The Bose-Einstein correlations in CDFII experiment

PhD Thesis

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Abstract

We present the results of a study of $p\bar{p}$ collisions at $\sqrt{s} = 1.96 TeV$ collected by the CDF-II experiment at Tevatron collider. The Bose-Einstein correlations of the $\pi^\pm\pi^\pm$ two boson system have been studied in the minimum-bias high-multiplicity events. The research was carried out on the sample at the size of 173761 events. The two pion correlations have been retrieved. The final results were corrected to the coulomb interactions. Two different reference samples were compared and discussed. A significant two-pion correlation enhancement near origin is observed. This enhancement effect has been used to evaluate the radius of the two-pion emitter source. We have used the TOF detector to distinguish between $\pi$ and $K$ mesons. The $C_2(Q)$ function parameters have also been retrieved for the sample containing only tagged $\pi$ mesons. A comparison between four different parametrizations based on two different theoretical approaches of the $C_2(Q)$ function is given.
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Chapter 1

Introduction

The method of photon intensity interferometry was invented in the mid-1950s by Hanbury-Brown and Twiss for the measurement of stellar dimensions and is sometimes called the HBT method. In 1959-1960 G. Goldhaber, S. Goldhaber, W. Lee and A. Pais discovered that identical charged pions produced in \( p - \bar{p} \) annihilation are correlated (the GGLP effect). Both the HBT and the GGLP effects are based on Bose-Einstein correlations (BEC). Afterward Fermi-Dirac correlations for nucleons have also been observed. Both these correlation effects can be viewed as a consequence of the symmetry (antisymmetry) properties of the wave function with respect to permutation of two identical particles with integer (half-integer) spin and are thus intrinsic quantum phenomena.

When the two particles are near in momentum space, the HBT and GGLP effects cause an enhancement in the two identical boson correlation function, what is result of the Bose condensation.

Detailed studies of the two-particle Bose-Einstein correlations allow to determination of the space and time characteristic of the source of bosons, what gives the possibility to analyze the characteristic of the hadronization region.

Let’s assume, that we have \( p - \bar{p} \) annihilation. We can imagine the point of this annihilation as a middle of the source, this source produces \( \pi \) mesons with some emission probability. For example, with the Gauss emission probability \( F(r) = \frac{1}{(2\pi R^2)^{3/2}} e^{-\frac{r^2}{2R^2}} \), where the \( R \) is the source radius. In this area, the arose \( \pi \) mesons, with the same charge, exchanges among themselves the part of their energy. These correlations are called the Bose-Einstein Correlations. The particles which leave this area are affected by this correlation and, in first order, do not correlate among
themselves anymore. Therefore the effect of BEC is frozen in the information which this particles are carrying with them. Some source characteristics, like the radius, can be found by the study of this correlation effect using the fit of the experimental data with, so called, $C_2$ function, derived on different theoretical approaches.

With the aim to explore the correlations of Bose-Einstein type we have used the excellent properties of the CDF detector especially its tracking system to study the soft hadron processes in the 1.96 TeV region.

The shapes of Bose-Einstein correlations functions for $\pi^{\pm}$ mesons were experimentally established in the FNAL E735-Collaboration experiment [16], CERN UA1-Minimum-Bias Collaboration experiment [17], LEP experiments ALEPH [18], ZEUS Collaboration at HERA [19], OPAL [20] and DELPHI [21] Collaborations took a look on Bose-Einstein correlations for $\pi^0$ and $K$ mesons respectively.

In Chapter 2 the theoretical background of the BEC is presented. The Chapter 3 describes the experimental setup: the Tevatron collider and the CDF-II detector. In the Chapter 4 the used data sample for this study and the selection cuts are described. In the Chapter 5 the result of this analysis are presented and compared with the results from other experiments. Finally a summary and conclusion are presented.
Chapter 2

Theoretical Background

2.1 Identical particles

In classical approach there are two ways in which one might distinguish between particles. The first method relies on differences in particle properties, such as mass, electric charge, and spin. Even if the particles have equivalent physical properties, there remains a second method for distinguishing between them, which is to track the trajectory of each particle.

The problem comes with quantum mechanic. The particles do not possess definite positions during the periods between measurements. They are governed by wave functions that give the probability of finding a particle at each position. As time passes, the wave functions tend to spread out and overlap. Therefore we cannot determine which of the particle position correspond to that measured earlier. Therefore identical quantum mechanical particles are completely indistinguishable from one another.

Suppose that we have two identical particles $n_1$ and $n_2$. The wave function for this two particle system is $\Psi(n_1, n_2)$. Because the identical particles cannot be distinguished by any interaction, any operator corresponding to a physical observable must treat all the particles in the same way. Such operators are called symmetric operators.

\[
A(n_1, n_2) = A(n_2, n_1)
\]  

(2.1)

Let’s define operator $P$ as a operator of particles permutation.
2.2 Bose-Einstein correlations

To illustrate the origin of Bose-Einstein correlations, we shall follow the explanation given in [4]. The correlation function, \( C_2 \), of two identical bosons is defined as

\[
C_2(p_1, p_2) = \frac{P(p_1, p_2)}{P(p_1)P(p_2)},
\]

where \( p_1 \) and \( p_2 \) are the momenta of the two particles. \( P(p_1, p_2) \) is the two-particle probability density and \( P(p_1) \) and \( P(p_2) \) represent single-particle probability densities.

Let us assume, that we have two identical bosons (for example, two like signed pions) which are emitted incoherently from the source (see Fig. 2.1). Let \( \Psi_{p_i}(r_i) \) be the wave function of an emitted particle. The probability to observe a particle with momentum \( p_i \) is

\[
P(p_i) = \int |F(r_i)\Psi_{p_i}(r_i)|^2 d^3r_i,
\]

where \( F(r_i) \) is the emission probability amplitude at space point \( r_i \). The inco-

\[P\Psi(n_1, n_2) = c\Psi(n_2, n_1)\]  

If we apply this operator twice, we have to bring up the same wave function. \( P^2\Psi(n_1, n_2) = \Psi(n_1, n_2) \) therefore \( c^2 = 1 \) so the \( c = \pm 1 \). Now let’s look what is the relation between \( \Psi(n_1, n_2) \) and \( \Psi(n_2, n_1) \):

\[
A(n_1, n_2)\Psi(n_1, n_2) = a\Psi(n_1, n_2), A(n_1, n_2)\Psi(n_2, n_1) = b\Psi(n_2, n_1)
\]

so \( a = \pm b \) and \( \Psi(n_1, n_2) = \pm\Psi(n_2, n_1) \) ‘+’ for symmetric wave function (bosons) and ‘-’ for antisymmetric wave function (fermions). Nothing between, for two identical particle system, does not exist. It’s obvious that operator \( P \) commute with operator \( A \) and also with Hamiltonian operator \( H \), which is symmetric. Therefore, if a particle is once boson it remains boson until its extinction.
herent summation in Eq. 2.5 means that the individual space points are independent sources of emission. Similarly, the probability to observe two particles with momenta $p_1$ and $p_2$ is

$$P(p_1, p_2) = \int \int |\Psi_{p_1, p_2}(r_1, r_2)|^2 f_{p_1}(r_1)f_{p_2}(r_2)d^3r_1d^3r_2,$$  

(2.6)

where $\Psi_{p_1, p_2}(r_1, r_2)$ is two particle wave function and $f_{p_i}(r_i) = |F(r_i)|^2$ is the source emission probability. Let us assume that the single particle wave function is a plane wave $\Psi_p(r) \sim exp(i p r)$. Assuming further that we deal with two identical bosons, we get a symmetrical two particle wave function

$$\Psi_{p_1, p_2}(r_1, r_2) = \frac{1}{\sqrt{2}}[e^{i(p_1r_1+p_2r_2)}+e^{i(p_1r_2+p_2r_1)}].$$  

(2.7)

With this wave function one obtains [4]

$$P(p_1, p_2) = |\tilde{f}(0)|^2 + |\tilde{f}(p_1 - p_2)|^2,$$  

(2.8)

where $\tilde{f}$ is the Fourier transform of $f$. If we denote $q$ to be the momentum difference $p_1 - p_2$. The second-order correlation function will take a form:

$$C_2(p_1, p_2) = \frac{P(p_1, p_2)}{P(p_1)P(p_2)} = 1 + |\tilde{f}(q)|^2.$$  

(2.9)

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure2.1.png}
\caption{Two identical bosons emitted from the same source.}
\end{figure}

The BEC experiments have shown from the very beginning that the extrapolation of the correlation function to $q = 0$ usually never led to the maximum value
$C_2(0)$ function permitted by Eq. 2.9. To take into account empirically this effect experimentalists introduced into the correlation function a correction factor $\lambda$. Afterwards the Eq. 2.9 has been modified as follows

$$C_2(p_1, p_2) = 1 + \lambda |\tilde{f}(q)|^2.$$  \hfill (2.10)

$\lambda$ has been postulated to be limited by $(0, 1)$ and called the incoherence factor, $\lambda = 0$ leads to a totally coherent source and $\lambda = 1$ to a totally chaotic one. In the case of coherent source instead of Eq. 2.6 with incoherent sum, the two particle emission probability reads:

$$\left| \int \Psi_{p_1,p_2}(r_1,r_2)f_{p_1}(r_1)f_{p_2}(r_2)d^3r_1d^3r_2 \right|^2.$$  \hfill (2.11)

In this case the $C_2(p_1,p_2)$ function will be equal to unit $C_2(p_1,p_2) = 1$ [10]. We have to stress here, that $\lambda$ is not just a pure factor carrying the information about the source coherence. There is a strong correlation between the source radii, presented in the density function ($\tilde{f}$), and this empirical factor [4].

The shape of the correlation function $C_2$ for Gaussian density distribution of the source will be

$$C_2(Q) = 1 + \lambda e^{-Q^2 R^2},$$  \hfill (2.12)

where $Q^2 = -q^2$ is a relativistic invariant, $Q^2 = M^2 - 4m^2$. One can obtain the $C_2$ function by measuring a single scalar quantity $Q$, what can by also a deficiency. The associated quantity $R$ in this conventional parametrization of the correlation function is neither a radius nor a lifetime, but a combination of these two quantities.

**The limitations of the wave function formalism.**

Up to now at the construction of $C_2(Q)$ function we have relied on two basic quantities: the source emission probability, which was usually assumed to have a Gaussian form, and the two boson plane wave function. In addition to that we have assumed an incoherent summing of emission sources (see Eq. 2.6) and only then we have corrected the expression of $C_2(Q)$ function on a possible coherence present in process.

There are several limitations of the wave function formalism presented in previous
section. First of all, the source coherence is introduced into this model empirically. The $C_2(Q)$ function was calculated for two extreme cases, for the pure chaotic emission source which led to $C_2(Q) = 1+e^{-R_0Q^2}$ and pure coherent source which resulted in $C_2(Q) = 1$, and the parameter $\lambda$ was introduced with a regard to the two above mentioned cases. We will see in the next section that this approach is not correct.

Assumption that two and only two bosons are produced in collision is made. In Eq. 2.7 the two bosons plane-wave function is assumed without any correction to the many body, strongly interacting system which is in real created. To obtain a correct wave function, one would have to solve the Schrodinger equation which describes the two bosons in presence of all other produced particles.

Another thing is that although at the beginning we are dealing with two independent variables, momenta $p_1$ and $p_2$, the $C_2(Q)$ function depends only on their difference $p_1 - p_2$. One may expect that the number of degrees of the freedom will remain unchanged and the $C_2(Q)$ function will depend also on their sum $p_1 + p_2$.

At the correlation function construction we have ignored isospin (the isospin part of wave function) and thereby the $C_2(Q)$ function will be the same for charged and neutral particles and will lead to correlations between particles and antiparticles.

### 2.3 Quantum optical methods in BEC

Quantum optical methods are based on coherent states $|\alpha\rangle$ and squeezed coherent states $|\beta\rangle_s$. A coherent state is a specific kind of quantum state of the quantum harmonic oscillator. Mathematically the coherent state is defined to be the eigenstate of the annihilation operator $a$

$$a|\alpha\rangle = \alpha |\alpha\rangle.$$  \hspace{1cm} (2.13)

$\alpha$ is a complex number which can be represented as $\alpha = |\alpha|e^{i\theta}$, where $|\alpha|$ and $\theta$ are called the amplitude and phase of the state. The eigenstate of the annihilation operator has a Poissonian number distribution. The solution of the eigenvalue Eq. 2.13, using the representation in the basis of Fock states can be written as:

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle,$$ \hspace{1cm} (2.14)
where $|n\rangle$ are energy (multiplicity) eigenvectors of the Hamiltonian. This particle multiplicity has a Poissonian distribution with the mean value equal to the $|\alpha|^2$ and variance $(\Delta n)^2 = |\alpha|^2$.

It should be noted that the $|\alpha\rangle$ does not refer to a Fock state with a definite multiplicity or energy, it represents a whole spectrum of states with different multiplicities. For example, if $\alpha = 1$, one should not interpret the state $|\alpha = 1\rangle$ as a single Fock state with the multiplicity 1, but as a superposition of the multiplicity states for which the particle multiplicity is distributed by the Poisson distribution with the mean value equal to 1.

