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A SEARCH FOR THE INCLUSIVE OSCILLATIONS OF MUON NEUTRINOS

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Abstract

A search for the inclusive oscillations of ν_μ and ν_μ^c has been performed in the narrow band neutrino beam at Fermi National Accelerator Laboratory. The energy dependence of the interaction rates for charged current neutrino scattering with a muon in the final state have been compared in two massive detectors running simultaneously at different distances from the neutrino source. The data show no positive signal and rule out oscillations of ν_μ and ν_μ^c into any other single type of neutrino for $20 < \Delta m^2 < 900 \text{ eV}^2$ and $\sin^2 2\theta > 0.02$ to 0.20 .

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Theory and Background

§1.1 Introduction

The neutrino is a neutral lepton. At present we recognize three varieties of neutrinos and their conjugate antiparticles. These three neutrinos are associated respectively with the three charged leptons, the electron (e), the muon (μ), and the tau (τ).

The neutrino was first discussed by Pauli¹ as a means to explain the continuous electron energy spectrum observed in neutron beta decay. In his original proposal Pauli recognized that the neutrino (in this case the electron antineutrino ν_e^c) was less massive than the electron and might even be massless. Generally, all neutrinos are assumed to be massless although a compelling theoretical reason for this has not been given. Experimentally, the neutrinos are known to be considerably less massive than their

corresponding charged leptons, ($M_e = 0.5110034 \pm 0.0000014$ MeV, $M_{\nu_e} < 60$ eV² (90% C.L.); $M_\mu = 105.6593 \pm 0.00029$ MeV, $M_{\nu_\mu} < 0.49$ MeV³ (90% C.L.); $M_\tau = 1784.2$ MeV, $M_{\nu_\tau} < 164$ MeV⁴ (95% (C.L.))).

The differentiation among neutrinos according to their association with the charged leptons is made because experimentally, particular neutrinos (antineutrinos) appear to only participate in reactions with the corresponding charged leptons. An example which is taken to imply a distinction between electron neutrinos and anti-electron neutrinos is the chain,

$$n \rightarrow p e^- \nu_e^c \quad (1.1)$$

followed by,

$$\nu_e^c p \rightarrow n e^+ \quad (1.2)$$

The antineutrino from neutron beta decay is not observed to induce the transition⁵,

$$\nu_e^c n \rightarrow p e^- \quad (1.3)$$

The muon neutrino is distinguished by the chain,

$$\pi^+ \rightarrow \mu^+ \nu_\mu \quad (1.4)$$

followed by,

$$\nu_\mu n \rightarrow \mu^- p \quad (1.5)$$

The muon antineutrino results from the negative pion decay and the quasi-elastic process then occurs with neutron and proton interchanged and the negative muon replaced by a positive muon.

These selection rules are formalized in two empirical principles of lepton number conservation⁶. The first principle reflects the differentiation between particles and antiparticles. We associate to these a total lepton "charge", or number,

$$L_{TOTAL} = \pm 1 \text{ particle (antiparticle)} \quad (1.6)$$

which is conserved in all interactions. By this, an interaction which changes a lepton into an antilepton would be forbidden. The second principle differentiates among leptons of different families (ie, electron, muon, and tau) and associates a lepton charge separately to each family. Here the separate lepton numbers,

$$L_e, L_\mu, L_\tau = \pm 1 \text{ particle (antiparticle)} \quad (1.7)$$

are conserved additively in all interactions. Because of this, an interaction involving only two leptons which are of different families would be forbidden.

In 1959 and 1967 Pontecorvo⁷ suggested that some violation of lepton number conservation (either total or separate) might lead to a quantum mechanical mixing among different neutrino states. In this case, for example, the familiar weakly interacting neutrinos (ν_e , ν_μ , and ν_τ), can be superpositions of another set of neutral lepton states. If these other states are both massive and non-degenerate then they each evolve in time with a different phase (see Section 1.3). The result is that the weakly interacting neutrinos would spontaneously transform among each other with harmonic probability. This conjectured process has been called neutrino oscillations. Its existence would indicate both lepton number nonconservation as well as nonzero neutrino mass. Although such a phenomenon would be very interesting in its own right, the implications of neutrino

mass and lepton number non-conservation are relevant to physics in a more general way. A sufficiently massive neutrino would have an important astrophysical significance for the overall mass density of the universe⁸, and may have a bearing on the missing light problem⁹, and the dynamics of galaxy formation¹⁰. Lepton number violating interactions and neutrino masses are also suggested by certain Grand Unified models of elementary particles¹¹. Recent experimental studies of electron energy spectra from neutron beta decay have indicated a non-zero electron antineutrino mass in the range 14–44 eV (90% C.L.)¹² (see Appendix B).

In this thesis is described an experiment which was performed to search for the oscillations of muon neutrinos into any other type of neutrino. The conjectured oscillatory time dependence of the neutrino type was measured by comparing event rates for charged current neutrino nucleon scattering with a final state muon in two massive detectors. These detectors collected data simultaneously at different positions along the neutrino beamline at Fermi National Accelerator Laboratory (Fermilab) in Batavia, Illinois. The two event rates were compared as a function of neutrino energy. The experiment (Fermilab E-701) was performed in 1982 by a collaboration of physicists from Columbia University, the University of Chicago, Fermilab, and the University of Rochester. Within the sensitivity of the experiment, the oscillation effect was not observed. This result¹³ implies certain correlated limits on possible neutrino mass differences and lepton number violation.

The thesis is organized in the following way. First this chapter gives some theoretical background to neutrino mass, lepton number violation, and neutrino oscillations. Second the experimental situation is discussed with regards to neutrino oscillations.

Finally we present an overview of the experiment. Chapter 2 describes the neutrino beam used in the experiment. The properties of this beam are particularly important for the measurement. Chapter 3 describes the two detectors used to observe the interactions of muon neutrinos. Chapter 4 is a discussion of the procedure used to analyze the events and extract their kinematical properties. Chapter 5 presents the event counting procedures used to compare flux received by the two detectors. Chapter 6 is a description of the results and a discussion of the statistical analyses used to restrict possible values of the oscillation parameters. Included also are a set of appendices which detail some of the topics in the thesis.

§1.2 Theory of Neutrinos, and Neutrino Mass

With a conventional view holding both the neutrino massless and lepton number conserved¹⁴, we would reasonably expect the oscillation effect to be absent. Before describing the experimental search it is thus relevant to discuss the assumptions which support this view. The notation used in this section is defined in Appendix A.

1.2.1 The Conventional View

Experimentally, the neutrino is observed to only participate in the weak interactions. The weak interactions are thought to come about through a coupling between a fermion current and one of a triplet of vector particles¹⁵, the charged $W^\pm$, ($M_{W^\pm} = 80.9 \pm 2.9 \text{ GeV}^{16}$), and the neutral Z^0 ($M_{Z^0} = 91.9 \pm 1.9 \text{ GeV}^{17}$). Weak interactions

between fermion currents, for example beta decay (Equation 1.1), or neutrino scattering, are mediated by the virtual exchange of one of these massive vector particles. When a massive virtual particle is exchanged, the effective strength of the interaction is determined by the relationship between the four momentum (q^2) it carries and its mass. This is because the propagator is proportional to,

$$\frac{g^2}{q^2 - M_{W,Z}^2} \quad (1.8)$$

where g is some universal coupling strength. Historically, the weak interactions were recognized as being weak because in all the processes studied, $q^2 \ll M_{W,Z}^2$.

The fermion currents which couple to the $W^\pm$ are charged while those which couple to the Z^0 are neutral. These currents may consist of either leptons (charged or neutral) or the "quarks"¹⁸, the fundamental hadronic constituents. The quarks are particles which experience strong, weak, and electromagnetic interactions. What will concern us here in particular are, the charged currents. Experimentally, their space-time structure is thought to be that of a vector minus an axial vector (V-A)¹⁹. Thus the leptonic charged current assumes the form,

$$j_\mu^{\text{leptons}} = \sum_{i=1}^{N_g} \bar{L}_i \gamma_\mu \frac{(1 - \gamma_5)}{2} N_i \quad (1.9)$$

The quantity N_g is the number of lepton generations, presently thought to be three. Here L_i refers to the i^{th} charged lepton, and N_i to the corresponding neutrino. The corresponding quark current is of a similar form but the fields which enter represent six different quark flavors,

$$j_\mu^{\text{hadrons}} = \sum_{i=1}^{N_g} \bar{U}_i \gamma_\mu \frac{(1 - \gamma_5)}{2} \sum_{j=1}^{N_g} V_{ij} D_j \quad (1.10)$$

Again there are thought to be three quark generations. The fields represented by U_i are the so called "up" quarks, the u (up), the c (charm) and t (top or truth). The other three fields, the "down" quarks, the d (down), s (strange), and b (bottom or beauty) which participate in weak interactions are mixed with respect to the down quarks, D_{ij} , which are strongly interacting. This mixing is parametrized in terms of the unitary 3×3 Kobayashi-Maskawa²⁰ matrix V_{ij} . (This mixing is analogous to the lepton family mixing which might lead to neutrino oscillations).

It was the observation of parity violation in the weak interactions^{21,22} which was taken to imply that the charged currents have the structure given by Equations 1.9 and 1.10. The implication is that the charged quanta of the weak interactions, $W^\pm$, couple only to charged currents with the above forms. By invoking $\frac{1}{2}(1 - \gamma_5)$ as the left handed spin projection operator it becomes apparent that these weak interactions *only* involve left handed fermions and right handed antifermions. This is known as the principle of maximal parity violation. Although, experimentally the neutrinos are known to be light, it is really because of this observed pattern of parity violation that the neutrino has earned its reputation of masslessness. Dirac theory endows a massive spin $\frac{1}{2}$ particle with four components. Such a particle is often called a Dirac particle. By the observed weak interactions of neutrinos, two components, the right handed neutrino and the left handed antineutrino, appear to be absent. Thus the neutrino is massless. The charged leptons and the quarks on the other hand are massive, and their right handed components exist, but only interact electromagnetically (or strongly).

1.2.2 Neutrino Mass and Total Lepton Number Conservation

At least two potential loopholes in the logic presented above can be considered. Their existence is to some extent a theoretical justification for the experiment described in this thesis as well as for numerous other searches for neutrino mass effects in different phenomena (see Appendix B).

The first argument allows the neutrino to remain a massive Dirac particle. We simply assume that its two unobserved components couple to matter much more weakly than the familiar neutrinos. A charged current composed of these other components would have the space-time structure of a vector *plus* an axial vector (V+A). The implication is that the $W_R^\pm$ which couple to these V+A charged currents are much heavier than the observed $W^\pm$. By Equation 1.8, these right handed weak interactions remain weak until a much higher q^2 is exchanged. Limits upon maximal parity violation, in particular a recent measurement of the muon polarization in the weak decay,

$$\pi^+ \rightarrow \mu^+ \nu_\mu \quad (1.11)$$

imply a lower bound, $M_{W_R^\pm} > 380 \text{ GeV}^{23}$ (assuming a light ν_R).

The second argument, which is perhaps more economical, dispenses with the distinction between particle and antiparticle. Because the neutrino is electrically neutral, its charge conjugation properties are in principle ambiguous. This ambiguity leads to the possibility that the neutrino is, as is usually assumed, a Dirac particle, different from its antiparticle, or a Majorana particle²⁴, equal to its antiparticle. In the second case, the neutrino is a single particle, and its two observed states can just be the two helicity states of a massive fermion.

In practice, most of the phenomenology of the massive Dirac neutrino of the first argument and the massive Majorana neutrino of the second need not be very different from the observed weak interactions²⁵. To see this we refer to the diagrams a and b in Figure 1.1. There we show two massive neutrinos initially at rest. One is accelerated up and strikes the ceiling approximately right handed. The other is accelerated down and strikes the floor approximately left handed. Let the neutrino mass be m , let the center of mass energy in the collisions be E , and let the strength of the $V+A$ weak interaction amplitude be reduced by the factor η relative to the $V-A$ amplitude. For the Dirac neutrino in (a) normal weak interactions occur at the floor, while at the ceiling, interaction amplitudes are suppressed by the factor η or by $\frac{m}{E}$, the probability for an helicity flip. For the massive Majorana neutrino in (b), except for order η and $\frac{m}{E}$ effects, what we would have called neutrino interactions occur at the floor and antineutrino interactions occur at the ceiling. But because the same neutrino makes electrons and positrons the total lepton conservation rule, Equation 1.6, is violated. We emphasize that the existence of a Majorana neutrino would imply other rare manifestly lepton number violating effects. These effects would be proportional to $\frac{m}{E}$ and thus most likely to occur at *low* energy. A striking example would be the neutrinoless double beta decay. There a nucleus decays emitting two electrons and emitting and absorbing the same virtual Majorana neutrino,

$$(Z, A) \rightarrow (Z + 2, A) + e^- + e^- \quad (1.12)$$

(See Appendix B for a discussion.) The nonobservation of this process at present implies correlated limits on Majorana neutrino mass and weak $V+A$ currents²⁶.

To understand the formal relationship between Dirac and Majorana fermions

consider first the generic four component Dirac spinor,

$$\Psi_D = \begin{pmatrix} \phi_R \\ \phi_L \end{pmatrix} \quad (1.13)$$

which is taken to represent a solution to the Dirac Equation,

$$(i\partial_\mu \gamma^\mu - m)\Psi_D = 0 \quad (1.14)$$

Historically, the Dirac Equation was first constructed to describe the relativistic electron. At that time it was recognized that the Dirac Equation admitted both "positive" and "negative" energy solutions. To each state Ψ_D , which satisfies the Dirac Equation there exists a conjugate "antiparticle" state Ψ_D^c . In the case of the physical electron, this state is the positron. Given the state Ψ_D it is possible to construct the state Ψ_D^c . This is most easily seen by considering a charged particle coupled to the electromagnetic potential A_μ . Here the Dirac Equation reads,

$$\{(i\partial_\mu - eA_\mu)\gamma^\mu - m\}\Psi_D = 0 \quad (1.15)$$

We expect Ψ_D^c to satisfy instead,

$$\{(i\partial_\mu + eA_\mu)\gamma^\mu - m\}\Psi_D^c = 0 \quad (1.16)$$

Taking the complex conjugate we find,

$$\{(-i\partial_\mu - eA_\mu)\gamma^{\mu*} - m\}\Psi_D^* = 0 \quad (1.17)$$

If an operator C exists such that,

$$C\gamma^{\mu*}C^{-1} = -\gamma^\mu \quad (1.18)$$

then,

$$\Psi_D^c = C \Psi_D^* \quad (1.19)$$

In the Weyl representation, C is given by,

$$C = \begin{pmatrix} 0 & \sigma_2 \\ -\sigma_2 & 0 \end{pmatrix} \quad (1.20)$$

and then,

$$\Psi_D^c = \begin{pmatrix} \sigma_2 \phi_L^* \\ -\sigma_2 \phi_R^* \end{pmatrix} \quad (1.21)$$

If we decouple the particle from the electromagnetic field by letting $e \rightarrow 0$, it is not obvious that we cannot equate Ψ_D and Ψ_D^c in which case we obtain the Majorana spinor,

$$\Psi_M = \begin{pmatrix} \phi_R \\ -\sigma_2 \phi_R^* \end{pmatrix} \quad (1.22)$$

Although the Majorana spinor is a four component object it has now only two degrees of freedom. The Majorana spinor is analagous to the real scalar field which represents the π^0 particle and is an eigenstate of charge conjugation. The charged pions are described by a complex field with twice as many components as the π^0 just as the Dirac spinor has twice as many independent components as the Majorana spinor.

As was noted in Appendix A, a Lorentz scalar may be constructed by taking certain bilinear combinations of left and right handed Weyl spinors. Some such bilinear combinations can account for the masses of the state being described in a Lagrangian function. Using the result of (A1.22), the combination $(\frac{1}{2}, 0) \otimes (\frac{1}{2}, 0)$ gives,

$$\phi_R^\dagger \phi_L + \phi_L^\dagger \phi_R \quad (1.23)$$

In terms of four component Dirac spinors this bilinear can be written

$$\bar{\Psi}_D \Psi_D \quad (1.24)$$

A term of the form $m\bar{\Psi}_D \Psi_D$, where m is a mass, is called a Dirac mass term. We note that it couples left and right handed spinors. This is sensible since in the case of non-zero mass the two helicity states are not independent but may be linked by a Lorentz transformation. If there is no obvious difference between particle and antiparticle we can also construct a scalar using the Majorana spinors. In this case,

$$\bar{\Psi}_{M_1} \Psi_{M_1} = (\phi_R^\dagger, -\phi_R^T \sigma_2) \gamma_0 \begin{pmatrix} \phi_R \\ -\sigma_2 \phi_R^* \end{pmatrix} \quad (1.25a)$$

or,

$$\bar{\Psi}_{M_1} \Psi_{M_1} = -\phi_R^\dagger \sigma_2 \phi_R^* - \phi_R^T \sigma_2 \phi_R \quad (1.25b)$$

and

$$\bar{\Psi}_{M_2} \Psi_{M_2} = -\phi_L^\dagger \sigma_2 \phi_L^* - \phi_L^T \sigma_2 \phi_L \quad (1.26)$$

This exact same expression can be written instead with Dirac spinors,

$$\bar{\Psi}_D \Psi_D^c + \bar{\Psi}_D^c \Psi_D \quad (1.27)$$

which makes explicit the coupling between charge conjugate states.

The term,

$$\mathcal{L}_{mass}^{Dirac} = m\bar{\Psi}_D \Psi_D \quad (1.28)$$

admits the phase invariance $\Psi_D \rightarrow e^{i\theta} \Psi_D$ while the Majorana term,

$$\mathcal{L}_{mass}^{Majorana} = m(\bar{\Psi}_D \Psi_D^c + \bar{\Psi}_D^c \Psi_D) \quad (1.29)$$

does not. Whenever the Lagrangian function is invariant under such a transformation it is possible to define a conserved "charge". In the Lagrangian for a neutral particle, such a charge may be postulated in which case only mass terms of the Dirac type are permitted. For massive Dirac neutrinos this conserved charge is identified with the total lepton number. In theories with Majorana neutrinos, we of course expect interactions which violate total lepton number conservation rules. Although Dirac neutrinos may never participate in total lepton number violating interactions, it is possible to postulate couplings between Dirac neutrinos associated with different lepton generations. In this case, interactions preserving the sign of the lepton number but violating a separate lepton number conservation rule may occur. The neutrino family oscillations described in the Introduction are of this variety. In Section 1.3 we outline other total lepton number violating oscillations which would signal the existence of massive Majorana neutrinos.

1.2.3 Separate Lepton Number Conservation

From the above discussion it is clear that the question of total lepton number conservation may be linked to the nature of the neutrino. Besides the question of total lepton number conservation however, the possibility of separate lepton family number nonconservation is also related to neutrino oscillations. If lepton family oscillations occur then certain other processes are likely at some level also. These are listed in Table 1.1 along with upper limits on their branching ratios. It is hard to relate these to neutrino oscillations in a model independent way. If we assume they come about through mixings of virtual neutrinos, as indicated in Figure 1.2a (for the muon electron conversion), then

their branching ratios may be estimated³¹ (see Appendix B). In this case they are expected to occur at rates which are roughly ten orders of magnitude below the measured limits. In Appendix B we mention other mechanisms which can cause these even in the absence of neutrino family mixing. In the case of neutrino mixing however, the oscillations, which are an interference effect, are a much more sensitive indication of separate lepton number violation than any of the transitions listed in Table 1.1.

§1.3 Theory of Neutrino Oscillations

The neutrino masses we have discussed above do not arise naturally in the theories which describe the presently observed weak interactions. If these masses exist, their origin may lie outside the familiar weak interactions. In the currently popular unified theories¹¹ for example, neutrino mass terms arise through Yukawa couplings of fermions with Higgs-like (scalar) particles. The Dirac mass terms come after symmetry breaking from an interaction like,

$$\Phi \bar{\nu}_R \nu_L + h.c. \rightarrow \langle \Phi \rangle \bar{\nu} \nu \quad (1.30)$$

while the Majorana mass can come from

$$\frac{1}{M} \Phi \Phi \{ (\bar{\nu}^c)_R \nu_L + (\bar{\nu}^c)_L \nu_R \} + h.c. \rightarrow \frac{\langle \Phi \rangle^2}{M} \bar{\nu}^c \nu + h.c. \quad (1.31)$$

Here $\langle \Phi \rangle$ is the expectation value of this scalar field, and M is some characteristic (large) unification mass.

Whatever the true origin of neutrino mass might be, it seems likely that the eigenstates of the weak interactions (ν_e, ν_μ, ν_τ etc.) would be different than the mass

eigenstates. In this case the neutrino associated with the i^{th} charged lepton, ν_{l_i} , can be expressed as a superposition of the mass eigenstates ν_j .

$$\nu_{l_i} = \sum_j U_{ij} \nu_j \quad (1.32)$$

U_{ij} is a unitary "neutrino mass mixing matrix". Here the conservation law for separate lepton family number is automatically violated. Assume initially that the m_j of these ν_j are of the Dirac type. If the m_j are at least partially nondegenerate then as a weak interaction eigenstate evolves with time, the mass eigenstates of which it is superposed will propagate with different phases. At some time t , the probability for a weak interaction eigenstate is given by,

$$\begin{aligned} P(\nu_{l_i}(t) = \nu_{l_i}(0)) &= |\langle \nu_{l_i}(t) | \nu_{l_i}(0) \rangle|^2 = \left| \sum_{k=1}^n \langle \nu_k | e^{iE_k t} U_{ik}^* \sum_{j=1}^n U_{ij} | \nu_j \rangle \right|^2 \\ &= \left| \sum_{j,k=1}^n e^{iE_k t} U_{ik}^* U_{ij} \langle \nu_k | \nu_j \rangle \right|^2 = \left| \sum_{j=1}^n e^{iE_j t} |U_{ij}|^2 \right|^2 \end{aligned} \quad (1.33)$$

which may differ from unity. The general time dependence of this probability is a complicated but harmonic function of time. This is the phenomenon of neutrino oscillations⁷. A neutrino born in the weak interaction has a non-zero harmonic probability to appear at a time t as another type of neutrino.

In a massive three Dirac neutrino (real?) world³², with the three charged leptons equal to their mass eigenstates we have for the mass part of the Lagrangian,

$$\mathcal{L}_{Mass}^{Dirac} = \sum_{l_i=1}^3 m_{l_i} \bar{l}_i l_i + \sum_{l_i=1}^3 \sum_{l_j=1}^3 m'_{l_i, l_j} \bar{\nu}_{l_i} \nu_{l_j} \quad (1.34)$$

Here the l_i refer to the three charged leptons, e, μ , and τ . The neutrino mass matrix has seven independent real elements. It is diagonalized by U_{ij} the unitary 3×3 mixing

matrix. The oscillation phenomenon will then be described by the three masses m_1, m_2 , and m_3 , and the four independent elements of U_{ij} , taken to be three angles $\theta_1, \theta_2, \theta_3$ and a phase δ . This is analagous to the Kobayashi-Maskawa three quark mixing scheme. The matrix U_{ij} then takes the form,

$$\begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ -s_1 s_2 & c_1 s_1 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix} \quad (1.35)$$

Here c_i and s_i are the cosine and sine of the angle θ_i . If δ is non-zero we may, as in the hadronic case, have CP violating effects whereby (for example)

$$P(\nu_e \rightarrow \nu_\mu) \neq P(\nu_e^c \rightarrow \nu_\mu^c) \quad (1.36)$$

while CPT still requires

$$P(\nu_e \rightarrow \nu_\mu) = P(\nu_\mu^c \rightarrow \nu_e^c) \quad (1.37)$$

A neutrino born in the decay (for example) $\pi^+ \rightarrow e^+ \nu_e$ will at time $t=0$ be described by

$$|\nu_e(0)\rangle = c_1 |\nu_1\rangle + c_3 s_1 |\nu_2\rangle + s_1 s_3 |\nu_3\rangle \quad (1.38)$$

and at time $t=\tau$ by,

$$|\nu_e(\tau)\rangle = c_1 e^{-iE_1\tau} |\nu_1\rangle + s_1 c_3 e^{-iE_2\tau} |\nu_2\rangle + s_1 s_3 e^{-iE_3\tau} |\nu_3\rangle \quad (1.39)$$

then,

$$\begin{aligned} P(\nu_e \rightarrow \nu_e) = & 1 - 2c_1^2 s_1^2 c_3^2 \{1 - \cos((E_1 - E_2)\tau)\} \\ & - 2c_1^2 s_1^2 s_3^2 \{1 - \cos((E_1 - E_3)\tau)\} - 2s_1^4 s_3^2 c_3^2 \{1 - \cos(E_2 - E_3)\tau\} \end{aligned} \quad (1.40)$$

If $E_i, E_j \gg m_i, m_j$, then we may approximate,

$$E_i - E_j \approx \frac{1}{2} \frac{m_i^2 - m_j^2}{p_\nu} \quad (1.41)$$

The momentum of the electron neutrino is p_ν and $E_i^2 = p_\nu^2 + m_i^2$. Replacing all the constants and letting $\tau = \frac{L}{c}$ we may define the oscillation length L_{osc} by

$$\begin{aligned} \frac{2\pi L}{L_{osc}} &= \frac{2\pi(m_i^2 - m_j^2)c^4 L}{2p_\nu \hbar c^2} \\ &= \frac{2.54(m_i^2 - m_j^2)L}{p_\nu} \end{aligned} \quad (1.42a)$$

or,

$$L_{osc} = 2\pi \frac{0.394 p_\nu}{(m_i^2 - m_j^2)} \quad (1.42b)$$

Here, m_i is in (eV^2/c^4) , p_ν is in (GeV/c) , and L is in kilometers. The preceding equation is central to understanding the experimental signatures of neutrino oscillations. It implies that the neutrino composition in a beam will vary with time (or distance from the point of creation) and with energy. In general, the three neutrino system is rather complicated unless a particular mass difference or a particular mixing angle is much larger than the others. With seven free parameters there are practical difficulties in comparing three neutrino oscillation schemes with data. The situation simplifies considerably when only two neutrinos (ν_e and ν_μ for example) are mixed. In this case $\theta_2 = \theta_3 = 0$ and U_{ij} becomes the real 2×2 matrix

$$\begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \quad (1.43)$$

and the probability,

$$P(\nu_e \rightarrow \nu_e) = 1 - 2 \sin^2 \theta \cos^2 \theta \left(1 - \cos\left(\frac{2\pi L}{L_{osc}}\right)\right) \quad (1.44)$$

which may also be written,

$$P(\nu_e \rightarrow \nu_e) = 1 - \sin^2 2\theta \sin^2\left(\frac{1.27 \Delta m^2 L}{E_\nu}\right) \quad (1.45)$$

where $\Delta m^2 = m_1^2 - m_2^2$ and we have made the approximation $E_\nu \approx p_\nu$. The probability for the electron neutrino to oscillate into a muon neutrino is given by,

$$P(\nu_e \rightarrow \nu_\mu) = 1 - P(\nu_e \rightarrow \nu_e) \quad (1.46)$$

Here there can be no CP violation without nonconservation of probability.

In the preceding discussion we have assumed that all neutrino masses are Dirac masses. An analysis of neutrino oscillations in the case where both Majorana and Dirac masses are present has been made³³. It happens that this system possesses a somewhat different phenomenology. The oscillations which we have discussed previously are known as first class or flavor oscillations. This is because they violate the separate lepton flavor or family number conservation rules. If the Majorana mass terms are present then there is also the possibility of total lepton number violating or so-called second class oscillations.

Second class oscillations can occur in a system where in addition to the usual Weinberg-Salam left handed doublet and right handed charged singlet,

$$\begin{pmatrix} \nu_{eL} \\ e_L^- \end{pmatrix} \quad e_R^- \quad (1.47)$$

we allow a weak interaction sterile, neutral singlet ν_{eR} also. The mass term, containing both Dirac and Majorana masses, is

$$\mathcal{L}_{Mass}^{M,D} = \sum_{i,j=1}^{Ng} M_{ij}^L (\bar{\nu}_{iL})(\nu_{jL})^c + M_{ij}^R (\bar{\nu}_{iR})^c \nu_{jR} + M_{ij}^D (\bar{\nu}_{iL} \nu_{jR} + (\bar{\nu}_{jR}^c) \nu_{iL}^c) + h.c. \quad (1.48)$$

Note that,

$$(\nu_L)^c = \nu_R^c \text{ familiar RH antineutrino}$$

$$(\nu_R)^c = \nu_L^c \text{ sterile LH "antineutrino"}$$

If we restrict this to one generation we can easily diagonalize the mass term,

$$(\overline{\nu_R} \ \overline{\nu_R^c}) \begin{pmatrix} m_{11} & m_{12} \\ m_{21}^* & m_{22} \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_L^c \end{pmatrix} + h.c. \quad (1.49)$$

Because of CPT invariance, $m_{11} = m_{22} = m$. If the off diagonal elements are real and equal then the eigenvectors are,

$$\Psi_M^\pm = \frac{1}{\sqrt{2}}(\nu \pm \nu^c) \quad (1.50)$$

which are Majorana neutrinos since they have no definite lepton number. The masses of these states are, $m_+ = m + m_{12}$ and $m_- = m - m_{12}$. The left and right handed projections are,

$$\begin{aligned} \Psi_{M_L}^\pm &= \frac{1}{2}(1 - \gamma_5)\Psi_M^\pm = \frac{1}{\sqrt{2}}(\nu_L \pm (\nu_R)^c) \\ \Psi_{M_R}^\pm &= \frac{1}{2}(1 + \gamma_5)\Psi_M^\pm = \frac{1}{\sqrt{2}}(\nu_R \pm (\nu_L)^c) \end{aligned} \quad (1.51)$$

Thus the state born at $t=0$ as ν_{eL} will evolve as

$$|\nu_{eL}(t)\rangle = e^{-iE_+t} |\Psi_L^+\rangle + e^{-iE_-t} |\Psi_L^-\rangle \quad (1.52)$$

and so has a non-zero probability of appearing as ν_{eL}^c which is weak interaction sterile. In the general case, with N generations and $2N$ massive Majorana neutrinos we may also have mixing between the different flavors. In this case there will be transitions between doublet

and singlet members of the same and different generations, the latter being reduced by the mixing angle.

There is an important practical distinction between first and second class oscillations. With first class oscillations, the total weak interaction probability is always divided among the various mixed flavors. A measurement of the total rate of all charged and neutral current weak interactions will be constant with time. In the case of second class oscillations there will instead be an observed variation in the total weak interaction probability. Operationally, for first class oscillations, the observed rate for a particular charged current channel will change with time while the neutral current rate will not change. It is impossible to distinguish neutral currents induced by different neutrino flavors. For second class oscillations, both charged and neutral currents will oscillate with time.

