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PRODUCTION OF TWO PSEUDOSCALAR MESONS
IN ANTIPROTON-PROTON ANNIHILATION AT $2.9 < \sqrt{s} < 3.6$ GeV

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Abstract

Experimental results for $\bar{p}p \rightarrow$ two pseudoscalar mesons have been obtained from Fermilab experiment E760. Specifically $\bar{p}p \rightarrow \pi^0\pi^0, \pi^0\eta, \eta\eta$ were studied. Angular distributions and the $\sqrt{s}$ dependence of the differential cross sections are presented. These data are in the range $\sqrt{s}$ from 2.9-3.6 GeV. For the high Q^2 region, where perturbative QCD theory works well, different models are compared to the data.

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Chapter 1

Introduction

Quantum Chromodynamics (QCD) is widely accepted as the best theory for describing the interaction of hadrons and their constituents. This thesis work concerns experimental data that are used to test some aspects of QCD. More specifically, it is the study of $\bar{p}p \rightarrow \bar{M}M$, where M is one of the pseudoscalar mesons; π^0, η .

The data are from Fermilab experiment E760. This experiment was designed to study charmonium spectroscopy and this defined the energy range of data, $\sqrt{s}$, from 2.9 to 3.6 GeV. The experimental design allowed for the inclusion of triggers to study exclusive neutral final states.

This thesis presents data from E760 primarily in the form of center-of-mass angular distributions, and for a given production angle, the differential cross section as a function of $\sqrt{s}$. The analysis includes a comparison of data to predictions made by Perturbative QCD (PQCD).

Chapter 2 provides theoretical motivation for the study of this type of reaction. Models that make predictions for high momentum transfer are discussed.

Chapter 3 presents details of E760. An overview of the experiment's goals and their bearing on the detector design is presented. This includes the experiment's interaction with the Fermilab Tevatron and Antiproton Accumulator. An overview of the E760 detector components is given, as well as a discussion of the triggering system. Finally, a data summary is given.

Chapter 4 discusses the method of analysis and data processing. The χ^2 method is explained and developed for this analysis. Data selection, and analysis using the χ^2 method, are described. Corrections made to the data to obtain final results are discussed.

Chapter 5 presents results. Angular distributions are shown. The energy dependence of differential cross sections is shown. A comparison to other experiments is given. A discussion of results is given, as they relate to ideas presented in chapter 2. Chapter 6 presents conclusions.

Chapter 2

Theoretical Motivation

2.1 Introduction

Testing of QCD provides the motivation of the study of $\bar{p}p \rightarrow \bar{M}M$. QCD is the theory used to describe the strong force, the basic interaction between hadron constituents; quarks and gluons. As such, it should also describe the collective behavior of these quanta, namely hadron-hadron interactions. Clearly, this is a formidable task, analogous to applying the theory of quantum electrodynamics (QED) to systems of electrons and protons (atoms).

In relative terms, bound states of $\bar{q}q$ are representative of simpler systems and QCD has had a great deal of success in understanding them[1–3]. This analysis is derived from antiproton interactions, where we have in the initial state a proton (p) consisting of uud valence quarks and an antiproton ($\bar{p}$) comprised of the corresponding anti-quarks. While $\bar{p}p \rightarrow \bar{q}q$ was the major thrust of the experiment [4], this work concerns reactions of the type $\bar{p}p \rightarrow \bar{M}M$ where M is one of the pseudoscalar mesons; π^0 , η . Understanding the production mechanism involved in terms of QCD is not simple and therefore provides a testing ground for theoretical models.

The strong force has the property that, as the distance between interacting constituents increases, so does the force. There are various potential models used to describe the strong force. One that nicely displays the qualitative features of the strong force is the so-called Cornell Model [5]:

$$V(r) = kr - \frac{4\alpha_s}{3r}. \quad (2.1)$$

In QCD the field itself carries color. This results in a screening effect, whereby the strong coupling constant, α_s , increases with distance. The closer one probes the quark the less

color one “sees”. This is called asymptotic freedom.

In this region where the force is weaker, calculations in QCD can make use of perturbation theory or PQCD. Therefore if one wants to make use of or test PQCD, one must probe small distances. It is not possible to determine directly the impact parameter for individual interactions. However, one can measure momentum transfer which is inversely proportional to the distance probed, $x = 1/q$, where x is the interaction distance and q is the momentum transfer. Here the units are such that Planck’s constant, $\hbar$, and the speed of light, c , are set equal to 1. The relationship has its roots in the uncertainty principle. The smaller the region the interaction is confined to, the greater the range of allowed momentum transfer.

The production through large momentum transfer is called “hard scattering”. How large is large? It must at least be large enough to begin to resolve hadronic constituents. A proton’s size (interaction region) is on the order of 1 Fm, which corresponds to a momentum transfer of 200 MeV/c. This is the Fermi motion of particles confined to 1 Fm. For these data, the range of momentum transfers is shown in table 2.1 along with the corresponding distances probed. The table lists these numbers for the range of $M\bar{M}$ production angles, $\theta_{\bar{p}M}^*$, that will be used in the analysis of hard scattering. The table shows that momentum transfers for which the range of data applies correspond to distances on-the-order-of 0.1 Fm. One would therefore expect that large angle scattering studied here involves physics about which PQCD has something to say.

There are in fact PQCD predictions that can be applied to our data. Two models that describe large angle products in $\bar{p}p \rightarrow \bar{M}M$ are Brodsky scattering [6], and Landshoff scattering [7]. They each make different predictions, and there are also predictions on an interference between the two. These ideas will now be discussed.

Table 2.1: Range of the square of the momentum transfer (t) for this experiment versus the cosine of the center-of-mass production angle ($\theta_{\bar{p}M}^*$) for the reactions shown. Also listed are the corresponding distances probed.

reaction	$\sqrt{s}$	$t \text{ (GeV/c)}^2$		$\hbar/\sqrt{t} \text{ (Fm)}$	
		$\cos \theta_{\bar{p}M}^* = 0.0$	$\cos \theta_{\bar{p}M}^* = 0.3$	$\cos \theta_{\bar{p}M}^* = 0.0$	$\cos \theta_{\bar{p}M}^* = 0.3$
$\pi^0 \pi^0$	2.9500	3.45	2.45	0.108	0.128
$\pi^0 \eta$	2.9500	3.31	2.34	0.110	0.131
$\eta \eta$	2.9500	3.17	2.24	0.112	0.134
$\pi^0 \pi^0$	3.6200	5.65	3.98	0.084	0.100
$\pi^0 \eta$	3.6200	5.51	3.87	0.085	0.102
$\eta \eta$	3.6200	5.37	3.77	0.086	0.103

2.2 Dimensional Counting Rules

Dimensional counting rules are derived by calculating amplitudes for individual quark scattering when they are far off the mass shell [6]. As discussed in reference [8], the differential cross section is

$$\frac{d\sigma}{dt} \propto \frac{1}{s^2} |M|^2, \quad (2.2)$$

where M is the scattering amplitude for going from the initial state to the final state of an individual quark scattering. The dimension of M is length^{n-4} , and $\sqrt{s}$ provides the length scale from which equation 2.3 follows. This can be seen as each hard gluon (propagator) contributing a factor $1/q^2$ to M .

Dimensional counting rules [6] have been used with great success to predict differential cross sections [9]. Applying the rules, the differential cross section can be written as,

$$\frac{d\sigma}{dt} \propto \frac{1}{s^{n-2}} f(\theta), \quad (2.3)$$

where σ is the cross section, t is the square of the momentum transfer, s is the center-of-mass energy squared, θ is the center-of-mass production angle, and n is the number of hadronic constituents in the interaction. Experimental data for numerous reactions have demonstrated the validity of equation 2.2, for example; $\pi^-p \rightarrow \pi^-p$, $\pi^+p \rightarrow \pi^+p$, $K^-p \rightarrow K^-p$, $K^+p \rightarrow K^+p$. An overview of the success of dimensional counting rules can be found in references [8], and [9]. For $\bar{p}p \rightarrow \bar{M}M$ we have 6 quarks in the initial state and 4 in the final state. Dimensional counting rules, therefore, predict the differential cross section to have an s^{-8} dependence.

For $\bar{p}p \rightarrow \bar{M}M$ this type of scattering is depicted in figure 2.1. Here, quarks are represented by smooth lines and gluons by curly lines. The connection between quark lines and gluons is represented by dots. An important feature of this diagram is that all quark lines are connected through gluons. For wide angle scattering to occur, therefore, they must interact at close range. As the figure indicates, the interaction is due to valence quarks which must be relatively close, meaning the hadron itself must be in a state where its size is smaller than normal (~ 1 Fm).

2.3 Landshoff Scattering

An alternative explanation for wide angle scattering was put forth by Landshoff [7]. This is represented by figure 2.2 for $\bar{p}p \rightarrow \bar{M}M$. A significant difference between figure 2.1 and figure 2.2 is that we now have quark lines that are not completely connected. That is, the top-most and bottom-most quark lines are connect to each other, but not to the other quarks. This means the constituents do not have to be off-shell [6]. For $pp \rightarrow pp$ and $\pi\pi \rightarrow \pi\pi$, for example, the reaction can take place with quark lines completely unconnected. For $\bar{p}p \rightarrow \pi\pi$ there must be some connected line since the number of quarks in the initial state does not equal the number in the final state. That is, some quarks from each baryon must interact since quarks are annihilated. The idea here is that scattering at the same angle

Figure 2.1: Diagram showing completely connected quark lines occuring in Brodsky scattering.

Figure 2.2: Diagram showing unconnected quark lines in Landshoff scattering.

occurs through independent scatterings. The hard scattering involving large momentum transfer is not necessary here, because the results of independent scattering are said to have their momentum aligned in such a way so as to “readily recombine to make up the final state hadron” [7].

This argument does not require that the proton be relatively small. On the contrary, the larger it is the more space there is available for independent scattering to occur.

This type of scattering predicts a smaller dependence on energy than dimensional counting rules. This comes from the fact that there are fewer propagators needed in the calculation of the scattering amplitude for the disconnected diagram of figure 2.2 than the connected diagram of figure 2.1.

The smaller s dependence means that this type of scattering should dominate at higher energies. There is, however, an argument that discusses the suppression of the Landshoff type scattering - the Sudakov effect [10]. For Landshoff type scattering, close range hard scattering is not needed. However, the near on-shell constituents now being considered may also interact via soft gluon exchange. In addition, loop diagrams may be involved. Figure 2.3 represent Landshoff scattering with an additional loop term. This is the gluon that begins and ends on the same quark line. These higher order terms are accounted for in what is called the Sudakov factor.

These Sudakov effects are also s dependent, and combine in a way that suppresses Landshoff type scattering [11]. Thus, at high s , dimensional counting rules should again dominate. This leaves the mid-range of s for Landshoff scattering to make a significant contribution. There are in fact predictions for, and some evidence for, an interference between the two types of scattering [12,13]. The data analyzed in this work concerns the region of s where this interference is expected. Below, the form of the Sudakov effect will be discussed as well as interference that is expected to result from it.

Figure 2.3: Example of Landshoff scattering with a loop term.

2.4 Deviations From Dimensional Counting Rules

2.4.1 Interference Between Brodsky Scattering and Landshoff Scattering

The Sudakov effect is derived from higher order terms in PQCD calculations [10,11]. These calculations are needed because when higher order diagrams are drawn (diagrams involving soft gluons and loops) divergent terms arise in the scattering amplitude. Interference between different terms in the PQCD expansion results in cancellations of these divergent diagrams. The result is a correction to Landshoff scattering amplitude. The total scattering amplitude for hard scattering and Landshoff scattering has the form

$$M = M_{QC} + e^{i\Phi(s)+i\delta} M_L. \quad (2.4)$$

Here M_{QC} is the amplitude for hard scattering previously discussed and M_L is the amplitude for Landshoff scattering [14]. The δ is an energy independent phase that is not calculable.

The $\Phi(s)$ factor is an energy dependent phase term and has the form

$$\Phi(s) = \frac{\pi}{0.06} \ln \ln \left(\frac{s}{\Lambda_{QCD}^2} \right). \quad (2.5)$$

Consequences of the Sudakov effect are a suppression of Landshoff type scattering, and, as can be seen from equation 2.4, an interference between the two types of scattering [12]. The suppression of Landshoff scattering is expected to allow hard scattering to dominate at high s . Hard scattering dominance at low s leaves a mid-range where Landshoff scattering can have a significant effect [13, 14]. An understanding of both types of scattering, therefore, can be obtained by looking for interference between them, realized in the form of oscillations in the differential cross section since $\frac{d\sigma}{dt} \propto |M|^2$.

These oscillations have been studied by Ralston *et al* [12, 14]. They discuss in quantitative terms the form of the interference and present data that are consistent with the expected oscillations. The data are from pp elastic scattering [15] and are shown in figure 2.4. Carlson *et al.* [13] have stated that $\bar{p}p \rightarrow \pi^0\pi^0$ should also be a good reaction for studying this behavior. In their work they discuss $\bar{p}p \rightarrow \pi^+\pi^-$ and present evidence for oscillations in the data. Studying these oscillations in $\bar{p}p \rightarrow \bar{M}M$ has an advantage over $pp \rightarrow pp$, since the scattering amplitude involves only two helicity amplitudes (versus 5 for $pp \rightarrow pp$). The result is a less “smeared” interference. In other words, the interference may be more pronounced for $\bar{p}p \rightarrow \pi^0\pi^0$. If the initial state particle polarizations are known the interference can be studied by measuring the analyzing power (the asymmetry seen in the final-state particle production angles). For Brodsky scattering, close interactions result in quark helicity conservation “carrying over” to hadron helicity conservation [13], or zero analyzing power. Landshoff scattering, on the other hand, does not require helicity conservation, and thus a large analyzing power is evidence for interference between the two processes. This also has been seen in pp elastic scattering [13]. For this analysis, there is no polarization information, so the asymmetry analysis cannot be done.

Figure 2.4: Data from elastic pp scattering, showing oscillations in the scaled differential cross section. Data taken from Ralston *et al* [14].

Data presented here will include $\bar{p}p \rightarrow \pi^0\pi^0$, as well as $\bar{p}p \rightarrow \pi^0\eta, \eta\eta$, where more complicated Fock states are expected to suppress the Landshoff amplitude and thus the oscillations [16]. That is, for $\bar{p}p \rightarrow \bar{M}M$ channels where the M is an η the quarks are less off-shell. Thus, higher terms in the PQCD expansion are needed and may result in detectable differences in the $\sqrt{s}$ dependence of the cross section for large production angle, $\theta_{\bar{p}M}^*$.

The data from E760 are in the range where oscillations are expected to occur. Dimensional counting rules say that the differential cross section for $\bar{p}p \rightarrow \bar{M}M$ should have an s^{-8} dependence. Landshoff scattering without Sudakov corrections has an s^{-7} dependence, which is not as strong since there are fewer hard scatterings of the interaction constituents involved. With the Sudakov corrections this becomes $s^{-7.7}$. Predictions for the oscillatory behavior for $pp \rightarrow \pi\pi$ come from Carlson *et al* [13]. The interference between the two due to the s dependent phase in the Sudakov correction term to the Landshoff scattering is expected to begin around $s = 10 \text{ GeV}^2$. This is based on what has been observed in pp elastic scattering. The energy dependent phase should be a function of the soft gluon scattering involved in the initial reaction [13]. In other words, according to these models $pp \rightarrow pp$ should exhibit the same interference effects as $\bar{p}p \rightarrow \pi^0\pi^0$ in terms of the period of

the oscillations (about 1 unit in $\ln(s)$), since the effect is caused by the interaction of the initial hadrons.

Carlson *et al* make no mention of other reactions that will be presented here ($\pi^0\eta$ and $\eta\eta$). The data however, allow a direct comparison between these reactions over the energy range $\sqrt{s} = 2.9\text{-}3.6$ GeV.

2.4.2 Resonance Formation

An alternative explanation for deviations from dimensional counting rules has been put forth by Brodsky and Teramond [17]. They have hypothesized the existence of resonances which account for structure in the scaled differential cross section. That is, rather than an interference phenomenon causing oscillations in the scaled differential cross section, resonant states are being formed. In reference [17] a model is presented to describe data from pp elastic scattering. Two resonances are proposed, which interfere with “hard” processes described by PQCD, from which dimensional counting rules are derived. The model presented is able to account for the analyzing power, and the deviations from fixed-angle scaling laws observed in pp elastic scattering. [17].

Brodsky and Teramond are able to fit data from pp elastic scattering to a model where the scattering amplitude consists of a PQCD component plus a sum of resonance terms having a Breit-Wigner form. The resonances are attributed to: (a) a multigluonic excitation, or (b) a hidden-color singlet excitation, or (c) a hidden-flavor excitation. This model is expected to apply to $\bar{p}p \rightarrow \bar{M}M$ channels [18]. Although no predictions exist for $\bar{p}p \rightarrow \bar{M}M$, the results of this thesis should motivate further development of the resonance hypothesis.

Chapter 3

Experiment E760

3.1 Introduction

This section describes details of experiment E760 at Fermilab. This includes a description of the apparatus, the event trigger scheme, the data acquisition and a summary of the data collected.

E760 was designed for studies of the resonant states of charm and anti-charm quarks, $\bar{c}c$, where production is from $\bar{p}p$. The center-of-mass energies therefore must cover the range from the lowest bound state (1S_0 or η_c) at about 2.98 GeV to the higher states around 4.20 GeV.

Production from $\bar{p}p$ has the advantage of making all $\bar{c}c$ states directly accessible, as opposed to e^+e^- experiments where only the states having the quantum numbers of the intermediate virtual photon are directly produced (J^{PC} of 1^{--}). Other states can be produced via intermediate photons, but this introduces a suppression in the scattering amplitude of $\alpha = 1/(137)$. In e^+e^- experiments the remaining states are studied through secondary decays and this introduces the difficulty of more complicated event topologies and greater dependence on calorimetry. In $\bar{p}p$ annihilation $\bar{c}c$ states can be produced via 2 or 3 intermediate gluons. Since the coupling constant is much stronger than α ($\alpha_s \sim 0.5$) suppression from the additional vertices in the Feynman diagrams is not as significant.

A primary consideration in the design of the detector and trigger was to be able to extract the signal from $\bar{c}c$ resonance decays from the rest of the $\bar{p}p$ interactions. This was accomplished to such a degree that it allowed other event types to be included in the trigger. This was done by running several triggers in parallel. One of these triggers was

derived from fast processing of calorimeter signals. This trigger was able to select exclusive neutral two-body final states - the subject of this thesis.

3.2 Fermilab and the Antiproton Accumulator

E760 is located in the Antiproton Accumulator at FNAL (see figure 3.1). This ring was designed to collect and provide 8.9 GeV/c antiprotons for injection into the main ring where they are accelerated and injected into the 1 TeV superconducting ring (Tevatron) for $\bar{p}p$ collider experiments. The Tevatron is also used to supply 1 TeV protons for “fixed target” experiments on beam lines external to the ring. In this second mode of operation the main ring is free for approximately 80% of the one minute cycle time to provide protons for production of antiprotons in the Accumulator. Roughly, the procedure is to inject 8.9 GeV/c protons from the booster, accelerate them to ~ 200 GeV/c in the main ring, and extract them to the target area (A0). Here they collide with a tungsten/rhenium target and the interaction of the protons (numbering $\sim 10^{12}$) with the target produce antiprotons (primarily through $pp \rightarrow ppp\bar{p}$). The antiprotons are focused using a lithium lens [19] and sent to the Accumulator.

The Accumulator [20, 21] consists of two rings - a debuncher, where each burst of particles coming from the target is circulated for ~ 2 -3 seconds, and the Accumulator. In the debuncher, pions have time to decay and the momentum spread is cooled from $\sim 3\%$ to 0.023% . The main ring is then ready to produce more antiprotons and the ones currently in the debuncher are transferred to the Accumulator. This cycle, from the linac, booster and main ring to the Accumulator, takes ~ 2 -3 seconds.

For E760, typically 3×10^{11} antiprotons were accumulated over a period of about 10 hours to define a stack. The procedure for scanning a charmonium resonance was to first decelerate to an energy above the resonance and cool the beam to $\frac{\Delta p}{p} < 2 \times 10^{-4}$. At this point the target, an H_2 gas jet, was turned on and data were taken with the detector. The

Figure 3.1: Schematic of the Fermilab Accelerator Complex.

beam momentum was lowered in steps of between $\sqrt{s} \sim 170 - 500$ KeV. The momentum spread in the center-of-mass was ~ 240 KeV (FWHM) [4]. Thus the initial state energy for the $\bar{p}p$ interaction was very well known and high resolution fine step scans were undertaken.

Accuracy in knowledge of the initial state energy (the beam energy plus the mass of the target proton) translates into an accuracy of the resonant mass being scanned through the relation

$$s = M_R^2 = (p_1 + p_2)^2, \quad (3.1)$$

where M_R is the resonant mass, and p_1 and p_2 are the 4-momenta of the antiproton and proton. Evaluating this in the lab system where the proton is stationary ($\vec{p}_2 = 0$) we get

$$M_R^2 = 2m_p(m_p + E_{beam}), \quad (3.2)$$

and from this we obtain

$$\delta M_R = \left(\frac{m_p}{M_R} \right) \delta E. \quad (3.3)$$

Calibration of the beam momentum was performed through scans of the J/ψ and ψ' resonances [22]. The beam energy is related to the frequency and orbit length of antiprotons by

$$E_{beam} = \frac{mc^2}{\sqrt{1 - \beta^2}} = \frac{mc^2}{\sqrt{1 - (fL_{orbit}/c)^2}}, \quad (3.4)$$

where f is the antiproton orbit frequency and L_{orbit} is the orbit length. These are quite well known (1 part in 10^7 for frequency and <1 mm in ~ 474.06 m) and the J/ψ and ψ' served as reference orbits for determining the momentum away from these points [22].