In terms of quantum oscillator coordinate ($x$) and momentum ($p$):

$$\hat{x} = \sqrt{\frac{\hbar}{2m\omega}} (a^+ + a)$$
$$\hat{p} = i \sqrt{\frac{\hbar m\omega}{2}} (a^+ - a)$$ (2.15)

the Heisenberg uncertainty relation reads: $\Delta p \Delta x = \frac{\hbar}{2}$. By $\hat{x}$, $\hat{p}$ we understand operators while their eigenvalues are denoted as by $x$, $p$.

Using Eq. 2.15 and 2.13 we can write the relation between quantum oscillator coordinates, position $x$ and momentum $p$, and the coherent state amplitude $|\alpha\rangle$ and phase $\Theta$:

$$X = |\alpha| \cos(\theta)$$
$$P = |\alpha| \sin(\theta),$$ (2.16)

where $X$ and $P$ are now dimensionless coordinates ($X \rightarrow x \sqrt{\frac{m\omega}{2\hbar}}$, $P \rightarrow p \sqrt{\frac{1}{2m\omega}}$). In terms of this dimensionless coordinates the Heisenberg uncertainty relation reads: $\Delta P \Delta X = \frac{1}{4}$.

From the Fig. 2.2, for a coherent state, one can retrieve the relation between the phase fluctuation and intensity amplitude: $\Delta \Theta |\alpha| \simeq \frac{1}{2}$ (Using the simple derivation $\Delta |\alpha|^2 = 2|\alpha|\Delta \alpha$ and facts that $\langle n \rangle = |\alpha|$ and $\Delta n = |\alpha|$ one obtains that $\Delta |\alpha| = \frac{1}{2}$. Now, at $\alpha \gg 1$ we can use approximation that the area $\Delta X \Delta P = \frac{1}{4}$ is equal to the area $\Delta |\alpha|\Delta \Theta = \frac{1}{2}|\alpha|\Delta \Theta$, where $|\alpha|\Delta \Theta$ is the length of the curve given by the radius $|\alpha|$ and angle difference $\Delta \Theta$). From this we can see that there is a tradeoff between multiplicity uncertainty and phase uncertainty $\Delta \Theta \Delta n \simeq \frac{1}{2}$ what can be sometimes interpreted as the multiplicity-phase uncertainty relation. This is not a formal Heisenberg uncertainty relation, as there
is no uniquely defined phase operator in quantum mechanics.

![Figure 2.2: Phase space plot of a coherent state.](image)

In general, the Heisenberg uncertainty relation reads: \( \Delta p \Delta x \geq \frac{\hbar}{2} \). In coherent states this uncertainty relation is saturated, i.e. the product of uncertainties reaches its minimum. In addition, the coherent states require that the uncertainties in dimensionless variables \( P \) and \( X \) are equal: \( \Delta P = \Delta X \).

If the uncertainty is not balanced between \( X \) and \( P \) (\( \Delta X \neq \Delta P \)), the state is now called a squeezed coherent state. It has been found in [5] that squeezed states appear naturally when one deals with rapid phase transitions (explosions).

It is assumed that the fast phase transition between the phase \( a \), described by the free field annihilation and creation operators \( a, a^+ \), and the phase \( b \), described by the corresponding squeezed operators \( b, b^+ \) is fast enough for the relation between the creation and annihilation operators remains unchanged. This can be mathematically expressed by postulating at the moment of this transition the following relations:

\[
\hat{x} = (1/\sqrt{2E_b})(b^+ + b) = (1/\sqrt{2E_a})(a^+ + a) \\
\hat{p} = i\sqrt{(E_b/2)}(b^+ - b) = i\sqrt{(E_a/2)}(a^+ - a) \tag{2.17}
\]

where, \( E_a \) and \( E_b \) are the characteristic oscillator mode constants reflecting the basic oscillator frequencies in phase \( a \) and \( b \), respectively (using \( c = 1 \) and \( h = 1 \), \( E_a = m\omega_a \), with \( \omega_a \) being a characteristic oscillator frequency in the phase \( a \)). From this equation, valid for each momentum \( p \), follows the relation between the \( a \) and \( b \)
operators.

\[ a = b \cosh(r) + b^+ \sinh(r) \]
\[ a^+ = b \sinh(r) + b^+ \cosh(r), \quad (2.18) \]

where \( r = r(p) = \frac{1}{2} \log(E_a/E_b). \)

*Expansions in terms of coherent states.* Coherent states form a complete set of states so any arbitrary state can be expressed in terms of these states. For a pure coherent state the density operator is

\[ \rho = |\alpha> <\alpha|. \quad (2.19) \]

For a squeezed state the density operator can be expressed in terms of coherent states:

\[ \rho = |\beta>_s <\beta>_s = \int P(\alpha) |\alpha> <\alpha| d^2\alpha, \quad (2.20) \]

where \( P(\alpha) \) is a weight function with the meaning of a probability. In this representation all correlation functions can be expressed in terms of the creation and annihilation operators \( a^+ \) and \( a \).

The knowledge of \( P(\alpha) \) practically means the knowledge of the density matrix and thereby it also describes the squeezed state in question. The exact form of \( P(\alpha) \) is not accessible and one has to choose it. The most privileged form of \( P(\alpha) \) is the Gaussian one, which introduces a mathematical simplification and is frequently met in many-body physics [4].

On high multiplicity production we can look as on a fast phase transition from a pion liquid, described by the operators \( b \) and \( b^+ \), to the free pion system, described by the free creation and annihilation operators \( a \) and \( a^+ \). Fast enough to have the relations between annihilation and creation operator unchanged (Eq. 2.17). If one has the density operator for the pion liquid (Eq. 2.20) we can say something about the space-time characteristic of the source. We can construct it using the known free pion system operators and the chosen weight function \( P(\alpha) \), which is taken to be Gaussian and its parameters, found in experiment, say what are the source dimensions.
2.4. MODEL SOURCES FOR PARTICLE EMISSION

**Correlation functions.** The first-order correlation function reads \[4\]

\[ C_1(r_1, r_2) = Tr[\rho \pi^+ (r_1), \pi(r_2)]. \quad (2.21) \]

The \( n \)^{th}-order correlation function \( C_n \) is defined as

\[ C_n(r_1, ..., r_n, r_{n+1}, ..., r_{2n}) = Tr[\rho \pi^+ (r_1), ..., \pi^+(r_n), \pi(r_{n+1}), ..., \pi(r_{2n})]. \quad (2.22) \]

Here the \( \rho \) is the density matrix defined by Eq. 2.20 and \( \pi(r) \) is the Fourier expansion of an arbitrary field in second quantization

\[ \pi(r) = \sum_p [a_p e^{-ipr} + a_p^* e^{ipr}]. \quad (2.23) \]

Pure coherent or pure chaotic fields are just extreme cases. In general one expects a superposition of the coherent and chaotic fields:

\[ \pi = \pi_{\text{coherent}} + \pi_{\text{chaotic}}. \quad (2.24) \]

This leads to the final shape of \( C_2 \) function for the Gaussian parametrization \[4\]

\[ C_2(Q) = 1 + 2p(1-p)e^{-R^2Q^2} + p^2 e^{-2R^2Q^2}, \quad (2.25) \]

where \( p \) is the chaoticity, which varies between 0 (for the purely coherent sources) and 1 (for the totally chaotic sources). We can now see that Eq. 2.12 is a particular case of Eq. 2.25 with \( p = 1 = \lambda \). The presence of coherence introduces a new term into the \( C_2(Q) \) function.

2.4 Model sources for particle emission

A lot of efforts at study of the BEC effect was devoted to investigation of emission sources \[10\] \[11\]. In this section a brief review some of them is done.

The most extensively used parametrization corresponds to particle emission from a Gaussian source, used by Goldhaber, Goldhaber, Lee and Pairs \[6\], which does not evolve in time. The distribution function reads:
2.4. MODEL SOURCES FOR PARTICLE EMISSION

\[ F(r) = \frac{1}{\sqrt{a_x^2 a_y^2 a_z^2 \pi^3 R^6}} e^{-\left[\left(\frac{x}{a_x}\right)^2 + \left(\frac{y}{a_y}\right)^2 + \left(\frac{z}{a_z}\right)^2\right]/R}, \]  

(2.26)

where the \(a_i\)'s are dimensionless constants. The correlation function for incoherent source is given by

\[ C_2(Q) = 1 + e^{-\frac{1}{2}(a_x^2 q_x^2 + a_y^2 q_y^2 + a_z^2 q_z^2)R^2}. \]  

(2.27)

Although in the experiment, for simplicity or due to limited statistic, the \(a_i\)'s are usually set to be the same \((a_x = a_y = a_z = \sqrt{2})\), so the source is considered to be spherical.

We can also correct the \(C_2(Q)\) function given in Eq. 2.12 for the effects that may arise from the existence of a halo of long-lived hadronic resonances, proposed by Lednicky and Podgoretskii [7]. The source emission probability function will now consist of two terms, one belongs to the small region corresponding to the core of the interaction and the second one to the long halo region:

\[ F(r) = \frac{\mu_1}{(2\pi R^2)^{3/2}} e^{-\frac{r_1^2}{2R^2}} + \frac{\mu_1}{(2\pi R_h^2)^{3/2}} e^{-\frac{r_1^2}{2R_h^2}}, \]  

(2.28)

where \(R\) and \(R_h\) are the core and the halo source radii (see fig. 2.3), \(\mu_1 + \mu_2 = 1\) is the normalization condition.

![Figure 2.3: Physical picture of core-halo model.](image)

The corresponding \(C_2(Q)\) function reads
2.4. MODEL SOURCES FOR PARTICLE EMISSION  

\[ C_2(Q) = 1 + \lambda_1 e^{-R^2 Q^2} + \lambda_2 e^{-R_h^2 Q^2}, \]  

(2.29)

where \( \lambda_1 \) and \( \lambda_2 \) are both correlated with \( R \) and \( R_h \). If the halo radius \( R_h \) is big enough the halo part of the \( C_2(Q) \) will go to zero unless \( e^{-R_h^2 Q^2} \to 0 \) and the only affected parameter will be \( \lambda_1 \). We can say, that although the coherence parameter \( \lambda \) in Eq. 2.12 was introduced empirically, it also includes a correction to this halo effect.

Another refinement of the Gaussian-source model is obtained by inclusion of the finite source lifetime (in works by Yano and Koonin [8]). The emission distribution function

\[ F(r) = \frac{1}{\pi^2 r_0^2} e^{-\frac{r^2}{r_0^2}} \]  

(2.30)

leads to the following correlation function

\[ C_2(p_1, p_2) = 1 + e^{-\frac{1}{2} |p_1 - p_2|^2 r_0^2 - \frac{1}{2} (E_1 - E_2)^2 r^2}. \]  

(2.31)

We have to say here, that nor \( q_x, q_y, q_z \) (Eq. 2.27) nor \( p_1 - p_2 \) (Eq. 2.31) are the relativistic invariants, so one has to choose the coordinate system which is usually in the laboratory frame.

Another source model introduced by Kopylov and Podgoretskii [9] considers emission of pions from the interior or the surface of an ellipsoid. The pion is produced in two steps. In the first step an oscillator (oscillators are distributed uniformly inside the boundary) is excited and in the second step it deexcites by emitting a pion. The decay is assumed to take place statistically. The correlation function reads

\[ C_2(q_t, q_0) = 1 + \frac{\lambda_4 J_1^2(q_t r_{KP})/(q_t r_{KP}^2)}{1 + (q_0 \tau)^2}, \]  

(2.32)

where \( J_1 \) is first-order Bessel function, \( q_0 \) is equal to \( |E_1 - E_2| \) and \( q_t = |q \times k|/|k| \), where \( k \) is sum of the two hadron momenta and \( q \) is difference between them. This correlation is not Lorentz invariant and its variables are calculated in the centre-of-mass system (CMS) of the final state hadrons.
2.5 Construction of $C_2(Q)$ function

In order to measure two-pion BEC correlation function $C_2(Q)$ experimentally, one has to find the $N(Q)$ distribution, corresponding to the signal, and divide it by the reference one, $N^{ref}(Q)$:

$$C_2(Q) = \frac{N(Q)}{N^{ref}(Q)},$$  \hspace{1cm} (2.33)

where $N(Q)$ is the number of like charged pion pairs $N^{\pm\pm}(Q)$ and $N^{ref}(Q)$ is the number of unlike charged pion pairs or the number of like charged pion pairs, using the pairs of pions from different events. Ideally, the reference distribution should have all other correlations except to the Bose-Einstein ones. In practice, some compromise has to be made. For example, when using a different charge reference sample, some areas of $Q$ have to be excluded from the fit because of the resonances. In both cases the correlation function has to be corrected for the final state Coulomb interaction. The mixed events technique has a disadvantage of not preserving the dynamical correlations (see Sec. 5.2).

We have to scale $N^{ref}(Q)$ distribution to have the same number of entries like the signal $N(Q)$ distribution to obtain a correct $C_2(Q)$ function.

2.6 Coulomb Correction

Among the charged particles acts a long range Coulomb force, which causes the momentum shift in our signal sample (see Fig. 2.4). We have to correct reference sample to this Coulomb interaction to make our final $C_2$ function free of the coulomb correlations. When using an unlike signed pions reference sample, the effect of the Coulomb interaction on the reference sample is just opposite to the Coulomb interaction effect on the data sample. Therefore the correlation functions of charged particles need to be corrected for these Coulomb interactions.

