2 GeV[41]. The Daya Bay experiment measured non-zero value $\sin^2(2\theta_{13})$ by monitoring the disappearance of electron antineutrinos[42] in the beam. The experiment has three pairs of detectors placed at 470 m and 576 m, and 1648 m. Therefore, all three experiments cover approximately the same $L/E$ region.

Another important physics goal of these experiments is the determination of $\delta_{CP}$. A non-zero value of $\delta_{CP}$ indicates that neutrinos and antineutrinos do not oscillate in the same way, and therefore there is Charge-Parity Violation (CPV) in the neutrino sector. The $T2K$ collaboration[43] has recently placed bounds favoring a non-zero $\delta_{CP} \in [-3.41, -0.03]$ for the case of normal ordering and $\delta_{CP} \in [-2.54, -0.32]$ for the case of inverse ordering, at a 3$\sigma$ confidence level. The next-generation experiments are designed to push these measurements beyond the 5$\sigma$ standard for claiming a discovery. Such discovery will lend credence to a class of theories that predict the matter-antimatter asymmetry in the Universe originates from an asymmetry in the leptonic sector in the decay of a CP-violating heavy sterile neutrinos produced in the early universe[18].

DUNE, Hyper-K, and JUNO experiments are successors to NOνA, T2K, and Daya Bay. These experiments will perform precision measurements of oscillation parameters. They are expected to have the sensitivity to determine the neutrino mass ordering and push measurements of $\delta_{CP}$ beyond the 5$\sigma$ standard for a new discovery.
1.3 Neutrino Energies and Their Interactions on the Nucleus

The most important quantities that we need to measure are the neutrino flavor and neutrino energy. Neutrino flavor can be unambiguously measured from the charged lepton flavor, the neutrino energy reconstruction is more complex.

Neutrinos primarily react with the nuclei in a target-detector. Nuclei other than hydrogen are composed of neutrons and protons bound by the strong nuclear force. Therefore a neutrino-nucleus scattering is not strictly a two-body process. The interaction cross section with nuclei is energy-dependent. In section 2.2 we discuss in more detail the how nuclear binding affects the reconstruction of neutrino energy. We begin with a brief discussion of how neutrinos react with nuclei in different ranges of energy.

Measurements of neutrino properties are performed by investigation of neutrino interactions in the detector. Just like photons excite atomic electrons in our eyes, neutrinos must interact with the nuclei (in the detector) and produce secondary particles that can be detected and measured.

The most fundamental rule about a neutrino interaction is that a neutrino must exchange either a charged $W^\pm$ boson or a neutral $Z^0$-boson with the target particle. When a $W$ boson is involved, the process is called a charged current (CC) interaction, and when a $Z$ boson is involved, the process is called a neutral current (NC) interaction. These are Standard Model weak processes with very small cross sections (the probabilities to scatter).

With that in mind, different interaction processes are important in different neutrino energy ranges,
1.3 Neutrino Energies and Their Interactions on the Nucleus

1.3.1 Very Low Energy

For neutrino energy on the order of $E_\nu \sim 0 - 1 \text{ MeV}$, two processes are important. One is neutrino capture by radioactive nuclei, similar to the reaction in Eq.(1.4). An example would be

$$\nu_e + ^{14}_6 \text{C} \rightarrow e^- + ^{14}_7 \text{N},$$

(1.27)

where an electron neutrino is captured on a radioactive carbon-14 to produce a nitrogen atom. Since this process releases energy, very little energy is needed to initiate the reaction.[44]. This process is often called the “stimulated” beta decay because the beta decay process has been initiated by the neutrino.

The other reaction is coherent elastic neutrino-nucleus scattering (CEvNS). It is an NC process with an enhanced total scattering amplitude. The enhancement comes from the quantum mechanical, in-phase addition of the scattering amplitudes from each nucleon in the nucleus[45]. When this happens, the scatter is coherent. The coherent elastic scattering proceeds as follows:

$$\nu + A_Z^N \rightarrow \nu + A_Z^N,$$

(1.28)

where the $A_Z^N$ on the right-hand side recoils against the neutrino. The differential cross section $d\sigma/dT$ as a function of the nucleus recoil energy $T$, can be written as

$$\frac{d\sigma}{dT} = \frac{G_F^2 Q_w^2 M_A(1 - \frac{M_A T}{2 E_\nu^2}) F(Q^2)^2}{4\pi},$$

(1.29)

where $G_F = 1.1663787(6) \times 10^{-5} \text{ GeV}^2$[46] is the Fermi coupling constant (the strength of the weak current on the leptonic side), $M_A$ is the mass of the target nucleus, $F(Q^2)$ is a quantity called the nuclear form factor that describes the degree...
of coherence across the nucleus. This quantity drops to zero when both the momentum transfer and the nuclear radius are very large. The variable $Q^2$ is defined as the negative of 4-momentum squared, which is introduced formally in section 2.1. $Q_W$, finally, is a term describing the strength of the weak current on the hadronic side and has the form

$$Q_W = N - Z(1 - 4\sin^2\theta_W),$$

where $\theta_W$ is the weak mixing angle. $Q_W$ is linearly dependent on the neutron number $N$ and proton number $Z$. The scatter from protons is suppressed by a factor of $1 - 4\sin^2\theta_W = 0.01084$. [47]

The maximum recoil energy allowed in an elastic collision is

$$T_{\text{max}} = \frac{E_\nu}{1 + M_A/2E_\nu}.$$  (1.31)

It has been suggested that the CEvENS process has important implications in the evolution of core-collapse supernova[45]. The final state nuclear recoil which has a very small energy in this process has recently been detected[48]. Because of the relatively large mass of the nucleus and the low recoil energy, the reconstruction of the direction of the recoil nucleus is very challenging. Here, the absence of visible second particle in the final state renders it impossible to reconstruct the full kinematics of events.

### 1.3.2 Low Energy

At the $1 - 100$ MeV energy scale, the Inverse Beta Decay (IBD) process becomes important. Neutrinos were first discovered by the observation of the IBD process[17].
The IBD process (Eq.(1.33)) is the inverse of the beta decay process (Eq.(1.32)).

\[ n \rightarrow p + e^- + \bar{\nu}_e, \quad (1.32) \]
\[ \bar{\nu}_e + p \rightarrow n + e^+. \quad (1.33) \]

Because the positron has non-zero mass (0.511 MeV/c²) and the neutron (939.565 MeV/c²) is heavier than the proton (938.272 MeV/c²), the neutrino must have at least \( E_\nu = 1.804 \text{ MeV} \) to trigger the reaction. The outgoing positron will be immediately annihilated by an electron to produce two photons with total energy \( E_{\gamma\gamma} = 1.022 \text{ MeV} \).

The neutron, if freed, could be later captured on another nucleus and excite the nucleus. When the excited nucleus drops to its ground state, it emits de-excitation photons that can be observed[44].

It is unfortunate that because of large density of electrons in the target-detector, the final state positron annihilates and its kinematic information is lost. The final state neutron is observable only if the neutrino reacts with a hydrogen or deuterium nucleus. For heavier nuclei, the neutron can remain bound and the final state is an excited nucleus with electric charge of \((Z-1)\).

As we continue to climb the energy ladder, we begin to have neutrinos that can transfer enough energy to the nucleon to free it from the bound nucleus. The energy required to liberate a neutron or a proton in carbon-12, for example, is 18.7 MeV and 16.0 MeV respectively[3]. Next, we discuss the various processes that produce final state hadrons when the neutrino crosses the few-hundred MeV to the GeV region.
1.3 Neutrino Energies and Their Interactions on the Nucleus

1.3.3 The GeV Region

In the pre-GeV region and the few-GeV region there are several processes with final states that allow for the reconstruction of the energy of the neutrino interactions in the detector. There are three basic types of charged current processes that are of particular interest to oscillation experiments. Although nuclear effects, to be discussed later, smear out the distinctions between these processes, they remain valuable starting points in understanding neutrino interactions.

The first process is the charged current quasielastic (CCQE) process:

\[ \nu(\bar{\nu}) + n(p) \rightarrow l^{- (+)} + p(n), \]  

CCQE is the simplest process relevant to oscillation experiments. Its Feynman diagram is shown in Fig 1.4a.

The incoming neutrino exchanges a W boson with a nucleon in the nucleus. The neutrino is transformed into a charged lepton. For electron and muon neutrinos, we observe either electrons or muons in the final state. These two light charged leptons are relatively stable and can be identified through their ionization of atomic electrons in the detector medium. Because the \( \tau \) lepton is massive, it decays immediately, often producing several hadrons (particles composed of quarks) in the final state.

On the hadronic side of the CCQE reaction, energy and momentum are transferred to the nucleon and the charge of the nucleon is also changed. The CCQE process is very similar to an elastic two-body billiard ball collision, except for the fact that the final states particles have masses which are different from the masses of the initial state particles.

At higher neutrino energies the W boson is sufficiently energetic to induce excita-
1.3 Neutrino Energies and Their Interactions on the Nucleus

Figure 1.4: The charged current (CC) processes involve the exchange of a $W$ boson. The neutrino $\nu$ is transformed into its charged counterpart $l$. In the figures $n(p)$ denotes a neutron (proton). a) The quasielastic process (QE) in which the neutrino knocks out a single nucleon. Here, the neutron (proton) is changed to a proton (neutron) for incident neutrinos (antineutrinos). b) Single pion production (RES), the nucleon is excited to a $\Delta$ resonance, which then decays to a nucleon and a pion. c) Deep inelastic scattering (DIS). Here, significant energy is transferred to quarks inside the nucleon. On the way out of the nucleon the quarks fragment into hadrons, and typically the nucleus is also fragmented. Each type of interaction dominates in a different energy range. The CCQE process is dominant around a neutrino energy of 0.5 GeV, the RES process at about 1.1 GeV, and the DIS process dominates at higher energies.
tion of protons and neutrons into a higher mass resonance. Many types of resonances exist, such as 14 $\Delta$ resonances [49] and 20 $N$ resonances [50]. The $\Delta$ resonances are the dominant resonance mode we encounter in the few-GeV region. The first $\Delta$ resonance has a mass of 1232 MeV/c$^2$, and decays into a nucleon and a pion 99.5% of the time (Fig 1.4b). This process, which is referred to as resonant pion production (RES), can proceed in several different ways. For neutrinos, we have

$$\nu + n \rightarrow \Delta^+ + l^-$$

(1.35)

$$\Delta^+ \rightarrow p + \pi^0$$

$$\rightarrow n + \pi^+,$$

and

$$\nu + p \rightarrow \Delta^{++} + l^-$$

(1.36)

$$\Delta^{++} \rightarrow p + \pi^+.$$ 

And for antineutrinos, we have

$$\bar{\nu} + p \rightarrow \Delta^0 + l^+$$

(1.37)

$$\Delta^0 \rightarrow p + \pi^-$$

$$\rightarrow n + \pi^0,$$
and

\[ \bar{\nu} + n \rightarrow \Delta^- + l^+ \]  
\[ \Delta^- \rightarrow n + \pi^- . \]  

As shown above several different final states are possible. Since the \( \Delta \) particle decays inside the nucleus, we treat it as an intermediate state which produces a nucleon and a pion in the final state.

The complexity of the neutrino-nucleus reaction increases at higher energy. This is the region in which the dominant reaction is the "deep inelastic scattering" (DIS) process. Here, the \( W \) boson has a large momentum and by the Heisenberg Uncertainty Principle, it can probe a size smaller than the size of the nucleon, and directly interact with quarks and gluons. The final state struck quarks are energetic and fragment into showers of hadronic particles (nucleons, \( \Delta s \), \( \pi/K \) mesons).

The DIS process has been used to test quantum chromodynamics (QCD)[51], which is the theory that describes the strong force and the interaction of quarks and gluons.

For accelerator based oscillation experiments, CCQE is the primary process used to measure neutrino oscillation parameters for two reasons. First, most oscillation experiments operate at energies for which the CCQE is dominant. For example, the flux of the muon neutrino beam in the T2K experiments peaks at about 0.6 GeV[39], and the neutrino beam in the NO\( \nu \)A experiment peaks at about 2 GeV[41]. The second reason lies in the simplicity of the two-body CCQE process.
1.3 Neutrino Energies and Their Interactions on the Nucleus

\[ \nu(\bar{\nu}) \rightarrow k_1 \rightarrow k_2 \rightarrow l^{-}(+) \]

\[ q \rightarrow W^{+}(-) \]

\[ n(p) \rightarrow p_1 \rightarrow p_2 \rightarrow p(n) \]

Figure 1.5: Feynman diagram of the CCQE process. Here, \( k_1, k_2 \) are the 4-momentum of the initial state neutrino \( \nu \) and final state charged lepton \( l \), respectively, and \( p_1 \) and \( p_2 \) are the 4-momenta of the initial state and final state nucleons, respectively. The exchanged \( W \) boson has 4-momentum \( q = (q_0, q) \).

### 1.3.4 Neutrino Energy Reconstructions in the Few-GeV Region

Figure 1.5 illustrates the energy and momentum (4-momentum) transfer in the CCQE reaction. The incoming neutrino, with 4-momentum \( k_1 \), interacts with the target nucleon which has 4-momentum \( p_1 \). A 4-momentum \( q \) is transferred to the initial state nucleon. The outgoing lepton has a 4-momentum \( k_2 = k_1 - q \), and the final state nucleon has a 4-momentum \( p_2 = p_1 + q \). The square of the 4-momentum transfer \( q \) is:

\[ q^2 = (k_1 - k_2)^2, \]
\[ Q^2 = -q^2, \]

where we have chosen the Minkowski metric to be \( \text{diag}(1,-1,-1,-1) \). In this metric, \( q^2 < 0 \), so we define \( Q^2 = -q^2 \) to be always positive for ease of calculation.

The location of the source of the neutrinos in reactor and accelerator neutrino
1.3 Neutrino Energies and Their Interactions on the Nucleus

experiments is well defined. The neutrino direction can then be determined from the location of the neutrino source and the detector. At the peak of the CCQE process, the neutrino energy can be reconstructed by the following expression,

\[ E_{\nu,\bar{\nu}} = \frac{W^2 - M'_{IS} - m^2_l + 2M'_{IS}E_l}{2(M'_{IS} - E_l + p_l \cos \theta_l)}, \]  

(1.41)

and

\[ Q^2_{QE} = 2E_{\nu}(E_l - p_l \cos \theta_l) - m^2_l. \]  

(1.42)

where \( W^2 \) is the invariant mass of the final state particles, \( M'_{IS} = M_{N,P} - \epsilon^{N,P} \) is the target nucleon mass reduced by the nuclear binding energy, \( E_l \) and \( p_l \) are the final state charged lepton energy and momentum, respectively, and \( \theta_l \) is the charged lepton angle with respect to the incoming neutrino direction. This equation assumes that the nucleon is at rest, which is generally not true in a nucleus with many nucleons. Fermi motion, to be discussed in Sec 2.2.1, gives bound nucleon momentum. Eq.(1.41) could not precisely reconstruct neutrino energy on an event-by-event basis, but it provides an estimate on a statistical sample where the Fermi motion averages to zero.

For CCQE, \( W^2 = M^2_{P,N} \) is simply the final state nucleon’s mass. Every parameter in Eq.(1.41) is well-defined for the CCQE reaction for which there is only a nucleon and a muon in the final state. The T2K experiment relies on the CCQE hypothesis to reconstruct the neutrino energy in their investigation of oscillations parameters using both muon neutrino disappearance and electron neutrino appearance analyses[37, 38].

While theoretically simple, the neutrino reconstruction using the CCQE hypothesis is actually more complicated experimentally. First, the detector technology
limits the energy range for which final state particles can be detected. The T2K experiment identifies charged particles in its far detector through the detection of Cherenkov light. However, a pion with an energy below 4.6 GeV is not in the detectable\cite{52} because it is below Cherenkov threshold. Therefore the detector cannot reliably distinguish between CCQE and pion production events. Consequently, reliable cross section models for the CCQE and RES processes are needed to estimate the fraction of actual CCQE events.

Secondly, nuclear effects wash out the distinctions between CCQE and RES processes because final state hadrons can interact with other nucleons in the nucleus, and thus change the hadronic final state which is observed in the detector. Therefore, we also need a reliable nuclear model (in addition to the modeling the neutrino-nucleon cross section) to correctly simulate the nuclear effects.

A detector that is sensitive to electromagnetically or strongly interacting particles is called a calorimetric detector. A calorimeter is efficient at measuring the energy of charged particles because they are able to produce tracks through ionizing the surrounding electrons. Neutral particles are usually invisible until they interact to produce charged secondary particles. A photon, for example, must convert into an electron-positron pair. A neutron, on the other hand, must interact on the target nucleus with enough energy to be seen. The NO$\nu$A experiment is a calorimetric detector and uses calorimetric energy reconstruction in the muon neutrino disappearance analyses\cite{53}. However, both the NO$\nu$A and the T2K detectors have difficulty in the detection of neutrons in the final state, Unlike protons or the muons, neutrons do not produce visible tracks, and the energy carried by final state neutrons is mostly unobserved. To account for this unobserved energy, these experiments must also rely on models of neutrino interactions with nuclei.
1.4 A Brief Layout of this Work and the Connection to MINER$\nu$A

In Chapter 2 we briefly discuss the theory of CCQE interactions and motivate the importance of measuring neutrino cross sections on free protons. We then follow with a discussion of nuclear effects and the introduction of neutrino-nucleus scattering Monte Carlo (MC) generators used by the wider neutrino community. We then present on a phenomenological study of one of the generators, GENIE$^{[54]}$, to illustrate the issues involved in the modeling of interactions with nuclei.

The need to study nuclear effects in different nuclei has motivated the construction of the MINER$\nu$A detector. MINER$\nu$A is an acronym for “Main-Injector Neutrino Experimen$\nu$ - $A$”. Here, $A$ stands for the nucleus. An overview of the MINER$\nu$A detector is presented in Chapter 3.

The work done by the author on MINER$\nu$A including phenomenological studies is presented in Chapter 4. The results motivate corrections to GENIE needed to facilitate the measurement of the antineutrino CCQE cross section on hydrogen which is discussed in chapter 5. We define new observables to pinpoint issues in the nuclear models which are used to place additional constraints on the detector’s neutrino-nucleus interactions. These constraints are needed to make a measurement on hydrogen possible. A novel neutron reconstruction algorithm has been developed for the measurement on hydrogen.

In chapter 7 we present new analysis techniques made possible by the novel neutron reconstruction algorithm, and then follow with concluding remarks.
Chapter 2

Neutrino Interactions – The Nuclear Mess

2.1 The Llewellyn Smith Model of the Quasi-Elastic Reaction

2.1.1 Theoretical Framework

Llewellyn Smith, in his 1971 seminal review, *Neutrino Energy at Accelerator Energies* [55], lays down the theoretical framework to calculate the CCQE cross section on free nucleon.

The CCQE differential cross section is written as

\[
\frac{d\sigma}{dQ^2} = \frac{M^2 G_F^2 \cos^2 \theta_c}{8\pi E_\nu^2} \left[ A(Q^2) \mp B(Q^2) \frac{(s-u)}{M^2} + C(Q^2) \frac{(s-u)^2}{M^4} \right].
\]

\[(2.1)\]

\(M = (M_n + M_p)/2\) is the average nucleon mass, \(G_F\) is the Fermi coupling constant.
that encode the strength of the weak current coupled to the leptons, its value is first shown on page 22, \( \theta_c = 13.04^\circ \) is the Cabibbo angle, \( E_\nu \) is the neutrino energy, \( s \) and \( u \) are the Mandelstam variables, defined as (see Fig. 1.5)

\[
s = (k_1 + p_1)^2 = (k_2 + p_2)^2, \quad (2.2)
\]

\[
u = (k_1 - p_2)^2 = (k_2 - p_1)^2, \quad (2.3)
\]

and

\[
s - u = 4ME_\nu - m^2 + q^2
\]

\[
= 4ME_\nu - m^2 - Q^2. \quad (2.4)
\]

The \( A, B, C \) terms are only dependent on \( Q^2 \). They encode information about how the nucleon responds to the weak current:

\[
A(Q^2) = \left( \frac{m^2}{M^2} + 4\tau \right) \left[ (1 + \tau) |F_A|^2 - (1 - \tau) |F_1|^2 + \tau (1 - \tau) |\xi F_2|^2 + 4\tau \text{Re} (F_1^* \xi F_2) - 4\tau (1 + \tau) |F_A^3|^2 \right. \\
\left. - \frac{m^2}{4M^2} \left[ |F_1 + \xi F_2|^2 + |F_A + 2F_P|^2 - 4(1 + \tau) (|F_3|^2 + |F_P|^2) \right] \right], \quad (2.5)
\]

\[
B(Q^2) = 4\tau \text{Re} [F_A^* (F_1 + \xi F_2)] \\
- \frac{m^2}{M^2} \text{Re} \left[ (F_1 - \tau \xi F_2)^* F_3 - (F_A - 2\tau F_P)^* F_A^3 \right], \quad (2.6)
\]

\[
C(Q^2) = \frac{1}{4} \left( |F_A|^2 + |F_1|^2 + \tau |\xi F_2|^2 + \tau |2F_A^3|^2 \right), \quad (2.7)
\]
and

$$\tau = \frac{Q^2}{4M^2}. \quad (2.8)$$

Note the term $\xi$ here is the difference between the proton and neutron anomalous magnetic moment

$$\xi = \mu_p - \mu_n = 3.71, \quad (2.9)$$

not to be confused with $\xi(\tau)$ that will appear in Sec. 2.1.2.

2.1.2 Form Factors

The parameterization of the $A, B, C$ terms involves nuclear form factors linked to the charge distributions for different physical processes. Form factors, in a nutshell, describe these distributions.

$F_1$ and $F_2$ are the vector form factors associated with the electric and magnetic charge distributions of the proton and neutron. $F_A$ and $F_P$ are the axial and the induced pseudoscalar form factors related to the distribution of the weak charge in the nucleon. $F_3^A$ and $F_3^V$ are the second-class currents, which are very small$^{[56]}$ if they exist at all.

In the simplest picture, a form factor is just the Fourier transform of an underlying charge distribution:

$$F(q) = \int \rho(x)e^{-iq\cdot x}dx^3.$$

The cross section for the scattering of a test charge from the charge distribution is then

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{point}} |F(q)|^2.$$
For an exponential charge distribution $\rho(r)$, $F(q)$ has a dipole structure:

$$\rho(r) = \frac{8\pi}{m^3} e^{-mr},$$  \hspace{1cm} (2.10)

$$F(q) = \left(1 - \frac{q^2}{m^2}\right)^{-2}. \hspace{1cm} (2.11)$$

The dipole form is in fact a very good starting point for the nucleon form factors, even after generalizing to the negative square of 4-momentum transfer $Q^2$

$$F(Q^2) = \left(1 + \frac{Q^2}{M^2_{V,A}}\right)^{-2}. \hspace{1cm} (2.12)$$

In Eq.(2.12), $M_V$ and $M_A$ are the vector and axial mass term similar to the $m$ term in Eq.(2.11).

The form factor’s interpretation as the Fourier transform of the charge density remains valid at the classical limit when the momentum transfer is very small compared to the nucleon mass, $|q|^2 \ll M^2$. We often derive a charge radius from the form factor at $Q^2 = 0$, therefore satisfying the low $|q|^2$ condition.

**Vector Form Factors**

Electron scattering processes are dominated by electromagnetic interactions. The EM interaction couples only to the vector form factors. The vector form factors $F_1$ and $F_2$ have been extensively fitted using data from electron scattering experiments[57]. They can be further expressed in terms of the Sachs electric and magnetic
form factors as

\begin{align*}
F_1(Q^2) &= \frac{[G^V_E(Q^2) + \tau G^V_M(Q^2)]}{1 + \tau}, \tag{2.13} \\
\xi F_2(Q^2) &= \frac{[G^V_M(Q^2) - G^V_E(Q^2)]}{1 + \tau}, \tag{2.14}
\end{align*}

where \( G^V_E \) and \( G^V_M \) are defined in terms of the electric form factors and the magnetic form factors on proton (\( G^p_E, G^p_M \)) and neutron (\( G^n_E, G^n_M \))[58]:

\begin{align*}
G^V_E &= G^p_E(Q^2) - G^n_E(Q^2), \tag{2.15} \\
G^V_M &= G^p_M(Q^2) - G^n_M(Q^2). \tag{2.16}
\end{align*}

To first order, the proton electric form factors can be modeled after the dipole form factor as

\[ G_D = \frac{g_V}{1 + \frac{Q^2}{M_V^2}}, \tag{2.17} \]

with the vector mass term \( M_V^2 = 0.71 \text{ GeV}/c^2[57] \) and the vector coupling constant \( g_V = 1 \).

Many attempts to improve upon the dipole form factor exist. Parameterizations for the vector form factors have been made[57–62] with increasing sophistication over the years. Figure 2.1 shows one such parameterization by Bodek et al[57], expressed in the Nachtman scaling variable \( \xi \):

\[ \xi = \frac{2}{1 + \sqrt{1 + 1/\tau}}, \tag{2.18} \]

The fits shown in Fig. 2.1 are consistent with data within 5% except in \( G^n_E/G_D \). In 2007 when the fits were made, the data for \( G^n_E/G_D \) were only available up to
Figure 2.1: Fits (BBBA2007) of $G_E^{n,p}$ and $G_M^{n,p}$ from Ref. [57]. The fits have been expressed as a ratio to $G_D$ for the proton(a) and neutron(c) electric form factors. The proton(neutron) magnetic form factor has been expressed as a ratio to $\mu_p(n)G_D$, the product of proton(neutron) magnetic moments. The solid line is the BBBA2007 fit, short dashed lines are the Kelly[60] fits (Galster[59] fit for $G_E^n$). The long dashed line is a fit based on a different constraint. Note that $\xi$ here is the Nachman scaling variable defined in Eq.(2.18). The two high $Q^2$ points shown in blue in the $G_E^n$ panel were measured later in 2010.

$Q^2 = 2.1$ GeV$^2$ ($\xi = 0.75$). Currently, more data is available[63].

The new measured values of $G_E^n/G_D$ at $Q^2 = 2.48$ GeV$^2$ ($\xi = 0.783$), and $Q^2 = 3.41$ GeV$^2$ ($\xi = 0.833$) are $0.404\pm0.050$ and $0.478\pm0.078$, respectively. These values which are shown in blue in the $G_E^n$ panel in Fig. 2.1 are consistent with the BBBA2007 parameterizations. Therefore, the BBBA2007 parameterizations are valid up to highest $Q^2$ for which current neutrino experiments[64, 65] have data.
Axial Form Factors

The axial form factors are new parameters introduced for the weak currents, as it has a “V-A” (vector-axial vector) current structure. A detailed treatment of the “V-A” structure can be found in Ref. [66].

Currently, most experiments treat \( F_A \) to have a dipole form

\[
F_A = g_A \left( 1 + \frac{Q^2}{M_A^2} \right)^{-2}. \tag{2.19}
\]

The constant \( g_A = -1.267 \) is the axial coupling strength determined from neutron \( \beta \)-decay experiments[67]. The detail of its derivation can be found in Ref. [68].

The axial form factor has been measured in the deuterium bubble chamber experiments in the 1970s and 1980s[69–75], with the reaction

\[
\nu + d \rightarrow \mu^- + p + p^S. \tag{2.20}
\]

Here \( d \) is a deuteron, composed of a neutron and a proton \((np)\) in the nucleus. \( p \) is the final state proton formed from neutrino interaction and \( p^S \) is the proton originally residing in the deuterium nucleus. For neutrino, CC interaction changes the bound neutron to a proton, while the original proton does not change flavor (but may gain momentum). About \( \sim 6000 \) neutrino quasielastic events were obtained at various \( Q^2 \). In addition, there are some measurements of the axial form factor from pion electroproduction experiments on hydrogen. In the BBBA2007 publication[57], in addition to studies of the EM form factors, an analysis of all available measurements of the axial form factors was done. Data from both neutrino experiments on deuterium and pion electroproduction experiments on hydrogen provide information
Figure 2.2: Axial form factor analysis from the BBBA2007 publication[57]. (a) $F_A(Q^2)$ re-extracted from all neutrino-deuteron data divided by dipole form (corrected for hadronic effects). Solid line – BBBA2007 duality based fit. short-dashed line assuming vector is equal to axial form factor. Dashed-dot line - a constituent quark model. The values of $\xi$ and the corresponding values of $Q^2$ are shown on the bottom and top axis.

on $F_A(Q^2)$. The average value of the axial mass term $M_A$ derived from the deuterium bubble chamber experiments is $1.015 \pm 0.026\text{GeV}/c^2$[57]. The current value used in the GENIE neutrino-nucleus interaction generator is $M_A = 0.99$. However, there is an indication that there are deviations of $F_A(Q^2)$ from the dipole form as seen in Fig 2.2 taken from the BBBA2007 publication.

We note that some nuclear corrections are needed in the analysis of the neutrino experiments on deuterium, and hadronic corrections are needed in the analysis of the pion electroproductions experiments on hydrogen. Therefore, a measurement of $F_A(Q^2)$ using the following antineutrino CCQE process

$$\bar{\nu}_\mu p \rightarrow \mu^+ n$$

(2.21)

on free proton would be of great interest.

An extraction of $F_A(Q^2)$ from this reaction using a hydrogen bubble chamber was
Figure 2.3: $Q^2$ distribution for quasielastic events from Ref. [76]. A theoretical cross section is shown for three values of $M_A$. The statistical uncertainty is large because the total number of events is only 13.

published in 1980[76] by Fanourakis et. al. However, the experiment accumulated 13 antineutrino CCQE events on hydrogen as shown in Fig 2.3.

Recently, a new parameterization based on the “z-expansion” formalism has been developed[77]. This formalism aims to make model independent form factor extraction with an expansion form

$$F_A(Q^2) = \sum_{k=0}^{k_{\text{max}}} a_k z(Q^2)^k,$$

$$z(Q^2) = \frac{\sqrt{t_{\text{cut}} + Q^2} - \sqrt{t_{\text{cut}} - t_0}}{\sqrt{t_{\text{cut}} + Q^2} + \sqrt{t_{\text{cut}} - t_0}},$$

where $t_{\text{cut}} = 9m^2$ is the leading 3-pion threshold and $t_0$ is a parameter chosen to minimize the range of $|z|$ for which data is present. In Ref. [77], $t_0$ is chosen to be
2.1 The Llewellyn Smith Model of the Quasi-Elastic Reaction

Figure 2.4: $Z$-expansion fit to deuterium data, compared with a dipole form factor with $M_A = 1.014$. Ref [77].

The result of the fit is shown in Fig 2.4 and compared with a dipole form factor with $M_A = 1.014\text{GeV}$. Deviation of $F_A$ from the dipole form could be potentially measured with more precise antineutrino measurements on hydrogen. This thesis is the first new measurement of the $\bar{\nu}_\mu + p \rightarrow \mu^+ + n$ quasielastic reaction on free proton since the initial results of Fanourakis in 1980[76].

Recent measurements of $F_A$ on complex nuclei were reported by the MiniBooNE[78] experiment (on carbon) and by the K2K[79] experiment (on oxygen). Nuclear effects (discussed in detail in Sec 2.2) such as Meson Exchange Current (MEC) introduce a bias that resulted in $M_A$ values of about $1.2\text{ GeV}/c^2$ when measured on nuclei. This is resolved by accounting for MEC in the simulation [80]. This example illustrates
the difficulty of measuring the $F_A$ on nuclei since extracting $F_A$ from neutrino data on heavier nuclei is highly model-dependent. In contrast, the measurement reported in this thesis is not model-dependent because it relies on interactions involving the hydrogen nuclei in the scintillator target only.

**The Pseudoscalar Form Factor**

Finally, the induced pseudoscalar form factor $F_P$ has not been measured. It is estimated to have the form[81]

$$F_P(Q^2) = \frac{4M^2F_A(Q^2)}{m_{\pi}^2 + Q^2}, \quad (2.24)$$

where $M$ and $m_\pi$ are the nucleon and pion masses, respectively. The form of the pseudoscalar form factor is derived from the partially conserved axial-vector current (PCAC) and the Goldberger-Treiman[82] relation. The PCAC[83]

$$F_P(Q^2) = \frac{2M^2F_A(0)}{Q^2} \times \left( \frac{F_A(Q^2)}{F_A(0)} - \frac{g_{\pi NN}(Q^2)}{g_{\pi NN}(0)} \frac{1}{Q^2/m_{\pi}^2} \right), \quad (2.25)$$

where $g_{\pi NN}$ is the pion nucleon form factor. The Goldberger-Treiman relation states

$$g_{\pi NN}(Q^2)F_\pi = F_A(Q^2)M, \quad (2.26)$$

where $F_\pi$ is the pion decay constant. Combining equations (2.25) and (2.26) gives us Eq. 2.24.

In the case of electron and muon neutrino reactions common in oscillation experiments, the term is suppressed by a factor of $m^2/M^2$, the square of the ratio between the charged lepton mass and the nucleon mass. Only in tau neutrino reactions will
$F_P$ become a significant contribution to the cross section.

## 2.2 Nuclear Effects

### 2.2.1 Fermi Motion

According to the spin-statistics theorem, all particles with half-integer spins are fermions, while particles with integer spins are bosons. The wave functions of a pair of identical bosons remain unchanged after swapping their positions. However, if a pair of identical fermions were to swap, their wave functions change sign.

The sum of the wave functions of two identical fermions is,

$$|\psi\rangle = |x_1x_2\rangle - |x_2x_1\rangle.$$  \hspace{1cm} (2.27)

If we set $x_1 = x_2$, $|\psi\rangle = 0$, both wave function becomes zero, and the particles cease to exist. This is the famous Pauli exclusion principle. For many particles system, the fermions must obey the Fermi-Dirac statistics and the average number of particles occupying a single particle state is

$$\langle n \rangle = \frac{1}{e^{(\epsilon_i - \mu)/kT} + 1},$$  \hspace{1cm} (2.28)

where $\epsilon_i$ is the energy of particle $i$, $\mu$ is a chemical potential, $k$ is the Boltzmann’s constant, and $T$ is the temperature of the system. The value of the average $n$ maximizes at 1, consistent with the requirement that only one fermion can occupy a single state.

We can immediately derive a simple model, the relativistic Fermi gas (RFG)
model for the initial state nucleons bound in the nuclei. Since nucleons from the same species are identical except for their momentum, they must be in different momentum states to obey the Pauli exclusion principle. Each particle can occupy a distinct momentum state \((k_x, k_y, k_z)\) with total momentum \(k = \sqrt{k_x^2 + k_y^2 + k_z^2} \leq K_F\). The constant \(K_F\) is the Fermi momentum of a nucleus, and it defines the upper bound on the total momentum a nucleon could attain. We can see that the momentum states essentially form the interior of a sphere, with radius \(K_F\). The probability for a particle \(i\) to have momentum \(k_i\) is then just the ratio of the volume of a thin spherical shell \(4\pi k_i^2\) with width \(dk\) to the total volume of the momentum sphere \(4/3\pi K_F^3\)

\[
P(k) = \frac{3k^2}{K_F^3} \, dk. \tag{2.29}
\]

For elements relevant to neutrino experiments, the size of \(K_F\) is well below the nucleon mass and non-relativistic. Table 2.1, taken from Ref. [3], shows the size of Fermi momentum on various elements measured from electron scattering data. For nuclei with equal number of protons and neutrons, the Fermi momentum is the same for both nucleons. When the proton number and neutron numbers are not equal, the Fermi momentum is different.