3.3 The Hydrogen Target, Luminosity, and Rates

As stated in the previous section, antiprotons collide with a hydrogen gas jet. The gas jet system [23] was designed to provide a localized interaction region with a high luminosity ($\sim$

$10^{31} \text{ cm}^{-2}\text{sec}^{-1}$), while minimizing effects on the circulating antiproton beam. A schematic of the gas jet system is shown in figure 3.2.

In the gas jet system the hydrogen gas starts in a region of high pressure and is allowed to expand after passing through a specially designed nozzle. It then traverses a series of chambers before being collected in the last chamber. As the hydrogen passes through the nozzle it undergoes a rapid expansion. The shape of the nozzle is designed to minimize the angular expansion region. That is, the fastest expansion takes place along the jet axis. Furthermore, the design is such that at a point on the jet axis away from the nozzle aperture temperature and pressure put the H_2 gas in a state of saturation. Further from this point as supersaturation occurs H_2 molecules will form molecular clusters with on-the-order-of 10^5 to 10^6 molecules per cluster. It is at the point of the cluster formation that the gas jet interacts with antiprotons as the jet crosses transverse to the beam. Typical densities achieved with the gas jet were on-the-order-of $3.5 \times 10^{13} \text{ atoms/cm}^2$ and with a width of $\sim 0.63 \text{ cm}$ over the region of the beam ($\sim 3.5 \text{ mm}$), or $\sim 10^{14} \text{ atoms/cm}^3$. With these conditions luminosities, defined as

$$L = N_{\bar{p}} \rho l f = (30 \times 10^{10} \bar{p}) \left(10^{14} \frac{p}{\text{cm}^3} \right) (0.63 \text{ cm}) \left(6.2 \times 10^5 \frac{1}{\text{sec}} \right) \simeq 10^{31} \frac{1}{\text{cm}^2 \text{ sec}}, \quad (3.5)$$

were achieved, where f is the orbiting frequency of the antiprotons in the Accumulator. As the numbers indicate, the interaction region is very well localized ($0.35 \times 0.35 \times 0.63 \text{ cm}^3$).

The chambers through which the jet passes have collimating apertures, and are pumped to help prevent the rest of the expanding H_2 from entering the beam pipe and deteriorating the beam. Energy losses and small angle scattering were dealt with by continuous stochastic cooling of the circulating beam.

Detectors used for a luminosity monitor [24] were placed 1.5 m from the interaction region at about 90 degrees from the beam axis. They consisted of eight 0.5 mm silicon detectors that stopped protons from small angle elastic $\bar{p}p$ scattering. The luminosity

Figure 3.2: Schematic of gas jet system.

monitor detected the elastic energy peak, and with this measurement and the known cross section for the elastic $\bar{p}p$ scattering, the luminosity was determined.

As stated, the luminosity was $\simeq 10^{31} \text{ cm}^{-2}\text{sec}^{-1}$. For the energies at which the experiment operated the total $\bar{p}p$ cross section is about 70 mb ($7 \times 10^{-26} \text{ cm}^2$), giving an interaction rate of

$$\frac{\text{events}}{\text{sec}} = L\sigma = 700 \text{ kHz}. \quad (3.6)$$

The lifetime of the beam was 50-90 hours, depending on the beam energy.

3.4 The E760 Detector

3.4.1 Overview

While the total $\bar{p}p$ cross section at energies used in the experiment is $\sim 70\text{mb}$, the cross section for $\bar{p}p \rightarrow \bar{c}c$ is at most on-the-order-of $1\mu\text{b}$. Thus charmonium production represents only a small fraction of the 700 kHz interaction rate. For a study of charmonium, therefore, the bulk of the interactions is seen as background. The detector was designed with the primary consideration of being able to filter out the charmonium signal by detecting electromagnetic decays of $\bar{c}c$. Two important detector components for this were the electromagnetic calorimeters and a threshold Cerenkov counter. The complete detector is shown in figure 3.3. It has no magnetic field (a constraint imposed by the restricted space in the Accumulator) and employs a pointing geometry.

The detector has complete azimuthal coverage and polar coverage from 70° down to about 10° for the central cylindrical detectors, and from 12° down to about 2° for the end cap detectors. The detector coordinate system is right-handed, with the origin at the interaction point and the positive z-axis in the direction of antiprotons. The positive x-axis is defined to be the direction of the protons in the gas jet. Thus the positive y-axis is pointing up.

The calorimeters are the detectors furthest from the interaction region. The detectors

Figure 3.3: Schematic of the E760 detector.

in between are designed with low mass considerations to minimize conversion of γ 's before their detection in the calorimeters. This includes γ 's from π^0 's and η 's. The idea is to reduce the background in the detection of e^+e^- from $\bar{c}c$ decays.

In the cylindrical portion of the detector two scintillation hodoscope detectors provide a tag for charged particles. The inner hodoscope, H1, consists of 8 segments in azimuth and is positioned just outside the beam pipe. H2 consists of 32 azimuthal segments at a radius of 17 cm.

Next out from H1 are inner straw tube chambers [25]. This detector consists of two concentric layers of straw tubes staggered in azimuth to cover gaps and resolve right-left ambiguities. The straws are made of aluminized mylar and are $\simeq 0.9$ cm in diameter and 25 cm in length. Charge division provides a z measurement. A time-to digital converter, TDC, readout provides an azimuth measurement.

After the inner straw tubes there is the inner tracking chamber [26]. This consists of an inner radial projection chamber (RPC), and an outer multiwire proportional counter (MWPC). The RPC provides up to 16 ionization measurements of a charged track. For each track ρ, z and, ϕ are determined, with z coming from charge division, ρ from drift time, and ϕ from the sense wire hit. The MWPC resolves right-left ambiguities. The large radial volume is used to avoid large gain variations due to space charge effects. Grid wires are used to direct the ionization to the field and sense wires for multiplication.

Outside the inner tracking chamber are the H2 detector and Cerenkov counter [27]. The Cerenkov detector is a threshold gas detector, where the index of refraction of the gas allows separation of electrons from higher mass particles. The radial size of the Cerenkov counter is from 17 cm to 590 cm. The Cerenkov counter has 8-fold segmentation in ϕ and 2-fold segmentation in θ with a septum at 38° . For the low angle region ($\theta < 38^\circ$) each azimuthal segment contains an ellipsoidal mirror with the interaction region at one focus and photomultiplier tubes at the other focus. The high angle region uses spherical mirrors. The two different regions use gases with different Cerenkov thresholds (index of refraction),

which compensate for the fact that forward particles tend to have higher energies.

Between the Cerenkov counter and the Central Calorimeter, CCAL, is the outer tracking chamber [28]. It is at a distance of 128 cm from the beam axis and runs 135 cm in length. This detector consists of two layers of Iarocci tubes [29]. The polar and azimuthal angles are measured by reading out aluminum strips running in both direction (θ and ϕ). These strips detect the induced charge from the wires in the Iarocci tubes.

The Forward Tracking Chamber [28], FTC, caps the cylindrical detectors, and is positioned just inside CCAL at a distance of 165 cm from the interaction point and covering the polar angles from 10° to 18.5° . It consists of three planes of MWPC's where the planes are aligned at 60° to the neighboring planes. It is used to track low angle particles into CCAL (those beyond the acceptance of the cylindrical tracking chambers).

The Veto counters consist of eight scintillating hodoscopes segmented in ϕ . They are perpendicular to the beam pipe and cover the polar angles from $\simeq 12^\circ$ down to the beam pipe at 2° . The kinematics of e^+e^- produced from J/ψ restrict both particles to CCAL. Thus, the Veto counters were used in the trigger to veto on forward going charged particles.

The Forward Straws are positioned directly before FCAL. They consist of 4 planes of 112, 1 cm diameter, aluminized mylar straw tubes. The length is such that they cover the face of FCAL. The Forward Straws are aligned at -45° , $+45^\circ$, 0° , and 90° , to the vertical. Drift times are measured with a TDC readout system.

3.4.2 Calorimetry

Calorimetry is provided by the central calorimeter [30], CCAL, and the forward calorimeter [31], FCAL. CCAL consists of 1280 lead glass blocks employing a pointing geometry. It covers the full azimuthal range and the polar range from $\theta = 10^\circ$ to 70° . There are 20 polar angle segments (rings) and 64 azimuthal segments (wedges). The polar segmentation is designed to be equal units of pseudorapidity ($-\ln(\tan(\theta/2))$). A major source of background in the detection of $\bar{p}p \rightarrow \bar{c}c$ comes from π^0 's. The granularity of CCAL provides sufficient

separation of the γ 's from the π^0 decays. For the same reason, the detector is required to detect soft γ 's from asymmetric π^0 decays.

The blocks are contained in a stainless steel honey-comb frame. Each segment of ϕ containing 20 blocks was assembled in separate structures called wedges. The 64 wedges were then assembled to form CCAL. The resulting structure contained minimal inter-block gaps. In ϕ the blocks were separated by 0.735 mm and in θ 0.254 mm. Each of the 1280 blocks were wrapped in a white paper cushion. A fiber optic was mounted to the back face for gain monitoring using a xenon flash lamp.

Energy is detected by photomultiplier tubes attached to each block that collect the Cerenkov light from the e^+e^- pairs generated in the electromagnetic showers. The lengths are designed to contain 90-95% of the shower, which translates into 12-15.5 radiation lengths (with a radiation length of 3.141 cm).

Studies with a set of assembled wedges were done at Brookhaven National Laboratory [30] in a test beam of $\sim 97\%$ pions and $\sim 3\%$ electrons. A pair of scintillators along the beam provided a charged particle trigger. A threshold Cerenkov counter was used to trigger on either pions or electrons. PWC's were used to track particles entering the block. Results of these studies showed energy losses in the inter-block gaps to be $\sim 5\%$ in θ and $\sim 20\%$ in ϕ . The average energy resolution was found to be $\frac{\sigma}{E} = 0.03/\sqrt{E(\text{GeV})}$. Showers centered close to inter-block gaps suffered a degradation in resolution up to $0.16/\sqrt{E}$.

From these data a ratio of the pulse height from minimum-ionizing pions to electrons was obtained for the various block shapes (20 in all). This was subsequently used to obtain a 10-15% level calibration of the remaining blocks from cosmic ray data. Final calibration for the assembled calorimeter was obtained from e^+e^- data and from $\bar{p}p \rightarrow \pi^0\pi^0$. From these data the calibration was found to be $\frac{\sigma}{E} = 0.01 + 0.06/\sqrt{E(\text{GeV})}$.

A cluster algorithm was developed to determine the centroids of electromagnetic showers. This algorithm uses a 3×3 grid of CCAL blocks centered on the block with the largest signal. Cluster centroids are determined by weighting the position of each block with the

energy deposition. Corrections are made to account for losses in the interblock “cracks” (the stainless steel honeycomb support structure). For close showers where two 3×3 grids overlap, a shower profile is used to determine energy sharing which is needed to obtain the correct weighting factors in determining the position of each cluster centroid. Using e^+e^- data from the J/ψ decays, position resolution was found to be 6 mrad in θ and 12 mrad in ϕ .

FCAL covers the polar angles from about 2° to a range that just overlaps the CCAL ($11\text{--}12^\circ$). FCAL was used to detect low energy γ 's from π^0 decay in order to veto on forward neutral events that were background for neutral decays of $\bar{c}c$. For example, $2\pi^0$ decays can mimic $\gamma\gamma$ final states where two clusters are found in the CCAL with the proper kinematics, but the soft γ 's from the pions have not hit CCAL. In addition, FCAL could detect the γ from χ decays where $\chi \rightarrow J/\psi\gamma \rightarrow e^+e^-\gamma$. Here, the J/ψ is close to stationary in the χ center-of-mass frame, meaning that the same J/ψ inclusive trigger which looked for $J/\psi \rightarrow e^+e^-$ in the CCAL could be used for this inclusive trigger.

Each FCAL block consists of stack of alternating lead and scintillator (figure 3.4). There are 148 1 mm thick lead plates measuring $10 \times 10 \text{ cm}^2$ transverse to the beam. The first 32 scintillator plates are 0.64 cm thick and the remaining 42 plates are alternating between 0.64 cm and 0.32 cm thick. The length of the blocks are 48.4 cm (14.7 radiation lengths). The electromagnetic shower is produced by e^+e^- pairs created in the lead. Light from the scintillator plates is readout by a wavelength shifter plate along one side of the block. At the end of the wavelength shifter plate a light guide brings light to a photomultiplier tube. Each scintillator plate and the entire stack are wrapped in aluminum foil to help avoid light losses. The entire stack is compressed and wrapped in opaque heat shrink tubing which provided mechanical support for each block. FCAL consists of 144 blocks stacked as shown in figure 3.5.

The blocks contained $\sim 95\%$ of the electromagnetic shower. A clustering algorithm was developed and the resolution was found to be $\frac{\sigma}{E} = 0.19/\sqrt{E(\text{GeV})}$ with a spatial resolution

of 3 cm on the face of the blocks.

Figure 3.4: Details of an FCAL module.

Figure 3.5: Beam view of the FCAL.

3.5 Triggering

Interaction rates for the experiment were typically $\simeq 700$ kHz. The data acquisition system was able to trigger on candidate events resulting in ranges of a few hundred Hz, and store them on magnetic media for subsequent analysis. The triggering system allowed several event types to be selected in parallel. The two main trigger types were those of charged and neutral events. The neutral trigger required there to be no signals from the charged particle detectors and sufficient energy in CCAL. The charged trigger consisted of an OR of several possible software defined triggers. There was, for example, a J/ψ inclusive trigger, which required associated tracks from the hodoscopes, tracking chamber, and Cerenkov counter, and sufficient energy in CCAL. The charged trigger will not be discussed in detail since it is not the relevant trigger for this analysis.

The neutral trigger for the experiment consisted of a hardware level [32], and a software level. CCAL signals were summed in hardware into forty overlapping regions on CCAL blocks. Energy thresholds were set on these regions with a θ dependence appropriate to two-body kinematics. That is, for a two-body final state the energy of each particle is a function of its polar angle. For the energies studied in the experiment these values ranged over several GeV for a given $\sqrt{s}$.

The hardware trigger consisted of two sub-triggers. Both triggers vetoed on signals from H1 and the forward veto counters. One trigger, called PBG1, required at least two energetic cluster separated by $> 90^\circ$ in azimuth. The other, called Etot, was a total energy trigger. This required $> 80\%$ of the total available energy to be seen in CCAL. Etot also required that there be no signal in the FCAL, which was necessary to manage the high data rates from the forward peaked background.

The data were read out from CAMAC crates using the Fermilab Smart Crate Controller [33] and ACP system [34]. The ACP's computed the invariant mass of all γ pairs and the total energy deposited in the calorimeters. An event was accepted if any γ pair had an

invariant mass $> 2 \text{ GeV}/c^2$, or if the total energy from CCAL and FCAL was $> 90\%$ the total available energy.

3.6 Data Summary

With the Fermilab fixed target mode of operation having a limited running time, the priorities of the experiment's charmonium spectroscopy determined the integrated luminosities achieved at the various energies studied. Table 3.1 shows the integrated luminosities and the energies used in this analysis. More data were taken than used in this analysis. Problems with the total energy trigger, or limited statistics, made some data unreliable for an analysis of neutral channels. As the table shows most data were taken at $\sqrt{s} \approx 3.526 \text{ GeV}$ where a search for the 1P_1 was undertaken. A signal was, in fact, seen [35]. The E760 collaboration has published several analyses from these data [4, 22, 35–41].

The cross sections for the neutral final states studied here decrease over the range of $\sqrt{s}$ by a factor of $\simeq 10$. However, cross sections for most of the reactions are large enough, that with the amount of luminosity obtained, there are enough events to produce small statistical errors.

Table 3.1: Summary of E760 data used in this analysis.

$\sqrt{s}$ GeV	integrated luminosity (nb ⁻¹)	
	total	total used in this analysis
2.911	53	0
2.950	197	197
2.975	429	424
2.979	165	165
2.981	393	376
2.985	200	200
2.990	513	513
2.994	309	309
3.005	511	293
3.050	87	0
3.098	1107	589
3.250	28	28
3.523	1470	1334
3.524	1605	0
3.525	1641	591
3.526	9589	7970
3.527	2153	1690
3.556	2575	0
3.591	924	924
3.594	836	779
3.613	1168	1017
3.616	1224	1063
3.619	575	568
3.621	1216	1216
3.667	372	0
3.687	995	0
3.852	85	0
3.855	170	0
3.830	85	0
3.865	89	0
4.274	333	0
total	31,097	20,333

Chapter 4

Method of Analysis

4.1 Introduction

This chapter presents the method used to analyze the data. Event selection is based on a χ^2 probability distribution. A general discussion of the χ^2 method is given, followed by the details of its use in analysis. Using the χ^2 that is developed, a description of event selection and processing is given, and finally there is a discussion of corrections applied to the data to account for such things as detector acceptance and trigger efficiency.

4.2 The χ^2 Method

The primary difficulty in the analysis of $\bar{p}p \rightarrow \bar{M}M$ is that of extracting the signal from background. Other reactions can produce similar values in the variables used to distinguish the reaction one is studying. An example is $\bar{p}p \rightarrow 3\pi^0$, where one of the three π^0 's has escaped detection. The four γ 's detected will measure the correct π^0 invariant masses. Therefore, this variable alone is insufficient to reject some of this background source. One needs a method to weigh other variables such as the total energy measured. This analysis used a χ^2 probability distribution as a measurement of event quality. Here, for each reaction, several variables are considered simultaneously. With the initial energy and momentum of the system well known, there are six constraints that can be applied to the measured final state. They are the energy, three-momentum, and the invariant masses of the final-state particles, $M\bar{M}$.

The χ^2 calculation [42] is the sum of the deviations in a set of variables from their

expected values, normalized to the root mean square deviations (r.m.s.):

$$\chi^2 \equiv \sum_{i=1, \#variables} \left(\frac{x_{i_{measured}} - x_{i_{expected}}}{\sigma_{x_{i_{expected}}}} \right)^2, \quad (4.1)$$

where each of the variables has a Gaussian distribution. For example, if the χ^2 were written as a function of the component of momentum along the z-axis, P_z , and the total energy, E_{total} , the corresponding Gaussian distribution would be given by

$$f(P_z, E_{total}) = \frac{1}{2\pi\sigma_{P_z}\sigma_{E_{total}}} e^{-\left(\left(\frac{P_z - P_{beam}}{\sigma_{P_z}}\right)^2 + \left(\frac{E_{total} - E_{lab}}{\sigma_{E_{total}}}\right)^2\right)}. \quad (4.2)$$

For a χ^2 distribution, the number of variables is called the number of degrees of freedom. For a given measurement of χ^2 , and the number of degrees of freedom, one can calculate the probability that a measurement will have a χ^2 equal to or greater than the value measured. This corresponds to the fractional area under a Gaussian curve integrated from the χ^2 cut-off value to infinity. A set of such measurements will produce a uniform probability distribution [42]. Background events that distort the Gaussian distribution will appear as an enhancement of low values in the probability distribution. Thus, it is possible to make a selection of events based on the probability distribution, with events being rejected below the values where the background excess appears. Correction for the signal events that are also rejected with the background is possible since these events will have a known distribution (uniform) in the same region.

A difficulty that arises in using the six constraints directly in equation 4.1 is that the errors for some variables have correlations with those of other variables. For example, the measured momentum along the beam axis, P_z , is strongly correlated with the total measured energy, E_{total} . The result of ignoring this correlation is to accept background events that clearly fall outside the range of correlation.

To develop the idea of correlated errors, one can start by writing the exponent of equation 4.2 as a matrix product,

$$\begin{pmatrix} E_{total} - E_{lab}, & P_z - P_{beam} \end{pmatrix} \begin{pmatrix} \frac{1}{\sigma_{E_{total}}} & 0 \\ 0 & \frac{1}{\sigma_{P_z}} \end{pmatrix} \begin{pmatrix} E_{total} - E_{lab} \\ P_z - P_{beam} \end{pmatrix}. \quad (4.3)$$

The exponent of equation 4.2 describes an ellipse in a two dimensional space with P_z and E_{total} as coordinates. The semi-major axes are given by σ_{P_z} and $\sigma_{E_{total}}$. A correlation between these two variables corresponds to a rotation of the axes. In this case, equation 4.2 takes the form

$$P(P_z, E_{total}) = Ae^{-\left(\frac{1}{2(1-\rho^2)}\left(\frac{P_z - P_{beam}}{\sigma_{P_z}}\right)^2 + \left(\frac{E_{total} - E_{lab}}{\sigma_{E_{total}}}\right)^2 + \left(\frac{2\rho(E_{total} - E_{lab})(P_z - P_{beam})}{\sigma_{E_{total}}\sigma_{P_z}}\right)\right)}, \quad (4.4)$$

where

$$A = \frac{1}{2\pi\sigma_{P_z}\sigma_{E_{total}}\sqrt{1-\rho^2}}.$$

The ρ in equation 4.4 is called the correlation coefficient, [42] and in general is given by

$$\rho = \frac{cov(x_i, x_j)}{\sigma_{x_i}\sigma_{x_j}} = \frac{\langle (x_i - \bar{x}_i)(x_j - \bar{x}_j) \rangle}{\sigma_{x_i}\sigma_{x_j}}, \quad (4.5)$$

where the $x_{i,j}$ are the variables and the $\bar{x}_{i,j}$ are the means of the variables. The term in brackets is the expectation value of $(x_i - \bar{x}_i)(x_j - \bar{x}_j)$. Equation 4.4 can be derived by applying a rotation to the coordinates of P_z and E_{total} .