The measured $N(Q)$ distribution for the like or unlike signed pion pairs in the presence of Coulomb interaction is given by: [12] [13] [14]

$$N^{meas}(Q) = G(Q)N(Q),$$  \hspace{1cm} (2.34)

where $N^{meas}(Q)$ is the measured distribution, $N(Q)$ is the distribution free of
2.6. COULOMB CORRECTION

Figure 2.4: Two identical bosons emitted from the same source, affected by coulomb force (left). Gamow penetration factor (right) for like-signed and unlike-signed pairs. Black line, on right plot, correspond to final Coulomb correction used for $C_2(Q)$ if $Q^{+-}$ as a reference sample is used.

Coulomb correlations and $G(Q)$ is the Gamow penetration factor given by:

$$G(Q) = \frac{2\pi\eta}{e^{2\pi\eta} - 1},$$

where the Sommerfeld parameter $\eta$ is defined as:

$$\eta = \pm \frac{\alpha m}{|Q|}.$$  \hspace{1cm} (2.36)

Here $\alpha$ is the electromagnetic fine structure constant (1/137) and $m$ is the mass of the single particle. The plus sign is for the like-signed pairs and minus for the unlike-signed pairs. The size of this correction is shown in Fig. 2.4 (right).
Chapter 3

Experimental Setup

3.1 Tevatron

The Tevatron, proton-antiproton collider, is designed for acceleration of proton and antiproton particles to their final energy of 980GeV at maximal luminosity up to $2 \times 10^{32} cm^{-2}s^{-1}$. The Tevatron is located at Fermi National Accelerator Laboratory (FNAL) near Chicago/Illinois (Fig. 3.1). Tevatron is the largest running project on the world surface (until the start of LHC).

![Fermilab Acceleration Complex](image)

**Figure 3.1:** The picture of Fermilab acceleration complex taken from airplane (left) with it’s scheme (right)

Its acceleration chain consist of multiple parts (see Fig. 3.1 right), where each of them has special purpose.
3.1. TEVATRON

The Cockcroft-Walton chamber provides the first stage of acceleration chain. Inside this device, additional electrons are added to the hydrogen atoms to produce negative ions. The resulting ions, each consisting of two electrons and one proton, are attracted to a positive voltage and accelerated to an energy of 750,000 electron volts. After leaving the Cockcroft-Walton chamber, the negative hydrogen ions enter a linear accelerator called the Linac. They are accelerated here to the energy of 400 million electron volts (MeV). Due to the geometry of the oscillating field, the ions are, at the end of the Linac, grouped into the bunches. Before entering the third stage, the Booster, the ions pass through a carbon foil, which removes the electrons, leaving only the protons. The Booster is a cycling synchrotron about 150 meters in diameter. The protons travel around the Booster about 20,000 times and their energy is raised up to 8 GeV. The intensity of the proton beam is increased by repetitive inject further bunches of protons. Protons are then extracted from the Booster into the Main Injector, a 3 km circumference synchrotron. Here the protons are accelerated to the energy of 120 GeV for antiproton production or up to 150 GeV for direct injection into the Tevatron.

Antiprotons are produced by directing the 120 GeV proton beam from Main Injector onto a nickel target where approximately 20 antiprotons with kinetic energy of 8 GeV are produced per $10^6$ protons. The antiprotons are then separated, focused and accumulated into the continuous beam using Debuncher Ring and Accumulator. This two devices are also called Antiproton source. After accumulation of sufficient number of antiprotons (about $9.0 \times 10^{10}$, this take about one day), they are redirected back into the Main Injector. The limiting factor of operating at high luminosities is production rate of antiprotons. The Recycle Ring uses the same tunnel as Main Injector, and it was built for storage residual protons from Tevatron. But the process of antiproton cooling and restorage shows to be very inefficient, so this idea was abandoned. Now, the Recycler is used as a antiproton storage. When a sufficient number of antiprotons is created, they are redirected into the Recycler and kept there, while the antiproton production can continue. The last part of acceleration chain as called Tevatron. Tevatron is another synchrotron accelerator, 6 km in circumference, wherein the protons and antiprotons are accelerated from 150 GeV up to 980 GeV.

The very important characteristic of each accelerator is luminosity. The luminosity tells you the intensity of interacting particles in unit of [number of particles per cm$^2$.s]. The luminosity of collisions can be written as:
\[ L = f F \frac{N_B N_p N_{\bar{p}}}{2\pi (\sigma^2_p + \sigma^2_{\bar{p}})}, \]  

(3.1)

where \( f \) is the revolution frequency, \( F \) is a form factor describing the geometric properties of a bunch, \( N_B \) the number of bunches, \( N_p, N_{\bar{p}} \) the number of protons/antiprotons per bunch and \( \sigma_p, \sigma_{\bar{p}} \) the RMS beam size at the interaction point.

The *integrated luminosity* is the integral of the luminosity with respect to time. Estimated number of events for a given process, with cross section \( \sigma \), can be evaluated as \( N = \sigma \times \int L dt \).

Table 3.1 summarize *Tevatron* parameters, Fig. 3.2 shows the integrated(top) and peak luminosity(bottom).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_B )</td>
<td>36</td>
</tr>
<tr>
<td>bunch length [m]</td>
<td>0.37</td>
</tr>
<tr>
<td>bunch spacing [ns]</td>
<td>396</td>
</tr>
<tr>
<td>protons/bunch ( (N_p) )</td>
<td>( \sim 3 \times 10^{11} )</td>
</tr>
<tr>
<td>antiproton/bunch ( (N_{\bar{p}}) )</td>
<td>( \sim 90 \times 10^{10} )</td>
</tr>
<tr>
<td>highest peak ( L ) [cm(^{-2})s(^{-1})]</td>
<td>( 3.30 \times 10^{32} )</td>
</tr>
</tbody>
</table>

Table 3.1: Accelerator parameters of the Tevatron.

### 3.2 CDF II detector

#### 3.2.1 Overview

The *CDFII* detector is one from two collider detectors on *Tevatron*. The second one is so-called *D0*. They are designed to detect and measure properties of particles being produced in \( p-\bar{p} \) collisions. Fig 3.3 shows the side view of the *CDFII* detector.

The CDFII detector, x meters high, y meters width, z ton weight device, consist of many subdetectors, each of them has its own purpose. We can divide them into 3 main subsystems, the tracking system, calorimeters and a muon detectors, more details about each part will be given in next sections.

The geometry of *CDFII* is cylindrical, the \( z \) axis goes through the middle of the detector and is parallel to the beam. The polar angle \( \theta \) is defined as a angle between the positive half-axle (the proton direction) and the pseudorapidity \( \eta \) is defined as:
3.2. CDF II DETECTOR

Figure 3.2: Integrated (top) and peak luminosity (bottom).

\[ \eta = - \ln \left( \tan \frac{\theta}{2} \right) \]  

Azimuthal angle \( \Phi \) is defined as is shown in Fig. 3.4 (left). The y axis goes from the bottom of the detector to the upper part and together with the perpendicular x axis forms \( x - y \) or \( r - \Phi \) plane. The plane perpendicular to this \( r - \Phi \) plane is also known as \( r - z \) plane. The \( r \) stands for distance between given point and the middle of the detector.
Figure 3.3: Side view of the CDFII detector.

Figure 3.4: The CDFII co-ordinate system (left). Three dimensional view of the CDFII detector (right).

Whole detection area can be partitioned into the 3 areas, the first one, which provides full COT coverage (see section 3.2.2) so-called central area, is within \(|\eta| < 1\). And two forward areas in \(|\eta| > 1\), the west (\(\eta < 0\)) one and the east (\(\eta > 0\)) one.
3.2.2 Tracking system

The tracking system is built cylindrically around the beam and contained within the 1.4 T magnetic field created by a superconducting solenoid. Tracking system is shown in Fig. 3.5.

![CDF Tracking Volume](image)

**Figure 3.5:** The CDFII tracking system.

The Layer00 detector is the innermost layer and is only 1.6cm far from the beam axis. It is single-sided silicon microstrip detector constructed to survive high radiation doses. This layer highly improves the $D_0$ parameter (the distance of extrapolated track from the interaction point).

The main part of silicon system, the SVX detector, consist of 3 barrels, each 29cm long. The inner radius is 2.44cm and the outer one is 10.6cm, the full acceptance is up to $|\eta| < 2$. There are 12 wedges in $\Phi$, each with 5 layers of double-sided silicon microstrip detector. The strips are aligned axially on one side, with 90-degree stereo on the other side for layers 0, 1 and 3 and small angle stereo (1.2 degrees) for layers 2 and 4. This is designed to permit good resolution in locating the $z$-position of secondary vertices and enhance the 3-D pattern recognition capability of the silicon tracker. The SVXII detector is shown on fig 3.6.

The SVXII silicon detector provides coverage to $|\eta| \sim 1$. In the region $|\eta| < 1$
the combination of SVXII and the Central Outer Tracker (COT) can provide full 3D tracking. For $|\eta| > 1$, SVXII can only perform 2D tracking. This problem is solved by the detector called Intermediate Silicon Layers (ISL), is shown on fig 3.7. To enhance 3D tracking in the central region, a single layer of double-sided silicon(axial on one side, 1.2 degrees stereo on other side) is placed at a radius of 22cm. In the region $1 < |\eta| < 2$, where the COT coverage is incomplete or missing, two layers of silicon are placed at radii of 20cm and 28cm. Precision space point measurements at these radii enables 3D tracking in the plug region and significantly improves the momentum resolution.

The next stage of tracking system is provided by the already mentioned Central Outer Tracker (COT). The COT is cylindrical drift chamber with inner radius of 40cm and outer of 137cm. It’s length is about 3.1m, so the full coverage is up to $|\eta| < 1$. The COT operates with Ar-Ethane-CF₄ gas, mixtured at the ration of 50:35:15. The drift velocity is $\sim 100\mu m/\text{ns}$ which provide sufficiently fast response to avoid the signal disposal. The COT contains 96 wire layers divided into 8 superlayers (12 wires in each), where in four superlayers the wires go along z-direction (axial superlayers) and the other four are tilted by 2 degrees(stereo superlayers). The geometry of the COT endplate and the superlayer layout is shown on fig 3.8.

The gap between the COT and the solenoid coil fulfill the Time of flight (TOF) detector. The whole TOF system consist of 216 scintillation bars, parallel to the $z$-axis. Each bar, 40x40mm in cross section and 2794mm in length, is on both
3.2. CDF II DETECTOR

![Diagram of the CDF II detector with labeled parts like Layer 00, SVXII, ISL, Forward ISL, etc.]

**Figure 3.7:** The cross-section of the ISL detector (top), endview of the whole silicon system (bottom): Layer 00, SVXII and ISL.

sides ended by photomultiplier (PMT), which measures time and the size of the impulse. The $dE/dx$ measurement provides one standard deviation separation between charged kaons and charged pions. The $COT$ maintains this $dE/dx$ performance for momenta greater than $2GeV/c$. The $TOF$ system, with the resolution of $100ps$, provides at least two standard deviation separation between $K^\pm$ and $\pi^\pm$ for momenta $p < 1.6GeV/c$, complementing the $dE/dx$ measurement from $COT$.

Particle identification with $TOF$ is performed by measuring the time of arrival of a particle at the scintillator with respect to the collision time, $t_0$. The particle mass $m$ can then be determined from the momentum $p$, the path length $L$, and the time-of-flight $t$: 
Figure 3.8: The 1/6 of the COT endplate (top), the second superlayer layout (bottom).

\[ m = \frac{p}{c} \sqrt{\frac{c^2t^2}{L^2} - 1} \]  

(3.3)

where \( p \) and \( L \) are measured by the tracking system.
3.2.3 Calorimeters

The CDFII calorimetry system consist of four calorimeters. The central ($|\eta| < 1$) electromagnetic and hadronic calorimeter and the end-plug (up to $|\eta| < 3.6$) electromagnetic and hadronic ones. In each of them the "calorimeter sampling type" technology was used (absorber volume alternating with scintillator).

The Central electromagnetic (EM) Calorimeter starts after the solenoid coil at the radii of $\sim 180\,\text{cm}$ and ends at the radii of $\sim 210\,\text{cm}$. It consists of 432 towers, 15 degrees in $\phi$ and 0.11 in $\eta$. Each wedge (one tower in $\phi$ 9 towers in $\eta$, see Fig. 3.9) has alternating lead and scintillator with an imbedded two dimensional readout strip chamber at shower maximum. The signal is from the scintillator transferred to the PMT’s by the wavelength shifters. The whole Central EM Calorimeter with thickness of $19X_0$ (radiation length) has energy resolution:

$$\frac{\sigma(E)}{E} = 14.0\% \frac{\sqrt{E}}{E} + 2\% \quad (3.4)$$

Central Hadronic Calorimeter consist of 23 layers of metal and scintillating plates. Its inner radius is $\sim 220\,\text{cm}$ and the outer one is about $\sim 360\,\text{cm}$. Projective geometry is the same as in the electromagnetic part. The hadronic calorimeter with thickness of $\sim 4.5\lambda_I$ (interaction length) has energy resolution:

$$\frac{\sigma(E)}{E} = 50.0\% \frac{\sqrt{E}}{E} + 3\% \quad (3.5)$$

The end-plug calorimeter covers the polar angle region from $37^\circ$ to $3^\circ$ (1.1 < $|\eta| < 3.6$). The top half of one plug is shown in Fig. 3.10(top).