The probability of finding a nucleon with momentum magnitude \(k\) increases quadratically for \(k < K_F\) then drops sharply to zero beyond that. Interactions for which the momentum transfer is \(|q| < 2K_F\) are suppressed because the resulting nucleon may end up with a momentum state occupied by another nucleon. This effect is Pauli blocking, which is a multiplicative suppression factor on the cross
### Table 2.1: Summary of Fermi momentum for different isotopes.

<table>
<thead>
<tr>
<th>Element</th>
<th>$K_F^p, K_F^n$ (MeV/c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^7$H</td>
<td>88</td>
</tr>
<tr>
<td>$^7$Li</td>
<td>169</td>
</tr>
<tr>
<td>$^{12}$C</td>
<td>221</td>
</tr>
<tr>
<td>$^{16}$O</td>
<td>225</td>
</tr>
<tr>
<td>$^{27}$Al</td>
<td>238, 241</td>
</tr>
<tr>
<td>$^{40}$Ar</td>
<td>251, 263</td>
</tr>
<tr>
<td>$^{40}$Ca</td>
<td>251</td>
</tr>
<tr>
<td>$^{59}$V</td>
<td>253, 266</td>
</tr>
<tr>
<td>$^{56}$Fe</td>
<td>254, 268</td>
</tr>
<tr>
<td>$^{197}$Au</td>
<td>275, 311</td>
</tr>
<tr>
<td>$^{208}$Pb</td>
<td>275, 311</td>
</tr>
</tbody>
</table>

The Fermi momentum for all except $^{40}$Ar and $^{208}$Pb have been experimentally measured. $K_F$ for $^{40}$Ar and $^{208}$Pb are derived from shell models. See Ref. [3] for the full summary. While plenty of data for lead exist, only 1 measurement on argon, the main detecting medium for DUNE, has been made [84].

Finally, a fixed removal energy value applies to all nucleons in the RFG model. Effectively, all nucleons of the same species in the nuclei must have the same initial state energy.

The RFG model is hardly a convincing model of the nucleus, but it is an important first step in modeling the neutrino-nucleus interactions. The RFG model has been utilized in common neutrino-nucleus Monte Carlo event generators such as GENIE [54], NuWro [85], and NEUT [86].

More sophisticated initial state models such as the Local Fermi Gas (LFG) [87], and the Benhar Spectral Function (SF) [88] models try to improve on the simplistic...
2.2 Nuclear Effects

Figure 2.5: (Left) the $r$ dependent $K_F$ for oxygen, argon, and iron. (Right) the momentum-dependent effective potential used in NuWro. Both figures taken from Ref. [89]

assumptions of the Fermi gas model by making the value of $K_F$ dependent on the local density at each space points in the nucleus (LFG) or modelling a 2-dimensional probability based on the nucleon energy and momentum (SF). These models have been implemented in NuWro and NEUT as well.

In the RFG and LFG models, it is possible to include non-constant potential that effectively modifies the nucleon energy. See Fig 2.5 for an implementation of the momentum dependent $K_F$ and the effective potential used in NuWro.

The spectral function model (SF), on the other hand, is defined as the probability $P(p, \epsilon)$ to remove a nucleon with momentum $p$ from the nucleus and leave the spectator nucleus with energy $E_R = M_A - M + \epsilon + p^2/(2M_{A-1})$, where $\epsilon$ is the removal energy. For carbon and heavier nucleus, the kinetic energy term $p^2/(2M_{A-1})$ is negligible due to the large spectator nucleus mass. NuWro implements an approximate spectral function by splitting the probability into a mean-field (MF) part and a correlated part. Nucleons in the mean-field do not interact with each other and are modeled using the shell model of nucleus. Nucleons in this model occupy energy levels, called “shell”, similar to how bound electrons move around the nucleus in orbitals with fixed energy. Nucleons in a shell accounts for approximately 80% of
the nucleons. The rest of the nucleons interact with one another through short-range correlations (SRC) and may result in instantaneous momentum exceeding the Fermi momentum cut-off. SRC occurs when the wave functions of two nucleons momentarily overlap. When this happens, their total momentum could be below the Fermi momentum, but each nucleon could acquire high momentum that anti-correlates with the other nucleon\[90]. $P(p, \epsilon)$ depends on the nucleon momentum $p$ and the removal energy $\epsilon$ that modifies the nucleon’s total energy. NuWro’s SF also implements the optical potential of the nucleus, to be discussed in the next section.

### 2.2.2 Energy Conservation at the Interaction Vertex

Neutrino interactions on the nucleus at energies relevant to the oscillation experiments are often modeled with the plane wave impulse approximation (IA). In the impulse approximation, the probe sees the target nucleus as a collection of independent nucleons. The momentum of the nucleon distributes according to the initial state model. The argument for the validity of the IA roots in the uncertainty principle, for the position and momentum conjugate variables, in units of GeV and set $\hbar$ to 1, we have

$$\Delta x \Delta q \geq \frac{1}{2}.$$

The proton charge radius, as measured by electron scattering experiments, is about 0.88 fm = 4.5 GeV$^{-1}$, so for the momentum transfer we have $\Delta p \geq 110$ MeV/$c$. So long as the probing momentum is on the order of 100 MeV and larger, it can resolve the individual nucleons in a nucleus.

Therefore, scattering on a nucleus can often be reduced to the scattering on an individual nucleon. For simplicity, consider an electron scattering experiment
in which the nucleon species does not change. The energy balance equation for scattering with 4-momentum transfer \((q_0, \mathbf{q})\) on a nucleus target of mass \(M_A\) with \(A\) nucleons is

\[
q_0 + (M_A - \sqrt{M_{A-1}^2 + k^2}) = \sqrt{(k + \mathbf{q})^2 + M_{n,p}^2} + V, \tag{2.31}
\]

\(M_{A-1}\) is the spectator nucleus with a proton or neutron removed since the interaction happened on a proton or neutron. The \(^*\) indicates that this spectator nucleus is in an excited state – it has more energy than a \(M_{A-1}\) ground state spectator nucleus. \(k\) is the momentum of the spectator nucleus. Because the nucleus’ net momentum must be zero, the interacting nucleon momentum is \(-k\) to balance out the spectator nucleus. Therefore the total momentum of the target nucleus is zero. Energy \(q_0\) is transferred to the nucleus, and on the right hand side we obtain an on-shell neutron or proton with mass \(M_{n,p}\) and initial momentum \(\mathbf{p} = k + \mathbf{q}\). The term \(V\) includes various potentials in the nucleus to conserve the total energy during an interaction. This equation uses the off-shell formalism since the interacting nucleon is not on the mass shell\(^1\).

While the interacting nucleon mass \(M\) does not come directly into the left hand side, it can be made to appear by using the following relation:

\[
M_A = M_{A-1} + M_{n,p} - S_{n,p}, \tag{2.32}
\]

\[
M_{A-1}^* = M_{A-1} + E_{x,n,p}.
\]

\(S\) and \(E_x\) are the separation energy and the excitation energy for a neutron or a

\(^1\)A particle is on-shell if it satisfies the classical equation of motion \(E^2 = M^2 + |\mathbf{p}|^2\). It is off-shell if otherwise.
Table 2.2: Summary of the separation energy $S$ and the excitation energy $E_x$ on different nuclei for an interacting neutron or proton. The Coulomb potential is denoted by an effective potential $|V_{\text{eff}}|$. Values taken from Ref. [3]

| Element | $S_{\text{p,n}}$(MeV) | $E_{x_{\text{p,n}}}$ (MeV) | $|V_{\text{eff}}|$ |
|---------|----------------------|-------------------------|------------------|
| $^1_2\text{H}$ | 2.2 | 0.0 | |
| $^6_3\text{Li}$ | 4.4, 5.7 | 12.2 | 1.4 |
| $^{12}_6\text{C}$ | 16.0, 18.7 | 10.1, 10.0 | 3.1 |
| $^{16}_8\text{O}$ | 12.1, 15.7 | 10.9, 1.2 | 3.4 |
| $^{27}_{13}\text{Al}$ | 8.3, 13.1 | 21.6 | 5.1 |
| $^{40}_{18}\text{Ar}^*$ | 12.5, 9.9 | 17.8, 21.8 | 6.3 |
| $^{40}_{20}\text{Ca}$ | 8.3, 15.6 | 19.4, 19.8 | 7.4 |
| $^{50}_{23}\text{V}$ | 8.1, 11.1 | 17.0 | 8.1 |
| $^{56}_{26}\text{Fe}$ | 10.2, 11.2 | 19.0 | 8.9 |
| $^{197}_{79}\text{Au}$ | 5.8, 8.1 | 19.5 | 18.5 |
| $^{208}_{82}\text{Pb}^*$ | 8.0, 7.4 | 14.7, 16.9 | 18.9 |

proton in the nucleus respectively. For the $M_{A-1}$ nucleus to gain a nucleon, it must release an energy $S$ to remain in the ground state. Similarly, the excited $M_{A-1}^*$ must release energy $E_x$ to transit into the ground state.

Since $|k| \ll M_{A-1}$ (think 200 MeV vs 5 GeV), we can expand the bracket term on the left hand side of Eq.(4.1) using the relations in Eq.(2.32) to read

$$\nu + M - S - E_x - \frac{k^2}{2M_{A-1}^2} = \sqrt{(k + q)^2 + M_{n,p}^2} + V.$$  \hspace{1cm} (2.33)

Both $S$ and $E_x$ have been measured or derived for various nuclei [3]. Table 2.2 shows the values. $p^2$ depends on the initial state model.

Finally, the $V$ term consists of 2 types of potentials. First, the proton charge distribution in the nucleus subjects any charged particle to an electric potential called the Coulomb potential. Due to electrostatic repulsion, an outgoing proton gains kinetic energy from the Coulomb potential as it leaves the nucleus (Table 2.2). A neutron, on the other hand, is unaffected.
The complex many-body interactions between the nucleons inside the nucleus are difficult to calculate. It is easier to describe the nucleon-nucleus scattering data with a phenomenological model instead, without going in-depth about its cause. The optical potential of the nucleus is such a model, and it can take on the form

\[ V = U + iW. \]

The term, optical potential, was coined in analogy to the refraction and absorption of light by a medium of complex refractive index\(^91\). As such, the real part of the potential \(U\) serves to change the energy of the nucleon, while the imaginary part \(W\) reduces incoming nucleon flux through nuclear capture or other inelastic processes that take the nucleons out of the incident nucleon flux. The optical potential is largely a phenomenological one—it could take on various functional forms to match the observation in nucleon-nucleus scattering data without explicitly modeling every underlying process (that is the realm of QCD!). The optical potential is present for the outgoing nucleon in a lepton-nucleus scattering as well, since the nucleon must travel a short distance inside the nucleus before exiting, effective subject it to a scattering process by the remnant nucleus with \(A - 1\) nucleons. Therefore, the optical potential needs to be accounted for in the energy conservation equations Eq.(4.1) and Eq.(2.33).

More recently, Cooper et al. [92, 93] have fitted both \(U\) and \(W\) in a form that depends on the incident nucleon energy. The author has also undergone an endeavor to fit the optical potential from a large ensemble of electron scattering data, published in Ref. [3]. Figure 2.6 shows example of fits on carbon, argon, iron and lead and Fig. 2.7 shows the trend of \(U_{\text{opt}}\) as a function of the square of the nucleon’s
initial momentum \((k + q)^2\).

While \(k\) for each event is unknown, we can derive an expression for the average Fermi motion. Taking the direction of the momentum transfer \(q\) to be the z-axis, \(k\) can be decomposed into the transverse and z components as \((k_T, k_z)\). Because the RFG model assumes the nucleons’ motions to be isotropic, on average \(k_z\) is zero. Let the magnitude of \(q\) be \(q_3\):

\[
\langle (k + q)^2 \rangle = \langle |k|^2 \rangle + |q|^2 + \langle k \cdot q \rangle
\]

\[
= \langle k^2 \rangle + Q^2 - q_0 + \langle k_z \rangle q_3^0
\]

\(\langle k^2 \rangle\) can be evaluated with Eq.(2.29) to be \(3K_F^2/5\). For carbon, it is 0.029 GeV/c².

2.2.3 Final State Interactions and Multi-nucleon Correlations

![Illustrations of final state interactions that can affect particles before they leave the nucleus](image)

Figure 2.8: Illustrations of final state interactions that can affect particles before they leave the nucleus[94].
Figure 2.6: Example fits for the optical potential in electron scattering data. Top left to right: $^{12}\text{C}$, $^{40}\text{Ar}$. Bottom left to right: $^{56}\text{Fe}$, $^{208}\text{Pb}$. The dashed red curves are the calculated cross sections without optical potential $U_{\text{opt}}$ and Coulomb potential $V_{\text{eff}}$. The shift in the peak position is due to the optical potential. Inputting the correct optical potential shifts the calculated peak to coincide with data.
Figure 2.7: Comparison of the extracted $U_{\text{opt}}(|k + q|^2)$ against the Cooper, et al fits. The size of the optical potential is largest at low nucleon initial momentum and gradually tends towards zero.
Final state particles may interact inside the nucleus before they leave. Such interactions are called the final state interaction (FSI). Examples of FSI modeled by MC generators such as GENIE are

<table>
<thead>
<tr>
<th>FSI Type</th>
<th>Effect:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic</td>
<td>particle elastically scatters on the whole nucleus</td>
</tr>
<tr>
<td>Inelastic</td>
<td>particle scatters on individual nucleon causing nucleon knockout</td>
</tr>
<tr>
<td>Charge exchange</td>
<td>the FS particle may gain or lose charge to the nucleus,</td>
</tr>
<tr>
<td></td>
<td>changing its species</td>
</tr>
<tr>
<td>Pion production</td>
<td>a pion is produced in the nucleus</td>
</tr>
<tr>
<td>Pion absorption</td>
<td>an FS pion is absorbed by the nucleus,</td>
</tr>
<tr>
<td></td>
<td>often causing nucleon knockout</td>
</tr>
</tbody>
</table>

Table 2.3: Types of FSI modelled by GENIE.

The presence of FSI could modify the topology of a neutrino-nucleus scattering event. The nucleon from an initial CCQE event could excite the nucleus to produce pion in the final state. This event will be subsequently misidentified because a pion and a nucleon in the final states are the signatures of RES. Pion absorption FSI reverses the process–with only nucleons in the final states, we could misidentify the event as CCQE.

Lastly, correlations among the nucleons in the nucleus will also drastically alter the energy reconstruction. While each nucleon spends most of its time evolving independently, its wave function could strongly overlap with another nucleon in the spatial dimensions for short periods, resulting in the short-range correlation. In this scenario, an interaction could knock out a nucleon with the correlated pair going in the opposite direction\cite{95}. In another scenario, two nucleons could be exchanging
a pion when a scatter occurs. Such events are called the meson-exchange current (MEC). Both particles get momentum transfer as well. When they leave the nucleus, two vacancies (called holes) are left. Events resulting in two-nucleon knockouts are called the two-particle-two-hole (2p2h) events. In the case of 2p2h, Eq.(1.41) fails because the initial state hypothesis—that the neutrino scatters on a single bound nucleon—breaks down.

Both the SRC and the MEC are short-range correlations that are modeled as initial state interactions. Outgoing particles from these models must still travel through the nucleus and experience FSI. The FSI is a form of long-range correlation (LRC) because its effect is over the entire width of a nucleus. Another form of the LRC is the RPA effects. The nucleons are held together by the strong nuclear force, and it could transmit long-range correlation (LRC) between the nucleon and the other nucleons in the nucleus, which has a net effect of reducing the scattering probability. The suppression in cross section is calculated using the random phase approximation (RPA)[96] and its effect on a RFG model is shown in Fig 2.9.

In summary, energy reconstruction based on Eq.(1.41) faces significant bias due to the FSI and correlations of the initial state particles. Therefore, to capture the connection between the particles’ content and the initial interaction type in neutrino scattering experiments, we need to rely on sophisticated Monte Carlo simulations of the neutrino-nucleus interactions and the nuclear effects. However, as no model could fully describe nature, our reliance on the MC models will invariably introduce neutrino measurements to modeling uncertainties. The wider neutrino community has adopted a signal definition based on the actual final state particle contents in the detector to reduce the model-dependence in measurements. The QELike signal definition captures any neutrino events with a lepton, any number of nucleons,
and no other hadrons in the detector. This definition takes away a measurement’s dependence on the initial and final state models to only rely on observable quantities in the detector. A more detailed analysis of the QELike samples allows model-builders access to information to create new models that better match nature.

2.3 The GENIE Neutrino-nucleus Interaction Model

While multiple models—such as nuWro[85] and neut[86]—exist, GENIE[54] is by far the most popular model used in neutrino experiments in the United States. GENIE3.0.6 is the most current public version available[98]. MINERνA, however, uses an older version GENIE2.12.10. All descriptions of GENIE in the body of this work shall base on GENIE2.12.10, unless otherwise stated. The nuclear physics model in
GENIE is based on the impulse approximation (IA). It is an RFG model with an extended high-momentum tail for $k > K_F$. The high-momentum tail should account for the effect of short-range correlations between the nucleons\cite{99}, but in GENIE, it just extends the RFG model beyond the Fermi momentum cut-offs for single nucleons. For scattering on a single nucleon bound in the nucleus, GENIE sums up the single nucleon cross section for all the nucleons and then apply a suppression factor to account for Pauli-blocking.

GENIE does not innately support the LRC, but MINER$\nu$A has been able to account for the phenomenon through a reweighing method, based on a calculation using the random phase approximation (RPA)\cite{97}. Figure 2.9 illustrates the $(q_0, q_3)$-dependent suppression on the CCQE cross section on carbon.

For single nucleon knockout, a binding energy term $\Delta^\text{nucleus}_{\text{GENIE}}$ is subtracted from the final state nucleon before exit. If its kinetic energy becomes negative, the nucleon momentum is set to 0 and does not appear in the detector. The values of $\Delta^\text{nucleus}_{\text{GENIE}}$ come from fits to binding energy based on an on-shell formalism by Moniz\cite{100–102}:

$$q_0 + M + \frac{k^2}{2M} + \Delta^\text{nucleus}_{\text{GENIE}} = M + \frac{(k + q)^2}{2M}$$  \hspace{1cm} (2.34)

There are a few things wrong with GENIE’s implementation. First, the binding energy should apply at the interaction vertex. Second, the values extracted come from a different functional form than GENIE’s off-shell energy balance equation. Third, GENIE does not model the Coulomb potential and the optical potential at the interaction vertex. There is an option to add or subtract the Coulomb potential from the final state particles after FSI. Finally, GENIE does not account for the excitation energy. Reference [3] contains detailed information on how to fix GENIE.
GENIE models the CCQE process with the Llewellyn Smith model described in Sec 2.1. The vector form factors are modeled by default using the BBBA05[62] fits. The axial form factor uses an axial mass with $M_A = 0.99 \text{ GeV}/c^2$. Both RES and DIS events, introduced in Sec 1.3.3, typically produce pions and other mesons in the final states. However, the nucleus could absorb the mesons to produce only nucleons in the final states through final state interactions. Such events end up in the QELike event definition. GENIE uses a modified Rein-Sehgal[103] model to simulate baryon resonance production for 16 unique resonances with invariant mass $W \leq 1.7 \text{ GeV}[104]$. GENIE uses a quark-parton model parameterized by the Bodek-Yang[105] structure functions to simulate DIS. The DIS model also includes 2 non-resonant pion production modes that the Rein-Sehgal model originally described. A reduction to the non-resonant pion production strength by 48% is implemented for MINER$\nu$A’s GENIE[104] to agree with external measurements of the non-resonant pion production in deuterium bubble chamber experiments from the Argonne National Laboratory and the Brookhaven National Laboratory.

Until 2.12.10, GENIE does not have an innate model for the 2p2h process. GENIE now incorporates two 2p2h models, including the IFIC Valencia group’s Valencia model. The Valencia model describes 2p2h processes that produce only nucleons in the final states. The cross section for this 2p2h model is dependent on $(q_0, q_3)$ as shown in Fig 2.10.

Work previously published by the MINER$\nu$A collaboration[107] has shown that even with the 2p2h model, there is a deficiency in the MC at the low $(q_0, q_3)$ region. Reference [107] analyzed a sub-sample of MINER$\nu$A’s neutrino-carbon scattering data, focusing on the double differential cross section as a function of the $q_3$ and $E_{\text{avail}}$, the available energy in the detector. The $E_{\text{avail}}$ variable accounts for the
energy of all the charged final states, which excludes neutrons. After applying both the RPA and Valencia 2p2h model, there is significant deficiencies in $E_{\text{avail}}$ across $q_3$, as shown in Fig 2.11. Subsequently, an empirical enhancement to the Valencia 2p2h model has been applied to account for the deficiency. This 2p2h enhancement has become a standard part of MINERνA’s modified version of GENIE that describes MINERνA’s data to a good degree.

Finally, GENIE has a few final state interaction models (INTRANUKE) to choose from. They can be classified into two types, the hN models that calculate the full intra-nuclear cascade for the rescattering of the outgoing hadrons on the nucleons in the nucleus. As it is a cascade model, there is knock-on effect from the secondary particles produced as well. The hN models are computationally expensive, an alternative is to use known hadron-nucleus cross sections and tune the final state particles to data. The hA models are designed in this spirit and GENIE defaults to using the INTRANUKE-hA models to simulate final state interactions.

Collectively, the RPA reweight, non-resonant pion reduction, inclusion of the Nieves 2p2h and MINERνA’s data-driven tune on the Nieves 2p2h model are called
Figure 2.11: Double differential cross section $\frac{d^2\sigma}{dq_3dE_{\text{avail}}}$ for $0.0 < q_3/\text{GeV} < 0.2$, $0.2 < q_3/\text{GeV} < 0.3$, and $0.3 < q_3/\text{GeV} < 0.4$. The short-dashed line is GENIE 2.8.4 with the non-resonant pion reduction, long-dashed line adds RPA correction, and the solid line contains additional 2p2h contribution. The deficiencies in the $q_3$ regions can be accounted for by increasing the 2p2h contribution.

MINvGENIE-v1, and it forms the foundation for all of MINERνA’s recent results.
Chapter 3

The MINER\(\nu\)A Experiment

The existence of nuclear effects presents an obstacle to reconstructing neutrino energy and bias the energy-dependent oscillation results. The MINER\(\nu\)A experiment aims to improve the shortfall in our understanding of the nuclear effects by measuring neutrino cross sections on various nuclear targets. Such measurements provide important constraints on the deficiencies in nuclear models and crucial evidence to support the inclusion of nuclear processes previously missing from generators. For example, the 2p2h process did not incorporate into \textsc{genie} until recently. MINER\(\nu\)A’s first measurements of the CCQE cross sections on the neutrino\cite{108}, and antineutrino\cite{109} in carbon hinted at a missing 2p2h contributions. Figure 3.1 shows the distributions of energy around the interaction vertex (to be described in more detail later). For neutrino scattering at \(Q_{\text{QE}}^2 < 0.2 \text{ GeV}^2\), there is a deficiency in the total energy near the vertex in the MC. The data prefers the addition of a proton with less than 225 MeV of kinetic energy 25% of the time. In antineutrino, data and MC are consistent with each other. Measurements of the SRC in electron scattering experiments show that \(np\) pairs are 10 times as likely than \(nn\) or \(pp\) pairs, result-
Figure 3.1: The energy around an interaction vertex for neutrino[108] (top) and antineutrino[109] (bottom). The left figures are for \( Q_{QE}^2 < 0.2 \text{ GeV}^2 \) and the right for \( Q_{QE}^2 > 0.2 \text{ GeV}^2 \). The excess in the MC in the first vertex energy bin in neutrino means data has more energy. The addition of a proton below 225 MeV for 25% of time would bring better data MC agreement.

...ing in the addition of a proton in neutrino and a neutron in antineutrino. These observations are consistent with missing 2p2h contributions.

The measurements illustrate the power of MINERνA to constrain nuclear models by leveraging the correlations between the leptonic and hadronic system. A
3.1 Description of the MINERνA Detector

3.1.1 NuMI Tunnel

Figure 3.2 shows the NuMI tunnel system. At the most upstream, pions produced in the NuMI beamline shown in Fig 1.3 decays along the decay pipe into muons and neutrinos. The absorber in the absorber hall stops any undecayed hadrons. Muons may pass through the absorber but later stopped by the thick layer of dolomite rock formation between the absorber and the detector. Neutrinos travel onward and arrive at the MINOS[110] hall, which is home to both the MINERνA detector and the MINOS near detector. The NOνA near detector is also close by.

3.1.2 Detector Components

Figure 3.3 shows the schematics of the MINERνA detector looked upon from the front (left) and the side (right). Neutrinos upstream of the detector come in from the left, pass through MINERνA and exit MINOS to the right. A veto wall sits...
Figure 3.2: Schematics of the NuMI tunnel. The pions are produced at the target, which subsequently decay into muons and neutrinos in the decay pipe before the absorber. The neutrinos then travel onward towards the MINOS[110] hall, where MINERνA is placed. Ref. [111].

Figure 3.3: Front view (left) and elevation view (right) schematics of the MINERνA detector. From Ref. [112]
upstream of the main detector. It consists of steel plates and scintillators to stop low-energy hadrons and tag any muons arriving from the detector’s upstream. The neutrinos could interact with the rocks between the absorber and the detector to produce muons and hadrons. The muons produced in this way are called the “rock muons,” and the veto wall allows MINER$_{\nu}$A to identify them. A liquid helium target sits behind the veto wall and before the MINER$_{\nu}$A detector.

The MINER$_{\nu}$A detector comprises 120 hexagonal modules (see the frontal view of MINER$_{\nu}$A) stacked horizontally along the beam direction. Each module, radially, consists of an inner detector (ID), a side electromagnetic calorimetry (ECAL), and an outer detector (OD) configured as hadronic calorimetry (HCAL). The ECAL and HCAL cause particles to shower so that their energies could be measured. The ID, where the majority of MINER$_{\nu}$A analyses happen, consists of a nuclear target region, the active tracker region, and the ECAL and HCAL stacked along the beam’s direction.

The nuclear target region comprises planes made of carbon, iron, and lead. These nuclear target planes sit in between modules of scintillators that can detect charged particles. A Kevlar bag containing water (oxygen and hydrogen) is also in the nuclear target region. Measurements done on the nuclear targets could compare the interaction cross sections of neutrinos on elements with different nucleons. The nuclear target planes and the water target are dead materials that do not measure any energy deposition. Instead, we rely on the scintillators in between to measure the final state particles from an interaction.

Behind the nuclear target region is the fully active scintillator region. The scintillator used in MINER$_{\nu}$A is polystyrene doped with 2,5-diphenyloxazole (PPO) and 1,4- bis(5-phenyloxazol-2-yl) benzene (POPOP). It emits blue light that subse-
3.1 Description of the MINERνA Detector

Table 3.1: The materials made up each tracker plane, their relative mass fractions in the tracker and the mass fraction of the elements in each material.

<table>
<thead>
<tr>
<th>Material</th>
<th>Scint.</th>
<th>Coating</th>
<th>WLS</th>
<th>Lexan</th>
<th>Grey Epoxy</th>
<th>Clear Epoxy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction</td>
<td>0.812</td>
<td>0.078</td>
<td>0.017</td>
<td>0.060</td>
<td>0.022</td>
<td>0.011</td>
</tr>
<tr>
<td>H</td>
<td>0.085</td>
<td>0.072</td>
<td>0.077</td>
<td>0.056</td>
<td>0.087</td>
<td>0.056</td>
</tr>
<tr>
<td>C</td>
<td>0.915</td>
<td>0.778</td>
<td>0.923</td>
<td>0.756</td>
<td>0.552</td>
<td>0.756</td>
</tr>
<tr>
<td>O</td>
<td>0</td>
<td>0.060</td>
<td>0</td>
<td>0.189</td>
<td>0.206</td>
<td>0.189</td>
</tr>
<tr>
<td>Al</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.031</td>
<td>0</td>
</tr>
<tr>
<td>Si</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.033</td>
<td>0</td>
</tr>
<tr>
<td>Cl</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.091</td>
<td>0</td>
</tr>
<tr>
<td>Ti</td>
<td>0</td>
<td>0.090</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Subsequently gets read out by a wavelength-shifting (WLS) fiber to green light. A white reflective coating made with 15% TiO2 by weight in polystyrene is co-extruded with each strip. The WLS fiber connects to photomultiplier tubes (PMTs) that transform the photons into electron showers digitized by an analog-to-digital converter (ADC) circuit. The digital signal is stored and processed into forms usable by analyzers. This process is very similar to how the eye-neuron-brain link works. Each scintillator module consists of 2 scintillator planes. Each plane is made of 127 scintillator strips glued together by a clear epoxy. A Lexan sheet covers each side of the plane and is glued to the plane by a grey epoxy. The epoxies comprise mostly of hydrogen, carbon, and oxygen. Both the epoxies also contain nitrogen and chlorine, but the grey epoxy also contain trace amount of metal such as aluminum and silicon. By taking into account the full set of materials (scintillator strips, coatings, Lexan covers and epoxies) used to build a plane we could derive the relative elemental mass fraction in the tracker region. Table 3.4 shows the relative mass fraction of each material comprising a tracker plane, and the mass fraction of the elements that made up of each material. Using these numbers we could evaluate the total mass fraction of each element in the tracker and the results are summarized in Table 3.2.
3.1 Description of the MINERνA Detector

<table>
<thead>
<tr>
<th>Element</th>
<th>Rel. Atomic Mass</th>
<th>Mass Fraction</th>
<th>Number Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>1.00794</td>
<td>0.0818</td>
<td>0.518</td>
</tr>
<tr>
<td>C</td>
<td>12.0107</td>
<td>0.8851</td>
<td>0.4705</td>
</tr>
<tr>
<td>O</td>
<td>15.9994</td>
<td>0.025</td>
<td>0.010</td>
</tr>
<tr>
<td>Al</td>
<td>26.982</td>
<td>0.0007</td>
<td>0.0002</td>
</tr>
<tr>
<td>Si</td>
<td>28.0855</td>
<td>0.0007</td>
<td>0.0002</td>
</tr>
<tr>
<td>Cl</td>
<td>35.453</td>
<td>0.002</td>
<td>0.0004</td>
</tr>
<tr>
<td>Ti</td>
<td>47.867</td>
<td>0.0047</td>
<td>0.00063</td>
</tr>
</tbody>
</table>

Table 3.2: The mass fraction by elements in the tracker region. The areal density of a scintillator plane is 1.9872/cm². The mass fraction was obtained after chemically analyzing the elemental content in the scintillators and the epoxies.

Figure 3.4: Each scintillator strip in MINERνA has a triangular base. Stacking the strips together, as shown on the right, maximizes the likelihood a charged particle may travel through more than 1 strip in a plane. Each strip emits blue light which is converted to green light by a WLS fiber. The scintillator strip in the image on the left is coated and those on the right are not. From Ref [112].

Each strip has a triangular base as shown in Fig 3.4. The strips are stacked together, the top of each scintillator strip points upstream or downstream alternatively along a scintillator plane. When a charged particle travels through a scintillator plane, it will deposit energy in at least two strips. Given the charge deposition and each scintillator’s coordinate in a plane, we could calculate a charge-weighted position for better spatial resolution.
The scintillator strips in each plane orient in one of the three $X$, $U$, and $V$ views as illustrated in Fig 3.5. Scintillator trips in the $X$-view points vertically upward. $U$ and $V$ views point either $\pm 60^\circ$ from the $X$-view. The first plane in each scintillator module is always an $X$ plane. The second plane alternates between $U$ and $V$ as we go down each module along the beam direction.

MINER$\nu$A is capable of 3D position reconstruction. The coordinate system in the MINER$\nu$A detector is as follows: the $z$ axis points from the upstream to the downstream of the detector, horizontal to the ground, the $y$ axis points vertically upwards, and the $x$ axis points to the right of the detector when we stand behind the detector and face the direction of the beam. Each plane in MINER$\nu$A has a known $z$ position (since we placed it there). The transverse position in an $X$-view plane corresponds to the $x$ position in the detector. When a charged particle travels more than one plane, it must leave a trace in an $X$ plane and either $U$ or $V$ plane. Let the transverse position in the $U$ and $V$ planes be $u$ and $v$ respectively, and we
can calculate the $y$ position as

\begin{align*}
    y_{xu} &= \frac{x - 2u}{\sqrt{3}}, \\
    y_{xv} &= -\frac{x - 2v}{\sqrt{3}}.
\end{align*}

Having three views instead of 2 (i.e., $X$ and $Y$ views) is advantageous when multiple particles travel through the detector. Suppose two particles travel on the same $yz$ plane, a detector with only $X$ and $Y$ views sees only one track. With three orientations, an additional view breaks the degeneracy.

The fully active scintillator (tracker) region spans 62 modules and has a length of 2.78 meters.

Behind the active scintillator region is an ECAL region consisting of 10 modules (Fig 3.3) very similar to the active tracking region. Each plane in the ECAL region is covered by a 2mm thick lead sheet allowing photons and electrons to shower. The HCAL is the last detector component. Instead of 2 planes of scintillators in each module, a module in the HCAL consists of only one scintillator plane and a 2.54 cm thick steel plane. Each scintillator plane in the HCAL alternates in the $XVXU$ pattern as well.

### 3.1.3 MINOS Near Detector

Finally, 2 meters behind the MINER$\nu$A detector is the MINOS[110] near detector. A muon originating in MINER$\nu$A could exit the detector and enter the MINOS ND. The MINOS ND acts as a muon spectrometer by bending the muon with its strong magnetic field. The MINOS ND measures a muon’s momentum by range if it is totally contained in the detector. When a muon is partially contained, MINOS uses
the curvature of the muon track to determine its momentum. We sum the muon’s
energy deposits in MINERνA and MINOS to obtain its total energy. The muon’s
bending direction also discloses its charge, and hence whether it is produced by a
neutrino or an antineutrino.

3.2 Detector Simulation

3.2.1 Simulation Toolkit

The MINERνA experiment uses GENIE to simulate the final state particles from a
neutrino-nucleus interaction. The final state particles must then propagate through
a detector model that simulates the detector response.

MINERνA simulates its detector using the GEANT4 toolkit[113]. A realistic
detector geometry, including the scintillator strips, the WLS fibers, the epoxy glues,
the outer coating, and metal components, is generated. GEANT4 has many physics
interaction models to simulate the passage of particles through different materials.
MINERνA uses the Bertini Cascade[114] model to simulate particle interactions in
the detector. GEANT4 propagates a particle through the detector in a series of
steps. In each step, GEANT4 advances the particle position in the detector accord-
ing to its current momentum and decides if an interaction occurs based on different
physical processes’ probabilities. For example, a particle could elastically scatter
on a nucleus, resulting in momentum change. The particle could also inelastically
scatter and break the nucleus apart to produce secondary particles. When an inter-
action produces secondary particles, the new particles will propagate through the
detector in the same fashion as the primary particle. As the position of interaction
is known, we can trace the energy deposited to the simulated scintillator strips.
MINERνA then simulate the scintillator strips’ optical responses and the electronic
responses from MINERνA’s PMT and data acquisition system. These simulated
electronic signals then process through the same software framework MINERνA
uses to process real signals to analyze the simulated events and real physical events
in an identical procedure.

3.2.2 Simulation Chain

MINERνA could simulate a neutrino interaction through the entire event chain –
from neutrino generation at the source to interactions in the detector and to propa-
gating the final state particles through the GEANT4 simulation. First, the GEANT4
simulation can tell GENIE what fraction of nuclei are in the detector. GENIE then
simulates neutrino interactions on each nucleus according to their abundance. The
final state particles from the simulated event can be put into a part of the detector
according to the probability of finding that particular nucleus. The final state
particles then propagate inside the detector to be reconstructed.