As equation 4.4 indicates, the degree of correlation between two variables is obtained from the coefficient of the cross-terms. As was done with equation 4.2, equation 4.4 can be written in matrix form:

$$P = Ae^{-\left(\mathbf{X}^T \mathbf{E}^{-1} \mathbf{X}\right)}, \quad (4.6)$$

where $\mathbf{X}$ is the matrix of variables and $\mathbf{E}$ is called the error matrix. The elements of the error matrix are given by

$$cov(x, y) = \langle (x_i - \bar{x}_i)(x_j - \bar{x}_j) \rangle. \quad (4.7)$$

The off-diagonal elements indicate the degree of correlation between variables.

If the error matrix is diagonal, it means there are no correlations and one can use equation 4.1 as is, for the six constraint variables. If there are correlations, one must form a χ^2 that accounts for the correlations. One way of doing this is to invert the error matrix and use a χ^2 of the form as in the exponent in equation 4.6. An alternative is to change the χ^2 variables in such a way as to diagonalize the error matrix (remove the correlations) and use a χ^2 of the form as in equation 4.1. A difficulty with the first method is that a matrix inversion is required for each measurement (i.e., each event). That is, as will be discussed further in the next section, some correlations may depend on variables that change from event to event. A consequence of this is that the processing time required for a matrix inversion becomes significant. For this analysis, the latter method is used. Using this method, the starting point is determining what correlations exist. To do this, a matrix of the correlations coefficients is constructed:

$$\rho(x_i, x_j) = \frac{cov(x_i, x_j)}{\sigma_{x_i} \sigma_{x_j}}. \quad (4.8)$$

Where a pair of variables shows a strong correlation, a transformation to a new set of variables is made having the form

$$\begin{pmatrix} x'_i \\ x'_j \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} x_i \\ x_j \end{pmatrix}, \quad (4.9)$$

where θ is the angle through which the x_i and x_j axes are rotated to minimize the correlation. Figure 4.1 shows this transformation graphically. The new coordinates, x'_i and x'_j , will describe the ellipse without the cross-terms as in equation 4.2. In this figure, the projections of the ellipse onto the axes of the old (unprimed) and new (primed) coordinates are shown. The projections demonstrate that the distributions for the new variables will be different from the old. That is, one of the new distributions will be more narrow than either distribution of the old variables, and one of the new distributions will be broader than either distribution of the old variables. The gain is not in producing more narrow distributions, but rather in producing projections that better isolate the signal from the background.

Once the correlation in these two variables has been removed, again a matrix of correlation coefficients is formed. If there are variables showing a strong correlation, another transformation is performed to remove this correlation, and this process is repeated until the matrix is diagonal. In practice, as will be seen in the next section, three changes of variables are sufficient to remove all strong correlations. The variables initially considered in this procedure are;

- $P_x = \sum_{i=1,4} E_i \sin \theta_i \cos \phi_i$, the residual momentum in the experiment x direction;
- $P_y = \sum_{i=1,4} E_i \sin \theta_i \sin \phi_i$, the residual momentum in the experiment y direction;
- $P_z = \sum_{i=1,4} E_i \cos \theta_i$, the momentum along the beam axis;
- $E_{total} = \sum_{i=1,4} E_i$, the total energy measured in the laboratory system;
- m_1, m_2 , the invariant mass of the γ pairs making up the final state particles ($M\bar{M}$).

In the laboratory frame the proton is stationary and the antiproton is moving in the positive z -direction in a right-handed coordinate system. Thus, the final state will have P_z equal to the initial momentum of the antiproton beam, p_{beam} , and therefore, $P_x = P_y = 0$. The total energy in the laboratory system will be a sum of the beam energy and the mass

of the target proton; $E_{lab} = E_{beam} + m_p = \sqrt{P_{beam}^2 + m_p^2} + m_p$. The invariant masses correspond to the masses of the π^0 and η . The correlations between these variables will be discussed next, and each change of variables used to remove the correlations will be treated in a separate section.

Figure 4.1: Representation of correlated variables. Also shown are the projections of the ellipse onto the axes where the variables are correlated (axes x and y), and uncorrelated (x' and y').

4.2.1 P_x and P_y Correlation

With the beam axis perpendicular to the experimental x and y axes, measured momenta perpendicular to the beam axis should be equal to zero:

$$P_x = \sum_{i=1,4} E_i \sin \theta_i \cos \phi_i = 0, \quad (4.10)$$

and

$$P_y = \sum_{i=1,4} E_i \sin \theta_i \sin \phi_i = 0, \quad (4.11)$$

where the sum is over the energies and angles of the four γ 's detected in the final state. (Here, $E_i = P_i$, since the mass of the γ is zero.) The difference in equations 4.10 and 4.11 is the ϕ dependence. The predicted variations in these variables will, likewise, depend on ϕ :

$$\delta P_x = \sum_{i=1,4} (\delta E_i \sin \theta_i \cos \phi_i + E_i \cos \theta_i \delta \theta_i \cos \phi_i + E_i \sin \theta_i \sin \phi_i \delta \phi_i), \quad (4.12)$$

$$\delta P_y = \sum_{i=1,4} (\delta E_i \sin \theta_i \sin \phi_i + E_i \cos \theta_i \delta \theta_i \sin \phi_i + E_i \sin \theta_i \cos \phi_i \delta \phi_i). \quad (4.13)$$

For a sample of events the predicted variations in the residual transverse momentum are shown in figure 4.2 as a function of the angle of the reaction plane. The reaction plane is defined by the beam axis and the momentum vectors of the two final-state particles. These momentum vectors are reconstructed from the final state γ 's. The angle of the reaction plane is measured from the positive x-axis to the azimuth angle of the softest (lowest energy) final-state particle. Figure 4.3 depicts a $2\pi^0$ final state showing a projection of the momentum vectors of the γ 's and the reconstructed π^0 's onto the x-y plane.

One can improve on characterizing the final state by replacing P_x and P_y with variables that account for the ϕ dependent variation shown in figure 4.2. That is, there exists a correlation between the angle of the reaction plane and each of the variables P_x and P_y .

Figure 4.2: Correlation between reaction plane and δP_x .

Figure 4.3: Vectors defining the reaction plane.

For many events one is “averaging out” the correlation when using P_x and P_y since one sees a sample of all ϕ angles. To remove this correlation, for each event the variables P_x and P_y are transformed to new variables, PT1 and PT2. These variables give the residual momentum parallel and perpendicular to the reaction plane. The transformation is in the form of a rotation given by

$$\begin{pmatrix} PT1 \\ PT2 \end{pmatrix} = \begin{pmatrix} \cos \phi_{rp} & \sin \phi_{rp} \\ -\sin \phi_{rp} & \cos \phi_{rp} \end{pmatrix} \begin{pmatrix} P_x \\ P_y \end{pmatrix}. \quad (4.14)$$

This rotation is shown graphically in figure 4.4.

For a typical data sample the distributions of these variables, before and after the transformation, are shown in figure 4.5a-d. Figure 4.5c shows the new variable PT2 with a significant peak, and a broad background. Using these new variables, therefore, to obtain the means and σ 's for the χ^2 calculation results in a better discrimination between the signal and the background events that is not possible when using P_x and P_y .

Figure 4.4: Axes of variables, PT1 and PT2, relative to the reaction plane.

Figure 4.5: Result of decorrelating ϕ dependent transverse momentum residuals. (a,b) Residual momentum along the experimental x and y axes. (c,d) Residual momentum using the decorrelated variables, PT1 and PT2.

Table 4.1: Matrix of correlation coefficients for PT1, PT2, P_z , E_{total} , m_1 m_2 .

$\frac{cov(x_i, x_j)}{\sigma_{x_i} \sigma_{x_j}}$ (diagonal elements are the variable σ 's)						
	PT1	PT2	P_z	E_{total}	m_1	m_2
PT1	0.085	0.043	-0.185	-0.091	-0.030	0.074
PT2	0.043	0.025	-0.090	-0.088	0.026	-0.029
P_z	-0.185	-0.090	0.153	0.991	0.061	-0.036
E_{total}	-0.091	-0.088	0.991	0.176	0.061	-0.029
m_1	-0.030	0.026	0.061	0.061	0.015	-0.005
m_2	0.074	-0.029	-0.036	-0.029	-0.005	0.016

4.2.2 P_z and E_{total} Correlation

Table 4.1 shows the matrix of correlation coefficients that results from using these variables. The data sample is taken at $\sqrt{s} = 3.526$ GeV. Similar results are found at all energies. The diagonal elements of the matrix are normally given as the square of the r.m.s values ($\sigma_{x_i}^2$). For the correlation matrices that will be presented, the r.m.s will be shown to provide easier reference to the corresponding histograms.

Table 4.1 shows a strong positive correlation between P_z and E_{total} . Figure 4.6 is a scatter plot of these two variables. For these data, the lab energy, E_{lab} , and beam momentum, P_{beam} , are marked on the figure. The data show several features. There is a cluster of events centered on E_{lab} and P_{beam} , marked region I. There is a cluster of events with low energy and high momentum, marked region II. Finally, there are events with low momentum and low energy marked region III. To understand why the data show these features, a Monte Carlo simulation was done with the purpose of understanding where background events would appear on such a scatter plot. Using the same energy as the data sample, $3\pi^0$ events were generated where the particles populate phase space uniformly. That is, there is no angular dependence as might be the case for real data.

Figure 4.6: Correlation between P_z and E_{total} .

Once the event is generated, it is accepted only if exactly four γ 's are within the range of CCAL. When processing the experiment data, events with exactly four clusters are selected. Once the Monte Carlo event passes these criteria it undergoes the same processing as the experiment data.

The results are shown in figure 4.7. The Monte Carlo data show features similar to the experimental data. The cluster of events in figure 4.7 in the region corresponding to region II of figure 4.6 are $3\pi^0$ events where at least one of the γ 's missing the CCAL is energetic and at a larger θ than CCAL covers. Thus, there is energy missing, and to conserve momentum, the measured momentum may need to be larger than the beam momentum. The rest of the Monte Carlo data fall on a band that includes regions I and II in figure 4.6. These are events where the more energetic of the two missing γ 's has gone forward and missed CCAL with a small θ . This band, therefore, ranges from $E=E_{lab}$ and $P=P_{beam}$ to very small values where the missing γ 's have increasing energy. In the experimental data the lower values do not extend as far because FCAL produces a veto signal for forward going γ 's. The Monte Carlo shows that exclusive four γ final states must lie in region I of figure 4.6.

The correlation coefficients in table 4.1 are those obtained from events that lie in region I of figure 4.6, that is, the region where exclusive final states should be. If P_z and E_{total} are the variables used in the χ^2 calculation their means and r.m.s values are obtained from the projection of these variables to the axes of figure 4.6. One can see that this results in accepting background events that fall within these projections. To avoid this, new variables are defined, PE1 and PE2, which correspond to a rotation of the P_z and E_{total} axes where the new variables are uncorrelated. The transformation is given by

$$\begin{pmatrix} PE1 \\ PE2 \end{pmatrix} = \begin{pmatrix} \cos \theta_{PE} & \sin \theta_{PE} \\ -\sin \theta_{PE} & \cos \theta_{PE} \end{pmatrix} \begin{pmatrix} E_{total} \\ P_z \end{pmatrix}, \quad (4.15)$$

where $\theta_{PE} = \tan^{-1}(\frac{P_z}{E_{total}})$. The rotation is shown graphically in figure 4.8. The distributions of the old and new variables, P_z , E_{total} , PE1, and PE2, are shown in figure 4.9a-d.

Figure 4.7: Distribution of P_z and E_{total} for $3\pi^0$ events generated from a Monte Carlo.

Figure 4.8: Orientation of axes for variables, PE1 and PE2, relative to P_z and E_{total} .

Figure 4.9: Results of decorrelating E_{total} and P_z for $E_{lab} = 6.89$ and $P_{beam} = 5.87$. Distributions of E_{total} and P_z (a,b). Distributions of the decorrelated variables, PE1 and PE2 (c,d).

Table 4.2: Matrix of correlation coefficients for PT1, PT2, PE1, PE2, m_1 m_2 .

$\frac{cov(x_i, x_j)}{\sigma_{x_i} \sigma_{x_j}}$ (diagonal elements are the variable σ 's)						
	PT1	PT2	PE1	PE2	m_1	m_2
PT1	0.085	0.043	-0.136	-0.711	-0.030	0.074
PT2	0.043	0.025	-0.073	0.034	0.026	-0.029
PE1	-0.136	-0.073	0.228	0.084	0.046	-0.038
PE2	-0.711	0.034	0.084	0.015	-0.046	-0.078
m_1	-0.030	0.026	0.046	-0.046	0.015	-0.005
m_2	0.074	-0.029	-0.038	-0.078	-0.005	0.016

Note that the change of variables corresponds to a rotation about the origin. When the data are projected onto the axes, PE1 is offset from zero as can be seen from figure 4.8.

The correlation matrix that results from this change of variables is given in table 4.2. Comparison with table 4.1 shows that the correlation between P_z and E_{total} has been removed.

4.2.3 PT1 and PE2 Correlation

Table 4.2 shows a correlation between the variables PT1 and PE2. In section 4.3 PT1 is defined to be the residual momentum in the reaction plane projected onto the x-y plane. Figures 4.10 and 4.11 show the experiment coordinates and the momentum vectors for a fictitious event with $2\pi^0$ as the final state. Figure 4.10 shows the events projected onto the x-y plane and shows the variables, PT1 and PT2, as defined in section 4.3. Figure 4.11 is the projection onto the z-y plane. Here the vector PT2 is perpendicular to the z-axis and is not shown. This figure shows that the residual momentum in the plane of the reaction, PT1, will have a component along the z-axis.

Figure 4.10: Momentum vectors and PT1 and PT2 vectors shown for a fictitious $2\pi^0$ event projected onto the experimental x-y plane.

Figure 4.11: Momentum vectors and PT1 vector shown for a fictitious $2\pi^0$ event projected onto the experimental y-z plane.

The magnitude of this vector is related to the production angle, $\theta_{\bar{p}M}^*$. This is the angle in the center-of-mass of the $\bar{p}p$, between the z-axis and the final-state particles, M , which are collinear for two-body final states. The reason for this is that for two-body exclusive final states, the production angle completely determines laboratory energies and angles of both particles. Therefore, since PT1 is related to the production angle it is possible for there to be a correlation with the measured momentum. In fact, the variable PE2 is, as defined in section 4.4, related to P_z through equation 4.15.

Tables 4.3 and 4.4 show correlation coefficients for the same data, but for different production angles. These tables show that the correlation between PT1 and PE2 is strongest for large $\cos\theta_{\bar{p}M}^*$. For both regions of $\cos\theta_{\bar{p}M}^*$, these variables are plotted in figures 4.12 and 4.13.

To remove this correlation, a transformation is made to new variables, PT12 and PE22. This corresponds to a rotation of the PT1 and PE2 axes through an angle ψ . The transformation is given by

$$\begin{pmatrix} PT12 \\ PE22 \end{pmatrix} = \begin{pmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{pmatrix} \begin{pmatrix} PT1 \\ PE2 \end{pmatrix}. \quad (4.16)$$

Since the correlation is dependent on $\cos\theta_{\bar{p}M}^*$, ψ is defined accordingly. From figures 4.12 and 4.13 it is seen that ψ must vary from 0 to $\simeq 19.5^\circ$. To meet this requirement, ψ is defined as

$$\psi = \left(\frac{\cos\theta_{\bar{p}M}^*}{N} \right) \theta_{max}, \quad (4.17)$$

where θ_{max} is the angle needed to remove the maximum correlation, 19.5° , and N is normalization term which is equal to the maximum value of $\cos\theta_{\bar{p}M}^*$ for a given $\sqrt{s}$. That is, if the detector covered the full polar range, the experiment would detect final states over the complete range of $\cos\theta_{\bar{p}M}^*$ from 0 to 1.

Table 4.3: Matrix of correlation coefficients for PT1, PT2, PE1, PE2, m_1 m_2 for $\cos\theta_{\bar{p}M}^* < 0.2$.

$\frac{cov(x_i, x_j)}{\sigma_{x_i} \sigma_{x_j}}$ (diagonal elements are the variable σ 's)						
	PT1	PT2	PE1	PE2	m_1	m_2
PT1	0.086	0.038	0.028	-0.184	-0.031	0.049
PT2	0.038	0.027	0.016	-0.013	-0.025	0.013
PE1	0.028	0.016	0.235	-0.025	0.016	0.005
PE2	-0.184	-0.013	-0.025	0.011	-0.013	-0.039
m_1	-0.031	-0.025	0.016	-0.013	0.016	-0.023
m_2	0.049	0.013	0.005	-0.039	-0.023	0.016

Table 4.4: Matrix of correlation coefficients for PT1, PT2, PE1, PE2, m_1 m_2 for $\cos\theta_{\bar{p}M}^* > 0.5$.

$\frac{cov(x_i, x_j)}{\sigma_{x_i} \sigma_{x_j}}$ (diagonal elements are the variable σ 's)						
	PT1	PT2	PE1	PE2	m_1	m_2
PT1	0.084	-0.008	-0.201	-0.887	0.139	0.116
PT2	-0.008	0.023	0.000	0.046	-0.009	0.083
PE1	-0.201	0.000	0.287	0.174	-0.010	-0.010
PE2	-0.887	0.046	0.174	0.030	-0.114	-0.091
m_1	0.139	-0.009	-0.010	-0.114	0.013	-0.004
m_2	0.116	0.083	-0.010	-0.091	-0.004	0.014

Figure 4.12: Correlation between PT1 and PE2 for small values of $\cos \theta_{pM}^*$.

Figure 4.13: Correlation between PT1 and PE2 for large values of $\cos \theta_{pM}^*$.

The CCAL covers from 11° to 70° in the lab and for $\bar{p}p \rightarrow \pi^0\pi^0$ this corresponds to $\cos\theta_{\bar{p}M}^*$ from 0 to $\simeq 0.5$. This range does not have a precise cut-off because the final state particles decay to two γ 's, where the opening angle will vary from event to event. That is, two events could have the same momentum vectors for the final state particles, but one of the events could have one or two γ 's missing the detector because of the γ opening angle differences in the two events. Furthermore, the range will have a small $\sqrt{s}$ dependence, where the same production angle for two different values of $\sqrt{s}$ will transform differently to the lab frame resulting in more acceptance for large values of $\sqrt{s}$.

It was found that θ_{max} does not depend on $\sqrt{s}$. However, since the acceptance does depend on $\sqrt{s}$, the normalization, N , needs to be defined for each $\sqrt{s}$. Therefore, N was evaluated for the range of $\sqrt{s}$ in the experiment and the number used for each energy studied is an interpolation of these results.

The distributions of the old and new variables, PT1, PE2, PT12, and PE22, are shown in figure 4.14a-d. The correlation matrix that results from this change of variables is again calculated for two regions of $\cos\theta_{\bar{p}M}^*$ and is given in tables 4.5 and 4.6. Comparison with table 4.3 and 4.4 shows that the correlation between PT1 and PE2 has been removed.

Figure 4.14: Results of decorrelating PT1 and PE2. (a,b) Distributions of the correlated variables, PT1 and PE2. (c,d) Distributions of the decorrelated variables, PT12 and PE22.

Table 4.5: Matrix of correlation coefficients for PT12, PT2, PE1, PE22, m_1 m_2 for $\cos\theta_{\bar{p}M}^* < 0.2$.

$\frac{cov(x_i, x_j)}{\sigma_{x_i} \sigma_{x_j}}$ (diagonal elements are the variable σ 's)						
	PT12	PT2	PE1	PE22	m_1	m_2
PT12	0.085	0.034	0.034	0.193	-0.028	0.052
PT2	0.034	0.027	0.017	-0.008	-0.026	0.014
PE1	0.034	0.017	0.235	-0.011	0.017	0.006
PE22	0.193	-0.008	-0.011	0.011	-0.038	-0.025
m_1	-0.028	-0.026	0.017	-0.038	0.016	-0.024
m_2	0.052	0.014	0.006	-0.025	-0.024	0.017

Table 4.6: Matrix of correlation coefficients for PT12, PT2, PE1, PE22, m_1 m_2 for $\cos\theta_{\bar{p}M}^* > 0.5$.

$\frac{cov(x_i, x_j)}{\sigma_{x_i} \sigma_{x_j}}$ (diagonal elements are the variable σ 's)						
	PT12	PT2	PE1	PE22	m_1	m_2
PT12	0.089	-0.017	-0.201	0.015	0.133	0.109
PT2	-0.017	0.024	0.001	0.022	-0.009	0.083
PE1	-0.201	0.001	0.287	-0.028	-0.010	-0.010
PE22	0.015	0.022	-0.028	0.011	-0.076	-0.091
m_1	0.133	-0.009	-0.010	-0.076	0.014	-0.004
m_2	0.109	0.083	-0.010	-0.091	-0.004	0.014

4.2.4 Invariant Masses

The two variables used in the χ^2 calculation not yet discussed are the two invariant masses. As with the other variables, the means and σ 's must be determined in order to form a χ^2 that most accurately represents the exclusive final states being studied. To demonstrate the existence of the invariant mass signals in the data, events were selected where the measured values of the total momentum and total lab energy were $\geq 80\%$ of P_{beam} and E_{lab} , respectively. In addition, it was required that the events lie in the region of figure 4.6 where the exclusive final states occur. Figure 4.15 is a scatter plot of the invariant masses of the $\gamma\gamma$ pairs. The method of choosing the pairing of the four final states γ 's will be discussed in the next section.

The scatter plot shows clusters occurring at mass of known particles, (π^0, η) which are marked on the figure. This plot demonstrates not only that the particles of known mass are found, but also that they are found in conjunction with other particles. For example, on both the horizontal and vertical axes the π^0 mass is associated with clusters at the π^0 and η masses, corresponding to events possibly of the type $\bar{p}p \rightarrow \pi^0\pi^0$ and $\bar{p}p \rightarrow \pi^0\eta$, respectively.

A projection of this scatter plot (with greater statistics) is shown in figures 4.16. The vertical scale has been magnified to remove the dominant π^0 peak for better viewing of the remaining masses. Here one can see clear signals at the π^0, η, ω , and η' masses.