§1.4 Neutrino Oscillation Experiments

A number of experiments have searched for neutrino oscillations³⁴. These experiments have been of two classes. The first, which is called an *exclusive* or appearance experiment, uses a neutrino beam of known composition. A search is made for the presence of interactions due to other types of neutrinos. Exclusive experiments are sensitive to particular oscillation channels such as $\nu_\mu \rightarrow \nu_e$ or $\nu_\mu \rightarrow \nu_\tau$. Here, the probability of another neutrino appearing at some point L along a ν_μ beam for example, is given in a two component scheme by Equation 1.46. (We will restrict the discussion to this two component model to simplify the analysis).

In the second type of measurement, which is called an *inclusive* or disappearance experiment, a search is made for any apparent depletion in the flux of a particular variety of neutrino. Such experiments are sensitive to all exclusive channels as well as oscillations into new exotic neutrinos. These may include the *second class* oscillations described in the preceeding section or first class oscillations into a neutrino associated with a new charged lepton which is too massive to be produced at present energy. For a given beam, (say ν_μ) the inclusive probability is given by Equation (1.45).

The inclusive and exclusive oscillation probabilities depend upon both the neutrino energy, E_ν , and the distance L , from the decay point of the parent particle to the detector. Oscillations may therefore be detected in either of two ways.

A) Compare the energy variation of measured event rates at a fixed distance from the neutrino source with some expected behavior. This method requires an *absolute* knowledge of the energy dependence of the interaction cross section, the detection efficiency, and the neutrino energy spectrum at the neutrino source ($L=0$). Because of these requirements, there may be important systematic limitations to this method. Unknown physics may effect the total cross section. Neutrino energy spectra are infamously difficult to calculate and in general impossible to check independent of neutrino oscillations.

B) Compare the energy variations of measured event rates at two distances from the neutrino source. (This would be equivalent to A if one measurement could be made at precisely the neutrino source.) This experiment is best done using two similar detectors simultaneously. Method B is free of uncertainties in the absolute flux and cross section. In principle it should have smaller systematic errors than method A. Dependences upon

the neutrino spectrum should enter at the level of small relative corrections if the two detectors receive the geometric flux with slightly different acceptance.

The general aim of an oscillation analysis is to identify the parameters $(\Delta m^2, \sin^2 2\theta)$ which describe the observed variations in the neutrino flux, or in their absence, specify a range of excluded values. The results are usually presented in the two dimensional $(\Delta m^2, \sin^2 2\theta)$ plane.

The sensitivity of an experiment to these parameters is set by geometry, neutrino energy, detector resolutions, and statistical and systematic errors. The value of Δm^2 sets the frequency of the oscillation. When in Equation (1.42), $L_{osc} \gg L$, the small angle approximation applies to Equations (1.45) and (1.46). Thus, the maximum value of $\frac{L}{E_\nu}$ attainable determines an absolute lower limit on sensitivity to Δm^2 . Table 1.2 lists some neutrino sources, typical maximum values of $\frac{L}{E_\nu}$, and the corresponding minimum Δm^2 sensitivity. In practice, this low Δm^2 sensitivity is further constrained by the size of statistical and systematic errors. With large errors, a very gradual oscillation will be impossible to resolve. Statistics also determine the minimum $\sin^2 2\theta$ sensitivity. When, $L_{osc} \ll L$, there are many oscillations over the physical layout of the experiment. If the neutrino spectrum at $L=0$ is known, then the detector just samples the the average oscillation probability. This is $\frac{1}{2} \sin^2 2\theta$ for exclusive, and $1 - \frac{1}{2} \sin^2 2\theta$ for inclusive experiments. Here, for Δm^2 large the sensitivity is independent of Δm^2 . If the $L=0$ spectrum is not known, so that measurements are instead made at two locations, there will be an upper Δm^2 cutoff in sensitivity when $L_{osc} < L_{source}$, the length of the neutrino source. High frequency oscillations will also be smeared by detector energy resolutions. A

rule of thumb is to keep

$$\frac{\Delta E_\nu}{E_\nu} < \frac{0.2 L_{detector}}{L_{osc}} \quad (1.53)$$

Here $L_{detector}$ is the average distance from source to detector. The successive effects of all these limitations is shown in Figure 1.3a. At top we plot Equation 1.45 for $L=1116$ meters (the average distance of Lab E from the neutrino decay region at Fermilab) and an oscillation hypothesis of $\Delta m^2 = 360 \text{ eV}^2$ and $\sin^2 2\theta = 0.2$. In the lower three curves we average over an experimental energy resolution of 10% and a 350 meter long decay region. In Figure 1.3b we plot the ratio of Equation 1.45 for the Lab E and Wonder Building distances ($L_{WB} = 715$ meters), for the ideal case and then again averaged over the experimental energy resolution and decay region length. Thus a real experiment must be sensitive to a considerably damped signal for a given hypothesis.

Below we review aspects and results of exclusive and inclusive measurements.

1.4.1 Exclusive Measurements

Exclusive measurements have been carried out at high energy accelerators where relatively pure muon and anti-muon neutrino beams are available ($\frac{N(\nu_\mu)}{all \nu} > 99\%$). For lower energy sources, such as a reactor, the energy of the ν_e^c are below the μ and τ production thresholds so that these sources are not useful for exclusive studies. The exclusive experiments utilize detectors with good pattern recognition capability, bubble chambers and emulsions, to isolate events with the unique topology of ν_e or ν_τ interactions. For ν_e in a ν_μ beam there is some expected rate already because of K_{e3} decays and this

must be well calculated. The accuracy to which this expected rate can be calculated is an important systematic limitation in $\nu_\mu \rightarrow \nu_e$ searches. At present energies, ν_τ contamination in a normal ν_μ beam is expected to be negligible. Another problem is misidentification. This is more important for ν_τ searches where a short τ track followed by the leptonic decay looks like a ν_μ interaction.

Exclusive searches have thus far failed to detect a positive signal for the $\nu_\mu \rightarrow \nu_e$, $\nu_\mu^c \rightarrow \nu_e^c$, $\nu_\mu \rightarrow \nu_\tau$, and $\nu_\mu^c \rightarrow \nu_\tau^c$ channels. These experiments have been performed with the 15 foot bubble chamber at FNAL, the BEBC and Gargamelle bubble chambers at CERN, and in a hybrid emulsion spectrometer at FNAL (E-531)^{35,36,37,38,39}.

If two detectors are used then presumably the analysis would be independent of the absolute background rate of ν_e or ν_τ . The fine grained electronic neutrino detector CHARM at CERN has good pattern recognition for electrons. It was divided into two subsections placed 123 meters and 903 meters from a low energy ($E_\nu \approx 1.5 \text{ GeV}$) bare target ν_μ beam. Limits on $\nu_\mu \rightarrow \nu_e$ oscillations have been set from this data⁴⁰. Figure 1.4 shows the present best limits from exclusive measurements.

1.4.2 Inclusive Measurements

Inclusive measurements have a much more contraversal history than exclusive measurements. They have been performed at high energy accelerators, nuclear reactors, and with extra-terrestrial neutrinos of solar and atmospheric origin. A number of these experiments have reported anomalous results which have been intepreted by some as evidence for neutrino oscillations. In terms of a general acceptance however, at present

the matter remains unsettled. Here we survey the various measurements and discuss their systematic limitations.

1) Solar Neutrinos: A long standing problem in solar astrophysics has been the discrepancy between the measured rate of solar neutrinos from the reaction ${}^8\text{B} \rightarrow {}^8\text{Be} + e^+ + \nu_e$ and calculations based upon the standard solar model⁴¹,

$$\frac{\Gamma_{\text{measured}}}{\Gamma_{\text{predicted}}} = \frac{2.1 \pm 0.3 \text{ SNU}}{7.9 \pm 1.5 \text{ SNU}} \quad (1.54)$$

where $1 \text{ SNU} = 10^{-36} \nu$ captured per second per atom of ${}^{37}\text{Cl}$. The interactions of the ${}^8\text{B}$ neutrinos in a chlorine target are detected radiochemically by the presence of ${}^{37}\text{Ar}$ produced in the reaction,

Interpreting this as an indication of neutrino oscillations implies $\Delta m^2 \approx 10^{-11} \text{ eV}^2$. There are uncertainties in the calculation of the ν_e flux from the ${}^8\text{B}$ reaction which is a rare branch of the solar energy producing radiochemistry. At present, these probably preclude any definitive conclusion about neutrino oscillations from observations of the solar neutrino flux.

2) Reactor Experiments: Three reactor experiments have been performed to date to search for inclusive oscillation of ν_e^c from the beta decay of fission products. They all are contradictory. The first experiment⁴², done at the Savannah River power reactor by a group from the University of California at Irvine (UCI) measured the ratio,

$$\frac{\Gamma_{CC}(\nu_e^c + d \rightarrow n + n + e^+)}{\Gamma_{NC}(\nu_e^c + d \rightarrow n + p + \nu_e^c)} \quad (1.56)$$

at 11.2 meters from the reactor core. This quantity is basically independent of the source spectrum. For first class oscillations, the NC (neutral current) rate is not expected to vary and so provides a normalization. The cross section ratio has been calculated, and the result,

$$\frac{(\frac{\sigma_{CC}}{\sigma_{NC}})_{exp}}{(\frac{\sigma_{CC}}{\sigma_{NC}})_{theory}} = 0.38 \pm 0.21 \quad (1.57)$$

has been reported. This was interpreted as 3σ evidence for neutrino oscillations. The allowed solutions are shown in Figure 1.5.

A series of experiments⁴³ measuring the rate for the more common inverse beta decay,

$$\nu_e^c + p \rightarrow n + e^+ \quad (1.58)$$

has been made by a group at the Institute Laue-Langevin and the Goesgen power reactor (ILL/Goesgen). These have included single and multiple position measurements. All results are consistent with no neutrino oscillations. Their best limits are also shown in Figure 1.5. The more restrictive one is derived from a combination of two single position measurements at Goesgen (37.9 meters and 45.9 meters). They completely exclude the UCI solution. In that experiment, the observed positron yield is compared to a calculation derived from predictions of the reactor antineutrino flux and the cross section for Equation 1.58. The reactor antineutrino flux is predicted by unfolding a measurement of the corresponding fission electron spectrum. The electron measurement is performed by exposing a sample of each fissile isotope to the reactor neutron flux in a test spectrometer⁴⁴ ("BILL"). The ratio of the data to the prediction is shown in Figure 1.6a. Below, in Figure 1.6b we show the two position ratios. The data taken at the ILL

reactor is also shown in Figure 1.6a. It was compared to a theoretical calculation of the reactor antineutrino spectrum and yielded limits similar to the Goesgen result.

There are systematic uncertainties in experiments which rely upon predictions of reactor antineutrino spectra. Originally, as was done in the ILL analysis, these predictions were made by summing the separate beta spectra of all the reactor fission products. Unfortunately there remained unknown contributions from uncataloged fission product beta decays to the total spectrum. The method used to analyze the Goesgen data is perhaps better since it is based upon a measurement. There are however still possible ambiguities in the unfolding procedure used to transform a sum of separate electron spectra into a sum of antineutrino spectra. In addition there may be differences in the core neutron flux during the test measurement and the actual oscillation search. In Figure 1.7 we compare the flux calculations of Davis⁴⁵ et al and Avignone⁴⁶ et al with the inference from the fission electron spectrum. While the electron data and the Davis calculation agree rather well there is a clear discrepancy with the calculation of Avignone et al. Klapdor⁴⁷ et al have done a reanalysis of the ILL data and has argued that a different choice of spectra could lead to a positive signal for neutrino oscillations.

Another systematic uncertainty in these normalized measurements originates from an ignorance of the correct neutron lifetime. The cross section for Equation 1.58 depends upon the neutron lifetime. At present, some experimental controversy exists as to what its true value is. Clearly, a shift in the neutron lifetime could directly alter the predicted positron spectrum. This uncertainty along with the flux calculations contribute to an overall normalization error which has been quoted at 5.3% for the Goesgen limit

(the total systematic error is 7.4% including other effects).

A third reactor experiment has been performed by a group at the Bugey power reactor in France⁴⁸. This is a two position (13.6 m, 18.3 m), high statistics measurement. As a result it is largely independent of the reactor neutrino flux except at the level of small corrections. The detector positions were very close to the reactor core and so there were important solid angle effects. One detector was utilized alternately at the two positions.

The Bugey group has reported evidence for neutrino oscillations with the allowed values of $(\Delta m^2, \sin^2 2\theta)$ shown in Figure 1.8. The data points are also shown with the most likely solution $(\Delta m^2 = 0.2 \text{ eV}^2, \sin^2 2\theta = 0.25)$ drawn in. Note that this is at the lower Δm^2 end of their sensitivity and so appears as a gradual offset. The frequency is too low to show any characteristic wiggles with energy. The experiment is continuing and so more results should be forthcoming. We note that such a small mass difference might be characteristic of two nearly degenerate Majorana neutrinos in which case the oscillation would be of the second class.

3) Accelerators: High energy neutrinos from accelerators have been utilized by three inclusive experiments besides E-701. At CERN, the bubble chamber BEBC was used to search for the process $\nu_e \rightarrow \nu_x$ in a muon neutrino beam³⁷. This was done by comparing the observed ν_e event rate to a calculation of the expected background ν_e rate from K_{e3} decays. An *excess* of electron events was found. The rate was,

$$\frac{N_{\text{observed}}}{N_{\text{predicted}}} = \frac{110 \pm 13}{91 \pm 10} \quad (1.59)$$

The BEBC group misinterpreted this result and placed too strong a limit on ν_e inclusive oscillations. In actuality the true observed rate should be *less than or equal to* the true

predicted rate with oscillations present. The correct limit can be found by evaluating,

$$\frac{\int_0^\infty dn_{pred} P(n_{pred}) \int_0^{R n_{pred}} dn_{obs} P(n_{obs})}{\int_0^\infty dn_{pred} P(n_{pred}) \int_0^{n_{pred}} dn_{obs} P(n_{obs})} = 0.1 \quad (1.60)$$

for the factor R . The 90% confidence limit is given by $2(1 - R)$, which yields the limit $\sin^2 2\theta < 0.38$ rather than the published limit of 0.07.

Also at CERN, was the two detector CDHS experiment⁴⁹. Here a low energy bare target ν_μ beam from the PS was used as a source of neutrinos with an expected $\approx \frac{1}{L^2}$ flux dependance. The beam crossed the two detectors, located 193 meters and 903 meters from the target at 22 degrees. The layout is shown in Figure 1.9. Rates for the quasi-elastic process (Equation 1.5) were compared in total and as a function of projected muon range in the two massive iron-scintillator calorimeters. The projected range is strongly correlated with the neutrino energy. Monte Carlo correction for non $\frac{1}{L^2}$ effects in the beam, solid angle differences, efficiencies, and background acceptances were applied to the data. The integrated analysis yielded,

$$\frac{(\frac{N_{back}}{N_{front}})_{data}}{(\frac{N_{back}}{N_{front}})_{MonteCarlo}} = 1.044 \pm 0.023 \quad (1.61)$$

where the error is statistical. This excludes oscillations for Δm^2 in the range $1 - 10 \text{ eV}^2$. For $\Delta m^2 > 10 \text{ eV}^2$ the projected range analysis was more sensitive and yielded the spectrum and limit shown in Figure 1.10. Total systematic errors were estimated to be 2.5%. This result also significantly increased the limits on $\nu_\mu \rightarrow \nu_\tau$.

The CHARM collaboration performed an inclusive search⁴⁰ in the same beam as CDHS. Their limit though, is completely covered by the CDHS limit. It is included in Figure 1.10.

§1.5 Overview of E-701

Experiment 701 was an inclusive search for oscillations of the type $\nu_\mu \rightarrow \nu_x$. The experiment was performed in a narrow band high energy ($E_\nu = 40 - 240 \text{ GeV}$) muon neutrino beam. In the narrow band beam it is possible to monitor the incident neutrino energy. Most of the important systematic effects related to neutrino cross sections and flux spectra were eliminated by running two similar detectors simultaneously at different positions along the neutrino beamline. Neutrinos originated from decays of mesons along a 350 meter long decay region. The first detector was set at 715 meters and the second 1116 meters from the center of the decay region. These locations are indicated on a site view of Fermilab shown in Figure 1.11. Care was taken to ensure very similar event and flux acceptances existed at the two locations. The oscillation effect depends upon both neutrino energy and distance. Because measurements were made at two locations, and the neutrino energy was known, the experiment was well suited to detect this effect.

In the analysis event rates for the process,

$$\nu_\mu \text{ nucleon} \rightarrow \mu^- X \quad (1.62)$$

where X is any hadronic state, were compared in the two detectors as a function of neutrino energy. With the neutrino energies available in the experiment, the wavelength of the neutrinos was smaller than the characteristic nucleon size. These high energy neutrino interactions with nucleons can be understood as individual scatters off constituent quarks. Here also however, the typical four momentum transfer squared was always much less than the mass of the weak quanta. Thus the neutrino quark scattering is effectively a point interaction. This leads to a total cross section which rises linearly with laboratory neutrino

energy. This cross section was measured in a previous experiment by the collaboration responsible for the work described in this thesis (Fermilab E-616)⁵⁰. These are,

$$\sigma_{\nu_\mu} = (.669 \pm .003 \pm .024) \times 10^{-38} \times E_\nu \text{ cm}^2 \quad (1.63a)$$

$$\sigma_{\nu_\mu^c} = (.340 \pm .003 \pm .020) \times 10^{-38} \times E_\nu \text{ cm}^2 \quad (1.64b)$$

Though these cross sections factor out of a comparison of rates in two detectors, they determine the statistical power available to the experiment in a fixed running period. Experiment 701 collected data continuously for ≈ 6 months and obtained roughly 38,000 events in each detector which could be used in the oscillation analysis. These events were chosen from a larger sample of approximately 200,000 interactions in each detector. The choice was based upon criteria designed ensure the incident neutrino energy could be measured and to maximize the probability that a given event would have been accepted by both detectors.

With the given geometry, neutrino energy, and average energy resolution of the detectors ($\approx 10\%$) the sensitivity of the experiment was limited to the range $10 \text{ eV}^2 < \Delta m^2 < 1000 \text{ eV}^2$. With the statistical errors on event rates, sensitivity to the lepton number nonconserving mixing angle was restricted to the region $\sin^2 2\theta > .05 - .10$.

The oscillation effect was not seen. A series of fits to the measured event rates were used to exclude a region of the $(\Delta m^2, \sin^2 2\theta)$ plane at a chosen confidence level.

Chapter 2

Neutrino Source

Experiment 701 was run in the Fermilab dichromatic neutrino beam⁵¹. An overview of the Fermilab neutrino area is shown in Figure 2.1. The dichromatic beam was produced by allowing approximately 10^{13} 400 Gev protons to impinge upon a 0.33 meter long beryllium oxide (BeO) target. The resulting secondaries, pions, kaons, and protons were sign and momentum selected by a series of magnets and collimators into a narrow momentum bite ($\frac{\Delta P}{P} = \pm 10\%$, parallel ($\Delta\theta \approx 0.0002$ radians), beam of charged hadrons. These hadrons were then allowed to traverse a 350 meter long evacuated decay region. There, some fraction of them decayed into muons and neutrinos. This decay region was followed by a 6.1 meter long beam dump which absorbed the remaining hadrons. This dump, which contained approximately 7.7 and 18.3 nuclear interaction lengths of aluminum and steel respectively, is detailed in Figure 2.2. A 545 meter long earth and steel muon shield separated the beam dump from the first neutrino detector. The second detector was placed after another 401 meters of earth. Access ports at two locations

in the decay region and after the beam dump contained intensity and profile monitors for hadrons and muons. The neutrino energy spectrum from this beamline is called dichromatic because the neutrinos which result from pion decay and the neutrinos which result from kaon decay fall into two distinct energy bands.

§2.1 Proton Beam

The synchrotron at Fermilab accelerated nominally a few 10^{13} protons to 400 GeV. The accelerator was pulsed with a period of 10-20 seconds. The acceleration and extraction cycle is shown in Figure 2.3 where the times listed were with respect to an accelerator reference time T1. Beam was extracted from the accelerator in three possible modes. Between T5 and T6 there was a slow extraction (slow spill). Just after T6 there was a fast extraction (fast spill) of 3 milliseconds duration. There were also up to six mini fast extractions distributed during the slow spill (pings). The experiment was only "live" during the fast spill and ping periods in order to minimize cosmic ray backgrounds.

The E-701 neutrino detectors had a readout deadtime of ≈ 50 milliseconds, and therefore could only record one event for each of the pings and fast spills. To minimize the loss of neutrino events, the beam intensity was therefore distributed among the pings and fast spill to keep the interaction rate less than one event per spill.

§2.2 The Dichromatic Neutrino Beam

In this section we discuss in more detail the properties of the dichromatic

neutrino beam. The production spectrum of pions, kaons, and protons in collisions between 400 GeV protons and beryllium have been measured at CERN⁵² as a function of secondary momentum and angle. Reasonable fluxes of secondaries are produced up to about 250 GeV. The neutrino spectrum which would result from decays off this bare target would be very broad and contain a large antineutrino component. For the oscillation experiment and for measurements of the energy dependence of the total neutrino interaction cross section, it is important to have a sign selected beam of well defined energy. The dichromatic neutrino beam satisfied these requirements.

The layout of the beam in the horizontal and vertical planes is shown in Figure 2.4. To understand the beam it is useful to consider the zeroth order narrow band beam shown in Figure 2.5. This beam consists of two bending magnets, a focusing system, and a collimator. The rays coming off the target have a wide angular distribution and the focusing will take them from point source to parallel. The first bending magnet will disperse different momenta to different angles. The lever arm between the first bend and the collimator allows different momentum rays to separate transversely in space. The collimator is set to then select off a particular momentum band with a particular bite. The second dipole recombines the remaining beam and points it toward the neutrino detector.

The actual dichromatic beam was more complicated. Focusing of the point source to parallel was accomplished by placing the production target at the focus of vertical and horizontal quadrupole doublets. Most of the momentum dispersion occurred in the horizontal plane and the momentum selection was facilitated by the collimator

0CTP before D4. The dipoles further downstream (D4 and D5) served to recombine the dispersed beam and direct it towards the neutrino experiments. The width of the momentum defining slit determined the interval about the central momentum contained in the beam. The actual momentum spectrum was a convolution of this acceptance with a falling production spectrum. As a result, this spectrum was slightly asymmetric with a tail extending to high momenta. Dumping of the primary proton beam occurred in a steel block which could be moved into the beam and in the first seven magnet apertures. To provide for this and for additional horizontal bending, the first five vertical focusing quadrupoles were set off the central beamline axis. The size of these apertures limited the angular acceptance of rays off the target. The angular dispersion in the beam in y was mainly constrained by this acceptance.

The major background under the narrow band momentum spectrum came from decays of secondaries before momentum and sign selection had occurred. This is known as the wideband background. This background is measured by closing the momentum defining collimator near the end of the magnet chain and observing interactions in the neutrino detectors. The dichromatic beam was designed to minimize this background. This was done by arranging the magnets so that the secondary beam followed a helical trajectory about the beamline axis. It never pointed towards the neutrino detectors until it entered the decay region. In this way most of the neutrinos which resulted from decays along the beamline never hit the neutrino detectors. Table 2.1 summarizes the properties of the dichromatic neutrino beam.

Because of the particular arrangement of beamline elements there were certain

correlations among coordinates describing the beam in phase space. For a particular ray, the phase space vector is given by the five elements $(x, y, \theta_x, \theta_y, P)$. Qualitatively, since momentum dispersion occurred in mainly the horizontal plane, one expects a strong correlation between x and P and between θ_x and P . To a lesser extent there were correlations among the other coordinates. In the vertical (y) plane, where mostly focusing occurred, the size of the beam as it exited the last magnet was very sensitive to the vertical beam spot size on the target. Measurements of the beam incident upon the target made with SWICs (Segmented Wire Ion Chambers) suggested the dimensions ($\Delta x_{incident} = 1 \text{ mm}, \Delta y_{incident} = .25 \text{ mm}$). A Monte Carlo simulation of the beamline (see section 2.4) made using the program DECAY TURTLE⁵³ predicts the correlations shown in Figure 2.6.

Neutrinos from decays of secondaries selected by the dichromatic train resulted from the processes,

$$\pi \rightarrow \mu \nu_\mu \quad B.R. = 99.999\% \quad (2.1)$$

$$K \rightarrow \mu \nu_\mu \quad B.R. = 63.5\% \quad (2.2)$$

$$K \rightarrow \pi^0 \mu \nu_\mu \quad B.R. = 3.2\% \quad (2.3)$$

$$K \rightarrow \pi^0 e \nu_e \quad B.R. = 4.82\% \quad (2.4)$$

The muon neutrino energy spectrum was dominated by two narrow bands corresponding to the two body decays. As can be seen in Figure 2.7 the neutrinos from the three body decays represented a small contamination to the dichromatic spectrum. The wideband

background, from decays before sign and momentum selection, only became significant at lower energies.

Below we will consider some of the features of the dichromatic neutrino spectrum in detail since they have a significant bearing on the systematics of the oscillation experiment. It is useful though to first provide a crude estimate of the neutrino flux from such a beam. As mentioned earlier, the incident proton intensity was about 10^{13} particles per spill. The beamline described above transmits roughly $10^{10} - 10^{13}$ secondaries. Of these, for example, at +165 GeV, approximately 30% are pions, 4% are kaons, and 66% are protons. At this energy, about 4% of the pions and 25% of the kaons decay in the 350 meter long decay region. The yields, $10^8 - 10^9$ neutrinos per accelerator spill.

Starting from a simple idealized model of the secondary beam we may attempt to understand the important features of the dichromatic neutrino energy spectrum. We will initially assume the secondary beam to have no divergence and be at fixed momentum. For the two body decays, kinematics imposes a correlation between the neutrino energy and the neutrino angle with respect to the parent particles direction in the laboratory. With four momenta and angles as defined in Figure 2.8 we have for the decay,

$$m \rightarrow \mu + \nu \quad (2.5)$$

$$p_m = p_\nu + p_\mu \quad (2.6)$$

so that,

$$E_\nu = \frac{m_m^2 - m_\mu^2}{2(E_m - \sqrt{E_m^2 - m_m^2} \cos \theta_\nu)} \quad (2.7)$$

If $m_m \ll E_m$ and $\sin \theta_\nu \approx \theta_\nu$ this relation may be written,

$$E_\nu = \frac{E_\nu^{max}}{1 + \gamma^2 \theta_\nu^2} \quad (2.8)$$

where, $\gamma = \frac{E_m}{m_m}$ and,

$$E_\nu^{max} = E_m \left(\frac{m_m^2 - m_\mu^2}{m_m^2} \right). \quad (2.9)$$

For parent kaons, $E_\nu^{max} = .97 E_{kaon}$, and for parent pions, $E_\nu^{max} = .41 E_{pion}$. If z is the distance from the decay point to the neutrino detector, and R is the radius of impact at the detector,

$$E_\nu = \frac{E_\nu^{max}}{1 + \frac{\gamma^2 R^2}{z^2}}. \quad (2.10)$$

Thus the energy of the incident neutrino can be determined from the interaction radius in the detector. Figure 2.9 shows a plot of Equation 2.10 for a set of parent energies at the Lab E location.

Since the parent mesons decay isotropically in their center of mass reference system, the number of neutrinos into some annular ring ($R, R+\Delta R$) may be easily calculated. The geometry is indicated in Figure 2.10a. The number into some solid angle in the center of mass is,

$$dN_\nu = \frac{N_d B d\Omega_{cm}}{4\pi} \quad (2.11)$$

where N_d is the number of secondaries which decay, and B is the two body branching ratio. The center of mass and lab solid angles are related by,

$$d\Omega_{cm} = \frac{d(\cos \theta^*)}{d(\cos \theta)} d\Omega_{lab} \quad (2.12)$$

where θ^* and θ are the center of mass and lab decay angles respectively. The lab neutrino energy is related to the lab decay angle by Equation 2.8 above. The Lorentz boost specifies

a relationship between the lab energy and the center of mass angle,

$$E_\nu^{lab} = \gamma E_\nu^{cm} (1 + \cos \theta^*) \quad (\beta \approx 1). \quad (2.13)$$

The center of mass energy is determined for a two body decay,

$$E_\nu^{cm} = \frac{m_m^2 - m_\mu^2}{2m_m} \quad (2.14)$$

so that $E_\nu^{lab\ max} = 2\gamma E_\nu^{cm}$. Therefore,

$$\frac{d(\cos \theta^*)}{d(\cos \theta)} = \frac{4\gamma^2}{(1 + \gamma^2 \theta^2)^2} \quad (2.15)$$

and,

$$N_\nu(R, R + \Delta R) = \frac{2N_d B \gamma^2}{z^2} \int_R^{R+\Delta R} \frac{R dR}{(1 + \frac{\gamma^2 R^2}{z^2})^2}. \quad (2.16)$$

As described in Section 1.5, the neutrino interaction cross section increases linearly with energy. If we call $\frac{\sigma_\nu}{E_\nu}$ the cross section slope (from Equation 1.63) then the total number of neutrino interactions at E_ν , in a detector of density N_{TGT} nucleons per cm^2 due to the incident flux N_ν is given by,

$$N_\nu^{int} = N_\nu \left(\frac{\sigma}{E_\nu} \right) E_\nu N_{TGT}. \quad (2.17)$$

Thus if we weight by an additional factor of the cross section slope times the lab energy, the number of neutrino interactions in the region between R and $R + \Delta R$ may also be computed.

$$N_\nu^{int}(R, R + \Delta R) = \frac{2N_d B \gamma^2 E_\nu^{max} N_{TGT} \left(\frac{\sigma}{E_\nu} \right)}{z^2} \int_R^{R+\Delta R} \frac{R dR}{(1 + \frac{\gamma^2 R^2}{z^2})^3}. \quad (2.18)$$

Both (2.15) and (2.16) may be integrated analytically. These forms are found in Appendix C. The energy vs. radius, flux vs. radius, and events vs. radius relationships are shown

in Figures 2.9, 2.11a, and 2.12a for five settings of the secondary beam energy at the Lab E location (1116 meters).

In practice, there are deviations from this ideal behavior which lead to an energy dispersion of neutrinos at a given radius, and a change in the event distribution there also. These deviations originate from a number of sources.