The MINERνA simulation can also simulate the passage of single-particle through
the detector through a “particle cannon” mode, enabling analyzers to understand
simulated particle response and to refine techniques to reconstruct a particle from
the clusters in the detector.
3.3 Targets

3.3.1 Number of Targets in Detector and Simulation

There is a small difference between the simulated and the real detector. For example, the scintillator strips are treated to have equal number of hydrogen and carbon according to Ref. [115]. The chemical compositions of the epoxies are slightly different that affect the amount of the heavy elements in the detector. These differences have a small effect on the particle responses and is on the order of 0.1%.

To evaluate the total number of targets in the tracker region, we must know the exact chemical compositions of the tracker’s components. The tracker is made of coated scintillator strips, green WLS fibers, Lexan sheets covering the front and back of a scintillator plane, and epoxies that hold various components together. Chemical analysis done on the scintillator strips found there are 10% more hydrogen atoms than carbon atoms. The MC detector simulation, which was developed before the chemical analysis, simulates the scintillator strip with equal parts of hydrogen and carbon, resulting in an 8% reduction in the number of hydrogen targets.

From Table 3.3 and Table 3.4, we can derive the mass fraction of hydrogen and carbon in the tracker region and calculate the total number of hydrogen targets in the tracker by

$$N_a = \frac{f_a M_{\text{det}} N_A}{M_a},$$

(3.2)

where $N_a$ is the number of atoms from element $a$, $f_a$ is the element’s mass fraction, $M_{\text{det}}$ is the detector’s mass in the tracker, $M_a$ is the atomic mass and $N_A$ is the Avagadro’s constant. Table 3.5 tabulates the elemental mass fractions and total targets for hydrogen and carbon in the tracker.
### 3.3 Targets

#### Table 3.3: The elemental mass fraction in each of the materials that makes up a tracker plane. There is a difference between the measured chemical composition and the simulated composition.

<table>
<thead>
<tr>
<th>Material</th>
<th>Detector</th>
<th>Simulated</th>
<th>H</th>
<th>C</th>
<th>O</th>
<th>Al</th>
<th>Si</th>
<th>Cl</th>
<th>Ti</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scintillator</td>
<td>0.085</td>
<td>0.915</td>
<td>0.0774</td>
<td>0.923</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WLS</td>
<td>0.077</td>
<td>0.923</td>
<td>0.0774</td>
<td>0.923</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coating</td>
<td>0.0723</td>
<td>0.778</td>
<td>0.0899</td>
<td>0.0601</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Epoxy (grey)</td>
<td>0.0873</td>
<td>0.552</td>
<td>0.206</td>
<td>0.0314</td>
<td>0.0326</td>
<td>0.0905</td>
<td>0.0166</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lexan</td>
<td>0.0555</td>
<td>0.7558</td>
<td>0.1887</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Epoxy (clear)</td>
<td>0.0555</td>
<td>0.7558</td>
<td>0.1887</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Table 3.4: The mass fraction of the materials making up the a tracker plane, ECAL plane and a HCAL module. The mass fraction are the same between the detector and the simulation.

<table>
<thead>
<tr>
<th>Property</th>
<th>Tracker Plane</th>
<th>ECAL Plane</th>
<th>HCAL Module</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness/cm</td>
<td>1.85</td>
<td>2.048</td>
<td>4.39</td>
</tr>
<tr>
<td>Areal mass/(g/cm²)</td>
<td>1.987</td>
<td>4.222</td>
<td>21.89</td>
</tr>
<tr>
<td>Average Density/(g/cm³)</td>
<td>1.074</td>
<td>2.062</td>
<td>4.985</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Material Composition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scintillator</td>
</tr>
<tr>
<td>WLS</td>
</tr>
<tr>
<td>Lexan</td>
</tr>
<tr>
<td>Coating</td>
</tr>
<tr>
<td>Epoxy(grey)</td>
</tr>
<tr>
<td>Epoxy(clear)</td>
</tr>
<tr>
<td>Steel(Fe+Mn)</td>
</tr>
<tr>
<td>Lead</td>
</tr>
</tbody>
</table>
Table 3.5: Elemental mass fraction and the number of hydrogen/carbon targets in the tracker region in both the detector and simulation.

### 3.3.2 Detector Mass Uncertainty

The total detector mass uncertainty at MINERνA has been determined to be 1.4%. This uncertainty is applicable to most of the analyses done at MINERνA, where we care about the cross section on each nucleon. The antineutrino hydrogen CCQE analysis requires cross section to be measured on hydrogen atom only and we need to evaluate an additional uncertainty on top of the total detector mass uncertainty.

In principle, each scintillator strip consists of equal number fraction of hydrogen and carbon[115]. In reality, the fraction of hydrogen and carbon may vary depending on the processes of making the polystyrene. A chemical analysis performed on a sample of MINERνA’s scintillator strips with the the coating shows it consists of larger number fraction of hydrogen than carbon. Table 3.6 shows the mass fraction of the elements comprising the strips. Table 3.7 compares the chemical analysis result to the current values assumed for the two types of epoxies.

Using the assumed mass fraction of the bare scintillator strip and the coating from Table 3.4, the hydrogen mass fraction of the coated scintillator strip is found to be 0.0839, 0.6% smaller than the averaged hydrogen mass fraction in Table 3.6. If we treat the two values as consistent, and only adjust the chemical compositions for the epoxy, the final discrepancies in the hydrogen mass fraction is only 0.2%,
3.3 Targets

<table>
<thead>
<tr>
<th>El.</th>
<th>Strip 1</th>
<th>Strip 2</th>
<th>Averaged</th>
<th>Normalized</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>0.0845</td>
<td>0.0843</td>
<td>0.0844</td>
<td>0.0829</td>
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<tr>
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<td>0.0061</td>
<td>0.0058</td>
<td>0.0057</td>
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<tr>
<td>Ti</td>
<td>0.0083</td>
<td>0.0091</td>
<td>0.0087</td>
<td>0.0085</td>
</tr>
</tbody>
</table>

Table 3.6: The result of chemical analysis on two strips of MINERνA's scintillator strip with the coating, titled Strip 1 and Strip 2. The average mass fraction is calculated in column 4. Due to uncertainties in the material assay, the total mass fraction for all elements exceeded 1. We normalize the total mass fraction to 1 and show it in the last column.

<table>
<thead>
<tr>
<th>Epoxy El</th>
<th>Grey CA norm</th>
<th>Grey Current</th>
<th>Clear CA norm</th>
<th>Clear Current</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
<tr>
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<td>0.1715</td>
<td>0.1887</td>
</tr>
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<td>0.0314</td>
<td>0.0002</td>
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</tr>
<tr>
<td>Si</td>
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<td>0.0326</td>
<td>0.0005</td>
<td></td>
</tr>
<tr>
<td>Cl</td>
<td>0.0005</td>
<td>0.0905</td>
<td>0.0048</td>
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<tr>
<td>N</td>
<td>0.0166</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.7: The chemical analysis result on the grey and clear epoxies. CA stands for chemical analysis and their results have been normalized to 1. The current values are used to evaluate their chemical contributions to the tracker.

well within in the 1.4% detector mass uncertainty.

The difference of the assumed coated strip composition and the normalized chemical analysis result is significant enough to warrant additional treatment. We assess the hydrogen fraction from the normalized result by adjusting both the bare scintillator composition and the mass fraction of TiO$_2$ in the coating. Since the coating is made up of bare scintillator and TiO$_2$, this adjustment is equivalent to solving a coupled equation. The result is as follows—hydrogen mass fraction in the bare scintillator strip is 0.0841 and the coating comprises 16.3% TiO$_2$ (used to be 15%) by mass. The tracker’s hydrogen mass fraction becomes 0.0811, a 0.9% reduction.
3.4 The NuMI Beam Line and the Neutrino Flux

3.4.1 Flux Simulation

Neutrinos from the NuMI beamline (Fig 1.3) enter MINERvA at a downward angle of 0.05887 radian. The muon neutrinos (antineutrinos) are the decay products of $\pi^+$ or $K^+$ from the secondary particles produced in the scattering of a 120 GeV proton beam on a graphite target approximately 1 m long[35]. The charged secondary particles can exit the target at a variety of an-
gles. Two sets of electromagnetic “horns”, one directly encircling part of the target and one 10 meters downstream of the target, focus positively (negatively) charged particles in the forward direction when passing forward (reverse) horn current. Figure 3.6 illustrates the trajectories of charged particles when the horns are subject to a forward current. The forward current goes from the left to the right at the inner horn surface and returns to the left on the outer surface. A consequence of this configuration is that magnetic flux exists only inside the horn. Outside the horn, the magnetic fields from opposing current on the horn’s inner and outer surfaces cancel. For the forward current, the magnetic field lines $\mathbf{B}$ are concentric circles pointing counterclockwise when viewed from the front. Very forward going pions and kaons are unfocused as they pass through the horns without experiencing any magnetic fields at all, as shown by the red kaon trajectory in Fig 3.6. Mesons produced at shallow angles will proceed to horn 2 and be focused towards the beam direction. Those mesons that are produced at deep angles may be first deflected by horn 1. The meson could be focused directly along the beam direction, at which point it will pass through horn 2 without deflection (dotted blue line in the middle of Fig 3.6). The meson may also end up overfocused to reach horn 2. Horn 2 will subsequently correct its trajectory towards the beam direction. The magnetic field bends particle trajectory towards or away from the beam axis according to

$$\hat{n} = \hat{i} \times \hat{B},$$

where $\hat{n}$ is the direction of bending, $\hat{i}$ is the direction of charge flow, and $\hat{B}$ is the direction of the magnetic field. Negatively charged particles will divert away from the beam direction. The situation reverses when the horn current goes in the
The focused pions and kaons subsequently decay in the decay pipe 675 meters in length with a 2-meter diameter. The long decay pipe allows more energetic particles to decay to producing higher energy neutrinos. The flexibility in the NuMI beamline to produce neutrinos of different energies has allowed MINER$\nu$A access to two different neutrino fluxes according to the physics priorities of the day.

The neutrino beam during the MINOS\cite{110} era has relatively low energy (LE), which spread broadly and peaks at 3 GeV to ensure the greatest flexibility in measuring unknown oscillation parameters. During the NO$\nu$A\cite{40} era, the focus was on the precision measurement of known oscillation parameters. NO$\nu$A needs access to a lower energy beam with narrower energy spread to maximize $\Delta m_{32}^2$ measurement while reducing background\cite{40}. NO$\nu$A achieves a narrow neutrino flux peaked at 2 GeV by situating 14 mrad to the left of the beamline axis (off-axis) when NuMI is running in the medium energy (ME) mode. Figure 3.7 also shows the ME beam. The ME beam peaks at 5.5 GeV with a broad energy profile reaching 10s of GeVs.

As we could not directly measure the incoming neutrino energy, we must rely on the beamline geometry and the physics of the particle decays to predict the neutrino flux arriving at MINER$\nu$A. These are managed by the G4NuMI simulation package based on the GEANT4 toolkit\cite{113}. G4NuMI provides a detailed simulation of the beamline configuration (within uncertainty) so that the effect of the beamline components on the propagation of particles can be estimated. Uncertainties on the beamline geometry are evaluated by varying parameteres associated with each part of the beamline, examples among them are the variations of the beam position, beam size and the positions of the horns.

Particle interactions such as protons hitting the graphite target, secondary parti-
3.4 The NuMI Beam Line and the Neutrino Flux

Figure 3.7: Predicted (Left) neutrino and (right) antineutrino fluxes at MINERνA. Each plot shows a low energy (LE) flux during the MINOS era and a medium energy (ME) flux during the NOνA era.[116]

cle production, reinteraction and decays are also simulated with the G4NuMI package. Collectively, the generation of pions and kaons either through primary proton interaction or multiple scatterings within the target are known as hadron production. The baseline hadron production model used by GEANT4 needs to be tuned to external data, since deviation of the models from external data imply possible bias in the predicted neutrino flux. NA49[117], a dedicated hadron production experiment as CERN, provides such data and we use the Package to Predict the FluX (PPFX) [118] package, developed by MINERνA, to predict the neutrino flux and propagates their associated uncertainties.

3.4.2 $\nu - e$ Flux Constraint

The total uncertainties on the flux prediction is on the order of 10% due to the uncertainties in the hadron production cross section and beam focusing. MINERνA has pioneered a technique to constrain the absolute flux normalization by measuring
the neutral current muon neutrino - electron scattering cross section[119, 120]. The reaction occurs between two leptons, both of which are point particles and the theoretical description of the process is precise. The \( (\nu_e^- \rightarrow \nu_e^-) \) cross section is given by

\[
\frac{d\sigma}{dy}(\nu_e^- \rightarrow \nu_e^-) = \frac{G_F^2 s}{\pi} \left[ C_{LL}^2 + C_{LR}^2 (1 - y)^2 \right],
\]

where \( G_F \) is the Fermi constant and \( s \) is the Mandelstam variable defined in Eq.(2.2). \( y \equiv T_e/E_\nu \) is the ratio of the electron kinetic energy \( (T_e) \) to the neutrino energy \( (E_\nu) \). For muon and tau neutrinos, \( C_{LL} = \frac{1}{2} - \sin^2 \theta_W \) and \( C_{LR} = \sin^2 \theta_W \), with \( \theta_W \) the Weinberg angle. For anti-muon or tau neutrinos, \( C_{LL} \) and \( C_{LR} \) are interchanged. For electron neutrinos, both \( C_{LL} \) and \( C_{LR} \) are \( \frac{1}{2} + \sin^2 \theta_W \) because the scatterings between the same flavor leptons contain contributions from both a neutral current and a charged current components.

The electron angle is given by

\[
1 - \cos \theta = \frac{m_e (1 - y)}{E_e}.
\]

At the few-GeV regions relevant to MINER\( \nu \)A, \( E_e \gg m_e \) and the electrons are very forward going. Therefore MINER\( \nu \)A loses the ability to determine neutrino energy since there is not enough angular resolution available. MINER\( \nu \)A is, however, able to measure the total number of \( \nu - e \) scatterings and constrain the normalization of the incoming flux.

MINER\( \nu \)A was able to reduce the flux uncertainty from 9% to 6% in the LE configuration[119], and from 7.5% to 3.8% in the ME configuration[120] in the neutrino mode. An antineutrino analysis is also near completion and is included in these results. The antineutrino result improves the flux uncertainty from 7.8% to 5%[121].
3.5 Particle Reconstruction in MINER$\nu$A

3.5.1 Charged Particle Reconstruction

When a muon neutrino undergoes the charged current interactions in the MINER$\nu$A detector, a muon is produced. The muons are charged particles traveling at relativistic velocity. As the muons pass through matter, they lose energy through collisions with bound electrons and occasionally with the nuclei. The electrons most frequently totally dislodge from the atom resulting in ionization, and the hit nuclei could become excited. For charged particles heavier than the electron, their average energy loss through matter can be summed up by the Bethe Equation according to Ref. [9].

$$\left\langle -\frac{dE}{dx} \right\rangle = K z^2 \frac{Z}{A} \beta^2 \left[ \frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 W_{\text{max}}}{I^2} - \beta^2 - \frac{\delta(\beta \gamma)}{2} \right]. \ (3.5)$$

where $-dE/dx$ is the particle’s energy loss per distance traveled; $K = 4\pi N_A r_e^2 m_e c^2$ with $N_A$ as Avogadro’s constant; $m_e$ is electron mass; $r_e = e^2/4\pi \varepsilon_0 m_e^2$ is classical electron radius; $Z$ and $A$ are the atomic number and atomic mass respectively; $z$ is the charge number of the incident particle; $W$ is the energy transfer to an electron in a single collision, and $W_{\text{max}}$ is the maximum energy transfer possible; $I$ is the mean excitation energy; $\beta = v/c$ is the velocity of the particle expressed as fraction of the speed of light; and $\gamma = 1/\sqrt{1 - \beta^2}$.

Figure 3.8 illustrates the average energy loss of charged particles such as the muons, pions, and protons in materials of various densities. In general, a charged particle’s energy loss rises sharply as its velocity slows. When its velocity is $\beta \gamma > 1.0$, its velocity has little effect on its average energy loss. In carbon, for example,
the average energy loss increases by 1 MeVg$^{-1}$cm$^2$ from 2.8 MeVg$^{-1}$cm$^2$ over the four orders of magnitude in momentum, $\beta\gamma \in (1.0, 10^4)$. Therefore, the energy deposit along a charged particle’s track is constant for most of the particle’s lifetime. MINER$\nu$A can measure the energy deposit because MINER$\nu$A’s scintillators can turn the ionized electrons into scintillation light.

The most basic building block of a MINER$\nu$A event is the signal from a single scintillator strip called a “hit”. When a charged particle passes through a plane, it could deposit energy into multiple strips due to the charge sharing between the triangular-shaped strips next to one another. Hits on adjacent strips could group into a more extended object called a “cluster”. The width of a cluster is the maximum transverse expanses of the hits comprising the cluster within a view. The cluster’s position along the plane is an energy-weighted average of each hit in the cluster. A charged particle often traverses multiple planes, creating a cluster on each plane it passes through. Clusters on multiple planes can group into a “track”. Not all clusters are eligible for track reconstruction, and we categorize each cluster according to its total energy and the distributions of the hits: **Low activity** clusters are those with less than 1 MeV in total energy deposits. They are usually ignored in an analysis. **Trackable** clusters are narrow with energy consistent with MIP signature. Each hit in a cluster has $1 \sim 8$ MeV and the total cluster energy cannot exceed 12 MeV. These clusters are used to reconstruct the muons in the detector. **Heavy Ionizing** are narrow but have high cluster energy. There is no upper bound on the single hit or cluster energy. These clusters are usually formed by decelerating hadrons in the detector and are included, along with the Trackable clusters, to create hadronic tracks. **SuperCluster** are broad clusters with width equal to at least 4 strips. They are usually signatures of showers. **CrossTalk** are
Figure 3.8: Calculated mean energy loss for charged particles in liquid hydrogen, gaseous helium, carbon, aluminum, iron, tin, and lead. Between $\beta\gamma \in (1.0, 10)$ the mean energy loss hits a minimum. Particles traveling in these energy regions are called minimum-ionizing particles (MIP). Figure from Ref. [9], chapter 34.
Figure 3.9: Arachne[122] event display for a reconstructed CCQE event in MINER$\nu$A. The forward going long track is a muon and the sideways shorter track is a proton. From Ref. [123].

clusters that have been recognized as “fake” hits due to the signal from an optical fiber leaking into adjacent fibers in MINER$\nu$A’s optics and electronics.

Only the Trackable clusters are used to create muon tracks, while HeavyIonizing clusters could be used to reconstruct tracks left behind by charged hadron.

Figure 3.9 is an example of MINER$\nu$A’s reconstructed CCQE candidate. Two tracks intersect at the interaction vertex. The longer track that passes through the end of the MINER$\nu$A detector is the muon track, while the shorter track at a steeper angle is a proton track. The darker and colder colors such as blue and purple denote more energy deposits, while warmer colors such as red and orange denote hits with less energy. The proton’s energy loss peaks abruptly towards the end of the track as it comes to rest according to Eq.(3.5) and this peak is called the Bragg peak.

The reconstruction of a muon in the MINER$\nu$A detector is relatively simple. Most muons in MINER$\nu$A’s analyses are required to pass into MINOS, where their
3.5 Particle Reconstruction in MINER$\nu$A

Charges and momenta are measured through the bending of their trajectories by the MINOS magnet. For events occurring in the tracker, the muons need to have at least 1.5 GeV to be reconstructed by MINOS. Reconstruction of the long track is done through a seeding and fitting algorithm. First, clusters are grouped according to their views. For each view and starting from the most downstream cluster, every combination of three clusters on three adjacent same-view planes that forms a straight line becomes a track seed. Track seeds from all views are then compared for overlaps in the $z$-positions and whether their directions are collinear. A Kalman filter determines the track momentum. The track's starting point in the upstream part of the detector is set as the interaction vertex. Events with other charged particles can sometimes make shorter tracks as well. These short tracks begin at the interaction vertex defined by the muon track. MINER$\nu$A uses an “anchored short tracking” algorithm to search for them. The anchored short tracker looks for seeds with 4-cluster combinations $XVXU$, $XUXV$, $VXUX$, and $UXVX$ (refer to Fig. 3.10 for illustration). Multiple seeds with consistent slopes are combined together to form longer (but still short compared to the muon) tracks. The shortest track made this way spans four planes. Both the pions and protons could form short tracks attached to the vertex. Protons over a momentum of 0.8 GeV/$c$ can span four planes, while most pions have enough momentum to span four planes. Therefore, we need to identify the particle causing the track. Two methods exist in MINER$\nu$A.

$dE/dx$ Test

The procedure to make a particle identification (PID) has been described in detail in Ref. [124]. Briefly, both proton and pion hypotheses are tested for each reconstructed track. First, we pre-calculate the $dE/dx$ profiles for proton and pions at different
Particle Reconstruction in MINERνA

Figure 3.10: Illustration of a charged particle passing through MINERνA’s planes.

initial momenta. These profiles are then stored for use later. When we reconstruct a track, its range provides the first estimate of the particle momentum under each hypothesis. For each hypothesis, the algorithm searches through the $dE/dx$ profiles to look for the one with the minimum $\chi^2$. Once the first minimization cycle is over, a second cycle proceeds after removing outliers in the reconstructed track. The outlier removal procedures are described in Ref. [124]. The particle profile with the minimum $\chi^2$ for each hypothesis gives us its momentum. Comparing the minimum $\chi^2$ for both hypotheses tell us what the most likely particle is. Figure 3.11 shows an example fit for a track identified as proton. On average, a proton deposits more energy per unit length than a pion.

Log Likelihood Ratio

The log likelihood ratio (LLR) test is another procedure applied in MINERνA to PID protons and pions. The material discussed in this subsection is obtained from
Figure 3.11: The $dE/dx$ distribution for a proton track compared to a proton and pion profile. From Ref. [124].

Ref. [125]. The likelihood for a particle $\alpha$ can be defined as

$$
\mathcal{L}_\alpha = \prod_x P(E_x|\alpha),
$$

(3.6)

where $P(E_x|\alpha)$ reads as the probability for position $x$ along the track to have energy deposit $E$ assuming it is particle $\alpha$.

The likelihood ratio (LLR) between the proton and the pion hypothesis is $R = \mathcal{L}_p/\mathcal{L}_\pi$, and the log likelihood ratio is

$$
\log R = \sum_x [\log P(E_x|p) - \log P(E_x|\pi)].
$$

(3.7)

We produce a large sample of proton and pion particle canon to statistically evaluate the energy distributions at fixed locations $x$ from the end of a track. Normalizing the energy distribution to the number of events in at each $x$, we obtain the probability $P(E_x|p, \pi)$ for protons and pions respectively. The result of the LLR
Figure 3.12: Illustration of the LLR score obtained for fully and partially tracked protons and pions. Very good separation exists for the fully tracked particles. Ref [125].

calculation[125] is shown in Fig 3.12.

3.5.2 Neutron Reconstruction

Neutrons, unlike the charged particles, do not ionize electrons during flight. Therefore they are invisible unless interactions that produce charged secondary particles happen. Neutrons undergoing secondary interactions often retain enough kinetic energy to bias calorimetric energy measurement. For oscillation experiments, neutrons are a source of uncertainties. Neutrons are produced as primary hadron final states, such as antineutrino CCQE, pion productions in Eq. (1.36,1.381.39), DIS and 2p2h. Final state interactions can also place neutrons into the detector.

The tracker region of the MINERνA detector is composed primarily of hydrogen and carbon. Neutrons could elastically or inelastically scatter on hydrogen and
carbon targets. Neutron capture on either nuclei could also happen, but happens to a much smaller extend. At the MeV-scale relevant to MINER\(\nu\)A, neutron captures are about 4 orders of magnitude smaller than the two dominant processes\cite{113}. Figure 5.28 compares the neutron scattering cross sections on hydrogen and carbon. Scattering on hydrogen occurs primarily through the elastic channel, with inelastic processes occurring after 500 MeV. The free proton in hydrogen could be easily dislodged to produce ionizing electrons. The elastic scattering on carbon is more difficult to observe because an carbon atom weighs 11.2 GeV/c\(^2\), the small energy transfer from a low energy neutron is invisible. Carbon atoms begin to undergo inelastic processes at 5.7 MeV, when the neutron is able to break a carbon into a beryllium nucleus and an \(\alpha\) particle. The \(\alpha\) particles are 4 times as heavy as a proton with two positive charges. They are able to deposit observable energies in the detector. Additional inelastic channels are available as the neutrons become more energetic, which usually includes various combinations of nucleus breakups. The broken nuclei could exist in excited states that deexcite through the production of photons or pions. For the majority of the applications in MINER\(\nu\)A, we are interested in events that produce proton knockouts because they deposits the most energy in the detector.

Figure 3.14 shows a simulated antineutrino CCQE reaction on hydrogen, with a \(\mu^+\) and a neutron \(n\) in the final state. The energy deposits from the neutron re-interaction are far away from the vertex. The black line in the figure is the neutron trajectory. The direction of the neutron changes after causing the energy deposits and bounces again before finally stops.

To mathematically understand neutron recoil, let us first consider a beam of generic particles with initial intensity \(I_0\) propagating through a material of atomic
Figure 3.13: The neutron elastic and inelastic scattering cross sections on hydrogen and carbon as a function of neutron kinetic energy used by GEANT4.

Figure 3.14: Example of a neutron candidate from antineutrino CCQE reaction on hydrogen. An antimuon exits the event vertex along with an outgoing neutron. The neutron does not deposit energy until it re-interacts in the detector to produce a proton. The proton spans two X-planes, creating a 21 MeV and a 30 MeV cluster respectively.
number density $\rho$, we can calculate the survival particle intensity $I$ after traveling a distance $x$ as

$$I = I_0 e^{-\rho\sigma x}$$

(3.8)

where $\sigma$ is the interaction cross section, depending on the particles’ energy. As the distance traveled increases, there will be fewer uninteracted particles. The mean free path $\lambda$ of a particle, which is a characteristic length scale at which the first interaction occurs, can be calculated as

$$\int_0^\infty x\rho\sigma e^{-\rho\sigma x} dx = \frac{1}{\rho\sigma}.$$  

(3.9)

As neutrons do not typically ionize atoms as they travel through material, their total interaction cross sections are low compared to charged particles and travel some distances before interacting.

We simulated events with a muon and a neutron in the MINER$\nu$A detector to understand how neutrons could deposit energy. In this simulation, the muon and neutron are placed at a fixed point in the detector and then given momentum. The muon is added because MINER$\nu$A’s event reconstruction program requires a vertex from a track to identify an event. Neutrons often do not leave enough energy for track reconstruction to proceed, so muons are added as a guarantee. The generated events contain detailed information about what particle goes in to make a hit.

Figure 3.15 shows the fraction of nuclei struck when the neutrons do interact in the MINER$\nu$A detector. At low kinetic energy $< 10$ MeV, most of the neutrons scatter on hydrogen. This is consistent with Fig. 5.28 because the hydrogen elastic cross section is about 10 times larger than carbon at low energy. As energy increases, carbon scattering becomes dominant. At higher energies, the carbon nuclei tend
Figure 3.15: In these plots, a neutron interaction is defined as a nuclear collision causing secondary particle(s). For the (left) the fraction of nuclear targets struck at given neutron kinetic energy and (right) the probability for the neutron interaction to produce clusters with at least 1 MeV energy.

to break and produce secondary particles, as shown in Fig. 3.16. Larger energy deposits tend to come from protons, with the most energetic protons ranging more than 300 MeV in kinetic energy, enough to produce hadronic tracks. Collisions on hydrogen in this energy range appear to be fully elastic. The neutron dislodges the free proton in hydrogen to produce a secondary proton in the detector. Collisions on carbon could produce a diverse set of secondary particles. A collision could be elastic, in which case the whole carbon nucleus gains a small momentum. An inelastic collision could proceed in a few ways. The neutron could excite the nucleus but leaves it intact. The nucleus will then fall back to the ground state emitting deexcitation photons. The neutron could also break the nucleus into individual nucleons and nucleus fragments. Pions and photons are sometimes produced in the process. While most energy deposits are due to single protons, the nucleus fragments, pions, and photons can also deposit energy. A small fraction of energy depositing events is recognized as “neutron” by GEANT4. The energy deposited in
Fragment Particles Type vs Energy Loss

Energy Loss is due to 
1. Sum due to energy loss processes 
2. Sum of energy loss due to low energy secondaries below tracking threshold

Particles Types
- Nuclei
- e
- n
- p
- \(\pi^+\)
- \(\pi^0\)
- \(\gamma\)

Energy Loss (MeV)

0 50 100 150 200 250 300 350 400

1 10 100 1000 10000 100000 1000000

Figure 3.16: Nucleus fragments that deposited energy in the MINER\(\nu\)A detector.

This category comes from secondary charged particles with very low energy. There is a cut on the minimum distance particles could travel in GEANT4. Particles with very short path lengths do not go through the full simulation. Energy deposits from these very soft particles are collectively attributed back to the neutron.

In essence, a neutron reconstruction algorithm must look for energy deposits away from the interaction vertex, and the energy signature should be proton-like. However, many neutron-induced secondary particles do not possess enough energy to produce a track necessary for PID in the MINER\(\nu\)A detector. Therefore, the algorithm should focus on reconstructing energy deposits away from the interaction vertex.
Figure 3.17: The schematic illustration of the neutron reconstruction algorithm.
Figure 3.17 shows a schematic of the reconstruction algorithm. The first step of the algorithm is to collect the Trackable and HeavyIonizing clusters in the tracker not associated with any tracks. Next, the algorithm separates clusters according to the views and sorts them in decreasing order of energy. The most energetic cluster in each view automatically becomes a seed. A seed has a transverse expanse $\Delta T$ given by the width of the cluster. Clusters on the adjacent planes in the view are added to the seed if they overlap with the seed’s transverse expanse. The width of the seed are updated to include the new clusters. Once a seed is formed, we calculate its centroid position $T$ on the plane. The algorithm removes a cluster from the list whenever it is added to a seed. Once no more cluster can be added to a seed, the remaining clusters in each view are automatically promoted into seeds. The algorithm loops through seeds in the $U$ and $V$ planes for overlaps in the $z$ position. Matching seeds provide an $x$ coordinate with $x = T_U + T_V$. The algorithm then loops through the $X$-view seeds to search for a seed that overlaps with the $UV$ seed in $z$ and $x = T_X$. When $\delta x$ between the two $x$-coordinate is small, all three seeds are combined to form a “blob”. This blob contains spatial information from all three views, and a 3D position can be calculated with $y = (T_V - T_U)/\sqrt{3}$.

The 3-view requirement minimizes the probability that a seed is added due to random chance. Once we exhaust seeds to form 3-view blobs, we have to combine the remaining seeds wherever possible. We consider clusters on adjacent planes. The combination could only be $UXU$, $XVX$, $XV$, $XUUX$, or $VX$ (Fig 3.5). 2-view blobs are more susceptible to clusters that may align accidentally, where each comes from distinctive processes. Therefore we only combine the clusters on adjacent planes when there is no more than 1 cluster in each plane. These 2-view blobs still contain enough spatial information to calculate a 3D position. The remaining seeds
become 2D blobs. When multiple blobs exist, we designate the most energetic 3D blob as the main candidate.

The probability of finding the main candidate increases with neutron energy. Figure 3.18 shows the efficiency of the algorithm when a forward-going neutron does interact in the detector. The efficiency reaches 70% when the neutron kinetic energy is above 400 MeV. Due to the randomness of how neutrons interact, there will always be neutrons that do not leave behind enough energy to make 3D candidates.

This algorithm forms the basis for the antineutrino hydrogen CCQE analysis. In the antineutrino CCQE reaction, the neutron is the dominant final state. Contamination by other neutrals could be reduced by cutting on the energies of clusters that do not form part of a track. This energy is the recoil energy in the detector. The re-
constructed 3D positions of the neutron candidates provide directional information crucial to separate the hydrogen and carbon events.

### 3.6 Overview of Analysis Procedure

The goal of a MINERνA analysis is to measure a cross section. A neutrino has a small probability of interacting on a nucleus, and we want to find out what this probability is.

The total number of interactions from a neutrino beam in a detector can be calculated from

$$N = \Phi T \sigma.$$  \hfill (3.10)

Where $N$ is the number of events in the detector, $\Phi$ is the total flux of neutrinos arriving at the detector per unit area, $T$ is the total number of targets in the detector, and $\sigma$ is the total cross section. Both $N$ and $T$ are dimensionless, so $\sigma$ has the unit of area.

We could gain more information about neutrino properties by looking at a *differential* cross section. The differential cross section for a quantity $x$ is

$$\frac{d\sigma}{dx} = \frac{dN}{(\Phi T)dx},$$  \hfill (3.11)

where $\frac{d\sigma}{dx}$ tells us how the cross section varies with quantity $x$, and $dN/dx$ is how the number of observed events changes with quantity $x$. $x$ can be any observable, such as the neutrino energy, $Q^2$, or any other kinematic variables of interest. This equation works in an ideal world with unlimited data and a detector with infinite precision and sensitivities. The unlimited data allows us to measure the number of
events in vanishingly small intervals along $x$, while the perfect detector reconstructs quantity $x$ in every neutrino interaction precisely and accurately. Our cross section measurements are limited by sample size in the real world, forcing us to make measurements in a finite interval $\Delta x$, called a bin, to accumulate statistics. Our detector has finite resolution so that the reconstructed physical quantities always have variation from their true value. Finally, we may have to throw away events because some interactions cannot be fully reconstructed. For instance, neutrino in a CCQE reaction produces a muon and a proton in the final state. The proton may re-interact in the detector, causing it to fail the proton $dE/dx$ selection. Neutron is another example where we must throw away events since there will always be neutrons leaving the detector without depositing enough energy to be seen.

Therefore, we must modify our cross section formula to account for the finite bin size, smearing from detector resolution, and inefficiencies in event selection.

$$
\left( \frac{d\sigma}{dx} \right)_i = \sum_j U_{ji} \left( N_{\text{data},j} - N_{\text{bkg},\text{data},j} \right) (\Phi T) \epsilon_i \Delta x_i.
$$

(3.12)

where $i$ is the index of a bin with size $\Delta x_i$. $N_{\text{data},j}$ and $N_{\text{bkg},\text{data},j}$ are the total number of events and the estimated background events in the $j$th bin respectively. The smearing from the detector has the effect of moving events in the $i$th bin into other bins, and $U_{ji}$ is the unfolding matrix element $ji$ that instruct us on how to statistically undo the smearing by moving events from $j$th bins back to the $i$th bin. Finally, $\epsilon_i$ is the efficiency correction, the probability that the event was reconstructed and found in the detector by the selection algorithm.

$N_{\text{data}}$, the total reconstructed event rate, is obtained from the data. We estimate the background fraction using MC prediction. The prediction needs to be con-
strained by looking at the background-dominated samples called sidebands. Weights can be applied to the prediction to make the data and MC agree in the sidebands. We then assume the background distribution receives the same weights in the signal region. We obtain the reconstructed signal event rate by subtracting the background estimates from the total. The unfolding matrix $U$ then corrects the detector distortions present in the reconstructed distribution to the “true” distribution.

The unfolding matrix $U$ is estimated from detector simulation, where we know both the true physics distribution and the smeared distribution after event reconstruction. We could construct a 2D square matrix. When the true value falls into the $i$th bin and the reconstructed value falls into the $j$th bin, we can add this event to the matrix element corresponding to the $i$th row and $j$th column. Therefore, each row corresponds to how the true value at the $i$th bin spreads in the reconstructed space. This matrix is called the migration matrix, and $U$ is its inverse estimated by the D’Agostini iterative unfolding procedure described in Ref. [126]. Refer to Fig 5.24 for the actual migration matrix we use in the hydrogen analysis. Essentially we want to “deconvolve” detector effects to get back the underlying true distribution.