Electromagnetic end-plug calorimeter is a lead-scintillator sampling detector type. There are 23 layers, each layer is composed of $4.5\,\text{mm}$ lead and $4\,\text{mm}$ scintillator. The total thickness is about $21X_0$ (radiation length). The detector is divided into 24 wedges ($\Delta\phi = 15^\circ$), each consist of 20 towers divided into 12 layers (see Fig. 3.10 bottom). A position detector at the depth of the EM shower maximum (approximately $6X_0$) is placed. This detector consist of two layers, made of scintillator strips read out by wavelength shifter fibers, angled by 45 degrees. The energy resolution of EM section is approximately:

$$\frac{\sigma(E)}{E} = 16.0\% \frac{\sqrt{E}}{E} + 1\% \quad (3.6)$$
Figure 3.9: One wedge of the central EM calorimeter (left). One wedge of the central calorimeter, electromagnetic calorimeter is seen at the bottom and the hadronic calorimeter at the top.

The hadron end-plug calorimeter is a 23 layer iron and scintillator sampling device with unit layers composed of 50\,mm iron and 6\,mm scintillator. The coverage of this detector is area of pseudorapidity $1.30 < |\eta| < 3.64$, the total thickness is $7\lambda_I$ and its resolution:

$$\frac{\sigma(E)}{E} = 80.0\% \frac{1}{\sqrt{E}} + 5\%$$ \hspace{1cm} (3.7)

3.2.4 Muon detectors

The muon system consists of four subsystems called: CMU (Central Muon), CMP (Central Muon Upgrade), CMX (Central Muon Extension) and IMU (Intermediate Muon).

The CMU muon drift chamber consists of 144 modules with 16 rectangular cells per module, located behind $\sim 5.5\lambda$ of absorber. Each cell is 6.35x2.68x226\,cm in size and has 50\,\mu m stainless wire in the center. The 16 cells in a module are arranged into the 4 layers in radial direction. The first and third cells, with half cell offset in $\phi$ direction against the second and fourth cells, are ganged together in the readout. The $z$ muon position is estimated via charge division, measured on the ends of the cell. The chambers run in proportional mode.

The CMP consists of a second set of muon chambers after additional 60\,cm of
3.2. CDF II DETECTOR

The chambers with fixed $z$ length form the box around the central detector. The pseudorapidity coverage thus varies in $\phi$ as is shown in Fig. 3.11 (left). The chambers arrangement is the same as for CMU detector, with only difference, the drift tube cross-section is $2.5\times15\times640\text{cm}$. At the top of CMP, with respect to the interaction point, is a scintillator, called CSP Central Scintillator Upgrade, placed. These counters improves the muon positioning and are necessary to identify beam crossing of muon track.

The pseudorapidity area $0.6 < |\eta| < 1.0$ covers the next muon detector CMX, with scintillator plates on the top of it known as Central Scintillator Detector (CSX).
The drift tubes for this section differs from those of the CMP only in their length. They are 180cm long.

In the forward region (1.0 < |\eta| < 1.5), the IMU detector is placed. For this detector the same technology is used, combination of the drift chamber with scintillator plates. A detailed section of the IMU barrel is shown on fig 3.11 right.

![Figure 3.11: The coverage of the muon system(left). A detailed section of the IMU barrel, showing several chamber cells and corresponding scintillator (right).](image)

### 3.2.5 Trigger

The trigger plays an important role in hadron collider experiments because the collision rate is much higher than the rate at which data can be stored on tape. In the CDFII experiment the collision rate is 7.6MHz (bunch spacing up to 132ns) while the tape writing speed is less than 50Hz. The CDF trigger system has a three level architecture with each level providing a rate reduction sufficient to allow processing in the next level with minimal dead time. The first two trigger levels consist of hardware, the third one is software based. The trigger scheme is shown in Fig. 3.12 left.

The Level-1 hardware consists of three parallel synchronous processing streams which are connected, as an input, to the global L1 decision unit (see Fig. 3.12 right). One stream find calorimeter based objects (L1CALO), second one finds muons (MUON PRIM., L1MUON) and the third one find tracks from COT (XFT,
XTRP, L1TRACK). The stream devoted to the COT subdetector starts with the XFT (eXtremely Fast Tracker) which is fast enough to process the data from each bunch crossing. The purpose of extrapolation unit XTRP is to receive tracks from XFT and distribute the tracks or information derived from the tracks (extrapolation of the track to the calor. and muon areas) to the L1 and L2 subsystems. The L1TRACK trigger is adjunct to the XTRP. The XTRP modules select tracks above a given \( P_T \) threshold and passes them to this L1 trigger module. The L1CALO stream watches for information about electrons, photons, jets, total event transverse energy (\( \sum E_T \)) and missing transverse energy (\( E_T \)). The calorimeter trigger is divided onto two boards: one for the object triggers (electrons, photons and jets) and one for global triggers (\( \sum E_T \) and \( E_T \)). The information from XTRP is used. The last L1 stream devoted to the muon subdetectors starts with MUON PRIM module. The muon primitives are derived from single hits or coincidences of hits with the muon system scintillators. This information together with the information from XTRP module goes to the L1 MUON module to provides the single and dimuon objects
for the global L1 trigger decision. The Global Level-1 card or "FRED" module is responsible for issuing the level-1 trigger decision. It suppress the data acquisition by factor $\sim 150$ what results in data frequency decrease from $7.6 MHz$ to $50 kHz$. The "FRED" is able to generates up to 64 Level-1 triggers, each based on a subset of 8 of 64 Level-1 inputs. FRED has also ability to prescale or rate limit each of the 64 Level-1 triggers separately.

Processing for a Level-2 trigger decision starts after the event is written into one of the four Level-2 buffers by the Level-1 accept. If all four buffers are full the experiment is in deadtime until one of the buffer is released. In order to keep L2 deadtime in the acceptable numbers the Level-2 is pipelined in two stages. The first stage of the pipeline is an event builder stage: all the data from Level-1 is collected into the memory of the Level-2 processors. Also the additional data from calorimeter shower maximum detector (XCES) is added. Simultaneously a hardware cluster finder (L2CALO) process a calorimeter data to add this information into the same memory. SVT (Silicon Vertex Tracker) preprocess the information from silicon detectors to find tracks and impact parameter. After all this data is stored in processor the event is examined to find out if the criteria for any of Level-2 trigger is satisfied (stage 2). So, while the data is analyzed in the L2 processor the data from the next event can be loaded into the memory. The Level-2 suppress factor is about $\sim 150$ what leads to L2 output frequency $300 Hz$.

If the event is accepted by the Level-2 the whole event information is reconstructed by the Level-3 software. It is C++ based software with the same reconstruction algorithms like the offline version. After the event is reconstructed, its characteristics are histogramed and checked if they fulfill different Level-3 cuts. Finally the event is written on the tape by the Level-3 accept at the speed of $\sim 50 Hz$.

3.2.6 Accessing software

The data are stored on tapes as a "raw data", but also in more useful (for physicists) ROOT [22] format. The data are preprocessed and divided, according some criteria to select only interesting events for each physics group at CDF, into the datasets.

The data are then accessed by the software known as a cdfsoft. This software is developed by the physicists and modified according the current needs. It’s organized into the releases, we have used the 6.1.4 one.

This software is written in object oriented C++. The code is stored within CVS
(Concurrent Versions System) management system. This system allows multiplier users to edit the same files, it saves all the versions of the code, it allows to return to previous versions of the code, etc.
Chapter 4

Dataset and event selection

The high multiplicity triggered samples, which we will refer to as \textit{gmbsbi} and \textit{gmbsbj}, are the primary datasets used in this analysis. Only the events passed _MULTIPLICITY_20_v-1 trigger path are used. The detailed description of _MULTIPLICITY_20_v-1 trigger path as is described in PHYSICS tables is shown in Table 4. These datasets contain data collected from 10.11.2005 up to 30.01.2007. The valid trigger tables are PHYSICS\_3\_08_v-2 up to PHYSICS\_5\_01_v-2. At Level-3 the full COT information is available. The \textit{gmbsbi} and \textit{gmbsbj} datasets contains together 298559 events passing through _MULTIPLICITY_20_v-1 trigger path.

4.1 Track selection

Additional cuts to the tracks were applied to remove main sources of background and reconstruction errors. The track must pass through a minimum number of layers in \textit{COT}, and has a minimum number of hits in each layer in order to reduce the number of tracks with reconstruction errors. In our case a good track should have

- at least 3 Axial and 3 Stereo \textit{COT} layers with at least 5 hits in each or
- at least 3 Axial and 2 Stereo \textit{COT} layers with at least 5 hits in each and at least 3 Stereo Silicon hits.

These cuts were found to have the best acceptance and efficiency [3]. Fake and secondary particle tracks are removed by requiring that \textit{COT} tracks must pass within 0.5\textit{cm} from the beam axis, and within 2\textit{cm} along the \textit{z} axis from the primary vertex. If there is a silicon flag (hits in \textit{SVX} detector) than more stringent criteria to the track distance from primary vertex position were applied:
4.2 Event selection

The events used in this analysis are selected as follows:

- The event must not have a cosmic tag.
- The event must have more or equal than 20 reconstructed tracks.

**Table 4.1**: The trigger path for MULTIPLICITY_20_v-1 trigger

<table>
<thead>
<tr>
<th>Level 1</th>
<th>L1_MB_CLC_PS100K_v-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Specific option MB_CLC_v-1 Instance of MINIMUM_BIAS.</td>
<td>MINIMUM = 1 integer</td>
</tr>
<tr>
<td>2. Specific option PRESCALE_100K_v-1 Instance of PRESCALE.</td>
<td>PRESCALE_FACTOR = 100003 integer</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Level 2</th>
<th>L2_RL3HZ_L1_MB_CLC_v-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Specific option L2_AUTO_v-1 Instance of L2_AUTO.</td>
<td>DUMMY_PARAMETER = 1 integer</td>
</tr>
<tr>
<td>2. Specific option L2_RATELIMIT_3HZ_v-1 Instance of RateLimit.</td>
<td>RATE_LIMIT = 3 Hz</td>
</tr>
<tr>
<td>3. Specific option L2_READOUT_ALL_v-1 Instance of ReadOutList.</td>
<td>READOUT_LIST = 7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Level 3</th>
<th>L3_TRACKMULT_20_v-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. L3 Instance: trackMult20_v1, of class L3TrackFilterModule_v-7</td>
<td>d0Max = 1.0</td>
</tr>
<tr>
<td></td>
<td>d0Min = 0.0</td>
</tr>
<tr>
<td></td>
<td>d0Significance = 0.0</td>
</tr>
<tr>
<td></td>
<td>dzMax = 5.0</td>
</tr>
<tr>
<td></td>
<td>eta = 1.0</td>
</tr>
<tr>
<td></td>
<td>nTracks = 20</td>
</tr>
<tr>
<td></td>
<td>pt = 0.40</td>
</tr>
<tr>
<td></td>
<td>vertexCollection = &quot;COT Standalone&quot;</td>
</tr>
<tr>
<td></td>
<td>z0 = 100.</td>
</tr>
</tbody>
</table>

- for track having only Axial hits in SVX
  \[|\Delta Z| < 5cm \text{ and } |d_0| < 0.1cm\]
- for track having also Stereo hits in SVX
  \[|\Delta Z| < 1cm \text{ and } |d_0| < 0.1 - (0.1 \times \Delta Z)cm \text{ (see Fig. 4.1)}\]

Only tracks with \(p_T \geq 0.40 \text{GeV/c}\) and within \(|\eta| < 1\) were used.
Figure 4.1: Corrected (to the beam position) $D_0$ vs. $\Delta Z$ of the track for (from top) tracks having only COT information, for tracks having also SVX information from axial strips and for tracks having SVX information also from stereo strips.
• The event must have only one primary vertex, with \( z < 30 \text{cm} \) and \( \text{symmetry} < 0.9 \).

We used \( ZVertexBlock \) data block for the cut to the number of primary vertexes. Additional cuts given to the primary vertex were applied. The primary vertex must not be farther from the center of the detector than 30\text{cm} in order to keep good tracking efficiency in \( COT \). We define the vertex symmetry as a difference of the number of track with positive and negative \( \eta \), where only tracks with \(|\Delta Z| < 5.\text{cm}\) and \(|BcD0| < 1.\text{cm}\) were counted (BcD0 stands for Beam constrained D0).

\[
\text{symmetry} = (\#	ext{ of tracks with } -1. < \eta < 0.) - (\#	ext{ of tracks with } 0. < \eta < 1.)
\]
\[
(\#	ext{ of tracks with } -1. < \eta < 0.) + (\#	ext{ of tracks with } 0. < \eta < 1.)
\]

We used only vertexes with \( |\text{symmetry}| \) smaller than 0.9.