Figure 4.15: Scatter plot of the two invariant mass calculations for each event.

Figure 4.16: Projection of figure 4.15 with vertical scale magnified to show signals at the π^0 , η , ω , and η' masses.

χ_0^2 :	P_x	P_y	P_z	E_{total}	M1	M2
	$\Downarrow$	$\Downarrow$				
χ_1^2 :	PT1	PT2	P_z	E_{total}	M1	M2
			$\Downarrow$	$\Downarrow$		
χ_2^2 :	PT1	PT2	PE1	PE2	M1	M2
	$\Downarrow$			$\Downarrow$		
χ_3^2 :	PT12	PT2	PE1	PE22	M1	M2

Figure 4.17: Summary of χ^2 method.

4.2.5 Summary of the χ^2 Calculation

The χ^2 method is used as a quantitative way to measure event quality. The χ^2 is developed for the six constraint variables, P_x , P_y , P_z , E_{total} , m1 and m2. Through a procedure of removing correlations between these variables, several variable transformations are made resulting in the final χ^2 consisting of six new variables, PT12, PT2, PE1, PE22, m1 and m2. Figure 4.17 represents these transformations, where the symbol “ $\Downarrow$ ” designates the transformations discussed in the previous sections. The next section will discuss the details of obtaining the parameters used in the χ^2 calculation - the means and σ 's.

4.3 Event Processing

Figure 4.18 shows the flow of the data processing. The use of several stages of data summary tapes, DST's, was necessitated by the large volume of data. There were approximately 700 experiment data tapes each containing almost 2 gigabytes of data. The Fermilab Computing Department provided 30 mips of CPU that were used over a period of 3-4 months to process the experiment data tapes and produce roughly 200 “neutral DST's”. These DST's contain the neutral events and had the data from the detector electronics decoded as well as CCAL clustering. Another level of DST's further reduced this number to about 30 tapes by storing only the energies and angles of CCAL clusters.

Figure 4.18: Flow of data analysis.

Since this analysis considered final states of the type $M\bar{M} \rightarrow \gamma\gamma\gamma\gamma$, another level of DST's were made selecting 4 cluster events.

To process a data run, each of which corresponds to one tape of the experimental data tapes, the runs were grouped into "scans". A scan is a set of data runs that were taken at one energy and used one stack of antiprotons. This corresponds to 2-3 days of data taking, yielding about 1 pb^{-1} .

At this point, a cut is made to discard events with $<80\%$ of the beam momentum and lab energy. Figure 4.19 shows these variables for $E_{lab} = 6.89 \text{ GeV}$ and $P_{beam} = 5.87 \text{ GeV}/c$. One can see peaks near the nominal values and significant tails extending to very low values. The experiment trigger, as has been stated, required neutral events to be above an 80% threshold. The low-end tail that occurs in the data is due to more than one event in the same trigger window. With an interaction rate as high as 700 kHz, and an ADC gate of 150 ns, one expects accidentals at a rate of 75 kHz, or $\simeq 21\%$ of the interactions. This rate can be larger due to pulses from the photomultiplier tubes having long tails. Subsequent analysis of signal timing allows for discrimination between clusters occurring "on-time" with the event trigger and those prior to, or following, the event trigger (referred to as "out-of-time"). Thus, an event can pass the trigger where the energy deposited from one interaction, plus that deposited from another interaction, is above the trigger threshold. When the "out-of-time" clusters are removed from an event, in making the last level of DST, the measured energy is reduced. Thus, a plot of the total energy and momentum, after removal of the "out-of-time" clusters for each event, can shift events to lower values. This produces the tails seen at low values in figure 4.19.

Once the 80% P_z and E_{total} cuts have been applied, the parameters for the χ^2 calculation are obtained - the σ 's and means for each of the six χ^2 variables. The parameters of the χ^2 need to represent the values that characterize the exclusive final states being studied. That is, for each of the six χ^2 variables, the means and σ 's for $\bar{p}p \rightarrow \bar{M}M$ (with $M\bar{M}$ being $\pi^0\pi^0$, $\pi^0\eta$, and $\eta\eta$) must be determined. In the previous section it was demonstrated that

Figure 4.19: Plots of P_z and E_{total} demonstrating low energy tails that appear when CCAL clusters from out-of-time interactions have been removed.

exclusive final states produce signal peaks in the variables. In the processing of the data sets, the means and widths of the peaks are determined using an automated procedure. This includes the variables involving the invariant masses. The reason for this is to manage the large volume of data and to adjust for differences in the data sets such as energy shifts due to miscalibration.

The procedure for obtaining χ^2 parameters is to sample events having small transverse momentum from a set of data runs from a stack, and that occur in the region of the P_z/E_{total} plot where exclusive final states lie. This was discussed in section 4.2.2. Figure 4.20 shows these variables again, as well as the function used to select the region of interest. The function used is derived from the functional dependence of P_z on E_{total} ,

$$E_{total} = E_{beam} + m_p = \sqrt{P_z^2 + m_p^2} + m_p, \quad (4.18)$$

or,

$$P_z = \sqrt{E_{total}^2 - 2E_{total}m_p} \equiv f(E_{total}, m_p). \quad (4.19)$$

This function is shown in figure 4.20, and passes through the cluster containing the exclusive final states. This function conveniently follows the trend of the exclusive final-state cluster.

To select these events, a new function is defined, which is shifted in such a way as to isolate the exclusive final-state cluster. This is done by changing the value of m used from 0.938 to 0.850:

$$f(E_{total}, m = 0.850) \equiv \sqrt{E_{total}^2 - 2E_{total}m}. \quad (4.20)$$

This function is also shown in figure 4.20. It should be emphasized that this function is no more than a tool used in an algorithm to automate the procedure for obtaining the χ^2 parameters. Thus, the value of 0.850 was chosen empirically in order to produce a shift in the function that would provide a boundary for the exclusive final states. With f defined, an event is used in the sample of events to determine the χ^2 parameters if

$$P_z < f(E_{total}, m = 0.850). \quad (4.21)$$

Figure 4.20: Plots of P_z versus E_{total} showing functions used to select sample events to determine initial widths and means of χ^2 parameters.

With this selection, each of the χ^2 variables is histogrammed and initial means and σ 's are extracted. To more accurately characterize the σ 's and means of each variables, six more χ^2 calculations are done where each calculation omits one parameter, and uses the initial values of the σ 's and means for the remaining 5 variables. For example, to obtain a more accurate σ and mean for PT12 a χ^2 calculation is done using the remaining 5 variables (PT2, PE1, PE22, m_π , and m_η). The final σ 's and means for the χ^2 variables come from fitting these distributions to a Gaussian plus a second order polynomial. This process is done for all six χ^2 variables. The χ^2 includes the invariant mass of each of the final state particles, m_1 and m_2 . Since three reactions are being studied, two mass plots are made for each of the particles being studied (π^0 and η). Histograms of these variables for a data sample taken at $\sqrt{s} = 2.981$ GeV are shown in figure 4.21 as well as the total fit, and the second order polynomial terms. The invariant mass plots are shifted to the respective masses, where a nominal distribution would be centered on zero. They are also modulated by $\frac{E_{total}}{E_{lab}}$ to offset any calibration errors. That is, the invariant mass calculation is a function of both energies and angles. This factor will partially cancel energy measurement fluctuations, relying more on the angle measurements.

With the χ^2 parameters determined for a scan, the next step in the processing is to calculate the χ^2 for each event. For the final states being studied there are 12 possible γ pairings to consider. They are listed in table 4.7.

In table 4.7, the notation $\pi^0(\gamma_1\gamma_2)$ means that γ_1 and γ_2 are hypothesized to be associated with a π^0 . The reactions $\pi^0\pi^0$ and $\eta\eta$ have half as many γ pairing combinations as $\pi^0\eta$ since the particles are indistinguishable. For each of the 12 possible γ pairing combinations the invariant mass for each pair is calculated according to

$$m_{ij} = \sqrt{2E_i E_j (1 - \cos \theta_{ij})}, \quad (4.22)$$

where i and j are the γ indices used for the pairing, E_i are the CCAL cluster energies and θ_{ij}

Figure 4.21: Histograms used to obtain the χ^2 σ 's and means. Each distribution is obtained by a χ^2 calculation using the remaining variables. The curves are the fit to a Gaussian (from which the χ^2 σ 's and means are extracted) plus a second order polynomial, and the polynomial.

Table 4.7: $\gamma\gamma$ pairing combinations.

reaction	γ pairing combinations	sub totals
$\pi^0\pi^0$	$\pi^0(\gamma_1\gamma_2)\pi^0(\gamma_3\gamma_4), \pi^0(\gamma_1\gamma_3)\pi^0(\gamma_2\gamma_4), \pi^0(\gamma_1\gamma_4)\pi^0(\gamma_2\gamma_3)$	3
$\pi^0\eta$	$\pi^0(\gamma_1\gamma_2)\eta(\gamma_3\gamma_4), \pi^0(\gamma_1\gamma_3)\eta(\gamma_2\gamma_4), \pi^0(\gamma_1\gamma_4)\eta(\gamma_2\gamma_3)$ $\pi^0(\gamma_2\gamma_3)\eta(\gamma_1\gamma_4), \pi^0(\gamma_2\gamma_4)\eta(\gamma_1\gamma_3), \pi^0(\gamma_3\gamma_4)\eta(\gamma_1\gamma_2)$	6
$\eta\eta$	$\eta(\gamma_1\gamma_2)\eta(\gamma_3\gamma_4), \eta(\gamma_1\gamma_3)\eta(\gamma_2\gamma_4), \eta(\gamma_1\gamma_4)\eta(\gamma_2\gamma_3)$	3
	total	12

is the $\gamma\gamma$ opening angle. For each of the 12 $\gamma\gamma$ combinations considered the χ^2 is calculated if the masses fall within the ranges; $0.05 < m_\pi < 0.30$ GeV, and $0.30 < m_\eta < 0.85$ GeV. These values are greater than 6σ from the peaks. The result of this criterion is that, on average, 1.5 $\gamma\gamma$ combinations are further considered in the analysis. For each of the surviving combinations a reaction plane is determined. This is for the χ^2 variables, PT1 and PT2, (the residual momentum parallel and transverse to the reaction plane). A χ^2 is then calculated for each combination and the event is labeled as the combination giving the smallest χ^2 . For this combination the production angle is calculated. This is the angle between the $\bar{p}p$ and $M\bar{M}$ in the center-of-mass system as shown in figure 4.22. The production angle is given by

$$\cos \theta_{\bar{p}M}^* = \left(\frac{1}{p^*\beta} \right) \left(\frac{E_2 E_1^* - E_1 E_2^*}{E_1 + E_2} \right), \quad (4.23)$$

where the superscript, “*” denotes center-of-mass values, p^* is the momentum of each particle in the center-of-mass frame, β is the boost from the lab to the center-of-mass frame, E_i^* is the energy in the center-of-mass frame of the i^{th} particle and E_i is the lab energy of the i^{th} particle. The lab energies used for this calculation are derived from the

Figure 4.22: Production angle, $\vartheta_{\bar{p}\pi}^*$, for $\bar{p}p \rightarrow \pi^0\pi^0$.

angle measurements of the reconstructed particles according to

$$p_i = P_{beam} \left(\frac{\sin \theta_i}{\sin(\theta_i + \theta_j)} \right), \quad (4.24)$$

and finally

$$E_i = \sqrt{p_i^2 + m_i^2}. \quad (4.25)$$

Here, θ_i are reconstructed laboratory angles of the final-state particle, and p_i is the laboratory momentum of particle i . One can see that it is possible to calculate the energies from the angles since, for a two-body final state, the angles are completely determined by the energies. The reason for doing the calculation this way is that CCAL angular resolution is much better than the energy resolution.

After the events are processed corrections to the measured cross sections are applied. They will now be described.

4.3.1 Confidence Level Distributions

Figures 4.23 and 4.24 show confidence level distributions for events processed as described above for the reactions, $\pi\pi$, $\pi\eta$, and $\eta\eta$ at $\sqrt{s} = 2.981$ GeV and 3.526 GeV. The distributions are shown for the full range of acceptance, and also for $\cos\theta_{pM}^* < 0.3$. The latter corresponds to the region of highest momentum transfer. The distributions are very nearly flat, down to values of about 0.20. The first two bins have been suppressed because of the large number of events occurring in this region that would dominate the plot. As described in section 4.2, events consistent with the χ^2 hypothesis should have a uniform distribution. Thus, if exclusive states have been characterized correctly by the mean's and σ 's, the enhancement in the confidence level distribution, at low values, corresponds to background events. To account for this background, events are discarded if their χ^2 probability value is below 0.20. The number of events accepted must, therefore, be corrected by this amount:

$$\#events = \frac{\#events\ with\ \chi^2(probability) > 0.2}{0.8}. \quad (4.26)$$

Confidence Level

Figure 4.23: χ^2 confidence level distributions at 2.950 GeV for (a) $\bar{p}p \rightarrow \pi^0\pi^0$, (b) $\pi^0\eta$, (c) $\eta\eta$, (d) $\bar{p}p \rightarrow \pi^0\pi^0$ for $\cos \theta^*_{\bar{p}\pi} < 0.3$, (e) $\pi^0\eta$ for $\cos \theta^*_{\bar{p}\pi} < 0.3$, and (f) $\eta\eta$ for $\bar{p}p \rightarrow \pi^0\pi^0$ $\cos \theta^*_{\bar{p}\eta} < 0.3$.

Confidence Level

Figure 4.24: χ^2 confidence level distributions at 3.526 GeV for (a) $\bar{p}p \rightarrow \pi^0\pi^0$, (b) $\pi^0\eta$, (c) $\eta\eta$, (d) $\bar{p}p \rightarrow \pi^0\pi^0$ for $\cos \theta^*_{p\pi} < 0.3$, (e) $\pi^0\eta$ for $\cos \theta^*_{p\eta} < 0.3$, and (f) $\eta\eta$ for $\bar{p}p \rightarrow \pi^0\pi^0$ $\cos \theta^*_{p\eta} < 0.3$.

4.3.2 Geometrical Efficiencies

A Monte Carlo simulation was used to correct for detector acceptance. The range of CCAL coverage is 2π in azimuth and from 11° to 70° in polar angle. As a consequence of the finite acceptance, even though the momentum vector of a particle may fall within the detector range, it is still possible for one of its γ 's to miss CCAL depending on the $\gamma\gamma$ opening angle.

To account for this loss, Monte Carlo events of the type $\bar{p}p \rightarrow \bar{M}M \rightarrow \gamma\gamma\gamma\gamma$ are generated for all $\cos\theta_{\bar{p}M}^*$, having a uniform $\cos\theta_{\bar{p}M}^*$ distribution. The acceptance correction is the ratio of the number of events generated at a given $\cos\theta_{\bar{p}M}^*$ divided by the number of these events where all four γ 's are in the range of CCAL. The number of events with all four γ 's in the range of CCAL and above the threshold used the CCAL clustering algorithm (25 MeV) will approach zero as the momentum vector of the parent particle moves to CCAL boundaries. Beyond this point, no corrections are applied.

Figures 4.25a, 4.25b and 4.25c show examples of acceptance efficiencies for the three final-states; $\pi^0\pi^0$, $\pi^0\eta$, and $\eta\eta$, along with data, where the ordinate is the cosine of the production angle, $\theta_{\bar{p}M}^*$. The plots for $\pi^0\pi^0$ and $\eta\eta$ range from $\cos\theta_{\bar{p}M}^* = 0 - 1$ since, in each case, the particles are indistinguishable. The figures show the detector “cut-off” at about $\cos\theta_{\bar{p}M}^* \simeq 0.6$. This is a function of $\sqrt{s}$ since, for a given $\cos\theta_{\bar{p}M}^*$, for higher $\sqrt{s}$ the transformation to the lab will result in a smaller laboratory polar angle, θ . The shape of the acceptance efficiency depends on the mass of the parent particle. The γ 's from a π^0 will have a smaller opening angle than for an η for the same energy. A consequence of this is the asymmetric efficiency shown in figure 4.25b. Tables 4.8, 4.9, and 4.10 show the acceptance efficiencies for the three reactions at several energies covering the range of energies studied.

Each of the decay particles, π^0 and η , has spin zero, and, therefore, in their center-of-mass frame (also called the helicity frame), will produce a uniform distribution of $\gamma\gamma$ opening angles. This factor can be used to study losses that occur for two reasons. First,

Table 4.8: Acceptance efficiencies (in percent) as a function of the production angle, $\cos \theta_{\bar{p}\pi}^*$, for $\pi^0\pi^0$ at several energies. The errors shown are due to the Monte Carlo statistics.

$\cos \theta_{\bar{p}\pi}^*$	$\sqrt{s}$ (GeV)					
	2.950	2.980	3.098	3.250	3.526	3.621
0.025	92.5 ± 1.6	92.6 ± 1.6	93.2 ± 1.6	94.5 ± 1.6	94.0 ± 1.9	94.5 ± 1.9
0.075	92.0 ± 1.6	92.1 ± 1.6	93.4 ± 1.6	93.6 ± 1.7	94.7 ± 1.9	94.1 ± 2.0
0.125	92.1 ± 1.7	92.0 ± 1.6	92.4 ± 1.6	93.6 ± 1.7	94.3 ± 1.9	93.8 ± 1.9
0.175	90.5 ± 1.7	92.0 ± 1.7	92.2 ± 1.7	93.3 ± 1.7	93.8 ± 1.8	93.9 ± 1.9
0.225	90.2 ± 1.7	90.3 ± 1.7	91.0 ± 1.7	92.0 ± 1.8	94.4 ± 1.9	94.6 ± 2.0
0.275	84.1 ± 1.7	86.2 ± 1.7	89.6 ± 1.8	92.1 ± 1.8	94.4 ± 1.9	93.8 ± 1.9
0.325	73.3 ± 1.6	75.5 ± 1.7	84.1 ± 1.8	90.4 ± 1.8	93.4 ± 1.9	93.4 ± 1.9
0.375	59.1 ± 1.5	60.6 ± 1.5	73.4 ± 1.7	88.1 ± 1.9	92.8 ± 1.9	93.0 ± 1.9
0.425	46.4 ± 1.3	48.8 ± 1.4	60.9 ± 1.5	82.3 ± 1.9	92.4 ± 1.9	92.4 ± 1.8
0.475	32.3 ± 1.1	35.3 ± 1.2	48.6 ± 1.4	69.8 ± 1.7	91.1 ± 1.9	94.0 ± 1.9
0.525	16.4 ± 0.8	18.9 ± 0.9	36.2 ± 1.2	52.3 ± 1.5	88.0 ± 1.9	90.8 ± 1.9
0.575	4.5 ± 0.4	5.5 ± 0.5	15.6 ± 0.8	33.7 ± 1.2	83.2 ± 1.9	87.6 ± 1.9
0.625	0.0 ± 0.0	0.1 ± 0.0	3.2 ± 0.4	13.4 ± 0.8	62.8 ± 1.6	78.5 ± 1.8
0.675	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	1.1 ± 0.2	26.0 ± 1.1	49.4 ± 1.4
0.725	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	2.0 ± 0.3	7.6 ± 0.6

Table 4.9: Acceptance efficiencies (in percent) as a function of the production angle, $\cos \theta_{\bar{p}M}^*$, for $\pi^0\eta$ at several energies. The errors shown are due to the Monte Carlo statistics.