We estimate the detector’s efficiency by taking the ratio of the true event distribution in the selected signal sample to the full generated sample. MINER$\nu$A has precisely modeled the detector geometry and the particle responses to emulate how the real detector throws away events.

Finally, the efficiency-corrected event rate is normalized by the total flux and the total number of targets to derive the cross section.
3.6 Overview of Analysis Procedure

3.6.1 Evaluate the Systematic Uncertainties in MINERνA

A physics measurement will always be limited by the sample size and the various systematic uncertainties from instrumentation and modeling. The limitation is especially true for neutrino analyses because we must rely on the detector, flux, and physics models to predict both the signal and the background events. The background prediction will then be subtracted from the data to obtain the signal distributions in the data. Although we could build elaborate detector models to reflect the real detector’s geometry and material properties, flux and neutrino physics models have large uncertainties built into them.

MINERνA uses a Monte Carlo method to evaluate the systematic uncertainties. There are uncertainties due to our imperfect knowledge of the detector, such as its mass, particle responses, and reconstruction efficiencies. There are also uncertainties due to the neutrino flux, or the underlying physics models used to predict background distribution. To evaluate the uncertainties, we could modify the predicted event distribution according to how a parameter affects the distribution through a weighting procedure. Each MC event receives a central value (CV) weight according to our best model prediction described in Sec 2.3. Selected CV events are stored in histograms for later analysis. We call this the CV universe because we believe in a specific value for each uncertain parameter.

For each uncertain parameter, we evaluate the weight using $\pm \sigma$ shifts in that parameter. For example, we could change the detector mass by by $\pm 1.4\%$ to effect a flat shift on the total number of events generated. We put each of the shifted distribution into a parallel histogram called an error band universe. The uncertain parameters could also come from the detector, such as errors in the reconstructed energy and direction of the muons. A 2% shift in the muon energy would shift $Q^2$
calculated by 2%. There are uncertainties in GEANT4’s particle response simulation, such as how the neutrons may interact in the detector. Neutrons could cause more energies, and we will select more neutron events in this universe. There are also various GENIE model uncertainties. For example, our knowledge of $M_A$ is accurate within 10%, so our generated cross section may increase or decrease. Uncertainties in the final state interactions may place more or fewer pions in the detector, modifying the background event rate. There are also uncertainties on the predicted flux that changes the number and energy of neutrinos arrived at the detector. All of these parameter shifts are evaluated using the MC method, and we call it the many-universe method because each parallel histogram is a universe in which we believe in an alternative set of physical parameter.

When we constrain the MC background using sidebands, both the CV and error bands universes need to fit the data. The fit provides each universe with a weight. Each universe is going to have a slightly different background prediction in the signal region. When we subtract the background from data, the differences will propagate to the extracted signal events. Finally, each universe has a specific migration matrix and efficiency correction as well. All of these will contribute to the systematic uncertainties in the final cross section.

We end this chapter with a toy example of the background constraint. Suppose we want to measure the total number of neutrino events with only a muon and proton in the final states. We reconstructed events with a muon track and a hadronic track. The hadronic track could descend from either a proton or a charged pion. We could cut on the proton score. We could also search and exclude the electron signature in the pion decay chain $\pi^+ \rightarrow \mu^+ + \nu_\mu$ and $\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e$. The electrons are commonly referred to as Michel electrons and often appear a few $\mu$s
after the main event. These selections should give us a fairly pure proton sample. Inefficiencies in both cuts would cause pion tracks to leak in. To constrain the size of the irreducible pion background, we could form a separate sample that specifically look for candidates with Michel electrons, these will be dominantly pion events. So now we have a signal region with more protons and a sideband with more pions. Now suppose there is more data in the pion sideband than MC, which tells us our MC under-predicts the number of pion events. Therefore, we increases the weight to pion events in both the signal and sideband region. The weighted pion fraction in the signal region is the predicted background fraction after sideband constraint. When multiple background event types exist (for example, single charged pion, single neutral pion, multiple pions), we could form additional sidebands sensitive to these final states and in effect turn the system into solving a system of linear equations. The sideband constraint method assumes GENIE models each of the event type consistently across the signal and sideband samples.
Chapter 4

Nucleon binding energy and transverse momentum imbalance in neutrino-nucleus reactions

The material attached in this chapter has been published in Ref [1]. The work describes the measurement of a set of transverse kinematic imbalance (TKI) variables, $\delta p_{Tz}$ and $\delta p_{Ty}$, which are sensitive to the Fermi motion of the nucleus and GENIE’s implementation of binding energy.

Section 4.1 motivates the importance of measuring removal energy. Section 4.2 describes briefly GENIE’s CCQE model in the impulse approximation framework. Section 4.3 formally introduces the transverse kinematic imbalances variables and define $\delta p_{Tz}$ and $\delta p_{Ty}$ as the projection of the total transverse momentum in the final state system. The analysis was performed on a sample of events called the QELike (referred to as QE-like in the following manuscript), based on the final state particle...
4.1 Introduction

Neutrino oscillation experiments measure the final state particles produced by neutrino-nucleus scattering processes. Models that accurately describe these interactions are crucial to reducing the uncertainties in the measurements of oscillation parameters.

Most neutrino-nucleus interactions are modeled through the impulse approximation (IA), where the probe sees the target nucleus as a collection of independent nucleons and the resulting particles then evolve independently. Important components of modeling in the IA picture include the initial state nucleon’s energy-momentum distributions, the nuclear potentials, and the final state interactions (FSIs) that modify the kinematics of the final-state particles as they propagate through the nucleus.
The leptonic system provides energy to the hadronic side of the reaction to bring a bound nucleon on-shell and separate it from the remnant nucleus. Such energy is often loosely referred to as “binding energy”, but Ref [3] draws a distinction between the different energy parameters in neutrino models and how their effects depend on the implementation details.

In this paper we refer to the average energy transferred to the target nucleus to bring a bound nucleon inside the target onto the mass shell as the “removal energy”, represented in this paper by $\epsilon_N(P)$ for the neutron (proton) initial state in neutrino (antineutrino) interactions. The energy associated with nuclear potentials is referred to as the nuclear potential energy. The combined effects of the “removal energy” and the nuclear potential energy is referred to as the “interaction energy”. The interaction energy implementation in the IA picture is discussed in detail in Sec. 4.2.

For many neutrino experiments, particularly at low energies like T2K, MicroBooNE, and the second oscillation maximum in DUNE, incorrect treatment of the interaction energy may significantly bias the reconstructed neutrino energy and will alter the expected kinematics of final state nucleons. Such effects are already a significant systematic in the measurement of $\Delta m^2_{23}$ in the T2K experiment [3, 128].

This paper will examine a set of new observable quantities that are sensitive to nuclear effects, and especially to the interaction energy implementation used in generators. The variables are extensions to the recent measurements of momentum imbalance in mesonless events with a muon and proton in the final state, here referred to as single transverse kinematic imbalance (single-TKI) [129] by the MINER$\nu$A [130] and T2K [131] experiments.

The new observables are derived from the single-TKI observable $\delta\mathbf{p}_T$. Specifi-
4.2 Impulse Approximation

We illustrate the effects of interaction energy with the charged-current quasielastic (CCQE) interaction in the IA picture shown in Fig. 4.1. In this picture, only a single nucleon is involved in the hard scattering, $\nu n \rightarrow \mu^- p$.

The neutrino with energy $E_0$ made 4-momentum transfer, $q = (q_0, q_3)$ to a bound
4.2 Impulse Approximation

Figure 4.1: A neutrino interaction with a bound neutron in the impulse approximation. $\nu$, $\mu$, $N$ and $P$ are the neutrino, muon, neutron and protons respectively. The incoming neutrino with 4-momentum $E$ interacts with the bound neutron with 3-momentum $k$ and removal energy $\epsilon^N$. The removal energy consists of the nucleon separation energy $S^N$, average excitation energy $\langle E_x^N \rangle$ and the kinetic energy of the remnant nucleus. $E^{P(\mu)}$ and $p^{P(\mu)}$ are the proton ($\mu$) total energy and momentum, $T^P$ is the proton kinetic energy, $|U_{opt}|$ and $|V_{eff}| (|V_{eff}|)$ are the magnitudes of the optical potential and the Coulomb potential experienced by proton (muon). The quantities with the subscript (vtx) are those immediately after 4-momentum transfer.

The neutron of mass $M_N$ inside a target nucleus with $A$ nucleons, where $q_0$ and $q_3$ are the energy and momentum transfer respectively. The target nucleus was initially at rest with mass $M_A$, the bound neutron has 4-momentum $E_i = (M_N - \epsilon^N, k)$ where $k$ is neutron’s Fermi momentum. The remnant nucleus with 4-momentum $P^*_{A-1}$ must have momentum $-k$ for the target nucleus to be at rest. The energy of the initial state neutron can be written as:

$$E_i = M_A - \sqrt{M^*_{A-1}^2 + k^2},$$ (4.1)
where \( M_{A-1}^* \) is the mass of the excited spectator nucleus. For the nuclear targets typically used in neutrino experiments (\(^{12}\text{C}, \,^{16}\text{O} \) and \(^{40}\text{Ar}\)), we have \( M_{A-1}^* - 1 \gg k^2 \).

Then we can expand the initial state nucleon energy as:

\[
E_i \approx M_A - M_{A-1}^* - \frac{k^2}{2M_{A-1}^*} = M_N - S^N - E_x^N - \frac{k^2}{2M_{A-1}^*}.
\] (4.2)

The removal energy parameter

\[
\epsilon^N = S^N + E_x^N + \langle T_{A-1} \rangle,
\] (4.3)

accounts for the neutron separation energy from the target nucleus \( S^N \),

\[
S^N = M_{A-1} + M_N - M_A,
\] (4.4)

and the excitation energy of the final state nucleus, \( E_x^N \), when the initial state nucleon is a neutron,

\[
E_x^N = M_{A-1}^* - M_{A-1}.
\] (4.5)

The average kinetic energy \( \langle T_{A-1} \rangle = k^2/(2M_{A-1}^*) \) of the excited remnant nucleus with \( A-1 \) nucleons affects the interaction only through its nuclear potentials. For neutrino QE interactions on \(^{12}\text{C}\), \( S^N = 18.7 \) MeV, \( E_x^N = 10.1 \) MeV and \( \langle T_{A-1} \rangle = 1.4 \) MeV [3]. The removal energy is the average energy needed to bring the neutron onto the mass-shell.

There are additional effects associated with the nuclear potentials that should be accounted for. For example, the nuclear optical potential describes the nucleus
as a medium with complex refractive index: the real part of the potential affects
the allowed kinematics of the initial state lepton-nucleon system in the IA while the
imaginary part is related to inelastic scattering as the outgoing nucleon is making
an exit from the nucleus [91]. Reference [3] fits inclusive electron scattering data to
determine the real part of the optical potential which depends on the 3-momentum
of the outgoing nucleon at the interaction vertex. This optical potential is denoted
as \( U_{\text{opt}}[(k + q_3)^2] \) in this work. The effect of the optical potential is largest at lower
momentum. For carbon, the parameterization of Ref [3] is

\[
U_{\text{opt}} = \min \left[ 0, -29.1 + \left( \frac{40.9}{\text{GeV}^2} \right)(k + q_3)^2 \right] \text{MeV.} \tag{4.6}
\]

In this analysis we use this parameterization, and it is on average 2 MeV for our
selected events.

Another potential, the Coulomb potential \( V_{\text{eff}} \) of the positively charged remnant
nucleus will modify the momenta of the outgoing charged particles as they propagate
through the nucleus. In Fig. 4.1, a distinction between the Coulomb potential expe-
rienced by muon (\( |V_{\text{eff}}| \)) and proton (\( |V_{\text{eff}}^P| \)) is made; however, for neutrino interactions
both particles experience the same Coulomb potential as the proton number in the
nucleus remains unchanged after the interaction. For carbon, \( |V_{\text{eff}}| \) is 3.1 MeV [3].

Figure 4.1 illustrates energy and momentum conservation between the initial and
final state. The total energy of the final proton and muon is equal to the total of
the initial neutron and lepton, less energy required to create the final state excited
nucleon in the reaction. The Coulomb potential affects any charged final state par-
ticles, but the optical potential affects the final state nucleon only. For example, the
4.2 Impulse Approximation

Muon with total energy $E_\mu = E_{\mu\text{vtx}}$ begins inside the Coulomb potential, with kinetic energy $E_\mu + |V_{\text{eff}}|$ and potential energy $-|V_{\text{eff}}|$, and is decelerated during transport inside the nucleus medium so that its kinetic energy is $E_\mu$ outside the nucleus. The proton experiences both the Coulomb potential and the optical potential, which modify its kinetic energy and momentum but conserve the total energy. The full energy conservation equation on the hadronic side is as follows [3]:

$$E_{\mu\text{vtx}} = q_0 + M_N - S^N - E^x_{\mu} - \frac{\langle k^2 \rangle}{2M^*_A - 1} = \sqrt{(k + q_3)^2 + M^2_P} - |U_{\text{opt}}[(k + q_3)^2]| + |V_{\text{eff}}| = E^P. \quad (4.7)$$

Here, the final state proton is assumed to be on-shell with energy $E_{\mu\text{vtx}} = E_f^P$, before and after exiting the nucleus. Its kinetic energy immediately after the 4-momentum transfer is

$$T_{\mu\text{vtx}} = q_0 + M_N - M_P - \epsilon^N, \quad (4.8)$$

and is modified by the nuclear potentials so that outside the nucleus the kinetic energy becomes

$$T^P = T_{\mu\text{vtx}} - |U_{\text{opt}}| + |V_{\text{eff}}|. \quad (4.9)$$

The removal energy used by neutrino Monte Carlo (MC) generators, such as GENIE [54], NEUT [86], and NuWro [85], are discussed in detail in Ref [3]. These generators use variants of spectral functions, mostly the Fermi gas model in the IA picture with removal energy constrained by inclusive electron scattering data [134]. However, they have distinct implementations of the IA model which affects the energy terms going into the removal energy parameter. For example, in GENIE’s IA
implementation, the off-shell bound initial nucleon is generated with Eq. (4.7), but
with $E_N^x$, $U_{\text{opt}}$, and $V_{\text{eff}}^P$ set to 0. GENIE subtracts an additional “binding energy”
parameter $\Delta_{\text{GENIE}}^{\text{nucleus}}$ from the final state protons in QE processes to account for the
removal energy. The implementation of this term is independent of the kinematics
at the interaction vertex, which causes the energy of the final state nucleons to be
biased. The values of $\Delta_{\text{GENIE}}^{\text{nucleus}}$ were measured by Ref [134] and referred to as the
“Moniz interaction energy” in Ref [3]. The Moniz interaction energy is an empirical
fit to the sum of the removal energy and the nuclear potentials, but for a non-
relativistic on-shell formalism. For $\nu + ^{12}\text{C}$ scattering, $\Delta_{\text{GENIE}}^{\text{C}} = 25$ MeV [54].

Table 4.1: Calculated energy corrections to the final state leptons and hadrons
from the GENIE generator for QE neutrino scattering on $^{12}\text{C}$, $\Delta_{\text{GENIE}}^{\text{C}} = 25$ MeV, $E_x = 10.1$ MeV. Other interaction channels are not altered.

<table>
<thead>
<tr>
<th>Correction</th>
<th>$E_P = E_{\text{GENIE}}^P + \delta P$ (MeV)</th>
<th>$E_{\mu} = E_{\text{GENIE}}^\mu + \delta \mu$ (MeV)</th>
<th>GENIE Baseline Shift $\langle \delta P \rangle, \langle \delta \mu \rangle$ (MeV)</th>
<th>QE Baseline Shift $\langle \delta p_{\text{Ty}} \rangle$ (MeV/c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0: Default (no corrections)</td>
<td>0</td>
<td>0</td>
<td>0,0</td>
<td>0</td>
</tr>
<tr>
<td>1: $U_{\text{opt}}$ only (w/ $E_x$ &amp; $\Delta_{\text{GENIE}}^{\text{C}}$)</td>
<td>$\Delta_{\text{GENIE}}^{\text{C}} -</td>
<td>U_{\text{opt}}</td>
<td>$</td>
<td>$</td>
</tr>
<tr>
<td>2: $U_{\text{opt}}$ and $V_{\text{eff}}$ (w/ $E_x$ &amp; $\Delta_{\text{GENIE}}^{\text{C}}$)</td>
<td>$\Delta_{\text{GENIE}}^{\text{C}} -</td>
<td>U_{\text{opt}}</td>
<td>+</td>
<td>V_{\text{eff}}</td>
</tr>
</tbody>
</table>

In this paper, we refer to the collective energy shifts due to removal energy
and the nuclear potentials as the “interaction energy”, in the spirit of the Moniz
interaction energy of Ref [3]. This interaction energy is specific to the off-shell
formalism described in Eq. (4.7).

We simulate the effects of the interaction energy implementations in GENIE by
modifying the final state muon and proton energies after a sample is generated
according to Table 4.1. The corrections outlined are motivated by the study in
Ref [3]. Comparisons between the default GENIE implementation ( 0 in Table 4.1)
and two different corrections (1 and 2) are made. For both sets of corrections, which
are applied to QE events, we add $\Delta_{\text{GENIE}}^C$ back to the exiting proton to undo the bias, we then subtract $E_x$ from the muon to account for the shift in momentum transfer in the leptons (derived in Appendix 4.8). In addition, the correction 1 applies an optical potential correction to both the muon and proton, while the correction 2 applies the Coulomb correction on top of correction 1. The average $|U_{\text{opt}}|$ is $\approx 2 \text{ MeV}$ for the proton and muon kinematics chosen.

The corrections are approximations which implement the leading effect of the nuclear potential. That potential will also cause changes, small for our events that have an energetic final state proton, in the four momentum transferred to the off-shell target nucleon. Appendix 4.8 provides the derivation of our corrections.

### 4.3 The single-TKI variables

The single-TKI measurements for QE-like events, which include a lepton, at least one proton and no mesons in the final state, are defined in Ref [129]:

\[
\delta p_T \equiv p_T^p + p_T^\mu, \tag{4.10}
\]

\[
\delta \alpha_T \equiv \arccos \left( -\hat{p}_T^\mu \cdot \hat{\delta p}_T \right), \tag{4.11}
\]

where $p_T^p$ and $p_T^\mu$ are the components of proton and muon momenta in the plane perpendicular to the neutrino direction. The single-TKI variable $\delta p_T$ and its decompositions along the Cartesian coordinate system defined with respect to the neutrino
4.3 The single-TKI variables

Figure 4.2: Schematics of the Single Transverse Kinematic Imbalances and their projections. The incoming neutrino interacts on the neutron \( N \) in the nucleus. The neutrino direction \( \vec{p}_\nu \) forms the \( z \) axis. A final state muon with \( p_\mu \) and a proton with \( p_p \) are produced. The muon transverse momentum is \( p_T^\mu \), and \( -\hat{p}_T^\mu \) defines the \( y \) axis. The proton transverse momentum is \( p_T^P \) and decomposed along \( x \) and \( y \) axis respectively. In this example, both \( \delta p_{Tx} \) and \( \delta p_{Ty} \) are negative, and only the distribution of \( \delta p_{Tx} \) for QE events is expected to be symmetric around zero.
Figure 4.3: $\delta p_{Ty}$ vs. $\delta p_{Tx}$ for CCQE events in GENIE. The contours, from outside towards the center, represent 0.1, 0.2, 0.3, and 0.7 of maximum. The angles correspond to $\delta \alpha_T$ values. The neutrino direction points out of the page. The dashed contour describes the default GENIE distribution with a MvGENIE-v1 tune, described in Sec. 4.5. The solid contour shows the shift in the distribution after correction is made to the interaction energy with correction 01 of Table 4.1. There is negligible deformation in the $\delta p_{Tx}$ direction compared to the shift in $\delta p_{Ty}$.

and muon kinematics are illustrated in Fig. 4.2 and mathematically defined as

$$\delta p_{Tx} = (\hat{p}_\nu \times \hat{p}_T^\mu) \cdot \delta p_T,$$

$$\delta p_{Ty} = -\hat{p}_T^\mu \cdot \delta p_T. \quad (4.12)$$

Here, $\hat{p}_\nu$ is the neutrino direction, $\delta p_{Ty}$ is anti-parallel to the muon transverse direction $\hat{p}_T^\mu$ while $\delta p_{Tx}$ is perpendicular to $\delta p_{Ty}$ along the normal of neutrino-muon plane. The coordinate system describing $\delta p_{Tx}$ and $\delta p_{Ty}$ is relative to the neutrino and
muon kinematics. Specifically $\hat{y}$ is along the transverse component of 3-momentum transfer, $\hat{z}$ is along the neutrino direction and there is no 3-momentum transfer in the $\hat{x}$ direction. Both $\delta p_{Tx}$ and $\delta p_{Ty}$ can be measured from the final state particles. Any interaction energy effect will mostly affect the 4-momentum transfer and $\delta p_{Ty}$. For CCQE events, $\delta p_{Tx}$ is expected to symmetrically distribute on both sides of the neutrino-muon interaction plane.

$(\delta p_{Tx}, \delta p_{Ty})$ can be defined in terms of $(\delta p_T, \delta \alpha_T)$ as:

$$|\delta p_{Tx}| = \delta p_T \sin \delta \alpha_T,$$

$$\delta p_{Ty} = \delta p_T \cos \delta \alpha_T.$$

(4.13)

Here $\delta p_{Ty}$ is positive if the proton has gained momentum along $-p_T^\mu$. Figure 4.3 illustrates the relationship between $(\delta p_{Tx}, \delta p_{Ty})$ and $(\delta p_T, \delta \alpha_T)$ as the different projections of $\delta p_T$ in the Cartesian and the polar coordinate systems respectively. The resulting distribution in the $\delta \alpha_T$ and $\delta p_T$ residuals provides insights into other nuclear effects affecting the cross section, such as FSIs, the Fermi motion and two-particle-two-hole (2p2h) processes[130].

4.4 Sensitivities to interaction energy implementation

The shapes of $\delta p_{Tx}$ and $\delta p_{Ty}$ are affected by nuclear effects. The non-zero width of $\delta p_{Tx}$ for the QE portion of the signal is largely due to the Fermi motion of the target nucleus. The average Fermi momentum in Carbon is approximately 221 MeV. In the absence of FSI effects, this is the only momentum available in the $x$ direction. FSIs
could alter the outgoing protons’ directions, but in Carbon, an outgoing nucleon typically exits without interacting with the nucleus, or interacts with the nucleus elastically producing a small change in direction.

The momentum transferred to the hadronic system is confined in the $yz$ plane. On an event by event basis, the nuclear potential may alter this momentum as well, and therefore the direction of the final state nucleon, but this effect averages to zero because the initial state nuclear momentum is on average zero. Therefore, changes to the interaction energy at the event vertex, on average, only alter $\delta p_{Ty}$. Mathematically, the effect of the interaction energy is as follows:

For an outgoing nucleon with energy $E_f'$ before it has left the region of nuclear potentials, its momentum $p_f$ as a function of an energy shift $\tau$ due to the interaction energy is:

$$|p_f(\tau)| = \sqrt{(E_f' - \tau)^2 - M^2_p},$$

(4.14)

where $E_f' = \sqrt{M_p^2 + p_0^2}$ and $p_0 = p_f(0) = k + q$. In the limit

$$\frac{\tau E_f'}{p_0^2} \ll 1,$$

(4.15)

we can approximate $p_f(\tau)$ by

$$p_f(\tau) \approx \left(1 - \frac{E_f'}{p_0^2} \tau\right) p_0.$$  

(4.16)

Defining $\alpha = \tau E_f'/p_0^2$, we can write the 4-momentum conservation equation without
FSI as:

\[
\begin{pmatrix}
q_0 \\
0 \\
q_T \\
q_L
\end{pmatrix} + \begin{pmatrix}
E_i \\
k_x \\
k_y \\
k_z
\end{pmatrix} \approx \begin{pmatrix}
E'_f \\
p_{0x} \\
p_{0y} \\
p_{0z}
\end{pmatrix} - \begin{pmatrix}
\tau \\
\alpha p_{0x} \\
\alpha p_{0y} \\
\alpha p_{0z}
\end{pmatrix},
\]

(4.17)

where \((0, q_T, q_L)\) are components of the 3-momentum transfer \(\mathbf{q}\), \((k_x, k_y, k_z)\) are components of Fermi motion \(\mathbf{k}\). In this picture, \(q_T\) is directly measurable as the transverse component of muon momentum, with magnitude \(p_\mu T\), but \(q_L\) cannot be directly measured and estimates depend on the model used to calculate neutrino energy.

The transverse components of the 3-momentum imbalance are

\[
\begin{align*}
\delta p_{T\times} &= (1 - \alpha)p_{0x} \\
&\approx k_x - \alpha p_{0x} = k_x - \tau \frac{E_f}{P_0} p_{0x}, \\
\delta p_{Ty} &= (1 - \alpha)p_{0y} + \mathbf{p}_\mu \cdot \hat{y} \\
&= p_{0y} - p'^T_T - \alpha p_{0y} = p_{0y} - q_T - \alpha p_{0y} \\
&\approx k_y - \alpha p_{0y} = k_y - \tau \frac{E_f}{P_0} p_{0y},
\end{align*}
\]

(4.18, 4.19)

where we have assumed \(k_x \approx p_{0x}\) and \(q_T + k_y \approx p_{0y}\) because the Fermi momentum is large compared to the interaction energy-induced change in momentum. In the limit \(\tau, \alpha \to 0\), \(\delta p_{T\times}\) and \(\delta p_{Ty}\) are the transverse components of the Fermi momentum, \((k_x, k_y)\). The effect of energy shift, \(\tau\), in each component of \((\delta p_{T\times}, \delta p_{Ty})\) is then proportional to that component of \(p_0\). When \(p'^T_T \gg k_y\), the shift in \(\delta p_{Ty}\) will be larger than the shift in \(\delta p_{T\times}\). In both components, the interaction energy effects
acting on the Fermi momentum will average to zero, whereas the effects on $\delta p_{TY}$ from $q_T$ will yield a net average shift in $\delta p_{TY}$. For events in GENIE2.12.10, there is approximately $+15$ MeV/c offset in $\delta p_{TY}$. The last column in Table 4.1 shows how applying energy corrections to the final state proton affects the average QE peak positions in $\delta p_{TY}$.

4.5 Apparatus and Methodology

The measurements of differential cross sections in $\delta p_{TX}$ and $\delta p_{TY}$ with the MINERvA detector [112] use the same sample and methodology of the measurements described in Ref [130]. The signal requires no pions, one muon, and at least one proton in the
Figure 4.5: Reconstructed event rate in the $\delta p_{TX}$ signal (top left) and a representative background sideband (top right); the $\delta p_{TY}$ signal (bottom left) and a background sideband (bottom right). The background fraction in the signal have been fitted with a data-driven constraint using the sidebands.

The final state, satisfying

\[
\begin{align*}
1.5 \text{ GeV/c} < p_\mu < 10 \text{ GeV/c}, \quad &\theta_\mu < 20^\circ, \\
0.45 \text{ GeV/c} < p_p < 1.2 \text{ GeV/c}, \quad &\theta_p < 70^\circ,
\end{align*}
\]

where $p_\mu$ and $\theta_\mu$ ($p_p$ and $\theta_p$) are the final-state muon (proton) momentum and opening angle with respect to the neutrino direction, respectively. The data set corresponds to $3.28 \times 10^{20}$ protons on target (POT) delivered between 2010 and 2012 by the NuMI beam line [35] at Fermilab. For this beam, the integrated $\nu_\mu$ flux is predicted to be $2.88 \times 10^{-8}$/cm$^2$/POT[118].
Neutrino interactions are simulated with GENIE 2.8.4 [54] in both a nominal form, and also with a MINERvA “tune” (mnvGENIE-v1.0.1). The nominal GENIE generates initial states with a modified Fermi Gas model containing contributions from the Bodek-Ritchie tail [99]. The CCQE cross section is produced by the Llewellyn Smith formalism [55], with a dipole axial form factor with axial mass $M_{\alpha}^{QE} = 0.99$ GeV/$c^2$. Resonant pion production is modeled by the Rein-Sehgal [103] model. Deep inelastic scattering is simulated with a Quark-Parton Model parameterized with the Bodek-Yang structure functions [105]. FSI is simulated with the GENIE hA model.

The tuning is based on mnvGENIE-v1, which has been applied in previous publications [130, 135, 136]. mnvGENIE-v1 includes the Valencia two-particle-two-holes (2p2h) model [106, 137, 138] for two-body current simulation. Furthermore, the interaction strength of this 2p2h model has been tuned to MINERvA inclusive scattering data [107], resulting in a significant enhancement relative to the Valencia model in a restricted region of energy-momentum transfer. MnvGENIE-v1 also includes a non-resonant pion reduction to 43% of the nominal as constrained by comparisons with bubble chamber deuteron data [104, 139]. There is also a modification to the collective excitations of the nucleus for the CCQE channel, approximated as a superposition of 1p1h excitations and calculated with the Random Phase Approximation (RPA) in Ref [140] and uncertainties in Ref [97]. The effects of non-resonant pion production and RPA in this analysis are negligible.

On top of the mnvGENIE-v1 tuning, mnvGENIE-v1.0.1 removes QE events with elastic nucleon-nucleus FSI, replacing them with events where there is no FSI, to remove the effect of a mistake in GENIE’s implementation of the elastic nucleon-nucleus FSI. The primary effect in the final state is in the angular distribution of outgoing protons. A detailed discussion of this mistake can be found in Appendix 4.9.
Reconstructed events with one muon and at least one proton in the MINERνA tracker satisfying Eq.(4.20)-(4.21) are selected. Figure 4.4 shows the reconstruction efficiencies of the muons and the protons due to event selection and detector acceptance.

Only the muons which exit from the back of the MINERνA detector and end up in the MINOS detector can be fully reconstructed. The muon momentum lost inside MINERνA is measured by energy deposits. The momentum in MINOS is estimated by range or curvature, which depends on whether the muon is contained in the MINOS spectrometer.

Proton identification is done with a track-based dE/dx algorithm which could reconstruct the proton energy (including rescattered protons) to 5% energy resolution[141]. An additional dE/dx selection is applied on these protons to favor ones that interact elastically and contained (ESC) within the CH tracker, which improves the energy resolution to 3% [130, 142]. The ESC requirement impacts the selection efficiencies for protons with higher momentum, which tend to rescatter inelastically more often.

The reconstructed proton energy and angular resolutions are 3% and 2°, while the reconstructed muon energy and angular resolutions are ~ 8% and 0.6°. The resolutions of the composite variables $\delta p_{TX}$ and $\delta p_{TY}$ have been evaluated to be 0.05 GeV/c and 0.06 GeV/c respectively.

After the event selection, background contributions are estimated using predictions from GENIE 2.8.4. The predicted background consists of events with pions in the final states, which mostly comes from RES and DIS interaction channels. The background is then constrained with a data-driven method with sidebands described in Ref [124]. The event rate in the signal region and in a representative sideband
for $\delta p_{Tx}$ and $\delta p_{Ty}$ are shown in Fig. 4.5. In this figure, the sideband sample shown contains events with off-track visible recoil energy between 0.06 and 0.385 GeV. Four sidebands with different visible recoil energy are used to constrain the backgrounds in bins of proton $Q^2_{QE}$ from 0.15 to 2.0 GeV$^2$. Separate weights are used for inelastic events with a baryon resonance events and for other, higher $W$, inelastic backgrounds.

After subtracting the fitted background, the signal fraction is treated with an iterative unfolding procedure [126] to account for the detector resolution [130]. Four iterations are chosen [130] to balance model bias and statistical uncertainties in the unfolded distribution. The stability of the unfolding with four iterations is studied by unfolding different pseudodata sets with model variations different from our assumed cross section model. As an extreme test, one of the variations we study for each of $\delta p_{Tx}$ and $\delta p_{Ty}$ puts in a large, non-physical, asymmetry in the relevant distribution. For each of these pseudodata studies, we compare the consistency of the unfolded pseudodata with the input model assumption as a function of number of iterations. For each pseudodata set, statistical uncertainties are added about the mean data prediction from the mode variation. One thousand pseudodata sets are created for each study. We find that four iterations of unfolding are sufficient to achieve good agreement, where the metric for agreement is the mean $\chi^2$ from the comparisons of unfolded pseudodata to its true distribution. We also verify that the mean $\chi^2$ fails to decrease significantly with additional iterations.

The unfolded data is corrected for the predicted efficiency calculated as ratio between the predicted and selected number of simulated events in each bin. The efficiencies for $\delta p_{Tx}$ and $\delta p_{Ty}$ in the QE region ($-0.5, 0.5$) GeV/c are constant at 0.1 with 10% relative variations and slowly falls by a factor of 2 over the regions
\( \pm (0.5, 0.1) \text{ GeV}/c \). The flux-averaged differential cross sections are then obtained by normalizing the efficiency-corrected distribution with the number of target nucleons \( (3.11 \times 10^{30}) \) and the predicted \( \nu_\mu \) flux.

Uncertainties on \( \delta p_{TX}(\delta p_{TY}) \) result from statistical fluctuations and uncertainties in the NuMI flux prediction, the \textsc{genie} modelling, and the detector response. The uncertainties are propagated throughout the cross section extraction procedure, and the results are summarized in Fig. 4.6.

The final differential cross sections in \( \delta p_{TX} \) and \( \delta p_{TY} \) are reported over \(-0.7 \text{ GeV}/c < \delta p_{TX}, \delta p_{TY} < 0.7 \text{ GeV}/c \). Each broad category of systematic uncertainties, neutrino flux, detector response, and assumed interaction model ("\textsc{genie}") ranges between 5\% to 10\% within this region. The largest contributing factor to uncertainty in the detector response is the tracking efficiency; the largest uncertainty in the neutrino interaction model is \textsc{genie}'s model of pion absorption in final state interactions.

![Diagram](image.png)

**Figure 4.6:** Uncertainties on the extracted \( \delta p_{TX}(\text{top}) \) and \( \delta p_{TY}(\text{bottom}) \) cross sections.
4.6 Results and Discussions

Model comparison is facilitated with the NUISANCE [143] neutrino interaction cross section comparison package. For the primary comparison with data, we use GENIE 2.12.10 with the Valencia 2p2h model replacing the default empirical 2p2h model. NUISANCE is used to apply the MnvGENIE-v1.0.1 tune that is described above. GENIE 2.12.10 and GENIE 2.8.4 have consistent model implementations. A careful internal MINERvA study indicates the main difference for this analysis is an increase of $S^N$ by 14.8 MeV from changes to the nuclear masses in GENIE.

The unfolded cross section results are shown in Fig. 4.7. The $\delta p_{Tx}$ and $\delta p_{Ty}$ cross sections are in the top and bottom panels respectively. There are significant non-QE contributions for both distributions. Of these about half are due to the tuned 2p2h. For each cross section, the QE distribution is broken down into the generated FSI modes. Here, the GENIE no FSI means the final state nucleon exited
Figure 4.8: The bin-by-bin asymmetry in the differential cross sections between \( \pm |\delta p_{Tx}| \) bins (Eq. 4.22). Data is compared to \textsc{MnGenie-v1.0.1}, which is representative of the other MC generators used in this study and exhibits no asymmetry.