### 4.3 Dataset properties

Fig. 4.2 shows the multiplicity distribution for our high multiplicity sample.

\[ -5 \leq n \leq 60 \]

\[ \begin{array}{cc}
\text{Entries} & 125745 \\
\hline
n & 20 25 30 35 40 45 50 \\
\hline
P(n) & 1 \times 10^{-5} \, 1 \times 10^{-4} \, 1 \times 10^{-3} \, 1 \times 10^{-2} \, 1 \times 10^{-1} \\
\hline
\chi^2/\text{ndf} & 358.1 / 26 \\
\text{a} & 671.7 \pm 15.4 \\
\text{b} & -0.3744 \pm 0.0010 \\
\end{array} \]

**Figure 4.2:** The multiplicity distribution in logarithmic scale.

In the Fig. 4.3 is shown the track \( p_t \) distribution.
4.3. DATASET PROPERTIES

Mass identification

All tracks stored in STnTuple storage system have by default a pion mass assigned, without any corrections to the TOF system. Using the TOF detector, one can calculate the mass of the particle, unfortunately the TOF detector is not able to measure the time-of-flight for all tracks, particularly in our high multiplicity region. As we found only $\sim 38\%$ of all tracks has, so called, TOF match, needed for track mass evaluation. The track mass is calculated from the momentum $p$, the path-length $L$, measured by the tracking system, and the time-of-flight $t$ measured by the TOF:

$$M = \frac{p}{c} \sqrt{\frac{c^2 t^2}{L^2 - 1}} \quad (4.2)$$

In Fig. 4.4 (left) is show the mass distribution in dependence on momentum, multiplied by track charge. The red lines shows cuts used for the selection whether the track is assigned to be a pion, Kaon or proton.

In Table 4.2 is shown the mass identification percentage for our high multiplicity dataset.

The particle energy was recalculated as $E = \sqrt{p_x^2 + p_y^2 + p_z^2 + m^2}$, where the mass was taken $0.13957\text{GeV}/c^2$ for pions, $0.49368\text{GeV}/c^2$ for Kaons and $0.93827\text{GeV}/c^2$ for protons. The TOF efficiency, in our high multiplicity region, vs. track $p_t$ is shown
4.3. DATASET PROPERTIES

Figure 4.4: The mass distribution in dependence on momentum, multiplied by track charge.

<table>
<thead>
<tr>
<th>Identified as</th>
<th>pion</th>
<th>Kaon</th>
<th>proton</th>
<th>track</th>
</tr>
</thead>
<tbody>
<tr>
<td>nr. of Tracks</td>
<td>710342</td>
<td>230719</td>
<td>110990</td>
<td>2795576</td>
</tr>
<tr>
<td>percentage</td>
<td>25.41%</td>
<td>8.25%</td>
<td>3.97%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Table 4.2: Mass identification percentage.

In Fig. 4.5. The efficiency was calculated as the number of tracks tagged as π, Kaon or proton divided by total number of tracks in a given $p_t$ interval. The data was fitted in 3 different intervals: by the linear function $\varepsilon_{TOF} = p[0] + p[1]p_t$ for $p_t$ smaller than 0.8GeV, by the quadratic function $\varepsilon_{TOF} = p[2] + p[3]p_t + p[4]p_t^2$ for $p_t$ from interval $0.8 - 6.0GeV$ and by the constant function $\varepsilon_{TOF} = p[5]$ for $p_t$ bigger than 6.0GeV.

Two track separation

In Fig. 4.6 is shown the two particle distribution in the $\Theta - Q$ plane, where $\Theta$ is the space angle between two tracks. As can be seen there is a peak at the origin of coordinate system.

To exclude this region of possible two track fake reconstruction a small $Q$ threshold was introduced, $Q < 0.01GeV$, as a minimal $Q$ between two tracks.

After all cuts, listen in the previous sections, 173761 events remains.
4.3. DATASET PROPERTIES

Figure 4.5: The TOF efficiency, in our high multiplicity region, vs. track $p_t$

Figure 4.6: Two particle distribution in the $\Theta - Q$ plane.
Chapter 5

Results

As was written in Sec. 2.2, the $C_2(Q)$ function is defined as a ratio of two $N(Q)$ distributions. The first one is the signal distribution constructed using the like-sign track pairs coming from the same event. The second one is reference one, which should be constructed using tracks without Bose-Einstein correlations. The $N(Q)$ distribution is used as a base distribution in this analysis, $Q$ stands for the 4-momentum difference, which is defined as:

$$\sqrt{(E_i - E_j)^2 - [(p_{x_i} - p_{x_j})^2 + (p_{y_i} - p_{y_j})^2 + (p_{z_i} - p_{z_j})^2]}$$

(5.1)

$i, j$ indexes correspond to different tracks. We are using all the tracks passing our cuts listen in the previous sections, except those tracks marked as a $K$ meson or as a proton by the TOF detector (see Fig. 4.4).

Before finding the correct $C_2(Q)$ function, some corrections should be applied to retrieve the correct $N(Q)$ distribution. We should take into account the track reconstruction efficiency, the corrections connected to the Coulomb interaction and the correction connected to the QCD effects [15]. The correction to the QCD peak will be discussed in Sec. 5.2.1.

5.1 Corrections applied

Correction to the track reconstruction efficiency

The track reconstruction efficiency for CDF tracking system has not the same value for the whole region of track $p_t$ used in this analysis. We are calculating $Q$ between
5.1. CORRECTIONS APPLIED

two tracks with different $p_t$, therefore we have to weight, the $Q$ value filled into the $N(Q)$ distribution, to the reco. efficiency as follows:

$$\text{weight} = \frac{1}{\varepsilon_1} \times \frac{1}{\varepsilon_2}, \quad (5.2)$$

where the $\varepsilon_i$ is the reconstruction efficiency of $i^{th}$ track at given transverse momentum $p_t$.

The reconstruction efficiency vs. track $p_t$ is shown in Fig. 5.1. This curve was taken from [3] and corrected after personal communication with authors. The function used for fit was

$$\varepsilon = \frac{p[0]}{p[1] + e^{p[2] \cdot p_t}} + \frac{p[3]}{p[4] + e^{p[5] \cdot p_t}} + p[6] \cdot \tanh(\log(p_t \cdot p[7])) . \quad (5.3)$$

![Figure 5.1: Track reconstruction efficiency vs. track $p_t$ (left).](image)

Coulomb correction

The coulomb correction was applied to all $N(Q)$ distributions according to the Eq. 2.34. Measured $N(Q)$ distribution was divided by the Gamow penetration factor (with respect to the tracks charges) to obtain $N(Q)$ distribution free of the coulomb interactions.
5.2 \( N(Q) \) distribution and reference sample

The \( N(Q) \) distribution for \( \pi^+ \pi^+ \) pairs sample combined with \( \pi^- \pi^- \) pairs sample (we will refer to this sample as \( N^{++--} \)) for different \( p_t \) track cuts is shown in Fig. 5.2. Similar distribution was obtained in Monte Carlo sample [15], where was shown that the peak at about 0.6 GeV comes from pairs of particles hadronized from the same final state parton of a "hard collision". The presence of this QCD peak is also seen in \( \pi^+ \pi^- \) pairs sample, see fig 5.2 (we will refer to this sample as \( N^{+-} \)), but not in \( \pi^+ \pi^+ \) pairs sample when \( \pi^+ \) mesons were taken from different events (the combination of this sample with the same one for \( \pi^- \) mesons will be referred to as \( N^{++--}_{EM} \) (event mixing)).

As can be seen in fig 5.2 the QCD peak in \( \pi^+ \pi^- \) pairs sample is a little bit bigger.
To retrieve the BEC effect it is inevitable to handle correctly the QCD effect. If we use for the $C_2(Q)$ function construction the $N_{EM}^{++--}(Q)$ pair distribution as a reference sample, then the QCD peak present in the signal $N^{++--}(Q)$ distribution will not be compensated and will affect the BEC effect that is expected to occur at low $Q$. In the case when the same event $N^{+-}(Q)$ distribution is used as the reference sample, its QCD peak is more profound than that presented in the signal distribution, hence a distortion of the BEC peak can be also expected.

To suppress this QCD peak we can use cut to the track $p_t$ as small as possible. We can also suppress QCD peak by cutting out tracks coming from the same final state parton. To do this, at last partially, we selected the events where jets were required to fulfill certain kinematic criteria - the events with leading jet $p_t$ over a chosen threshold were excluded. The used jets were taken from JetCluModule07 data block. The $N(Q)$ distribution for different jet $p_t$ cuts is shown in Fig. 5.3. The events containing a jet with $p_t$ over the cut were excluded from analysis. As can be seen from upper and middle left plots in Fig. 5.3 the smaller $p_t$ cut is applied on leading jet the stronger is the suppression of the QCD peak.

The nature of the QCD peak is well demonstrated in the right (upper and middle) plots in Fig. 5.3, where the $N^{++--}(Q)$ and $N^{+-}(Q)$ distributions are shown for the events with leading jet $p_t$ over the appropriate cut. The plots clearly demonstrate that with increasing leading jet $p_t$ the QCD peak is getting more profound.

As a reference sample we can use the same event like-sign pair $N^{+-}(Q)$ distribution or some of the event mixing $N_{EM}(Q)$ distributions. As has been shown (see Fig. 5.2-5.3), the samples created using the event mixing technique does not contain large QCD background correlations which are present in the numerator. The $N^{+-}$ sample is considered to have the same characteristic as the signal sample, except of the coulomb interaction and slightly different QCD peak.

In Fig. 5.4 are shown the $C_2^{++--}(Q)$ functions constructed using different reference samples, for different track and leading jet $p_t$ cuts. Although the cut on leading jet $p_t$ suppresses the QCD peak in $N(Q)$ distribution, the shape of $C_2^{++--}(Q)$ function exhibits only a weak dependence on the leading jet $p_t$ cut if the same event $N^{+-}(Q)$ distribution is used as the reference one - it is well demonstrated by the left middle and bottom plots in Fig. 5.4 where the $C_2(Q)$ functions for different leading jet $p_t$ cuts are shown for the track $p_t$ cut 0.4$GeV$ (bottom) and 1.5$GeV$ (middle). At the same time, even if we put track $p_t$ cut as small as possible, there is still some drop in the $C_2^{++--}$ function for $Q < 1$GeV in the case if $N^{+-}(Q)$ distribution is
5.2. \(N(Q)\) DISTRIBUTION AND REFERENCE SAMPLE

Figure 5.3: \(N(Q)\) distribution for different leading jet \(p_t\) cuts. The track \(p_t\) cut was taken to be 1.5GeV to better see QCD peak behavior. \(N^{++--}(Q)\) sample top, \(N^{+-}(Q)\) sample middle, \(N_{EM}^{+-}(Q)\) sample bottom. Events which doesn’t pass required leading jet \(p_t\) cut was not used in the analyze. These distributions are normalized to have the same integral, equal to the unit.
5.2. \( N(Q) \) DISTRIBUTION AND REFERENCE SAMPLE

used as the reference one. This drop can be explained by more profound QCD peak in the \( N^{+-}(Q) \) distribution in comparison with the \( N^{++--}(Q) \) one.

In the case of \( N^{EEEM}_{EM}^{++--}(Q) \) distribution there is no QCD peak and if this distribution is used as the reference one the corresponding \( C_{2EM}^{++--}(Q) \) function does not have a drop at \( Q < 1\text{GeV} \), but instead of that, it reaches so big values.

5.2.1 Reference sample QCD peak correction

It was shown in [15] that the QCD peak comes from pairs of particles hadronized from the same final state parton and it is well demonstrated on the real CDF data shown in Fig. 5.3. At the same time this figure also shows that the QCD effects are bigger in the \( N^{+-}(Q) \) distribution than in the \( N^{++--}(Q) \) one. This circumstance should be taken into account if both the distributions are used for \( C_{2EM}^{++--}(Q) \) function construction.

The fact that the QCD peak is more profound in the case of \( N^{+-}(Q) \) than in the case of \( N^{++--}(Q) \) can be explained by different track pair statistics in jets for the LS and US track pairs. Let us now suppose, that we have a jet with \( n_1 \) positive and \( n_2 \) negative tracks. Into the \( N^{+-}(Q) \) sample will go \( n_1n_2 \) pair combinations from this jet, while into the combined \( N^{++--}(Q) \) sample it will be only \( \frac{n_1(n_1-1)+n_2(n_2-1)}{2} \) pair combinations going in to the QCD peak. The ratio, \( R_{QCD} \), of the number of the US track pair combinations to the LS ones, coming from the jet,

\[
R_{QCD} = \frac{2n_1n_2}{n_1(n_1-1)+n_2(n_2-1)},
\]

is always bigger than unit. This excess of the unlike-signed track pair combinations causes, that the QCD peak seen in the \( N^{+-}(Q) \) reference sample is bigger than that in the signal sample \( N^{++--}(Q) \) (see Fig. 5.2 and 5.3). To negotiate this difference we have weighted the US jet track pairs with the weight factor \( 1/R_{QCD} \). Only the tracks passing our selection cuts were counted. In addition, we have attached zero weight to the pairs filled into the \( N^{++--}(Q) \) distribution, if only positive or only negative tracks were in jet.