	$\sqrt{s}$ (GeV)					
$\cos \theta_{\bar{p}\pi}^*$	2.950	2.980	3.098	3.250	3.526	3.621
-0.725	0.0 $\pm$ 0.0	0.0 $\pm$ 0.0	0.0 $\pm$ 0.0	0.0 $\pm$ 0.0	1.5 $\pm$ 0.3	6.7 $\pm$ 0.5
-0.675	0.0 $\pm$ 0.0	0.0 $\pm$ 0.0	0.0 $\pm$ 0.0	0.9 $\pm$ 0.2	21.0 $\pm$ 0.9	35.6 $\pm$ 1.2
-0.625	0.0 $\pm$ 0.0	0.1 $\pm$ 0.1	2.7 $\pm$ 0.3	12.1 $\pm$ 0.7	48.4 $\pm$ 1.4	60.4 $\pm$ 1.6
-0.575	5.6 $\pm$ 0.5	7.4 $\pm$ 0.6	16.9 $\pm$ 0.9	30.4 $\pm$ 1.1	67.7 $\pm$ 1.7	71.9 $\pm$ 1.7
-0.525	17.9 $\pm$ 0.9	20.3 $\pm$ 0.9	31.7 $\pm$ 1.2	48.2 $\pm$ 1.4	77.8 $\pm$ 1.8	80.8 $\pm$ 1.8
-0.475	32.2 $\pm$ 1.1	33.6 $\pm$ 1.1	47.5 $\pm$ 1.4	64.6 $\pm$ 1.7	80.5 $\pm$ 1.8	82.1 $\pm$ 1.8
-0.425	46.2 $\pm$ 1.3	47.4 $\pm$ 1.4	58.6 $\pm$ 1.5	74.4 $\pm$ 1.7	82.6 $\pm$ 1.8	85.5 $\pm$ 1.8
-0.375	57.6 $\pm$ 1.4	58.1 $\pm$ 1.5	69.8 $\pm$ 1.6	80.6 $\pm$ 1.8	86.2 $\pm$ 1.8	85.9 $\pm$ 1.8
-0.325	70.2 $\pm$ 1.6	70.6 $\pm$ 1.6	77.7 $\pm$ 1.7	82.8 $\pm$ 1.8	86.3 $\pm$ 1.8	87.9 $\pm$ 1.8
-0.275	77.5 $\pm$ 1.6	80.1 $\pm$ 1.7	82.9 $\pm$ 1.7	85.2 $\pm$ 1.8	87.8 $\pm$ 1.8	88.1 $\pm$ 1.8
-0.225	83.2 $\pm$ 1.6	82.7 $\pm$ 1.6	85.0 $\pm$ 1.7	85.9 $\pm$ 1.7	88.6 $\pm$ 1.8	90.8 $\pm$ 1.9
-0.175	85.2 $\pm$ 1.6	84.2 $\pm$ 1.6	84.2 $\pm$ 1.6	87.0 $\pm$ 1.7	89.0 $\pm$ 1.8	90.7 $\pm$ 1.9
-0.125	85.9 $\pm$ 1.6	85.1 $\pm$ 1.6	86.3 $\pm$ 1.6	86.7 $\pm$ 1.6	89.8 $\pm$ 1.8	89.9 $\pm$ 1.9
-0.075	84.0 $\pm$ 1.6	85.9 $\pm$ 1.6	86.5 $\pm$ 1.6	88.0 $\pm$ 1.6	89.4 $\pm$ 1.8	89.8 $\pm$ 1.9
-0.025	84.6 $\pm$ 1.5	86.9 $\pm$ 1.5	87.5 $\pm$ 1.5	88.2 $\pm$ 1.5	89.9 $\pm$ 1.8	90.7 $\pm$ 1.9
0.025	84.3 $\pm$ 1.5	84.9 $\pm$ 1.5	86.4 $\pm$ 1.5	88.0 $\pm$ 1.6	90.0 $\pm$ 1.8	90.9 $\pm$ 1.9
0.075	80.8 $\pm$ 1.5	81.3 $\pm$ 1.5	83.7 $\pm$ 1.5	86.6 $\pm$ 1.6	90.8 $\pm$ 1.8	90.1 $\pm$ 1.9
0.125	75.2 $\pm$ 1.5	76.6 $\pm$ 1.5	79.8 $\pm$ 1.5	85.3 $\pm$ 1.6	89.6 $\pm$ 1.8	90.5 $\pm$ 1.9
0.175	67.3 $\pm$ 1.4	69.3 $\pm$ 1.4	73.4 $\pm$ 1.5	81.0 $\pm$ 1.6	89.0 $\pm$ 1.8	89.8 $\pm$ 1.9
0.225	60.9 $\pm$ 1.3	61.1 $\pm$ 1.4	67.5 $\pm$ 1.5	76.2 $\pm$ 1.6	87.9 $\pm$ 1.8	89.5 $\pm$ 1.9
0.275	49.5 $\pm$ 1.3	52.5 $\pm$ 1.3	59.6 $\pm$ 1.4	71.7 $\pm$ 1.5	86.7 $\pm$ 1.8	88.8 $\pm$ 1.9
0.325	36.5 $\pm$ 1.1	39.6 $\pm$ 1.1	52.3 $\pm$ 1.3	66.1 $\pm$ 1.5	82.4 $\pm$ 1.7	86.5 $\pm$ 1.8
0.375	28.6 $\pm$ 1.0	31.0 $\pm$ 1.0	41.3 $\pm$ 1.2	56.0 $\pm$ 1.5	79.8 $\pm$ 1.7	85.0 $\pm$ 1.8
0.425	19.1 $\pm$ 0.8	21.4 $\pm$ 0.9	31.1 $\pm$ 1.1	44.9 $\pm$ 1.3	73.8 $\pm$ 1.7	80.6 $\pm$ 1.7
0.475	12.8 $\pm$ 0.7	13.3 $\pm$ 0.7	21.2 $\pm$ 0.9	35.0 $\pm$ 1.2	67.0 $\pm$ 1.6	74.4 $\pm$ 1.7
0.525	5.4 $\pm$ 0.4	7.2 $\pm$ 0.5	13.2 $\pm$ 0.7	23.5 $\pm$ 1.0	53.9 $\pm$ 1.4	63.5 $\pm$ 1.5
0.575	1.3 $\pm$ 0.2	1.9 $\pm$ 0.3	5.6 $\pm$ 0.5	14.1 $\pm$ 0.8	42.6 $\pm$ 1.3	50.0 $\pm$ 1.4
0.625	0.0 $\pm$ 0.0	0.1 $\pm$ 0.0	1.0 $\pm$ 0.2	4.2 $\pm$ 0.4	26.1 $\pm$ 1.0	36.6 $\pm$ 1.2
0.675	0.0 $\pm$ 0.0	0.0 $\pm$ 0.0	0.0 $\pm$ 0.0	0.4 $\pm$ 0.1	10.6 $\pm$ 0.6	19.5 $\pm$ 0.9
0.725	0.0 $\pm$ 0.0	0.0 $\pm$ 0.0	0.0 $\pm$ 0.0	0.0 $\pm$ 0.0	1.0 $\pm$ 0.2	2.9 $\pm$ 0.3

Table 4.10: Acceptance efficiencies (in percent) as a function of the production angle, $\cos \theta_{\bar{p}\eta}^*$, for $\eta\eta$ at several energies. The errors shown are due to the Monte Carlo statistics.

	$\sqrt{s}$ (GeV)					
$\cos \theta_{\bar{p}\eta}^*$	2.950	2.980	3.098	3.250	3.526	3.621
0.025	76.2 ± 1.4	77.5 ± 1.4	80.1 ± 1.4	81.5 ± 1.5	85.6 ± 1.7	86.3 ± 1.9
0.075	74.0 ± 1.4	74.8 ± 1.4	78.1 ± 1.4	82.0 ± 1.5	85.2 ± 1.7	85.5 ± 1.8
0.125	68.3 ± 1.4	69.6 ± 1.4	74.2 ± 1.4	79.2 ± 1.5	83.5 ± 1.7	85.7 ± 1.8
0.175	60.9 ± 1.3	63.3 ± 1.3	69.5 ± 1.4	75.0 ± 1.5	83.3 ± 1.7	85.0 ± 1.8
0.225	55.3 ± 1.3	54.9 ± 1.3	64.3 ± 1.4	71.7 ± 1.5	83.1 ± 1.7	83.6 ± 1.8
0.275	45.8 ± 1.2	48.3 ± 1.2	55.7 ± 1.3	66.1 ± 1.5	81.1 ± 1.7	83.0 ± 1.8
0.325	36.9 ± 1.1	39.8 ± 1.1	48.8 ± 1.3	60.5 ± 1.5	76.6 ± 1.7	78.4 ± 1.7
0.375	26.1 ± 0.9	28.3 ± 1.0	37.6 ± 1.2	52.7 ± 1.4	72.9 ± 1.6	77.2 ± 1.7
0.425	19.3 ± 0.8	20.8 ± 0.9	29.2 ± 1.0	42.6 ± 1.3	67.7 ± 1.6	72.2 ± 1.6
0.475	13.3 ± 0.7	14.0 ± 0.7	21.3 ± 0.9	30.1 ± 1.1	57.1 ± 1.5	64.8 ± 1.6
0.525	6.9 ± 0.5	7.4 ± 0.5	11.4 ± 0.7	22.4 ± 1.0	47.8 ± 1.4	55.0 ± 1.4
0.575	1.3 ± 0.2	2.3 ± 0.3	4.5 ± 0.4	13.9 ± 0.8	34.7 ± 1.2	40.7 ± 1.2
0.625	0.0 ± 0.0	0.0 ± 0.0	0.7 ± 0.2	4.6 ± 0.4	20.9 ± 0.9	27.5 ± 1.0
0.675	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.3 ± 0.1	7.8 ± 0.5	14.7 ± 0.7
0.725	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.9 ± 0.2	2.4 ± 0.3

for a given particle momentum, $\vec{p}$, if a $\gamma\gamma$ pair is produced at 90° in the helicity frame relative to $\vec{p}$, the $\gamma\gamma$ opening angle observed in the lab will be a minimum. These are called “symmetric” decays. If the minimum angle is small enough, the showers produced in CCAL will coalesce to the point where they cannot be resolved by the clustering algorithm. Thus, they are considered a three-cluster event and are excluded from this analysis. The second case occurs when the γ ’s are produced near 0° and 180° in the helicity frame. The transformation to the lab frame results in a large opening angle, where the larger the opening angle the larger the difference in the lab energies of the γ ’s. These are called “asymmetric” decays.

$$\cos \theta_{p\pi}^*$$

Figure 4.25: Production angular distributions showing detector acceptance efficiencies for (a) $\pi^0\pi^0$, (b) $\pi^0\eta$, and (c) $\eta\eta$. The solid histogram is data, and the dashed is Monte Carlo. The horizontal line marked "full acceptance" is the distribution of Monte Carlo events disregarding detector acceptance. The data are taken at 3.098 GeV.

For asymmetric decays, the least energetic γ may miss detection either by missing CCAL, or by being below the CCAL energy threshold for a cluster (25 MeV). These are corrected for in the Monte Carlo simulation.

To determine losses from symmetric decays, γ decay angles are studied. This is the angle between the momentum vector of M and the daughter γ in the helicity frame. This angle is defined as, $\theta_{\gamma M}$. Figures 4.26 through 4.30 show these distributions for the data (solid line) and for the Monte Carlo simulation described above (dotted line). To decrease statistical fluctuations, all data have been combined for $\sqrt{s} \leq 3.250$ GeV (labeled "Low E"), and $\sqrt{s} \geq 3.523$ GeV (labeled "High E"). Within statistical errors, except for two plots, all distributions, agree quite well with the Monte Carlo simulation. All plots show a depletion of events for large values of $\cos \theta_{\gamma M}^*$, where the particles are decaying asymmetrically. The Monte Carlo distributions simulate this loss well for all plots and thus it is accounted for in the "geometrical acceptance" correction. The two exceptions are for the symmetric decays in $\pi^0\pi^0$ and $\pi^0\eta$ at high energy, and in the central region of CCAL ($\cos \theta_{\bar{p}\pi}^* < 0.3$). In figure 4.26c, 4.28a, and 4.28b, one can see a depletion of events for the more symmetric decays ($\cos \theta_{\gamma M}^* < 0.5$). It is reasonable to assume these losses are due to coalescing showers. At $\sqrt{s} = 3.523$ GeV, a π^0 in the central region of CCAL will have an energy of $\simeq 3.6$ GeV, and a minimum $\gamma\gamma$ opening angle of $\simeq 4.3^\circ$. The CCAL block separation in this region is 3.1° in θ . The greater discrepancy seen between data and Monte Carlo in the central region of CCAL is consistent with coalescing showers because the ratio of the distance between the γ 's at the block faces and the largest diagonal on the front face of the blocks is smallest for the central region [43]. That is, if showers are to coalesce to the point where they are unresolvable, the effect will be greatest in the central region of CCAL.

The losses occurring for each particle are independent since the $\gamma\gamma$ opening angles of each particle's decay are independent. The losses due to symmetric decays are determined by averaging the bin contents where the distribution is level and calculating the fractional area with no events, omitting the region of large $\cos \theta_{\gamma M}^*$, where the asymmetric decays

occur ($0.6 < \cos \theta_{\gamma M}^* < 0.9$). The loss seen in figure 4.26c is 13.7%, in figure 4.28a is 7.2%. figure 4.28b is 11.3%. These numbers are used to correct for losses due to coalescing showers.

Defining the loss from each particle decay as $LOSS_i$, correction to the data for the losses of symmetric decays is then given by

$$corrected \#events = \frac{\#events \text{ measured}}{(1 - LOSS_1)(1 - LOSS_2)}. \quad (4.27)$$

Thus, for $\pi^0\pi^0$ for $\sqrt{s} \geq 3.523$ GeV, for $\cos \theta_{\bar{p}\pi}^* < 0.3$, $LOSS_1 = LOSS_2 = 0.137$. For $\pi^0\eta$ for $\sqrt{s} \geq 3.523$ GeV, for $-0.3 < \cos \theta_{\bar{p}\pi}^* < 0.0$, $LOSS_1 = 0.072$ and $LOSS_2 = 0.0$. For $\pi^0\eta$ for $\sqrt{s} \geq 3.523$ GeV, for $0.0 < \cos \theta_{\bar{p}\pi}^* < 0.3$, $LOSS_1 = 0.113$ and $LOSS_2 = 0.0$. For all other cases $LOSS = 0.0$.

$$\cos \theta_{\gamma\pi}^*$$

Figure 4.26: Comparison of $\gamma\gamma$ decay angle, $\theta_{\gamma\pi}$, for π^0 in $\pi^0\pi^0$ for data (solid line) and Monte Carlo (dotted line) for (a) low energy (≤ 3.250 GeV) and $\cos \theta_{p\pi}^* < 0.3$, (b) low energy (≤ 3.250 GeV) and $\cos \theta_{p\pi}^* > 0.3$, (c) high energy (≥ 3.523 GeV) and $\cos \theta_{p\pi}^* < 0.3$, (d) high energy (≥ 3.523 GeV) and $\cos \theta_{p\pi}^* > 0.3$.

$$\cos\theta_{\gamma M}^*$$

Figure 4.27: Comparison of $\gamma\gamma$ decay angle, $\theta_{\gamma M}$, where M is the the π^0 or η in $\pi^0\eta$ as indicated in each figure for data (solid line) and Monte Carlo (dotted line) for low energy (≤ 3.250 GeV), (a) π^0 with $-0.3 < \cos\theta_{\bar{p}\pi}^* < 0.3$, (b) π^0 with $\cos\theta_{\bar{p}\pi}^* > 0.3$, (c) π^0 with $\cos\theta_{\bar{p}\pi}^* < -0.3$, (d) η with $-0.3 < \cos\theta_{\bar{p}\pi}^* < 0.3$, (e) η with $\cos\theta_{\bar{p}\pi}^* > 0.3$, (f) η with $\cos\theta_{\bar{p}\pi}^* < -0.3$.

Figure 4.28: Comparison of $\gamma\gamma$ decay angle, $\theta_{\gamma\pi}$, for the π^0 in $\pi^0\eta$ for data (solid line) and Monte Carlo (dotted line) for high energy (≥ 3.523 GeV), (a) π^0 with $-0.3 < \cos\theta_{p\pi}^* < 0.0$, (b) π^0 with $0.0 < \cos\theta_{p\pi}^* < 0.3$, (c) π^0 with $\cos\theta_{p\pi}^* > 0.3$, (d) π^0 with $\cos\theta_{p\pi}^* < -0.3$.

$$\cos\theta_{\gamma\eta}^*$$

Figure 4.29: Comparison of $\gamma\gamma$ decay angle, $\theta_{\gamma\eta}$, for the η in $\pi^0\eta$ for data (solid line) and Monte Carlo (dotted line) for high energy (≥ 3.523 GeV), (a) η with $-0.3 < \cos\theta_{\bar{p}\pi}^* < 0.3$, (b) η with $\cos\theta_{\bar{p}\pi}^* > 0.3$, (c) η with $\cos\theta_{\bar{p}\pi}^* < -0.3$.

Figure 4.30: Comparison of $\gamma\gamma$ decay angle, $\theta_{\gamma\eta}$, for η in $\eta\eta$ for data (solid line) and Monte Carlo (dotted line) for (a) low energy (≤ 3.250 GeV) and $\cos \theta_{p\pi}^* < 0.3$, (b) low energy (≤ 3.250 GeV) and $\cos \theta_{p\pi}^* > 0.3$, (c) high energy (≥ 3.523 GeV) and $\cos \theta_{p\pi}^* < 0.3$, (d) high energy (≥ 3.523 GeV) and $\cos \theta_{p\pi}^* > 0.3$.

4.3.3 Trigger Efficiencies

A correction for the neutral trigger efficiency must be applied. The neutral trigger consists of a CCAL total energy trigger, an H1 veto, and a forward charged veto. The CCAL total energy trigger efficiency was measured to be $\simeq 95\%$ [44]. The loss of efficiency is attributed to energy threshold levels used to obtain data rates that could be managed by the data acquisition system (i.e., to reduce electronic dead-time). Inefficiencies in the charged veto detectors, H1, and the forward charged veto counters, FCH, can be attributed to δ -rays ($\bar{p}e$ interactions) in the gas jet and possibly out-of-time annihilation products. Both processes may result in signals in these detectors vetoing exclusive all-neutral events and causing a decrease in the measured cross section. The inefficiency of FCH was measured to be $\simeq 3.5\%$, and within errors is independent of the interaction rate [44]. The H1 veto inefficiency was observed to be rate dependent [45]. To account for the rate dependence of veto inefficiencies, cross sections for $\pi^0\pi^0$ at a given energy as a function of the measured instantaneous luminosity were studied. Figure 4.31 shows data from $\simeq 230$ data runs at $\sqrt{s} \simeq 3.526$ GeV, where the measured cross section is plotted as a function of the measured instantaneous luminosity, L . The data were fit to a Poisson distribution describing the probability for no accidentals (i.e., no pile-up). That is, the number of events fitted to $\bar{p}p \rightarrow \bar{M}M$ is given by

$$\text{number of events fitted to } \bar{p}p \rightarrow \bar{M}M = \sigma \cdot L \cdot P(0), \quad (4.28)$$

where σ is the cross section for the final-state, and $P(0)$ is given by

$$P(0) = e^{-\mu\tau}. \quad (4.29)$$

Equation 4.29 is a Poisson distribution giving the probability for no accidentals occurring in time, τ , for an interaction rate of μ . Since the interaction rate is proportional to the instantaneous luminosity, and the number of events fitted to $\bar{p}p \rightarrow \bar{M}M$ is proportional to the differential cross section, the data were fit to

$$d\sigma/d\Omega = Ae^{-BL}, \quad (4.30)$$

where A and B are constants (see figure 4.31). The fitted χ^2 per degree of freedom is 3.8. This relatively poor value is attributed to systematic errors (see section 4.3.5) not included in the fit. The result is used to correct the measured cross sections for each data run according to

$$\epsilon_{rate} = 2.41e^{-0.057L}. \quad (4.31)$$

4.3.4 Residual Background

To determine possible residual background after applying the 20% confidence level cut, a χ^2 calculation was done using the four χ^2 variables, PT12, PT2, PE1, and PE22. Applying a 10% confidence level cut, and restricting the invariant mass of one γ pair to be 0.100-0.170 GeV/c² for π^0 and 0.450-0.650 GeV/c² for η , the invariant mass of the remaining γ pair is studied. The invariant mass plots are shown in figures 4.32 and 4.33. The invariant mass distributions were fit to a Gaussian plus a linear term. The results were found to be independent of energy within errors. The distributions for π from $\bar{p}p \rightarrow \pi\pi$, η from $\bar{p}p \rightarrow \eta\eta$, and π from $\bar{p}p \rightarrow \pi\eta$, are shown in figure 4.32. The fractional areas under the linear curves, out to 3σ of the Gaussian fit, are 1.89%, 12.50%, and 2.70%, respectively. For the η 's from $\bar{p}p \rightarrow \pi\eta$, the background level was found to be dependent on $\cos\theta_{\bar{p}\pi}^*$. The background level was determined for four regions of $\cos\theta_{\bar{p}\pi}^*$ (-1.0 to -0.3, -0.3 to 0.0, 0.0 to 0.3, and 0.3 to 1.0) as indicated in figure 4.33. The background levels for the four regions shown are 6.86%, 9.27%, 10.41%, and 17.93%, respectively. To correct for the trend of increasing background for larger values of $\cos\theta_{\bar{p}\pi}^*$ these four data points were fit to a line giving;

$$background_{\eta,\pi\eta} = 0.00828 \cos\theta_{\bar{p}\pi}^* + 0.1118, \quad (4.32)$$

where the subscript refers to the background for η in $\bar{p}p \rightarrow \pi\eta$. The results are used to correct the cross sections for background according to

$$corrected \# \text{ events} = (\# \text{ events measured})(1 - background_{M,MM}), \quad (4.33)$$

with

$$background_{\pi,\pi\pi} = 0.0189\%, \quad (4.34)$$

$$background_{\eta,\eta\eta} = 0.1250\%, \quad (4.35)$$

Figure 4.31: Dependence of measured cross section of $\pi^0\pi^0$ on interaction rate. The error bars are the statistical errors. Systematic errors are discussed in section 4.3.5.

and

$$background_{\pi,\pi\eta} = 0.0270\%. \quad (4.36)$$

Invariant Mass (GeV/c^2)

Figure 4.32: Invariant mass distributions for (a) π^0 from $\bar{p}p \rightarrow \pi^0\pi^0$, (b) η from $\bar{p}p \rightarrow \eta\eta$, and (c) π^0 from $\bar{p}p \rightarrow \pi\eta$. The complete fit and the linear fit functions are shown separately.

Invariant Mass (GeV/c^2)

Figure 4.33: Invariant mass distributions of η from $\bar{p}p \rightarrow \pi\eta$ for (a) $-1.0 < \cos\theta_{\bar{p}\pi}^* < -0.3$, (b) $-0.3 < \cos\theta_{\bar{p}\pi}^* < 0.0$, (c) $0.0 < \cos\theta_{\bar{p}\pi}^* < 0.3$, and (d) $0.3 < \cos\theta_{\bar{p}\pi}^* < 1.0$. The complete fit and the linear fit functions are shown separately.

4.3.5 Corrections for Branching Ratios to $\gamma\gamma$ and γ Conversions

Corrections to the cross section are also needed to account for the branching ratios of $M \rightarrow \gamma\gamma$. For π^0 , the branching ratio for Dalitz decay is 0.012. For η the branching ratio for other channels is 0.611. In addition, corrections are needed to account for γ conversions in the beam pipe, i.e., before the the H1 scintillator which provides the neutral charged veto signal. The probability for one of the four γ 's to convert is 0.011. A correction of $(1 - 0.011)^4$ must, therefore, be applied.

4.3.6 Systematic Errors

The systematic error in the results was determined by studying $\pi^0\pi^0$ cross sections at a production angle of 90° for several scans taken at the same energy (3.526 GeV). Table 4.11 shows these data with the statistical errors given. These data show fluctuations that are typically 2-3 times larger than statistical errors. The luminosity measurements have a systematic error of 4% [46], which is comparable to statistical errors, and, therefore, cannot account for the fluctuations seen. The statistical errors in the luminosity measurements are quite small and are included in the statistical error given in table 4.11.