The nucleus without interaction; elastic FSI refers to elastic nucleon-nucleon scattering which typically involves scattering angles less than 10°; and inelastic FSI refers to events with knockout of one or more additional nucleons. The other FSI category includes channels such as charge exchange multi-nucleon knockout, and pion production/absorption during nucleon transport. Appendix 4.9 describes an error in \textsc{Genie}'s implementation of elastic FSI, the fix we implemented, and the effect of the fix on the predictions and the analysis.

4.6.1 Distribution in \( \delta p_{Tx} \)

The measured differential cross-sections in \( \delta p_{Tx} \) and \( \delta p_{Ty} \) exhibit a QE peak near 0. If the interaction occurred on a free nucleon, then we would expect a delta function at 0 because the muon and proton final states must balance. The width of the QE peak mostly results from Fermi motion.

The measured cross-section in \( \delta p_{Tx} \) in the peak region is wider in the data than in the reference model. While our correction to simulation of elastic FSI does not
4.6 Results and Discussions

Table 4.2: $\chi^2$ of asymmetries ($A_{Tx}$) against no asymmetry case for regions of $\delta p_{Tx}$ distributions calculated with the covariance matrix.

<table>
<thead>
<tr>
<th>$\delta p_{Tx}$ Range (GeV)</th>
<th>$\chi^2$/ndf</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00 $\sim$ 0.40</td>
<td>19.9/8</td>
</tr>
<tr>
<td>0.40 $\sim$ 0.70</td>
<td>4.95/7</td>
</tr>
<tr>
<td>0.00 $\sim$ 0.70</td>
<td>21.6/15</td>
</tr>
</tbody>
</table>

precisely reproduce a “fixed” elastic FSI, the width of the predicted no FSI contribution itself is larger than the data. If we assume no significant deviation in the non-QE distributions, then the discrepancy could imply an overestimation of the Carbon Fermi momentum, or a reduction in the total fraction of the no FSI contribution, or both.

Besides the width discrepancy, the data distribution in $\delta p_{Tx}$ visually leans towards the positive side.

To measure the significance of the asymmetry, we define the bin-by-bin asymmetry between the positive and negative sides of the differential cross section in $\delta p_{Tx}$ as:

$$A_{Tx}(|\delta p_{Tx}|) = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-}, \quad (4.22)$$

where $\sigma_{\pm}$ is the cross section at either $\pm|\delta p_{Tx}|$ bin. The resulting distribution is reported in Fig. 4.8, where observation of bin-by-bin asymmetries in the data and their significances in different ranges of $\delta p_{Tx}$ are reported in Table 4.2. None of the generators used in this study reproduce the asymmetric feature, where mvGENIE-v1.0.1 is shown as an example.
The total asymmetry is defined as:

\[ A_{LR} = \frac{N_- - N_+}{N_- + N_+}, \]

(4.23)\]

with \( N_-/+ \) being the integrated cross sections on the left/right side of the neutrino-muon plane. The result is

\[ A_{LR} = -0.05 \pm 0.02, \]

(4.24)

where the uncertainty is calculated from the covariance matrix in the Supplemental Material.

Such an asymmetry has been suggested to result from the pion absorption contributions to the signal [2]. Measurements of single-pion production at low energy in deuterium [144] and single-\( \pi^0 \) production by MINER\( \nu \)A [127] have seen positive pion asymmetries about the neutrino-muon plane. The correlated proton angular distributions in this measurement, from baryon resonance production with an unobserved absorbed pion, could exhibit an opposite asymmetry.

### 4.6.2 Distribution in \( \delta p_{Ty} \)

Unlike in the \( \delta p_{Tx} \) distribution, we observe a non-QE tail towards the negative \( \delta p_{Ty} \) values. Inelastic events such as 2p2h, resonance and DIS are inefficient at transferring the lepton momentum to the final state nucleons, since multiple initial states particles are often involved. Therefore the protons tagged in the non-QE events will in general have less momenta then the muons, and are shifted to the left.

The two sets of corrections proposed in Table 4.1 are made to the final states muons and protons in MNVGENIE-v1.0.1’ CCQE contribution in the MC sample.
Figure 4.9: Differential cross sections in $(\delta p_{T_x}, \delta p_{T_y})$ compared with MnvGENIE-v1.0.1 interaction energy corrections, defined in Table 4.1. The corrections minimally affect $\delta p_{T_x}$, while bringing the $\delta p_{T_y}$ peak region into closer agreement with data. Note the similar trends in $\delta p_{T_y}$ ratios between the corrections and data.

The effect of correction 1, with $U_{opt}$ only, and correction 2, with both $U_{opt}$ and $|V_{eff}|$ corrections, are shown in Fig. 4.9. The effects of $U_{opt}$ is on the order of 2 MeV as it mainly affects nucleons at low kinetic energies.

Almost all of the shift comes from adding the Moniz interaction energy for Carbon ($\Delta_{GENIE}^C$) back to the final state proton, and removing the average excitation energy ($\langle E_x \rangle$) from the muon. These corrections alone shift the $\delta p_{T_y}$ peak 34.2 MeV/c to the right. Application of the optical potential partly cancels the shift, resulting in a net shift of 29.4 MeV/c. However, the addition of the Coulomb effect shifts the peak back, nearly canceling the effect of the optical potential, for a net shift including both effects of 33.9 MeV/c.

The ratios, in the lower panels of Fig. 4.9, of the corrected models and the data to MnvGENIE-v1.0.1 show the same upward-going trend in the QE peak region between $|\delta p_{T_y}| < 0.2$ GeV. This trend is characteristic of a peak shift, and the similarities
lend confidence to the validity of the theoretically motivated corrections.

Figure 4.10 and Fig. 4.11 compares NuWro Local Fermi Gas (LFG), NuWro Spectral Function (SF), GiBUU, the nominal GENIE, MnvGENIE-v1.0.1, as well as NEUT SF and LFG, distributions normalized to data cross sections. In terms of $\delta p_{Ty}$, the nominal GENIE with Nieves 2p2h does not depart much from the overall peak offset seen in MnvGENIE-v1.0.1, the ratio between which is nearly flat. The modifications to the 2p2h fraction, the non-resonant pion reweighting and RPA introduced by the MINERνA tune have little effect on the position of the peak, since their effects are nearly constant at the QE peak. Data to MnvGENIE-v1.0.1 ratio, and in fact the ratios of all other models to MnvGENIE-v1.0.1, except NuWro LFG follow very similar trends. The NuWro SF and GiBUU models both have better agreements with data while NuWro LFG has overall disagreement in cross section.

The NuWro models include nuclear effects such as Pauli blocking and the Coulomb potential. The NuWro SF model, in particular, includes an effective potential simulating the optical potential. The effective potential is validated against electron scattering data on targets including $^{16}$O [90], a nucleus similar to $^{12}$C [3]. The NuWro LFG has larger disagreement with the data. However, the average Fermi motion of the typical LFG models produces a narrower width in the QE peak than that of the Fermi momentum in regular Fermi gas models. This produces a narrower peak more suggestive of the data.

The NEUT SF describes the QE peak location well, while the LFG shifts the peak location by more than 1σ. In fact NEUT SF describes both $\delta p_{Tx}$ and $\delta p_{Ty}$ very well near the peak regions. Unlike the NuWro variant, NEUT LFG predicts wider QE width in $\delta p_{Ty}$ while at the same time produces width in $\delta p_{Tx}$ comparable to that of the data.
GiBUU models the initial state nucleons with a local Thomas-Fermi approximation, and the nucleons are bound in a mean-field potential, where Pauli blocking is naturally simulated. The final state particles propagating through the nuclear medium are subject to a scalar potential that usually depend on both the nucleon momentum and nuclear density [145]. These features of GiBUU do not contribute to an especially superior description of the QE peak. Unrelated to the description of the peak, the tail distributions of the single-TKI quantities are sensitive especially to the 2p2h component and pion production followed by pion absorption with proton knockout. With a lower proton threshold than this analysis, it could include significant amounts of QE events followed by FSI. GiBUU seems to be quite adept at predicting three of the four tails of these signal distributions, while the other generators systematically overestimate these regions.

We investigate the agreement of the $\delta p_{Ty}$ measurement with model predictions using a weighted average, $\langle \delta p_{Ty} \rangle$, defined as

$$\langle \delta p_{Ty} \rangle = \frac{\sum_i \sigma_i \delta p_{Ty,i}}{\sum_j \sigma_j},$$

(4.25)

$$V = \frac{\sum_{i,j} \delta p_{Ty,i} C_{ij} \delta p_{Ty,j}}{(\sum_k \sigma_k)^2},$$

(4.26)

where $\sigma_i$ and $\delta p_{Ty,i}$ are the cross section and position in the $i$-th $\delta p_{Ty}$ bin, $i, j$ span over the summed range. The calculation of the variance $V$ takes into account the covariance matrix $C_{ij}$, which contains the correlated errors between the $i$th and $j$th bins. The covariance matrices for $\delta p_{Tx}$, $\delta p_{Ty}$ and the variables reported in Ref [130] are available as digital data release.

The computation of $\langle \delta p_{Ty} \rangle$ is sensitive to the range selected due to the underlying non-QE contribution. The $(-0.20, 0.20)$ GeV/$c$ momentum range is chosen because
it is dominated by the QE events. The results are summarized in Fig. 4.12.

For each model, a p-value is calculated under the assumption of normally distributed uncertainties on the data. The average peak positions of MnvGENIE-v1.0.1 lie outside $1\sigma$ uncertainty range of the data. Measurable shifts to larger $\langle \delta p_{TY} \rangle$ are observed when interaction energy corrections are applied. The shifts are on the order of 15 to 20 MeV/c, consistent with corrections made to the underlying model. The measurements disfavor the default GENIE removal energy implementation, but does not distinguish between the nuclear potential corrections. Among the models NuWro SF, NEUT SF and GIBUU models are comparable to the data average, while NuWro and NEUT LFGs have larger disagreement with the data. Between them, NEUT LFG peaks outside the measurement uncertainties.

Next, we calculate $\chi^2$ distributions in four consecutive, disjoint $\delta p_{TY}$ ranges dominated by QE interactions to illustrate the mismodelling in the MnvGENIE-v1.0.1
Figure 4.11: Model comparisons for NEUT SF and NEUT LFG. The $\delta p_{Ty}$ distribution in NEUT SF describes the data peak well, while NEUT LFG over-predicts the left side of the peak, leading to a wider peak similar to GENIE.

Figure 4.12: $\langle \delta p_{Ty} \rangle$ calculated from the differential cross section within $|\delta p_{Ty}| < 0.20$ GeV/c. The p-value is the probability, assuming normal distribution, that the observed result would have been produced by change from this model.
simulations. Table 4.3 summarizes the results. The $\chi^2$ in $\delta p_{Ty}$ for minvGENIE-v1.0.1 is not symmetric about 0, where the falling side $(0, 0.2)$ GeV/c, with $\chi^2 = 13.7$, is in much better agreement with the data than the rising side $(-0.2, 0)$ GeV/c with $\chi^2 = 89.0$.

The corrections for minvGENIE-v1.0.1 reduce the model asymmetry, bringing the $\chi^2$ at the left edge from 89.0 to the order of 30. The $\chi^2$ for the right edge increases from 13.7 to 18.8 and 30.2 between corrections 1 and 2. The total $\chi^2$ between $(-0.2, 0.2)$ GeV/c is reduced by more than 50% after the corrections are applied. The overall $\chi^2$s for minvGENIE-v1.0.1 is 72.5, while both its corrections are 111 for 28 degrees of freedom.

Other Fermi gas-based models, such as NuWro LFG and neut LFG, in general have better $\chi^2$ than the GENIE variations. The NuWro LFG $\chi^2$ in the edges are more consistent with each other, at 25.1 and 15.5 respectively. The neut LFG, on the other hand, seems to suffer from model asymmetry similar to minvGENIE-v1.0.1, but the cause might be due to a systematic excess in cross section predicted in the negative tail of the $\delta p_{Ty}$ distribution, as shown in Fig. 4.11. The tail is dominated by non-QE interactions. In contrast, spectral function models and GiBUU predict $\delta p_{Ty}$ very well.

We also show the $\chi^2$ distributions for $\delta p_{Tx}$ in Table 4.4. Across all of the models we observe bias in the $\chi^2$ as a result of the asymmetry in data that we previously characterized by the measurement of $A_{LR}$ in Eqn. 4.24.
4.7 Summary and outlook

The variables $\delta p_{T_x}$ and $\delta p_{T_y}$ are measured on the CH target in MINER$\nu$A. We expect $\delta p_{T_x}$ to be sensitive to the Fermi momentum in QE and there is tension between data and MC. The data is narrower than the GENIE model, as is true of most models other than a simple Fermi gas. The measurement also shows a statistically marginal proton asymmetry in $\delta p_{T_x}$ of $-0.05 \pm 0.02$. This asymmetry, if truly non-zero, might be attributed to pion absorption events included in the signal. No model in current event generators predicts an asymmetry. Future measurements could verify the presence of this asymmetry.

The observable $\delta p_{T_y}$ shows sensitivity to the interaction energy implemented in nuclear models. In particular, the measurement, which is based on GENIE, disfavors the default GENIE implementation of the interaction energy on Carbon. This implementation lacks the excitation energy while subtracts an extra Moniz interaction energy from the final state proton. The average peak positions between GENIE and data differ by more than $1.5\sigma$. Approximate corrections accounting for the excitation energy and Moniz interaction energy bring the average peak position within $1\sigma$ of the data. This measurement is not precise enough to distinguish the more subtle nuclear effects such as the optical potential and the Coulomb potential. To first order more statistics could reduce the overall uncertainties in the distributions. Further improvements in the overall uncertainties need to come from better constrained flux, detector response and signal model, especially in the modelling of pion absorption in the nucleus.

We have compared different Monte Carlo models with respect to $\delta p_{T_x}$ and $\delta p_{T_y}$. The measurements are based on the MnvGENIE-v1.0.1 tune of GENIE, which re-
moves the elastic FSI components in GENIE on top of the mvngenie-v1 base tune. This modification subsequently impacts the single-TKI measurements performed in Ref [130]. The elastic FSI is discussed in Appendix 4.9. The Supplemental Material to this paper contains an update to the single-TKI results presented in Ref [130] based on this modification.

Future MINERνA analysis using the medium energy [146] dataset will benefit from higher statistics, which will enable examination of correlations between $\delta p_{Tx}$, $\delta p_{Ty}$ and other variables. In particular, probing the correlation of the asymmetry in $\delta p_{Tx}$ with other variables may shed light on its origin. Other targets in MINERνA and future liquid argon experiments could make measurements of $(\delta p_{Tx}, \delta p_{Ty})$, and the single-TKI variables in general, to test models on other nuclei.

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### Table 4.3: $\delta p_T y$: $\chi^2$ comparisons, POT normalized.

<table>
<thead>
<tr>
<th>POT Normalized</th>
<th>$-0.2 \sim -0.1$ GeV</th>
<th>$-0.1 \sim 0.0$ GeV</th>
<th>$0.0 \sim 0.1$ GeV</th>
<th>$0.1 \sim 0.2$ GeV</th>
<th>$-0.2 \sim 0.2$ GeV</th>
<th>$-0.7 \sim 0.7$ GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>GENIE Nominal</td>
<td>41.1/2</td>
<td>19.0/2</td>
<td>0.743/2</td>
<td>13.5/2</td>
<td>52.9/8</td>
<td>69.5/28</td>
</tr>
<tr>
<td>MINVGENIE-v1.0.1</td>
<td>89.0/2</td>
<td>38.9/2</td>
<td>0.184/2</td>
<td>13.7/2</td>
<td>100/8</td>
<td>72.5/28</td>
</tr>
<tr>
<td>1: $U_{opt}$ only</td>
<td>32.4/2</td>
<td>22.5/2</td>
<td>2.73/2</td>
<td>18.8/2</td>
<td>38.2/8</td>
<td>111/28</td>
</tr>
<tr>
<td>2: $U_{opt}$ and $V_{eff}$</td>
<td>27.7/2</td>
<td>19.6/2</td>
<td>3.71/2</td>
<td>30.2/2</td>
<td>45.6/8</td>
<td>111/28</td>
</tr>
<tr>
<td>NuWaRO LFG</td>
<td>25.1/2</td>
<td>159/2</td>
<td>130/2</td>
<td>15.5/2</td>
<td>50.7/8</td>
<td>131/28</td>
</tr>
<tr>
<td>NuWaRO SF</td>
<td>10.6/2</td>
<td>8.87/2</td>
<td>1.46/2</td>
<td>0.296/2</td>
<td>6.66/8</td>
<td>60.0/28</td>
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<tr>
<td>NEUT 5.40 LFG</td>
<td>43.6/2</td>
<td>113/2</td>
<td>82.6/2</td>
<td>0.842/2</td>
<td>52.6/8</td>
<td>75.9/28</td>
</tr>
<tr>
<td>NEUT 5.40 SF</td>
<td>7.31/2</td>
<td>9.03/2</td>
<td>0.397/2</td>
<td>0.302/2</td>
<td>4.41/8</td>
<td>54.3/28</td>
</tr>
<tr>
<td>GiBUU</td>
<td>1.50/2</td>
<td>3.81/2</td>
<td>6.85/2</td>
<td>6.04/2</td>
<td>7.70/8</td>
<td>45.0/28</td>
</tr>
</tbody>
</table>

### Table 4.4: $\delta p_T x$: $\chi^2$ comparisons, POT normalized.

<table>
<thead>
<tr>
<th>POT Normalized</th>
<th>$-0.2 \sim -0.1$ GeV</th>
<th>$-0.1 \sim 0.0$ GeV</th>
<th>$0.0 \sim 0.1$ GeV</th>
<th>$0.1 \sim 0.2$ GeV</th>
<th>$-0.2 \sim 0.2$ GeV</th>
<th>$-0.7 \sim 0.7$ GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>GENIE Nominal</td>
<td>26.0/2</td>
<td>31.6/2</td>
<td>3.40/2</td>
<td>4.03/2</td>
<td>26.4/8</td>
<td>69.5/28</td>
</tr>
<tr>
<td>MINVGENIE-v1.0.1</td>
<td>38.6/2</td>
<td>40.4/2</td>
<td>4.00/2</td>
<td>9.11/2</td>
<td>34.5/8</td>
<td>67.2/28</td>
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<tr>
<td>01: $U_{opt}$ only</td>
<td>36.3/2</td>
<td>35.2/2</td>
<td>4.02/2</td>
<td>9.40/2</td>
<td>35.0/8</td>
<td>67.4/28</td>
</tr>
<tr>
<td>02: $U_{opt}$ and $V_{eff}$</td>
<td>36.2/2</td>
<td>34.4/2</td>
<td>4.03/2</td>
<td>9.55/2</td>
<td>35.2/8</td>
<td>67.8/28</td>
</tr>
<tr>
<td>NuWaRO LFG</td>
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<td>85.5/2</td>
<td>31.4/2</td>
<td>4.72/2</td>
<td>58.7/8</td>
<td>132/28</td>
</tr>
<tr>
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<td>20.1/2</td>
<td>0.831/2</td>
<td>1.48/2</td>
<td>16.6/8</td>
<td>63/28</td>
</tr>
<tr>
<td>NEUT 5.40 LFG</td>
<td>21.4/2</td>
<td>73.3/2</td>
<td>19.0/2</td>
<td>5.82/2</td>
<td>43.5/8</td>
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<tr>
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<tr>
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<td>11.7/2</td>
<td>7.69/2</td>
<td>1.27/2</td>
<td>11.9/8</td>
<td>40.6/28</td>
</tr>
</tbody>
</table>
The following sections for the appendix of Ref. [1].

4.8 Derivations of GENIE corrections

The purpose of this correction is to modify the prediction of the GENIE event generator for a different value of $E_N^N$. In general, this correction could modify both the energy and three-momentum transferred to the nucleus, but have the freedom to pick some quantity which should be conserved event-by-event in this correction. We choose the magnitude of the three momentum transfer, $q_3$. Changes to $q_0$, $Q^2$, and angles and energies of the final state muon and proton follow. Denote the change in $q_0$ to be

$$\tau = E_N^N - |U_{opt}[(k + q_3)^2]| + |V_{eff}|, \quad (4.27)$$

and let

$$M'_N = M_N - S^N - \frac{\langle k^2 \rangle}{2M^2_{A-1}}. \quad (4.28)$$

Then the energy conservation at the vertex in GENIE is

$$q_{0_{GENIE}} + M'_N = \sqrt{(k + q_3)^2 + M^2_{P}}, \quad (4.29)$$

comparing to Eq.(4.7), we obtain

$$q_0 \approx q_{0_{GENIE}} + \tau. \quad (4.30)$$

The difference in energy transfer manifests on the outgoing muon energy:

$$E_\nu - E_\mu = E_\nu - E^_{\mu_{GENIE}} + \tau. \quad (4.31)$$
we obtain the energy correction to the GENIE muon:

\[ E_\mu = E_\mu^{\text{GENIE}} - \tau \]  

(4.32)

The outgoing proton energy in GENIE is

\[ E_P^{\text{GENIE}} = \sqrt{(k + q_3)^2 + M_P^2 - \Delta_{\text{nucleus}}^{\text{GENIE}}}. \]  

(4.33)

Comparing to the right hand side of Eq.(4.7), we obtain

\[ E_P = E_P^{\text{GENIE}} + \Delta_{\text{nucleus}}^{\text{GENIE}} - |U_{\text{opt}}| + |V_{\text{eff}}|. \]  

(4.34)

As noted above, this correction conserves energy, and assumes \(|q_3|\) is constant. The fractional change to the \(Q^2\) of the system is approximately \(\Delta Q^2/Q^2 = \tau/M_N\). For our sample, \(\tau \approx 10\text{MeV}\), produces a 1\% shift in \(Q^2\), which causes changes to the hard scattering cross-section \(\lesssim 1\%\).

We can also evaluate how changes in angle of the muon and the proton that are neglected in the GENIE correction would affect the prediction. The muon momentum before and after the correction are:

\[
P_\mu = p_\mu \begin{pmatrix} 0 \\ \sin (\theta) \\ \cos (\theta) \end{pmatrix},
\]

(4.35)

\[
P'_\mu = (p_\mu - \tau) \begin{pmatrix} 0 \\ \sin (\theta + \delta) \\ \cos (\theta + \delta) \end{pmatrix}.
\]

(4.36)
Solving the equation $|p_\nu - p_\mu|^2 = |p_\nu - p_\mu'|^2$ for $\delta$ to first order in $\tau$, we have:

$$\delta \approx \tau \left( \frac{1}{E_\nu \sin(\theta)} - \frac{1}{p_\mu \tan(\theta)} \right).$$

Note that the effect on the angle could become significant at small $\theta$, but in this region $E_\nu - E_\mu$ becomes small, and in this region the recoiling protons in quasielastic events are also soft that such they do not enter our sample. For our events, $\delta \lesssim 1.5$ mrad.

For an interaction, the $p_\mu$ and $p_P$ are balanced at the vertex. Changing the $\theta$ must elicit a compensating change in the proton angle to conserve the momentum. In our correction, we neglect the small changes of angles above. This introduces a very small error in the calculation of $\delta p_{Ty}$ for events that pass our selections, particularly the proton momentum cut. This error decreases with $Q^2$ and is at most 0.25 MeV for $Q^2 = 0.2$ GeV$^2$. Therefore the simplifying assumption in our modification of GENIE that the muon and proton angles do not change is justified.

## 4.9 GENIE elastic FSI simulation

This section will discuss the elastic FSI prediction and the fixes to it in more detail.

The prediction from mnvGENIE-v1 in $\delta p_{Tx}$ has three distinct regions shown in Fig. 4.13: a non-CCQE tail beyond $|\delta p_{Tx}| \gtrsim 0.2$ GeV/$c$, a no FSI CCQE dominated region in $0.2 \gtrsim |\delta p_{Tx}| \gtrsim 0.1$ GeV/$c$, which reflects the Fermi momentum, and an elastic FSI peak at $|\delta p_{Tx}| \lesssim 0.1$ GeV/$c$. The GENIE elastic FSI is sharply peaked and much narrower than the underlying Fermi gas distribution. Since the protons in the elastic FSI peak follow the no FSI distribution before the FSI simulation, we
Figure 4.13: GENIE FSI modes breakdown for CCQE events. The non-interacting fraction is symmetric and preserves the Bodek-Ritchie Distribution while the GENIE elastic FSI appears accelerated with respect to the transverse momentum transfer $q_T$.

Figure 4.14: Comparison of the effect of fixing the GENIE code and comparing key QE proton distributions. Left: Proton acceleration showing the old code produced less than 2 MeV shift. Middle: the small energy shift has negligible effect on the momentum distribution. Right: a major distortion of the angle distribution is what affects the single-TKI analyses; the correct angle distribution is similar to protons (not shown) which had no FSI. Before the fix, this and other distributions based on proton angle such as Fig. 4.13 are too narrowly peaked. [147]
expect the width of the elastic FSI distribution to be at least as large as that of the
no FSI distribution.

Hints of the unphysical nature of the angular distribution already appeared in
the original single-TKI analysis reported in Ref [130]. MINER$\nu$A uses the default
GENIE configuration of version 2.12 which uses the “hA” model for FSI. In this
model, every nucleon experiences exactly one of the following fates: 1) no FSI, 2)
charge exchange with single nucleon knockout, 3) elastic hadron+nucleus scattering,
4) inelastic single nucleon knockout, 5) multi-nucleon knockout (including pion ab-
sorption) and 8) pion production. An advantage of this model is that a reweighting
technique can be used to modify the relative mix of fates without fully regenerating
the Monte Carlo samples. This is convenient for studying FSI systematic effects
with an analysis, similar to the existing FSI uncertainties available with the GENIE
hA model.

The routine used to calculate all FSI reactions involving a two body scatter
contains (in GENIE versions 2.6 to version 3.0.6) a mistake that affects hA fates
“2” and “4” (nucleon knockout, with and without charge exchange) and fate “3”
(elastic hadron nucleus scattering) for both protons and pions. Fate “3”, combined
with quasielastic events and single-TKI variables, create the largest in observable
distributions[147].

The primary effect is on the angular distribution of the scattered hadrons. In
the QE case the original code causes too few of the most highly-transverse protons,
which have low efficiency to be tracked in the MINER$\nu$A planar design. It also
produces a population, especially of QE events, with a very narrow angle distribu-
tion, and in quantities derived from those angular distributions, like many of the single-TKI observables. The angular distribution relative to the lepton and other hadrons are separately affected. This combination affects the predicted distributions presented in Ref [130] in multiple ways. In addition, the resulting hadrons pick up an acceleration of up to 2MeV. This is smaller than most hadronic energy uncertainties and has negligible role in selection or calorimetry. Instead it appears as an unphysical population in all single-TKI populations in Ref [130]. The largest effects of the distorted input model, after performing the iterative unfolding procedure, are on the acceleration angle and the coplanarity angle.

The study reported in Ref [147] suggests reweighting up the no FSI fate and removing the elastic fate contributions will sufficiently mimic the proton distributions in a fixed code without having to regenerate all MC. Figure 4.14 shows the effect of fixing the GENIE code and comparing key QE proton distributions. The reason is that the intended elastic scattering angle for protons and neutrons is always small: 90% would be less than 8°. For this analysis, the weights are only applied to GENIE quasielastic events; the distortion of angles for non-quasielastic events with multiple hadrons has a small effect on these distributions.

Figure 4.15 and 4.16 shows $\delta\phi_T$ and $p_n$ respectively. The left plot shows the comparison between data extracted using the mngenie-v1 (Old Data) and the mngenie-v1.0.1 (Updated Data). The right plot compares the two MC models. All model distributions are modified significantly, but the extracted cross section shifts are only significant in the first bins of $\delta\phi_T$ and $p_n$.

Table 4.5 compares the old and updated data to the mngenie-v1.0.1 model. The two data extractions are consistent within 1 unit of reduced $\chi^2$. The two data for $\delta\phi_T$ and $p_n$ differ significantly only in the first bin, but the effect on $\chi^2$ is small.
Figure 4.15: $\delta \phi_T$ is the angular projection of proton transverse momentum. The original FSI code produces a peak at 0 (rad) indicating protons were being produced in the reaction plane more often than Fermi-motion would give. The first cross section data point is the only one that shifted by more than 2$\sigma$.

Table 4.5: $\chi^2$ of the old and updated data compared to MnvGENIE-v1.0.1

<table>
<thead>
<tr>
<th>Variables</th>
<th>Old Data</th>
<th>Updated Data</th>
<th>DOF</th>
<th>$\Delta$ in Reduced $\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_n$</td>
<td>103</td>
<td>96.8</td>
<td>26</td>
<td>0.24</td>
</tr>
<tr>
<td>$\delta \alpha_T$</td>
<td>25.6</td>
<td>26.4</td>
<td>13</td>
<td>-0.062</td>
</tr>
<tr>
<td>$\delta \phi_T$</td>
<td>100</td>
<td>77.1</td>
<td>24</td>
<td>0.95</td>
</tr>
<tr>
<td>$\delta p_T$</td>
<td>48.1</td>
<td>30.0</td>
<td>26</td>
<td>0.70</td>
</tr>
</tbody>
</table>
Figure 4.16: The inferred initial neutron momentum, $p_n$, extracted before and after the elastic FSI reweight. Only the first bin differ more than 2σ.
The Supplemental Material to this paper contains an update to the single-TKI results presented in Ref [130] based on this modification.

Citation of the new cross sections should include this paper and the original paper [130] describing the full method.
Chapter 5

Measuring Anti-neutrino

Charged-Current Quasielastic Cross Section on Hydrogen and Extracting the Axial Form Factor

This analysis measures the charged current quasielastic (CCQE) cross section between the scattering of antineutrino on the hydrogen nucleus in the detector. Specifically, the reaction of interest is

\[ \bar{\nu}_\mu + p \rightarrow \mu^+ + n. \]  \hspace{1cm} (5.1)

As the proton in hydrogen is not bound in a composite nucleus, there is no Fermi motion, binding energy, or the final state interactions to distort final state content. The four-momentum transfer is well defined because the scattering on the
hydrogen nucleus is a two-body process. Equation (1.41) and Eq. (1.42) calculates the neutrino energy and $Q^2$ with the binding energy term set to zero.

An unconstrained system involving the incoming neutrino, the final state muon, and the final state neutron contains nine degrees of freedom (DOF), three from each particle due to its momentum. In the antineutrino CCQE hydrogen reaction, however, there are only two DOF. The neutrino flux and its incoming direction specify the neutrino degree of freedom. The neutron momentum can be unambiguously determined from the neutrino and muon kinematics. Of the three muon DOF, one corresponds to the rotational symmetry around the neutrino direction that we typically integrate over, leaving behind only two DOF – the magnitude of muon momentum $p_l$ and the polar angle $\theta_l$ to the neutrino axis.

Therefore, we could completely specify the interacting system with the charged lepton’s energy and the opening angle:

\[
E_\nu = \frac{M_n^2 - M_p^2 - m_l^2 + 2M_p E_l}{2(M_p - E_l + p_l \cos \theta_l)}, \quad (5.2)
\]

\[
Q^2 = 2E_\nu (E_l - p_l \cos \theta_l) - m_l^2. \quad (5.3)
\]

We could also rewrite Eq.(5.2) and Eq.(5.3) in terms of $E_\nu$ and $Q^2$ to establish the equivalency between the two variable systems

\[
E_l = \frac{2E_\nu M_p - M_n^2 + M_p^2 - Q^2}{2M_p}, \quad (5.4)
\]

\[
\cos \theta_l = \frac{M_p (2E_\nu^2 - m_l^2 - Q^2) - E_\nu (M_n^2 - M_p^2 + Q^2)}{E_\nu \sqrt{(M_n^2 - M_p^2 + Q^2 - 2M_p (E_\nu - m_l)) (M_n^2 - M_p^2 + Q^2 - 2M_p (E_\nu + m_l))}}. \quad (5.5)
\]
Finally, our neutrino energy distribution has been constrained by external measurements so that $Q^2$ is the only variable needed to understand the system. (Refer to Sec 5.7.2).

The outgoing neutron’s momentum can be derived unambiguously. The longitudinal component along the neutrino direction is

$$p_{n}^{L} = \frac{M_{n}^{2} + p_{T}^{\mu 2} - (p_{L}^{\mu} - E_{\mu} + M_{p})^{2}}{2(p_{L}^{\mu} - E_{\mu} + M_{p})^{2}}$$

(5.6)

where $M_{n(p)}$ is the neutron (proton) mass, $p_{T}^{\mu}$ is the muon transverse momentum perpendicular to the neutrino direction, $p_{L}^{\mu}$ is the muon longitudinal momentum, and $E_{\mu}$ is the muon energy. The neutron and muon transverse momenta are balanced, adding up to 0:

$$p_{n}^{T} = -p_{T}^{\mu}.$$ 

(5.7)

The final neutron momentum under the hydrogen assumption is

$$p_{H} = (-p_{T,x}^{\mu}, -p_{T,y}^{\mu}, p_{L}^{n}),$$

(5.8)

when the neutrino points along the $z$-axis.

With both $E_{\nu}$ and $Q^2$ known, we could directly derive the nucleon axial form factor ($F_{A}$) from this reaction according to Eq.(2.1). $F_{A}$ contains information about the weak charge distribution and is also the first parameter used to generate the nuclear model in the simulation of neutrino-nucleus reaction.

Measurements of $F_{A}$ have been made on deuterium bubble chamber experiments [69–75]. Only one experiment[76] measured the $F_{A}$ in hydrogen bubble chamber, obtaining only 13 events. The measurement described in this chapter would signifi-
cantly expand the sample size.

The hydrogen atoms in the MINERνA detector are in a sea of carbon. Chemical analysis done on MINERνA’s scintillator strips show that hydrogen only accounts for 8.5% of their masses (see Sec. 3.3). The remaining nuclei are predominantly carbon with a trace amount of titanium and oxygen. The neutrino cross section scales with the number of targets. We expect approximately 5 reactions from carbon for each hydrogen reaction since hydrogen only contributes to 16% of the proton content in the scintillator. To separate hydrogen events from the carbon background, we must fully utilize a crucial difference between them – nuclear effects. Fermi motion from carbon imparts the bound proton with non-zero initial momentum. The neutron at the interaction vertex inherits the initial state momentum,

\[ \mathbf{p}_n = \mathbf{k} + \mathbf{q}, \]

where \( \mathbf{k} \) is the proton’s Fermi motion and \( \mathbf{q} \) is the 3-momentum transferred. For the free proton in the hydrogen, \( \mathbf{k} = \mathbf{0} \).

Additionally, final state interactions (FSI) could alter the outgoing neutron’s direction or remove it from the final state altogether. The strategy to reduce the carbon background requires reconstructing the neutron in the detector and then comparing its direction to the expected neutron momentum \( \mathbf{p}_H \) under the hydrogen hypothesis.

As discussed in Section 3.5.2, however, neutrons could exit our detector without interacting at all. Only 70% of the interacting neutrons at the highest kinetic energy can be reconstructed with 3D information. Therefore, we drop any tracking requirement on the reconstructed neutron candidate in favor of cuts that pre-select
what we refer to as the QELike sample.

5.1 Signal Definition

5.1.1 QELike Signal Definition

MINERνA’s very first CCQE papers[64, 65] have used the true CCQE definitions in its signal selection. The advantage of the selection lies in the simplicity of the physical process we measure. However, the disadvantage is FSI that often alters the final state particle contents. In this definition, CCQE events that produced pions through FSI are also a signal process. The analyses thus must rely solely on the FSI model for background prediction. MINERνA has since moved away from the true CCQE signal definitions in favor of the quasielastic-like (QELike) signal definition[130, 135, 148]. The QELike signal definition relies solely on the observed final states in the detector. For example, resonant and DIS events that, through FSI, produce only nucleons in the final states are also part of the QELike signal definition. CCQE events that produced pions through FSI are part of the background.