(i) Finite Decay Region Length (Parallax): The meson decays occurred over a 350 meter long region (see Figure 2.1). This was an appreciable fraction of the separation between the decay region and the neutrino detectors. For fixed radius at the detectors there are therefore contributions from a range of angles, and thus from a range of energies. This geometry is indicated in Figure 2.10b. The effect of the decay path on the energy dispersion at a setting of 250 GeV is shown as a function of radius by the dotted curve in Figure 2.13. There we plot the ratio,

$$\left(\frac{\Delta E_\nu}{E_\nu}\right)_z = \frac{E_\nu(z = 1290 \text{ meters}) - E_\nu(z = 940 \text{ meters})}{E_\nu(z = 1116 \text{ meters})} \quad (2.19)$$

corresponding to the fractional difference between the maximum and minimum neutrino energies at fixed radius. These deviations are also tabulated for each setting in Table 2.2. The effect upon the flux and event distributions may be calculated by integrating (2.17) and (2.18) above times the decay probability distribution over the decay region. The probability distribution function for decays is given by,

$$\frac{dN_\nu}{dz} = \frac{\exp(-\frac{z}{L_0})}{L_0} \quad (2.20)$$

where $L_0 = \beta\gamma c\tau$, τ is the meson lifetime. If

L_1 = distance from the beginning of the decay region to the detector

L_2 = distance from the end of the decay region to the detector

then equation (2.16) becomes,

$$N_\nu(R, R + \Delta R) = \frac{2N_s B \gamma^2}{L_0} \int_{L_1}^{L_2} \frac{dz \exp(-\frac{(z-L_1)}{L_0})}{z^2} \int_R^{R+\Delta R} \frac{R dR}{(1 + \frac{\gamma^2 R^2}{z^2})^2} \quad (2.21)$$

and equation (2.18) becomes,

$$N_\nu^{int}(R, R + \Delta R) = \frac{2N_s B \gamma^2 E_\nu^{max} N_{TGT}(\frac{\sigma}{E_\nu})}{L_0} \int_{L_1}^{L_2} \frac{dz \exp(-\frac{(z-L_1)}{L_0})}{z^2} \int_R^{R+\Delta R} \frac{R dR}{(1 + \frac{\gamma^2 R^2}{z^2})^3} \quad (2.22)$$

where, N_s is the number of secondaries. With the approximation, $\exp(-\frac{(z-L_1)}{L_0}) = 1 - \frac{z-L_1}{L_0}$ both may also be integrated analytically. These integrals are found in Appendix C. Figures 2.11b and 2.12b show the distributions.

(ii) Angular Dispersion of the Secondary Beam: The beamline magnets were designed to focus the secondary rays into a parallel beam. In practice, this focusing was not perfect, and the secondary rays had some net angular divergence. Assume some ray makes an angle θ_S with the beamline axis. When the secondary decays, the resulting neutrino will have some angle θ_ν with respect to the direction of the secondary. The neutrino detector is nominally aligned with the beamline axis. Thus, the radius at the detector will be determined by a combination of the secondary angle and the decay angle. This effect is indicated in Figure 2.10c. If the neutrino decay angle is of the same order as the secondary angle, the energy/radius relationship described above will be altered. As the parent angle increases, the neutrino appears to originate at the entrance to the decay region rather than from the decay point. In Table 2.3 we list the mean lab neutrino decay angle as a

function of parent energy and mass. Below we will discuss how the secondary angular divergence was determined. The r.m.s. spread of the projected angular distribution was however roughly 0.15-0.20 milliradians. The numbers in Table 2.3 indicates the angular divergence was only an effect for the higher energy pion decays. The contribution of the angular divergence to the neutrino energy dispersion may be estimated by the expression,

$$\left(\frac{\Delta E_\nu}{E_\nu}\right)_{\theta_S} = \frac{2\gamma^2 R \Delta\theta_S}{z(1 + \frac{\gamma^2 R^2}{z^2})} \quad (2.23)$$

This is shown for the 250 GeV setting in Figure 2.13 where we assume $\Delta\theta_S = 0.00025$ radians (space angles) and listed in Table 2.2. Also, to the extent that the secondary momentum was correlated to the secondary angle, there could also have been a systematic shift in the mean neutrino energy at a particular radius.

(iii) Momentum Spread of the Secondary Beam: The $\pm 10\%$ spread in the momentum of the secondary beam contributed to an energy spread at fixed radius. This effect can be estimated by the expression,

$$\left(\frac{\Delta E_\nu}{E_\nu}\right)_{\Delta p} = \Delta E_{meson} \left(\frac{1}{E_{meson}} - \frac{2E_\nu E_{meson} R^2}{E_\nu^{max} m_m^2 z^2} \right) \quad (2.24)$$

which is plotted for 250 GeV mesons in Figure 2.13 and listed in Table 2.2. Note that at each setting this dispersion will always vanish at some radius. The secondary momentum spread was the major contribution to the kaon neutrino energy dispersion.

The Figure 2.14 shows scatter plots of the interacting neutrino energy versus radius at the Lab E detector for a set of secondary beam energies. To evaluate the contributions from (ii) and (iii) on the flux and event yields requires the integrals of be made also over the meson angular and momentum distribution functions (which are

correlated). Rather than parametrize these by guessing the (unknown) functional form, this plot is made from a Monte Carlo calculation which takes into account all the above effects (see Section 2.4). Note that even with the energy dispersion, a clear separation exists at all radii between pion and kaon neutrino events. For 250 GeV, Figure 2.15 shows the radial projections for the flux and events superimposed on the curves of Figures 2.11b and 2.12b. (Note that the Monte Carlo calculation shown includes a square cut on event vertices outside ± 60 inches. This is the source of the abrupt fall off at 60 inches in the figure.) The most significant feature is that the pion decay neutrinos get pushed out to larger radii because of the angular dispersion of the secondary beam. This effect will be relevant to the oscillation analysis discussed in Chapters 5 and 6.

§2.3 Beam Monitoring System

This neutrino oscillation measurement is performed by comparing the event rates recorded simultaneously in the two detectors. There is no necessity to have a detailed knowledge of the *absolute* neutrino flux spectrum since it is common to both event samples. There do though exist dependences upon the flux which enter at the level of a few percent systematic corrections. These corrections are discussed in detail in Sections 5.5 and 5.8., and have their origin in the requirement that both neutrino detectors subtend the same time integrated and geometric flux. The beam related quantities which have a bearing on these corrections are the following:

- 1) total protons on target,
- 2) protons on target gated on detector livetime,

- 3) beam steering,
- 4) beam size and angular divergence.

In this section we describe those elements of the beam monitoring system which were used to measure these quantities. An overview of the system is seen in Figure 2.16. Beam monitors were placed around the primary proton target, at two locations in the decay region, and downstream of the secondary beam dump in the muon shield.

2.3.1 Proton Monitors

The primary proton beam was targeted in the area called Neuhall. An inductive pickup toroid, NTOR, was placed there. The proton beam passed through the center hole of this toroid immediately before striking the production target. The current induced in the windings was related to the beam current. Fast cables linked Neuhall and the neutrino experiment. The toroid signal was used to gate the neutrino detectors on. In addition, the signal was also converted to a pulse train and scaled at Neuhall. The scaler was recorded both over the entire spill (2-3 milliseconds) and up to the time an event trigger occurred in each neutrino detector. In this way total number of protons on target and the total live protons on target for each detector could be determined.

The NTOR readout and gating is diagrammed in Figure 2.17. A significant delay must be introduced before the livetime scaler to compensate for the transit time of the beam to the detectors and the return of the event trigger gate. For the Lab E and Wonder Building livetime scalers, the delays were 13.7 and 6.9 microseconds respectively.

The accuracy of the livetime gating system was tested. This was done by exciting the toroid with a pulser and scaling the output both on a local trigger and on a trigger sent down and back from Lab E. Different pulse widths and shapes were tried. The difference was always less than .1%. If the delay was removed altogether, a difference of up to 3% was observed.

2.3.2 Beam Steering

Beam steering could be adjusted from the neutrino experiments by changing the currents in vertical and horizontal bending dipoles just upstream of the production target. The position of the primary beam near the target was measured by a set of SWICs. The readout from these SWICs was accessible to the experimenters online. The position of the secondary beam in the decay region was also monitored by SWICs in the Expansion Port (136 meters from the beginning of the decay region) and at the Target Manhole (290 meters from the beginning of the decay region). In addition to SWICs there were also split plate ion chambers at these two locations. These devices contained two circular readout electrodes one of which was split top and bottom and the other, right and left. Though not linear in their dependence upon the beam position, the quantities,

$$\frac{LEFT - RIGHT}{LEFT + RIGHT} \quad (2.25a)$$

and,

$$\frac{TOP - BOTTOM}{TOP + BOTTOM} \quad (2.25b)$$

could be used to judge how well the beam was centered. Distributions in these quantities, and in the SWICs were ascertained such that the neutrino beam center remain within a

4 inch square at Lab E. Constant attempts were made to maintain the steering of the neutrino beam near its nominal setting.

2.3.3 Beam Size and Divergence

The angular divergence of the secondary beam as it exited the magnet chain was measured using a device called the hole collimator (HC). Its location is marked in Figure 2.16. The hole collimator was a steel block 4 feet long (7.3 nuclear interaction lengths) and 10 inches $\times$ 6 inches transversely, with a $\frac{1}{2}$ inch square hole running down its axis parallel to the beam. The transverse size of the beam at this point was approximately 4 inches $\times$ 1.5 inches, determined mainly by the magnet apertures. This block was swept transversely, both horizontally and vertically, across the beam sampling small segments at a given position. A special SWIC (XT) was set in the Expansion Port during the hole collimator sweeps. Charge was collected on copper strips etched upon G-10 rather than through thin wires so as to eliminate any gain at the wires which might introduce nonlinearities in the signal. By considering the $\frac{1}{2}$ inch square in the hole collimator to be a point source of particles, the spatial distributions in the XT SWIC were transformed into an angular distribution of rays from that region of the beam. This is indicated in Figure 2.18. The fraction of beam transmitted at a particular collimator position was determined by comparing the response of an ion chamber in the decay region with hole collimator out measurements normalized to NTOR. The separate angular distributions were combined, weighted by the transmission, to produce a total angular spectrum for the beam.

The hole collimator sweeps were carried out with the dichromatic train set to

transmit negative secondaries of momentum $-250, -200, -165, -140$ GeV. The negatives were used because the positive beam composition had a very large proton component. The point was to measure the distribution of particles which give rise to neutrinos and the negative beam is mostly pions and kaons. Monte Carlo calculations indicate that the angular divergence of the pion and kaon components are similar. The negative beam is believed to give a good representation of the actual beam divergence.

The results of a hole collimator measurement of the angular divergence for the dichromatic beam set for a number of settings are shown as the solid curves in Figure 2.19.

A further handle on the beam divergence was available by observing the spatial distribution of muons after the secondary beam dump. A long room (≈ 150 meters) called Enclosure 100 separated the beam dump from the muon shield. Two large (2 meter $\times$ 2 meter) segmented strip ion chambers⁵⁴ (muon SWICs) were placed at two positions in this room. The positions were at 5.5 meters and 29 meters from the end of the beam dump. Distributions in these devices could be compared with Monte Carlo calculations based upon models of the secondary beam properties (see Section 2.4). Figure 2.20 shows measured muon distributions at the two locations in Enclosure 100 for a $+165$ GeV setting of the dichromatic beam. Each of these profiles corresponds to one spill of the accelerator and contains $\approx 10^9$ muons. The horizontal profiles are offset because of space limitations in Enclosure 100. The narrow peak corresponds to muons from pion decay while the wide tail corresponds to muons from kaon decay. In addition, there is a broad background under the profiles due soft spray and decays emerging from the beam dump. This background

is about 15% of the profile area at the close location at +200 GeV and drops to 3.3% at +120 GeV. It is roughly half at the far location. These devices were initially installed to cross check absolute flux measurements made with ion chambers in the decay region for normalized experiments.

2.3.4 Other Monitors

In addition to the components described above there were other elements in the beamline monitoring system⁵⁵ which were used indirectly in the oscillation experiment. A differential gas Cerenkov counter could be moved into the beam at the Expansion Port. The Cerenkov counter was used to measure the fraction of pions, kaons, and protons in the secondary beam in the decay pipe. These particle fractions were used as input to the decay Monte Carlo programs such as the one which simulated muon distributions in the shield. Between +100 and +250 GeV respectively they ranged from 60% to 7% for pions and 6% to 1% for kaons. Total intensity monitors, used primarily for normalized measurements, included helium filled ionization chambers in the Expansion Port and Target Manhole, and a radio frequency cavity at the Expansion Port. The RF cavity was tuned to the radio frequency of the accelerator. During hole collimator studies, these total intensity monitors were used in conjunction with NTOR to measure the relative beam transmission.

§2.4 Beam Monte Carlo

2.4.1 Beam Models

A Monte Carlo program was developed to produce a neutrino beam which could be used to simulate the oscillation experiment. The beam Monte Carlo consisted of a model of the secondary beam which specified the coordinates $(x, y, \theta_x, \theta_y, p)$ of a given secondary particle, pion, kaon, or proton at the entrance to the decay region. The mesons were decayed according to the proper decay distribution for the given momentum. Neutrinos were traced to the detectors and muons were further subjected to multiple coulomb scattering in the beam dump at the end of the decay region and traced through the muon SWICs. Another Monte Carlo program discussed in Section 5.7 simulated neutrino interactions in the detectors.

Two approaches to the problem of specifying the secondary beam model were tried. The first approach was to use secondary particle production spectra based upon the CERN⁵² measurements as input to a magnetic ray tracing program, DECAY TURTLE⁵³. This program used surveyed positions of beamline elements (magnets and collimators) and measured magnet currents to produce files of pions, kaons, and protons at the entrance to the decay region. Each particle was specified by the five values of its phase space coordinates $(x, y, \theta_x, \theta_y, p)$. Arbitrarily many particles could be traced. This is the method which has traditionally been used in past analyses.

The second approach was to rely more directly upon measurements and use the hole collimator data discussed in Section 2.3 to construct files of secondary rays. To do this, the following approach was adopted. A transverse point at the location of the hole collimator was chosen at random according to the measured spatial distribution of

the beam. The angles, θ_x and θ_y , were chosen according to the angular distributions measured by the hole collimator. This procedure produced four of the five phase space coordinates required to specify a beam particle, $(x, y, \theta_x, \theta_y)$. Again, any number of beam particles could be specified in this manner. As described in Section 2.2, the momentum dispersion of the dichromatic beam results in complicated correlations between momentum and other coordinates. Since the momentum spectrum of the beam transmitted at each separate collimator position was not measured, no quantitative experimental handle on these correlations existed. No simple function was known which could uniquely assign a momentum to the vector $(x, y, \theta_x, \theta_y)$. The beam produced by DECAY TURTLE, based completely upon a calculation may quantitatively misrepresent the behavior of certain trajectories, especially those far from the central ray. In a qualitative way however, it probably does correctly describe the momentum correlations in the beam. We used the form of the correlations predicted by the DECAY TURTLE program as a guide in assigning momenta to rays produced from the hole collimator data. A subset of DECAY TURTLE beam phase space corresponding to (x, y, θ_x) was divided into a $10 \times 10 \times 10$ bin grid. In each bin were stored the DECAY TURTLE predicted momentum spectrum. For each hole collimator ray we randomly sampled the corresponding momentum spectrum.

2.4.2 Comparison of Beam Models

Both the DECAY TURTLE beam and the hole collimator beam were used to predict distributions in the decay region, the muon shield, and the neutrino detectors. These distributions could be compared to measurements made with the beamline monitors

and with the neutrino detectors. These measurements include the spatial and angular profiles of secondaries in the decay region, the muon distributions, and energy verses radius distributions for pion decay neutrino interactions in the detectors. All of these quantities reflect, in varying degree, the secondary beam phase space. Here we describe these comparisons.

1) Decay Region Distributions These are the measurements most closely coupled to the beam phase space. They by definition agree with the HC beam since it was produced from these measurements. In Figure 2.21 we show the x and y spatial profiles as measured by the XT SWIC at the Expansion Port. Superimposed, and normalized to the peak, are the DECAY TURTLE predictions for the corresponding momentum settings. The obvious feature is that the DECAY TURTLE beam is broader in y. In this regard we note that the beam size in y is strongly coupled to the proton spot size in y at the production target. This is because the magnets mainly focus in the vertical plane (while dispersing in the horizontal). Though this discrepancy may appear large, the transverse beam size has negligible effect on the distributions of neutrinos at the distant detectors. There, the lever arm is large (700-1100 meters) and the beam grows considerably in size because of angular dispersion.

In Figure 2.19 we show the θ_x and θ_y distributions for secondaries as measured by the hole collimator technique. Superimposed again are the corresponding DECAY TURTLE distributions. Here, reasonable agreement is seen in the θ_y comparison, and in the central peak of the θ_x distribution. This is born out quantitatively in Table 2.4 where we list the full width at half maximum for these distributions. The major discrepancy is

in the asymmetric large angle tail on the θ_x plots. This tail probably arises because the magnets momentum disperse the beam in the horizontal plane.

2) Muon Distributions: The muon distributions are coupled to the secondary beam phase space primarily through the beam divergence. These distribution are however taken after non-negligable multiple coulomb scattering has occurred in the decay region beam dump. For the amount of material present, the gaussian sigma on the multiple coulomb scattering distribution is roughly 0.0017 radians at 165 GeV. The muon distributions are not strongly coupled to the momentum dispersion of the secondary beam. In Figure 2.22 we show the vertical and horizontal profiles measured 29 meters downstream of the beam dump. Here, background spray from the dump should be small. The increase in width as the beam momentum decreases is clearly seen. Superimposed are predicted profiles calculated with the beam Monte Carlo described in above. Again, reasonable agreement is observed in the vertical view for both beam models while the DECAY TURTLE beam is too narrow in the horizontal. The HC model beam is seen to agree well with the measured muon distributions.

3) Neutrino Distributions: This discussion is somewhat premature since we have not yet described the neutrino detectors. For the sake of simplicity we assume that the detectors provide a measurement of the neutrino interaction vertex and the incident neutrino energy. From the discussion in Section 2.2 it is clear that the radial and energy distributions of events at the neutrino detector may depend upon the secondary beam phase space. There we indicated this to be particularly true for neutrinos which result from the pion decays. These distributions are coupled in more than one way to the secondary beam phase space.

This is because the number of interactions at some radius depends both upon the number of neutrinos there and also their energy, through the total cross section. Thus, an observed discrepancy between the measured distributions and the Monte Carlo predictions is not simply correlated to one particular feature of the model phase space.

In Figure 2.23 we show the ratios of the radial event distributions in 5 inch bins for the pion decay neutrino data and the two beam models for settings of $+200$ GeV and $+165$ GeV. The normalization is arbitrary. Pions are shown because as mentioned in Section 2.2 they are most closely coupled to the beam phase space. In Figure 2.24 we show the corresponding mean radial energy ratios. Both sets are in Lab E. Here the agreement between the data and the DECAY TURTLE model is reasonable. The HC model beam is however seen to be considerably and systematically broader than the data. As we mentioned above, it is not clear whether this is caused by the divergence or the choice of momentum assignment. In light of the nice agreement between the HC model and the data for the muon distributions this is somewhat disturbing.

To simulate the neutrino oscillation experiment some choice between the DECAY TURTLE and HC beam models had to be made. Considering all the comparisons, we used as a standard, the DECAY TURTLE beam wherein momenta have been assigned in a natural way. The predictions of the HC model beam, in particular the observed discrepancies between it and the DECAY TURTLE model for the decay region, were used as an estimate of systematic uncertainties in our knowledge of the beam divergence. These estimates were a major contribution to the ultimate systematic error on the oscillation measurement. These issues are discussed again in Sections 5.8 and 5.9.

Chapter 3

Neutrino Detectors

§3.1 General Criteria

The oscillation measurement relied on a comparison of event rates for the charged current deep inelastic neutrino nucleon scattering process,

$$\nu_{\mu} \text{ nucleon} \rightarrow \mu^{-} X \quad (3.1)$$

where X is any hadronic final state. The neutrino detectors used in E-701 were optimized to study this reaction.

The first detector, located at Lab E, 1116 meters from the center of the decay region was a reduced version of the Caltech-Columbia-Fermilab-Rochester-Rockefeller (CCFRR)^{56,57,58,59} neutrino detector previously used in a series of high energy neutrino scattering experiments (E-482,E-356,E-616). The second detector, which was constructed

specifically for E-701, was located at the Wonder Building, 715 meters from the center of the decay region.

The cross section for reaction (3.1) is small ($\approx 10^{-38} \text{ cm}^2$ at 100 GeV). As a result, the neutrino detectors are required to be massive in order that there be a reasonable interaction rate. To observe one neutrino interaction from a flux of 10^9 neutrinos at 100 GeV, $\approx 10^{31}$ nucleons/cm² are required. In the neutrino detectors used in this experiment the mass was provided by steel plates. The Lab E and Wonder Building detectors had total masses of 1000 and 200 tons respectively. The major demands upon the individual detectors were that they be able to reconstruct certain kinematical quantities which characterized the reaction 3.1, in particular the incident neutrino energy, the interaction vertex in space, and the muon trajectory. To determine the incident neutrino energy, a measurement of the muon momentum and the energy of the hadronic state X was required.

Figure 3.1a and b are scale drawings of the two detectors, their detailed geometry is indicated in Figures 3.2 and 3.3. Both consisted of two components. Upstream was a combined neutrino interaction target and hadronic calorimeter which consisted of alternate sheets of steel and scintillator. These were followed by magnetized steel toroidal spectrometers to measure the muon momentum. The magnet current was set to focus positive muons. The transverse position of the muon was tracked with spark chambers distributed in the target and about the spectrometer.

In the steel target calorimeters, the process (3.1) was characterized by a local deposition of hadronic energy just downstream of the interaction vertex and a single highly penetrating minimum ionizing particle, the outgoing muon. At the interaction vertex,

the energy of the incoming neutrino was divided between the scattered muon and the fragmented target nucleon. In iron, the muon, which interacts only electromagnetically, will in general travel many tens of meters before losing its energy and ranging out. The target nucleon fragments, strongly interacting, will immediately initiate a hadronic cascade with a characteristic longitudinal extent of $\approx 1 - 2$ meters. This hadronic cascade was sampled by the interleaved scintillator planes. The detector properties are summarized in Table 3.1

To minimize systematic effects in the oscillation analysis, such as biases in acceptance, the two detectors were geometrically very similar. By a choice of restrictions upon the target fiducial volumes, an exact correspondence was established between the surface density of target nucleons at the two detectors. This ensured an equal interaction rate for an equal incident neutrino flux.

In the following sections we describe in more detail the various features of the neutrino detectors.

§3.2 Calorimetry

The reaction (3.1) is displayed pictorially in Figure 3.4. The primary nucleon fragmentation has been observed in bubble chamber experiments⁶⁰. From these it is seen that the nucleon breaks up into 5-10 hadronic particles. In steel, these primary hadrons will quickly reinteract and initiate a hadronic cascade. Through this process the initial hadronic energy will eventually be absorbed in the steel. A large fraction of it will appear as soft particles which lose their energy through ionization. This ionizing energy

may be determined by the technique of sampling calorimetry. In this technique, a low Z material such as scintillator, is interleaved in the shower. Therein some fraction of the ionizing shower energy will be deposited. The intensity of light (or pulse height) emitted by the scintillator is proportional to this ionizing energy. The sum of the pulse height from all the samples is proportional to the total shower energy. This proportionality may be calibrated by placing the sampling calorimeter in a hadron beam of known energy. Hadronic energy was measured in both the Lab E and Wonder Building detectors by sampling calorimeters. These consisted of scintillator layers between 10 centimeter lengths of iron. In these devices the iron acted as both a heavy target for neutrino interactions and a passive absorber for the hadronic shower development.

The energy resolution of a sampling hadronic calorimeter is limited by the effects of sampling fluctuations and shower fluctuations. Sampling fluctuations refer to the statistical variations in the number of ionizing particles crossing a scintillator segment. In a calorimeter with vanishingly thin sheets of (iron) absorber, these fluctuations are negligible. In the case of the 10 centimeter calorimetry of the Lab E and Wonder Building detectors, they are a major contribution to the energy resolution. Besides the sampling fluctuations, the shower fluctuations represent an intrinsic limitation to the resolution of hadronic calorimeters. If the showering process were to proceed by only electromagnetic interactions, all the shower energy would eventually appear in the form of photons, electrons, and positrons. In hadronic interactions there are large probabilities for nuclear excitations, production of short range nuclear fragments, and production of leptons which subsequently decay into muons and neutrinos. Through these processes, some shower energy can go undetected by the calorimeter. The fraction of unmeasured

energy is subject to statistical fluctuations. This is easily visualized by considering a shower initiated by a single charged pion. If in its very first interaction the pion undergoes a charge exchange, all the shower energy will appear in an electromagnetic cascade. If instead, the pion participates in a series of nuclear interactions, some of the initial energy will go unmeasured.

The hadronic energy resolution of the Lab E calorimeter has been directly measured in a test calibration beam (see Section 4.2.1) to be,

$$\frac{\sigma_E}{E} = \frac{0.89}{\sqrt{E}} \quad (3.2)$$

Other studies of the behavior of hadronic calorimeters discussed in the literature⁶¹ suggest that this resolution is determined roughly equally by sampling and shower fluctuations. With identical sampling, the resolution of the Wonder Building calorimeter is equal to this also. This calibration test, carried out in conjunction with an earlier neutrino experiment (E-616), was also used to determine the absolute relationship between pulse height from the Lab E calorimeter and hadronic energy. No such measurement was made for the Wonder Building calorimeter but a cross calibration using the properties of the dichromatic beam was possible. This is discussed in Section 4.2.2.

3.2.1 Lab E Calorimeter

The Lab E calorimeter was divided into four carts. Each cart consisted of twenty-eight 5 centimeter steel plates, fourteen scintillation counters, and six spark chambers. The counters, each placed after 10 centimeters of steel, were 3.05 meter $\times$ 3.05 meter

×2.54 centimeter plexiglas tanks of liquid scintillator. The pattern of material in the target is displayed in Figure 3.5a. A detail of a calorimetry counter is shown in Figure 3.6a. Primary ionization initiated the emission of UV light which was reabsorbed after approximately 1 mm. Isotropic light shifted blue was reemitted by a secondary fluor. The attenuation length of the blue light was much longer and this light travelled to the edges of the counter. There it crossed an air gap and entered a BBQ doped wavelength shifter bar. The tertiary reemitted light was green and travelled along the bar by multiple internal reflections. Eight shifter bars, each 1.53 meter long surrounded the counters. Pairs met at each corner and were there coupled to one of four RCA 6342A photomultiplier tubes. A typical minimum ionizing particle produced 16 photoelectrons in the four tubes together. Light was collected with an efficiency of roughly 12%.

By using wavelength shifter technology a large area could be read out with a minimum of photomultiplier tubes. Since a large area was being read out, the response of the counter versus transverse position was not uniform. It could vary up to 40%. Spatial response maps were measured and used to correct light signals for these variations. These maps are discussed in more detail in Chapter 4.2.1

Over the course of running an experiment, which typically lasts many months, the response of the calorimeter can vary. These variations can originate from chemical changes in the scintillator or changes in the gains of the photomultiplier tubes. Time variations in overall counter response were monitored by observing the pulse height in a counter due to a single minimum ionizing particle (a muon). Time variation in photomultiplier tube gains in the Lab E calorimeter were monitored by a light flasher

system. Light from a spark gap was transmitted via fiber optics to the center of each counter. There the light was dispersed through a translucent white spot to the four tubes. The varying response of the four tubes to this standard signal was periodically recorded and gains were adjusted. In case the flasher signal itself varied, the flasher light was simultaneously recorded by a special reference photomultiplier tube and a reference photodiode. The photodiode in particular was supposed to be a stable light monitor.

Once time variations are known it is also important to measure the relative gains of the four tubes in a counter. This was done by observing the response of the tubes to an isotropic source of light at the counter center. This light was produced both by placing a small radioactive source at the counter center and by hadron showers initiated near the center. In Chapter 5 we discuss the hadron energy calculation and return to these issues.

The readout front end electronics for the calorimetry is depicted in Figure 3.7a. Individual tubes as well as counter sums were digitized and discriminated as trigger latches. Since the calorimeter was required to respond to both minimum ionizing particles (single muons) and hadron showers consisting of many particles a large dynamic range was needed. Historically at Lab E both a "high" gain ADC for muons and a "low" gain ADC for hadron showers were used. The "high" ADC digitized the sums of four tubes from each counter. The "low" ADC digitized the signal of each tube. In E-701 the low ADC's were 15 bit Lecroy model 2280's. With such a large dynamic range, only this system was needed for all energy measurements. It could maintain most hadron showers below saturation and still resolve single particles. Thus the highs were unnecessary. For events

in which particular lows saturated, a special "superlow" was available. The superlow was a sum of tubes from widely separated parts of the apparatus. Since the hadron showers were localized in the detector, it was unlikely that more than one tube in a superlow combination would see the same large light signal.

3.2.2 Wonder Building Calorimeter

The Wonder Building calorimeter consisted of fifty-six, 1.53 meter $\times$ 1.53 meter $\times$ 2.54 cm acrylic scintillator planes between 10 centimeter lengths of steel. A detail is shown in Figure 3.5b. The plastic scintillator was cast by Polycast Inc. and was a mixture of 83.99% PMMA acrylic, 15% Napthalene, 1% Butyl PBD and 0.01% BBOT. This formula yields about 70% of the light output of NE110. The typical attenuation lengths for this scintillator were measured to be $\approx$ 1 meter. The counters were surrounded by four BBQ doped wavelength shifter bars, each 1.53 meters in length. Pairs met at two opposite corners and light was measured by two RCA 6342A photomultiplier tubes. The other ends of the bars were coated with reflective tape. Figure 3.6b shows the layout of one counter. The corners near the phototubes were masked with reflective tape to prevent large pulse heights resulting from showers very near the edge. If the two tubes were summed, then the spatial variation across the counter was observed to be less than 15 %.

As in Lab E, photomultiplier tube gains in the Wonder Building detector were tracked with a flasher system. The flasher signal was monitored by a reference tube which simultaneously viewed light from an NaI crystal containing an impregnated ^{241}Am source.

The output of the reference tube was fanned out to an ADC for the flasher light as well as to a pulse height analyzer for the NaI light.

The readout electronics in the Wonder Building were similar to the Lab E system. Both high, low, and superlow signals were digitized. Here though all the ADC's were 10 bit LeCroy 2249's. With this smaller dynamic range, all three sets of ADC's were necessary. A diagram of the front end is shown in Figure 3.8a.