In Fig. 5.5 are shown the \( N^{+-}(Q) \) and \( N^{++--}(Q) \) distributions and the new \( N_{\text{corrected}}^{+-}(Q) \) and \( N_{\text{corrected}}^{++--}(Q) \) distributions obtained by the above mentioned approach for the case when the track \( p_t > 1.5\text{GeV} \) and there is no cut on leading jet \( p_t \). The track \( p_t \) cut was taken to be \( 1.5\text{GeV} \) to see the QCD peak behavior more
5.2. $N(Q)$ DISTRIBUTION AND REFERENCE SAMPLE

Figure 5.4: $C_{2}^{++−−}(Q)$ function with $N^{++−}(Q)$ reference sample (left) and $N_{EM}^{++−−}(Q)$ reference sample (right) for different track $p_t$ cuts (top) and different leading jet $p_t$ cuts where cut on track $p_t$ is 1.5$GeV$ (middle) and 0.4$GeV$ (bottom).

clearly.

The $C_{2}^{++−−}(Q)$ function constructed using this new reference sample for different
5.3. BOSE-EINSTEIN $\pi^-\pi^-$ AND $\pi^+\pi^+$ CORRELATIONS

We have used $N^+_{\text{corrected}}(Q)$ and $N^+_{\text{EM}}(Q)$ distributions as a reference samples for the construction of the final $C_2(Q)$ functions. The $N^+_{\text{corrected}}(Q)$ reference sample has a resonance peaks in the area of 0.4-0.8GeV, which corresponds to the $K^0$ and track $p_t$ cuts is shown in Fig. 5.6.

The $N^+_{\text{corrected}}(Q)$ distribution does depend on track $p_t$ cut, but the weight of the QCD peak is almost the same in $N^{++--}(Q)$ and $N^{+--}_{\text{corrected}}(Q)$ distributions, therefore, the final $C_2^{++--}(Q)$ function approximately doesn’t depend on track $p_t$ cut. We didn’t remove the QCD peak, we just made the result more stable on track $p_t$ cuts.

**Figure 5.5:** $N^+-(Q)$, $N^{++--}(Q)$ distributions and the new $N^{+--}_{\text{corrected}}(Q)$ distribution for different jet blocks, jetBlock (top-left), jetCluModule07 (top-right) and jetCluModule10 (bottom).
$\rho/\omega$ resonances. The sharp rise in the region of $Q \lesssim 0.3 GeV$ corresponds to the Bose-Einstein correlations.

One can see, that the $C_2(Q)$ function constructed using this reference sample is hard to fit with the theoretically obtained $C_2$ function (see 2.12). To obtain the meaningful fit, we had to exclude this resonance area, extended to the 0.2-0.8$GeV$, from the fitting procedure. We had also varying the boundaries of the excluded interval to find the best fit and to find the systematic error caused by the choice of this interval.

We always compare the $C_2^{++--}(Q)$ function based on the same event $N_{corrected}^{++--}(Q)$ reference distribution with the $C_2^{++--}_{EM}(Q)$ function, constructed using the reference sample, obtained by the event mixing technique. Although the latter function does
not reflect all the kinematic properties of the signal sample, we want to show the
differences between the use of this two reference samples and at the same time we
want to be able to compare our results with those obtained by other experiments,
which are using event mixing reference distribution.

We have found that the best suppression of the QCD peak has been obtained for
the track $p_t$ cut taken at 0.4$GeV$ (see Fig. 5.4). The cut on leading jet $p_t$ was taken
to be 20$GeV$ with the aim to exclude the events with very energetic jets, though no
strong influence of the leading jet $p_t$ cut on the final shape of the $C_2(Q)$ function
has been noticed.

In Fig.5.7 is shown the final $C_2^{++--}(Q)$ function fitted with two different $C_2(Q)$
function parametrization:

$$C_2(Q) = C_0(1 + \lambda e^{-R^2Q^2})$$
$$C_2(Q) = C_0(1 + \lambda e^{-RQ}),$$ (5.5)

where $C_0$ is a normalization constant, $\lambda$ is so called coherence factor and $R$ stands
for the source size. The first function was obtained assuming the plane wave approx-
imation of particle wave function and the Gaussian shape of the particle emission
source, the second one is an empirical function introduced in [17] to obtain better fit
of the experimental data. It should be noted that the second function cannot be ob-
tain by a simple substitution of the Gaussian density distribution of the source with
the exponential one due to the Fourier transformation performed in Eq. 2.10. One
can obtain this function by substituting the expressions for the Gaussian correlator
$(e^{-R^2Q^2})$ with the exponential correlator $(e^{-RQ})$ without any theoretical support to
this substitution. We can write a similar expression for Eq. 2.25:

$$C_2(Q) = C_0(1 + 2p(1-p)e^{-R^2Q^2} + p^2 e^{-2R^2Q^2})$$
$$C_{empirical}^{++--}(Q) = C_0(1 + 2p(1-p)e^{-RQ} + p^2 e^{-2RQ}).$$ (5.6)

The $C_2^{++--}(Q)$ function fitted with Eq. 5.6 is shown in Fig. 5.8. Table 5.1 sum-
marizes the fitted parameters.

The $C_2^{++--}(Q)$ and $C_2^{++--}_{EM}(Q)$ functions for the tracks reconstructed as pions
and as kaons are shown in Fig. 5.9. The information from the TOF detector was used to distinguish between the individual particle types. The parameters of the $C_2(Q)$ function for the case that only the tracks tagged as $\pi$ mesons were used (we will mark this case with subscript $\pi$) are shown in Table 5.1.

### 5.3.1 Influence of the applied correction to the $N(Q)$ distribution and $C_2(Q)$ function.

In the Sec. 5.1 are the track reconstruction efficiency and the Coulomb correction discussed. In the Fig. 5.10 (top) is shown the shape of the $N(Q)$ distribution (normalized to unit), for the track $p_t$ cuts equal to 0.4$GeV$ and 1.5$GeV$, for different cases of the correction level. In the case of the track $p_t$ cut 0.4$GeV$, there is a
shift after the correction to the track reconstruction efficiency, what is caused by
the shape of the correction function (see Fig. 5.1) and the fact, that there is more
tracks with smaller $p_t$ (see Fig. 4.3). At the same figure (5.10 bottom) is shown
the ration of the corrected distributions with the uncorrected one. The errors in the
bottom-right plot are so big due to the lower statistic.

The influence of these corrections to the $C_2(Q)$ functions is shown in Fig. 5.11.
One can seen that the shift observed in the $N(Q)$ distribution caused by the track
reconstruction efficiency has disappeared, while the Coulomb correction comes to
light. The parameters of the $5.5_{\text{Gauss}}$ function are shown in Table 5.2.
5.4. **DEPENDENCIES OF $C_2(Q)$ FUNCTION PARAMETERS**

<table>
<thead>
<tr>
<th>fit function</th>
<th>Param.</th>
<th>$C_{2}^{++--}(Q)$</th>
<th>$C_{2}^{+-++}(Q)$</th>
<th>$C_{2}^{++-+}(Q)_\pi$</th>
<th>$C_{2}^{+-+-}(Q)_\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.5$_{Gauss}$</td>
<td>$C_0$</td>
<td>1.006 ± 0.0</td>
<td>0.982 ± 0.0</td>
<td>1.008 ± 0.0</td>
<td>0.987 ± 0.0</td>
</tr>
<tr>
<td></td>
<td>$\lambda$</td>
<td>0.293 ± 0.010</td>
<td>0.276 ± 0.003</td>
<td>0.502 ± 0.036</td>
<td>0.463 ± 0.016</td>
</tr>
<tr>
<td></td>
<td>$R$</td>
<td>7.53 ± 0.17</td>
<td>2.565 ± 0.019</td>
<td>9.06 ± 0.39</td>
<td>4.63 ± 0.12</td>
</tr>
<tr>
<td></td>
<td>$\chi^2/ndf$</td>
<td>414/45</td>
<td>1223/57</td>
<td>108/45</td>
<td>410/57</td>
</tr>
<tr>
<td>5.5$_{exp}$</td>
<td>$C_0$</td>
<td>1.006 ± 0.0</td>
<td>0.979 ± 0.0</td>
<td></td>
<td>0.985 ± 0.0</td>
</tr>
<tr>
<td></td>
<td>$\lambda$</td>
<td>0.502 ± 0.023</td>
<td>0.519 ± 0.006</td>
<td></td>
<td>0.889 ± 0.034</td>
</tr>
<tr>
<td></td>
<td>$R$</td>
<td>12.23 ± 0.43</td>
<td>4.495 ± 0.042</td>
<td></td>
<td>8.46 ± 0.24</td>
</tr>
<tr>
<td></td>
<td>$\chi^2/ndf$</td>
<td>399/45</td>
<td>605.8/57</td>
<td></td>
<td>208/57</td>
</tr>
<tr>
<td>5.6$_{Gauss}$</td>
<td>$C_0$</td>
<td>1.006 ± 0.0</td>
<td>0.982 ± 0.0</td>
<td>1.008 ± 0.001</td>
<td>0.987 ± 0.0</td>
</tr>
<tr>
<td></td>
<td>$p$</td>
<td>0.161 ± 0.006</td>
<td>0.152 ± 0.002</td>
<td>0.31 ± 0.03</td>
<td>0.256 ± 0.012</td>
</tr>
<tr>
<td></td>
<td>$R$</td>
<td>7.39 ± 0.16</td>
<td>2.525 ± 0.018</td>
<td>8.83 ± 0.37</td>
<td>4.52 ± 0.12</td>
</tr>
<tr>
<td></td>
<td>$\chi^2/ndf$</td>
<td>401/45</td>
<td>1180/57</td>
<td>105/45</td>
<td>395/57</td>
</tr>
<tr>
<td>5.6$_{exp}$</td>
<td>$C_0$</td>
<td>1.006 ± 0.0</td>
<td>0.979 ± 0.0</td>
<td></td>
<td>0.984 ± 0.0</td>
</tr>
<tr>
<td></td>
<td>$p$</td>
<td>0.309 ± 0.018</td>
<td>0.328 ± 0.005</td>
<td></td>
<td>0.80 ± 0.05</td>
</tr>
<tr>
<td></td>
<td>$R$</td>
<td>11.42 ± 0.36</td>
<td>4.213 ± 0.037</td>
<td></td>
<td>5.91 ± 0.27</td>
</tr>
<tr>
<td></td>
<td>$\chi^2/ndf$</td>
<td>412/45</td>
<td>629.4/57</td>
<td></td>
<td>200/57</td>
</tr>
</tbody>
</table>

**Table 5.1:** The parameters obtained by the fit of the $C_{2}^{++--}(Q)$ and $C_{2}^{+-++}(Q)$ functions. In two right columns are shown the parameters for the case when only tagged $\pi$ mesons were used. The empty cells in this table mean, that it was unable to perform fit.

<table>
<thead>
<tr>
<th>$C_{2}^{++--}(Q)$</th>
<th>$C_{2}^{+-++}(Q)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>$R$</td>
</tr>
<tr>
<td>0.13±0.01</td>
<td>7.39±0.37</td>
</tr>
<tr>
<td>0.13±0.01</td>
<td>7.43±0.30</td>
</tr>
<tr>
<td>0.29±0.01</td>
<td>7.53±0.17</td>
</tr>
<tr>
<td>0.252±0.003</td>
<td>2.45±0.02</td>
</tr>
<tr>
<td>0.248±0.003</td>
<td>2.52±0.02</td>
</tr>
<tr>
<td>0.276±0.003</td>
<td>2.56±0.02</td>
</tr>
</tbody>
</table>

**Table 5.2:** The dependence of the $C_2(Q)$ function parameters on different level of correction applied. The 5.5$_{Gauss}$ parametrization of the $C_2(Q)$ function was used for the fit.

## 5.4 Dependencies of $C_2(Q)$ function parameters

We have investigated behavior of $C_2(Q)$ function in different physical conditions. We have looked on a possible impact of the accelerator luminosity on stability of the $C_2(Q)$ function as well as on its possible dependence on pseudorapidity ($\eta$) region and multiplicity interval.
5.4. DEPENDENCIES OF $C_2(Q)$ FUNCTION PARAMETERS

![Graphs showing $C_2(Q)$ function for pions and kaons.](image)

**Figure 5.9:** The $C_2(Q)$ function for pions (top) and for kaons (bottom). On the left plots the $N^{+-}$ reference sample is used, on the right ones the $N^{++--}_{EM}$.

Instantaneous luminosity

At the high instantaneous luminosities ($L$) the pile-up effect can pose a problem. For this purpose we looked for a possible effect of the $L$ on the obtained correlation functions. In order to study dependence of $C_2^{++--}(Q)$ function parameters on $L$ we have defined the following intervals (in units of $10^{30} cm^{-2}s^{-1}$): [0, 30, 50, 70, 90, 120](see Fig. 5.12 top-left). Fig. 5.12 shows the $C_2^{++--}(Q)$ (top-right) and $C_2^{++--}_{EM}(Q)$ (bottom) functions for the different luminosity intervals. All the $N(Q)$ distributions, including the reference one, were calculated with respect to the individual luminosity intervals.

From Fig. 5.12 we are justified to state that no noticeable dependence of the $C_2(Q)$ function on the beam luminosity is observed. No dependence on luminosity was expected, because the prescale factor at the trigger level 1 was set to 100k and
5.4. DEPENDENCIES OF $C_2(Q)$ FUNCTION PARAMETERS

we took the events with only one primary vertex.