The systematic error is determined by calculating a χ^2 of the measurements in table 4.11 according to

$$\chi^2 = \sum_{i=1,9} \frac{(y_i - \bar{y})^2}{\sigma_i^2 + S^2}, \quad (4.37)$$

where the y_i and σ_i are taken from table 4.11 and $\bar{y}$ is the average of the y_i . For the 9 data points the expected χ^2 is 8, which corresponds to $S=0.12$ nb/sr, and therefore a systematic error of $S/\bar{y} = 5\%$. This value is used for all data, and errors in cross section are calculated by combining the systematic error in quadrature with statistical errors.

The exact source of systematic errors is unknown. To check the stability of the χ^2 method of analysis discussed in section 4.2, all data from table 4.11 were analyzed using fixed values of the χ^2 parameters, versus determining the χ^2 parameters separately for

Table 4.11: Flucuations in the measured $\pi^0\pi^0$ differential cross section at a production angle of 90° for data taken at 3.526 GeV.

$\sqrt{s} = 3.526 \text{ GeV}$	
$\frac{d\sigma}{d\Omega} \text{ (nb/sr)}$	statistical error
2.58	± 0.113
2.14	± 0.162
2.95	± 0.084
2.73	± 0.072
2.67	± 0.067
2.50	± 0.058
2.59	± 0.056
2.38	± 0.049
2.57	± 0.055

each scan. The result was that changes in the calculated cross sections were small in comparison to the fluctuations being studied. A study of data rates [46] indicates that trigger instabilities cannot account for the fluctuations. One possible explanation is due to the position of the interaction region, which is known to change in the range of 1-2 mm for different data runs [47]. Such shifts in beam position cause errors in event reconstruction. That is, when reconstructing particle vectors from CCAL clusters a nominal beam position is assumed.

As will be presented in chapter 5, an angular-dependent systematic error has been indentified by studying symmetry in the $\pi\eta$ reaction. This error is approximately $\pm 10\%$. The overall systematic error, therefore, is approximately $\pm 15\%$ after combining all errors.

4.3.7 Summary of Corrections to the Measured Cross Sections

Cross sections that will be presented include corrections for efficiencies discussed above. A summary of these efficiencies is shown in table 4.12. A correction factor is generated by taking the reciprocal of the efficiency.

Table 4.12: Summary of efficiencies used to correct measured cross sections.

$\sigma_{measured} = \sigma \cdot \epsilon_{\chi^2(prob)} \cdot \epsilon_{accept} \cdot \epsilon_{coalesceLOSS} \cdot \epsilon_{Etot} \cdot \epsilon_{FCH} \cdot \epsilon_{rate} \\ \cdot \epsilon_{back_{M,MM}} \cdot \epsilon_{conversions} \cdot \epsilon_{BR}$	
ϵ_{prob}	$(1 - cut\chi^2(probability)) = (1 - 0.2) = 0.800$
ϵ_{accept}	from Monte Carlo, see tables 4.8-4.10
$\epsilon_{\gamma\gamma LOSS}$	$(1 - LOSS_1)(1 - LOSS_2)$
	$\pi\pi$ for $\sqrt{s} \geq 3.523$ GeV, for $\cos\theta_{\bar{p}\pi}^* < 0.3$, LOSS= 0.137
	$\pi\eta$ for $\sqrt{s} \geq 3.523$ GeV, for $-0.3 < \cos\theta_{\bar{p}\pi}^* < 0.0$, LOSS= 0.072
	$\pi\eta$ for $\sqrt{s} \geq 3.523$ GeV, for $0.0 < \cos\theta_{\bar{p}\pi}^* < 0.3$, LOSS= 0.113
	for all other cases LOSS= 0.0
ϵ_{Etot}	0.95
ϵ_{VETO}	$(1 - 0.032) = 0.937$
ϵ_{rate}	$2.412e^{-0.0576L}$
$\epsilon_{back_{M,MM}}$	$\epsilon_{back_{\pi,\pi\pi}} = 1 - 0.0189 = 0.9811$
	$\epsilon_{back_{\eta,\eta\eta}} = 1 - 0.1250 = 0.8750$
	$\epsilon_{back_{\pi,\pi\eta}} = 1 - 0.0270 = 0.9730$
	$\epsilon_{back_{\eta,\pi\eta}} = 1 - 0.00828 \cos\theta_{\bar{p}\pi}^* + 0.1118$
$\epsilon_{conversions}$	$(1 - 0.011)^4 = 0.957$
ϵ_{BR}	branching ratio of $M \rightarrow \gamma\gamma$
	$\epsilon_{BR(\pi^0\pi^0 \rightarrow \gamma\gamma\gamma\gamma)} = (1 - 0.012)^2 = 0.976$
	$\epsilon_{BR(\pi^0\eta \rightarrow \gamma\gamma\gamma\gamma)} = (1 - 0.012)(1 - 0.611) = 0.384$
	$\epsilon_{BR(\eta\eta \rightarrow \gamma\gamma\gamma\gamma)} = (1 - 0.611)^2 = 0.151$

Chapter 5

Results

Tables 5.1 through 5.15 and figures 5.1 through 5.15 show angular distributions for the center-of-mass energies indicated, and for the three final states; $\pi^0\pi^0$, $\pi^0\eta$, and $\eta\eta$. These plots give the differential cross section, $\frac{d\sigma}{d\Omega}(\frac{nb}{sr})$, as a function of the center-of-mass production angle, $\cos\theta_{\bar{p}M}^*$, where M is a π^0 or η . This is the angle between the $\bar{p}p$ and $M\bar{M}$ axis in the center-of-mass frame, demonstrated in figure 4.22.

Each plot shows a cut-off due to the detector acceptance. Data are not shown where the acceptance efficiencies are $\leq 10\%$ due to the large statistical errors at the detector boundaries from both Monte Carlo simulation (on-the-order-of 10%) and small number of events seen in this region of the detector due to the decrease in acceptance efficiency. This cut-off is a function of $\sqrt{s}$, where the larger $\sqrt{s}$ the greater the acceptance. This is because, for a given production angle, $\cos\theta_{\bar{p}M}^*$, the stronger the boost (β) the smaller laboratory polar angles will be. For two-body final states, the acceptance cut-off of CCAL occurs as a result of the backward boundary ($\theta_{lab} = 70^\circ$) rather than the forward boundary ($\theta_{lab} = 11^\circ$). Thus, particles that escaped detection at a given $\sqrt{s}$, by “going out the back”, will have their momentum vectors shifted toward the CCAL boundary (lower θ) as $\sqrt{s}$ increases. For example, plots of $\cos\theta_{\bar{p}\pi}^*$ (figures 5.1 through 5.5) show this. Here, for $\sqrt{s} \leq 3.250$ GeV, the cut-off occurs at about $\cos\theta_{\bar{p}\pi}^* = 0.50$, and for $\sqrt{s} \geq 3.250$ GeV, the cut-off occurs at about $\cos\theta_{\bar{p}\pi}^* = 0.65$.

The plots for $\pi^0\pi^0$ and $\eta\eta$ range, in principle, from $\cos\theta_{\bar{p}M}^* = 0 - 1$, while the plots for $\pi^0\eta$ range from $\cos\theta_{\bar{p}M}^* = -1$ to $+1$. This is because, for $\pi^0\pi^0$ and $\eta\eta$, the particles are indistinguishable. The plots of $\pi^0\pi^0$ and $\eta\eta$ are referred to as “folded”, since one is double-counting events for a given $\cos\theta_{\bar{p}M}^*$. For the $\pi^0\eta$ plots, the angle that is plotted is

Table 5.1: Differential Cross Sections for $\pi^0\pi^0$ from 2.950 to 2.981 GeV.

$d\sigma/d\Omega(nb/sr)$ vs. π^0 production angle ($\cos\theta_{\bar{p}\pi}^*$) for $\pi^0\pi^0$				
	$\sqrt{s}$ (GeV)			
$\cos\theta_{\bar{p}\pi}^*$	2.950	2.975	2.979	2.981
0.025	22.73 ± 1.18	16.99 ± 0.81	17.43 ± 1.01	16.57 ± 0.80
0.075	17.60 ± 0.97	13.16 ± 0.66	11.58 ± 0.76	12.72 ± 0.64
0.125	9.75 ± 0.64	6.82 ± 0.40	6.04 ± 0.50	7.11 ± 0.42
0.175	8.82 ± 0.60	5.08 ± 0.33	4.77 ± 0.44	4.70 ± 0.32
0.225	19.64 ± 1.07	13.67 ± 0.69	14.02 ± 0.88	13.29 ± 0.68
0.275	52.23 ± 2.32	38.68 ± 1.64	38.84 ± 1.89	37.84 ± 1.64
0.325	114.60 ± 4.35	85.99 ± 3.20	84.98 ± 3.49	82.73 ± 3.11
0.375	211.01 ± 7.05	157.00 ± 5.08	164.20 ± 5.75	154.64 ± 5.05
0.425	325.64 ± 9.85	251.26 ± 7.25	233.15 ± 7.49	244.82 ± 7.16
0.475	448.21 ± 12.88	355.18 ± 9.60	342.09 ± 10.31	348.39 ± 9.53
0.525	527.27 ± 15.53	410.43 ± 10.97	434.49 ± 13.23	394.48 ± 10.71

Table 5.2: Differential Cross Sections for $\pi^0\pi^0$ from 2.985 to 3.005 GeV.

$d\sigma/d\Omega(nb/sr)$ vs. π^0 production angle ($\cos\theta_{\bar{p}\pi}^*$) for $\pi^0\pi^0$				
	$\sqrt{s}$ (GeV)			
$\cos\theta_{\bar{p}\pi}^*$	2.985	2.990	2.994	3.005
0.025	15.27 ± 0.88	15.01 ± 0.70	14.63 ± 0.77	13.80 ± 0.74
0.075	11.72 ± 0.73	10.92 ± 0.54	10.94 ± 0.62	10.21 ± 0.59
0.125	6.45 ± 0.50	5.95 ± 0.34	5.60 ± 0.39	4.38 ± 0.34
0.175	3.92 ± 0.37	3.54 ± 0.24	3.80 ± 0.31	2.87 ± 0.26
0.225	12.62 ± 0.78	11.55 ± 0.58	10.79 ± 0.62	9.67 ± 0.58
0.275	36.54 ± 1.74	34.54 ± 1.44	33.64 ± 1.52	31.10 ± 1.41
0.325	77.55 ± 3.13	73.17 ± 2.68	73.43 ± 2.83	64.54 ± 2.56
0.375	152.70 ± 5.28	139.01 ± 4.40	139.76 ± 4.59	116.44 ± 3.96
0.425	215.36 ± 6.80	211.26 ± 6.02	204.93 ± 6.21	186.38 ± 5.61
0.475	299.65 ± 8.86	304.24 ± 8.04	314.02 ± 8.78	257.08 ± 7.29
0.525	376.98 ± 11.13	385.85 ± 9.89	354.73 ± 9.81	345.98 ± 9.50

Table 5.3: Differential Cross Sections for $\pi^0\pi^0$ from 3.098 to 3.525 GeV.

$d\sigma/d\Omega(nb/sr)$ vs. π^0 production angle ($\cos\theta_{p\pi}^*$) for $\pi^0\pi^0$				
	$\sqrt{s}$ (GeV)			
$\cos\theta_{p\pi}^*$	3.098	3.525	3.523	3.525
0.025	5.25 ± 0.30	1.60 ± 0.54	2.82 ± 0.15	2.71 ± 0.23
0.075	3.63 ± 0.23	1.97 ± 0.60	2.85 ± 0.16	3.07 ± 0.25
0.125	1.57 ± 0.14	1.62 ± 0.54	3.06 ± 0.16	2.68 ± 0.23
0.175	0.88 ± 0.10	1.62 ± 0.54	2.78 ± 0.15	2.98 ± 0.24
0.225	4.60 ± 0.28	1.83 ± 0.58	2.71 ± 0.15	2.44 ± 0.22
0.275	14.20 ± 0.67	4.20 ± 0.89	2.59 ± 0.15	2.57 ± 0.23
0.325	30.56 ± 1.29	5.77 ± 1.06	2.73 ± 0.15	2.69 ± 0.23
0.375	48.31 ± 1.83	5.54 ± 1.05	3.63 ± 0.18	3.60 ± 0.28
0.425	76.80 ± 2.55	13.28 ± 1.72	5.14 ± 0.24	5.07 ± 0.35
0.475	118.33 ± 3.57	29.61 ± 2.85	7.58 ± 0.32	7.48 ± 0.46
0.525	182.47 ± 5.05	53.00 ± 4.27	8.73 ± 0.33	7.22 ± 0.41
0.575	364.57 ± 9.70	114.17 ± 7.84	14.17 ± 0.48	12.74 ± 0.63
0.625	0.00 ± 0.00	130.50 ± 12.76	30.45 ± 0.82	25.52 ± 1.01
0.675	0.00 ± 0.00	0.00 ± 0.00	53.75 ± 1.35	49.24 ± 1.81

Table 5.4: Differential Cross Sections for $\pi^0\pi^0$ from 3.526 to 3.594 GeV.

$d\sigma/d\Omega(nb/sr)$ vs. π^0 production angle ($\cos\theta_{p\pi}^*$) for $\pi^0\pi^0$				
	$\sqrt{s}$ (GeV)			
$\cos\theta_{p\pi}^*$	3.526	3.527	3.591	3.594
0.025	3.26 ± 0.08	3.36 ± 0.20	2.82 ± 0.20	2.65 ± 0.20
0.075	3.36 ± 0.08	3.57 ± 0.21	2.71 ± 0.19	2.37 ± 0.19
0.125	2.99 ± 0.07	3.34 ± 0.20	2.97 ± 0.21	3.00 ± 0.22
0.175	2.97 ± 0.07	3.24 ± 0.19	2.99 ± 0.21	2.87 ± 0.21
0.225	2.83 ± 0.07	2.96 ± 0.18	2.93 ± 0.20	2.99 ± 0.22
0.275	2.85 ± 0.07	3.00 ± 0.18	3.23 ± 0.22	2.94 ± 0.22
0.325	2.97 ± 0.07	3.23 ± 0.19	3.12 ± 0.21	2.74 ± 0.21
0.375	3.82 ± 0.09	4.24 ± 0.23	3.32 ± 0.22	3.31 ± 0.24
0.425	5.32 ± 0.11	5.38 ± 0.28	4.48 ± 0.27	3.83 ± 0.26
0.475	7.70 ± 0.15	8.61 ± 0.41	6.25 ± 0.35	5.62 ± 0.35
0.525	8.22 ± 0.15	9.35 ± 0.41	6.39 ± 0.34	5.94 ± 0.34
0.575	13.29 ± 0.21	14.55 ± 0.58	10.32 ± 0.50	9.10 ± 0.46
0.625	26.66 ± 0.35	28.92 ± 0.98	20.60 ± 0.83	18.30 ± 0.77
0.675	50.38 ± 0.58	56.10 ± 1.75	40.40 ± 1.30	36.54 ± 1.26

Table 5.5: Differential Cross Sections for $\pi^0\pi^0$ from 3.613 to 3.621 GeV.

$d\sigma/d\Omega(nb/sr)$ vs. π^0 production angle ($\cos\theta_{\bar{p}\pi}^*$) for $\pi^0\pi^0$				
	$\sqrt{s}$ (GeV)			
$\cos\theta_{\bar{p}\pi}^*$	3.613	3.616	3.619	3.621
0.025	2.91 ± 0.20	2.94 ± 0.20	2.38 ± 0.22	2.69 ± 0.18
0.075	3.49 ± 0.23	3.35 ± 0.22	2.75 ± 0.24	3.25 ± 0.21
0.125	3.57 ± 0.24	3.66 ± 0.23	2.68 ± 0.24	3.19 ± 0.20
0.175	3.77 ± 0.24	3.69 ± 0.24	2.89 ± 0.25	2.96 ± 0.19
0.225	3.72 ± 0.24	3.55 ± 0.23	2.89 ± 0.25	3.38 ± 0.21
0.275	3.77 ± 0.24	3.70 ± 0.23	3.05 ± 0.26	3.40 ± 0.21
0.325	3.56 ± 0.23	3.46 ± 0.22	2.77 ± 0.24	3.16 ± 0.20
0.375	3.84 ± 0.25	3.78 ± 0.24	3.46 ± 0.28	3.36 ± 0.21
0.425	4.55 ± 0.28	4.37 ± 0.26	4.25 ± 0.32	4.42 ± 0.25
0.475	6.30 ± 0.35	6.00 ± 0.34	5.34 ± 0.37	5.33 ± 0.30
0.525	6.08 ± 0.32	5.92 ± 0.31	4.86 ± 0.31	5.50 ± 0.28
0.575	9.92 ± 0.47	9.66 ± 0.47	8.63 ± 0.48	9.24 ± 0.44
0.625	18.40 ± 0.76	19.23 ± 0.78	14.61 ± 0.70	17.52 ± 0.71
0.675	36.94 ± 1.20	35.46 ± 1.14	29.34 ± 1.13	32.52 ± 1.04

Table 5.6: Differential Cross Sections for $\pi^0\eta$ from 2.950 to 2.981 GeV.

$d\sigma/d\Omega(nb/sr)$ vs. π^0 production angle ($\cos\theta_{\bar{p}\pi}^*$) for $\pi^0\eta$				
	$\sqrt{s}$ (GeV)			
$\cos\theta_{\bar{p}\pi}^*$	2.950	2.975	2.979	2.981
-0.525	228.80 $\pm$ 17.20	191.54 $\pm$ 13.38	176.44 $\pm$ 13.93	190.05 $\pm$ 13.30
-0.475	140.26 $\pm$ 10.41	102.14 $\pm$ 7.18	110.25 $\pm$ 8.66	109.26 $\pm$ 7.69
-0.425	70.00 $\pm$ 5.49	55.66 $\pm$ 4.08	50.63 $\pm$ 4.36	50.09 $\pm$ 3.73
-0.375	36.52 $\pm$ 3.13	23.18 $\pm$ 1.88	27.33 $\pm$ 2.62	24.82 $\pm$ 2.01
-0.325	20.81 $\pm$ 1.96	16.78 $\pm$ 1.41	14.29 $\pm$ 1.57	13.22 $\pm$ 1.19
-0.275	21.52 $\pm$ 1.98	15.11 $\pm$ 1.29	13.63 $\pm$ 1.48	16.48 $\pm$ 1.40
-0.225	24.96 $\pm$ 2.20	18.74 $\pm$ 1.53	17.10 $\pm$ 1.72	18.29 $\pm$ 1.51
-0.175	27.60 $\pm$ 2.37	20.36 $\pm$ 1.63	18.47 $\pm$ 1.81	20.21 $\pm$ 1.64
-0.125	26.96 $\pm$ 2.31	22.19 $\pm$ 1.75	20.98 $\pm$ 1.98	23.29 $\pm$ 1.84
-0.075	28.79 $\pm$ 2.43	23.08 $\pm$ 1.80	19.98 $\pm$ 1.90	23.29 $\pm$ 1.83
-0.025	26.88 $\pm$ 2.29	21.20 $\pm$ 1.67	23.68 $\pm$ 2.15	22.97 $\pm$ 1.80
0.025	23.73 $\pm$ 2.03	19.21 $\pm$ 1.49	17.60 $\pm$ 1.67	20.81 $\pm$ 1.63
0.075	23.08 $\pm$ 1.99	19.66 $\pm$ 1.52	19.93 $\pm$ 1.85	21.05 $\pm$ 1.64
0.125	25.50 $\pm$ 2.16	19.57 $\pm$ 1.51	19.42 $\pm$ 1.83	20.77 $\pm$ 1.62
0.175	25.02 $\pm$ 2.16	18.87 $\pm$ 1.47	17.54 $\pm$ 1.73	19.91 $\pm$ 1.57
0.225	24.68 $\pm$ 2.18	15.69 $\pm$ 1.28	15.43 $\pm$ 1.64	18.18 $\pm$ 1.48
0.275	21.04 $\pm$ 2.04	14.30 $\pm$ 1.23	15.57 $\pm$ 1.73	15.41 $\pm$ 1.34
0.325	27.86 $\pm$ 2.70	16.41 $\pm$ 1.44	19.21 $\pm$ 2.18	17.81 $\pm$ 1.57
0.375	38.24 $\pm$ 3.62	30.33 $\pm$ 2.46	26.33 $\pm$ 2.90	28.31 $\pm$ 2.38
0.425	74.17 $\pm$ 6.50	57.93 $\pm$ 4.40	63.31 $\pm$ 5.90	59.77 $\pm$ 4.62
0.475	137.96 $\pm$ 11.39	119.68 $\pm$ 8.64	126.52 $\pm$ 11.07	120.86 $\pm$ 8.78

Table 5.7: Differential Cross Sections for $\pi^0\eta$ from 2.985 to 3.005 GeV.