In general, the QELike signal definition looks for a muon and any number of nucleons (proton and neutron) in the final state. While this definition favors QE event type, 2p2h with 2 nucleons in the final states is part of the QELike definition. No mesons, such as pions and kaons, should be present in the final state. RES and DIS events could end up in the QELike signal definition if their primary mesons are absorbed in the nucleus through FSI.

The QELike signal definition has a slightly different meaning in the antineutrino case. Because we require only a muon track in the detector, any event with energetic protons greater than 120 MeV in kinetic energy is not part of the sample (our
5.1 Signal Definition

tracking threshold is 110 MeV). Therefore, the QELike definition in the antineutrino case states that no proton could have kinetic energy greater than 120 MeV.

The QELike definition of signal events allows MINERνA’s results to be relevant for other experiments with minimal model dependence and ambiguity. For example, the T2K experiment measures neutrino cross sections using the \textsc{neut}\cite{86} event generator, which almost certainly has a different FSI implementation than \textsc{genie}. The same proton kinematics could produce different final state contents. The T2K experiment also operates at a lower energy neutrino beam, resulting in very different kinematic distributions for the final state particles, affecting how FSI plays out. Therefore, a cross section result based on the true CCQE definition will be different depending on the generator used to predict the background (non-CCQE events could produce nucleons only final states through FSI). By looking only at the detector’s final state contents, MINERνA’s results are model-independent.

5.1.2 Hydrogen CCQE Signal Definition

The definition of CCQE on the reaction $\bar{\nu}_\mu + H \rightarrow \mu^+ + n$ has no ambiguity, and it forms a subset of the QELike signals. Our task at hand is then to pick out this reaction from all the QELike events and measure its cross section as a function of $Q^2$. 
5.2 Data and Simulation Samples

5.2.1 Run Periods

For the antineutrino hydrogen analysis, we use data from the Medium Energy (ME) run in the RHC mode from June 29, 2016, to July 7, 2017. Table 5.1 summarizes the data-taking periods and the total protons on target (POT) accumulated. In total, we have acquired $1.124 \times 10^{21}$ POT. We also generated MC samples corresponding to $4.96 \times 10^{21}$ POT.

<table>
<thead>
<tr>
<th>Playlist</th>
<th>Run Periods</th>
<th>Runs and Subruns</th>
<th>Data POT</th>
<th>MC POT</th>
</tr>
</thead>
<tbody>
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<td>ME-5A</td>
<td>06/29/2016-07/29/2016</td>
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<td>5.449E19</td>
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<tr>
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<td>Total</td>
<td></td>
<td></td>
<td>1.124E21</td>
<td>4.962E21</td>
</tr>
</tbody>
</table>

Table 5.1: Data run period and the accumulated POT
Figure 5.1: Figure shows the weight applied to the no FSI component of the final state interaction in GENIE’s INTRANUKE-hA code, as a function of nucleon’s kinetic energy.

5.3 Tuning genie

Further work has been done to analyze the FSI error’s effect in the ME era and extend the reweighting mechanism to neutrons.

Figure 5.1 shows the size of the reweight as a function of the proton’s and neutron’s kinetic energy on carbon. We could reconstruct proton tracks when the protons have at least 110 MeV in kinetic energy. While we could identify neutron candidates at any kinetic energy, the likelihood of finding one increases with the neutron’s kinetic energy (Fig. 3.18). From the work described in Chapter 4, we know that the RFG model used by GENIE is not the ideal model to describe the initial state particle distributions in the nucleus. NuWro’s spectral function model
5.3 Tuning genie

Figure 5.2: Initial nucleon momentum distribution for the GENIE RFG model, the NuWro SF and LFG models. The RFG model nominally has an abrupt cut-off at the Fermi momentum $K_F$ of carbon, but due to the Bodek-Ritchie tail, the initial nucleon momentum is extended beyond $K_F$. The NuWro LFG model is cut off after 250 MeV. The LFG is a variation of the Fermi gas model. The NuWro SF model has a smooth distribution extending far beyond $K_F$.

does a much better job at describing carbon in the neutrino mode. As the physics describing the initial state neutrons also describes the protons, we build a better carbon model by reweighting GENIE’s RFG model into NuWro’s SF model.

Figure 5.2 shows the initial state nucleon momentum distributions from GENIE RFG model, NuWro LFG and SF models. Nominally, the RFG model should drop to zero sharply at the Fermi momentum $K_F = 220$ MeV in carbon. However, GENIE
added an extended momentum distribution from the Bodek-Ritchie tail\cite{99} that works to our advantage. While the original purpose of the Bodek-Ritchie tail was to describe transverse-enhanced events, primarily from $2p2h$, GENIE used the Bodek-Ritchie tail to generate events on single nucleons. The nuwro LFG model uses different $K_F$ values depending on where the interaction happens inside the nucleus, but it has a cut-off as well. For nuwro SF, we have a smooth and continuous distribution that coincides with the Bodek-Ritchie tail after $350$ MeV. The fact that both GENIE's RFG model and the nuwro SF model predict events in the same range of initial state momentum allows us to reweigh the GENIE model.

Because we will measure the differential cross section in $Q^2$, we generate a 2D reweighing histogram with $Q^2$ in the $x$ axis and the initial nucleon momentum in the $y$ axis.

Figure 5.3 shows the double-differential cross section $d^2\sigma/dQ^2 dp_i$ in GENIE and nuwro SF respectively, where $p_i$ is the magnitude of the initial nucleon momentum. In both the GENIE RFG and nuwro SF, the initial nucleon is moving isotropically. Figure 5.4 shows the ratio of nuwro SF to GENIE in each of the $Q^2$-$p_i$ bin. We obtain the true $Q^2$ and $p_i$ information during the analysis run time and search for the bin corresponding to the particular $Q^2$ and $p_i$ combination. The value from the bin is the weight for that particular event.

Finally, a tune\cite{149} based on MINER$\nu$A's pion production data \cite{150–152} has been performed, resulting in the suppression of pion production at the low $Q^2$ region.

In summary, the MINER$\nu$A tunes applied to the nominal GENIE are the 1) long range correlation through the RPA tune\cite{137}, 2) reduction of nonresonant pion production\cite{139}, 3) inclusion of the Valencia $2p2h$ model\cite{138}, 4) data-driven tune on the Valencia $2p2h$ model\cite{107}, 5) low $Q^2$ suppression of resonant pion produc-
Figure 5.3: Double-differential QE cross section for carbon in $Q^2$ and the initial nucleon momentum $p_i$. (top) is GENIE RFG, (bottom) is NuWro SF.
Figure 5.4: The weight in each $Q^2-p_i$ bin is obtained by dividing the NuWro SF with GENIE RFG model. The weight is applied at run time when we have access to truth-level information on the $Q^2$ and $p_i$.

Collectively, tunes 1)-4) are called mnvGENIE-v1, the addition of 5) is mnvGENIE-v2. The elastic FSI and GENIE RFG reweights are reweighting mechanism specific to this analysis that would provide better consistency in carbon’s modeling.
Figure 5.5: Annotated event display with a muon track, a proton track, and untracked energies away from the vertex[154].

### 5.4 Quasielastic-Like Event Selection

#### 5.4.1 Basic Event Selection

Figure 5.5 shows an event display of a neutrino scattering. A muon comes out of the interaction vertex and passes into MINOS which measures its charge and momentum. A proton forms a track, and its identity can be estimated using one of the two proton PID methods described in Sec 3.5.1. The position of the vertex must be inside the tracker region and away from the edge of the hexagonal plane,

\[
\begin{align*}
5980 \text{ mm} & \leq z \leq 8422 \text{ mm}, \\
\text{Apothem} & = 850 \text{ mm},
\end{align*}
\]

where the apothem is the distance from the center of the detector towards the edge of the detector. Since we require the muon to pass into MINOS, we must place
additional requirement on the muon’s direction and momentum:

\[
\begin{align*}
\theta_\mu &< 20^\circ, \\
1.5 \text{ GeV} &< p_\mu < 20 \text{ GeV}.
\end{align*}
\]

(5.10)

Neutral particles and $\pi^0$ photons could deposit additional energy away from the vertex in the detector, and this energy is added to the non-vertex recoil energy. Besides summing over their energies, we also perform a "blobbing" reconstruction similar to the first stage of the neutron reconstruction algorithm to count the total numbers of isolated energy deposits with enough energy to span 3 views. This kind of detached blobs could come from a few sources. The primary hadron could inelastically scatter in the detector to produce secondary neutral particles that travel a distance before depositing energy. A primary neutron could scatter to form the deposits. A neutral pion ($\pi^0$) could decay in the nucleus into two photons. They will appear as isolated clusters in Fig. 5.5. The photons then produce a pair of electromagnetic showers that may be separable if they are far enough.

The final state content is typical of the neutrino QELike ($\nu + n \rightarrow \mu + p$) event selection as we expect a proton to be the primary final state nucleon. In the case of antineutrino, we must select events with only a muon track ($\bar{\nu} + p \rightarrow \mu^+ + n$) since we expect a neutron in the final state. The muon must have a positive charge as well.

Protons below the energy of 110 MeV[124] cannot create a track and may become part of the selection. Very low-energy charged pions may become part of the selection as well. In neutrino reactions (Eq. (1.36,1.37)), a stopped $\pi^+$ decays into a $\mu^+$ which then decays into a positron (Michel electron) 2.2 $\mu$s later. The positron will travel in
the detector to produce visible energy. We could look for Michel electrons near the vertex region 2.2\( \mu \)s after the main event for the definitive sign of a \( \pi^+ \). Identifying an electron from \( \pi^- \) in the antineutrino mode is ineffective because the \( \pi^- \) captures on nucleus before it decays.

Additionally, we limit the amount of non-muon recoil energy in the detector as a function of the \( Q^2 \). The recoil energy is the sum of energy from clusters that are 1) within a time window of \((-25, 30)\) ns around the muon track time, 2) not part of the muon track, and 3) outside a 100 mm circle around the vertex in each view. The functional form of the recoil energy cut is

\[
E_{\text{recoil max}} = \begin{cases} 
0.04 + (0.43/\text{GeV}^2)Q_{\text{QE}}^2 \text{ GeV}, & \text{if } Q_{\text{QE}}^2 < 0.3 \text{ GeV}^2 \\
0.08 + (0.3/\text{GeV}^2)Q_{\text{QE}}^2 \text{ GeV}, & \text{if } Q_{\text{QE}}^2 < 1.4 \text{ GeV}^2 \\
0.50 \text{ GeV thereafter} 
\end{cases}
\] (5.11)

The recoil energy cut is based on the one used in MINER\(\nu\)A's LE antineutrino QELike measurement[136], but with the maximum \( E_{\text{recoil}} \) in each \( Q^2 \) region increased by 50 MeV to increase the raw number of QELike events. Figure 5.6 illustrates the separation of the signal region with the recoil energy cut. The total QELike event fraction is \( \sim 60\% \) within this expanded cut region. For the LE result, the recoil energy cut is the only background rejection criteria. This analysis has applied new selection criteria to create sidebands useful for constraining the background fraction for the hydrogen analysis.

A neutron is the only final state particle in a hydrogen antineutrino CCQE, and we, therefore, reject any events with more than one three-view blobs in the tracker region. Next, we take all the clusters not associated with the muon, including those
in the three-view blob, and pass them to the neutron reconstruction algorithm described in Section 3.5.2. The neutron reconstruction algorithm uses a more inclusive set of clusters, including those in the vertex region. The resulting collection of blobs may contain more than one 3-view blobs. The main neutron candidate is the most energetic blob, and it must also be a 3D blob (3-view or 2-view). We reject an event if a leading 3D blob is not found. We then form a cylinder around the muon track and reject any event where the main candidate is closer than 100 mm to the muon. Lastly, we apply a neutron energy cut. The energy transferred to the proton in the hydrogen reaction is approximately

$$q_0 = \frac{Q^2}{2M_n}$$  \hspace{1cm} (5.12)
A neutron has definite kinetic energy \( q_0 \), and any main candidate with more energy than this cannot be a signal event.

Finally, the detector should not have any dead time when the interaction occurs.

A summary of the selections is shown in Table 5.2.

The number of 3-view blobs allows us to separate our samples into a QE-Rich sample \((N \leq 1)\) and QE-Deficit \((N > 1)\) regions.

<table>
<thead>
<tr>
<th>QELike Selection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Detector does not have dead time</td>
</tr>
<tr>
<td>event vertex inside the fiducial region</td>
</tr>
<tr>
<td>the primary vertex has only a muon track</td>
</tr>
<tr>
<td>muon is positively charged</td>
</tr>
<tr>
<td>muon satisfy phase-space constraint</td>
</tr>
<tr>
<td>the event passes the ( Q^2 ) dependent recoil energy cut</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Neutron Selection</th>
</tr>
</thead>
<tbody>
<tr>
<td>The leading neutron candidate is 3D</td>
</tr>
<tr>
<td>100 mm away from the muon track</td>
</tr>
<tr>
<td>Main candidate energy cut ( E &lt; \frac{Q^2}{2M_n} )</td>
</tr>
</tbody>
</table>

Table 5.2: Event selection criteria.
5.4.2 Angular Variables

Figure 5.7: Illustration of the perpendicular angle $\delta \theta_P$ and the reaction plane angle $\delta \theta_R$. The grey incoming beam is the neutrino which interacts with a nucleus target to produce an outgoing muon. We could calculate the outgoing neutron’s direction under the hydrogen hypothesis, denoted by the black arrow. The outgoing neutron travels a distance before interacting, creating a main neutron candidate. We for the reaction plane using the neutrino-muon vector, and we form a plane perpendicular to the reaction plane, intersecting the reaction plane along the predicted neutron vector. We calculate the angular separation of the measured neutron direction and the predicted direction in these planes. If we could reconstruct the final state particle’s momentum (i.e., proton), we could also calculate $\Delta p_R$ and $\Delta p_P$ using the same coordinate system.
Figure 5.8: Distributions of $\delta\theta_p$ (top) and $\delta\theta_R$ (bottom) in the QE-Rich sample. Before sideband fits.
Figure 5.9: Distributions of $\delta \theta_p$ (top) and $\delta \theta_R$ (bottom) in the QE-Deficit sample. Before sideband fits.
Figure 5.7 shows the set of the coordinate system defined by the neutrino-muon plane and the predicted neutron direction using the hydrogen hypothesis. We could calculate 2 angular variables in this coordinate system, the $\delta \theta_R$ and $\delta \theta_P$. We expect a hydrogen neutron’s $\delta \theta_P$ and $\delta \theta_R$ angles to be simultaneously small because the neutron must start in the expected direction. The uncertainties in the neutrino direction and muon resolutions are the only limiting factor on the predicted neutron direction. Neutrons originating from heavier elements, such as carbon, deviate from the predicted direction due to Fermi motion and FSI. This distinction allows us to separate the samples into a hydrogen-rich region centered around $\delta \theta_P = \delta \theta_R = 0$ and the carbon-dominated regions elsewhere.

Figure 5.8 shows the raw distributions of $\delta \theta_P$ and $\delta \theta_R$ after event selection in the QE-Rich sample. There are some crucial differences in the shape of these two angular distributions. First, we note that $\delta \theta_P$ is symmetric about zero because the variable is essentially tagging neutron candidates on both sides of the neutrino-muon reaction plane, similar to $\delta p_{T_x}$ described in Chapter 4. Events on carbon experience Fermi motion, which is isotropic and should distribute equally on both sides of the reaction plane. The $\delta \theta_R$ has a tail towards the negative end of the angle, which is defined as closer to the muon. Neutrons in the detector are more likely to be reconstructed if it is more forward-going inside the detector. Side-going neutrons travel through less material path length and may exit the detector without interaction.

Hydrogen events (red) mostly populate the region $\pm 10^\circ$ around the origins in both plots. The MC predicts 6945 hydrogen events. Non-hydrogen components of the QE (green) primarily consist of events on carbon. However, we do have other elements such as oxygen, nitrogen, aluminum, silicon, chlorine, and titanium from the coatings and epoxies, discussed in Sec 3.3. 2p2h events form the largest fraction
of non-QE events in the QELike event category. There is a small fraction of resonant pion production and DIS events. The $Q^2$-dependent neutron candidate energy cut is very effective at removing non-QE events. Finally, very few events with pions and other mesons have leaked into the QE-Rich sample.

The angular distributions in the QE-Deficit sample follow much of the same story, except we have an excess of events with pions in the final states. A large fraction in the QE-Deficit events is those producing $\pi^0$. $\pi^0$s are produced either through the primary interaction $\bar{\nu} + p \rightarrow n + \pi^0 + \mu^+$ or through the FSI. A $\pi^0$ will decay into two photons, and each of the photons could convert to an electron-positron pair in the detector, which we may observe. The QE-Deficit samples have more than one reconstructed neutron candidate outside the vertex region, which is the majority of the $\pi^0$ final state multiplicity. The primary $\pi^-$ production channels in the antineutrino mode are $\bar{\nu} + n(p) \rightarrow n(p) + \pi^- + \mu^+$. Both are capable of producing tracks disfavored by the single muon track requirement. $\pi^-$ events could become part of the selection if the charged pion reinteracted near the vertex and produce additional neutrons. When additional neutrons were not reconstructed, the event becomes part of the QE-Rich sample. There are pion events in this QE-Rich sample, but the majority of these are charged pion events.

5.4.3 Angular Sidebands

The non-hydrogen events dominate even at $\pm 10^\circ$ hydrogen-rich regions. At first glance, these angular variables are not very useful in separating hydrogen events from the rest of the QELike cohort.

However, we could utilize the correlations between the two angular variables in creating control regions.
5.4 Quasielastic-Like Event Selection

We separate the angular regions into

<table>
<thead>
<tr>
<th>Region</th>
<th>Selection</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$</td>
</tr>
<tr>
<td>1</td>
<td>$</td>
</tr>
<tr>
<td>2</td>
<td>$</td>
</tr>
<tr>
<td>3</td>
<td>$-40^\circ \leq \delta \theta_R &lt; -30^\circ$ and $</td>
</tr>
<tr>
<td>4</td>
<td>$-40^\circ \leq \delta \theta_R &lt; -30^\circ$ and $20^\circ &lt;</td>
</tr>
<tr>
<td>5</td>
<td>$-55^\circ \leq \delta \theta_R &lt; -40^\circ$ and $</td>
</tr>
<tr>
<td>99</td>
<td>Everything else.</td>
</tr>
</tbody>
</table>

Table 5.3: Angular regionings used to constrain non-signal background. Regions are defined in ascending region number and each region excludes the preceding region.

Figure 5.10 shows the ratio of CCQE hydrogen, QE carbon, 2p2h, and events with mesons in the final state to the full MC sample. Region 0 in the QE-Rich sample is the signal region, and all other regions are sidebands. Specifically, Region 1 and 2 are QE carbon-dominated regions. Region 3, 4, and 5 contain more fractions of 2p2h and RES. Only Regions 1, 5, and 99 in the QE-Rich sample and the full samples from the QE-Deficit regions participate in the background constrain. Regions 2, 3, and 4 are set to be validation regions. Results of the constrain will then be applied to Regions 2, 3, and 4 to ensure our fit results also work in those regions. Finally, we apply the constrain to Region 0 and extract the hydrogen event rate.

For each region, we plot the $Q^2$ distribution. The $Q^2$ bins have been chosen to coincide with MINERvA’s other QELike publications such as Ref. [148]. Table 5.4 shows the bin edges and bin center values.
Figure 5.10: 2D angular distributions of CCQE hydrogen, QELike QE carbon, 2p2h, and background leaked into the signal selection in the QE-Rich sample. The gray boxes and number indicates the regions defined for sideband constraints. The fractions from each angular bins sum to 1.

5.5 Background Constraint

5.5.1 Sidebands and Event Categories for Fitting

We choose the Region 1, 5, 99 in the QE-Rich sample and the full angular region of the QE-Deficit sample as sidebands to constrain the backgrounds.

The non-hydrogen categories participating in the fits are the QE, 2p2h, RES, DIS in the QELike category. Events with $1\pi^0$, $1\pi^\pm$, multiple pions, and no pions,
Figure 5.11: Event rate distributions in the QE-Rich regions.
Figure 5.12: Ratios of data and MC components to the tuned GENIE distribution. There is a consistent upward trend in data throughout the different QE-Rich regions. The error bar on the data has 2 segments, the inner segment contains the systematics error and the outer segment shows the statistical uncertainties. The systematics error is obtained when the data is divided by each individual universe, resulting in a range of possible ratios. The systematic variations from the CV are added up in quadrature.
Figure 5.13: Event rate distributions (top) and ratio (bottom) in the QE-Deficit sample.
Table 5.4: $Q^2$ bins used in analysis. Bin 0 and 19 also exist as the underflow and overflow bins. The predicted hydrogen event rate in the signal region is shown to the right, with the associated error.

but some other meson are in the QELikeNot (Not QELike) category.

### 5.5.2 Fitting Function

Our sidebands fitting mechanism involves the minimization of a Pearson’s $\chi^2$ test function

$$\chi^2 = \sum_{s,i} \left( \frac{\left(\sum_{C,i} w_{C,s,i} m_{C,S,i} - data_{S,i}\right)}{data_{S,i}} \right)^2 + \lambda \sum_{C} \sum_{j=1}^{N-2} (w_{C,j} + w_{C,j+2} - 2w_{C,j+1})^2.$$  

(5.13)

Where $m_{C,S,i}$ is the value of the MC in the category $C$, sideband $S$, in the $i$th fitting bin. In each sideband, we sum up the values of MC in each category. Each
category participating in the fit receives a weight $w_{C,i}$ in the particular fitting bin. We add a regularization term to the $\chi^2$ function to ensure the weights across the bins changes smoothly. $\lambda$ is the regularization strength and is a meta parameter we can tune. Finally, we add in an additional constrain on the $\chi^2$ function to make sure the weight of the single neutral pion events never fall below 0.9. The CV and each systematic universe are fitted individually to data.

We obtain the value of $\lambda$ by testing different $\lambda$ in the fit to Regions 1, 5, 99 with CV, and then applying the fit results to Regions 2, 3, and 4. For regions other than Region 0, we compute a $\chi^2$ using the Pearson’s $\chi^2$ statistics

$$\chi^2 = \sum_{i=6}^{15} \frac{(MC_i - Data_i)^2}{Data_i},$$

where $i$ is the bin number corresponding to $0.05 \leq Q_{QE}^2 \leq 1.60$ GeV$^2$ to avoid regions with low event rate. We average the reduced $\chi^2$ for regions 2, 3 and 4, weighted by the event rate, over each $\lambda$ and find the value of $\lambda$ with the minimum $\chi^2$. Figure 5.14 shows the $\chi^2$ as a function of $\lambda$. The minimum $\chi^2$ is first reached at $\lambda = 200$.

Figure 5.15 shows the weight for each category. We find that in general, QELike categories tend to be suppressed at lower reconstructed $Q^2$ ($Q_{QE}^2$) but increase at higher $Q_{QE}^2$. The trend is reversed for the QELikeNot categories. The tuning in the QELike shape is relatively flat despite the data favoring a stronger QE component at higher $Q_{QE}^2$. The 2p2h distribution sees a suppression at low $Q_{QE}^2$ and an enhancement at higher $Q_{QE}^2$. MINERνA’s 2p2h tune was initially extracted for neutrino mode[107], and the CV weight was applied as a mixture of the neutron-neutron (nn) and neutron-proton (np) initial state. For antineutrino, the proton-proton (pp)
initial state replaces the (nn) initial state, and the final state particles are a proton and neutron, respectively. Since we discard any event with a hadron track, the sample records fewer 2p2h from pp initial state events.

MINERνA’s 2p2h tune was extracted from the LE results and a more “inclusive” sample, in the sense that it did not account for the final state particle’s directions. Therefore, the 2p2h weight’s deviation from the MINERνA tune may also stem from the analysis’s sensitivity to the angular distribution of the final state particles. Finally, both QELike RES and events with pion final states only account for less than 20% of the events. Regions of large shifts in these categories are $Q_{\text{QE}}^2 < 0.1 \text{ GeV}^2$ and $Q_{\text{QE}}^2 > 1 \text{ GeV}^2$, which have low statistics.

We apply these weights to Region 0 in Fig. 5.11 and subtract non-QE hydrogen
Figure 5.15: The result of the fits in the QELike QE other, 2p2h, RES and not QELike single charged pion and single neutral pion category. The error band in each figure comes from the weight extracted from systematic universes, to be described in full detail in Sec 5.7.3. The lower and upper ends of $Q^2$ spectrum are sparsely populated, refer to Fig. 5.11 for the event rate in each bin.
5.5 Background Constraint

contributions. The fitted MC in these regions are consistent with data as seen in Fig. 5.12. Region 2, another QE-dominant region, is especially well constrained in the $Q^2_{QE} \in (0.1, 1) \text{GeV}^2$ range. Table 5.5 summarizes the $\chi^2$ in each region in the QE-Rich region for $\lambda = 200$.

<table>
<thead>
<tr>
<th>Region</th>
<th>$\chi^2/DOF$, $DOF = 9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.81</td>
</tr>
<tr>
<td>2</td>
<td>11.4</td>
</tr>
<tr>
<td>3</td>
<td>20.0</td>
</tr>
<tr>
<td>4</td>
<td>21.8</td>
</tr>
<tr>
<td>5</td>
<td>10.3</td>
</tr>
<tr>
<td>99</td>
<td>18.0</td>
</tr>
</tbody>
</table>

Table 5.5: Pearson’s $\chi^2$ with 9 degrees of freedom for $Q^2_{QE} > 0.05 \text{ GeV}^2$ and $Q^2_{QE} < 1.6 \text{ GeV}^2$, in regions 1 to 99 with $\lambda = 200$. Regions 1, 5, 99 are used in the fits while 2, 3 and 4 are used to assess the validity of the fits.

The event rate and compositions in each QE-Rich region are shown in Table 5.6. With comparable sample sizes, Regions 0, 1, and 2 are the most populated regions other than Region 99. QE carbon dominates in the three regions, with the 2p2h category forming the sub-leading background. Region 2 has a higher contribution from the 2p2h category at 17%, compared with 10% in Region 1 and 4% in Region 0. The consistency in Region 1 and 2 after the fit indicates that the background weights are valid in constraining the background shape and rate. Regions 3 and 4 are more susceptible to the statistical effects from a smaller MC sample size.
Table 5.6: Fraction of event types in each region.

<table>
<thead>
<tr>
<th>Region</th>
<th>N Events</th>
<th>QE Hydrogen</th>
<th>QE Carbon</th>
<th>2p2h</th>
<th>QELike RES</th>
<th>1π⁰</th>
<th>1π±</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>14603.2</td>
<td>0.34</td>
<td>0.57</td>
<td>0.04</td>
<td>0.01</td>
<td>0.01</td>
<td>0.02</td>
</tr>
<tr>
<td>1</td>
<td>16603.8</td>
<td>0.11</td>
<td>0.66</td>
<td>0.10</td>
<td>0.02</td>
<td>0.02</td>
<td>0.05</td>
</tr>
<tr>
<td>2</td>
<td>11888.9</td>
<td>0.07</td>
<td>0.55</td>
<td>0.17</td>
<td>0.04</td>
<td>0.03</td>
<td>0.08</td>
</tr>
<tr>
<td>3</td>
<td>1754.7</td>
<td>0.04</td>
<td>0.46</td>
<td>0.22</td>
<td>0.08</td>
<td>0.04</td>
<td>0.13</td>
</tr>
<tr>
<td>4</td>
<td>1519.3</td>
<td>0.04</td>
<td>0.43</td>
<td>0.30</td>
<td>0.05</td>
<td>0.05</td>
<td>0.09</td>
</tr>
<tr>
<td>5</td>
<td>3099.3</td>
<td>0.03</td>
<td>0.36</td>
<td>0.34</td>
<td>0.08</td>
<td>0.04</td>
<td>0.11</td>
</tr>
<tr>
<td>99</td>
<td>35290.3</td>
<td>0.04</td>
<td>0.34</td>
<td>0.28</td>
<td>0.07</td>
<td>0.04</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Figure 5.18 and Figure 5.19 shows the fitted distributions before and after background subtraction. The QE carbon category is still the dominant background. However, we can achieve nearly 1 : 1 hydrogen to carbon ratio at low $Q_{QE}^2$, but at $Q_{QE}^2$ larger than 0.3 GeV², carbon starts to dominate. Contributions from the non-QE background are highly suppressed. There is a trace of 2p2h across the entire spectrum of $Q_{QE}^2$. The non-QELike background primarily consists of single charged pions (light green) that did not make MINERνA’s tracking threshold. Collectively these background events make up to less than 15% of the total event rate. Therefore, Region 1 has an out-sized impact on constraining the background fraction because QE carbon is dominant in this region. Figure 5.20 shows $\delta \theta_P$ and $\delta \theta_R$ after applying the fits. The fitted MC distributions are in general agreement with the data. As the fit is extracted from regions symmetric around $\delta \theta_R = 0$ and $\delta \theta_P = 0$, asymmetries in each $|\delta \theta_R|$ and $|\delta \theta_P|$ are will still be present after the fit. There is a slight asymmetry in $\delta \theta_P$. Comparing to $\delta p_{TX}$ in Sec 4.6.1, we note that the asymmetry is in the opposite direction ($\delta \theta_P$ and $\delta p_{TX}$ have the same x axis).
Figure 5.16: Fitted event rate distributions in the QE-Rich regions.
Figure 5.17: Ratios of data and MC components to the tuned GENIE distribution after fitting.
Figure 5.18: The fitted event rate (top) and ratio to MC (bottom) in the signal region. The background is dominated by QE on carbon, which is constrained mostly by Region 1. The other QELike contributions and the non-QELike background contribute less than 15% to the total event rate between $Q^2_{QE} \in (0.1, 1)$ GeV$^2$. 
Figure 5.19: The signal only event rate (top) and ratio to MC (bottom).
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Figure 5.20: Event rate distributions (top) and the ratio to MC (bottom) of $\delta \theta_\nu$ (left) and $\delta \theta_R$ (right) in the QE-Rich sample. After sideband fits.

5.5.3 Testing the Model Prediction and Fit Mechanism on Neutrino Events

Another test of the model tuning and fit mechanism we have performed was to fit event distributions for the equivalent QELike events in the neutrino mode. Instead of searching for $\mu^+$, we require a $\mu^-$ in the final state. We drop all neutron selections in favor of reconstructed proton tracks. QE distribution is only dominant in regions 0 and 1 before being taken over by 2p2h, RES, and the QELikeNot contributions. The inefficiency in proton track identification allows more QELikeNot background to enter the selection. The angular resolution of the proton track is less affected
by proton rescattering since we have access to more hits from the proton ionizing the detector material, enabling us to average the effect of the rescatters. Neutrons, on the other hand, could rescatter silently before we observe them. This is why we have more QE events in the outlying angular regions in the antineutrino mode (Fig. 5.11).

Finally, we fit two model implementations. Both models contain the \textsc{mnvgenie-v2} and the elastic FSI weight, one of the models has the additional NuWro SF tune, described in Sec. 5.3, and the other uses the default RFG model. Figure 5.21 shows the neutrino mode event distributions in Regions 0, 1 and 2 with the NuWro SF model.

Comparing to the antineutrino mode (Fig. 5.16), QE carbon is no longer the dominant event type in region 2. 2p2h and QELikeNot contributions dominate in the lowest and highest $Q^2$ bins in the neutrino mode. This is also in contrast to the antineutrino mode, where QE carbon is dominant across the entire $Q^2$ spectrum. While antineutrino fits prioritize QE carbon, the neutrino mode fitting emphasizes all event types participating in the fit.

The 2p2h events picked up in Regions 0 and 1 are those with a forward going proton. For most of the 2p2h events, the proton will have more transverse momentum since the other particle counterbalances it. 2p2h events in Region 0, therefore, represent a small sub-sample with unconventional proton kinematics. MINERνA’s 2p2h tune[107], therefore, may not be able to guarantee the correct weighting for 2p2h events in the forward regions.

Figure 5.22 shows the ratio in the same regions after fitting. The NuWro SF model under-predicts data for $Q^2 < 0.3$ GeV$^2$ in Region 0. Compared with the fitted antineutrino mode distributions in Fig. 5.16 and Fig. 5.17, there is also an
5.5 Background Constraint

excess of 2p2h and QELikeNot background in this low $Q^2$ region. We could bring better data to MC agreement by increasing the 2p2h strength in Region 0 by 100%. Doing that, however, will cause the NuWro SF model to strongly over-predict the data in Regions 1 and 2. There are issues with both the 2p2h model and the underlying QELikeNot backgrounds in the low $Q^2$ regions. A possible cause of this behavior is GENIE mismodelled the angular distributions of the charged particles in the 2p2h and QELikeNot categories, such as the same Elastic FSI mistake that has been fixed for QE. In general, we may expect about 100% uncertainties in both the 2p2h and the QELikeNot models in neutrino mode.

In antineutrino mode the fits on the 2p2h and the QELikeNot categories yield $\sim 100\%$ uncertainties in the low $Q^2$ region, indicating the models are not well constrained. This is consistent with the observation in the neutrino fits. The lack of constraint is not so detrimental to the antineutrino analysis because the 2p2h and QELike strength is $\sim 15\%$ of the signal, resulting in larger uncertainties. Beyond the background-dominated $Q^2$ ranges in the neutrino mode, the agreement between data and the NuWro SF model is good, indicating the QE model is consistent between regions. The same cannot be said about the RFG neutrino model. Figure 5.23 compares the Region 0 distributions between the NuWro SF and the RFG model. The RFG model under-predicts data by 50% even for $Q^2 > 0.3$ GeV$^2$ where QE is dominant.

In conclusion, the prediction of the NuWro SF model provides a consistent description of the data where QE is dominant. As such, this validates the crucial extrapolation made by our simulation of the carbon QE events under the hydrogen peak for $Q_{QE}^2 > 0.3$ GeV$^2$. Below this, we are unable to validate this prediction because of the very different 2p2h contributions. A change in 2p2h of the same size
Figure 5.21: Before fitting neutrino mode event distribution across regions 0, 1 and 2. Event rate on the left and ratio to MC on the right. The MC tune applied are MinvGENIE-v2, elastic FSI reweight and NuWro SF. QE carbon is no longer the dominant event type in region 2. 2p2h and QELikeNot contributions dominates in the lowest and highest $Q^2$ bins in the neutrino mode. This is in contrast to the antineutrino mode (Fig .5.11 where QE carbon is dominant across the entire $Q^2$ spectrum.

required to bring the data into an agreement for the neutrino mode would be within the systematic uncertainties of the hydrogen cross section analysis.
Figure 5.22: After fitting neutrino mode event distribution across regions 0, 1 and 2. Event rate on the left and ratio to MC on the right. The MC tune applied are MINVGENIE-v2, elastic FSI reweight and nuWro SF. QE carbon is no longer the dominant event type in region 2. 2p2h and QELikeNot contributions dominates in the lowest and highest $Q^2$ bins in the neutrino mode. This is in contrast to the antineutrino mode (Fig. 5.11 where QE carbon is dominant across the entire $Q^2$ spectrum.
Figure 5.23: Comparisons of fitted events for models with (left) and without (right) NuWro SF reweight. The discrepancy in the NuWro SF model has some correlation with the strength of the 2p2h contribution while disagreement in the alternative model is stronger below $Q^2 = 1$ GeV$^2$ and also large in regions dominated by QE carbon.