Pseudorapidity

We have also studied how $C_2(Q)$ function depends on $\eta$ region with the aim to see if the $C_2(Q)$ function parameter $R$ remains unchanged. One can speculate that if we have a hadronization region with spherically symmetric emission of particles, then no dependence of $C_2(Q)$ function parameters on $\eta$ would be present. On the other hand, if the hadronization region exhibits a deflection from the spherical symmetry then it can project into an $\eta$ dependence of the $C_2(Q)$ parameters. We have divided the $\eta$ region ($-1 < \eta < 1$) into the following intervals: $[0.0, 0.2, 0.4, 0.6, 0.8, 1.0]$. Fig. 5.13 shows the $C_2^{++--}(Q)$ function for different $\eta$ ranges. No major differences

**Figure 5.10:** The $N(Q)$ distribution (normalized to unit) for track $p_t$ cuts equal to 0.4GeV (top-left) and 1.5GeV (top-right) and different level of applied correction. The ratio of corrected distributions to the uncorrected one (bottom).
can be seen there, also the fitted parameters shows no \( \eta \) dependence.

The study of the source shape in two-dimensions, where we have replaced the source radius \( R \) in \( C_2(Q) \) function by its \( R_{xy} \) and \( R_z \) components, can be found in Sec. 5.5.

**Multiplicity**

For the study purposes the track multiplicity area was divided into the 5 following intervals: 20, 21, 22-23, 24-27, 28 and more. The multiplicity distribution is shown in Fig. 4.2. The \( C_2^{++--}(Q) \) functions for these intervals are compared in Fig. 5.14. There are slight differences in the shapes of the \( C_2(Q) \) functions obtained for different multiplicity intervals, but the errors of the fitted parameters are bigger than the differences between the parameters themselves. We can conclude that if we restrict to the processes with the track multiplicity 20 and more tracks, then there is no noticeable dependence of \( C_2 \) parameters on track multiplicity.

\[ 5.5 \quad q_{xy} \text{ and } q_z \]

The \( C_2(Q) \) function as is defined by Eq. 2.12 assumes that the source is characterized by the spherical emission probability, what should corresponds to creation of a hadronic system with the thermodynamical equilibrium. This does not have to be always true in the collider like experiments (like CDF-II). At so high energies the two
partons (one parton from $p$ and one from $\bar{p}$) interaction dominates. This collision exhibits features of the deep inelastic process where the final particles hadronize almost independently instead of creation common hadronizing system which we are looking for. For this purpose we have investigated the angular distribution of the reconstructed tracks. Fig. 5.15 (left) shows the $\cos(\theta)$ distribution of the tracks. This distribution should be uniform in the case of tracks produced by a thermodynamical system. As can be seen from Fig. 5.15 (left) the produced tracks are not uniform in $\cos(\theta)$ and prefer the forward backward directions, hence it cannot be said that they are fully produced by a thermodynamical system. In addition, one can see that also the mean tracks energy shows thermal equilibrium distortion (see Fig. 5.15 (right)) as the mean track energy increases with $|\cos(\theta)|$.

To see the influence of track non uniform $\cos(\theta)$ distribution to the $C_2(Q)$ func-
5.5. $Q_{XY}$ AND $Q_Z$

Figure 5.13: The $C_2^{++--}(Q)$ (left) and $C_2^{++--}_{EM}(Q)$ (right) functions for the different $\eta$ intervals.

Figure 5.14: The $C_2^{++--}(Q)$ (left) and $C_2^{++--}_{EM}(Q)$ (right) functions for the different multiplicity intervals.

The parameters we have replaced the source radius $R$ by its transverse and longitudinal components, $R_{xy}$ and $R_z$, looking for possible source deformations. One can write:

$$C_2(Q) = 1 + \lambda e^{-Q^2 R^2} = 1 + \lambda e^{-q_{xy}^2 - q_z^2 |R|^2} = 1 + \lambda e^{-q_{xy}^2 R_{xy}^2 - q_z^2 R_z^2 - q_0^2 \tau^2},$$  \hspace{1cm} (5.7)

where $q_{xy}^2 = (p_{1x} - p_{2x})^2 + (p_{1y} - p_{2y})^2$ and $q_z^2 = (p_{1z} - p_{2z})^2$ are the transverse and longitudinal components of $q$ and $q_0^2 = (E_2 - E_1)^2$ is the energy difference square. $R_{xy}$ ($R_z$) is the radius corresponding to the $q_{xy}$ ($q_z$) and $\tau$, which is not important
5.5. $Q_{XY}$ AND $Q_{Z}$

for us, has the meaning of the time. If we put $q_0 = 0$, we can find the single components of the source size. In this analysis we have used the cut $q_0 < 0.2 GeV$ to have a data sample of finite size. For the purpose of Coulomb correction (see Sec. 2.6) we have used the $Q' = \sqrt{q_{xy}^2 + q_z^2}$ value in the Gamow penetration factor, as the $q_0$ is requested to be zero, in the ideal case.

The $C_{2}^{++--}(q_{xy}, q_{z})$ and $C_{2}^{++EM}(q_{xy}, q_{z})$ functions are shown in Fig. 5.16. We fit these functions with the exponential and Gaussian parametrization of $C_2(q_{xy}, q_z)$ function:

$$C_2(q_{xy}, q_z) = C_0 (1 + \lambda e^{-R_{xy}^2 q_{xy}^2 - R_z^2 q_z^2})$$

$$C_{2}^{empirical}(q_{xy}, q_{z}) = C_0 (1 + 2p(1-p)e^{-R_{xy} q_{xy} - R_z q_z} + p^2 e^{-2(R_{xy} q_{xy} + R_z q_z)})$$

We have also used a two-dimensional equivalent of the $C_2(Q)$ function obtained in the quantum optic approach (see Sec.2.3):

$$C_2(q_{xy}, q_z) = C_0 (1 + \lambda e^{-R_{xy}^2 q_{xy}^2 - R_z^2 q_z^2} + p^2 e^{-2(R_{xy} q_{xy} + R_z q_z)})$$

$$C_{2}^{empirical}(q_{xy}, q_{z}) = C_0 (1 + 2p(1-p)e^{-R_{xy} q_{xy} - R_z q_z} + p^2 e^{-2(R_{xy} q_{xy} + R_z q_z)})$$

Table 5.3 summarizes the parameters obtained from 2-dimensional fit of the $C_2(q_{xy}, q_z)$ functions. To perform a fit of the $C_{2}^{++--}(q_{xy}, q_z)$ function (see Fig. 5.15)

Figure 5.15: The $\cos(\theta)$ distribution of the tracks (left). The mean energy of tracks vs. $\cos(\theta)$ (right).
5.6 Study of systematics

The systematic uncertainties were estimated by comparing the $C_2(Q)$ function parameters for different approaches.

The systematic uncertainty due to primary vertex block selection

In the CDF experiment there are two methods used for finding interaction primary vertex and the results of finding are stored in two data blocks named $Z\text{VertexBlock}$ and $\text{VertexBlock}$.

We have used the $Z\text{VertexBlock}$ data block to select the events with only one primary vertex. We checked how much are our results affected, if we would use the $\text{VertexBlock}$ data block instead of the $Z\text{VertexBlock}$ one. This can pose a problem in the case when two primary vertices are presented in event. The presence of second interaction can decrease the BEC signal, due to two track combinations from different interactions.

We have constructed $C_{2}^{++--}(Q)$ and $C_{2EM}^{++--}(Q)$ functions for each case and compared the fitted parameters. The estimated systematic errors are shown in Table 5.4.
### Table 5.3: The result of the two-dimensional fit of the $C_2(Q)$ functions shown in Fig. 5.16

<table>
<thead>
<tr>
<th>fit function</th>
<th>Param.</th>
<th>$C_2^{++--}(Q)$</th>
<th>$C_2^{EM}(Q)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.8$\text{Gauss}$</td>
<td>$C_0$</td>
<td>$1.004 \pm 0.0$</td>
<td>$0.9884 \pm 0.0$</td>
</tr>
<tr>
<td></td>
<td>$\lambda$</td>
<td>$0.326 \pm 0.017$</td>
<td>$0.275 \pm 0.004$</td>
</tr>
<tr>
<td></td>
<td>$R_{xy}$</td>
<td>$5.38 \pm 0.27$</td>
<td>$2.10 \pm 0.02$</td>
</tr>
<tr>
<td></td>
<td>$R_z$</td>
<td>$7.69 \pm 0.33$</td>
<td>$2.99 \pm 0.04$</td>
</tr>
<tr>
<td></td>
<td>$\chi^2/\text{ndf}$</td>
<td>$2416/1387$</td>
<td>$4313/1596$</td>
</tr>
<tr>
<td>5.8$\text{exp}$</td>
<td>$C_0$</td>
<td>$1.004 \pm 0.0$</td>
<td>$0.9854 \pm 0.0$</td>
</tr>
<tr>
<td></td>
<td>$\lambda$</td>
<td>$0.57 \pm 0.04$</td>
<td>$0.558 \pm 0.010$</td>
</tr>
<tr>
<td></td>
<td>$R_{xy}$</td>
<td>$6.27 \pm 0.44$</td>
<td>$3.048 \pm 0.044$</td>
</tr>
<tr>
<td></td>
<td>$R_z$</td>
<td>$10.07 \pm 0.74$</td>
<td>$4.169 \pm 0.072$</td>
</tr>
<tr>
<td></td>
<td>$\chi^2/\text{ndf}$</td>
<td>$2470/1387$</td>
<td>$4366/1596$</td>
</tr>
<tr>
<td>5.9$\text{Gauss}$</td>
<td>$C_0$</td>
<td>$1.004 \pm 0.0$</td>
<td>$0.988 \pm 0.0$</td>
</tr>
<tr>
<td></td>
<td>$p$</td>
<td>$0.18 \pm 0.01$</td>
<td>$0.152 \pm 0.002$</td>
</tr>
<tr>
<td></td>
<td>$R_{xy}$</td>
<td>$5.23 \pm 0.26$</td>
<td>$2.072 \pm 0.023$</td>
</tr>
<tr>
<td></td>
<td>$R_z$</td>
<td>$7.49 \pm 0.32$</td>
<td>$2.95 \pm 0.04$</td>
</tr>
<tr>
<td></td>
<td>$\chi^2/\text{ndf}$</td>
<td>$2417/1387$</td>
<td>$4273/1596$</td>
</tr>
<tr>
<td>5.9$\text{exp}$</td>
<td>$C_0$</td>
<td>$1.004 \pm 0.0$</td>
<td>$0.985 \pm 0.0$</td>
</tr>
<tr>
<td></td>
<td>$p$</td>
<td>$0.37 \pm 0.04$</td>
<td>$0.377 \pm 0.010$</td>
</tr>
<tr>
<td></td>
<td>$R_{xy}$</td>
<td>$5.82 \pm 0.36$</td>
<td>$2.856 \pm 0.036$</td>
</tr>
<tr>
<td></td>
<td>$R_z$</td>
<td>$9.27 \pm 0.63$</td>
<td>$3.921 \pm 0.063$</td>
</tr>
<tr>
<td></td>
<td>$\chi^2/\text{ndf}$</td>
<td>$2473/1387$</td>
<td>$4331/1596$</td>
</tr>
</tbody>
</table>

As can be seen, the primary vertex block choice has no influence to the result.

### Table 5.4: The systematic errors of the $C_2(Q)$ functions for the different fit functions, caused by a different choice of the used primary vertex block.

<table>
<thead>
<tr>
<th>fit function</th>
<th>$C_0$ sys. error</th>
<th>$\lambda/p$ sys. error</th>
<th>$R$ sys. error</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.5$\text{Gauss}$</td>
<td>$\pm 0.0$</td>
<td>$\pm 0.0006$</td>
<td>$\pm 0.006$</td>
</tr>
<tr>
<td>5.5$\text{exp}$</td>
<td>$\pm 0.0$</td>
<td>$\pm 0.002$</td>
<td>$\pm 0.02$</td>
</tr>
<tr>
<td>5.6$\text{Gauss}$</td>
<td>$\pm 0.0$</td>
<td>$\pm 0.0005$</td>
<td>$\pm 0.006$</td>
</tr>
<tr>
<td>5.6$\text{exp}$</td>
<td>$\pm 0.0$</td>
<td>$\pm 0.002$</td>
<td>$\pm 0.01$</td>
</tr>
<tr>
<td>5.5$\text{Gauss}$</td>
<td>$\pm 0.0$</td>
<td>$\pm 0.001$</td>
<td>$\pm 0.04$</td>
</tr>
<tr>
<td>5.5$\text{exp}$</td>
<td>$\pm 0.0$</td>
<td>$\pm 0.004$</td>
<td>$\pm 0.02$</td>
</tr>
<tr>
<td>5.6$\text{Gauss}$</td>
<td>$\pm 0.0$</td>
<td>$\pm 0.0005$</td>
<td>$\pm 0.04$</td>
</tr>
<tr>
<td>5.6$\text{exp}$</td>
<td>$\pm 0.0$</td>
<td>$\pm 0.004$</td>
<td>$\pm 0.02$</td>
</tr>
</tbody>
</table>
5.6. STUDY OF SYSTEMATICS

The systematic uncertainty in jet block selection

For the QCD peak correction we have used the jets taken from the jetCluModule07 data block. The other possibilities, for the construction of the $N_{\text{corrected}}^+$ distribution, were to use jetCluModule10 or jetBlock data blocks (see Fig. 5.5). We have constructed the $C_2^{++-}(Q)$ functions for each of the cases and found the systematic uncertainties by comparing of the fitted parameters. For estimation of the systematic errors, we took into account the $\chi^2$ of each fit. The obtained systematic errors of the $C_2^{++-}(Q)$ function parameters are shown in Table 5.5.