§3.3 Muon Tracking

Muons were tracked with spark chambers. In the Lab E target there were twenty-four, 10×10 foot (3.05 meter) chambers, located after every 20 centimeters of steel. Each of three field free gaps between magnetic toroids contained five, 10×10 foot chambers also. Additional high magnetic permeability shielding served to isolate the spectrometer chambers from stray magnetic fields. The Wonder Building target contained seventeen, 5×10 foot spark chambers. These were larger than the transverse size of the Wonder Building steel. Occasionally tracks would leave the target but still enter the spectrometer. As a result the full chamber could be used for tracking. Upstream of the magnetic toroid, but completely outside the target steel, were placed four planes of 5×10 foot chambers (2 per plane) covering an area of 10×10 feet. Downstream, this configuration was repeated with six additional planes. Figures 3.2 and 3.3 show the exact detector layouts.

The spark chambers were constructed of two 1 inch thick sheets of aluminium hexcel panels separated by a $\frac{1}{2}$ inch plexiglas spacer. The gap was sealed and a gas

mixture, 90/10% helium and neon, with $\approx 1\%$ ethyl alcohol circulated inside. The two inner faces were covered with mylar backed sheets of 1 mm spaced wires running horizontally on one and vertically on the other. Upon passage of an ionizing particle across the gap, a 10 KV pulse could be applied between the wire planes. This resulted in a breakdown of the gas at the location of the ionization. A current pulse travelled up the nearest wire. This wire passed under a magnetostrictive line running orthogonal to it along the edge of the chamber. The arrival time of an acoustic pulse at the end of the magnetostrictive line indicated the location of the spark along that direction. In addition to the spark induced acoustic pulse, every firing of the chamber was accompanied by the generation of two reference fiducial pulses from the two ends of the wire. These set the distance scale. The pulse arrival times and fiducials were digitized and recorded on magnetic tape. Up to 16 hits per view in each chamber could be accommodated by the electronics. The transverse position resolution for hits in the spark chambers was 0.5 mm.

The 10 KV pulse on the spark chambers was 200 ns long. Electrical energy was stored in banks of 24 (per chamber) coiled 120 foot long high voltage charge lines. The voltage was switched across the chamber by a thyratron tube. After firing, the ionization was swept away in approximately 30 ms by a 90 volt DC and a 600 volt pulsed clearing field.

The coordinate system imposed upon the detectors was right handed, with z running downstream. The z position of each spark chamber was surveyed to 0.01 inch. The transverse position of each chamber was fit for using tracks of muons which passed through the entire apparatus. At Lab E, rotations about the z axis were measured with

a plumb bob and these angles were included in the alignment fit.

§3.4 Muon Spectrometer

Muon momentum and sign were measured by observing the particle's trajectory with spark chambers as it passed through a magnetized steel toroid. Muons entered the spectrometer from a direction normal to the toroid face. They saw an axial magnetic field. The general effect was to focus or defocus depending upon the charge and field direction. For neutrino running (positive secondaries), the magnet focused negative muons. If upon entering the field, the particle had some axial component to its momentum, then in addition to being focused, it would also execute a spiralling trajectory about the toroid axis.

Small air gaps were set in the steel to allow the insertion of Hall probes for magnetic field measurement. These measurements were used as input to numerical magnetic field calculations using the POISSON⁶² code. In general the field was known to $\approx 1\%$.

The Lab E spectrometer consisted of three 1.6 meter long iron toroids with an outer radius of 1.8 meters and an inner radius of 12.7 centimeters. The field in the iron was roughly 17 kilogauss and each of the three toroids delivered a transverse momentum kick of 0.8 GeV (total $p_T=2.4$ GeV). The Wonder Building spectrometer consisted of only one toroid, 2.4 meters in length with an outer radius of 1.5 meters and an inner radius of 12.7 centimeters. The spectrometer delivered a transverse momentum kick of 1.2 GeV.

The average momentum resolution of the spectrometers was determined by the amount of steel the muon had to pass through and the strength of the magnetic field. For Lab E and the Wonder Building, $\frac{\Delta P_{\text{muon}}}{P_{\text{muon}}}$ were 11% and 16% respectively.

§3.5 Triggers

The event sample used in the oscillation experiment consisted of a subset of all the events for which the total neutrino energy could be measured. Because of the size of the target calorimeters, and the acceptance of the spectrometers, only some fraction of all the neutrino interactions observed will be completely measurable. The primary trigger used to define the oscillation analysis sample was therefore designed to select only those events for which the total energy could be determined. This trigger, which was arranged in a similar way in Lab E and the Wonder Building, was known as TRIGGER 1, or the MUON trigger. In addition to TRIGGER 1, a number of other triggers were also configured. They included TRIGGER 2 or the neutral current (NC) trigger, TRIGGER 3 or the PENETRATION trigger, TRIGGER 4 or the EFFICIENCY trigger, and TRIGGER 6 or the STRAIGHT-THROUGH trigger. Both TRIGGER 3 and TRIGGER 4 were useful for determining efficiencies. TRIGGER 6 was used to select muons which entered from upstream. These muons were useful for aligning the detectors.

Available to the fast trigger logic were a beam spill gate from the accelerator (fast spill or PINGS), single and twice minimum ionizing latches from each calorimetry counter, crude total hadronic energy latches, signals from veto counters upstream of the detectors, and signals from trigger counters up and downstream of the spectrometers. The

trigger and veto counters were planes of NE110 plastic scintillator viewed by wavelength shifter bars and RCA 56AVP photomultiplier tubes. The position of the veto and trigger counters is shown in Figures 3.2 and 3.3. The veto counter in Lab E was composed of twelve 10×2.5 foot sections and covered a total area of $\approx 15 \times 20$ feet. Each section was viewed by two photomultiplier tubes. The veto counter in the Wonder Building was composed of two 5×10 foot panels of plastic scintillator, and covered an area of 10×10 feet. Each section was viewed by two photomultiplier tubes. The two Lab E trigger counters, T2 and T3 each covered an area of 10×10 feet. For mechanical reasons they were segmented in half with the top and bottom on opposite walls of the toroid gap as shown in Figure 3.2. The Wonder Building trigger counters T0, T2, and T3 were each 10×10 feet. The T0 counter was composed of two 2.5×10 foot sections and one 5×10 foot section. The T2 counter was composed of four, 2.5×10 foot sections. The T3 counter was composed of four 5×5 foot sections.

Because of the slightly different layout of the two detectors, the triggers varied somewhat in their implementation at the two locations. Here we state first the the general requirements that the four triggers fulfilled and then describe the particular arrangements at Lab E and the Wonder Building.

1) TRIGGER 1 (MUON): This trigger required the accelerator spill gate, the veto to be off, a well defined minimum ionizing track in the target, and a signal from a trigger counter after the spectrometer. The last requirement ensured that the muon could be momentum analyzed.

The detailed TRIGGER 1 logic from Lab E is shown in Figure 3.7b. The

target track requirement was fulfilled by requiring a minimum ionizing track to extend through the entire most downstream cart. While there was no minimum hadronic energy requirement in TRIGGER 1, the placement of the counter T2 after the first iron toroid forced the muon to have a minimum momentum at the toroid front face. Nominally this would be approximately 4 GeV due to 90 inches of steel but because of the magnetic field it becomes effectively 6.5 GeV.

The TRIGGER 1 logic for the Wonder Building is shown in Figure 3.8b. Here the target track requirement is fulfilled by signal from groups of four adjacent counters as indicated in the schematic. Trigger counters T1 and T2 surrounded the spectrometer and ensured a momentum analyzed muon. Since there is only one toroid in the Wonder Building detector, TRIGGER 1 here implies a somewhat stiffer muon momentum cut of $p_\mu > 7.5$ GeV.

TRIGGER 2 (NC): This trigger was effectively determined by a minimum amount of localized hadronic energy in the target calorimeter. Its purpose was to select candidate weak neutral current events. TRIGGER2 was never used in the oscillation analysis and we therefore do not discuss it further here.

TRIGGER 3 (PENETRATION): This trigger was designed to select probable charged current interactions which were not necessarily accepted by the spectrometer. In particular, these events may have low momentum muons which range out before the T2 counter or be at high y ($= \frac{E_{\text{hadron}}}{E_\nu}$) where the muon angle is large. Because of the properties of the dichromatic beam, the total energy of most TRIGGER 3 only events can be estimated on a statistical basis. In a total neutrino cross section analysis of the

data they are used. In a similar way they could have been used in the oscillation analysis event sample, but they were not. Here only the subset of TRIGGER 3 events which do not also fire TRIGGER 1 but extrapolate to within the T2 counter were used to estimate the efficiency of TRIGGER 1 (see section 5.5).

The Lab E TRIGGER 3 logic is shown in Figure 3.7c. The basic TRIGGER 3 requirement was that $E_{hadron} > 5$ GeV and at least 16 target counters fire. The hadron energy requirement served to eliminate most beam gate cosmic ray muon triggers.

The Wonder Building TRIGGER 3 logic is shown in Figure 3.8c. The hadron energy requirement here was also 5 GeV but the track was required to pass 14 counters. In this trigger and in Lab E a counter multiplicity of 12 was included in the logic also to better fix the trigger timing.

TRIGGER 4 (EFFICIENCY): This trigger was meant to be redundant with a subset of TRIGGER 1's to provide a monitor of TRIGGER 1 efficiency.

For Lab E, the TRIGGER 4 logic is shown in Figure 3.9a. Here, the veto was replaced by a set of calorimetry counters in an upstream region of the target which was outside the Lab E fiducial volume. The trigger required: no veto, $E_{hadron} > 5$ GeV, one of four calorimetry counters in the downstream target, and the T3 trigger counter after the second toroid. This last requirement meant that the effective muon momentum cut was larger than for TRIGGER 1. This restrictive choice for the TRIGGER 4 logic was designed to ensure that all TRIGGER 4 events would be both TRIGGER 1 also and real charged current muon neutrino interactions, not cosmic ray muons. Except for the regions of the event sample with range out muons and with very low y , an efficiency could

be measured using TRIGGER 4.

The Wonder Building TRIGGER 4 logic is shown in Figure 3.9b. The veto was substituted for by the two most upstream calorimetry counters. In addition, this trigger required a penetration of greater than 20 counter. The spectrometer acceptance was indicated by the trigger counter plane T3. The question of TRIGGER 1 efficiency and the use of TRIGGER 4 are discussed again in Section 5.5.

§3.6 Target Mass and Fiducial Volume

The number of neutrino interactions in a detector is proportional to the surface density of target nucleons. A requirement of the oscillation experiment is that a well defined relationship exist between the active target mass in the two detectors. Since both target calorimeters were constructed out of 5 centimeter steel plates, a straightforward correspondence could be established by including the same number of plates in each longitudinal fiducial region. To first approximation then both target surface densities should be the same. Small corrections entered because of the presence of extra material in the target calorimeters. This material was in the form of counters and spark chambers. An explicit accounting of this material is listed in Table 3.2. From this accounting there is $3.6 \pm 0.013\%$ more material in the Lab E target than in the Wonder Building. Of this difference, approximately 75% is water. This water is in polyethylene bags sandwiched between the steel and plexiglas counters in Lab E. Its function is to support the counters from bulging outwards. A small correction (.0017) included in this ratio originates from

the fact that water has a different neutron to proton ratio than iron. Because of this the interaction probability per nucleon is different (by 6%) in water and iron.

The full transverse dimensions of the two detectors were different, and not totally optimum for the beamline geometry at Fermilab. In the analysis the transverse fiducial dimensions were cut to scale between the two detectors to ensure that similar solid angles were accepted. Events which occurred outside a particular region were rejected from the data sample. These fiducial volumes were determined by the smaller Wonder Building size. A considerable number of events which occurred at the outer edges of the Lab E detector, though still well contained in the apparatus, were not used in the oscillation analysis. In the data taking however, these interactions were triggered upon and thus the Lab E apparatus had a correspondingly larger event induced deadtime.

§3.7 Data Acquisition System

Separate, though similar data acquisition systems were operated at the two locations. Event information collected included trigger latches, various bits, spark chamber data, digitized counter pulse heights, and ADC pedestals. These were read in through standard CAMAC to Digital Equipment Corporation PDP 11/50 computers and written to magnetic tape. Separate tapes were written at Lab E and the Wonder Building. Status latches indicating the state of the other detectors operation were available at each location. A sample of data also entered an online data processing program based upon MULTI⁶². This monitored the operation of many of the detector and beamline components and signaled failures of them. The beamline system including magnets and monitors (see Chapter

2) was controlled by a Lockheed MAC computer. Data from the MAC was available at the experiments and a separate MAC (monitor) record was written to tape after each spill of the accelerator. This information included the responses of various intensity monitors gated both upon the spill and event triggers, SWIC profiles, beam steering, and accelerator PULSE, DATE, and TIME. The MAC was linked to the experiment PDP 11/50's so that monitor information was available to both online processing programs.

The two detectors operated independently, and no attempt was made to correlate run numbers. Incident flux could still be correlated on a spill by spill basis using the common monitor information included in the MAC record at each location.

Chapter 4

Event Reconstruction

§4.1 Introduction

Experiment 701 was preceded by a series of neutrino experiments with the Lab E apparatus (E-616, E-482, E-356) and with an earlier detector at the Wonder Building (E-21). The software used to reconstruct neutrino interactions in the Lab E detector was largely developed during the analyses of these other experiments. Much of the software has been described in detail in previous theses^{54,55,56,57}. Here we will not attempt to entirely repeat these discussions, but instead summarize the procedures used and concentrate on features unique to E-701. The codes used to analyze events in the Wonder Building detector had this same origin but to some extent evolved differently in response to problems which emerged there.

The event analysis effort was divided geographically. The Lab E event analysis was coordinated from Nevis Laboratory while the Wonder Building analysis was coordinated from Fermilab. At a later stage, described in Chapter 5, monitor, Lab E, and Wonder Building reconstructed event records were merged onto a single summary tape for use in the oscillation analysis.

The goal of the event reconstruction is to take measurements recorded from the detectors, spark positions, and counter pulse heights, and recover the kinematical variables which specify an event. For the oscillation analysis we will be interested particularly in the incident neutrino energy. To calculate this requires a knowledge of the muon momentum (P_μ) and the hadronic energy (E_{hadron}). In addition, it is necessary to know the muon target track which is described by a space angle, (θ_μ) and an interaction vertex.

In Figures 4.1 and 4.2 we display computer drawn plots of typical charged current interactions in the Lab E and Wonder Building detectors which satisfy the TRIGGER 1 requirement. Crosses indicate sparks and the "thermometers" above the target indicate pulse height output by the scintillation counters. As suggested by these plots, the event reconstruction can be divided into three tasks: calorimetric reconstruction, track finding, and track fitting. In the first task, scintillation counter information alone is used to find a longitudinal interaction vertex and the hadronic energy deposited in the target. With a longitudinal vertex located, the track finding algorithms can begin to choose likely spark candidates upon the muon track, first in the target, then in the spectrometer. Finally, the track is fit for a muon trajectory, defined by a vertex, a space angle, and a momentum. In practice, the procedure is iterative. For example, the track fit χ^2 can be used as the

basis of a reselection of spark candidates. In this way, the track selection can be improved and a better determination of the muon angle and momentum made.

In practice, the raw data tapes from the experiment were first copied and compressed. The compressed tapes were read by a preprocessing program called the STRIPPER. The main reconstruction occurred in a program called the CRUNCHER. Certain correction to the data which had to be calculated required the data to be crunched first. These corrections were then applied in a second processing program call the DST WRITER. This program also wrote a summary tape of all legitimate events. We return to this description again in Section 5.2 where event accounting for the oscillation analysis is discussed.

In the following sections we describe in more detail the event reconstruction procedures.

§4.2 Calorimetry

Here we discuss the procedures used to extract energy measurements from the calorimeters. These devices were described in Section 3.2. There we indicated that some fraction of the energy of a hadron which interacts and showers in the calorimeter will trigger the emission of scintillation light. This light is converted into charge into the photomultiplier tubes. It is the goal of the calorimetric reconstruction to convert the observed light intensity into an estimate of the total energy deposited in the calorimeter.

In an ideal situation the calorimeter's energy response would be measured in

a calibration beam of known energy. This measurement would provide the conversion constant from charge to energy. In actuality, the process is complicated by a number of effects. We list these here.

- 1) Readout: Photomultiplier tube signals are digitized by ADCs. The ADC outputs may be nonlinear and have nonzero offsets (pedestals).
- 2) Photomultiplier Tube Gain: The photomultiplier tubes have large gains which may vary with time. These variations, or drifts, can be caused by changes in the tube itself or changes in the high voltage supplying the tube.
- 3) Scintillator Response: The overall response of the scintillator to a standard amount of ionization may vary with time. This can be due to chemical or physical changes in the scintillator. The variation may differ from counter to counter.
- 4) Spatial Response: The scintillation counters have transverse dimensions which are on the order of their optical attenuation lengths. As a result, equal light at different places on the counter will give rise to different response from the photomultiplier tube.

As a result of these effects, certain corrections must be applied to the raw ADC outputs which are recorded with each event. The determination and application of these corrections differed slightly between the Lab E and Wonder Building analysis and we discuss them separately below.

Before discussing the event reconstruction, it is useful to describe the so-called "minimum ionizing" signal. This is a convenient standard energy unit. Such a standard unit can be used both to track the calorimeter's response with time, and also gauge

the magnitude of hadronic energy deposition in a physically meaningful way. In the discussion of the reconstruction procedure we will give it an exact definition, but it roughly corresponds to the energy deposited by a high energy muon as it crosses (for example) a scintillator plane. Thus the signal from a single muon would be "one times minimum", from two, "two times minimum" and so forth. The concept of minimum ionizing is physically useful because each particle within the hadron shower deposits approximately the minimum ionizing energy in a scintillator plane. As a result, the shower energy can be readily expressed in terms of "times minimum".

4.2.1 Lab E Hadronic Energy Determination

The Lab E calorimeter was calibrated in hadron and muon beams of known energy. This calibration was done in conjunction with the previous run of the dichromatic neutrino beam (E-616). The calibration and analysis are described in an earlier thesis⁵⁸. The results are shown in Figures 4.3a and b. They indicate that a minimum ionizing particle in a hadronic shower deposits 0.216 GeV in each Lab E counter and that the hadronic energy is measured with the resolution given by Equation 3.2.

The process of converting the Lab E raw ADC counts into GeV consists of a number of steps designed to overcome the complications listed above. As discussed in Section 3.2.1, photomultiplier tube signals were digitized by three sets of ADCs, the highs, the lows, and the superlows. In the Lab E calorimetric energy determination only the low ADC signals are used. They have sufficient dynamic range to cover all signals. ADC pedestals were measured during a gate which opened between each accelerator spill.

In the STRIPPING program, a running average of these pedestals is kept. They are subtracted from the raw count values.

During the experiment, the flasher system described in Section 3.2.1 illuminated each scintillation counter. These flasher signals are used to monitor the drift in each photomultiplier tube relative to the other tubes in that counter. In the STRIPPING program, for each tape (≈ 3 days of running time), an average of each tube's response to the flasher signal is kept. An early tape was chosen as a reference and the flasher averages are normalized to that tape.

The overall response of each counter to a standard minimum ionizing signal is tracked on a tape by tape basis with signals from single muons. For each scintillation counter, TRIGGER 1 events are chosen for which the counter is clearly downstream of any hadronic activity. The minimum ionizing signal is defined as the mean of the muon pulse height distribution between 0 counts and twice the minimum ionizing value. Such a definition necessarily implies that the mean is determined iteratively. The iteration continues until the difference in the mean from two consecutive iterations is less than 1% of its value. To avoid the effects of spatial variations in the counter response, only events for which the muon falls within a 30 inch radius from the counter center are included in these distributions. The variations in this overall gain with time, over the course of the experiment, averaged about 5%. With the value of minimum ionizing defined, all observed pulse heights can be converted into times minimum units.

The final correction that must be determined compensates for the spatial response variations across the counter. A model of the counter is constructed which

depends upon five parameters. These are the horizontal and vertical optical attenuation length, and the three relative photomultiplier tube gains. These parameters are then found, in each counter, by making fits to the model using data from hadronic showers. The model function depends only upon the fractional response of each of the four tubes to the shower. The absolute (unknown) level of the hadronic energy is not needed to calculate the spatial maps. In Figure 4.4 we show a typical map. The fit indicates that the response could vary up to 40% across the counter.

Once all the corrections have been determined and applied, the hadronic energy of the event in GeV can be found using the calibration constant. Operationally, the algorithms which calculate the hadronic energy start by correcting for tube drifts and overall gains. These corrected signals are expressed in times minimum. In practice, pulse height due to a shower will be localized in a few counters downstream of the interaction vertex. A search is made for the most upstream of two counters with more than 0.25 times minimum pulse height. This counter is defined as PLACE. The longitudinal interaction vertex is assumed to be located in the steel just upstream. The less abrupt trailing edge of the shower is defined as SHEND. It is determined to be the most upstream of six counters with pulse height less than 3 times minimum. In general, the muon deposits ≈ 1 times minimum in each counter downstream of PLACE. The hadronic energy is then given by,

$$E_{hadron}(\times min) = \sum_{i=PLACE}^{SHEND} PH_i(\times min) - (PLACE - SHEND + 1) \times (1 \times min) \quad (4.1)$$

It is possible that the choice made for SHEND may be incorrect and result in a bias in the hadronic energy determination. The total pulse height observed in the first six

counters downstream of SHEND is found to be 0.74 ± 0.15 times greater than in the next six counters. This amount is added back into the hadronic energy sum for each event.

In practice the level of analysis described above has to be completed before the spatial maps themselves can be calculated. This is because hadron shower data itself is needed to fit for the map parameters. Because of this, the final map corrections are applied in the DST writing program, where the positions of PLACE and SHEND are also recalculated.

4.2.2 Wonder Building Hadron Energy Determination

The Wonder Building hadronic energy is determined by a procedure which is similar to the one used in Lab E. The major difference is that the Wonder Building calorimeter was never calibrated in a beam of known energy. Instead, a cross calibration with Lab E is performed. Here we describe the Wonder Building energy reconstruction to indicate the differences.

As in Lab E, the raw ADC counts are preprocessed by performing an averaged pedestal subtraction. In addition, small nonlinearities in the ADC responses (absent in Lab E) are corrected for. We may parametrize the nonlinearity by the constant α . Then the ADC counts are related to the input voltage as,

$$N_{counts} = aV^{1+\alpha} \quad (4.2)$$

where a is some calibration constant. The magnitude of α is observed to vary between 0.01 and 0.03.

Drifts in separate photomultiplier tubes are removed by the improved technique described in Section 3.2.2. Time variations in the reference tube's response are tracked with the standard output of the Am source/NaI system. The drift in the i^{th} tube with respect to its response at the start of the data taking is given by,

$$drift_i(t) = \frac{d_i(t)}{d_i(0)} \quad (4.3)$$

where,

$$d_i(t) = \frac{PH_i^{flasher}(t)}{Ref^{flasher}(t)/Ref^{NaI}(t)} \quad (4.4)$$

Here $PH_i^{flasher}$ is the pulse height resulting from the flasher signal, $Ref^{flasher}(t)$ is the reference tube pulse height resulting from the flasher, and $Ref^{NaI}(t)$ is the pulse height analyzed signal due to the NaI as viewed by the reference tube. This drift is tracked for each photomultiplier tube. Typical variations over a week of running are less than $\pm 5\%$. Figure 4.5 illustrates the value of the NaI in following drifts in the reference tube. Over the entire experiment the reference varied up to 10%.

In the Wonder Building detector it is necessary to use all three ranges of ADC for energy reconstruction. The Wonder Building lows have insufficient dynamic range to cover all energies. The highs cover the interval 0-4.5 GeV, the lows cover the interval 0-45 GeV, and the superlows cover the interval 10-1000 GeV. For channels in which a particular range ADC malfunctioned, one of the others can be substituted for it. In practice, all the calorimetry is expressed in terms of "equivalent high channels". The conversion factors from lows to highs and from lows to superlows were found with overlapping energy data. If we express the relationship between lows and highs for the proverbial i^{th} counter as,

$$Hi_i = \alpha_{1i}Lo_{1i} + \alpha_{2i}Lo_{2i} \quad (4.5)$$

then the coefficients are generally ≈ 10 . The superlows, sums of six widely separated lows, are defined as,

$$SLo_i = \sum_{j=n}^{n+5} \beta_{ij} Lo_j . \quad (4.6)$$

The superlow coefficients, which are generally $\approx \frac{1}{6}$ can be found, with enough events, by solving the system of coupled equations.

The pulse height obtained is corrected for spatial variations by a somewhat different map than used in the Lab E analysis. Each scintillation counter was mapped, in a vertical orientation, with a small radioactive source. One such map is shown in Figure 4.6.

Pulse height is converted to times minimum using the muon signals as described in the Lab E section above. In this case a minimum ionizing signal is found at about 40 channels in a high. The muon peaks were again determined for each counter on each tape separately.

The procedure for summing counters in the shower and locating the longitudinal vertex were the same in Lab E and the Wonder Building. The final conversion from times minimum to GeV relied upon a cross calibration done with respect to Lab E. For this, the mean hadron energy of ν_K only events with vertices inside a 25 inch (39 inch) square at the Wonder Building (Lab E) detector were adjusted to match. Naturally, this was an iterative procedure. The resulting conversion factors are listed in Table 4.1. These differ considerably from the Lab E value of 0.216 GeV per minimum. This has been attributed to differences in the number of photo-electrons produced by the two different types of scintillator.

§4.3 Muon Track Finding

4.3.1 Lab E Tracking

Once a Lab E longitudinal vertex is found, the task of choosing sparks upon the muon trajectory is initiated. The first step is to locate a transverse spark vertex. Since the target calorimeter is unmagnetized, the muon will follow an undeflected path except for random multiple coulomb scatters. We therefore expect sparks on the muon track to lie at some fixed angle as seen from the vertex. Other sparks, associated with the hadron shower will flare out in a conical distribution. A crude transverse vertex is first located by extrapolating a least squares fit to all the sparks in the first ten chambers downstream of PLACE. Given this vertex, the target track is improved by searching for sparks at a fixed angle as seen from the vertex. This is done by histogramming the angles of sparks in the x and y views of each chamber plane downstream. These distributions would be expected to peak at an angle corresponding to the muon track. Another least squares fit is done to sparks within the peaks of these distributions. Sparks which deviate from this fit by more than 3.5σ are thrown away and the process is iterated. The transverse vertex is also relocated in this procedure. At last, any remaining sparks within 2σ of the final fit are added back onto the track. In this way a target track is established.

The problem of finding a toroid track is complicated by the (as yet) undetermined bend caused by the magnetic field. Fortunately, the axial symmetry of the toroidal magnetic field gives rise to a constant of the motion,

$$L_\phi = r^2 \frac{d\phi}{dz} \quad (4.7)$$

where ϕ is the angle about the toroid axis of the track point at radius r and longitudinal point z . Multiple coulomb scattering, energy loss, and any radial component of the magnetic field will alter the value of L_ϕ . It has, however, proven possible to successfully find track sparks by stepping through the toroid section while updating the value of L_ϕ defined by the target track with each new found spark. The power of this technique is illustrated by the somewhat pathological event shown in Figure 4.7. Some fraction of the time we observe be multiple delta ray showers accompanying the muon as it leaves the toroid steel. Although globally tracks within these showers appear to diverge from the muon track trajectory, it is within a particular gap somewhat ambiguous which sparks are spurious. The toroid track finding algorithm is observed to usually choose the proper sparks even in this class of events. In those cases where it fails, the subsequent momentum fit will generally have a large χ^2 per degree of freedom. Based upon this the event enters a sample which is visually scanned and refit interactively (see section 5.4.3). (Later in this chapter we shall mention software attempts to improve this class of events.)

4.3.2 Wonder Building Tracking

The Lab E tracking algorithms were not optimum for the task of track finding in the Wonder Building. The target spark chambers had poor multi-spark efficiency. As a result, the region of the interaction often did not contain a cluster of sparks in spite of a large deposition of hadronic energy. This foiled attempts to locate the muon trajectory with respect to the vertex by the peak finding methods described above. Sparks chambers

were also observed to contain spurious isolated hits. In response to these problems the Wonder Building track finding was modified. The new algorithm makes maximum use of the fact that the muon track follows a linear trajectory both before and after the magnet. The two track segments are found by fitting a series of straight lines to all possible combinations of sparks. In each fit only one spark per chamber is used. The best lines are chosen based upon the value of the fit χ^2 . The line chosen in the target, coupled with the determination of PLACE by the counter algorithms, is also used to locate the transverse interaction vertex.

§4.4 Momentum Reconstruction

4.4.1 Lab E

The trajectory of a charged particle in a magnetic field is in principle determined by the incoming track and the momentum. In the case of muons traversing the magnetized steel spectrometer in Lab E, energy loss and multiple coulomb scattering will cause the real trajectory to deviate significantly from the ideal expectation. A calculation of the muon momentum based upon the observed track must account for these effects.

While both energy loss and multiple coulomb scattering are examples of stochastic processes, they effect the trajectory in different ways. The average energy loss per unit distance of a charged particle traveling through matter, due to electromagnetic interactions, has been studied extensively. While the exact energy loss is unpredictable, the

average may be estimated. The energy loss will always cause the track to deflect more than would be predicted in the absence of material. The multiple coulomb scattering will however always deflect the track in a random way and so may in principle decrease or increase the total bend. To calculate the muon momentum in Lab E, we form a χ^2 function between the observed track and a prediction. The predicted track is calculated by guessing a trial momentum and tracing a ray, beginning with the target track, through the spectrometer. This is done assuming an average predicted energy loss per unit distance. The random multiple coulomb scatters are regarded as correlated statistical errors upon the observed trajectory and enter into the denominator of the χ^2 function along with the intrinsic position errors of the spark chambers. By varying the input momentum, the χ^2 function may be minimized, and the most likely estimate of the momentum recovered.

To initiate the χ^2 calculation, a trial momentum is chosen by a crude circular arc fit to the observed trajectory. This choice is based upon the angle between the muon tracks before and after the magnetic field. That angle is related to the momentum P_μ in MeV through the relation,

$$P_\mu = \frac{0.3B_{avg}z}{\Theta} \quad (4.8)$$

where B_{avg} is the field at the center of the magnet in kilogauss and z is the distance travelled in centimeters. With this guess, a predicted trajectory is traced. This momentum is also used to calculate the statistical errors due to multiple coulomb scattering. Here we follow Fermi⁶⁴ and assume these scatters originate from a Gaussian distribution with σ_ϕ , (ϕ is the projected angle in the x or y plane and not the space angle) given by,

$$\sigma_\phi = \frac{14.1 \text{ MeV}}{P_\mu} \sqrt{\frac{L}{L_R}} \quad (4.9)$$

with L the distance travelled in centimeters through a material of radiation length L_R . The use of these errors in the calculation is complicated by the fact that multiple coulomb scattering introduces correlations between transverse track offsets at different depths in the material. We form a χ^2 which may be symbolically written,

$$\chi^2(P_\mu) = \sum_{x,y} \sum_{i,j=1}^{N_{chmbrs}} (x_i^{measured} - x_i^{predicted}(P_\mu)) M_{ij}^{-1}(P_\mu) (x_j^{measured} - x_j^{predicted}(P_\mu)) \quad (4.10)$$

where x_i, x_j are the transverse measured or predicted track positions in the i^{th} and j^{th} chambers respectively and N_{chmbrs} is the number of spark chambers containing observed hits. We sum separately over the x and y views to allow for the possibility that a spark may be omitted in one view. The symmetric matrix M_{ij} is the sum of the error matrix which is determined by the multiple coulomb scattering and the diagonal position measurement errors. The fit is done including spark positions in up to fifteen chambers so that M_{ij} may be as large as 15×15 . And because of the correlations mentioned above it will, in general, not be diagonal.