5.6 Unfolding

The detector we use to measure particle kinematics is inherently imperfect. A charged particle causes scintillation light in the scintillator by ionizing electrons from the atoms. The light signal must pass through a WLS fiber before a PMT converts the photons into charges that MINERνA’s ADC can digitize (see Sec. 3.1). The digitized signal from each strip is converted back into energy by applying a constant determined by calibration on a large sample of energy deposits. Each step of the conversion process is probabilistic, and the reconstructed muon kinematics will always deviate from its “true” momentum. Finally, the muon momentum in MINERνA is measured jointly by the MINERνA detector and the MINOS near detector[110]. The uncertainty in the MINOS muon reconstruction is 1% when reconstructed by range, and 0.6(2.5)% by curvature if the muon’s energy is greater(less) than 1 GeV.

Similarly, the muon direction is reconstructed by fitting the clusters we believe to be from the muon. We could inadvertently add or omit clusters due to reconstruction inefficiency. The muon could also elastically scatter inside the detector so that what
Figure 5.24: The migration matrix in truth and reconstructed $Q^2$. Off-diagonal terms in each row come from true $Q^2$ that migrated into the surrounding reco $Q^2$ bins due to detector smearing.

We measure is not its original direction. The same effect could happen on protons or pions and even neutrons. Therefore the image of the particles obtained from the detector will always be blurry. The reconstructed quantities are always “smeared”. Fortunately, we have simulated our detector as precisely as possible. The same smearing inside the detector is present in the simulation. We could plot the truth distribution against the reconstructed distribution to form a migration matrix (the true distribution migrates into off-diagonal reconstructed bins). For the hydrogen signal, we construct a migration matrix in $Q^2$ as shown in Fig 5.24.

We want to undo the smearing from the detector distortion in MINER$\nu$A so that our cross section results can be directly useful to model builders and other
5.6 Unfolding

experiments to make a direct comparison. The procedure to undo the smearing is unfolding, and in essence, we want to estimate the inverse of the migration matrix. At MINERνA, we use an iterative unfolding procedure[126, 155], as implemented in the RooUnfold[156] package. Let \( \hat{y}_i^{(k)} \) and \( y_j \) be the expected event rate on the truth and the reconstructed quantity. Let \( \lambda_i^{(k)} \) be the probability of the truth distribution. The subscript indicates the bin number, and the superscript, when present, indicates the number of iteration. Finally, let \( S_{nm} \) be the migration matrix with \( n \) and \( m \) corresponding to the reconstructed and truth bin, respectively. The unfolding equation is

\[
\hat{y}_i^{(k+1)} = \frac{\lambda_i^{(k)}}{\sum_n S_{ni} \sum_j S_{ji} \lambda_j^{(k)}} \sum_j S_{ji} y_j \sum_l S_{jl} \lambda_l^{(k)} \quad (5.15)
\]

We need to choose the number of iterations \( k \) that terminates the unfolding procedure. This is our regularization in an ill posed unfolding problem. As discussed in Ref [155], too many iterations may result in unphysical distribution resulting from the forward feedback from the statistical fluctuations in the distributions. Therefore, we must choose the iterations carefully.

At MINERνA we obtain the optimal regularization strength by unfolding toy experiments with statistical modification using different number of iterations. The toy experiments use a "warped" signal with statistical modifications. The best \( k \) occurs when the reduced \( \chi^2 \) between the fake data and fake truth distribution reaches 1/DOF. We take the migration matrix from our signal selection and subject the truth distribution to a multiplicative factor

\[
w = (2i/N - 1)^3 + r(i), \quad (5.16)
\]
where \( i \) is the bin number, \( N = 18 \) is the total number of bins and \( r(i) \) is a random number uniformly distributed between 0.95 and 1.05. The effect of the warping on truth is propagated to the reconstructed quantity through the migration matrix. Fig. 5.25 shows the ratio between the warped and the original MC in the reconstructed space.

We test iterations from 0 to 10. In each iteration, we generate 1000 “statistical universes” to simulate the effect of statistical fluctuations. The bin content in each statistical universe is generated with a Poisson number generator with its mean set to the value of the fake data bin. We unfold the fake data, along with the 1000 universes generated, and calculate a \( \chi^2 \) between each unfolded statistical universe and the truth. Figure 5.26 shows the \( \chi^2 \) distributions in the statistical universes for each iteration. It also shows the average \( \chi^2 \) and the median \( \chi^2 \) in each iteration. 4 iterations resulted in the minimum mean and median \( \chi^2 \).

We apply the iterative unfolding procedure to the background-subtracted data,
5.6 Unfolding

Figure 5.26: Distribution of $\chi^2$ between the true and the unfolded fake data distribution across the number of iterations (top). Each iteration contains 1000 statistical universes. (bottom left) The average $\chi^2$ and (bottom right) median $\chi^2$. Four iterations suffice to achieve the lowest $\chi^2$. At higher iterations, we start to see universes occupying high $\chi^2$ region because the unfolding amplifies the effect of statistical fluctuation, resulting in the “runaway” experiments. We have chosen 4 iterations for the analysis.
and this is the numerator to the cross section extraction procedure in Eq. (3.12).

5.7 Efficiency, Targets, and Flux

The cross section denominator comprises efficiency correction, the number of hydrogen targets, and the total flux (Eq. 3.12).

5.7.1 Efficiency

The efficiency in the $i$th bin is defined as

$$
\epsilon_i = \frac{N_{\text{truth, Sel}}}{N_{\text{truth, All}}}.
$$

The selection efficiency shown in Fig. 5.27 first is largest (0.083) between $Q^2$ of 0.2 GeV$^2$ and 0.3 GeV$^2$, then begins to decline with higher $Q^2$. The downward trend at high $Q^2$ seems to contradict the neutron selection efficiency shown in Fig 3.18, where we see a relatively flat neutron reconstruction efficiency at higher neutron kinetic energy. However, this is only the efficiency given that the neutron interacts somewhere in the detector. Neutrons tend to have larger opening angles and more side-going at higher $Q^2$. With less material path length, the neutrons may leave the
Figure 5.27: The efficiency of selecting hydrogen events with observable neutrons.

detector without interaction. At lower $Q^2$, neutrons tend to have low kinetic energy $E = Q^2 / 2M_n$ and therefore low reconstruction efficiency as shown in Fig. 3.18.

The reportable bins are chosen to be from bin 3 to bin 17 because they have reasonable efficiencies, $\epsilon > 0.2\%$.

### 5.7.2 Total Flux

This analysis uses the antineutrinos generated by the ME reversed horn current (RHC) configuration. The flux prediction (Fig. 3.7) has been evaluated using the fiducial volume in the tracker region so that we could directly use the predicted flux
and multiply with the total protons on target (POT), $1.12 \times 10^{21}$, to derive the total number of antineutrinos passing through the tracker region.

### 5.7.3 Evaluation of Systematic Uncertainties

**GENIE Systematics**

There are inherent uncertainties associated with the models we use in our measurements. GENIE has a set of weights associated with each tunable parameter, called a knob, and provides reweighting to event distributions when we need to access an alternate universe. Two classes of knobs corresponding to the cross section models and the FSI models exist. Modifying the cross section model knobs changes the normalization and shape of CCQE, CC-RES, and CC-DIS events, first described in Sec 1.3.3. Parameters related to the FSI change how outgoing nucleons and pions interact in the nucleus. The set of GENIE universes relevant to this analysis is shown in Table 5.7.

The values of the $1\sigma$ shifts for the cross section model are motivated by external studies of these uncertainties at MINOS and later by T2K. The values for the hadronic interaction are motivated by Ref. [157].

FSI such as nucleon (\_N) and pion (\_pi) absorption (Abs), charge exchange (CEx), elastic (Elas), inelastic (Inel), and pion production (PiProd) are important FSI knobs that change the likelihood a nucleon or a pion undergo these kinds of FSI. The MFP\_N and MFP\_pi are the mean free path of hadrons such as the nucleon and pion in the nucleus. The shorter the MFP, the more likely a hadron may undergo rescattering and fall into one FSI fate.

The cross section model parameters are associated with the individual models
### 5.7 Efficiency, Targets, and Flux

<table>
<thead>
<tr>
<th>Cross Section Model</th>
<th>1σ</th>
<th>Hadronic Interaction</th>
<th>1σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>AhtBY</td>
<td>±25%</td>
<td>AGKYxF1pi</td>
<td>±20%</td>
</tr>
<tr>
<td>BhtBY</td>
<td>±25%</td>
<td>FrAbs_N</td>
<td>±20%</td>
</tr>
<tr>
<td>CCQEPauliSupViaKF</td>
<td>±30%</td>
<td>FrAbs_pi</td>
<td>±30%</td>
</tr>
<tr>
<td>CV1uBY</td>
<td>±30%</td>
<td>FrCEx_N</td>
<td>±50%</td>
</tr>
<tr>
<td>CV2uBY</td>
<td>±40%</td>
<td>FrCEx_pi</td>
<td>±50%</td>
</tr>
<tr>
<td>EtaNCEL</td>
<td>±30%</td>
<td>FrElas_N</td>
<td>±30%</td>
</tr>
<tr>
<td>NormCCQE</td>
<td>(+20, −15)%</td>
<td>FrElas_pi</td>
<td>±10%</td>
</tr>
<tr>
<td>MaCCQEshape</td>
<td>±10%</td>
<td>FrInel_N</td>
<td>±40%</td>
</tr>
<tr>
<td>MaNCEL</td>
<td>±25%</td>
<td>FrPiProd_N</td>
<td>±20%</td>
</tr>
<tr>
<td>MaRES</td>
<td>±20%</td>
<td>FrPiProd_pi</td>
<td>±20%</td>
</tr>
<tr>
<td>MvRES</td>
<td>±10%</td>
<td>MFP_N</td>
<td>±20%</td>
</tr>
<tr>
<td>NormCCRES</td>
<td>±20%</td>
<td>MFP_pi</td>
<td>±20%</td>
</tr>
<tr>
<td>NormDISCC</td>
<td></td>
<td>RDecBR1gamma</td>
<td>±50%</td>
</tr>
<tr>
<td>NormNCRES</td>
<td>±20%</td>
<td>Theta_Delta2Npi</td>
<td>on/off</td>
</tr>
<tr>
<td>Rvn1pi</td>
<td>±50%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rvn2pi</td>
<td>±50%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rvp1pi</td>
<td>±50%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rvp2pi</td>
<td>±50%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>VecFFCCQEshape</td>
<td>BBA05 to dipole</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.7: GENIE systematics from the cross section model and the hadronic interaction model (FSI).

Implemented in GENIE to simulate various neutrino-nucleus interactions. There are different parameters for the different parts of the interaction models: the QE, RES, and DIS first introduced in Section 1.3.3.

AhtBY, BhtBY, CV1uBY, and CV2uBY are parameters controlling the Bodek-Yang quark-Parton model used to simulate the DIS events. EtaNCEL and MaNCEL are parameters that control the neutral current cross section. MaRES and MvRES control resonant pion production implemented in the Rein Sehgal model[103]. These are the axial and vector form factors implemented for the RES events similar to QE. Rvn1pi and Rvp1pi correspond to the nonresonant single pion production. Rvn2pi and Rvp2pi control nonresonant double pion production. MINERνA’s nonresonant
pion production tune uses the Rvn1pi and Rvp1pi knobs and assigns a different uncertainty after constraining these GENIE knobs on external data[139].

**MnvGENIE-v1 Systematic Uncertainties**

Each of MnvGENIE-v1’s tunes (Sec 2.3) has a prescription of evaluating systematic uncertainty. These are described in turn.

**Nonresonant Pion Weight:** Nonresonant pion production can occur through both NC and CC channels. We group together the NC and CC knobs together into two knobs, Rvn1pi and Rvp1pi, representing production of a single pion in $\nu n(\bar{\nu}p)$ or $\nu p(\bar{\nu}n)$ interactions. Applying the resonant pion reduction tune results in 4% uncertainties.

**Low_Recoil_2p2h_Tune:** The tune applies an empirical fit to increase the contribution of the Valencia 2p2h model based on Ref. [107]. Correlations in nucleons could come from neutron-neutron (nn) in neutrino mode, or proton-proton (pp) pairs in antineutrino mode, and proton-neutron (pn) pairs for both modes. Enhancements are applied on nn, pp, and pn pairs together, with nn and pp pairs receiving the same treatment. The systematic universes in this tune are evaluated by assuming that either nn(pp), pn, or QE with a single particle contributed to all of the enhancement required to resolve the data-MC differences. The uncertainty from each variation contributes equally to the total uncertainty by taking their RMS.

**RPA Weight:** Similar to the 2p2h weight, the RPA systematic uncertainty is evaluated by applying alternate models. The CV weight is replaced by variations with $\pm 1\sigma$ shift in the low-$Q^2$ events and the high-$Q^2$ events, forming four systematic universes. The low-$Q^2$ and high-$Q^2$ RPA systematic uncertainties form part of the cross section model uncertainties.
Table 5.8: Muon Uncertainties

<table>
<thead>
<tr>
<th>Muon Systematic Uncertainties</th>
<th>σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>BeamAngle_X</td>
<td>1 mrad</td>
</tr>
<tr>
<td>BeamAngle_Y</td>
<td>0.9 mrad</td>
</tr>
<tr>
<td>Muon_Energy_MINOS(range)</td>
<td>1%</td>
</tr>
<tr>
<td>Muon_Energy_MINOS(curvature, &gt; 1 GeV)</td>
<td>0.6%</td>
</tr>
<tr>
<td>Muon_Energy_MINOS(curvature, &lt; 1 GeV)</td>
<td>2.5%</td>
</tr>
<tr>
<td>Muon_Energy_MINERvA</td>
<td>32 MeV</td>
</tr>
<tr>
<td>Muon_Resolution</td>
<td>0.4%</td>
</tr>
</tbody>
</table>

Detector Uncertainties

There are a few uncertainties associated with MINERνA’s energy and position resolutions. This analysis requires a reconstructed muon in the MINERνA detector, which must then pass into the MINOS near detector. A muon’s momentum is reconstructed by either range or curvature depending on whether it is contained. The total muon momentum is the sum of the energy deposited in MINERνA and MINOS. To evaluate muon systematics, we shift the muon energy by the uncertainty in MINERνA and MINOS separately. The MINERνA detector has a 32 MeV uncertainty in the muon energy due to material assay. The MINOS uncertainties are evaluated based on whether the muon is reconstructed by range or by curvature. The shifted muon energy propagates to the $Q^2$ calculation, and the result is filled in a systematic universe. Similarly, there are uncertainties associated with the beam angle (BeamAngle.{X,Y}). We evaluate those as shifts in the muon’s directions because all kinematic quantities are calculated in the beam frame. The shifts are propagated to $Q^2$ calculations (see Eq.(1.42)). The error budgets are shown in Fig. 5.8.
Figure 5.28: (left) Comparisons of neutron cross sections on carbon as a function of neutron kinetic energy. The dashed line denotes cross sections from GEANT4 4.9.4, used by MINERνA, while the solid line and the green line denotes the newer cross section found in GEANT4 4.10.3. The new cross section has been tuned to the Abfaltrer cross section measured by Ref. [158]. Both GEANT4 cross sections are broken down into the elastic and inelastic components. The dashed cross section has irregular turns and drops sharply between 30 and 35 MeV. (right) Neutron inelastic cross section from Ref. [159]. The GEANT4 inelastic neutron cross section on carbon is consistent with the measurement.

GEANT4 Neutron Uncertainties

We use GEANT4 to simulate particle response in the MINERνA detector. MINERνA uses an older version (4.9.4) of GEANT4, and the neutron cross sections have been updated in newer GEANT4 (4.10.3) to more closely match a number of measured interaction cross sections.

Figure 5.28 compares the neutron cross sections on carbon between the two GEANT4 versions and the measurement performed by Abfaltrer in Ref. [158]. The neutron inelastic cross section has been published previously, such as in Ref. [159]. GEANT4’s inelastic cross section is in good agreement with the data. GEANT4
4.10.3 has been tuned to the Abfalterer data, while the older GEANT4 4.9.4 has not. GEANT4 4.9.4 disagrees with the measurement significantly for neutron kinetic energy, $T_n > 30\text{MeV}$, in the elastic channel.

Although we would like to generate events directly with the new cross section, the reality is that MINER$\nu$A must keep using GEANT4 4.9.4 because our software environment needs to be compatible with the MINOS software base. To solve this problem, MINER$\nu$A employs a reweighting mechanism in which we look for GEANT4 true neutron trajectories in our simulation, and weigh the neutrons to the new cross section according to their kinetic energies, energy losses, and the elasticity,

$$w = \frac{1 - e^{-\rho \sigma_{\text{total}}^{\text{new}}}}{1 - e^{-\rho \sigma_{\text{total}}^{\text{old}}}} \times \frac{\sigma_i^{\text{new}} \sigma_{\text{total}}^{\text{new}}}{\sigma_i^{\text{old}} \sigma_{\text{total}}^{\text{old}}}, \quad (5.18)$$

where $\sigma$ is the cross section averaged over the neutron's initial $T_n$ and final $T_n$ in an interaction, old and new refers to the two GEANT4 cross sections and $i$ is either elastic or inelastic.

When multiple neutrons exist, we take the product of all the weights for each neutron. We then calculate the total event rate before and after applying the weight and apply a normalization factor to all the weights to preserve the event rate.

<table>
<thead>
<tr>
<th>Neutron Kinetic Energy</th>
<th>Elastic</th>
<th>Inelastic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$&lt; 10\text{MeV}$</td>
<td>$\pm 25%$</td>
<td>$\pm 25%$</td>
</tr>
<tr>
<td>$&lt; 25\text{MeV}$</td>
<td>$\pm 10%$</td>
<td>$\pm 20%$</td>
</tr>
<tr>
<td>$&gt; 25\text{MeV}$</td>
<td>$\pm 3%$</td>
<td>$\pm 15%$</td>
</tr>
<tr>
<td>$&gt; 100\text{MeV}$</td>
<td>$\pm 3%$</td>
<td></td>
</tr>
<tr>
<td>$&gt; 200\text{MeV}$</td>
<td>$\pm 2%$</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.9: Uncertainties in neutron’s elastic and inelastic cross sections.
Figure 5.29: Distribution of measured energy deposit $E_{\text{dep}}$ per neutron candidate, normalized by the total number of events. The lowest bin only contains candidates down to the 1.5 MeV threshold. The lower panels contain ratios to the reference simulation for the data and for modifications to the GENIE and GEANT4 simulations. Reference [160]

Table 5.9 shows the size of the ±1σ uncertainties in neutron cross section after reweighting to the new GEANT4 cross section, resulting in two systematic universes.

Ad Hoc Neutron Interaction Uncertainties

MINERνA has performed a neutron multiplicity measurement in the LE era with an algorithm optimized to tag low energy neutrons[160]. The result from that analysis indicates an over-prediction by 25% of neutrons with energy deposit below 10 MeV, as shown in Fig. 5.29. The preference for higher multiplicities could come from the mismodellings in the low-energy proton and $\alpha$ production cross sections when neutrons interact on carbon.

Figure 5.30 shows the ratio between data and MC in the main neutron candidate energy. Each small histogram contains the distribution in a region of the reconstructed $Q^2$ denoted by $Q^2_{\text{QE}}$. We observe an over-prediction in the number of neutron candidates with energy deposits below 10 MeV, similar to the observation in Ref. [160]. Reference [160] posits over-prediction by GENIE and GEANT4
Figure 5.30: Ratio of main candidate energy in the event selection between data and MINERνA prediction in bins of reconstructed $Q^2$ (denoted $Q_{QE}^2$), after background constraints. Bins in the regions $Q_{QE}^2 \in (0, 0.05)$ GeV$^2$ and $Q_{QE}^2 > 2$ GeV$^2$ have limited statistics. Data has statistic uncertainty and MC has systematic uncertainties.
as equally likely. A prescription used to study the over-prediction in that analysis randomly drops 25% of neutron candidates below 10 MeV. In this analysis, we cover the excess MC events below 10 MeV with a systematic uncertainty attributed to the uncertainty in neutron production from our models. Instead of removing candidates, we apply analytical approximation to weight each 3D candidate in an event up or down by 20%. Since the neutron algorithm developed for this analysis only uses the leading 3D candidates, the weight is only active when the main candidate is below 10 MeV. The expression for the weight is

$$w = 1 \pm \sum_{i=1}^{N} r^i,$$

(5.19)

where \( r \) is the rate of discarding a neutron candidate, \( N \) is the total number of 3D candidates satisfying the drop criteria. In the limit \( N \) becomes large, \( w = 1 \pm 0.25 \).

### 5.7.4 Flux Uncertainties

As discussed previously in Sec 3.4, MINERνA uses an \textit{a priori} flux prediction based on G4NuMI and PPFX. Figure 5.31 shows the size of the systematic uncertainties in the \textit{a priori} flux. The flux uncertainties can be categorized into the beam focusing and the hadron production component. Beam focusing uncertainties are due to the variations in the positions of the beamline components. Variations could come from proton beam positions and horn positions, for example. The hadron production uncertainties are related to the proton interactions in the targets creating secondary hadrons. Some of the hadrons, such as \( \pi \) and \( K \)-meson, may decay to produce neutrinos directly. Other interactions could produce nucleons that may undergo additional scatters before creating neutrino-producing mesons. The hadron production
Figure 5.31: Systematic uncertainties on the *a priori* flux into beamline focusing (Flux BeamFocus) and hadron production (ppfx1_Total) categories for (left) neutrino and (right) antineutrino mode in the ME beam configuration.

uncertainties dominate over most of the energy regions in both the FHC and RHC configurations. The antineutrino analysis in this result benefits from the measured $\nu e^- \rightarrow \nu e^-$ constraint of the absolute flux normalization discussed in Sec. 3.4.2.

### 5.8 Results

#### 5.8.1 Cross Section and Systematic Uncertainties

We report the result of cross section measurement in Fig. 5.32. The top plot shows the cross section data compared with MC cross section and theoretical calculation based on the BBBA05[62] and BBBA07[57] vector form factors, with the fractional uncertainties broken into systematic groups at the bottom. Statistical uncertainties dominate the cross section because we measure a sample with a significant background. The background fraction in most $Q^2$ bins is greater than 50%, with increasing background fraction at higher $Q^2$ reaching more than 80% beyond $Q^2 = 1$ GeV$^2$.

To illustrate the dire effect of the background fraction on the statistical error in
a bin, let us assume there are $N$ events in the bin. The background prediction is generated with $n$ times more MC than data, so that the background prediction is $B_n/n$ after normalizing MC to data. The signal estimate is $S = N - B_n/n$. Since the number of events in a bin follows a Poisson distribution, the total uncertainty is then $\delta S = \sqrt{N} \oplus \sqrt{B/n} = \sqrt{N + B/n}$, where $B = B_n/n$. In the (impossible) limit in which we have infinite MC, $n \to \infty$, the statistical error on the sample becomes simply $\sqrt{N}$. If the fraction of the signal in the event is $f$, then $S = fN$ and $B = (1 - f)N$. The fractional uncertainty for signal, still in the infinite Monte Carlo limit, is $dS/S = 1/(f\sqrt{N})$. The signal fraction at high $Q^2$ is $1/5$, so that $dS/S = 5/\sqrt{N}$, even before accounting for finite MC. At MINER$\nu$A we generate MC with approximately $4.4 \times$ the data POT. The final statistical error is then $dS/S = \left(\sqrt{1 + (1 - f)/n}\right)/(f\sqrt{N})$.

The largest systematic errors come from those related to muon reconstructions (Fig. 5.36 and neutron interactions (Fig. 5.35). At the lowest $Q^2$, systematic uncertainties related to the neutrino beam’s direction and the reconstructed muon energy dominates. BeamAngleX and BeamAngleY account for the dispersion in the neutrino beam, and they form the largest systematic uncertainties at $Q^2 < 0.05$ GeV$^2$. In the Other category, the ad-hoc Neutron Interaction uncertainties dominate at the low $Q^2$ because neutron energy scales with $Q^2$. Reweight_Neutron is the uncertainties on the GEANT4 neutron cross section, and is large at low $Q^2$ (see Table 5.9).

Finally, Fig. 6.1 shows the covariance matrix and Fig. 5.39 shows the correlation matrices of the extracted cross section. The covariance matrix is a $N \times N$ matrix where $N$ corresponds to the total number of bins. Each element of the covariance matrix $V_{ij}$ is the extent of correlation in the uncertainties between the $i$th and $j$th bin. By definition, the covariance matrix is symmetric with $V_{ij} = V_{ji}$. 
The value of the covariance matrix is calculated as

\[
V_{ij} = \frac{\sum_{k=1}^{n} (x_{i,k} - \langle x_i \rangle) (x_{j,k} - \langle x_j \rangle) w_k}{\sum_{k=1}^{n} w_k},
\]

where \(V_{ij}\) is the covariance matrix element corresponding to the \(i\)th and \(j\)th bins; \(k\) is the enumeration of a systematic universe \(k\) with \(n\) the total number of systematic universes; \(w_k\) is the weight on the \(k\)th universe; \(x_{i,k}\) is the event count at the \(i\)th bin in the \(k\)th universe; \(\langle x_i \rangle = \sum w_k x_{i,k} / \sum w_k\) is the mean event count in the \(i\)th bin.

\[
\sigma_i = \sqrt{V_{ii}},
\]

is the uncertainty in the \(i\)th bin.

The correlation between any pair of bins is derived from the covariance matrix by

\[
\rho_{ij} = \frac{V_{ij}}{\sigma_i \sigma_j},
\]

where \(\rho_{ij}\) is the element of the correlation matrix. The element of a correlation matrix can vary between -1 to 1. If \(\rho_{ij} = 1\), the \(i\)th and \(j\)th bins are completely correlated, and a change in the \(i\)th bin causes a proportionally equal change in the \(j\)th bin in the same direction. If \(\rho_{ij} = -1\), the bins are completely anti-correlated, so that a change in the \(i\)th bin causes a proportionally equal change in the opposite direction.

Positive correlation in the systematic uncertainties usually occurs when we change the normalization of the event count. For example, \textit{GENIE\_NormCCQE} shifts the entire distribution up(down) by 20%(15%). Anti-correlations usually occur when we change the shape of the distribution or when the systematic universe increases.
the rate in one bin but decreases it in another bin.

The uncertainties in the cross section are almost entirely positively correlated. There are weak anti-correlation between the first bins (5,6,7,8) and bin 17, where there is little data. We observe a strong correlation in the systematic uncertainties between adjacent bins. Significant uncertainties from \textit{Reweight\_Neutron, Neutron\_Interaction} which change the normalization result in largely positive corrections of systematic uncertainties between bins.

<table>
<thead>
<tr>
<th>Bin</th>
<th>Bin Center $Q^2$ GeV$^2$</th>
<th>Cross Section/$10^{-38}$</th>
<th>Error/$10^{-38}$</th>
</tr>
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<tr>
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<td>2.4</td>
<td>1.7</td>
</tr>
<tr>
<td>4</td>
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<td>1.1</td>
<td>0.7</td>
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<td>0.8</td>
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</tr>
<tr>
<td>6</td>
<td>0.075</td>
<td>1.4</td>
<td>0.4</td>
</tr>
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<td>1.28</td>
<td>0.21</td>
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<td>0.74</td>
<td>0.07</td>
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<td>0.40</td>
<td>0.05</td>
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Table 5.10: Measured cross section and the associated uncertainties in each bin.
Table 5.11: Covariance matrix of the measured cross section

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<th>Bin($\times 10^{-80}$)</th>
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<td>700</td>
<td>490</td>
<td>250</td>
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<td>90</td>
<td>64</td>
<td>17</td>
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<td>1.1</td>
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<td>430</td>
<td>260</td>
<td>370</td>
<td>200</td>
<td>120</td>
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<td>44</td>
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<td>200</td>
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<td>1.6</td>
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<td>0.79</td>
<td>0.8</td>
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<td>0.19</td>
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<td>-0.99</td>
<td>-0.1</td>
<td>0.32</td>
<td>0.023</td>
<td>0.047</td>
<td>0.096</td>
<td>-0.0047</td>
<td>-0.0051</td>
<td>0.0024</td>
<td>0.27</td>
</tr>
</tbody>
</table>
Figure 5.32: Extracted cross section (top) and the fractional uncertainties (bottom). The regions bounded by the red arrows are the reportable cross section range based on efficiency.
Figure 5.33: FSI uncertainties

Figure 5.34: Cross section model uncertainties
Figure 5.35: Other uncertainties. This category includes particles response and target normalization.

Figure 5.36: Muon reconstruction uncertainties.
Figure 5.37: 2p2h and Flux uncertainties

Figure 5.38: The covariance matrix of the extracted cross section. Refer to Table 5.4 for the $Q^2$ to bin mapping.
Figure 5.39: The full (left) and systematic-only (right) correlation matrices of the extracted cross section. Refer to Table 5.4 for the $Q^2$ to bin mapping.
Chapter 6

$F_A$ Extraction and Analysis

6.1 Theoretical Cross Section

Besides data and MC cross sections, Fig. 5.32 also shows the theoretical cross section calculated with Eq.(2.1) using the BBBA05[62] and the BBBA07[57] elastic vector form factors (see Sec 2.1.2). In the BBBA parameterizations, the axial form factor $F_A$ takes on a dipole form with $M_A = 0.99 \text{ GeV}/c^2$ in accordance to GENIE’s simulation.

For each $Q^2$, we must also apply a phase space correction because both the muon angle cut of 20° and the muon momentum cut restricts the range of neutrino energy. The maximum neutrino energy is given by solving Eq.(5.4) for $E_l = 20 \text{ GeV}$. The minimum neutrino energy is affected by the minimum muon energy and the 20° muon phase space selection.

At low $Q^2$, solving Eq.(5.4) with $E_l = 1.5 \text{ GeV}$ gives the lower bound on the neutrino energy. At $Q^2 = 0.28 \text{ GeV}^2$, the angular selection takes over and the minimum neutrino energy is solved with Eq.(5.5) for $\theta = 20^\circ$. The phase space cuts
Table 6.1: The lower and upper bound on the neutrino energy available for interaction with the muon phase space cut. The minimum neutrino energy increases as a function of $Q^2$, while the maximum energy is relatively stable around 20 GeV$^2$. As a result, the fraction of the total flux available for interaction decreases as a function of $Q^2$. As a secondary effect, the average $Q^2$ weighted by neutrino flux shifts from the bin center values. The shift caps at 5% for the highest $Q^2$ bin.

result in a net reduction of the flux averaged cross section because the fraction of energy available to generate event decreases with increasing $Q^2$. Table 6.1 shows the lower and upper bounds in neutrino energy and the fraction of neutrino flux available for interactions at various $Q^2$.

### 6.2 $F_A$ Extraction and Covariance Calculation

The values of the axial form factor are extracted from the hydrogen cross section using either the BBBA05[62] or the BBBA07[57] vector form factors. At each $Q^2$
6.2 $F_A$ Extraction and Covariance Calculation

Figure 6.1: The covariance matrices for (left) BBBA05 and (right) BBBA07 vector form factors.

bin, we obtain the flux weighted $Q^2$ and use it to compute the $F_A$ needed to generate the cross section with Eq.(2.1). The results from the two vector form factors differ by at most 2%.

The covariance in the extracted $F_A$ are obtained by following the error propagation formula

$$\sigma^2 = AVA^T,$$

where $V$ is the data covariance matrix (see Fig. 6.1) and $A$ is the partial derivative matrix

$$A_{ii} = \left( \frac{\partial F_A}{\partial d\sigma/dQ^2} \right)_i = \left( \frac{\partial d\sigma/dQ^2}{\partial F_A} \right)^{-1}_i,$$

$$A_{ij} = 0, \text{ if } i \neq j.$$

where $i$ and $j$ are the index at each $Q^2$ bin. The diagonal elements of the partial derivative matrices for the BBBA05[62] and BBBA07[57] are shown in Table 6.2.
Table 6.2: Diagonal elements of the Jacobian matrices. Refer to Table 5.4 for the $Q^2$ corresponding to each bin.

<table>
<thead>
<tr>
<th>Bin</th>
<th>BBBA2005</th>
<th>BBBA2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>$-2.03 \times 10^{35}$</td>
<td>$8.22 \times 10^{35}$</td>
</tr>
<tr>
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<td>$-4.49 \times 10^{37}$</td>
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</tr>
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</tr>
<tr>
<td>6</td>
<td>$-2.37 \times 10^{38}$</td>
<td>$-2.12 \times 10^{38}$</td>
</tr>
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<td>7</td>
<td>$-6.62 \times 10^{37}$</td>
<td>$-6.57 \times 10^{37}$</td>
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<td>$-6.90 \times 10^{37}$</td>
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</tr>
<tr>
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<tr>
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<tr>
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<td>$-9.15 \times 10^{38}$</td>
</tr>
</tbody>
</table>

Table 6.3: Values of $F_A$ extracted with the BBBA05[62] and the BBBA07[57] elastic vector form factors at each $Q^2$ weighted by flux acceptance.
6.3 Comparisons and Z-Expansion Fits

Figure 6.4 shows the extracted $F_A$ with the BBBA05[62] and the BBBA07[57] vector form factor formulation, compared to the Z-expansion fit by Meyer[77] a Z-expansion fit using our extracted $F_A$ (Hydrogen), the joint fit between Meyer and Hydrogen fit, and finally the duality constrained fit performed in BBBA07[57].

We fit the hydrogen data with the Z-expansion formalism described Sec. 2.1.2. The formula for z-expansion are shown in Eq.(2.22) and Eq.(2.23) and reproduced below:

$$F_A(Q^2) = \sum_{k=0}^{k_{\text{max}}} a_k z(Q^2)^k,$$

$$z(Q^2) = \frac{\sqrt{t_{\text{cut}} + Q^2} - \sqrt{t_{\text{cut}} - t_0}}{\sqrt{t_{\text{cut}} + Q^2} + \sqrt{t_{\text{cut}} - t_0}}.$$ 

As in Sec 2.1.2, $t_{\text{cut}} = 9 m_\pi^2$. The value of $t_0$ needs to be optimized for the data range available according to Ref. [77]:

$$t_0^\text{opt} = t_{\text{cut}} (1 - \sqrt{1 + Q^2_{\text{max}}/t_{\text{cut}}}). \quad (6.1)$$

Equation 6.1 will constrain the range of $|z|$ to fit data, Fig. 6.2 shows the the extracted $F_A$ as a function of $z$.

The extracted $F_A$ goes out to $Q^2 = 5 \text{ GeV}^2$, we therefore choose $t_0 = -0.78$. The Meyer fit chose $k_{\text{max}} = 8$ and performed a 4 parameter fit on $[a_1, a_2, a_3, a_4]$ Parameters $a_0$ and $a_5$ to $a_8$ are constrained with $F_A(0) = g_A$ and that at high
### Table 6.4: $F_A$ covariance matrix, extracted with the BBBA05[62] vector form factor.
Table 6.5: $F_A$ covariance matrix, extracted with the BBBA07[57] vector form factor.
6.3 Comparisons and Z-Expansion Fits

Figure 6.2: $F_A$ as a function of $z$ for $t_0 = -0.78$, $|z| < 0.4$.