<table>
<thead>
<tr>
<th>fit function</th>
<th>$C_0$ sys. error</th>
<th>$\lambda/p$ sys. error</th>
<th>$R$ sys. error</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.5 Gauss</td>
<td>$\pm 0.002$</td>
<td>$\pm 0.005$</td>
<td>$\pm 0.37$</td>
</tr>
<tr>
<td>5.5 exp</td>
<td>$\pm 0.002$</td>
<td>$\pm 0.007$</td>
<td>$\pm 0.81$</td>
</tr>
<tr>
<td>5.6 Gauss</td>
<td>$\pm 0.002$</td>
<td>$\pm 0.003$</td>
<td>$\pm 0.37$</td>
</tr>
<tr>
<td>5.6 exp</td>
<td>$\pm 0.002$</td>
<td>$\pm 0.006$</td>
<td>$\pm 0.77$</td>
</tr>
</tbody>
</table>

**Table 5.5:** The systematic errors of the $C_2^{++-}(Q)$ function for different fit functions, caused by the choose of the used jet block.

The systematic uncertainty due to the choice of fit interval

In the case of the $C_2^{++-}(Q)$ function, where we exclude from fit the interval $0.2\text{GeV} < Q < 0.8\text{GeV}$, we have introduced a systematic error connected with our choice of boundaries for this interval. In order to find the systematic error, we have varied the lower boundary of the interval in the range from $0.15\text{GeV}$ to $0.6\text{GeV}$ and the upper boundary from $0.6\text{GeV}$ to $1.2\text{GeV}$. In the fit procedure only the parameter $\lambda (p)$ was restricted in its natural boundaries: $(0, 1)$. In some cases the fit failed or gave the resulting parameter at its limit. These cases were not used for the systematic errors evaluation. We have weighted the results of particular fits by the $\chi^2$ values. The estimated systematic errors of the $C_2^{++-}(Q)$ function parameters are shown in Table 5.6.

One can see, that the systematic error of the parameter $R$, for the fit function of the exponential form, is bigger than that for the Gaussian form. That is due to a poor stability of this parameter when the excluded $Q$ interval boundaries are varied.

In Table 5.7 are shown the systematic errors of the fitted parameters, for the
5.7. COMPARISON WITH OTHER EXPERIMENTS

<table>
<thead>
<tr>
<th>fit function</th>
<th>$C_0$ sys. error</th>
<th>$\lambda/p$ sys. error</th>
<th>$R$ sys. error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5.5_{Gauss}$</td>
<td>±0.002</td>
<td>±0.005</td>
<td>±0.11</td>
</tr>
<tr>
<td>$5.5_{exp}$</td>
<td>±0.002</td>
<td>±0.06</td>
<td>±1.45</td>
</tr>
<tr>
<td>$5.6_{Gauss}$</td>
<td>±0.002</td>
<td>±0.003</td>
<td>±0.14</td>
</tr>
<tr>
<td>$5.6_{exp}$</td>
<td>±0.002</td>
<td>±0.12</td>
<td>±1.19</td>
</tr>
</tbody>
</table>

Table 5.6: The systematic errors of the $C_{2}^{++--}(Q)$ function parameter for the different fit functions, caused by choice of the boundaries of the interval excluded from fit.

two-dimensional $C_{2}^{++--}(q_{xy}, q_{z})$ function, evaluated in the same way.

<table>
<thead>
<tr>
<th>fit function</th>
<th>$C_0$ sys. error</th>
<th>$\lambda/p$ sys. error</th>
<th>$R_{xy}$ sys. error</th>
<th>$R_z$ sys. error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5.5_{Gauss}$</td>
<td>±0.002</td>
<td>±0.02</td>
<td>±0.35</td>
<td>±0.15</td>
</tr>
<tr>
<td>$5.5_{exp}$</td>
<td>±0.002</td>
<td>±0.15</td>
<td>±1.77</td>
<td>±1.52</td>
</tr>
<tr>
<td>$5.6_{Gauss}$</td>
<td>±0.002</td>
<td>±0.01</td>
<td>±0.36</td>
<td>±0.15</td>
</tr>
<tr>
<td>$5.6_{exp}$</td>
<td>±0.002</td>
<td>±0.30</td>
<td>±0.71</td>
<td>±1.24</td>
</tr>
</tbody>
</table>

Table 5.7: The systematic errors of the $C_{2}^{++--}(q_{xy}, q_{z})$ function parameters for the different fit functions, caused by choice of the boundaries of the interval excluded from fit.

The systematic uncertainty as a result of bin size

We have investigated the influence of bin size on the $C_2(Q)$ functions parameters. The number of bins, in the interval from 0.01GeV (we have excluded the region of possible two track fake reconstruction, see Fig. 4.6) to 3.0GeV, has been varied from 30 to 90. The estimated systematic errors of the $C_2(Q)$ functions parameters are shown in Table 5.8.

5.7 Comparison with other experiments

The $E735$ collaboration [16], operating on Tevatron collider (Fermilab) at $\sqrt{s} = 1.8TeV$, obtained the result of $r_T = 1.06 \pm 0.07fm$. The event mixing reference sample was used. The cut to the $q_0$ was set to be greater than 0.2GeV, so the fit by the $C_2(q_T) = 1 + \lambda' e^{-r_T^2 q_T^2}$ function gives directly the transverse radius of the source. The longitudinal component is, in this case, part of the $\lambda' = \lambda e^{-r_L q_L}$. Only
the tracks tagged as $\pi$ mesons were used.

The UA1-MINIMUM BIAS Collaboration [17] presented the result obtained on $p-\bar{p}$ events at $\sqrt{s} = 630\, GeV$ and $\sqrt{s} = 900\, GeV$. Here the $|\eta|$ region was analyzed up to 3. As the reference sample, the one obtained by the event mixing technique, was used. For the fit, they have used the $C_2(Q)$ function obtained in the quantum optics approach (see, 2.3). The found parameters are shown in Table 5.9.

<table>
<thead>
<tr>
<th>fit function</th>
<th>$C_2$ sys. error</th>
<th>$\lambda/p$ sys. error</th>
<th>$R$ sys. error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5.5_{Gauss}$</td>
<td>$\pm 0.0$</td>
<td>$\pm 0.016$</td>
<td>$\pm 0.08$</td>
</tr>
<tr>
<td>$5.5_{exp}$</td>
<td>$\pm 0.0001$</td>
<td>$\pm 0.066$</td>
<td>$\pm 0.71$</td>
</tr>
<tr>
<td>$5.6_{Gauss}$</td>
<td>$\pm 0.0$</td>
<td>$\pm 0.011$</td>
<td>$\pm 0.09$</td>
</tr>
<tr>
<td>$5.6_{exp}$</td>
<td>$\pm 0.0$</td>
<td>$\pm 0.056$</td>
<td>$\pm 0.51$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$C_2^{+\bar{EM}}$</th>
<th>$C_2$ sys. error</th>
<th>$\lambda$ sys. error</th>
<th>$R$ sys. error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5.5_{Gauss}$</td>
<td>$\pm 0.0$</td>
<td>$\pm 0.003$</td>
<td>$\pm 0.0008$</td>
</tr>
<tr>
<td>$5.5_{exp}$</td>
<td>$\pm 0.0002$</td>
<td>$\pm 0.015$</td>
<td>$\pm 0.081$</td>
</tr>
<tr>
<td>$5.6_{Gauss}$</td>
<td>$\pm 0.0$</td>
<td>$\pm 0.0004$</td>
<td>$\pm 0.0008$</td>
</tr>
<tr>
<td>$5.6_{exp}$</td>
<td>$\pm 0.0002$</td>
<td>$\pm 0.015$</td>
<td>$\pm 0.075$</td>
</tr>
</tbody>
</table>

Table 5.9: The UA1 Collaboration results.

The ALEPH Collaboration [18] analyzed $e^+e^-$ collisions at $\sqrt{s} = 91.2\, GeV$. To remove the hadronic background, they have used events with more than 8 charged tracks. Two different reference samples were used, the one obtained by the event mixing technique and the one constructed using the opposite charged tracks from the event. The data were fitted to the $C_2(Q)$ function in the most used parametrization $5.5_{Gauss}$ multiplied, for a better data description, by an empirical $(1+\delta Q)$ factor. The parameters yield the values: $R = 0.53 \pm 0.01\, fm$, $\lambda = 0.36 \pm 0.01$ for the event mixing reference sample and $R = 0.78 \pm 0.01\, fm$, $\lambda = 0.44 \pm 0.01$ for the second one, the event opposite charged tracks combinations reference sample.

The ZEUS Collaboration [19], where the electron beam at the energy of 27.6$GeV$ was collided with the proton beam at the energy of 820(920)$GeV$, have used the ref-
erence sample calculated using unlike-charged pairs, corrected by the Monte Carlo simulations, to remove the effect of resonances. The parameters of the $C_2(Q)$ function ($5.5_{\text{Gauss}}$) are: $R = 0.67 \pm 0.01 \text{fm}$, $\lambda = 0.48 \pm 0.1$.

The OPAL Collaboration operating on the LEP collider shows the Bose-Einstein correlations between the $\pi^0$ mesons coming from the $Z_0$ decays. The event mixing reference sample was used and the found parameters of the $5.5_{\text{Gauss}}$ function (here multiplied by the $(1 + \delta Q + \epsilon Q^2)$ factor) are: $R = 0.59 \pm 0.08 \text{fm}$, $\lambda = 0.55 \pm 0.1$.

In the DELPHI experiment the kaon correlations were studied. The DELPHI operated on LEP, the $e^+e^-$ annihilation at $\sqrt{s} = 91.2 \text{GeV}$. The unlike-signed pairs, for the reference sample construction, were used. The standard $C_2(Q)$ function parametrization $5.5_{\text{Gauss}}$ was used and the found parameters yield: $R = 0.48 \pm 0.1 \text{fm}$, $\lambda = 0.82 \pm 0.3$ for the $K^{++--}$ sample and $R = 0.55 \pm 0.1 \text{fm}$, $\lambda = 0.61 \pm 0.2$ for the $K^0$ correlations.
Chapter 6

Final Summary

The results of study of the \( p\bar{p} \) collisions at \( \sqrt{s} = 1.96\)TeV collected by the \textit{CDF-II} experiment using the high multiplicity trigger are presented. The data sets coded as \textit{gmbsbi} and \textit{gmbsbj} datasets have been used in this study. The events contained in these datasets were collected from 10.11.2005 up to 30.01.2007 and contain 298559 events. After all the cuts, needed to remove main sources of background and reconstruction errors, 173761 events remains.

We have investigated the Bose-Einstein correlations using two particle \( C_2(Q) \) function. Two different reference samples, needed for it’s construction, were used: the \( N^{++}-(Q) \) one, obtained as a 4-momentum difference between positive and negative tracks taken from the same event, and the \( N_{EM}^{++--}(Q) \) one, obtained from the same-signed pairs combination of tracks from different events. The parameters of the \( C_2(Q) \) functions were retrieved. We have used the most used parametrization of the \( C_2(Q) \) function obtained by the plane-wave approach and also the one obtained in quantum optical approach to show the possible misinterpretation of the parameter \( \lambda \) from plane-wave approach. Also the 2-dimensional equivalents of those \( C_2(Q) \) functions were retrieved and the corresponding parameters summarized. Two parametrizations of the source emission probability were used. The first one has the Gaussian shape, and the second one, introduced to obtain a better fit of the experimental data, has the exponential form after Fourier transform. Although, the latter parametrization has fitted the data more precisely, it was less stable in the choice of the resonance area exclusion.

We have used the \textit{Time-of-flight} detector for particle identification and constructed the \( C_2(Q) \) functions exclusively for the \( \pi \) and \( K \) mesons. The \( C_2(Q) \) func-
tion parameters are presented only for the 1-dimensional case due to a lower statistic.

The values for the radius $R$ of the 2-pion emitter and the 'so called' coherence factor $\lambda$ (mostly used parametrization of $C_2(Q)$ function $5.5_{Gauss}$) of the BEC effect were measured to be (for $C_{2EM}^{++--}(Q)$ function):

$$R = 2.565 \pm 0.019(stat.) \pm 0.004(sys.) \text{fm}$$
$$\lambda = 0.276 \pm 0.003(stat.) \pm 0.003(sys.),$$

and (for $C_{2}^{++--}(Q)$ function):

$$R = 7.53 \pm 0.17(stat.) \pm 0.39(sys.) \text{fm}$$
$$\lambda = 0.293 \pm 0.01(stat.) \pm 0.017(sys.)$$

where the uncertainties are dominated by the systematic errors.
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Very special thanks also go to the CDF collaboration physicists, technical and engineering staff and students, who spent many years of dedicated work in this experiment. Without their work it would not be possible to perform any analyzes in this part of physics.

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Bibliography


[3] N. Moggi, M. Mussini, F. Rimondi *Track selection and counting efficiency in Minimum Bias*, CDF-note 8593