To understand the construction of M_{ij} we refer to Figure 4.8. There we depict the track undergoing a series of adjacent multiple coulomb scatters in layers of length L . If we regard this process as shown, the scattering in the i^{th} layer is parametrized by two numbers. The first is a net angle ϕ_i with respect to the initial track direction (before the material) for that layer. The second is a displacement δ_i with respect to the total track offset extrapolated from the previous layer. In terms of these quantities, the fundamental

correlation matrix is given by,

$$\langle \phi_i \phi_j \rangle = \sigma_\phi^2 \delta_{ij} \quad (4.11a)$$

$$\langle \phi_i \delta_j \rangle = \langle \phi_j \delta_i \rangle = \frac{L_i}{2} \sigma_\phi^2 \delta_{ij} \quad (4.11b)$$

$$\langle \delta_i \delta_j \rangle = \frac{L_i^2}{3} \sigma_\phi^2 \delta_{ij} \quad (4.11c)$$

where L_i is the length of the i^{th} segment and the σ_ϕ is given by Equation 4.9. Multiple coulomb scatters from different layers are independent. Correlations arise because the elements of M_{ij} refer to *net* displacements at a particular depth in the material. These depend upon the scattering in all previous layers. From the fundamental elements, the general element of M_{ij} may be built up. By combining all the contributions to the displacement after layer i we obtain,

$$x_i = \sum_{k=1}^i \delta_k + \sum_{k=1}^{i-1} \phi_i \sum_{m=k+1}^i L_m \quad (4.12)$$

If we define the lever arm for the k^{th} scattering angle,

$$z_{ki} = \sum_{m=k+1}^i L_m \quad (4.13)$$

then Equation 4.11 becomes,

$$x_i = \sum_{k=1}^i (\delta_k + \phi_k z_{ki}) \quad (4.14)$$

And,

$$x_i x_j = \sum_{k=1}^i \sum_{l=1}^j (\delta_k + \phi_k z_{ki})(\delta_l + \phi_l z_{lj}) \quad (4.15)$$

where i is less than or equal to j . Substituting the fundamental elements from Equation 4.10 we have,

$$M_{ij} = \langle x_i x_j \rangle = \sum_{k=1}^i \sigma_{\phi k}^2 \left(\frac{L_k^3}{3} + \frac{L_k^2}{2} (z_{kj} + z_{ki}) + z_{kj} z_{ki} \right) + \sigma_{spark} \delta_{ij} . \quad (4.16)$$

Here, σ_{spark} is the 0.5 millimeter transverse position resolution of the spark chambers. We emphasize that through the σ_{ϕ} the elements of M_{ij} are momentum dependent. In principle, once M_{ij} is inverted, the χ^2 can be minimized by varying the momentum. Care must be exercised here because of the momentum dependence of M_{ij} . Since the multiple scattering errors are inversely proportional to the momentum, the χ^2 has one minimum at very small momentum. To avoid this, we fit for the actual momentum by varying the predicted trajectory but *not* the error matrix. Once a minimum is found we recalculate M_{ij}^{-1} with this new momentum and minimize again. This iterative procedure is followed to convergence.

In Figure 4.9 we histogram the $\sqrt{\frac{\chi^2}{d.f.}}$ distribution for the momentum fit of real events and a sample of Monte Carlo generated events. The tail on the real data at large χ^2 has its origin in the inclusion of some bad sparks on the track. As mentioned in Section 4.3, occasional delta ray showers led to spurious hits near the muon track. These badly fit events are visually scanned and reconstructed interactively. We mention here for completeness an unsuccessful attempt which was made to address this problem in the reconstruction software. The toroid spark chambers were grouped in sets of five in each toroid gap. The gaps were free of both magnetic fields and material. The muons should travel undeflected there. This fact, in principle, provides a strong constraint upon the proper track sparks. We attempted to use this constraint as a criteria for adding and

removing sparks from the track. In addition we calculated the momentum determining χ^2 function in terms of slopes and intercepts in the three toroid gaps rather than with spark positions in the fifteen chambers. This reduces the error matrix to a 6x6 form with a decrease in reconstruction computer time. Such a technique is more optimally suited for the layout of the Lab E spectrometer in E-701. Unfortunately time constraints prevented this program from reaching a successful completion.

4.4.2 Wonder Building

The problem of determining momentum in the Wonder Building detector was somewhat simpler since there was only one toroid. Again a crude fit to the track bend was made and then a χ^2 minimization ensued. Here the χ^2 was calculated using the straight line constraint. In this case the error matrix was just 4x4. Besides this, the minimization was approached similarly to Lab E.

Chapter 5

Event Counting

§5.1 General Aim

In this chapter we shall discuss the event counting procedures which are used to search for the existence of neutrino oscillations. The oscillation effect was described in Chapter 1. There we indicated that the probability for the disappearance of a particular neutrino is a function of both its energy and the distance travelled from the point of its creation. In Experiment 701 the two neutrino detectors were set at fixed distances from the decay region. The first detector, at the Wonder Building, was 715 meters from the decay region center. The second detector, at Lab E, was 1116 meters from the decay region center. Data were collected with the secondary beam transport set to select secondaries of six momenta. The settings were +250, +200, +165, +140, +100, and -165 GeV. The neutrino energies spanned the interval $30 < E_\nu < 240$ GeV. The aim of the analysis

is therefore to compare the number of neutrinos passing the two locations as a function of energy. In practical terms, this is of course achieved by comparing interaction rates at fixed energy since the interaction rate is always proportional to the incident flux.

With the linearly rising total interaction cross section, the number of neutrinos N_ν passing through a target of density ρ_{tgt} nucleons per unit volume and length L_{tgt} is related to the number of interactions at E_ν ,

$$N_{int}(E_\nu) = N_\nu(E_\nu) \left(\frac{\sigma}{E_\nu} \right) E_\nu \rho_{tgt} L_{tgt} . \quad (5.1)$$

Here $(\frac{\sigma}{E_\nu}) = 0.669 \times 10^{-38} \text{ cm}^2 / \text{ GeV}$ is the cross section slope. In the ideal two detector oscillation experiment, both apparati would sample the same flux, N_ν , of neutrinos and each interaction would be efficiently measured. The ratio,

$$R(E_\nu) = \frac{N_{int}^{Lab E}(E_\nu)}{N_{int}^{WB}(E_\nu)} \quad (5.2)$$

would, in the absence of neutrino oscillations, be constant with energy and equal to the ratio of the detector's density times length. In the real experiment, a number of factors besides neutrino oscillations can contribute to an energy dependence in the ratio of Equation 5.2. There are two goals to the event counting procedures used in the oscillation analysis. The first is to isolate a sample of neutrino interactions in Lab E and the Wonder Building for which these factors are small. The second is to determine any corrections which might compensate for remaining differences. The level of systematic error in such an analysis is set by the ultimate uncertainties in these corrections.

To recognize the separate factors which might contribute to an energy dependence in the ratio of Equation 5.2 it is useful to expand it explicitly in these factors.

We understand here that what will be included in the real ratio is some subset of the total interaction sample which has satisfied the hardware trigger requirements and has been further chosen and corrected to minimize all the differences. We denote these event samples as N_{select} . The ratio is then given by,

$$R(E_\nu) = \frac{N_{select}^{Lab E}}{N_{select}^{WB}} = \left(\frac{N_\nu^{Lab E}(E_\nu^{Lab E})}{N_\nu^{WB}(E_\nu^{WB})} \right) \left(\frac{E_\nu^{Lab E}}{E_\nu^{WB}} \right) \left(\frac{A_\nu^{Lab E}(E_\nu^{Lab E})}{A_\nu^{WB}(E_\nu^{WB})} \right) \left(\frac{\rho_{tgt}^{Lab E} L_{tgt}^{Lab E}}{\rho_{tgt}^{WB} L_{tgt}^{WB}} \right) \left(\frac{\epsilon^{Lab E}}{\epsilon^{WB}} \right). \quad (5.3)$$

The quantities which determine this ratio may each be the source of anomalous differences in event rates at Lab E and the Wonder Building. Below we define and consider these explicitly.

1) Neutrino Flux N_ν : This quantity is most directly coupled to the event rates in the two detectors. For the flux dependences in the numerator and denominator to cancel, a careful accounting of all events which trigger the detectors must be made. In addition, both detectors must be exposed to an equal number of incident neutrinos. Thus the relative neutrino fluxes corresponding to the event samples must be monitored. If for example, one detector suffers a larger event related deadtime than the other, the ratio will shift unless the extra flux is accounted for.

2) Neutrino Energy E_ν : The ratio is evaluated at a chosen energy point E_ν . The events which enter the ratio for this point have energies which have been determined, by some procedure, to lie within an interval about the chosen energy. On the right hand side of Equation 5.3 we denote these energies with separate superscripts for the two detectors. This is done because different biases may be introduced in the energy determination

process. As a result the true energies of the event samples from the two detectors may be different from E_ν and different from each other. If for example, all other things being equal, we choose for Lab E a group of events corresponding to a somewhat higher neutrino energy than the Wonder Building group, the ratio will increase since the cross section is proportional to the neutrino energy

Because of the properties of the dichromatic beam, we have a choice of determining the neutrino energy from the interaction radius or from the event parameters measured by the calorimeter and the spectrometer. In both cases biases can arise. For example, with a beam determined energy a rule associating interaction radii for equal energy between the two apparati is needed. With a detector measured energy, a reliable cross calibration of the two measurements is required.

3) Acceptance A_ν : Both detectors must subtend the same geometrical (solid angle) flux. In addition, they must have similar acceptances to reconstruct neutrino interactions. This may be understood in the following way. At a fixed neutrino energy, kinematical constraints will determine the possible values of the variables which specify a neutrino nucleon interaction, P_{muon} , θ_{muon} , and E_{hadron} . In general, a detector will only trigger on events which correspond to some subset of the allowed values. This is because the trigger contains explicit or effective energy and geometrical cutoffs. For example, the transverse size of the apparatus and the trigger counters will limit the acceptance in θ_μ . The presence of material, through which the muon must travel, will determine a lower P_μ cutoff. In this experiment, the maximum TRIGGER 1 acceptance of the detectors determine the subset of neutrino interactions for which the incident neutrino energy can

be reconstructed. In the oscillation experiment the two detectors must be able to trigger on and reconstruct overlapping subsets of the interaction samples. If this is true, events can be chosen for the oscillation analysis so as to equalize the acceptance.

4) Target Density $\rho_{tgt}L_{tgt}$: Density and target length differences will lead to differences in interaction rates for equal accepted flux. The two detectors need not be equal, but the target density differences must be known.

5) Other Systematic Normalization Factors ϵ : Other overall normalization differences can also cause differing relative event rates. These could include (differences in) trigger efficiencies, reconstruction efficiencies, and deadtimes.

In the following sections we describe in detail the procedures used to minimize the differences in Lab E and Wonder Building counting rates which might mimic or distort neutrino oscillations.

§5.2 Flux

5.2.1 Bookkeeping

Since event rates at two separate detectors are compared it is important to have a careful accounting of these events so that periods of running can be reliably correlated between Lab E and the Wonder Building. As described in Section 3.7, separate data tapes were written at Lab E and the Wonder Building. Because common beamline information

from the MAC was written with each accelerator spill it was possible to correlate data in the two detectors on a spill by spill basis.

As noted in Section 4.1, the raw data tapes were read by a program called the STRIPPER. Basic preprocessing was done on the event data such as ADC pedestal subtractions. A series of STRIPPED event tapes was written. The STRIPPER also separated off monitor records and these were written to a series of monitor tapes. The separate monitor tapes from Lab E and the Wonder Building were then merged into a single set of monitor tapes. The relevant information in the combined monitor record for the oscillation analysis is listed and defined in Table 5.1. If one detector was not taking data during some set of spills, default values were entered for it. The words PULSE, DATE, and TIME, provided by the accelerator allowed the correct monitor records to be matched in the merging program.

The neutrino events on the STRIPPED event tapes were reconstructed by separate CRUNCHER programs as described in Chapter 4. The reconstructed events were filtered at this point to remove cosmic ray interactions which occurred during the beam gate. Any events with a good track whose vertex fell outside the apparatus were removed as cosmic rays. Events occurring during the COSMIC RAY gate which passed this constraint were written out so that a surviving cosmic ray rate could later be calculated. The events output on the CRUNCHED tapes underwent a second series of reconstruction programs which refit events to remove bad sparks and correct hadron energies by the spatial maps. A reduced summary data record was then written to a set of Data Summary Tapes (DST's), one for Lab E and one for the Wonder Building. The merged monitor

tapes and the two DST's were input to a super merging program, the SUPERWRITER. This created a combined set of records for each spill of the accelerator during which at least one detector was taking data. All events were included on this SUPERDST except those cut at the CRUNCHER level. All event counting for the oscillation analysis is done by programs which read these SUPERDST's. The SUPERDST's were separated according to momentum setting of the hadron beam transport.

The pattern of monitor and event records on the SUPERDST for the i^{th} machine pulse, P_i , DATE=D, and TIME=T is of the form,

MONITOR RECORD : $P_i, D, T, \dots$ monitor information.....

LE EVENT RECORD : $P_i, D, T, CR\ GATE, LE\ RUN\#, \dots$ event information.....

LE EVENT RECORD : $P_i, D, T, P1\ GATE, LE\ RUN\#, \dots$ event information.....

LE EVENT RECORD : $P_i, D, T, P2\ GATE, LE\ RUN\#, \dots$ event information.....

. . .

LE EVENT RECORD : $P_i, D, T, FS\ GATE, LE\ RUN\#, \dots$ event information.....

WB EVENT RECORD : $P_i, D, T, CR\ GATE, LE\ RUN\#, \dots$ event information.....

WB EVENT RECORD : $P_i, D, T, P1\ GATE, WB\ RUN\#, \dots$ event information.....

WB EVENT RECORD : $P_i, D, T, P2\ GATE, WB\ RUN\#, \dots$ event information.....

. . .

WB EVENT RECORD : $P_i, D, T, FS\ GATE, WB\ RUN\ \#, \dots event\ information \dots$

We have depicted the maximum sequence. In general there would not be an event during all the PING gates and FAST SPILL gate in both detectors.

5.2.2 Incident Flux

At each momentum setting of the secondary beam, the neutrino flux is proportional to the number of protons incident on the production target. For the oscillation analysis, neutrino interactions in each detector should be accepted over time periods corresponding to an equal number of protons on target. The time periods of data taking could be different for two reasons. The obvious difference arises from possibly different periods of maintenance and repairs. In practice, the Lab E and Wonder Building detectors usually ran simultaneously. No conscious attempt however, was made to maintain a strict correspondence in data taking periods. The second difference is caused by possibly different livetimes in the two detectors. As discussed in Chapter 3, the spark chambers experienced an ≈ 50 millisecond deadtime following an event trigger. The trigger rate in one detector might be higher if it responds to interactions over a larger target area or length than the other. In this case, integrated over some period of running, that detector will have intercepted fewer incident neutrinos.

This time integrated flux requirement is approached in two ways. The first approach to maintaining equal incident fluxes is known as the SIMULTANEOUS LIVE analysis. In this approach we accept events into the oscillation analysis which occurred only when *both* detectors were live. Here then we discard all running periods when only

one detector operated *and* all events which occurred after an event trigger in the other detector. This procedure results in a loss of event statistics since only that single first event is allowed to enter the analysis per ping and fast spill. By requiring SIMULTANEOUS LIVE however, both detectors by definition intercept the same temporal beam.

The second approach is known as the SIMULTANEOUS DATA TAKING analysis.

This requires only that both detectors be functioning and recording events on tape. In this case we are neglecting the livetime difference and allowing events into the analysis from both detectors in each spill. To compensate for the difference in livetimes, a relative livetime correction must be applied to the ratio of events seen in the two detectors. The SIMULTANEOUS DATA TAKING analysis has more statistical power than SIMULTANEOUS LIVE but includes an added systematic error related to this correction. The relative livetime correction is determined by accumulating sums of protons on target as measured by the NEUHALL toroid NTOR (see Section 2.3). These sums are gated on each detector's event trigger. The total proton flux was known as NTB, and the fluxes gated on the Lab E and Wonder Building event triggers were NTE and NTW respectively. The relative livetime correction (RLT) is then,

$$RLT = \frac{\sum_{spills} NTE}{\sum_{spills} NTW} \quad (5.4)$$

(The separate absolute livetimes would be $LT_{Lab E} = \frac{\sum NTE}{\sum NTB}$ and $LT_{Wonder Bldg} = \frac{\sum NTW}{\sum NTB}$). In the analysis these livetime factors are applied as systematic normalization corrections and we return to them in the discussion of Section 5.5.

Certain internal consistency checks are imposed upon these sums. If PULSE, DATE, or TIME in the monitor and event records fail to match or if $NTE, NTW > NTB$,

the events and all flux since the last good match are thrown out. The live flux taken at each secondary momentum setting is listed in Table 5.2.

In practice, the oscillation analysis uses the SIMULTANEOUS DATA TAKING event sample since it has more statistical power than SIMULTANEOUS LIVE. The SIMULTANEOUS LIVE analysis is used as a consistency check on the livetime correction. The comparison will be shown in Section 5.6

Certain preliminary cuts are applied to both the flux sums and the data. Entire runs are rejected based upon information in the logbooks which were kept during the experiment. Criteria for eliminating a run might include high voltage problems, gas problems, or failures of components of the beamline monitoring system etc. Cuts on the flux and the data, where the steering of the hadron beam deviated from some nominal setting, are also made. The nominal steering is found by studying the distribution of response from the split plate ion chambers in the decay region. Flux sums after these cuts are also included in Table 5.2. The large discrepancy between the live flux in the two detectors after the good runs cut at 250 GeV results from a long period of spark chamber problems in the Wonder Building.

§5.3 Energy

5.3.1 Energy and $\pi \setminus K$ Separation

As discussed in the introduction, the oscillation analysis will consist of a com-

parison of event rates in the two detectors as a function of the incident neutrino energy. Because of this, we must be able to separate groups of events according to the energy carried by the incident neutrino. This separation must be done consistently between the two detectors. For the purpose of comparing the data to various oscillation hypotheses, this energy determination must be calibrated absolutely.

In this experiment there can be two approaches to defining the incident neutrino energy. The first approach is to equate the energy reconstructed by the neutrino detector with the incident neutrino energy. This is done by summing the energy deposited in the calorimeter and the muon momentum. This approach has a number of disadvantages. First of all, such a reconstruction can only be performed for that subset of the data which satisfies the TRIGGER 1 requirements. As a result, only a portion of the interaction sample is available for the analysis. Another disadvantage is that the detectors have intrinsic measurement resolutions and so will smear the true energy. The total measured energy resolution varies with the separate fractions of muonic and hadronic energy. If we combine the muon and hadronic energy resolutions indicated in Chapter 3, the total measured energy resolution may be written in terms of E_ν and the scaling variable y as,

$$\frac{\Delta E_\nu}{E_\nu} = \{(1 - y)^2 \left(\frac{\Delta P_\mu}{P_\mu}\right)^2 + (0.89 \text{ GeV}) \frac{y}{E_\nu}\}^{\frac{1}{2}} . \quad (5.5)$$

This quantity is shown in Figure 5.1a and b for a range of neutrino energies in the Lab E and Wonder Building detectors. The total resolution differs in the two detectors because of the different muon momentum resolutions. In addition to this difference, the definition of the measured energy in the two detectors, is also sensitive to the cross calibration of the calorimeters. The Lab E calorimeter was calibrated in an external hadron and muon

beam. The Wonder Building calorimeter was however cross calibrated with respect to Lab E.

The second approach is to utilize the properties of the dichromatic neutrino beam. As discussed in Chapter 2, in this beam the interaction radius is correlated with the incident neutrino energy up to an ambiguity between ν_π and ν_K . This approach has the advantage that the energy can be defined by the Monte Carlo simulation of the dichromatic beam and is thus independent of the detailed characteristics of the separate detectors. In this method, the incident energy is not smeared by the measurement resolution. Nor is the determination sensitive to a small miscalibration in the measured energy scales between Lab E and the Wonder Building. There is however an energy dispersion at fixed radius because of the properties of the real dichromatic beam. This was discussed in Section 2.2, for ν_π this dispersion is greater than the measured energy resolution. In spite of this we opt to use the second method rather than be sensitive to the unequal measured energy resolution and possible relative miscalibrations. The ability to define the neutrino energy globally is a very positive feature for an experiment which is based upon a comparison of two separate detectors.

Operationally, the approach we use cannot be completely free of the detector energy measurements since they are needed to unfold the ν_π/ν_K ambiguity at fixed radius. This is however a rather crude use of the measured energy since the pion and kaon events are well separated in energy. We note however, that since the measured energy is used for the separation, the event sample is still restricted to only the TRIGGER 1 interactions. To see how readily the separation is done we refer to Figure 5.2a and b which summarizes

the entire E-701 data sample. Shown are scatter plots of measured neutrino energy against interaction radius for the six momentum settings of the secondary beam transport. These plots are projected onto the energy axis in Figure 5.3a and b. The measured energy separation of pion decay (ν_π) and kaon decay (ν_K) neutrinos at fixed radius is achieved in practice by defining a separation function,

$$E_i^{SEP}(R) = \frac{\alpha_i}{1 + (\beta_i R)^2} \quad (5.6)$$

where α_i and β_i for the i^{th} momentum setting are listed in Table 5.3. Events for which $E_{total} > E_{SEP}$ are from kaon decays, while events for which $E_{total} < E_{SEP}$ are from pion decays. The separation curves are superimposed in Figure 5.2. It is clear, even visually, that the measured energy allows a good separation between ν_π and ν_K events.

5.3.2 Method of Radial Bins

Once events are distinguished as ν_π or ν_K , they are assigned to one of three radial bins about the beam center. The beam centers are found by histogramming the x and y vertex distributions of ν_π events for which $E_\nu > 0.2E_{beam}$. The maximum ν_π energy is $0.41E_{beam}$. By choosing higher energy pions only, we eliminate tails on these histograms from large angle decays which are rather decoupled from the original hadron beam direction. The beam centers found in this way are all close to the nominal detector axis and are listed in Table 5.4.

The radial bins, shown in Figure 5.4, are defined in the Lab E apparatus to be 0-20 inches and 20-40 inches for ν_π and 0-50 inches for ν_K . Because of the dichromatic

beam, these radial bins are effective energy bins also. The mean energies for these bins are determined by the Monte Carlo beam simulation described in Section 2.4. Although the measured neutrino energy is *not* used to define these means, for consistency we may compare on a statistical basis the Monte Carlo and measured neutrino energies. This comparison is shown in Table 5.5. With the choice of these radial effective energy bins made in the Lab E detector it is necessary to determine the corresponding radial bins in the Wonder Building detector. The radial bins in the closer detector are determined by scaling radii by the ratio of distances from the detectors to the decay region center,

$$\begin{aligned}
 SCALE &= \frac{\text{Distance : WB to decay region center}}{\text{Distance : LE to decay region center}} \\
 &= \frac{715 \text{ meters}}{1116 \text{ meters}} = 0.6407 . \quad (5.7)
 \end{aligned}$$

This choice is central to the oscillation analysis. It is based upon the assumption that the effective mean decay position is at the decay region center. Clearly, at some level this is incorrect. The assumption will lead to a set of systematic corrections which will be discussed in Section 5.8.

Having chosen to define corresponding radial bins in the two detectors by the rule 5.5, we may compare the mean measured energies of events in these bins between the two detectors. In Figures 5.5a,b,c we plot the ratios of the mean total energy, hadronic energy, and muon momentum for each radial bin at each secondary setting. We note that with the cross calibration of hadronic energy made for the Wonder Building detector, the hadronic energy ratios for some subset of the kaons are fixed already by definition. Recognizing this, there is agreement at the $\pm 2.5\%$ level for the total energy. There is

somewhat more scatter in the separate muon and hadron energies. This indicates again that $\nu_\pi \setminus \nu_K$ may be identified consistently in the two detectors.

The mean energies and standard deviations for these radial bins as calculated by the Monte Carlo are listed in Table 5.5. Figure 5.6 shows the energy intervals spanned by these bins. The choice of three bins at each setting allows us to span the entire region between 30 GeV and 240 GeV and also roughly matches the energy resolution available in the dichromatic beam.

§5.4 Acceptance and Event Selection

As discussed in Section 5.1, we seek an event sample for the oscillation analysis which would have an equal acceptance in both detectors. In this section we discuss the criteria which were applied in selecting a sample of interactions which satisfies this requirement. We also discuss cuts on the events which were made to ensure the integrity of the sample.

5.4.1 Cuts

Although the two detectors were geometrically similar, a set of fiducial and acceptance cuts are applied to the event sample. These cuts are designed to fulfill several requirements. The set of fiducial cuts ensures the interactions which enter the analysis are well contained within the detectors. One set of acceptance cuts is applied to ensure the two detectors subtend very similar solid angles with respect to the neutrino source.

Another set is designed to minimize acceptance differences in the triggers between the two detectors. This last group serves to define a simple and efficient geometrical trigger. Transverse geometrical cuts are determined by whichever detector has more restrictive dimensions and then scaled to the other detector by the rule Equation 5.7. Here we list these cuts.

1) TRIGGER 1: In Section 5.1 and 5.3 we emphasized the importance of the measured neutrino energy in the oscillation analysis. The most basic requirement on the event sample was that all events satisfy the TRIGGER 1 logic. Only for these events can the incident neutrino energy always be determined by a measurement.

2) PLACE: A one to one correspondence existed between the number of steel plates in the two detectors. Events are required to have their vertex inclusively between counter 54 and counter 18 in both. This equates the longitudinal targets and ensures longitudinal containment of hadronic showers. The target track finding efficiency was studied with respect to the downstream location of this cut. The choice of counter 18 results from that investigation.

3) VERTEX: A square cut centered on the apparatus is applied to contain hadronic showers transversely. This cut is determined by the smaller Wonder Building detector where it is set to ± 25 inches $\times$ ± 25 inches. At Lab E, the scaled square is ± 38.82 inches $\times$ ± 38.82 inches.

4) RADIUS: A beam centered cut on the radius of the event vertex was set at 22.5 inches for neutrinos from pion decay and 25 inches for neutrinos from kaon decay in the Wonder Building. The Lab E radius was scaled from this. The relationship between the VERTEX

and RADIUS cuts is shown in Figure 5.7. The RADIUS cut together with the VERTEX CUT ensure the equality of solid angle subtended by the two detectors within the validity of the scaling assumption.

5) THETA MU: A muon angle cut of 200 milliradians is imposed on the target track. This cut is determined from an earlier (E-616) study of muon trigger efficiency versus muon angle. In general the efficiency will depend upon PLACE and the vertex position, but 200 milliradians is found to be an efficient cut for all parts of the apparatus.

6) ZT2: A straight line extrapolation of the muon track to the trigger counter T2 is made. Tracks are required to fall within the interval ± 58 inches $\times$ ± 58 inches in Lab E and the Wonder Building.

7) HOLE: The magnetic toroids had cylindrical holes down their central axis. At Lab E and the Wonder Building, the hole diameter was 10 inches. The tracking programs can follow the muon trajectory; if it remains within the hole for more than 20% of its path in the spectrometer, the event is cut.

8) TOROID RADIUS: A cut based upon the radius of the muon at the toroid front face is applied, 69 inches at Lab E and 58 inches at the Wonder Building.

9) PMIN: A minimum reconstructed momentum cut is applied. It is determined by the most probable energy loss for a high energy muon travelling through the steel toroidal spectrometer (11.6 MeV/cm). At Lab E, where the first toroid contains 2.032 meters of steel, $P_{\mu}^{min} = 2.38$ GeV for a straight track parallel to the toroid axis. At the Wonder Building, with only one toroid of 2.4 meters, $P_{\mu}^{min} = 2.78$ GeV. The presence of the magnetic field will however cause the track to curl up. Because of this, the PMIN cut is

set at 7.5 GeV. This value is tailored to the Wonder Building toroid which is longer and has a higher magnetic field. Occasionally, a very high momentum track might be misfit as having a high negative momentum. Such events will be cut by this PMIN requirement also. The events which fail PMIN are later scanned to retrieve this last category.

10) TRAVEL: This is a containment cut on the length of the Wonder Building track. It is required to be at least 132 inches (335.3 centimeters) long inside the detector.

Each event is in addition translated in software from the detector of occurrence to the corresponding steel plate in the other detector. This translation is along the line joining the vertex with the center of the decay region. In the other detector the event is required to pass all the above cuts also. For example, the PMIN cut, set for the more restrictive Wonder Building apparatus removes a class of Lab E events. Different acceptances of the T2 counters in each detector are removed in this manner also. Care was taken in the event translation to respect the different beam centers in the two detectors. Figure 5.8 details this translation geometry. The important point about the translation procedure is that it effectively cancels any geometrical acceptance difference between Lab E and the Wonder Building. The two acceptances are forced to be equal.

In addition to the above geometrical and acceptance cuts, three reconstruction cuts on the muon track are applied also.

11) PMAX: If the reconstruction program fails to find a minimum χ^2 in a fixed number of iterations, the momentum is tagged at 400 GeV. These events are cut.

12) CHISQUARED: Events for which the toroid track $\sqrt{\frac{\chi^2}{d.f.}} > 5$ for Lab E and 8 for the Wonder Building are cut. This value was determined by visually scanning events.

13) IPK: This cut requires that the reconstruction programs succeeded in determining a total measured energy for the event. This was essentially a safety flag which in practice, all events passing the above 12 cuts always passed.

The PMAX and CHISQUARED cuts are not true analysis cuts. Most of these events are failures of the track finding algorithms which cause the momentum finding algorithm to go astray. Corrections must be made for these failures. They are later separately scanned, manually reconstructed, and added back into the event sample. The PMIN failures with $P_\mu < 0$ are also included in the scanned sample. This is discussed below in section 5.4.3.

As mentioned earlier, only events satisfying TRIGGER 1 and good steering are included in the analysis. The evolution of the total event sample for each setting, through the cuts, is displayed in Table 5.6. Note that each geometric cut is being applied at both detectors. This clearly indicates where the important geometrical and acceptance differences are.