$d\sigma/d\Omega(nb/sr)$ vs. π^0 production angle ($\cos\theta_{\bar{p}\pi}^*$) for $\pi^0\eta$				
	$\sqrt{s}$ (GeV)			
$\cos\theta_{\bar{p}\pi}^*$	2.985	2.990	2.994	3.005
-0.525	196.35 $\pm$ 14.78	178.19 $\pm$ 12.22	189.36 $\pm$ 13.64	161.40 $\pm$ 11.71
-0.475	105.16 $\pm$ 8.00	98.06 $\pm$ 6.77	106.04 $\pm$ 7.68	82.88 $\pm$ 6.16
-0.425	49.16 $\pm$ 4.10	47.59 $\pm$ 3.45	47.41 $\pm$ 3.69	41.61 $\pm$ 3.32
-0.375	23.67 $\pm$ 2.25	23.06 $\pm$ 1.81	24.37 $\pm$ 2.08	19.44 $\pm$ 1.76
-0.325	13.38 $\pm$ 1.43	13.04 $\pm$ 1.11	11.49 $\pm$ 1.14	10.85 $\pm$ 1.11
-0.275	15.04 $\pm$ 1.51	12.32 $\pm$ 1.05	13.19 $\pm$ 1.24	12.16 $\pm$ 1.18
-0.225	18.72 $\pm$ 1.76	15.49 $\pm$ 1.26	17.10 $\pm$ 1.50	14.33 $\pm$ 1.32
-0.175	20.48 $\pm$ 1.88	19.03 $\pm$ 1.50	20.95 $\pm$ 1.76	16.85 $\pm$ 1.49
-0.125	22.02 $\pm$ 1.98	20.10 $\pm$ 1.56	23.64 $\pm$ 1.94	18.50 $\pm$ 1.59
-0.075	23.70 $\pm$ 2.09	19.92 $\pm$ 1.54	23.05 $\pm$ 1.89	18.15 $\pm$ 1.57
-0.025	24.98 $\pm$ 2.17	20.12 $\pm$ 1.55	22.35 $\pm$ 1.84	19.99 $\pm$ 1.68
0.025	21.13 $\pm$ 1.84	16.25 $\pm$ 1.25	17.96 $\pm$ 1.48	15.95 $\pm$ 1.35
0.075	20.16 $\pm$ 1.78	16.67 $\pm$ 1.28	17.94 $\pm$ 1.48	17.29 $\pm$ 1.45
0.125	19.34 $\pm$ 1.73	17.13 $\pm$ 1.31	19.25 $\pm$ 1.57	16.31 $\pm$ 1.39
0.175	18.12 $\pm$ 1.68	16.83 $\pm$ 1.29	17.45 $\pm$ 1.47	15.61 $\pm$ 1.36
0.225	17.34 $\pm$ 1.67	14.39 $\pm$ 1.15	17.12 $\pm$ 1.47	13.40 $\pm$ 1.24
0.275	17.09 $\pm$ 1.73	12.33 $\pm$ 1.04	13.42 $\pm$ 1.27	12.13 $\pm$ 1.21
0.325	16.00 $\pm$ 1.77	15.21 $\pm$ 1.29	15.13 $\pm$ 1.47	13.61 $\pm$ 1.39
0.375	31.63 $\pm$ 3.11	24.65 $\pm$ 1.99	26.60 $\pm$ 2.42	23.18 $\pm$ 2.21
0.425	56.28 $\pm$ 5.13	49.25 $\pm$ 3.65	59.16 $\pm$ 4.78	44.70 $\pm$ 3.82
0.475	131.84 $\pm$ 10.77	110.15 $\pm$ 7.71	102.35 $\pm$ 7.83	98.50 $\pm$ 7.70

Table 5.8: Differential Cross Sections for $\pi^0\eta$ from 3.098 to 3.525 GeV.

$d\sigma/d\Omega(nb/sr)$ vs. π^0 production angle ($\cos\theta_{\bar{p}\pi}^*$) for $\pi^0\eta$				
	$\sqrt{s}$ (GeV)			
$\cos\theta_{\bar{p}\pi}^*$	3.098 3.250	3.523	3.525	
-0.675	0.00 $\pm$ 0.00	0.00 $\pm$ 0.00	16.22 $\pm$ 1.20	16.33 $\pm$ 1.80
-0.625	0.00 $\pm$ 0.00	43.20 $\pm$ 12.77	11.45 $\pm$ 0.74	12.25 $\pm$ 1.15
-0.575	148.15 $\pm$ 10.43	43.04 $\pm$ 8.32	4.90 $\pm$ 0.37	5.25 $\pm$ 0.58
-0.525	98.85 $\pm$ 6.84	15.34 $\pm$ 3.85	1.81 $\pm$ 0.18	1.60 $\pm$ 0.26
-0.475	48.47 $\pm$ 3.48	8.76 $\pm$ 2.50	0.68 $\pm$ 0.10	0.83 $\pm$ 0.18
-0.425	20.30 $\pm$ 1.59	0.58 $\pm$ 0.59	0.77 $\pm$ 0.11	0.59 $\pm$ 0.14
-0.375	8.62 $\pm$ 0.78	1.62 $\pm$ 0.94	0.83 $\pm$ 0.11	0.82 $\pm$ 0.17
-0.325	6.13 $\pm$ 0.59	5.27 $\pm$ 1.70	0.94 $\pm$ 0.12	1.08 $\pm$ 0.20
-0.275	7.32 $\pm$ 0.67	3.58 $\pm$ 1.37	0.91 $\pm$ 0.12	0.82 $\pm$ 0.17
-0.225	10.64 $\pm$ 0.90	4.06 $\pm$ 1.46	0.53 $\pm$ 0.09	0.50 $\pm$ 0.13
-0.175	11.33 $\pm$ 0.94	1.50 $\pm$ 0.87	0.38 $\pm$ 0.07	0.36 $\pm$ 0.11
-0.125	10.45 $\pm$ 0.88	2.01 $\pm$ 1.01	0.26 $\pm$ 0.06	0.23 $\pm$ 0.09
-0.075	11.08 $\pm$ 0.92	1.98 $\pm$ 1.00	0.27 $\pm$ 0.06	0.20 $\pm$ 0.08
-0.025	11.37 $\pm$ 0.93	0.99 $\pm$ 0.70	0.08 $\pm$ 0.03	0.23 $\pm$ 0.09
0.025	9.55 $\pm$ 0.78	1.39 $\pm$ 0.81	0.10 $\pm$ 0.03	0.05 $\pm$ 0.03
0.075	9.79 $\pm$ 0.80	0.47 $\pm$ 0.47	0.18 $\pm$ 0.04	0.12 $\pm$ 0.05
0.125	10.94 $\pm$ 0.88	1.91 $\pm$ 0.96	0.31 $\pm$ 0.06	0.22 $\pm$ 0.08
0.175	11.08 $\pm$ 0.89	1.51 $\pm$ 0.88	0.39 $\pm$ 0.07	0.35 $\pm$ 0.10
0.225	9.76 $\pm$ 0.81	3.21 $\pm$ 1.33	0.60 $\pm$ 0.09	0.35 $\pm$ 0.10
0.275	8.29 $\pm$ 0.73	2.85 $\pm$ 1.29	0.65 $\pm$ 0.09	0.64 $\pm$ 0.13
0.325	7.61 $\pm$ 0.71	2.47 $\pm$ 1.24	0.63 $\pm$ 0.09	0.83 $\pm$ 0.16
0.375	10.36 $\pm$ 0.94	2.91 $\pm$ 1.47	0.78 $\pm$ 0.10	0.52 $\pm$ 0.13
0.425	26.00 $\pm$ 2.01	5.45 $\pm$ 2.25	0.71 $\pm$ 0.10	0.51 $\pm$ 0.13
0.475	56.68 $\pm$ 4.06	5.83 $\pm$ 2.63	0.92 $\pm$ 0.13	0.56 $\pm$ 0.14
0.525	123.92 $\pm$ 8.46	24.27 $\pm$ 6.64	1.69 $\pm$ 0.19	1.59 $\pm$ 0.27
0.575	0.00 $\pm$ 0.00	28.88 $\pm$ 9.29	4.93 $\pm$ 0.40	5.11 $\pm$ 0.60
0.625	0.00 $\pm$ 0.00	0.00 $\pm$ 0.00	12.91 $\pm$ 0.89	14.66 $\pm$ 1.40
0.675	0.00 $\pm$ 0.00	0.00 $\pm$ 0.00	29.28 $\pm$ 2.11	25.59 $\pm$ 2.74

Table 5.9: Differential Cross Sections for $\pi^0\eta$ from 3.526 to 3.594 GeV.

$d\sigma/d\Omega(nb/sr)$ vs. π^0 production angle ($\cos\theta_{\bar{p}\pi}^*$) for $\pi^0\eta$				
	$\sqrt{s}$ (GeV)			
$\cos\theta_{\bar{p}\pi}^*$	3.526	3.527	3.591	3.594
-0.675	18.69 ± 0.60	21.09 ± 1.54	12.96 ± 1.17	11.95 ± 1.15
-0.625	11.92 ± 0.35	13.65 ± 0.92	8.72 ± 0.76	6.22 ± 0.62
-0.575	4.85 ± 0.17	5.92 ± 0.44	3.29 ± 0.35	3.35 ± 0.38
-0.525	1.86 ± 0.08	2.25 ± 0.21	1.24 ± 0.18	1.25 ± 0.19
-0.475	0.80 ± 0.05	0.89 ± 0.12	0.86 ± 0.14	0.37 ± 0.10
-0.425	0.86 ± 0.05	0.78 ± 0.11	0.71 ± 0.13	0.65 ± 0.13
-0.375	1.13 ± 0.06	0.96 ± 0.12	0.60 ± 0.11	0.45 ± 0.10
-0.325	0.95 ± 0.05	1.01 ± 0.12	0.73 ± 0.13	0.85 ± 0.15
-0.275	0.90 ± 0.05	1.07 ± 0.13	0.75 ± 0.13	0.74 ± 0.14
-0.225	0.83 ± 0.05	0.98 ± 0.12	0.48 ± 0.10	0.41 ± 0.10
-0.175	0.51 ± 0.04	0.48 ± 0.08	0.27 ± 0.07	0.35 ± 0.09
-0.125	0.32 ± 0.03	0.44 ± 0.08	0.20 ± 0.06	0.15 ± 0.06
-0.075	0.24 ± 0.02	0.35 ± 0.07	0.08 ± 0.04	0.16 ± 0.06
-0.025	0.23 ± 0.02	0.21 ± 0.05	0.29 ± 0.08	0.27 ± 0.08
0.025	0.16 ± 0.02	0.14 ± 0.04	0.23 ± 0.06	0.13 ± 0.05
0.075	0.22 ± 0.02	0.23 ± 0.05	0.19 ± 0.06	0.20 ± 0.06
0.125	0.30 ± 0.02	0.35 ± 0.06	0.17 ± 0.05	0.15 ± 0.05
0.175	0.38 ± 0.03	0.40 ± 0.06	0.27 ± 0.07	0.34 ± 0.08
0.225	0.53 ± 0.03	0.49 ± 0.07	0.30 ± 0.07	0.36 ± 0.09
0.275	0.70 ± 0.04	0.78 ± 0.10	0.47 ± 0.09	0.31 ± 0.08
0.325	0.76 ± 0.04	0.93 ± 0.11	0.66 ± 0.11	0.41 ± 0.09
0.375	0.76 ± 0.04	0.78 ± 0.10	0.83 ± 0.13	0.55 ± 0.11
0.425	0.70 ± 0.04	0.91 ± 0.11	0.50 ± 0.10	0.49 ± 0.11
0.475	0.85 ± 0.05	0.96 ± 0.12	0.89 ± 0.14	0.42 ± 0.10
0.525	2.21 ± 0.10	1.82 ± 0.20	1.41 ± 0.20	1.15 ± 0.19
0.575	5.78 ± 0.20	6.29 ± 0.48	3.90 ± 0.41	3.26 ± 0.38
0.625	13.77 ± 0.41	16.96 ± 1.12	9.04 ± 0.80	8.03 ± 0.78
0.675	35.46 ± 1.05	41.65 ± 2.76	22.46 ± 1.93	19.79 ± 1.84

Table 5.10: Differential Cross Sections for $\pi^0\eta$ from 3.613 to 3.621 GeV.

$d\sigma/d\Omega(nb/sr)$ vs. π^0 production angle ($\cos\theta_{p\pi}^*$) for $\pi^0\eta$				
	$\sqrt{s}$ (GeV)			
$\cos\theta_{p\pi}^*$	3.613	3.616	3.619	3.621
-0.675	17.76 ± 1.48	13.02 ± 1.12	12.16 ± 1.28	13.65 ± 1.12
-0.625	8.15 ± 0.71	6.92 ± 0.62	6.62 ± 0.72	7.13 ± 0.61
-0.575	3.17 ± 0.34	3.01 ± 0.32	2.10 ± 0.32	3.02 ± 0.31
-0.525	1.14 ± 0.17	0.85 ± 0.14	0.78 ± 0.18	0.93 ± 0.14
-0.475	0.95 ± 0.15	0.59 ± 0.11	0.82 ± 0.18	0.48 ± 0.09
-0.425	0.72 ± 0.13	0.59 ± 0.11	0.92 ± 0.19	0.65 ± 0.11
-0.375	0.90 ± 0.14	0.93 ± 0.14	0.68 ± 0.16	0.89 ± 0.13
-0.325	0.82 ± 0.13	0.67 ± 0.12	0.71 ± 0.16	0.78 ± 0.12
-0.275	0.66 ± 0.12	0.28 ± 0.07	0.54 ± 0.15	0.65 ± 0.11
-0.225	0.55 ± 0.11	0.43 ± 0.09	0.61 ± 0.15	0.43 ± 0.09
-0.175	0.42 ± 0.09	0.23 ± 0.07	0.35 ± 0.11	0.38 ± 0.08
-0.125	0.44 ± 0.10	0.24 ± 0.07	0.30 ± 0.10	0.26 ± 0.07
-0.075	0.10 ± 0.05	0.31 ± 0.08	0.14 ± 0.07	0.33 ± 0.07
-0.025	0.25 ± 0.07	0.12 ± 0.05	0.13 ± 0.07	0.19 ± 0.06
0.025	0.16 ± 0.05	0.15 ± 0.05	0.13 ± 0.06	0.19 ± 0.05
0.075	0.21 ± 0.06	0.19 ± 0.05	0.23 ± 0.08	0.25 ± 0.06
0.125	0.29 ± 0.07	0.20 ± 0.05	0.15 ± 0.06	0.17 ± 0.05
0.175	0.22 ± 0.06	0.18 ± 0.05	0.28 ± 0.09	0.18 ± 0.05
0.225	0.26 ± 0.06	0.30 ± 0.07	0.21 ± 0.07	0.41 ± 0.08
0.275	0.51 ± 0.09	0.39 ± 0.08	0.52 ± 0.12	0.46 ± 0.08
0.325	0.72 ± 0.11	0.48 ± 0.09	0.58 ± 0.13	0.49 ± 0.08
0.375	0.68 ± 0.11	0.49 ± 0.09	0.53 ± 0.12	0.45 ± 0.08
0.425	0.47 ± 0.09	0.48 ± 0.09	0.52 ± 0.13	0.61 ± 0.10
0.475	0.67 ± 0.12	0.54 ± 0.10	0.43 ± 0.12	0.66 ± 0.11
0.525	1.37 ± 0.19	0.96 ± 0.15	1.01 ± 0.20	0.99 ± 0.14
0.575	3.59 ± 0.37	2.73 ± 0.30	3.17 ± 0.42	3.29 ± 0.33
0.625	7.84 ± 0.69	6.22 ± 0.58	5.97 ± 0.71	7.39 ± 0.64
0.675	21.90 ± 1.80	14.87 ± 1.31	13.38 ± 1.47	17.17 ± 1.39

Table 5.11: Differential Cross Sections for $\eta\eta$ from 2.950 to 2.981 GeV.

$d\sigma/d\Omega(nb/sr)$ vs. η production angle ($\cos\theta_{\bar{p}\eta}^*$) for $\eta\eta$				
	$\sqrt{s}$ (GeV)			
$\cos\theta_{\bar{p}\eta}^*$	2.950	2.975	2.979	2.981
0.025	12.22 $\pm$ 1.71	10.95 $\pm$ 1.26	9.20 $\pm$ 1.52	8.52 $\pm$ 1.07
0.075	14.25 $\pm$ 1.91	11.24 $\pm$ 1.29	11.17 $\pm$ 1.74	10.96 $\pm$ 1.30
0.125	15.98 $\pm$ 2.11	12.87 $\pm$ 1.46	11.15 $\pm$ 1.79	12.44 $\pm$ 1.45
0.175	23.84 $\pm$ 2.86	15.45 $\pm$ 1.69	12.92 $\pm$ 2.02	15.00 $\pm$ 1.69
0.225	23.72 $\pm$ 2.94	18.64 $\pm$ 2.01	16.70 $\pm$ 2.50	13.81 $\pm$ 1.64
0.275	26.79 $\pm$ 3.38	17.84 $\pm$ 2.02	15.02 $\pm$ 2.45	16.83 $\pm$ 1.98
0.325	33.37 $\pm$ 4.20	27.37 $\pm$ 2.93	17.51 $\pm$ 2.91	24.72 $\pm$ 2.80
0.375	58.35 $\pm$ 6.85	39.00 $\pm$ 4.09	43.81 $\pm$ 5.84	43.78 $\pm$ 4.52
0.425	73.12 $\pm$ 8.79	69.45 $\pm$ 6.77	67.85 $\pm$ 8.65	85.24 $\pm$ 8.16
0.475	122.06 $\pm$ 14.03	115.97 $\pm$ 11.33	107.18 $\pm$ 13.37	136.77 $\pm$ 13.12

Table 5.12: Differential Cross Sections for $\eta\eta$ from 2.985 to 3.005 GeV.

$d\sigma/d\Omega(nb/sr)$ vs. η production angle ($\cos\theta_{\bar{p}\eta}^*$) for $\eta\eta$				
	$\sqrt{s}$ (GeV)			
$\cos\theta_{\bar{p}\eta}^*$	2.985	2.990	2.994	3.005
0.025	9.58 $\pm$ 1.47	8.58 $\pm$ 1.00	11.30 $\pm$ 1.43	8.79 $\pm$ 1.22
0.075	11.60 $\pm$ 1.68	10.47 $\pm$ 1.16	11.70 $\pm$ 1.47	7.36 $\pm$ 1.10
0.125	15.51 $\pm$ 2.09	13.08 $\pm$ 1.40	13.35 $\pm$ 1.65	8.10 $\pm$ 1.20
0.175	16.52 $\pm$ 2.24	10.36 $\pm$ 1.20	14.51 $\pm$ 1.80	11.02 $\pm$ 1.51
0.225	16.81 $\pm$ 2.35	14.01 $\pm$ 1.53	11.31 $\pm$ 1.56	10.67 $\pm$ 1.52
0.275	18.35 $\pm$ 2.61	15.61 $\pm$ 1.73	13.93 $\pm$ 1.91	11.16 $\pm$ 1.65
0.325	25.69 $\pm$ 3.49	21.59 $\pm$ 2.33	26.72 $\pm$ 3.19	18.26 $\pm$ 2.46
0.375	46.37 $\pm$ 5.83	42.12 $\pm$ 4.14	41.30 $\pm$ 4.63	32.53 $\pm$ 3.92
0.425	80.71 $\pm$ 9.02	63.85 $\pm$ 6.09	74.00 $\pm$ 7.67	58.31 $\pm$ 6.51
0.475	152.63 $\pm$ 16.65	125.78 $\pm$ 11.37	130.61 $\pm$ 13.19	117.51 $\pm$ 12.14

Table 5.13: Differential Cross Sections for $\eta\eta$ from 3.098 to 3.525 GeV.

$d\sigma/d\Omega(nb/sr)$ vs. η production angle ($\cos\theta_{\bar{p}\eta}^*$) for $\eta\eta$				
	$\sqrt{s}$ (GeV)			
$\cos\theta_{\bar{p}\eta}^*$	3.098	3.250	3.523	3.525
0.025	4.58 ± 0.61	0.00 ± 0.00	0.21 ± 0.07	0.26 ± 0.13
0.075	4.59 ± 0.62	1.05 ± 1.05	0.24 ± 0.08	0.65 ± 0.21
0.125	4.22 ± 0.60	0.00 ± 0.00	0.59 ± 0.13	0.49 ± 0.18
0.175	4.99 ± 0.68	0.00 ± 0.00	0.47 ± 0.11	1.20 ± 0.30
0.225	4.45 ± 0.65	0.00 ± 0.00	0.78 ± 0.15	0.64 ± 0.21
0.275	5.25 ± 0.76	1.30 ± 1.31	0.38 ± 0.10	0.54 ± 0.20
0.325	10.02 ± 1.21	1.42 ± 1.43	0.33 ± 0.11	0.41 ± 0.17
0.375	28.49 ± 2.77	1.64 ± 1.64	0.59 ± 0.14	0.59 ± 0.21
0.425	49.63 ± 4.54	6.08 ± 3.54	0.86 ± 0.18	0.88 ± 0.27
0.475	84.01 ± 7.40	11.47 ± 5.80	1.63 ± 0.28	1.32 ± 0.36
0.525	126.96 ± 11.29	14.43 ± 7.29	2.28 ± 0.32	2.84 ± 0.52
0.575	0.00 ± 0.00	34.89 ± 14.47	3.02 ± 0.43	4.35 ± 0.76
0.625	0.00 ± 0.00	0.00 ± 0.00	6.96 ± 0.85	6.86 ± 1.23

Table 5.14: Differential Cross Sections for $\eta\eta$ from 3.526 to 3.594 GeV.

$d\sigma/d\Omega(nb/sr)$ vs. η production angle ($\cos\theta_{\bar{p}\eta}^*$) for $\eta\eta$				
	$\sqrt{s}$ (GeV)			
$\cos\theta_{\bar{p}\eta}^*$	3.526	3.527	3.591	3.594
0.025	0.34 ± 0.04	0.49 ± 0.11	0.30 ± 0.11	0.31 ± 0.12
0.075	0.49 ± 0.05	0.42 ± 0.10	0.46 ± 0.14	0.41 ± 0.14
0.125	0.59 ± 0.06	0.77 ± 0.14	0.52 ± 0.14	0.22 ± 0.10
0.175	0.74 ± 0.06	0.65 ± 0.13	0.64 ± 0.16	0.61 ± 0.18
0.225	0.67 ± 0.06	0.76 ± 0.14	0.67 ± 0.17	0.63 ± 0.18
0.275	0.64 ± 0.06	0.48 ± 0.11	0.15 ± 0.08	0.43 ± 0.15
0.325	0.60 ± 0.06	0.41 ± 0.10	0.32 ± 0.12	0.35 ± 0.13
0.375	0.82 ± 0.07	0.98 ± 0.17	0.40 ± 0.13	0.47 ± 0.16
0.425	1.10 ± 0.09	1.12 ± 0.19	0.36 ± 0.13	0.57 ± 0.19
0.475	1.94 ± 0.13	2.09 ± 0.30	1.29 ± 0.27	1.07 ± 0.26
0.525	2.64 ± 0.15	3.10 ± 0.36	1.60 ± 0.30	1.58 ± 0.32
0.575	4.53 ± 0.23	4.39 ± 0.51	3.28 ± 0.52	4.42 ± 0.66
0.625	7.40 ± 0.37	10.50 ± 1.05	5.47 ± 0.82	4.44 ± 0.75
0.675	0.00 ± 0.00	0.00 ± 0.00	10.55 ± 1.75	7.64 ± 1.48

Table 5.15: Differential Cross Sections for $\eta\eta$ from 3.613 to 3.621 GeV.