As $Q^2 \to \infty$, $F_A$ is expected to fall as $1/Q^2$ according to Ref [161].

$$
\begin{align*}
\sum_{k=0}^{k_{\text{max}}} a_k z(0)^k &= g_A, \\
\left. \frac{d^n F_A(z)}{dz^n} \right|_{z=1} &= 0, \quad n = 0, 1, 2, 3. 
\end{align*}
$$

Reference [77] proceeds to minimize a regularized $\chi^2$ function of the form

$$
\chi^2 = (x_{\text{data}} - x_{\text{fit}})^T C^{-1} (x_{\text{data}} - x_{\text{fit}}) + \lambda R, 
$$

where $x_{\text{data}}$ and $x_{\text{fit}}$ are the extracted $F_A$ and Z-expansion $F_A$ respectively, $C$ is the data covariance matrix, $\lambda$ is the strength of the regularization term $R$ that enforces
the bound

\[ |a_k/a_0| \leq 5, k \leq 5, |a_k/a_0| \leq 25/k, k > 5. \] (6.5)

We obtain, for \( N_a \) fit parameters

\[ R = \sum_{k=1}^{4} \left( \frac{a_k}{5a_0} \right)^2 + \sum_{k=5}^{k_{\text{max}}} \left( \frac{ka_k}{25a_0} \right)^2. \] (6.6)

Our choice of \( k_{\text{max}} \) differs from the fits presented in Reference [77]. Like Ref. [77], the number of free parameters in each \( k_{\text{max}} \) is \( N_a = k_{\text{max}} - 4 \). Reference [77] examined \( N_a \) between 3 and 5, and apparently set \( \lambda = 1 \). In our case, varying the values of \( \lambda \) causes the shape of the fit to change significantly, especially near \( Q^2 = 0 \). Figure ?? shows an L-curve study of the regularization term (Eq. 6.6) as a function of data \( \chi^2 \) (Eq. 6.4 with \( \lambda \) set to 0). For \( N_a = 4 \) the values of data \( \chi^2 \) is dependent on \( \lambda \) chosen. This affects both the values and the errors when extracting the axial charge radius from the fits (Sec. 6.4). Table 6.6 tabulates the mean squared axial charge radii \( \langle r_A^2 \rangle \) for a range of \( \lambda \) chosen. The uncertainty is highly dependent on the choice of \( \lambda \). The uncertainties on the extracted \( F_A \) incorporate both the statistical and systematic effects so that a rigorous fit should provide consistent results. The same \( \lambda \)-dependence is present for \( N_a = 3 \) and 5.

However, for \( N_a \leq 2 \), the data \( \chi^2 \) and the value of \( \langle r_A^2 \rangle \) are independent of \( \lambda \) and the error is constant for \( \lambda < 1 \). Larger values of \( \lambda \) reduce the total uncertainty on \( \langle r_A^2 \rangle \) due to the regularization term becoming the dominant contribution to the minimization function (6.4). In such case, the minimization is simply minimizing the values of Eq. (6.6) without data. Therefore, for \( N_a = 2 \), we choose \( \lambda = 0 \).
Table 6.6: The mean squared axial charge radii over a range of $\lambda$. The error is clearly dependent on the choice of $\lambda$.

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$\langle r_A^2 \rangle N_A = 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00001</td>
<td>-0.1(9)</td>
</tr>
<tr>
<td>0.0001</td>
<td>-0.1(8)</td>
</tr>
<tr>
<td>0.001</td>
<td>-0.2(5)</td>
</tr>
<tr>
<td>0.01</td>
<td>0.0(4)</td>
</tr>
<tr>
<td>0.1</td>
<td>0.34(24)</td>
</tr>
<tr>
<td>1.</td>
<td>0.48(15)</td>
</tr>
<tr>
<td>10.</td>
<td>0.47(8)</td>
</tr>
<tr>
<td>100.</td>
<td>0.39(4)</td>
</tr>
<tr>
<td>1000.</td>
<td>0.322(19)</td>
</tr>
</tbody>
</table>

Figure 6.3: The regularization term (Eq. 6.6) against the data $\chi^2$ term (Eq. 6.4 with $\lambda$ set to 0) for $N_A = 2$ and $N_A = 4$. Each point is labeled by $\log_{10} \lambda$. For $N_A = 8$, the data $\chi^2$ depends on the values of $\lambda$ chosen but the data $\chi^2$ for $N_A = 2$ does not.
The result of the $\chi^2$ minimization, here referred to as the Hydrogen Fit is

$$[a_0, a_1, ..., a_6] =$$

$$[-0.552113, 1.45(24), -0.8(4), -0.301583, -0.556628, 1.25953, -0.503362]$$

and the covariance matrix for the independent coefficients in the fit, $a = (a_1, a_2)$, is

$$\begin{pmatrix}
0.056 & -0.051 \\
-0.051 & 0.19
\end{pmatrix}$$

Next, we compare the extracted $F_A$ and the fits to a dipole form factor with $M_A = 1.015 \text{ GeV}/c^2$ and show the results in Fig. 6.5. The extracted $F_A$ has a strong tail component at high $Q^2$ comparing to the dipole. The duality-based BBBA07\cite{57} fit better describes the hydrogen data for $Q^2 < 1 \text{ GeV}^2$ than the Z-expansion fit by Meyer et al. \cite{77}, both of which were derived from external data sets. By eye, the BBBA07 fit undulates in the same direction as the extracted $F_A$ data, a feature the Meyer fit lacks. Similarly, the hydrogen does not follow the data trend below $Q^2 < 1 \text{ GeV}^2$. We compute $\chi^2$ between the extracted $F_A$ and the central values of the fits, summarized in Table 6.7. Finally, in Fig. 6.6, we apply the Z-expansion fits to the cross section model using the BBBA05\cite{62} electromagnetic form factors. We compare the theoretical cross sections with a dipole axial form factor ($M_A = 1.015 \text{ GeV}/c^2$) and the hydrogen cross section data. The uncertainty bands on the Z-expansion cross sections are only due to the Z-expansion uncertainties.


\[
\frac{1}{F_A(0)} \frac{dF_A}{dQ^2} = -\frac{1}{6} \langle r_A^2 \rangle. \tag{6.9}
\]

The form of Eq. (6.9) follows from the interpretation of the nucleon form factor as the Fourier transform of the “axial” charge distribution in the non-relativistic limits (see Sec 2.1.2 and Reference [162]). \( \langle r_A^2 \rangle \) is the mean square radius of the axial charge cloud responsible for scattering involving the weak current--an analogy to the electric charge cloud in electron scattering experiments.

The axial charge radius derived from the hydrogen fit is

\[
\langle r_A^2 \rangle = (0.50 \pm 0.22) \text{ fm}^2, \tag{6.10}
\]

\[
r_{A,\text{rms}} = (0.71 \pm 0.16) \text{ fm}, \tag{6.11}
\]

where \( r_{A,\text{rms}} \) is the root-mean-squared radius of the charge distribution.

This value is smaller than the world average for the proton electric charge radius \( 0.8409 \pm 0.0004 \text{ fm}[9] \). In the limit \( Q^2 \to 0 \), the form factor can be interpreted
Figure 6.4: Z-expansion fits to the $F_A$ extracted with the BBBA05 vector form factors. Fits performed with the correlated covariance and a $\chi^2$ minimization are shown.

to be the Fourier transformation of the charge distribution. In this limit we could interpret the smaller axial radius as interactions occurring more frequently towards the center of the nucleon.
Figure 6.5: Ratios of extracted $F_A$ distributions, the Z-expansion fit by Meyer et. al.\cite{77}, and the duality-constrained BBBA07($F_A$)\cite{57} fit to a dipole form factor of $M_A = 1.015$ GeV/$c^2$. The inset expands the y-scale to show the last two points.
Figure 6.6: Cross section models with the Z-expansion fits compared to data and a dipole ($M_A = 1.015$ GeV/$c^2$).
Chapter 7

Initial Study of the Muon, Proton, and Neutron Final States

7.1 Motivation

The neutron reconstruction algorithm presented in Sec. 3.5.2, and fully utilized in Chapter 5 can be extended to study events with multiple nucleons in the final states. An interesting application of the technique is to select QELike events with a muon, a proton, and a neutron in the final state. As neutrons only deposit a small fraction of their total kinetic energy in the detector, oscillation experiments must reply on the interaction model to account for the missing energy. This analysis aims to measure an important sub-sample to constrain the uncertainties associated with the (nearly) invisible neutrons.

The muon-proton-neutron (MPN) topology can be naturally produced by the 2p2h model. In these reactions, neutrino scattering on a neutron-neutron (nn) pair, and antineutrino scattering on a proton-proton (pp) pair, will produce a proton and
a neutron in the nucleus. When these nucleons exit the nucleus, we have a proton-neutron final state. Other reaction models, such as QE, RES, and DIS (see Sec 1.3.3), may also produce the MPN topology through final state interactions (FSI). For example, outgoing nucleons could inelastically scatter within the nucleus to knock out other nucleons. The energy from outgoing pions absorbed by the nucleus would also liberate additional nucleons. When nucleons are the only hadronic final state particles in the detector, the final appears "like" a CCQE reaction, and we call the event QELike.

7.2 Event Selection

In this chapter, we select events based on the neutrino QELike selection similar to those cataloged in Table 5.2, with the crucial distinction being on the number of reconstructed charged particle tracks. We require a muon track and a hadronic track filtered by the \( \frac{dE}{dx} \) proton score cut, and a constant 500 MeV unattached-recoil energy cut (recall the antineutrino recoil energy cut is \( Q^2 \) dependent, see Fig. 5.6). Finally, we require a negatively charged muon in the neutrino mode. The neutron candidate selection remains the same as in the hydrogen analysis. The analysis of the MPN final states is a demonstration of the neutron reconstruction techniques first developed in Chapter 5.

In events with a proton and neutron in the final state, we can measure the transverse kinematic imbalance (TKI) variables from the reconstructed muon and proton. (Refer back to Chapter 4 for a description of the TKI variables.) This study allows us to observe the imbalance from an unreconstructed final state neutron on those variables. Any deviation between the simulation and the measurement
expose areas of improvement for neutron-generating models to be used in oscillation analyses.

### 7.2.1 Sidebands for background estimation

We form control samples to constrain the size of the QELikeNot background categories using the sideband definition in Ref. [148, 163].

<table>
<thead>
<tr>
<th></th>
<th>Pass Michel Cut</th>
<th>Fail Michel Cut</th>
</tr>
</thead>
<tbody>
<tr>
<td>N Blobs &lt; 2</td>
<td>Signal</td>
<td>MichelSideBand</td>
</tr>
<tr>
<td>N Blobs ≥2</td>
<td>BlobSideBand</td>
<td>MicBlobSideBand</td>
</tr>
</tbody>
</table>

**Table 7.1: Sideband Definitions**

Classification of the sidebands is controlled by two cuts – the “Michel” cut (for MichelSideBand) and the “N Blobs” cut (for BlobSideBand). The Michel cut looks for charged pions in the final state by searching for the Michel electrons from the decay chain \( \pi^+ \rightarrow \mu^+ \nu_\mu, \mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu / \). As muons decay on average 2.2 \( \mu s \) after their creation, the Michel electron signals will come after the main event. For neutrino interactions with single pion production, \( \pi^+ \) is the dominant pion in the final state.

Neutral pions in the final state often decay into a pair of photons. Each photon may convert inside the detector to be reconstructed as a “blob” (see Sec. 5.4 for a more detailed discussion). Therefore, each cut is efficient at removing a specific single pion background.

When both cuts fail, we select events with both neutral and charged pions, and this sideband (MicBlobSideBand) therefore favors events with multiple pions in the final state.
While these selections are effective at removing events with pions, they are not useful for removing QELike events with “fake” neutrons. This chapter is a feasibility study to obtain a relatively pure sample of QELike events with both a proton and neutron in the final state. An important background in this study is the set of QELike events without a neutron in the final state. Such events may be selected when the proton or pion re-interacts in the detector releasing secondary particles. The neutron reconstruction algorithm may subsequently tag the energy deposited by these particles. Charged final state hadrons with energies too low to form a reconstructed track could also be tagged close to the interaction vertex (see Sec. 3.5.1 for a discussion of tracking threshold). We investigate the geometric relationships between the track and reconstructed neutron candidate to form selections that can reduce these backgrounds.

### 7.2.2 Candidate Geometric Distributions

#### Opening Angles

Figure 7.1 showed the signal and sideband distributions for the opening angle between the reconstructed proton track and the neutron candidate. There are very few QELike events are present in the sideband.

The signal region is contaminated by events without final state neutrons (0-neutron). At low $\theta_{p,n}$, the 0-neutron fraction is dominated by QE which produces a single proton at the vertex. The proton may experience FSI before exiting the nucleus, but clearly did not eject neutrons into the final state. The large number of observed events in the Monte Carlo simulation at $\theta_{p,n} \sim 0^\circ$ means the neutron candidates are collinear with the proton. These candidates could be misreconstructed
Figure 7.1: Opening angle distributions before fitting. (top left) Signal, (top right) BlobSideBand, (middle left) MichelSideBand, (middle right) MicBlobSideBand, and (bottom) magnified legend. Legend entries above the dashed box are QELike event categories with proton and neutron in the final states. Legend entries inside the dashed box are those QELike categories without final state neutrons. Entries below the dashed box are QELikeNot categories
hits from protons after elastic or inelastic scattering in the detector. Inelastic scattering in the detector medium can also produce neutrons which could be mistaken as originating from the neutrino interaction.

At $\theta_{p,n} \sim 120^\circ$, the 0-neutron sample consists predominantly of 2p2h events with multiple proton final states. The two protons from 2p2h events should exit the nucleus back-to-back. There are also some 0-neutron QE contribution, in which case the fake neutron candidate could be caused by subleading protons from FSI.

**Opening Angle Against Blob-Vertex Distance**

Figure 7.2 shows the 2D event rate distributions in $\theta_{p,n}$ and the distance to vertex $R$ for the QELike MPN signal, the 0-neutron QE, and the 0-neutron 2p2h on the left. Three tentative cuts can be formed from these distributions that will reduce 0-neutron contamination: 1) $\theta_{p,n} > 30^\circ$, 2) $R > 400$ mm, and 3) $R < 1800$ mm.

The effects of the cuts are as follows. The $\theta_{p,n}$ cut largely removes the 0-neutron QE contamination for which the misreconstructed final state neutron candidates are collinear with the hadron tracks. As discussed previously, these candidates could be either energy deposits misreconstructed from the inelastically scattered charged hadron, or secondary neutrons that interacted very close to the end of the track. These events must be rejected since the neutrino interactions did not produce final state neutrons. Beyond $R = 1800$ mm, we observe another region relatively rich in 0-neutron QE. These candidates are 2 ~ 3 meters away from the vertex and could be secondary neutrons reinteracted further downstream in the detector, or they could be neutron candidates from another interaction in the detector, uncorrelated with the neutrino interaction. MINERvA’s tracker and ECAL region (Fig. 3.3) have a combined length of 3.2 m, the apothem of the inner detector
Figure 7.2: 2D Distributions in $\theta_{p,n}$ and blob distance $R$ to the vertex. Event rate (left) and ratio to all QELike (right) for MPN signal (top), 0-neutron QE (middle) and 0-neutron 2p2h (bottom). The horizontal dashed line in each plot marks $\theta_{p,n} = 30^\circ$ and the vertical lines define $R = 400$ mm and $R = 1800$ mm.
is only approximately 850 mm. QE protons are generally forward-going, and the secondary neutrons from the rescatter have a higher probability of traveling along the detector and be reconstructed. Events from 0-neutron 2p2h do not produce as many faraway neutron candidates because the final state particles often travel at wide angles towards the side of the detector and the neutrons they produce are more likely to exit without interacting at all.

Better understanding the nature of the events in these regions are necessary and will be part of the next steps to obtain a cross section result. Another investigation that may reduce 0-neutron contamination is to apply an elastically scattered constrained (ESC) cut similar to that used in Chapter 4 and described in Ref. [142]. The cut would preferentially select protons that did not inelastically scatter to reduce the fake neutron rate. However, such a cut may severely reduce the proton selection efficiency (final proton efficiency is \( \sim 10\% \) after the cut in Ref. [142]). Coupled with the already low neutron selection efficiency, we may not have enough statistics for the measurements. Some trade-offs clearly need to be made.

The effects of the angular cut and the distance cuts on event rate are shown below. The MPN signals are also referred to as the n-neutron sample since they contain at least one final state neutron, i.e., \( n \geq 1 \).
The combined cut removes nearly 2/3 of the 0-neutron contamination while reducing the QELikeNot background by nearly 60%. Ultimately, the cuts can increase the signal purity in the QELike category from 60% to 73%, at the expense of decreasing signal efficiency by half. While significant QELikeNot events remain, we have a good way to control them through the sidebands described in Sec. 7.2.1.

### 7.3 Sideband Constraints

We constrain the sideband distributions with the muon transverse momentum ($p_T^\mu$), similar to Ref. [148, 163]. Fitting the MC distributions to the data in each sideband with the $\chi^2$ metrics defined in Eq.(5.13) is a good starting point. For this study the regularization strength is $\lambda = 0$ as each sideband corresponds to a specific background type and we expect a simple $\chi^2$ minimization to suffice. This initial investigation is performed on the best simulation ”central value” estimate, and the effects of the other systematic “universes” in the multi-universe framework (refer

<table>
<thead>
<tr>
<th>Cat/Cut</th>
<th>No Cut</th>
<th>$\theta_{p,n}$</th>
<th>Min $R$</th>
<th>Max $R$</th>
<th>All Cuts</th>
</tr>
</thead>
<tbody>
<tr>
<td>QELike n-neutron</td>
<td>10730.</td>
<td>9727.</td>
<td>6157.</td>
<td>10510.</td>
<td>5196.</td>
</tr>
<tr>
<td>QELike 0-neutron</td>
<td>7231.</td>
<td>5273.</td>
<td>3805.</td>
<td>6864.</td>
<td>1892.</td>
</tr>
<tr>
<td>QELikeNot</td>
<td>12450.</td>
<td>10700.</td>
<td>6641.</td>
<td>12320.</td>
<td>5204.</td>
</tr>
<tr>
<td>QELike QE 0-neutron</td>
<td>3679.</td>
<td>2036.</td>
<td>2729.</td>
<td>3387.</td>
<td>1112.</td>
</tr>
<tr>
<td>QELike 2p2h 0-neutron</td>
<td>2741.</td>
<td>2492.</td>
<td>915.8</td>
<td>2671.</td>
<td>657.1</td>
</tr>
<tr>
<td>MC</td>
<td>30410.</td>
<td>25700.</td>
<td>16600.</td>
<td>29700.</td>
<td>12290.</td>
</tr>
<tr>
<td>Data</td>
<td>29810.</td>
<td>24700.</td>
<td>15980.</td>
<td>29030.</td>
<td>11330.</td>
</tr>
</tbody>
</table>

Table 7.2: Event rate after adjusting each cut.
Section 7.3 Sideband Constraints

Figure 7.3: MPN fit on the single neutral pion (top left), single charged pion (top right), and multi-pions (bottom) events.

to Sec. 5.7.3 for a discussion of the systematics) have not been studied. . The $p_T^\mu$ distributions and the fitting parameters for the central value (CV) universe are shown in Fig. 7.4 and Fig. 7.3 respectively. Figure 7.6 shows the $\theta_{p_T}$ distributions before and after the fit.

As can be seen in Fig. 7.3, the weight required at the highest $p_T^\mu$ bins ($p_T^\mu > 0.5 \text{ GeV}/c$) is less than 25% from unity for most sidebands, while the single charged pion category is unchanged from the fit. Below $p_T^\mu > 0.5 \text{ GeV}/c$, the weights modify our MC models by less than 50%. The fitted $p_T^\mu$ sideband distributions (Fig. 7.5) are in good agreement within uncertainties. There is one outlier data point in the
fitted MicBlobSideBand at $p_T^\mu \sim 1.5$ GeV/c. We could increase the multi-pion category strength by 50% to achieve better agreement in MicBlobSideBand, but doing that will cause the MC to over-predict data by $20 \sim 30\%$ in the BlobSideBand and MichelSideBand. The increase won’t affect the Signal since the multi-pion contribution account for only 10% in that bin. It is likely that the inclusion of systematic uncertainties will help to explain the oulying data points in the fit.

### 7.3.1 Sidebands Discussion

The opening angle distributions in Fig. 7.6 and Fig. 7.7 show good agreement in the sidebands. The MC in the sidebands over-predicts the data at $\theta_{p,n} < 40^\circ$. Decreasing the multi-pion contributions in the sidebands in that region will bring better agreements. In the Blob and Michel sidebands for $\theta_{p,n} > 100^\circ$, we see an approximately linear trend in the ratio between data and MC, after fitting. As the constraints on $p_T^\mu$ do not directly control the opening angle, the trends indicate that the MC prefers to have wider angles between the reconstructed track and the (fake or not fake) neutron candidates. A visual inspection indicates that a reduction in the $1\pi^\pm$ strength as a function of $\theta_{p,n}$ beyond 100° might improve agreement.

In general, we may have to improve the fit by including information from the opening angles in the next steps because the muon-kinematics based constraints cannot fix issues with the hadronic interaction models in the background. For now, the discrepancies in $\theta_{p,n}$ sidebands have a limited impact on the signal region since both the $1\pi^\pm$ and $n\pi$ backgrounds are small.

In the $p_T^\mu$ signal distribution, the MC over-predicts data by $\sim 60\%$ below 0.5 GeV/c. In this region the dominant contributions are the n-neutron 2p2h, RES, and 0-neutron 2p2h. At the lowest $p_T^\mu$, their contributions have to be reduced by 75%.
There are mismodellings in this region, and further study needs to be done to pinpoint whether the problem lies in the interaction model, or the detector model. In the $\theta_{p,n}$ variable the excess MC events occur beyond $60^\circ$. 
Figure 7.4: $p_T^\mu$ event rates. From top to bottom: Signal, BlobSideBand, MichelSideBand, MicBlobSideBand. From left to right: before fit and after fit.
Figure 7.5: $p_T^\mu$ ratio to MC. From top to bottom: Signal, BlobSideBand, MichelSideBand, MicBlobSideBand. From left to right: before fit and after fit.
Figure 7.6: $\theta_{p,n}$ event rates. From top to bottom: Signal, BlobSideBand, MichelSideBand, MicBlobSideBand. From left to right: before fit and after fit.
Figure 7.7: $\theta_{p,n}$ event rates. From top to bottom: Signal, BlobSideBand, MichelSideBand, MicBlobSideBand. From left to right: before fit and after fit.
7.4 Transverse Kinematic Imbalances

A deliverable for this analysis is measuring the single-TKI variables analogous to Chapter 4 and Ref. [130], with a neutron in the final state. For demonstration, we have chosen to present the $\delta p_T$ variable in this section. We also form an additional variable, $\delta \phi_{TT}$, that measures the angular deviation between $-\delta p_T$ and the transverse direction of the neutron candidate. For a perfectly balanced system, we expect $\delta \phi_{TT} = 0^\circ$, non-zero $\delta \phi_{TT}$ is produced due to nuclear effects such as FSI. Figure 7.8 shows the definitions of both $\delta p_T$ and $\delta \phi_{TT}$.

While many Single-TKI variables exist (Ref. [164] and Chapter 4 in this thesis), we show only $\delta p_T$. Measurements on other Single-TKI variables are straightforward once we have a framework to measure $\delta p_T$ reliably.

7.4.1 $\delta p_T$

The transverse kinematic imbalances (TKI) are a powerful set of variables capable of exposing shortcomings in our models of the nucleus ([1, 164]). $\delta p_{Tx}$ and $\delta p_{Ty}$, the main measurement results of Chapter 4, are the projections of the $\delta p_T$ vector into and perpendicular to the reaction plane containing the neutrino and muon momenta. Figures 7.9 to 7.16 show $\delta p_T$ distributions for the Signal, BlobSideBand, MichelSideBand, and MicBlobSideBand, before and after fits.

For the signal distributions in Fig. 7.9 and 7.10, we observe higher rates for events whose hadronic candidates are on opposite sides of the neutrino-muon plane. These are the events with balancing particles on both sides of the neutrino-muon plane. For events with same-side hadronic final states $(++, --)$, there must be unobserved particles exiting on the other side to balance total momentum. In general, for $+-$
\[ \delta \vec{p}_T = \vec{p}_T^p + \vec{p}_T^\mu \]

Figure 7.8: $\delta p_T$ and $\delta \phi_{TT}$. The neutrino momentum points out of the page, and the plane perpendicular it forms the transverse plane. $p_T^\mu$ and $p_T^p$ are the transverse components of muon and proton momenta. The sum of the two momenta is $\delta p_T$. Hadronic final states could exit the nucleus on either side of the neutrino-muon plane. The "+" side is defined by the direction of $\vec{p}_\nu \times \vec{p}_T^\mu$, with the other side designated as "-". For muon-proton-neutron final states, we could thus define four sub-samples separated into quadrants: "++", "+-", "-+" and "--". $\delta \phi_{TT}$ is the angle between the neutron candidate and $-\delta p_T$. Again the sample is separated into "+" and "-" sub-samples depending on the side $-\delta p_T$ resides. The values of $\delta \phi_{TT}$ are negative if the neutron candidate is closer to the muon, regardless of which side $-\delta p_T$ is on.
and $-+$ events, we do not expect the neutron momentum to exactly balance the proton momentum. Fermi motion would bias the final state proton’s momentum. This bias will propagate to the FSI neutrons as well.

In each quadrant, 0-neutron QE events dominate at low $\delta p_T$ as the single proton transverse momentum should nearly balance the transverse muon momentum. While the MC predicts nearly the same event rates in $++$ and $-+$ quadrants, there are fewer data events for proton exiting into the $-$ direction. The observation is consistent with the asymmetry in $\delta p_{T\mu}$ presented in Chapter 4, where there are more data towards the $+$ side of $\delta p_{T\mu}$. Pion-absorption FSIs could cause the asymmetry in the initially pion production processes to manifest through QELike RES events. The sign in the two measurements is defined in the same way for consistent comparisons.

We observe the same MC excess in the ($-+$) quadrant in the BlobSideBand (Fig. 7.11 and 7.12), but not so much in the MichelSideBand and the MicBlobSideBand (Fig. 7.13 to 7.16). The crucial difference between the BlobSideBand and the others is its abundance of $1\pi^0$ events. A major source of $1\pi^0$ final state is the $\Delta^+(1232)$ production reaction in Eq. (1.36), reproduced below:

$$\nu + n \rightarrow \mu + p + \pi^0.$$  

In contrast, $1\pi^+$ events are formed by the $\Delta^{++}(1232)$ production reaction in Eq. (1.37):

$$\nu + p \rightarrow \mu + p + \pi^+.$$  

If the source of asymmetry is due only to the $\pi^0$ events, it implies there are differences between the production or the decay of $\Delta^+$ and $\Delta^{++}$. That said, we cannot rule out the possibility that $\Delta^{++}$ also produces an asymmetry that is suppressed by event
Figure 7.9: $\delta p_T$, before fitting. Signal distribution broken down into the quadrants. (top) Event Rate, (bottom) Ratio to MC.
Figure 7.10: $\delta p_T$, after fitting. Signal distribution broken down into the quadrants. (top) Event Rate, (bottom) Ratio to MC.
Figure 7.11: $\delta p_T$, before fitting. BlobSideBand distribution broken down into the quadrants. (top) Event Rate, (bottom) Ratio to MC.
7.4 Transverse Kinematic Imbalances

![Graphs showing event rate and ratio to MC against delta p_t (GeV/c)]

Figure 7.12: $\delta p_T$, after fitting. BlobSideBand distribution broken down into the quadrants. (top) Event Rate, (bottom) Ratio to MC.
Figure 7.13: $\delta p_T$, before fitting. MichelSideBand distribution broken down into the quadrants. (top) Event Rate, (bottom) Ratio to MC.
Figure 7.14: $\delta p_T$, after fitting. MichelSideBand distribution broken down into the quadrants. (top) Event Rate, (bottom) Ratio to MC.
Figure 7.15: $\delta p_t$, before fitting. MicBlobSideBand distribution broken down into the quadrants. (top) Event Rate, (bottom) Ratio to MC.
Figure 7.16: $\delta p_T$, after fitting. MicBlobSideBand distribution broken down into the quadrants. (top) Event Rate, (bottom) Ratio to MC.
Finally, a linear growth at high $p_T^\mu$ in the ratio of data to MC exists for all four quadrants in the multi-pion dominated MicBlobSideBand. A hint of the same trend exists in the BlobSideBand, where there is a sizeable muti-pion background.

### 7.4.2 $\delta\phi_{TT}$

The final variable we investigate in this thesis is $\delta\phi_{TT}$. Figures 7.17 to 7.22 are the equivalents of Figs. ?? to ?? shown earlier in the discussion of $\delta p_T$. They show distributions for the Signal, BlobSideBand, MichelSideBand and the MicBlobSideBand samples, respectively.

First, we observe the same deficiency in data on the $-\delta\phi$ side in the BlobSideBand that is not obvious in other sidebands. Second, all samples contain longer tails towards negative $\delta\phi_{TT}$. Negative $\delta\phi_{TT}$ means the neutron candidate is on the side of $\delta p_T$ towards the muon. Some neutron candidates could be very close to the transverse muon vector; therefore, some of the candidates may be misreconstructed Bremsstrahlung photons or ionized electrons and $\delta$-rays from the muon (“muon fuzz”). As a next step, muon fuzz-induced neutrino candidates need to be identified and eliminated.
Figure 7.17: $\delta \phi_{TT}$, before fitting. Signal distributions on each side of neutrino-muon plane (top) Event Rate, (bottom) Ratio to MC.

7.5 Next Steps in this MPN Analysis

This section outlines the steps needed to measure the cross section. Effort can be broadly categorized as either removing backgrounds, or creating more elaborate background constraints.

7.5.1 Background reduction

To achieve stronger background reduction we could apply a more stringent criteria on the clusters used to reconstruct neutron candidates. We need to exclude clusters found near the muon to avoid reconstructing clusters induced by the muon fuzz. We
would investigate the proton ESC cut to eliminate events whose proton has inelastically scattered in the detector. This will improve the proton energy resolution as well as reducing the number of secondary neutrons in the detector. The strength of the cut needs to be determined to strike a balance between the purity and the efficiency of the event selection. We also need to better understand the characteristics of the 0-neutron QE events to find new ways to remove them.
Figure 7.19: $\delta \theta_{\pi \pi}$, before fitting. BlobSideBand distributions on each side of neutrino-muon plane (top) Event Rate, (bottom) Ratio to MC.

### 7.5.2 Improve Background Constraints

We need to fix the $\theta_{p,n}$-dependence of the $1\pi^\pm$ (mostly $\pi^+$) background. A possible solution could be reweighting the true $\pi^+$-proton angular separation according to the reconstructed $\theta_{p,n}$. The background model also needs to account for the asymmetry in the $1\pi^0$ events, which we may be able to model with a single correction factor in each quadrant. Alternatively, we could perform corrections in the 2D $p_T\pi$-TKI space for each TKI variable in each quadrant. Such a fit might also remove the $\delta p_T$-dependence of the multi-pion background in the $\delta p_T$ MicBlobSideBand. It will also be instructive to study the final state composition of each background category.
Figure 7.20: $\delta \phi_{TT}$, after fitting. BlobSideBand distributions on each side of neutrino-muon plane (top) Event Rate, (bottom) Ratio to MC.

across the sidebands to ensure the extrapolation from the sidebands to the Signal is reasonable.

Like the hydrogen analysis, unfolding, efficiency correction, flux normalization, and target correction can follow after reliable event selection can be established.
7.5 Next Steps in this MPN Analysis

Figure 7.21: $\delta\phi_{TT}$, before fitting. MichelSideBand distributions on each side of neutrino-muon plane (top) Event Rate, (bottom) Ratio to MC.
Figure 7.22: $\delta \phi_{TT}$, after fitting. MichelSideBand distributions on each side of neutrino-muon plane (top) Event Rate, (bottom) Ratio to MC.
Figure 7.23: $\delta \phi_{TT}$, before fitting. MicBlobSideBand distributions on each side of neutrino-muon plane (top) Event Rate, (bottom) Ratio to MC.
Figure 7.24: $\delta\phi_{TT}$, after fitting. MicBlobSideBand distributions on each side of neutrino-muon plane (top) Event Rate, (bottom) Ratio to MC.
Chapter 8

Conclusion

A phenomenological study on the GENIE's implementations of the binding energy has been introduced in Chapter 2, which established the need to use the correct excitation energy and nuclear potential energy in modeling neutrino-nucleus interactions on the complex nuclei.

Chapter 4 presented an analysis published in peer-reviewed journal. In the analysis, we presented the measurement on a set of transverse variables $\delta p_{Tx}$ and $\delta p_{Ty}$, that are sensitive to different nuclear effects. The sample selection is based on the quasielastic-like (QELike) definition in the neutrino mode. This selection requires a negatively charged muon and a proton with momentum between 0.8 GeV/c and 1.2 GeV/c, and there should not be any mesons in the final states.

Both variables are projections of the single transverse kinematic imbalances (TKI). The single-TKI is the sum of the muon's and proton's momentum in the plane perpendicular to the incoming neutrino’s direction. $\delta p_{Tx}$ is sensitive to the Fermi motion of the nucleus, while $\delta p_{Ty}$ is sensitive to how GENIE, our MC simulation generator of choice, implements binding energy. Our results disfavor GENIE's
binding energy implementation. We also made comparisons to NEUT, NuWro, and the GiBUU models. Of all the models, spectral function (SF) models such as the NuWro SF provide the best agreement in $\delta p_{Ty}$. Additionally, we observe an asymmetry in $\delta p_{Tx}$, where we expect perfect symmetry. We believe the asymmetry comes from the underlying resonant pion production events, whose pions are absorbed by the nucleus through FSI.

In Chapter 5 and Chapter 6, we have presented the first measurement of antineutrino quasielastic reaction on the free protons in hydrogen since the 1980s. We utilized the QELike signal definition in the antineutrino mode as the basis of this analysis. In the QELike definition, we select events with a single positively charged muon, no protons with kinematic energy beyond 110 MeV, and no mesons in the final states. Our target material is the hydrocarbon scintillators in the tracker region in MINERvA. The number of hydrogen to carbon atoms in the scintillator is approximately 1:1, and we increase the fraction of selected hydrogen events by selecting the directions of the outgoing neutrons. To do so, we developed a novel neutron reconstruction and tagging algorithm and calculated the direction of the outgoing neutron. Antineutrino interactions on the free proton experiences no nuclear effects and we could precisely calculate the direction of the neutron. Selecting events whose neutron is close to the predicted direction boosts the fraction of hydrogen interactions. The MC model used for the analysis took lesson from the previous analysis and apply a reweight that transform GENIE’s initial state model for carbon to NuWro’s SF model.

We measured the single differential cross section $\frac{d\sigma}{dQ^2}$ from hydrogen and extracted the axial form factor $F_A$ based on Eq. 2.1. The extraction needs parameterizations of the electromagnetic (EM) form factors of the proton and neutron as input
parameters. We extracted $F_A$ using both the BBBA2005\cite{62} and BBBA2007\cite{57} EM form factors. We observe deviation in the axial form factor in the highest $Q^2$ region and shapes that deviate from a simple dipole parameterization of the $F_A$. The new result will provide constraints on the normalization and shape of the axial form factor, which were previously extracted from deuterium bubble chambers.

The methods developed in the Chapter 5 allows the extraction of axial form factors on free protons from hydrogen embedded in material with a significant number fraction of heavier nuclei. The same technique is applicable to future detectors and experiments with a mixture of hydrogen and heavier nuclei to make more precise measurement of the axial form factor.

In Chapter 7, we presented a preliminary event selection for QELike events with proton and neutron final states. We plot the TKI variables calculated using the muon and proton. We also define a new TKI variable $\delta\phi_{TT}$, the transverse angle between the neutron candidate and the negative of the combined proton-muon momentum in the transverse plane. Finally, we propose a list of steps necessary to make cross section measurements.
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Appendices
Appendix A

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