5.4.2 Backgrounds

Having discussed the event selection criteria, it is relevant to describe the effectiveness of these in discriminating against possible backgrounds under the charged current signal. In practice, the TRIGGER 1 and PMIN cuts are restrictive enough to eliminate essentially all backgrounds. The potential sources are the following:

1) Cosmic ray muons which stop in the detector: Most cosmic ray muons approach the

detector from the vertical. The most likely way for a cosmic ray muon to fire TRIGGER 1 is for it to enter the detector vertically, downstream of the T2 counter and then bend to the horizontal. If it ranges out in the target it will look like a charged current neutrino interaction with small hadron energy. Such muons will however reconstruct with a negative momentum and thus fail the PMIN cut. Alternately, a very stiff muon could enter with a small angle to the horizontal and also fire TRIGGER 1. The rate for such events was measured during a no-beam gate prior to each accelerator spill. They have been determined to be a negligible source of background.

2) Neutrinos from K_{e3} decay: About 4.8% of the kaons in the secondary beam decay into electron neutrinos. When these interact in the detector they make electrons. In steel, the electrons immediately shower. Thus, such interactions will never fire TRIGGER 1.

3) Neutrinos from $K_{\mu 3}$ decay: About 3.2% of the kaons in the secondary beam decay by the three body muon channel. The interactions of these neutrinos produce muons and are thus indistinguishable from the dichromatic neutrino interactions. As indicated in Figure 2.7, the energy of these neutrinos lie between the $\pi_{\mu 2}$ and $K_{\mu 2}$ bands. As a result, the energy separation procedure of Section 5.3 will assign them to one of the dichromatic groups. The effect of these neutrinos on the oscillation measurement is compensated for by a Monte Carlo simulation of the oscillation experiment. This Monte Carlo is described in Section 5.5.

4) Neutral current interactions: Approximately one third of the high energy neutrino interactions observed in the detectors are neutral current induced. In general such interactions are characterized by a large localized deposition of hadronic energy without the

penetrating muon track. Consequently they will not fire TRIGGER 1. The way a neutral current can result in TRIGGER 1 is if one of the shower particles punches through the toroid to T2 or if a shower particle decays leptonically. The first case is unlikely since there are at least 24 nuclear interaction lengths of steel between PLACE=18 and T2. The second case has been estimated by Monte Carlo shower calculations and fits to Bubble Chamber data. These suggest that the percentage of neutrino interactions which are accompanied by decay muons with momentum greater than 4 GeV ranges between 0.09% at $E_\nu=40$ GeV and 0.3% at $E_\nu=200$ GeV⁶⁵.

5) Wide band neutrino interactions: Decays upstream of the decay region result in neutrino and antineutrino interactions in the detectors. The latter are easily distinguished by the PMIN cut. The former are measured during a special run with the decay region blocked off and subtracted. This is discussed in Section 5.8.1.

6) External Muons: Muons produced in the external shielding will fire the veto counter and therefore not TRIGGER 1.

7) Neutrons: If these enter the detector from upstream, they may not fire the veto counter before interacting. They will not in general fire TRIGGER 1 since their interactions will not result in a penetrating muon.

5.4.3 Reconstruction Efficiency and Event Scanning

Approximately half the events which failed the PMAX, CHISQUARED, and $P_\mu < 0$ cuts were visually scanned. One cause of these failures was occasional electromag-

netic showers accompanying the muon as it left the toroid steel. These caused spurious sparks which in general clustered about the true trajectory. If the track finding algorithm picked any of these, the fit χ^2 tended to skew to higher values. Occasionally, completely spurious sparks far from the trajectory are also included. When this occurs, the momentum finding algorithm often does not converge at all. Typical rates for these reconstruction failures are; PMAX, 0.2%; CHISQUARED, 6.3%; $P_\mu < 0$, 0.04%.

Using an interactive track finding program, a given event can be visually displayed on a computer terminal and new sparks can be chosen. This event is reconstructed anew and saved on a file. These fixed events are then added back into the data sample. Since only a statistically significant sample was scanned (for practical reasons) these events are included with a weight equal to the fraction refit at each setting. This technique is better than simply assigning a global reconstruction efficiency correction since events return to the proper kinematic regions of the data.

A second important inefficiency can occur if an event receives a good χ^2 and yet its momentum is miscalculated. In this case, ν_π and ν_K can be confused. By visual scanning and Monte Carlo studies, this is determined to be a negligible effect.

In the Lab E event sample a small class of events were observed for which the calorimetry and trigger requirements which defined TRIGGER 1 were satisfied while the target track finding algorithms failed to locate a target track. A subset of this sample was scanned. That study indicated half of these events were good charged current interactions while the other half were (probably) cosmic ray induced firings of the trigger. Lab E events which lack an upstream track are flagged and an inefficiency factor weighted by 0.5

is applied to the events sums at each setting. This tracking inefficiency was absent in the Wonder Building analysis. The weighted Lab E correction factors are listed in Table 5.7.

§5.5 Systematic Normalization Differences

Besides the acceptance differences minimized by the cuts described above, a set of relative normalization differences exists. These are described here.

1) Target Mass Correction An equal number of steel plates was included in the fiducial volumes at Lab E and the Wonder Building. In spite of this, a careful accounting of material in the detectors yields a small target mass difference. This is specified in Section 3.6. The overall factor multiplying $\frac{Lab\ E}{Wonder\ Bldg}$ is 1.036 ± 0.003 .

2) Relative Livetime This correction is discussed in Section 5.2. Table 5.8 lists these setting by setting and they vary between 1.0 and 1.13. The relative livetime correction only applies to the SIMULTANEOUS DATA TAKING analysis.

3) Trigger Efficiency Corrections TRIGGER 1 efficiency in each detector is strongly coupled to the veto deadtime there since singles in the veto will suppress the TRIGGER 1 rate. To a large extent then the TRIGGER 1 efficiency correction is included in the veto deadtime correction which is discussed directly below. Any inefficiency remaining because of the T2 counter can be estimated by considering the sample of TRIGGER 3 only events which straight line extrapolate to within T2 but fail to fire TRIGGER 1. This analysis suggests that TRIGGER 1 is consistent with being 100% efficient.

4) Relative Veto Deadtime Veto counters described in Section 3.5 were set upstream of the

detectors to prevent triggers on charged tracks entering from outside. A deadtime in the trigger sensitivity is associated with this veto. Differences in the veto deadtime existed at Lab E and the Wonder Building. The Wonder Building was separated from the secondary beam by ≈ 500 meters of shielding. The Lab E detector followed another 300 meters. Because of these differences there could be higher singles rates in the Wonder Building veto counters. Sources of such singles might include, soft radiation from upstream hadronic interactions and a higher punchthrough and a higher rate of neutrino produced muons at the upstream location. Muons also entered the Wonder Building detector from the west where the N1 muon beam ended. During part of the running period a slow spill experiment operated in the N1 beam. Unacceptable muon fluxes were observed in the Wonder Building detector unless the N1 magnets ramped off at least 300 milliseconds before the beginning of the fast spill.

The veto deadtime is estimated in more than one way. Assuming most of the TRIGGER 1 inefficiency to be caused by the veto deadtime, the quantity,

$$1 - \frac{TRIGGER1 \cdot TRIGGER4}{TRIGGER4} \quad (5.8)$$

provides a direct measure. This quantity, listed by setting in Table 5.7 was a factor of three larger in the Wonder Building detector. The other techniques for ascertaining the veto deadtime are discussed in Section 5.8 which concerns the estimation of systematic errors in this experiment. The other techniques are used essentially to bracket this uncertainty.

Relevant to the veto deadtime correction is the spill structure of the neutrino beam. If most of the beam came during a fraction of the maximal spill gate (≈ 2 milliseconds) then a possibly large veto deadtime could result. In actuality, the spill structure

was carefully monitored by observing the analog signal from the NTOR pickup coil and the length of the accelerator spill gate. If the spill gate was shorter than 1 millisecond, then the instantaneous rate was too high and certain beamline monitors appeared to saturate. For this reason the experimenters continually instructed the accelerator operators to maintain a spill gate which averaged about 1.2 milliseconds.

§5.6 Result from Event Sample

The event sums in the SIMULTANEOUS DATA TAKING analysis which have passed all the analysis cuts are corrected for the normalization factors described above and for reconstruction inefficiency as,

$$\frac{N_{corr}^{LE}(i, j)}{N_{corr}^{WB}(i, j)} = \left(\frac{N_{select}^{LE}(i, j) + \alpha_{ij} N_{SK}^{LE}(i, j)}{N_{select}^{WB}(i, j) + \beta_{ij} N_{SK}^{WB}(i, j)} \right) \left(\frac{\rho_{tgt}^{WB} L_{tgt}^{WB} \epsilon_j^{WB} LT_j^{WB}}{\rho_{tgt}^{LE} L_{tgt}^{LE} \epsilon_j^{LE} LT_j^{LE}} \right). \quad (5.9)$$

Here i and j refer to the i^{th} radial bin from the j^{th} momentum setting. The only factors which have not been defined explicitly in Section 5.1 or 5.2 are the interactively reconstructed event sums denoted with the subscript "SK" and the statistical weights for the scanned sample, α for Lab E and β for the Wonder Building. The event sums from the SIMULTANEOUS LIVE analysis are similarly corrected except for the absence of the livetime factor.

The final event sample, after cuts and normalization corrections for the positive momentum settings, is listed in Table 5.9a for the SIMULTANEOUS DATA TAKING analysis. The corrected ratios $\frac{Lab E}{Wonder Building}$ for each radial bin are also plotted in Figure 5.9a. The errors are only statistical. In Figure 5.9b and Table 5.9b we show the ratio

of the SIMULTANEOUS LIVE events ratios and the SIMULTANEOUS DATA TAKING ratios. Again, the errors are statistical. The χ^2 for this set of points to be all unity is 7.5. The consistency is reasonable. As mentioned earlier, the remaining discussion will only consider the SIMULTANEOUS DATA TAKING analysis. In Table 5.9a and Figure 5.10 we also show the selected data for the -165 setting of the secondary beam. We separate the positive and negative samples since they correspond to neutrino and antineutrino oscillations respectively. In Chapter 6 we discuss the assumptions needed to combine them.

§5.7 Monte Carlo

The neutrino beam Monte Carlo files discussed in section 2.4 are used as input to event Monte Carlo programs for the Lab E and Wonder Building detectors. These programs have a number of uses. The entire experiment can be simulated and the effects of cuts and acceptance may be studied. Track finding and reconstruction algorithms are debugged with the Monte Carlo. Effects due to the properties of the dichromatic beam can be gauged. In particular, the systematic uncertainty created by the scaling assumption used in defining transverse cuts may be estimated. This last point is discussed in more detail in Section 5.7. Here we describe the implementation of the event Monte Carlo programs.

The neutrino ray files are extrapolated to the larger Lab E detector. Transverse vertices within a ± 50 inch $\times \pm 50$ inch square about the detector axis are retained. A longitudinal vertex (PLACE) is chosen randomly from a flat distribution in the range $56 >$

PLACE> 8. Charged current interactions are generated according to models of nucleon structure functions parametrized by Buras and Gaemers⁶⁰. A data record containing all geometrical and kinematical variables describing the interaction as well as the incident neutrino type, energy, angle, and decay point is written to a file.

The generated event files are input to event propagation routines for the two detectors. At the detectors, transverse vertex positions are determined from the incident neutrino information. The propagation routines smear the hadron energy according to the measured resolution. Muons are directed through the steel targets are spectrometers. Random multiple coulomb scattering impulses are applied along their paths. Sparks are generated modulo the 0.5 mm position resolution and spark buffers are filled.

The Monte Carlo events are reconstructed by the identical code used to reconstruct the data. DST's and SUPERDST's are written from these events. Monitor information was not simulated and so dummy monitor records were included in the SUPERDST. For each setting of the hadron beam transport, 30,000 events at each detector were generated.

In the oscillation analysis, ratios of events in the two detectors are formed. By propagating the *same* set of generated events in the two detectors an economical decrease in the error on the Monte Carlo ratios is achieved. The Lab E and Wonder Building event sums are strongly correlated. The ratio of events in the i^{th} radial bin may be written,

$$R_i^{Monte Carlo} = \frac{N_{iSAMB} + N_{iDIFFERENT}^{LE}}{N_{iSAMB} + N_{iDIFFERENT}^{WB}} \quad (5.10)$$

where,

$$N_{iSAMB} = \text{events accepted in this bin in both detectors}$$

$$N_{DIFFERENT}^{LE} = \text{events accepted in this bin in Lab E only}$$

$$N_{DIFFERENT}^{WB} = \text{events accepted in this bin in W.B. only}$$

$N_{DIFFERENT}^{LE}$ and $N_{DIFFERENT}^{WB}$ will be non-zero only because of acceptance differences and smearing. The error on R_i^{MC} originates only from these terms.

§5.8 Scaling Corrections

The assumption that the effective mean decay point ($\langle z_{decay} \rangle$) is at the center of the decay region (z_{center}) determines the scaling rule applied in Section 5.3. If the true location of $\langle z_{decay} \rangle$ is at another point there will be systematic shifts in the two detector event ratios for radial bins defined by the scaling rule. Referring to Figure 5.11a we can see that if,

$$\langle z_{decay} \rangle \text{ is greater than } z_{center} : \frac{N_{LE}}{N_{WB}} < 1$$

while if,

$$\langle z_{decay} \rangle \text{ is less than } z_{center} : \frac{N_{LE}}{N_{WB}} > 1 . \quad (5.11)$$

The scaling assumption results in a set of corrections which must be applied to the event ratios. There are a number of factors which determine $\langle z_{decay} \rangle$. These include decays upstream of the decay region (wide band background), the exponential decay distribution in the decay pipe, parallax and solid angle, and the secondary beam divergence. In this section we discuss the effect of each factor on $\langle z_{decay} \rangle$.

5.8.1 Wide Band Background

The source of neutrinos which originate before momentum and sign selection occurs was discussed in Section 2.2. For these wide band neutrinos, the mean decay position is far upstream of the decay region center. The transverse areas of the two detectors are defined by the scaling rule Equation 5.7. If we approximate the wide band decays as a point source originating at $\Delta=208$ meters upstream of the decay region center, then the solid angle ratio subtended by the transverse areas is given by,

$$\frac{\Omega_{Lab\ E}}{\Omega_{Wonder\ Building}} = \left(\frac{Z_{LE}}{Z_{WB}}\right)^2 \frac{(\Delta + Z_{WB})^2}{(\Delta + Z_{LE})^2} = 1.182 . \quad (5.12)$$

Here Z_{LE} and Z_{WB} are the respective distances from the decay region center to the detectors. Because of the choice of transverse cuts, the wide band background should result in an enhanced rate at Lab E relative to the Wonder Building.

To correct for these upstream decays, the wide band signal was measured. This measurement was made at each setting by closing the momentum defining collimator OCTP and counting interactions in the two detectors. The wide band neutrinos do not have the dichromatic energy spectrum. They can however be assigned to the pion and kaon sample by their measured energies. The closed slit data sample is listed in Table 5.10. Unfortunately not enough closed slit statistics were accumulated to allow a setting by setting correction at each detector. Instead, the following procedure has been adopted. For each setting the events are divided between the pion and kaon bands at each detector using their measured energy. These events as well as the corresponding proton flux are summed. In this way a total rate is computed over all energies. For pions and kaons, the

Lab E and Wonder Building rates are then averaged. The result is,

$$3.13 \pm 0.35 \nu_{\pi}/10^{16} \text{ protons} \quad (5.13a)$$

and,

$$0.20 \pm 0.09 \nu_K/10^{16} \text{ protons.} \quad (5.13b)$$

This approach minimizes statistical errors. These rates are then used to compute the fraction of the total Wonder Building event sample due to wide band background. The Lab E fraction is computed by multiplying the Wonder Building fraction by the factor of Equation 5.12. The corrected event ratios are found by subtracting the appropriate number of events from the total Lab E and Wonder Building sums for each radial bin and energy setting. Operationally, we compute correction factors, to be multiplied by the $\frac{\text{Lab E}}{\text{Wonder Building}}$ event ratios of Equation 5.9. All correction factors are less than 1.1% and are listed in Table 5.11. Typically, 2-5% of the Wonder Building pion neutrino sample and less than 1% of the kaon sample was due to wide band background. At Lab E, the calculated fractions were higher by the factor of 1.182.

5.8.2 Decay Region Decays

Most of the neutrinos originate in the decay region. A set of effects dependent upon the parent particle type and beam phase space determine the $\langle z_{decay} \rangle$ there. All these effects were calculated using the Monte Carlo simulation of the neutrino beam and detectors. In Chapter 2 we considered the origin of deviations from the ideal dichromatic

neutrino spectrum. Some of the factors which alter the dichromatic spectrum also determine $\langle z_{decay} \rangle$.

1) Exponential Decay: The decay distribution Equation 2.20 implies that $\langle z_{decay} \rangle$ will be shifted upstream. This effect is largest for low momentum kaons. The point, given by,

$$\langle z_{decay} \rangle = \frac{\int_0^{350 \text{ meters}} z \exp(-\frac{z}{L_0}) dz}{\int_0^{350 \text{ meters}} \exp(-\frac{z}{L_0}) dz} \quad (5.14)$$

is listed in Table 5.12 for different beam momentum and particle type. Here L_0 is the decay length for the meson, equal to $\beta\gamma c\tau$, where τ is the particle lifetime.

2) Parallax: Since the meson decays occur over a 350 meter long decay region a given annular segment $(R, R + \Delta R)$ at a detector will receive flux from different decay angular intervals $(\theta, \theta + \Delta\theta)$ depending upon where the decay point is. The scaling rule Equation 5.7 relates an annular region in the Wonder Building detector to a corresponding one at Lab E. For decays which occur at the downstream end of the decay region, flux through the Wonder Building will extrapolate to smaller radii at Lab E than predicted by the scaling rule. For decays at the upstream end the opposite effect will occur. For *fixed* ΔR , the downstream decays originate over a larger angular interval than the upstream decays. In terms of flux then, more neutrinos come from downstream than from upstream and thus $\langle z_{decay} \rangle$ is shifted down. This is indicated in Figure 5.11b. The neutrino event rates at the detectors are proportional to the neutrino energy and this energy is in turn related to the decay angle (Equation 2.8). Because of this, the upstream decays into $(R, R + \Delta R)$, though fewer in number, have a larger interaction probability since they come with smaller decay angles. The effects of parallax on event rates are therefore complicated by a set of influences which oppose each other. In general $\langle z_{decay} \rangle$ will end

up somewhere other than the center of the decay region but intuitively it is not possible to say which influence will dominate. These effects though are implicitly contained in both the radial event distribution integrals (Equations 2.21 and 2.22) and the Monte Carlo simulations. These calculations indicate that $\langle z_{decay} \rangle$ shifts downstream by roughly 5–25 meters depending upon the meson type and momentum.

3) Secondary Beam Divergence: The effect of the secondary beam divergence on the dichromatic spectrum was discussed in Section 2.2. There it was indicated that for $\theta_{beam} > \theta_{decay}$ the energy angle correlation would be disrupted. The secondary beam divergence has an important effect upon the event ratios in the oscillation experiment also. Since the beam divergence appears to originate at the entrance to the decay region, $\langle z_{decay} \rangle$ will shift upstream. For fixed radius at the near detector, rays will be shifted to smaller radii in the far detector than the scaling rule would predict. Table 2.3 lists the mean decay angle for pions and kaons over the momentum range in E-701. The angular spectrum of the secondary beam was measured with the hole collimator scan (Section 2.3). With an r.m.s. deviation of ≈ 0.0002 radians, the effect is strongest for the higher momentum pions.

To isolate the decay effects from possible acceptance differences in Lab E and the Wonder Building, a simplified Monte Carlo study was made. A beam model was constructed with Gaussian angular and momentum spectra. The width of the momentum spectrum was taken to be $\pm 0.1 \times P_{beam}$. Decays occurred over the length of the decay region according to Equation 2.20. Neutrinos were summed, weighted by their energy, in the usual radial bins at the two detectors. This simulation therefore included the three effects listed above. Figures 5.12a, b, c, and d show the ratio plot for choices of the

gaussian projected beam divergence, 0, 0.1, 0.2, and 0.3 milliradians. Note the similarity with Figure 5.9. This calculation predicts an enhancement in the ratio, $\frac{Lab E}{W_{ond} Bldg}$, for the higher energy pions which can reach 5-6% at 0.3 milliradians. All the kaon points appear low. The zero beam divergence calculation shows that parallax wins over exponential decay to move $\langle z_{decay} \rangle$ downstream.

To determine a set of scaling corrections for the data, the full Monte Carlo described in Section 5.4 is used. Neutrino rays are created with the DECAY TURTLE program. At the level of these corrections, some systematic uncertainty exists. This is due to discrepancies between the angular spectrum predicted by DECAY TURTLE and measured by the hole collimator and MUON SWICs. These discrepancies were discussed in Section 2.4.1. The basic conclusion was that the DECAY TURTLE beam was narrower in divergence than the actual secondary beam. This systematic uncertainty is not just a normalization error since it correlates different radial bins.

To bracket the influence of beam uncertainties we calculate a second set of scaling corrections. These are made from a different DECAY TURTLE beam. This beam had been created for Monte Carlo studies of an earlier neutrino experiment (E-616). There, locations of various beamline elements had been adjusted differently. This beam was initially narrower than the DECAY TURTLE beam calculated for E-701 but qualitatively matched the wide horizontal angle tail seen in the data. We use this beam in a totally ad hoc manner here, just to estimate the influence of these effects. Since the beam was narrow, we artificially increased its angular divergence. Meson angles were multiplied by 2.0 and 1.6 for θ_x and θ_y respectively. These factors were chosen by minimizing a χ^2 function

between the angular distributions of this "stretched" beam and the data. In Figure 5.13a we show a comparison between the two beam models and the hole collimator measured angular distribution for 165 GeV. The same comparison is found in Figure 5.13b for the muon distributions. Scaling corrections calculated with the nominal DECAY TURTLE beam and the "stretched" DECAY TURTLE beam are shown in Figures 5.14a and b for the positive and 5.15a and b for the negative secondary beam. They are listed in Table 5.13. The difference between the two calculations are less than 1% in the corrections. This suggests the wide angle tail is of negligible effect.

§5.9 Final Corrected Ratios

The final ratios for the i^{th} radial bin and j^{th} energy setting are corrected for the scaling assumption and for wide band background,

$$R_{ij}^{final} = \left(\frac{N_{corr}^{LE}(i,j)}{N_{corr}^{WB}(i,j)} \right) \frac{(1 - \Omega_R R_{ij}^{wideband})}{(1 - R_{ij}^{wideband})} \left(\frac{N_{MC}^{WB}(i,j)}{N_{MC}^{LE}(i,j)} \right). \quad (5.15)$$

Here, $R^{wideband}$ is the Wonder Building wide band fraction and N_{MC} are the Monte Carlo calculated event sums. These final ratios are shown for the two beam models in Figure 5.16a and b for the positive settings and in Figure 5.17a and b for the negative setting. The corrected ratios are listed in Table 5.14.

§5.10 Systematic Errors

Two types of systematic error enter this analysis. The first type is uncorrelated errors which shift groups of ratio points for each setting rigidly up or down. An example

is the error on the relative target mass correction discussed in Section 5.5 These may be considered as overall normalization errors between the two detectors. The second type is the correlated error discussed in the preceeding section. These result from uncertainties in the scaling corrections. In this section we discuss the separate sources of systematic error and estimate their contribution to the overall systematic uncertainty.

5.10.1 Uncorrelated Errors

1) Relative Veto Deadtime: The systematic error in the veto deadtime ratio is gauged using alternate measurements than obtained with Equation 5.8. Another estimate comes from the quantity,

$$1 - \frac{TRIGGER1 \cdot (VETO_{delayed})}{TRIGGER1} \quad (5.16)$$

where the delay is greater than the TRIGGER 1 event gate. This quantity is a direct measurement of the veto deadtime and is especially sensitive to fluctuations close in time with the event gate. Unfortunately this scaler was unavailable at Lab E because of a hardware failure. For the Wonder Building, the difference between it and the quantity in Equation 5.8, listed in Table 5.15, is taken as an estimate of the systematic error on the veto deadtime. At both Lab E and the Wonder Building, both the veto singles and a clock gated on the beam spill were scaled. With these an average singles rate could be measured for each detector. The alternate Lab E veto deadtime is estimated by assuming the Wonder Building deadtime, given by Equation 5.16, can be scaled the ratio of the average veto rates. If this is true then the Lab E deadtime is given by,

$$\frac{(VETO)_{LIVE}^E}{(CLOCK)_{LIVE}^E} \frac{(CLOCK)_{LIVE}^{WB}}{(VETO)_{LIVE}^{WB}} \left(1 - \frac{(TRIGGER1)^{WB} \cdot VETO_{delayed}^{WB}}{(TRIGGER1)^{WB}} \right) . \quad (5.17)$$

The difference between this quantity and Equation 5.8 for Lab E is listed in Table 5.14 and is taken to bracket the systematic uncertainty in that veto deadtime. Combining all these estimates we take 0.003 as the systematic error in the relative veto deadtime correction for the neutrino data and 0.0045 for the antineutrino data.

2) Relative Detector Livetimes: These corrections, which range between 1.0 and 1.13, were discussed in Sections 5.5 and 5.2. An alternate estimate of the the livetime may be obtained by comparing the total number of events due to a set of triggers with the total number of these triggers. We note that the triggers are scaled over the entire spill gate. A systematic error of 0.01 in the livetime correction for the neutrino data and .0125 for the antineutrino data is estimated by calculating the relative livetime as the ratio of,

$$\frac{N_{TR1} + N_{TR2} + N_{TR3} \text{ events}}{TR1 + TR2 + TR3 \text{ triggers}} \quad (5.18)$$

for each detector in addition to the way described in Section 5.2. Here, the numerator is the total number of events recorded due to the three triggers. The denominator is the total number of triggers.

3) Target Surface Density: This was discussed in Sections 3.6 and 5.5. The quoted error is 0.003.

4) Beam Centers: The detector beam centers are determined to 0.25 inches (0.64 centimeters). This uncertainty propagates as systematic uncertainty of a 0.003 to 0.004 in the radially binned event ratios. This uncertainty is estimated by varying the beam centers by the quoted error and observing the effect upon the event ratios.

5) Longitudinal Position of Detectors: The longitudinal position of the detectors with respect to the decay region determines the value of the scaling factor. These positions were determined through civil engineering surveys carried out by Fermilab and by aerial photographs of the neutrino beam line. The quoted error on these distances is ± 1 meter. This is found to propagate as an uncertainty of 0.001 in the event ratios.

6) Wide Band Background: The uncertainty in the wide band background correction is estimated from the statistics of the closed slit event sample. This is discussed in Section 5.7. The error on the correction is taken to be 0.003.

7) Target Tracking Efficiency: A small correction for target track finding failures in the Lab E detector was discussed in Section 5.4.3. The error on the correction is statistical but should be included in the total systematic error on the event ratios. The average error is 0.0004 for both the neutrino and antineutrino data.

The above seven factors contribute to the uncorrelated systematic error on the normalization at each setting. Combining them in quadrature yields a total estimated systematic error of $\pm 1.2\%$ for the neutrino data and $\pm 1.5\%$ for the antineutrino data. The systematic errors which are included in fits of the data to neutrino oscillation hypotheses are set at 1.5% for the neutrinos and 1.8% for the antineutrinos.

5.10.2 Correlated Scaling Error

The error is estimated by computing the scaling corrections with the two beam models as described in Section 5.7. The point to point difference in data corrected by the

two models measures the effect of this error and the difference is always less than about 0.01 (the correction range from 0 to 0.03). In Chapter 6 we will discuss a calculation of oscillation parameters excluded by this data. The correlated error is included in this calculation by performing it twice, using both models. The systematic error corresponds to the difference between the two excluded regions.

Chapter 6

Oscillation Analysis

§6.1 General Aim

Most of the ratios shown in Figure 5.16 and 5.17 are consistent with unity. This data cannot therefore be used to claim finite values for the oscillation parameters $(\Delta m^2, \sin^2 2\theta)$ within the region of experimental sensitivity. The statistical power of the results can however be used to exclude a definite region of the $(\Delta m^2, \sin^2 2\theta)$ plane with a given probability. In this chapter we describe the procedure used to extract limits on the inclusive oscillations of muon neutrinos.

§6.2 Fitting Procedure

6.2.1 Use of Monte Carlo

To find the probability that a particular hypothesis describe the data, the expected values of the $\frac{Lab\ E}{Wond\ Bldg}$ event ratios for that hypothesis must be calculated. At the most basic level, the two state inclusive oscillation probability for a muon neutrino of energy E_ν , having travelled a distance L is given by Equation 1.45,

$$P(\nu_\mu \rightarrow \nu_\mu) = 1 - \sin^2 2\theta \sin^2\left(\frac{1.27 \Delta m^2 L}{E_\nu}\right) \quad (1.45)$$

for the hypothesis $(\Delta m^2, \sin^2 2\theta)$. For two distances, the relative probability is just the ratio of two quantities of the form (1.45) with the corresponding distances substituted. In the actual oscillation experiment, the measured data points are ratios of event sums in discrete radial bins. By Equation 5.3 these are proportional to the flux ratios. Into a given radial bin enter neutrinos spanning the energy ranges shown in Figure 5.6. These neutrinos have travelled distances which can be as different as the length of the decay region. Furthermore, neutrino interactions in the detectors are sampled with finite measurement resolution. For these reasons, it is impossible to just use Equation 1.45 as a prediction of the outcome of the experiment for a given hypothesis. The calculation must in addition take into account the particular experimental details such as the geometry, the neutrino energy spectrum, and the experimental resolutions.

To calculate the predicted ratios for a given hypothesis, we use the Monte Carlo simulation discussed in Sections 2.4 and 5.7. The Monte Carlo event sample, which was used to find the scaling corrections, pass the same analysis cuts as the measured events. A file of these selected events for each energy setting is made which includes their generated energy and decay point. For a particular hypothesis $(\Delta m^2, \sin^2 2\theta)$, the Monte

Carlo experiment is reanalyzed just as in Section 5.8 but all the radial bin sums are made with weighted events. The weighting factors, ranging between 0 and 1, are the inclusive oscillation probabilities from Equation 1.45. Because the generated energy and decay point are available, the correct probability may be calculated for each event separately. The result is a set of ratios for each radial bin analogous to Table 5.9.