$d\sigma/d\Omega(nb/sr)$ vs. η production angle ($\cos\theta_{\bar{p}\eta}^*$) for $\eta\eta$				
	$\sqrt{s}$ (GeV)			
$\cos\theta_{\bar{p}\eta}^*$	3.613	3.616	3.619	3.621
0.025	0.53 ± 0.14	0.29 ± 0.10	0.35 ± 0.16	0.32 ± 0.10
0.075	0.27 ± 0.10	0.34 ± 0.11	0.38 ± 0.16	0.45 ± 0.12
0.125	0.26 ± 0.10	0.49 ± 0.14	0.20 ± 0.12	0.42 ± 0.12
0.175	0.47 ± 0.14	0.25 ± 0.10	0.34 ± 0.16	0.55 ± 0.14
0.225	0.46 ± 0.14	0.37 ± 0.12	0.70 ± 0.23	0.50 ± 0.13
0.275	0.54 ± 0.15	0.18 ± 0.08	0.54 ± 0.20	0.37 ± 0.11
0.325	0.46 ± 0.14	0.25 ± 0.10	0.51 ± 0.20	0.43 ± 0.12
0.375	0.36 ± 0.12	0.32 ± 0.11	0.30 ± 0.15	0.44 ± 0.13
0.425	0.97 ± 0.22	0.44 ± 0.14	0.48 ± 0.20	0.51 ± 0.14
0.475	0.87 ± 0.22	0.87 ± 0.21	1.16 ± 0.33	0.75 ± 0.18
0.525	1.93 ± 0.32	0.80 ± 0.19	1.27 ± 0.32	1.44 ± 0.25
0.575	2.90 ± 0.45	1.91 ± 0.34	1.56 ± 0.41	2.68 ± 0.40
0.625	5.83 ± 0.82	3.33 ± 0.56	4.79 ± 0.90	4.30 ± 0.62
0.675	8.45 ± 1.33	5.74 ± 1.08	7.51 ± 1.57	8.42 ± 1.20

Figure 5.1: Production angular distributions for $\bar{p}p \rightarrow \pi^0\pi^0$ at 2.950, 2.975, 2.979, and 2.981 GeV. Each data point is centered on bins of width 0.05.

Figure 5.2: Production angular distributions for $\bar{p}p \rightarrow \pi^0\pi^0$ at 2.985, 2.990, 2.994, and 3.005 GeV. Each data point is centered on bins of width 0.05.

Figure 5.3: Production angular distributions for $\bar{p}p \rightarrow \pi^0\pi^0$ at 3.098, 3.250, 3.523, and 3.525 GeV. Each data point is centered on bins of width 0.05.

Figure 5.4: Production angular distributions for $\bar{p}p \rightarrow \pi^0\pi^0$ at 3.526, 3.527, 3.591, and 3.594 GeV. Each data point is centered on bins of width 0.05.

Figure 5.5: Production angular distributions for $\bar{p}p \rightarrow \pi^0\pi^0$ at 3.613, 3.616, 3.619, and 3.621 GeV. Each data point is centered on bins of width 0.05.

Figure 5.6: Production angular distributions for $\bar{p}p \rightarrow \pi^0\eta$ at 2.950, 2.975, 2.979, and 2.981 GeV. Each data point is centered on bins of width 0.05.

Figure 5.7: Production angular distributions for $\bar{p}p \rightarrow \pi^0\eta$ at 2.985, 2.990, 2.994, and 3.005 GeV. Each data point is centered on bins of width 0.05.

Figure 5.8: Production angular distributions for $\bar{p}p \rightarrow \pi^0\eta$ at 3.098, 3.250, 3.523, and 3.525 GeV. Each data point is centered on bins of width 0.05.

Figure 5.9: Production angular distributions for $\bar{p}p \rightarrow \pi^0 \eta$ at 3.526, 3.527, 3.591, and 3.594 GeV. Each data point is centered on bins of width 0.05.

Figure 5.10: Production angular distributions for $\bar{p}p \rightarrow \pi^0\eta$ at 3.613, 3.616, 3.619, and 3.621 GeV. Each data point is centered on bins of width 0.05.

Figure 5.11: Production angular distributions for $\bar{p}p \rightarrow \eta\eta$ at 2.950, 2.975, 2.979, and 2.981 GeV. Each data point is centered on bins of width 0.05.

Figure 5.12: Production angular distributions for $\bar{p}p \rightarrow \eta\eta$ at 2.985, 2.990, 2.994, and 3.005 GeV. Each data point is centered on bins of width 0.05.

Figure 5.13: Production angular distributions for $\bar{p}p \rightarrow \eta\eta$ at 3.098, 3.250, 3.523, and 3.525 GeV. Each data point is centered on bins of width 0.05.

Figure 5.14: Production angular distributions for $\bar{p}p \rightarrow \eta\eta$ at 3.526, 3.527, 3.591, and 3.594 GeV. Each data point is centered on bins of width 0.05.

Figure 5.15: Production angular distributions for $\bar{p}p \rightarrow \eta\eta$ at 3.613, 3.616, 3.619, and 3.621 GeV. Each data point is centered on bins of width 0.05.

that between the π^0 and $\bar{p}p$ axis in the center-of-mass frame. One can compare figures 5.6 through 5.10 with figure 4.25 to get an idea of effects of acceptance corrections.

All plots have the general feature of a strong forward-peak. This dramatic increase in cross section is a function of $\sqrt{s}$ and starts at about $\cos \theta_{\bar{p}M}^* = 0.25$ at $\sqrt{s} = 2.950$ GeV, and $\cos \theta_{\bar{p}M}^* = 0.4$ at $\sqrt{s} = 3.526$ GeV. Symmetric production, i.e., particles occurring nearer to $\cos \theta_{\bar{p}M}^* = 0$, shows several different features. For $\pi^0\pi^0$, there is a clear enhancement of events at large center-of-mass angles for $\sqrt{s} \leq 3.250$ GeV that all but disappears at higher energies. For the $\eta\eta$ channel there is no such structure for the entire range of energies presented. For $\pi^0\eta$ there is an enhancement in the large angle region up to $\sqrt{s} = 3.250$ GeV. For the higher energies the distribution has a minimum in the same region. There is also an indication of second minima occurring at around $\cos \theta_{\bar{p}M}^* = +0.5$ and -0.5 . The structure in these distributions is indicative of Brodsky scaling law violations. That is, if dimensional counting rules hold true for a range of $\cos \theta_{\bar{p}M}^*$, then the differential cross section for that range should scale accordingly. For $\pi^0\pi^0$, the structure produces a “dip” in angular distributions at $\cos \theta_{\bar{p}\pi}^* \simeq 0.25$, or $\theta_{\bar{p}\pi}^* = 75^\circ$. Thus, from angular distributions alone, one sees limits to dimensional counting rules.

The angular distributions for $\pi\eta$ should be symmetric about $\cos \theta_{\bar{p}\pi}^* = 0.0$ due to CP invariance. (“C” is the charge conjugation operator and “P” is the parity operator.) To study this aspect of the data the difference between the positive (forward) and negative (backward) values of $\cos \theta_{\bar{p}\pi}^*$ are divided by the sum, $(\sigma_+ - \sigma_-)/(\sigma_+ + \sigma_-)$. These data are shown in figures 5.16 through 5.20. These plots show a systematic nonuniformity, ranging from approximately -10% near $\cos \theta_{\bar{p}\pi}^* = 0$ to $+10\%$ near the larger values of $\cos \theta_{\bar{p}\pi}^*$. The origin of this nonuniformity is unknown, but is presumably instrumental. However, results quoted in this chapter are not sensitive to these $\pm 10\%$ systematics. Such effects, presumably, may also exist in the $\pi\pi$ and $\eta\eta$ channels.

Figure 5.16: Asymmetry plot for $\bar{p}p \rightarrow \pi\eta$. σ_+ and σ_- refer to the cross section, $d\sigma/d\Omega$, for positive and the corresponding negative values of $\cos \theta_{p\pi}^*$. The plots are at energies 2.950, 2.975, 2.979, and 2.981 GeV.

Figure 5.17: Asymmetry plot for $\bar{p}p \rightarrow \pi\eta$. σ_+ and σ_- refer to the cross section, $d\sigma/d\Omega$, for positive and the corresponding negative values of $\cos \theta_{\bar{p}\pi}^*$. The plots are at energies 2.985, 2.990, 2.994, and 3.005 GeV.

Figure 5.18: Asymmetry plot for $\bar{p}p \rightarrow \pi\eta$. σ_+ and σ_- refer to the cross section, $d\sigma/d\Omega$, for positive and the corresponding negative values of $\cos \theta_{\bar{p}\pi}^*$. The plots are at energies 3.098, 3.250, 3.523, and 3.525 GeV.

Figure 5.19: Asymmetry plot for $\bar{p}p \rightarrow \pi\eta$. σ_+ and σ_- refer to the cross section, $d\sigma/d\Omega$, for positive and the corresponding negative values of $\cos \theta_{\bar{p}\pi}^*$. The plots are at energies 3.526, 3.527, 3.591, and 3.594 GeV.

Figure 5.20: Asymmetry plot for $\bar{p}p \rightarrow \pi\eta$. σ_+ and σ_- refer to the cross section, $d\sigma/d\Omega$, for positive and the corresponding negative values of $\cos \theta_{\bar{p}\pi}^*$. The plots are at energies 3.613, 3.616, 3.619, and 3.621 GeV.

To study dimensional counting rules, as well as predictions for oscillatory behavior resulting from interference between Brodsky scattering and Landshoff scattering, differential cross sections for various values of $\cos \theta_{\bar{p}M}^*$ are shown in figure 5.21 through figure 5.26. These plots include all three final states, $\pi^0\pi^0$, $\pi^0\eta$, and $\eta\eta$ and cover the range $|\cos \theta_{\bar{p}M}^*|$ from 0.0 to 0.3, or $\simeq 70^\circ$ - 90° . The plots are limited to this region where the momentum transfer is largest (where small distances are probed), and PQCD works best. Figure 5.21 through figure 5.23 show $\frac{d\sigma}{d\Omega}$ versus $\sqrt{s}$. Figure 5.24 through figure 5.26 show the differential cross sections scaled by $s^8 \cdot 10^{-9}$ versus $\ln(s)$. Thus, an s^{-8} dependent cross section would appear as a constant, energy-independent function.

Predictions for an interference between Brodsky scattering and Landshoff scattering, on the other hand, expect the oscillatory behavior of the scaled differential cross section discussed in chapter 2. The period of the oscillations is predicted for $\pi\pi$ to be about one unit of $\ln(s)$. These data cover about half that range, meaning the full amplitude of such oscillations should be apparent.

The $\pi^0\pi^0$ data, shown in figure 5.24, do, in fact, show a clear deviation from Brodsky scaling, (i.e., deviation from a horizontal line), that is not explained by measurement errors. The errors shown in figure 5.21 through figure 5.26 are statistical and systematic errors added in quadrature. (The statistical error for the $\pi^0\pi^0$ data is $\simeq 5\%$ and the systematic error is $\simeq 5\%$.) The data range from $\sqrt{s} = 2.950$ - 3.621 GeV, or $\ln(s)$ from 2.16 to 2.57. From $\ln(s)=2.1$ to 2.35 the data show a negative slope, and the differential cross sections from $\ln(s)=2.1$ to 2.5 show a strong positive slope. Therefore, the s dependence of the differential cross section has changed in this region. The variation seen is consistent with the predicted oscillation of half-period of $\simeq 0.5$ units in $\ln(s)$. However, since the data do not range over a full wavelength, there is no conclusive evidence for oscillations, i.e, no two maxima in the cross section.

For the remaining two channels, $\pi^0\eta$ and $\eta\eta$ (figures 5.25 and 5.26), the data show strong deviations from Brodsky scaling. For the $\pi\eta$ channel, there is also a significant change in

Figure 5.21: Differential cross sections as a function of energy for $\pi^0\pi^0$.

Figure 5.22: Differential cross sections as a function of energy for $\pi^0\eta$.

Figure 5.23: Differential cross sections as a function of energy for $\eta\eta$.

Figure 5.24: Scaled differential cross sections as a function of energy for $\pi^0 \pi^0$.

Figure 5.25: Scaled differential cross sections as a function of energy for $\pi^0 \eta$.

Figure 5.26: Scaled differential cross sections as a function of energy for $\eta\eta$. The data points with the horizontal line and vertical arrow correspond to a region in the angular distribution where there are no events. The horizontal line is placed where the cross section would be for one event, and the arrow represents an upper limit.

the s dependence. However, the data are not inconsistent with a half period seen between $\ln(s)$ from 2.2 to 2.6. Recalling chapter 2, the predictions for oscillations allow for suppression from higher Fock states. The $\eta\eta$ channel also shows a significant change in scaling. For the range of data studied, the $\pi\pi$ and $\eta\eta$ channels have similar features. Namely, a significant change in scaling, and a minimum in the scaled differential cross section at around $\ln(s) = 2.3$. For the $\pi\eta$ channel this is not the case. It has been proposed by F. Myhrer [16] that this difference may be due to the fact that the $\pi\eta$ channel has isospin one ($I=1$), whereas $\pi^0\pi^0$ and $\eta\eta$ have $I=0$. The different isospin may result in a different period and phase of the oscillations.

Predictions for oscillations in the scaled differential cross section for $\pi^0\pi^0$ come from Carlson *et al* [13]. They consider data from the charged pion channel, $\bar{p}p \rightarrow \pi^+\pi^-$. These data are from Eisenhandler *et al*, and Buran *et al* [48,49], and are shown in figure 5.27. The plot is taken from reference [13], where the data points for Buran *et al* come from an interpolation of their data to the 90° center-of-mass region. For the $\pi^0\pi^0$ channel there is a limited amount of data from other experiments [50,51]. These data are shown in figures 5.28 and 5.29 along with those from E760 (repeated from figure 5.24). For figures 5.27, 5.28 and 5.29 the data are for $\cos\theta_{\bar{p}M}^*$ in the 90° region. The data from Dulude *et al* are taken at lower energies than the other two data sets. These data all have relatively large values of $s^8 \frac{d\sigma}{dt}$. While each data point carries a relatively large error, the combined data-set shows values that are larger than those seen at higher energies. The other two data sets, for E760 and from Baglin *et al*, cover the same region of $\ln(s)$. The data from Baglin *et al* are consistently lower than those from E760; however, the trends in the data are closely matched. That is, the structure seen by both is large in comparison to the differences.

Taken as a whole, the $\pi^0\pi^0$ data show a peak in the same region of $\ln(s)$ as the $\pi^+\pi^-$ at $\ln(s) = 1.65$. For a period of one unit of $\ln(s)$, the next peak should occur at around $\ln(s) = 2.65$. The $\pi^0\pi^0$ data extend to about $\ln(s) = 2.59$, and show increasing values after

Figure 5.27: Scaled differential cross sections as a function of energy for $\bar{p}p \rightarrow \pi^+\pi^-$. Data from Eisenhandler *et al* [48] are from $0 < \cos \theta_{\bar{p}\pi}^* < 0.05$. Data from Buran *et al* [49] are an interpolation of data to 90° .

Figure 5.28: Scaled differential cross sections as a function of energy for $\bar{p}p \rightarrow \pi^0\pi^0$ for this experiment, Baglin *et al* [51], and Dulude *et al* [50]. Data are from $0 < \cos \theta_{\bar{p}\pi}^* < 0.05$.

Figure 5.29: Enlargement of figure 5.28 in the region $2 < \ln(s) < 3$.

Figure 5.30: Scaled differential cross sections as a function of energy for $\bar{p}p \rightarrow \pi\eta$ for this experiment and Dulude *et al* [51]. Data are from $0 < \cos \theta_{\bar{p}\pi}^* < 0.05$.

$\ln(s) \simeq 2.35$. The characteristics are, again, consistent with the predictions for oscillations.

Figure 5.30 shows $\pi^0\eta$ data from E760 and Dulude *et al* taken at 90° . The Dulude *et al* data, which are at a lower $\sqrt{s}$, show larger values of $s^8 \frac{d\sigma}{dt}$, and are consistent with scaling behavior seen in the E760 data. These data range over approximately one unit of $\ln(s)$, and, in contrast to the $\pi^0\pi^0$ data in figure 5.28, do not show a significant change in scaling. Therefore, from $\ln(s)1.6$ to 2.0 there are no clear indications of interference. This means that if theory is correct in understanding the phenomena underlying reactions, the period and phase of the oscillations for $I=1$ states is different from that seen in $I=0$ states.

As discussed in chapter 2, Brodsky and Teramond have put forth a different explanation for deviations from the dimensional counting rules [17]. They are able to fit cross section data from pp elastic scattering to a model where the scattering amplitude is a sum of the PQCD term, giving dimensional counting rules, and two resonance terms, each having a Breit-Wigner form. Although their model does not address $\bar{p}p \rightarrow \bar{M}M$, Brodsky has said that these ideas should apply to these channels as well [18]. He has pointed out that the stronger structure seen in the $\pi^0\pi^0$ and $\eta\eta$ channels, as compared to the $\pi^0\eta$ channel (i.e., the more significant change in the scaled differential cross section) appears to be associated with states having $I = 0$. According to Brodsky, “it is quite reasonable to associate the $I = 0$ structure with the onset on charm in the intermediate state” [18]. Furthermore, the apparent peak in the scaled differential cross section seen at $\ln(s) \simeq 1.7$ (see figures 5.27, 5.28, and 5.30) may be attributed to intermediate states associated with strange baryons. Thus, if these conjectures are correct, the scaled differential cross sections shown here are consistent with processes described by PQCD, taking into account resonant structure due to heavy quarks.

To summarize these results;

- All three channels show clear deviations from Brodsky scaling;

- The $\pi^0\pi^0$ and $\pi^0\eta$ data show a trend that is consistent with data from other experiments;
- The $\pi^0\pi^0$ data are consistent with predictions for interference between Brodsky scattering and Landshoff scattering, where there is a change in scaling behavior over the energy range studied;
- The structure seen in $\eta\eta$, unlike $\pi\eta$, is similar to that seen in $\pi\pi$. This may be an indication that the phase and period for I=1 states ($\pi\eta$) is different from that of I=0 states ($\pi\pi$ and $\eta\eta$);
- The data appear to be consistent with a model proposed by Brodsky and Tearmond for pp elastic scattering, where deviations from dimensional counting rules are attributed to resonance structures. This model has been applied to pp elastic scattering, but not to $\bar{p}p \rightarrow \bar{M}M$.

Chapter 6

Conclusions

As was demonstrated in chapter 4, the χ^2 method used to analyze these data successfully extracts the signal for exclusive final-state, two-body pseudoscalar mesons reactions, $\pi^0\pi^0$, $\pi^0\eta$, and $\eta\eta$. An η' signal was also shown in figure 4.16. The strong background apparent in this plot makes it difficult to include it in the study of final-state channels (e.g, $\pi^0\eta'$, $\eta\eta'$). With further study it could, perhaps, be included.

From the channels that were analyzed, one sees a clear deviation from the predicted Brodsky scaling of s^{-8} expected for $\bar{p}p \rightarrow \bar{M}M$, which totals 10 constituent quarks. The $\pi^0\pi^0$ data display a change in scaling that is consistent with oscillations predicted from interference between Brodsky scattering and Landshoff scattering. The oscillations are expected to have a period of one unit in $\ln(s)$. The data are consistent with this. Furthermore, the data are consistent with those from previous experiments [48,50,51]. The $\eta\eta$ data do show a change in scaling over the same energy range, and structure similar to the $\pi\pi$ channel. The $\pi^0\eta$ data, while showing deviations from Brodsky scaling, do not show a change in scaling in the range of energies studied. Furthermore, considering that the data from this experiment and that from Dulude *et al* are consistent in trend, for the data to be consistent with interference the phase and period of the oscillations for an I=1 state must be different from an I=0 state.

Although the data for $\pi^0\pi^0$ are consistent with the predictions for oscillations, one needs to see data over a full (predicted) wavelength to convincingly demonstrate interference. Furthermore, the issue of oscillations with reactions involving higher masses could benefit from an increase in the range of measurement. This possibility exists with the E760 data taken. There are data at higher energies, ($\ln(s)= 2.67$ and 2.87), that need further analysis.

If this is successful, presumably a stronger statement could be made on scaling behavior.

The data may also be consistent with a model proposed by Brodsky and Tearmond for pp elastic scattering, where deviations from dimensional counting rules are attributed to resonance structures. The fact that there is less structure in scaled differential cross sections for $\pi^0\eta$ seems to indicate association with an $I = 0$ state. According to Brodsky, the hypothesis that the increasing scaled differential cross section seen in these data at $\ln(s) \simeq 2.4$ is due to the onset of open charm, can be tested by measuring the charm meson production.

Finally, a new data run is expected for this experiment in 1996. One can be hopeful that, in light of these results, additional pursuit of understanding these phenomena will be undertaken.

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