Originally, the Monte Carlo beam was used to calculate the scaling corrections to the data since it accounts for the effects of exponential decay, parallax, solid angle, and secondary beam divergence. Since this Monte Carlo beam model is used to calculate the oscillation predictions, the resulting ratios contain the beam effects also. Because of this, it is appropriate to compare these predicted ratios with the data *uncorrected* for the scaling assumption (Table 5.9 and Figures 5.9 and 5.10). (Wideband background corrections are to be applied as we do not consider changes in these for the case of neutrino oscillations.)

Using the Monte Carlo method described above, sets of predicted ratios may be calculated for any hypothesis ($\Delta m^2, \sin^2 2\theta$). A discrete grid of points in the ($\Delta m^2, \sin^2 2\theta$) plane is made which contains the region of sensitivity for E-701. At great expense of computer time, the predicted ratios are determined for each grid point. A statistical test can then be applied to a set of predicted ratios to gauge its probability of coinciding with the data. In this manner a region of the plane may be excluded with a chosen probability.

6.2.2 χ^2 Function

In this analysis, the probability that a particular set of event ratios, correspond-

ing to the pair $(\Delta m^2, \sin^2 2\theta)$ coincide with the data is determined by the value of a χ^2 function. The validity of using the χ^2 has its basis in the principle of maximum likelihood. The χ^2 function may be put into correspondence with the natural logarithm of the likelihood function when the measured quantities are Gaussian distributed random variables. In the case of the oscillation measurement, the event sums recorded at each detector would qualify as Gaussian distributed quantities. The analysis is however done with respect to ratios of these sums. The ratios in general may not be gaussian distributed. For the following discussion we will however assume that it is correct to form the χ^2 with these ratios. Later we describe a study which indicates this to be true.

The χ^2 function depends upon the particular pair $(\Delta m^2, \sin^2 2\theta)$ and also upon a set of normalization factors, possibly different at each energy setting. These normalization factors have their origin in the uncorrelated systematic errors discussed in Section 5.9. Because of these errors, sets of ratios for each energy may shift up or down within the quoted 1.5% (1.8%) uncertainty for the neutrino (antineutrino) data. The χ^2 function is minimized with respect to these normalizations factors. The χ^2 function is given by:

$$\chi^2(\Delta m^2, \sin^2 2\theta, f_1, \dots, f_N) = \sum_{i=1}^N \sum_{j=1}^3 \frac{(R_{ij}^{measured} - f_i R_{ij}^{MC}(\Delta m^2, \sin^2 2\theta))^2}{\sigma_{ij}^2} + \sum_{i=1}^N \frac{(f_i - 1)^2}{\sigma_{norm}^2} \quad (6.1)$$

where,

$$\sigma_{ij}^2 = (\sigma_{ij}^{DATA})^2 + (\sigma_{ij}^{MC})^2$$

and,

$$\frac{\partial \chi^2}{\partial f_i} = 0$$

Here, $R_{ij}^{measured}$ is the measured event ratio $\frac{LabE}{WondBldg}$ from the i^{th} energy setting and j^{th} radial bin and $R_{ij}^{MC}(\Delta m^2, \sin^2 2\theta)$ is the corresponding Monte Carlo calculated prediction for that ratio. The error σ_{ij} includes a contribution from the statistical error of the data and the Monte Carlo. As described in Section 5.7, 30,000 Monte Carlo events were generated at each energy to calculate the scaling corrections listed in Table 5.12. The Monte Carlo statistical errors on the predicted ratios were typically a factor of six less than the statistical errors on the data. The normalization factor for the i^{th} setting is f_i and σ_{norm} is the normalization error discussed in Section 5.10. Typically the f_i vary between 0.99 and 1.01.

The question of the validity of using this χ^2 function to analyze ratios of Gaussian variables was addressed by recalculating the χ^2 in terms of logarithms of the ratios. Bodek⁶⁶ has discussed the case of a ratio where the errors on the numerator and denominator are similar. He points out that in this case the natural logarithm of the ratio is gaussian distributed. We then expect that the χ^2 function is in correspondence with the likelihood function. We have calculated the χ^2 over a wide range of values for $(\Delta m^2, \sin^2 2\theta)$ and find that the result obtained using Equation 6.1 is consistent with the analogous calculation in terms of logarithms of ratios.

As mentioned in the preceeding subsection, predicted ratios of events were found on a grid in the $(\Delta m^2, \sin^2 2\theta)$ plane. To gauge the influence of the correlated systematic uncertainty (from the beam divergence) we found predicted ratios using both

the nominal DECAY TURTLE and "stretched" DECAY TURTLE beam models. Tables 6.1a and b, 6.2a and b, and 6.3a and b, display numerically the calculated χ^2 grids for the neutrino, antineutrino, and combined data sets as calculated with the nominal DECAY TURTLE beam model. They were taken over a range extending from 4 eV^2 and 1500 eV^2 for Δm^2 and 0 to 1 (in 0.02 and 0.03 steps) for $\sin^2 2\theta$.

As noted, the corrected ratios are consistent with unity. Accordingly, the χ^2 for the case of no oscillations ($\sin^2 2\theta = 0$) is acceptable. In Table 6.4 we list these values and the corresponding minimum χ^2 for the neutrino, antineutrino and combined data sets. The best solutions always appear at the edges of the experimental sensitivity for the data sets with many degrees of freedom. This might be because it is easy for the fit to pick out either a very gradual or very rapidly oscillating solution. (For the antineutrinos, with only three data points there may also be good solutions within the region of experimental sensitivity.) In Figure 6.1b and b we show the neutrino and antineutrino data ratios and superimpose the best fits as indicated by the nominal DECAY TURTLE Monte Carlo beam simulation. Here we have divided out the no oscillation prediction. The fit allows the data points from each setting to shift within the quoted systematic normalization error.

§6.3 Tests of Oscillation Hypothesis

Two different statistical tests are applied to compare the predicted ratios with data. In general these tests give different probabilities for any particular $(\Delta m^2, \sin^2 2\theta)$ pair. The two different statistical tests applied to a given result can lead to different

interpretations of the measurement. We will discuss below the somewhat complimentary features of these tests.

In any statistical test based upon the value of the χ^2 function, the χ^2 is assumed to originate from a particular probability distribution. To gauge the probability that a certain hypothesis coincide with the data we must find what percentage of the χ^2 probability distribution lies above or below the χ^2 for that hypothesis. In this analysis we may locate an equi- χ^2 contour in the $(\Delta m^2, \sin^2 2\theta)$ plane such that we would expect to obtain that value of χ^2 or less a certain percentage of the time. Let this percentage be ninety (for example). In this case, we may say that pairs $(\Delta m^2, \sin^2 2\theta)$ for which χ^2 is greater than this contour value are excluded by the data at the 90% confidence level. Of course this value is determined by the particular distribution we choose to sample. The two statistical tests we apply are based upon sampling different types of χ^2 probability distributions.

In determining these χ^2 contours, we include the effects of correlated systematic uncertainties due to ignorance of the true beam divergence. These were discussed in Section 5.9. As noted above, the χ^2 surface has been calculated for both Monte Carlo beam models. For both of these χ^2 surfaces, a given confidence level contour may be drawn. To include the effect of the beam divergence we take as the excluded region, the intersection of the set of points in the $(\Delta m^2, \sin^2 2\theta)$ plane *excluded* in both surfaces. In other words, at a given Δm^2 we take the *more conservative* of the two limits.

6.3.1 Pearson's χ^2 Test

The first test is the standard Pearson χ^2 test⁶⁷. Therein the χ^2 is regarded as originating from the universal χ^2 distribution for some number of degrees of freedom. Since we are not fitting for the parameters $(\Delta m^2, \sin^2 2\theta)$, the number of degrees of freedom is just the number of data points, eighteen, fifteen, and three, for the combined, neutrino, and antineutrino data sets respectively. For eighteen, fifteen, and three degrees of freedom, the 90% point of this χ^2 distribution is given by 25.99, 22.31, and 6.25 respectively.

The Pearson χ^2 test may be viewed as an absolute test in that it returns the probability of a particular fit with respect to all possible fits. We had for convenience chosen to analyze the results of this measurement in terms of a two component oscillation formalism. In general we expect that a fit in terms of more than two components (three masses, four masses, etc.) might be better. The Pearson test as applied to this analysis measures a particular two component fit against any N component fit.

The contours determined by the Pearson test for a 90% confidence level are shown in Figure 6.2a, b, and c from the neutrino, antineutrino and combined data set for both beam models. Here, as in a direct comparison of the scaling corrections calculated with the two models, it is seen that the effect of the divergence uncertainty is small. Limits combining these contours to account for the beam divergence error, for the three data sets, are shown as the dashed curves in Figures 6.3, 6.4, and 6.5.

6.3.2 Likelihood Ratio Test

An alternative test measures a particular two component fit against all other two component fits. Usually such a test will yield more restrictive limits than the Pearson χ^2 test. This test is based upon the probability distribution for the ratio of the likelihood function⁶⁸ $\mathcal{L}$,

$$R_L = \frac{\mathcal{L}(\Delta m_0^2, \sin^2 2\theta_0)}{\text{MAXIMUM}\{\mathcal{L}(\Delta m^2, \sin^2 2\theta)\}} \quad (6.2)$$

for a given hypothesis $(\Delta m_0^2, \sin^2 2\theta_0)$ to its maximum value in the two parameter space. Such a probability distribution is determined both by the complicated dependence of the measured and predicted event ratios upon the the oscillation parameters and also by the event statistics. As a result, it may not in general be given in an analytical form though it can often be calculated by Monte Carlo methods. In the present case, since we have argued that the χ^2 function may be put into correspondence with the likelihood function, we may define,

$$\lambda \equiv -2 \ln R_L = \chi^2(\Delta m_0^2, \sin^2 2\theta_0) - \chi_{\text{minimum}}^2 \quad (6.3)$$

The distribution of λ indicates a χ^2 contour in the $\Delta m^2, \sin^2 2\theta$ plane with respect to the minimum χ^2 which excludes some region at a given confidence level.

We have determined the λ distribution by the following Monte Carlo method.

- 1) An underlying hypothesis, $(\Delta m_0^2, \sin^2 2\theta_0)$ is chosen.
- 2) Predicted ratios of events for each energy in the experiment are calculated assuming the above hypothesis. This calculation includes the experimental effects such as the decay distribution and measurement resolutions.

- 3) The event sums in 2) are smeared by the measured statistical errors of the E-701 data to produce fake data.
- 4) A χ^2 is calculated for each $(\Delta m^2, \sin^2 2\theta)$ point in the region of sensitivity between the fake data and a prediction.
- 5) The minimum χ^2 in the plane is found, χ^2_{minimum} , and the difference, λ , between it and the χ^2 for the chosen underlying hypothesis, $(\Delta m_0^2, \sin^2 2\theta_0)$, is determined.
- 6) Steps 3), 4), and 5) are repeated many (≈ 1000) times and a $\lambda = \chi^2(\Delta m_0^2, \sin^2 2\theta_0) - \chi^2_{\text{minimum}}$ distribution is formed.

The λ distributions, computed as described, for eighteen degrees of freedom and a set of three different underlying hypotheses are shown in Figures 6.6a, b, and c. In general it is observed that both the shape of the λ distribution and the 90% probability point vary with the underlying hypothesis. In Table 6.5 we list the 90% points for a set of these and for 3, 15, and 18 degrees of freedom.

The likelihood ratio test has been discussed in the literature. The λ distribution is expected to asymptotically approach a χ^2 distribution for a number of degrees of freedom equal to the number of parameters in the fit (here 2). Included in Figure 6.6 is the χ^2 distribution for 2 degrees of freedom. The Monte Carlo technique for calculating the λ distribution is necessary for the likelihood ratio test to be useful in an analysis of the type done in this thesis. This is because the calculated distribution both differs from the asymptotic one and is dependent upon the underlying hypothesis.

The use of the likelihood ratio test to determine whether the data exclude a

certain hypothesis is complicated by the fact that the λ distribution varies for different underlying oscillation parameters. In practice, once the minimum χ^2 for the real data is located, a value for λ can be determined for each possible $(\Delta m^2, \sin^2 2\theta)$ pair. This value should be checked against the correct λ distribution for that pair. In such a way, an excluded region may be found. The λ distribution actually varies slowly with $(\Delta m^2_0, \sin^2 2\theta_0)$ so it is sufficient to calculate the distribution for a few scattered values. The likelihood ratio test limits presented in this work were found using only the distributions for which the parameters are listed in Tables 6.5. The minimum χ^2 values for the various data sets are listed in Table 6.4, and the regions excluded at the 90% confidence level are shown by the solid curves in Figures 6.3, 6.4, and 6.5. Again these limits combine the beam divergence uncertainty as described above. In Figure 6.7 we show the 90% limits calculated with the two beam models.

6.3.3 Discussion of Statistical Analysis

A criticism of the likelihood ratio test is that it is particularly sensitive to the statistical fluctuations in the data which determine the value and location of χ^2_{minimum} . Ignoring the antineutrino data which poorly constrains the oscillation limit (since there are only three data points) we note that the minimum χ^2 is found for $(\Delta m^2, \sin^2 2\theta)$ at the edge of the experimental sensitivity. With the Monte Carlo simulation used to find the λ distribution we have constructed a scatterplot of the χ^2_{minimum} location in the $(\Delta m^2, \sin^2 2\theta)$ plane for an underlying hypothesis of no oscillations. In this calculation we have allowed the data points to shift within the the 1.5% systematic error in the

generation of the starting hypothesis. For each set of fake data, the value of $(\Delta m^2, \sin^2 2\theta)$ that minimized the χ^2 was found. The result is shown in Figure 6.8. Values of Δm^2 near the edge of the experimental sensitivity do not appear to be favored in particular. The minimum χ^2 does not tend to the small value found in the real experiment. Having done only a single experiment it is hard to say whether we are seeing a statistical fluctuation or an unknown systematic effect. Partially for these reasons, we opt to show contours determined with the likelihood ratio method and also with the (usually) less restrictive Pearson's χ^2 test.

§6.4 Results and Conclusions

6.4.1 This Experiment

Limits upon the oscillations parameters for muon neutrinos and antineutrino may be read off the plots in Figures 6.3, 6.4, and 6.55. Certainly Δm^2 may be nonzero within the experimental sensitivity if the mixing is smaller than indicated on the limit plots. By adopting certain assumptions about the (supposed) neutrino mass spectrum we may reach more specific conclusions. If the mixing is actually larger than indicated on the limit plots, ($\sin^2 2\theta > 0.03 - 1$) we may consider two extremes. In a two neutrino scenario, if one neutrino is massless and the other is not, then the excluded masses may be read off the limit plot by taking the square root of Δm^2 . This roughly corresponds to excluding the interval, $5 \text{ eV} < m_\nu < 30 \text{ eV}$ for neutrinos and $7 \text{ eV} < m_{\nu c} < 14 \text{ eV}$ for

antineutrinos. If on the other hand, two neutrinos are massive but their mass difference is small compared to their masses, we may approximate,

$$\Delta m^2 \approx 2m\Delta m \quad (6.4)$$

In this case we may exclude a particular interval depending upon Δm . This interval is depicted versus Δm in Figure 6.9 for neutrinos and antineutrinos corresponding to the 90% confidence level.

In this analysis, we have presented limits on oscillations from both neutrino data and antineutrino data, and from a combined set. The practice of combining data from neutrino and antineutrino interactions in an inclusive experiment warrants some justification⁶⁹. This is because these particles are related to each other by the CP transformation. Experimentally, CP violation has been observed in kaon decays⁷⁰ and is believed to come about through mixing in the weak hadronic (quark) current²⁰. As in that system, CP violation could only occur among the neutrinos if three or more states were mixed. As discussed in Chapter 1, the *exclusive* oscillation probabilities are related by the CP transformation as,

$$P(\nu_{l_i} \rightarrow \nu_{l_j}) = P(\nu_{l_i}^c \rightarrow \nu_{l_j}^c) \quad (6.5)$$

and by the CPT transformation as,

$$P(\nu_{l_i} \rightarrow \nu_{l_j}) = P(\nu_{l_j}^c \rightarrow \nu_{l_i}^c) \quad (6.6)$$

Here, ν_{l_i} and ν_{l_j} are two weak interaction neutrinos for which $i \neq j$. In the case of inclusive neutrino oscillations we are measuring a "non-transition" probability which involves a coherent sum over all possible exclusive channels. For this reason CP conservation

requires,

$$P(\nu_{l_i} \rightarrow \nu_{l_i}) = P(\nu_{l_i}^c \rightarrow \nu_{l_i}^c) \quad (6.7)$$

In addition to this however, CPT conservation implies the equality of (Dirac) neutrino and antineutrino mass eigenstates and thus also the relation 6.7. Therefore, inclusive neutrino and antineutrino oscillations are required to be the same by CP *or* CPT invariance. If this experiment had detected different (and nonzero) rates for neutrino and antineutrino oscillations this would indicate that both CP *and* CPT were violated. The only way to search for separate breakdown of CP or CPT in the neutrino system is to compare the exclusive oscillation channels described above and in Chapter 1. It is therefore with the assumption of CP or CPT conservation that we construct the combined neutrino and antineutrino oscillation limits.

6.4.2 Comparison With Other Oscillation Measurements

The results of this experiment may be compared both with other oscillation experiments and also with direct measurements or limits on neutrino masses. Prior to the completion of the E-701 analysis, the exclusive muon neutrino and antineutrino oscillations into $\nu_\mu^{(c)}$ and $\nu_\mu^{(c)}$ had been constrained according to the limits shown in Figure 1.4. While this experiment does not add to the regions excluded by those measurements it does constrain these oscillations with very different systematic limitations. All the exclusive experiments in this region were performed with single detectors and relied heavily on visual pattern recognition for the identification of neutrino interactions (bubble chambers and emulsions). The exclusive experiments are also sensitive to estimations of

background neutrinos which can mimic the oscillation signal. Other muon neutrino and antineutrino inclusive searches have been performed with similar techniques to E-701. In particular we refer to the measurements made at CERN by the CDHS⁴⁹ and CHARM⁴⁰ collaborations. Both these were performed with low energy (≈ 3 GeV) mixed polarity beams and consequently probe lower values of Δm^2 . Figure 6.10 is a composite of the three muon neutrino inclusive limits. The CHARM limit for this channel is completely enclosed by the CDHS limit. The intrinsic sensitivity of the CDHS experiment to that of E-701 may be seen by comparing Figure 6.11 and Figure 1.11. Therein we show the corresponding energy dependent ratio plots for the two measurements. Superimposed are predicted oscillation curves well within the respective regions of sensitivity for both small and full ($\sin^2 2\theta=1$) mixing.

6.4.3 Relationship with Other Mass Measurements

An oscillation measurement alone cannot constrain the possible values of a particular neutrino mass since the effect is only sensitive to mass differences. As discussed in Appendix B however, there are tentative constraints upon single masses from decay experiments and from cosmological arguments. It is in fact just the Δm^2 region probed by this experiment in which we might expect to find the effects of cosmologically useful neutrinos. The relationship between the results of this experiment and these other constraints may be seen only in the context of certain assumptions. To show the relationships implied by this measurement we shall assume a two massive neutrino world in which the mass eigenstates are close to the electron and muon neutrinos. In this two neutrino sys-

tem, CP is automatic and therefore we take the recent non-zero electron antineutrino mass measurement¹² as a possible range of electron neutrino masses. The cosmological arguments constrain various sums of masses. We note that the calculation of the constraint based upon the missing light problem⁹ starts with the assumption that the dark matter consists entirely of neutrinos. In Figure 6.12 we plot m_1 which corresponds to the muon neutrino against m_2 which corresponds to the electron neutrino and show the various excluded regions. If we assume the mass spectrum mimics the charged leptons then $m_1 > m_2$. Depending upon the Hubble constant, there remains just the region above that excluded by E-701 (with $\sin^2 \theta = 0.2$) and a small area below, down to the line $m_1 = m_2$. The curve based upon the expansion rate of the universe⁸ assumes a Hubble constant of 75 km per second per megaparsec. If this value is actually 50 or 100 km per second per megaparsec the curve shifts to one of the two indicated dashed curves.

6.4.4 Final Conclusion

Neutrino oscillations were not observed within the region of sensitivity of E-701. The restrictions implied by this measurement may be used to constrain certain cosmologically useful neutrino masses. There have been many neutrino oscillation searches in recent years but E-701 is unique in that it is the only inclusive measurement performed which probes the large Δm^2 region.

Appendix A

Spin $\frac{1}{2}$ Notation

In this appendix we define some notational conventions and discuss the description of spin $\frac{1}{2}$ in the Lorentz group⁷¹. Unless otherwise noted we shall take the Dirac matrices to be in the Weyl representation,

$$\begin{aligned}\gamma_0 &= \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \\ \gamma_i &= \begin{pmatrix} 0 & -\sigma_i \\ \sigma_i & 0 \end{pmatrix} \\ \gamma_5 &= \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}\end{aligned}\tag{A1.1}$$

where σ_i are the 2 x 2 Pauli matrices.

The Weyl representation is related to the more common Pauli-Dirac representation by the interchange of γ_0 and γ_5 , and γ_i is taken to $-\gamma_i$. We will use the symbol

Ψ_D to represent a general four component Dirac spinor and Ψ_M to represent a four component Majorana spinor (defined in section 1.2). Particular fields will be denoted in four component form by e or ν etc., while the i^{th} charged lepton will be L_i and the neutrino N_i . The right and left handed projections are $e_{R,L}$ or $\nu_{R,L}$ and anti-particle states will always be denoted with a superscript $\bar{\nu}_R$ (the familiar right handed anti-neutrino). The Pauli adjoint is given by,

$$\overline{\Psi_D} = \Psi_D^\dagger \gamma_0 \quad (A1.2)$$

$$\bar{\nu} = \nu^\dagger \gamma_0 \quad (A1.3)$$

The symbols ϕ_R and ϕ_L will always mean *two* component Weyl spinors.

The generators of Lorentz Transformations (LT) can be expressed in terms of separate rotation J_i and boost K_i generators which obey the commutation relations,

$$[K_i, K_j] = -i\epsilon_{i,j,k} J_k \quad (A1.4a)$$

$$[J_i, K_j] = i\epsilon_{i,j,k} K_k \quad (A1.4b)$$

$$[J_i, J_j] = i\epsilon_{i,j,k} J_k \quad (A1.4c)$$

where $\epsilon_{i,j,k}$ is the Levi-Civita symbol, totally anti-symmetric in all it's indices and $\epsilon_{1,2,3} =$

1. A common practice is to define,

$$N_i = (J_i + iK_i)/2$$

$$N_i^\dagger = (J_i - iK_i)/2 \quad (A1.5)$$

which satisfy,

$$[N_i, N_j^\dagger] = 0$$

$$[N_i, N_j] = i\epsilon_{i,j,k} N_k \quad (A1.6)$$

$$[N_i^\dagger, N_j^\dagger] = i\epsilon_{i,j,k} N_k^\dagger$$

This is the algebra of SU(2) with the two Casimir operators $N_i N_i$ and $N_i^\dagger N_i^\dagger$. The respective eigenvalues are $n(n+1)$ and $m(m+1)$ with $m, n = 0, \frac{1}{2}, 1, \dots$. The Lorentz group decomposes into two SU(2) subgroups. The SU(2) representations are labelled by the values (m, n) . We note that the two representations can be interchanged by $J_i \mapsto J_i$ and $K_i \mapsto -K_i$ (parity) and that since $J_i = N_i + N_i^\dagger$, the angular momentum of a representation is given by $m+n$. Here we will in particular be concerned with the spin $\frac{1}{2}$ "spinor" representations labelled $(\frac{1}{2}, 0)$ left handed, and $(0, \frac{1}{2})$ right handed. We will denote the two component spinors associated with these representations by ϕ_L and ϕ_R . The reason for the titles, right and left handed may be understood by considering the familiar four component Dirac spinor

$$\Psi_D = \begin{pmatrix} \phi_R \\ \phi_L \end{pmatrix} \quad (A1.7)$$

which may be constructed by taking a linear combination of these Weyl spinors. This Dirac spinor represents a solution to the Dirac equation,

$$(i\partial_\mu \gamma^\mu - m)\Psi_D = 0 \quad (A1.8)$$

If we allow the mass m to equal zero then this four component equation decouples into two two component equations,

$$(i\partial_0 + i\vec{\sigma} \cdot \vec{\nabla})\phi_L = 0 \quad (A1.9a)$$

$$(i\partial_0 - i\vec{\sigma} \cdot \vec{\nabla})\phi_R = 0 \quad (A1.9b)$$

In a momentum representation of the operators these equations describe the action of the helicity operator on the states ϕ_R and ϕ_L .

$$\frac{\vec{\sigma} \cdot \vec{p}}{E} \phi_L = -\phi_L \quad (\text{A1.10a})$$

$$\frac{\vec{\sigma} \cdot \vec{p}}{E} \phi_R = \phi_R \quad (\text{A1.10b})$$

These two spinors are polarized parallel (right handed) and anti-parallel (left handed) to their motion.

The action of the parity operator on the Dirac spinor is to interchange left and right. In the Weyl representation, $P\Psi_D$ is given by,

$$\gamma_0 \Psi_D = \begin{pmatrix} \phi_L \\ \phi_R \end{pmatrix} \quad (\text{A1.11})$$

The right and left components of a four component spinor may also be projected out by the operators

$$\frac{1}{2}(1 \pm \gamma_5) \quad (\text{A1.12})$$

By taking the Kroneker product of the spinor representations, we may obtain representations with other LT properties. The product,

$$\left(\frac{1}{2}, 0\right) \otimes \left(\frac{1}{2}, 0\right) = (0, 0) \oplus (1, 0) \quad (\text{A1.13})$$

for example. Here $(0,0)$ is a the Lorentz scalar and is particularly important because the Lagrangian function is always a scalar. The actual matrices which take ϕ_R and ϕ_L under a LT are given by

$$\Lambda_R = \exp\left(\frac{i}{2} \vec{\sigma} \cdot (\vec{\omega} + i\vec{v})\right)$$

and

$$\Lambda_L = \exp\left(\frac{i}{2}\vec{\sigma} \cdot (\vec{\omega} - i\vec{\nu})\right) \quad (\text{A1.14})$$

where ω_i are rotation parameters and ν_i are boost parameters. Because of the fact that

$$\sigma_2 \sigma_i \sigma_2 = -\sigma_i^* \quad (\text{A1.15})$$

we have the relation,

$$\sigma_2 \Lambda_L \sigma_2 = \Lambda_R^* \quad (\text{A1.16})$$

Under a LT the quantity, $\sigma_2 \phi_L^*$ will go to $\sigma_2 \Lambda_L^* \phi_L^*$ which therefore is,

$$\sigma_2 \Lambda_L^* \sigma_2 \sigma_2 \phi_L^* = \Lambda_R \sigma_2 \phi_L^* \quad (\text{A1.17})$$

so that $\sigma_2 \phi_L^*$ transforms as ϕ_R . From equation A1.14 it follows that,

$$\Lambda_L^{-1} = \Lambda_R^\dagger \quad (\text{A1.18})$$

therefore by A1.16 we have,

$$\Lambda_L^T = \sigma_2 \Lambda_L^{-1} \sigma_2 \quad (\text{A1.19})$$

so that,

$$\sigma_2 \Lambda_L^T \sigma_2 \Lambda_L = 1 \quad (\text{A1.20})$$

or

$$\Lambda_L^T \sigma_2 \Lambda_L = \sigma_2 \quad (\text{A1.21})$$

A scalar under LT is then given by,

$$(\sigma_2 \phi_R^*)^T \sigma_2 \phi_L = -\phi_R^\dagger \phi_L \quad (\text{A1.22})$$

Other Neutrino Mass and Lepton Number Nonconserving Effects

Here we discuss other effects of neutrino mass and lepton number non-conservation which are related to neutrino oscillations and might be expected to exist also.

§B.1 Decay Kinematics

The kinematics of leptonic and semileptonic weak decays will be modified by the presence of neutrino mass. By studying the energy spectrum of the charged decay products it is possible to obtain limits on the neutrino mass.

Although no direct evidence on the existence of the tau neutrino (ν_τ) exists its mass has been constrained to be less than 164 MeV at the 95% C.L.⁴ by measuring the invariant four pion mass distribution in the decay

$$\tau \rightarrow 3\pi^\pm \pi^0 \nu_\tau \tag{B1.1}$$

Here ν_τ mass effects are enhanced since the four pion mass can approach the τ mass.

The most restrictive limit on the muon neutrino mass has come from measurements of the muon momentum in $\pi_{\mu 2}$ decay at rest³. Here the muon neutrino mass determination is dependent upon the pion rest mass. The accuracy of the determination is limited by this input. The limit is $m_\mu < 0.49 \text{ MeV}$ at the 90% C.L.,

The tritium beta decay,

$${}^3\text{H} \rightarrow {}^3\text{He} + e^- + \nu_e^c \quad (\text{B1.2})$$

has a low Q value, a low electron endpoint energy ($T_e \approx 18.6 \text{ KeV}$), and a large transition probability. The shape of the electron spectrum near its endpoint is particularly sensitive to the anti-electron neutrino mass.

The best upper limit had been from Bergvist² who reported $m_{\nu_e^c} < 55 \text{ eV}$ (90% C.L.). In recent years, a new series of experiments performed by Lubimov¹² et al have reported results consistent with a finite non-zero ν_e^c mass, $14 \text{ eV} < m_{\nu_e^c} < 44 \text{ eV}$ (90 % C.L.). The radioactive source used in these measurements consisted of tritiated valine ($\text{C}_5\text{H}_{11}\text{NO}_2$). The experiment has been criticized on the basis of possible uncertainties in the atomic final state spectra of the ${}^3\text{He}$ in this complicated molecular environment. At present, approximately 10 experiments are underway to check this result. The findings of Lubimov et al have been one of the major causes of the current renewed experimental interest in neutrino mass and neutrino oscillations.

If the usual weak interaction eigenstates are mixed with other heavy neutrino then in addition to the kinematical effects discussed above, we expect secondary kinks

(3 body) or peaks (2 body) to appear in decay spectra. This phenomenon has been discussed by Shrock⁷² who has shown how to extract very restrictive correlated limits on the mixings of heavy neutrinos. Experimental work has been done on decays involving muon and electron neutrinos as the primary products.

§B.2 Cosmological Limits on Neutrino Mass

The standard (Big Bang) cosmological model predicts that as the temperature of the universe dropped below 1 MeV ($\approx 10^{10}$ Kelvin), the neutrinos, which had been in thermal equilibrium with matter, decoupled. Today, there is believed to be a relic sea of these primordial neutrinos which have cooled as the universe expanded. If these neutrinos have mass, they may have important dynamical effects. These effects may imply measurable constraints on neutrino mass. We consider some of these here.

1) Number of Light Neutrinos⁷³: A cosmological abundance of helium which is roughly 25% originated in an era of nucleosynthesis which occurred at a temperature of 10^9 Kelvin. This abundance depends upon certain conditions which existed at that time. These include the relative fractions of neutrons and protons, the reaction cross sections for those nuclear processes in which helium was built up from nucleons, the baryon to photon ratio, and the expansion rate of the universe. The expansion rate is related to the energy density of the universe. At high temperatures, especially, light particles will make a large contribution to the energy density. Because of this, the known helium abundance can be used to constrain the total number of heavy leptons and their associated light

neutrinos. The derived bound is $N_\nu \leq 4$. If at least one neutrino is greater than $\mathcal{O}$ (10eV) this bound is larger and may be as great as 10.

2) Present Rate of Expansion⁸: In an expanding universe, the Hubble constant, relates the velocity of celestial bodies to their distance from the observer. If the mass density of the universe is ρ , there is some critical density, ρ_c , such that if $\Omega = \frac{\rho}{\rho_c} < 1$ the universe will continue to expand forever, while if $\Omega > 1$ the universe will eventually recontract.

Depending upon the value taken for the Hubble constant, $\rho_c \approx 0.5 \times 10^{-29} - 3.2 \times 10^{-29}$ grams-cm⁻³. Baryonic, or luminous matter indicate that $\Omega_{Baryonic} \approx 0.1$. A suggestion is that primordial neutrinos with mass may comprise the mass density necessary to close the universe.

Cowsik and McClelland have considered this and have used Fermi-Dirac statistics to calculate the number density of massive (2 helicity) neutrinos in thermal equilibrium at 1 MeV. The number of neutrinos is conserved due to the small interaction cross section so that the present number density may be inferred. They find now, $n_\nu \approx 150(2s + 1)\text{cm}^{-3}$. This number is 10^{10} the number density of hydrogen atoms. Constraining the neutrino mass density,

$$\rho_\nu = \sum_i m_{\nu_i} n_{\nu_i} < \rho_c(1 - \Omega_{Baryon}) \quad (B1.3)$$

they find,

$$\sum_i (2s + 1)m_i < 66\text{eV} \quad (B1.4)$$

3) Missing Light Problem⁹: Radial velocity profiles of luminous matter in galaxies have been measured. These curves are flat outside the galactic center suggesting a mass

distribution which falls like $\frac{1}{R^2}$. The density of luminous matter falls much faster than this suggesting dark massive halos at distant radii. If this matter is massive neutrinos then a lower bound on their mass has been derived such that they be gravitationally bound to the galaxies. With a local density of dark matter $\approx 10^{-24}$ gram-cm $^{-3}$ this implies,

$$\sum_i m_i^4 > (20\text{eV})^4 \quad (B1.5)$$

4) Galaxy Formation¹⁰: Many authors have considered the effect of massive neutrinos on galaxy formation. Sometimes these conclusions have conflicted. This is a complicated, highly model dependent problem, so that here no unambiguous conclusion can be reached.

§B.3 Neutrinoless Double Beta Decay

Certain nuclei lie energetically lower than the isobar with $Z+1$ so that beta decay is forbidden, while a transition to the isobar with $Z+2$ is still energetically allowed. Naively one might imagine this process occurring through the chain shown in Figure B1.2a, or through the reabsorption of a virtual neutrino, as in Figure B1.2b. This latter process ($\beta\beta_{0\nu}$) which enjoys a considerable phase space advantage over the first ($\beta\beta_{2\nu}$) process violates lepton number conservation by two units. It requires the emission and absorption of the same neutrino. In the context of standard massless V-A weak interactions, the first decay results in a normal right handed antineutrino which can never be absorbed by the reaction

$$\nu_e^c n \not\rightarrow p e^- \quad (B1.6)$$

The neutrinoless double beta decay is of considerable interest because its existence would imply that the neutrino is a Majorana particle and that either $m_\nu \neq 0$ or that there is some right handed mixture in the weak current ($\eta \neq 0$). Assuming a Majorana neutrino, right handed currents would allow the absorption of the wrong helicity particle and a mass would allow an helicity flip at order m_ν/E to occur. The rate for $(\beta\beta_{0\nu})$ is then proportional to both η and m_ν . We note that if $\eta = 0$ and the neutrino mass is Dirac then $(\beta\beta_{0\nu})$ will be forbidden. Analyses of measured limits on $(\beta\beta_{0\nu})$ indicate that $m_{\nu M} < 5 \text{ eV}^{28}$.

§B.4 Wrong Helicity Weak Interactions

Related to neutrinoless double beta decay, but involving a real rather than a virtual neutrino are the wrong helicity weak interactions. These include using antineutrinos from the reaction (A)

$$n \rightarrow p e^- \nu_{eR}^c \quad (B1.7)$$

to interact as⁵

$$\nu_{eL}^{37} Cl \rightarrow e^- {}^{37}Ar \quad (B1.8)$$

or using neutrinos produced by the meson decays (B)

$$\pi^+ \rightarrow \mu^+ \nu_{\mu L} \quad (B1.9)$$

$$K^+ \rightarrow \mu^+ \nu_{\mu L} \quad (B1.10)$$

to induce,

$$\nu_{\mu R}^c \text{ nucleon} \rightarrow \mu^+ X \quad (B1.11)$$

the production of wrong sign single muons by left handed neutrinos⁷⁴ (see Figure 1.1). Both these chains violate lepton number conservation by two units and so require the neutrino to be a Majorana particle. If $m_\nu \neq 0$ then the amplitudes for these processes are of order $(\frac{m_\nu}{E})$ whereas if $m_\nu = 0$ they are proportional to η . Assuming $m_\nu \neq 0$ and taking known limits on neutrino masses we can bound the rates,

$$\Gamma_A < \left(\frac{60}{1\text{MeV}}\right)^2 \approx 4 \times 10^{-9} \quad (B1.12)$$

$$\Gamma_B < \left(\frac{500\text{KeV}}{10\text{GeV}}\right)^2 \approx 2.5 \times 10^{-7} \quad (B1.13)$$

Unfortunately, such effects are largest at low energy where neutrino interaction cross sections are very small. For $\eta \neq 0$ the limits on η from $(\beta\beta_{0\nu})$ decay apply rather than those from muon decay since these processes require a Majorana neutrino. These limits suggest $\eta < 10^{-5}$, (see Figure B1.2).

§B.5 Radiative Muon Electron Transition $\mu \rightarrow e\gamma$

If lepton flavour oscillations occur we expect also some other processes which violate separate lepton number conservation laws. The most well known example is the transition $\mu \rightarrow e\gamma$ which might proceed through the graph shown in Figure 1.2a. The rate with respect to $\mu \rightarrow e\nu_e\bar{\nu}_\mu$ has been calculated in the Weinberg- Salam theory³¹ and is given by,

$$\frac{\Gamma(\mu \rightarrow e\gamma)}{\Gamma(\mu \rightarrow e\nu_e\bar{\nu}_\mu)} = \frac{3}{32} \frac{\alpha}{\pi} \left(\frac{m_1^2 - m_2^2}{M_W^2}\right) \sin^2\theta \cos^2\theta \quad (B1.14)$$

If $m_1 = 35\text{eV}$ and $m_2 = .65\text{MeV}$ then, $R < 4.8 \times 10^{-37}$, while the experimental limit is $R < 1.7 \times 10^{-10}$. The predicted rate may however be enhanced even in the absence of neutrino flavor mixing. If there exist other heavy neutral leptons N_1, N_2 with $M_1, M_2 \gg m_1, m_2$ which also mix, as indicated in Figure 1.2b, then the rate is still given by (B1.14) but with the masses and mixing angles appropriately replaced. This can now be much larger. Alternatively, outside the standard model, there might be new types of neutral currents which change the familiar charged leptons into other heavy charged leptons as in Figure 1.2c.

Neutrino Flux Integrals

The integrals used to calculate the distribution of neutrino flux and interacting neutrinos at a detector defined in Section 2.2 have been evaluated. The indefinite forms are listed here for reference. In the calculations we assume a collimated beam of mesons of mass M_m , energy E_m , momentum p_m , and proper lifetime τ where,

$$N_s = \text{number of mesons}$$

$$B = \text{branching ratio for two body leptonic decay}$$

$$\gamma = \frac{E_m}{M_m}$$

$$L_0 = \beta \gamma c \tau$$

$$\beta = \frac{p_m}{E_m}$$

$$E_\nu^{max} = \text{maximum lab neutrino energy given by Equation 2.9}$$

N_{TARGET} = number of target nucleons per unit area

$(\frac{\sigma_\nu}{E_\nu})$ = neutrino interaction cross section slope, (Equation 1.63 divided by E_ν)

The results are,

Equation 2.16: The number of neutrinos incident on a detector, a distance z from the the decay point, between the radii R_1 and R_2 is given by,

$$N_\nu(R_1, R_2; z) = 2N_s B \left(\frac{-1}{2(1 + \frac{\gamma^2 r^2}{z^2})} \right) \Big|_{R_1}^{R_2} \quad (C1.1)$$

Equation 2.18: The number of neutrinos interacting in a detector, a distance z from the the decay point, between the radii R_1 and R_2 is given by,

$$N_\nu^{int}(R_1, R_2; z) = 2N_s B E_\nu^{max} N_{TARGET} \left(\frac{\sigma}{E_\nu} \right) \left(\frac{-z^2}{4(1 + \frac{\gamma^2 r^2}{z^2})^2} \right) \Big|_{R_1}^{R_2} \quad (C1.2)$$

Equation 2.21: The number of neutrinos incident on a detector, a distance z_1 from the beginning of the decay region and z_2 from the end of the decay region, between the radii R_1 and R_2 is given by,

$$N_\nu(R_1, R_2; z_1, z_2) = \frac{2N_s B \gamma^2}{L_0} \left(\left(1 - \frac{z}{L_0}\right) A + \frac{1}{L_0} B \right) \Big|_{z_1}^{z_2} \Big|_{R_1}^{R_2} \quad (C1.3a)$$

where,

$$A = \frac{1}{2\gamma} \left(\frac{z \log(r^2 \gamma^2 + z^2)}{2\gamma} + r \operatorname{atan}\left(\frac{z}{r\gamma}\right) \right) - \frac{1}{4\gamma^2} (z \log(2r^2 \gamma^2 + 2z^2)) \quad (C1.3b)$$

and,

$$B = \frac{1}{4\gamma^2} ((r^2 \gamma^2 + z^2) \log(r^2 \gamma^2 + z^2) - r^2 \gamma^2 - z^2)$$

$$+\gamma^2\left(\frac{z^2}{4\gamma^2}-\frac{z^2\log(r^2\gamma^2+z^2)}{4\gamma^4}\right) \quad (C1.3c)$$

The expression must be evaluated between the limits R_1, R_2 and z_1, z_2 .

Equation 2.22: The number of neutrinos interacting in a detector, a distance z_1 from the beginning of the decay region and z_2 from the end of the decay region, between the radii R_1 and R_2 is given by,

$$N_{\nu}^{int}(R_1, R_2; z_1, z_2) = \frac{2N_s B \gamma^2 E_{\nu}^{max} N_{TARGET}(\frac{\sigma}{E_{\nu}})}{L_0} \left(\left(1 - \frac{z}{L_0}\right) C + \frac{1}{L_0} D \right) \Big|_{z_1}^{z_2} \Big|_{R_1}^{R_2} \quad (C1.4a)$$

where,

$$C = \frac{3r \operatorname{atan}\left(\frac{z}{\gamma r}\right)}{8\gamma} - \frac{3\gamma^2 z^3}{16\gamma^4 z^2 + 16\gamma^6 r^2} + \frac{5z^3}{16\gamma^2 z^2 + 16\gamma^4 r^2} \quad (C1.4b)$$

and

$$D = \frac{r^2 \log(z^2 + \gamma^2 r^2)}{4} - \frac{3\gamma^4 z^4}{8\gamma^6 z^2 + 8\gamma^8 r^2} + \frac{4\gamma^2 z^4}{8\gamma^4 z^2 + 8\gamma^6 r^2} - \frac{z^2}{4\gamma^2} + \frac{r^2}{8} \quad (C1.4c)$$

The expression must be evaluated between the limits R_1, R_2 and z_1, z_2 .

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Table 1.1

Limits on Separate Lepton Number Violation		
Reaction	Rate	Reference
$\mu^+ \rightarrow e^+ \gamma$	$< 1.7 \times 10^{-10}$	27
$\mu^{-32}S \rightarrow e^{-32}S$	$< 7 \times 10^{-11}$	28
$\tau^+ \rightarrow e^+ \mu^+ \mu^-$	$< 3.3 \times 10^{-4}$	29
$K_L^0 \rightarrow e \mu$	$< 8 \times 10^{-6}$	30
$\mu^{-127}I \rightarrow e^+ {}^{127}Sb^*$	$< 3 \times 10^{-10}$	28

Table 1.2

Sources of Neutrinos				
Source	Neutrino	E_ν (MeV)	$(\frac{D}{E_\nu})(\frac{mtr}{MeV})$	Δm_{min}^2 (eV ²)
Sun	ν_e	0—4	10^{11}	10^{-11}
Reactor	ν_e^c	0—8	1—2	.01
Accelerator (PS,AGS)	ν_μ, ν_μ^c	500—3000	.1—.03	.50
Accelerator (SPS,FNAL)	ν_μ, ν_μ^c	$3 \times 10^3 - 3 \times 10^4$	.03—.003	10

Table 2.1

Average Properties of the N0 Dichromatic Beam	
Primary Proton energy	400 GeV
Primary Intensity	$10^{12} - 3.14 \times 10^{13}$ protons/pulse
Target	.3302 meters BeO
Target Angles	$\theta_x = .01196$ radian $\theta_y = .001125$ radian
Ω acceptance	$11.5 \mu SR$
Target to Exit	53 meters
Transmission	.1%
Maximum Transmitted Momentum P_{beam}	350 GeV
$\Delta P_{beam}/P_{beam}$	10%
Exit Δx	± 5.7 centimeters
Exit Δy	± 1.9 centimeters
Exit $\Delta \theta_x$	.00015 radians
Exit $\Delta \theta_y$	.00020 radians

Table 2.2a

Contributions to Energy Dispersion* : PIONS						
Energy	$\Delta P_{beam}(5'')$	$\Delta P_{beam}(30'')$	$\Delta\theta(5'')$	$\Delta\theta(30'')$	$\Delta z(5'')$	$\Delta z(30'')$
250	0.092	0.020	0.174	0.438	0.026	0.375
200	0.095	0.003	0.113	0.357	0.017	0.307
165	0.097	0.021	0.078	0.288	0.012	0.249
140	0.097	0.037	0.056	0.233	0.008	0.202
100	0.099	0.062	0.029	0.141	0.004	0.124

* $\frac{\Delta E_\nu}{E_\nu}$ from each effect at R

Table 2.2b

Contributions to Energy Dispersion* : Kaons						
Energy	$\Delta P_{beam}(5'')$	$\Delta P_{beam}(40'')$	$\Delta\theta(5'')$	$\Delta\theta(40'')$	$\Delta z(5'')$	$\Delta z(40'')$
250	0.099	0.065	0.015	0.096	0.002	0.113
200	0.100	0.076	0.009	0.066	0.001	0.078
165	0.100	0.083	0.006	0.047	0.001	0.055
140	0.100	0.088	0.005	0.034	0.001	0.041
100	0.100	0.093	0.002	0.018	0.000	0.022

* $\frac{\Delta E_\nu}{E_\nu}$ from each effect at R

Table 2.3

Mean Meson Decay Angles		
Energy	$\langle \theta_{decay} \rangle_{\pi}$	$\langle \theta_{decay} \rangle_K$
250	.00056	.00196
200	.00070	.00247
165	.00085	.00299
140	.00100	.00353
100	.00140	.00494

Table 2.4

FWHM Secondary Beam Divergence				
Energy	θ_x^{HC}	θ_y^{HC}	θ_x^{TURTLE}	θ_y^{TURTLE}
200	0.25	0.28	0.23	0.37
165	0.25	0.39	0.24	0.36
140	0.27	0.36	0.25	0.37

Table 3.1

Detector Properties		
Item	Lab E	Wonder Building
Total Mass	≈ 1000 Ton	≈ 200 Ton
Fiducial Mass	≈ 116 Ton	≈ 48 Ton
Total Transverse Size	$120'' \times 120''$	$60'' \times 60''$
Fiducial Transverse Size	$78.04'' \times 78.04''$	$50'' \times 50''$
Fiducial $\bar{\rho}L$	2994.34 gr/cm^2	2883.16 gr/cm^2
Magnetic p_T Kick	$3 \times 0.8 \text{ GeV}$	1.2 GeV
Muon Momentum Resolution $\Delta P_\mu/P_\mu$	11%	16%
Hadron Energy Resolution $\Delta E_{hadron}/E_{hadron}$	$0.89/\sqrt{E_{hadron}}$	$0.89/\sqrt{E_{hadron}}$
Spark Chamber Position Resolution	0.5 mm	0.5 mm
Avg. Distance from Decay Region	1116 meters	715 meters

Table 3.2a

Accounting of Lab E Fiducial Mass				
Material	Avg. Density (gr/cm ³)	Avg. Length Each (cm)	Number	$\bar{\rho}L$
Steel	7.85	≈ 5.17	68	2757.77
Scintillator	0.8	2.54	33	67.06
Lucite	1.18	0.64	33	24.73
H ₂ O	1.00	≈ 2.54	33	83.82
Mylar	1.20	0.76	33	30.10
Spark Chambers	≈ 0.27	6.35	18	30.86
Total				2994.34

Table 3.2b

Accounting of Wonder Building Fiducial Mass				
Material	Avg. Density (gr/cm ³)	Avg. Length Each (cm)	Number	$\bar{\rho}L$
Steel	7.84	≈ 5.18	68	2762.42
Counters	1.18	2.54	34	101.90
Spark Chambers	≈ 0.27	6.35	11	18.86
Total				2883.16

Table 4.1

Wonder Building Hadron Calibration	
Setting	CALBR: GeV/minimum
+100	0.152 ± 0.010
+140	0.161 ± 0.007
+165	0.171 ± 0.005
+200	0.165 ± 0.005
+250	0.169 ± 0.004
-165	0.171 ± 0.005
Average	0.164 ± 0.003

Table 5.1

Information in the Monitor Record for the Oscillation Analysis

DATE: Calendar date.

TIME: Time.

PULSE: Accelerator wide counter, resets after ≈ 8000 counts.

CYCLE: Counts spill from the beginning of a run separately in each detector.

NTS: NTOR gated on a predata about the spill time.

NTB: NTOR gated on the beam.

NTE: NTOR gated on the Lab E trigger.

NTW: NTOR gated on the Wonder Building trigger.

XTB: Expansion Port up/down split plate ion chamber.

XEW: Expansion Port east/west split plate ion chamber.

MTB: Manhole up/down split plate ion chamber.

MEW: Manhole east/west ion chamber.

Table 5.2

Flux in Protons on Target $\times 10^{16}$							
	+250	+200	+165	+140	+100	-165	$\Sigma \times 10^{18}$
Lab E							
Total Live	96.3	57.2	44.9	53.6	41.5	188.0	4.8
Good Runs	88.9	54.0	40.8	53.3	40.2	143.2	4.2
Good Steering	87.3	54.0	39.4	52.7	39.8	141.3	4.1
W.B. Takes Data	48.4	44.4	30.9	40.2	32.7	117.8	3.2
Wonder Building							
Total Live	101.2	61.6	51.2	54.8	44.2	208.4	5.2
Good Runs	66.4	61.4	43.8	50.3	38.3	158.6	4.2
Good Steering	65.4	61.3	42.7	50.0	38.2	154.3	4.1
L.E. Takes Data	51.5	47.4	35.4	40.2	34.9	126.7	3.4

Table 5.3

Measured Beam Centers (inches)				
Energy	$X_{L.E.}$	$Y_{L.E.}$	$X_{W.B.}$	$Y_{W.B.}$
250	2.515	2.091	-2.037	-0.523
200	2.882	2.277	-1.639	-0.280
165	4.007	2.683	-1.431	-0.572
140	3.260	2.075	-1.419	-0.757
100	3.706	1.615	-0.880	-0.215

Table 5.4

Energy Separation Factors		
Energy	α	β
250	151.71	0.0177
200	123.57	0.0155
165	104.39	0.0140
140	87.95	0.0126
100	68.25	0.0098

Table 5.5

Energy: Lab E Measured and Beam Monte Carlo Generated					
Setting	Radial Bin	E_{data}	σ	$E_{M.C.}$	σ
250	π 0"-20"	82.86	19.22	84.30	16.42
250	π 20"-40"	63.00	19.37	63.26	19.09
250	K 0"-50"	209.60	32.03	213.40	24.45
200	π 0"-20"	70.51	14.75	72.14	12.34
200	π 20"-40"	55.05	14.71	55.58	13.31
200	K 0"-50"	176.25	25.70	181.91	20.34
165	π 0"-20"	61.05	12.47	59.84	9.01
165	π 20"-40"	49.83	11.51	48.52	9.93
165	K 0"-50"	151.57	22.48	153.20	17.12
140	π 0"-20"	53.62	9.54	54.70	7.95
140	π 20"-40"	45.20	9.09	45.29	7.40
140	K 0"-50"	128.50	18.35	134.91	14.90
100	π 0"-20"	40.76	7.19	41.15	5.22
100	π 20"-40"	36.42	6.87	36.35	5.28
100	K 0"-50"	96.08	13.50	99.17	10.80
-165	π 0"-20"	59.84	11.81	59.76	9.12
-165	π 20"-40"	48.03	11.21	48.06	9.98
-165	K 0"-50"	145.87	20.78	146.48	17.90

Table 5.6

Event Sample Through Cuts*						
Energy	+250	+250	+200	+200	+165	+165
CUT	Lab E	Wond B	Lab E	Wond B	Lab E	Wond B
T1	19610	17303	27870	23916	20505	17729
STEER	17118	15125	24661	21198	18622	16131
PLACE	12226	11012	17352	15510	12966	11921
VERTEX	9150	9037	13049	12808	9912	9878
RADIUS	8136	7926	11448	11130	8556	8423
THETA MU	8073	7882	11317	11062	8443	8361
TRAVEL	8073	7882	11317	11062	8443	8361
ZT2	7599	7412	10478	10278	7663	7663
HOLE	6930	6710	9605	9418	7002	7008
TOR E	6930	6710	9605	9418	7002	7008
TOR W	6928	6708	9599	9410	6995	6996
PMAX	6901	6705	9573	9408	6967	6995
χ^2	6580	6506	9014	9163	6514	6867
PMIN	6401	6354	8753	8867	6279	6627
IPK	6401	6354	8753	8867	6279	6627

* Lab E numbers multiplied by normalization factor but

Table 5.6 (continued)

Event Sample Through Cuts*						
Energy	+140	+140	+100	+100	-165	-165
CUT	Lab E	Wond B	Lab E	Wond B	Lab E	Wond B
T1	25871	22088	15783	13384	17867	12948
STEER	23061	19698	15140	12895	15674	11274
PLACE	16184	14540	10567	9390	8918	8397
VERTEX	12042	11934	7451	7382	6831	7047
RADIUS	10194	9959	5981	5917	5842	5983
THETA MU	10043	9894	5868	5845	5832	5970
TRAVEL	10043	9894	5868	5845	5832	5970
ZT2	9040	9034	5262	5233	5558	5709
HOLE	8382	8304	4849	4847	4998	5139
TOR E	8382	8304	4849	4847	4998	5139
TOR W	8372	8294	4845	4836	4992	5133
PMAX	8366	8294	4828	4836	4973	5129
χ^2	7670	8138	4188	4763	4782	5058
PMIN	7334	7769	3924	4414	4563	4859
IPK	7334	7769	3924	4414	4563	4859

* Lab E numbers multiplied by normalization factor

Table 5.7

Efficiencies: multiplies Lab E/Wonder Bldg					
Setting	$\epsilon_{TRACK}^{L.E.}$	σ	$\epsilon_{VetoDead}^{L.E.}$	$\epsilon_{VetoDead}^{W.B.}$	R_ϵ
+250	0.9986	0.0005	0.9923	0.9838	0.9928
+200	0.9989	0.0004	0.9954	0.9885	0.9942
+165	0.9989	0.0004	0.9964	0.9802	0.9948
+140	0.9991	0.0003	0.9969	0.9915	0.9955
+100	0.9989	0.0004	1.0000	0.9844	0.9854
-165	0.9994	0.0004	0.9948	0.9874	0.9931

Table 5.8

Livetime Ratios: W.B./L.E.	
Setting	Ratio
250	1.0580
200	1.0673
165	1.1470
140	1.0011
100	1.0668
-165	1.0749

Table 5.9a

SIMULTANEOUS DATA TAKING SAMPLE*						
Setting	Bin	Events _{L.E.}	Events _{W.B.}	Ratio	σ_R	NORM
250	π 0"-20"	2413	2279	1.0591	0.0310	1.0136
250	π 20"-40"	1430	1452	0.9848	0.0367	"
250	K 0"-50"	2865	2806	1.0201	0.0273	"
200	π 0"-20"	3684	3522	1.0456	0.0245	1.0239
200	π 20"-40"	2595	2567	1.0109	0.0283	"
200	K 0"-50"	2989	3007	0.9944	0.0259	"
165	π 0"-20"	2204	2157	1.0219	0.0317	1.0900
165	π 20"-40"	1915	1929	0.9928	0.0328	"
165	K 0"-50"	1478	1581	0.9347	0.0348	"
140	π 0"-20"	2983	3063	0.9738	0.0248	0.9617
140	π 20"-40"	3168	3115	1.0166	0.0254	"
140	K 0"-50"	1727	1729	0.9994	0.0337	"
100	π 0"-20"	1646	1670	0.9853	0.0343	1.0144
100	π 20"-40"	2130	2090	1.0189	0.0314	"
100	K 0"-50"	705	703	1.0030	0.0539	"
-165	π 0"-20"	2189	2131	1.0267	0.0321	1.0301
-165	π 20"-40"	1999	2104	0.9498	0.0306	"
-165	K 0"-50"	666	723	0.9210	0.0502	"

*Lab E sums have been multiplied by the factor NORM.

Table 5.9b

Ratio of Simultaneous (Live/Data) Taking			
Setting	Bin	Ratio	σ_R
250	π 0"-20"	0.9999	0.0324
250	π 20"-40"	0.9654	0.0359
250	K 0"-50"	1.0063	0.0286
200	π 0"-20"	0.9801	0.0314
200	π 20"-40"	0.9976	0.0361
200	K 0"-50"	1.0195	0.0348
165	π 0"-20"	0.9822	0.0250
165	π 20"-40"	0.9815	0.0257
165	K 0"-50"	1.0106	0.0279
140	π 0"-20"	1.0883	0.0575
140	π 20"-40"	1.0433	0.0515
140	K 0"-50"	1.0550	0.0732
100	π 0"-20"	0.9622	0.0559
100	π 20"-40"	1.0215	0.0588
100	K 0"-50"	0.9459	0.0826

Table 5.10

Closed Slit Data Sample								
Energy	LE Run	NTE $\times 10^{13}$	π	K	WB Run	NTW $\times 10^{13}$	π	K
+250	824	2168	9	0	729	2288	6	0
+250	825	2598	3	1	730	1674	6	0
+250	-	-	-	-	731	908	3	0
+200	817	519	3	1	724	1737	2	0
+200	818	1115	4	1	725	69	0	0
+200	819	313	2	0	726	316	1	0
+165	392	2134	9	0	362	2095	11	0
+165	820	1467	4	0	727	2192	8	1
+140	821	635	3	0	-	-	-	-
+140	822	132	2	0	728	1388	0	0
+100	823	1191	3	1	-	-	-	-
TOTALS	-	12272	42	4	-	12667	36	1

Table 5.11

Wide Band Background Corrections		
Setting	Bin	Correction*
+250	π 0"-20"	1.008
+250	π 20"-40"	1.011
+250	K 0"-50"	1.001
+200	π 0"-20"	1.005
+200	π 20"-40"	1.007
+200	K 0"-50"	1.000
+165	π 0"-20"	1.004
+165	π 20"-40"	1.004
+165	K 0"-50"	1.000
+140	π 0"-20"	1.004
+140	π 20"-40"	1.004
+140	K 0"-50"	1.000
+100	π 0"-20"	1.007
+100	π 20"-40"	1.005
+100	K 0"-50"	1.001
-165	π 0"-20"	1.001
-165	π 20"-40"	1.006
-165	K 0"-50"	1.000

*Factors divided data ratios.

Table 5.12

Mean Decay Points for $z_{max}=350$ meters		
Energy	$\langle z \rangle_{\pi}$	$\langle z \rangle_K$
100	173.18	161.47
140	173.70	165.32
165	173.80	166.78
200	174.09	168.22
250	174.27	169.57

Table 5.13

Monte Carlo Scaling Corrections*					
Setting	Bin	$R_{nominal}$	σ_R	$R_{stretched}$	σ_R
250	π 0"-20"	1.0233	0.0051	1.0347	0.0051
250	π 20"-40"	1.0036	0.0085	1.0147	0.0085
250	K 0"-50"	0.9918	0.0045	0.9957	0.0045
200	π 0"-20"	1.0223	0.0056	1.0316	0.0056
200	π 20"-40"	1.0136	0.0081	1.0207	0.0081
200	K 0"-50"	0.9744	0.0053	0.9842	0.0053
165	π 0"-20"	1.0008	0.0060	1.0318	0.0062
165	π 20"-40"	1.0265	0.0080	1.0255	0.0080
165	K 0"-50"	0.9743	0.0065	0.9590	0.0065
140	π 0"-20"	1.0217	0.0069	1.0268	0.0069
140	π 20"-40"	1.0252	0.0080	1.0324	0.0080
140	K 0"-50"	0.9743	0.0075	0.9901	0.0075
100	π 0"-20"	1.0133	0.0080	1.0133	0.0075
100	π 20"-40"	0.9998	0.0081	0.9998	0.0081
100	K 0"-50"	0.9643	0.0110	0.9643	0.0110
-165	π 0"-20"	1.0070	0.0050	1.0400	0.0050
-165	π 20"-40"	1.0070	0.0060	1.0040	0.0060
-165	K 0"-50"	0.9750	0.0100	0.9590	0.0100

* These factors are divided into the data ratios.

Table 5.14

DATA Fully Corrected for Normalization and Scaling					
Setting	Bin	$R_{nominalcorr}^{final}$	σ_R	$R_{stretchcorr}^{final}$	σ_R
250	π 0"-20"	1.0267	0.0305	1.0157	0.0301
250	π 20"-40"	0.9706	0.0371	0.9602	0.0367
250	K 0"-50"	1.0283	0.0279	1.0242	0.0277
200	π 0"-20"	1.0177	0.0247	1.0087	0.0245
200	π 20"-40"	0.9904	0.0288	0.9845	0.0286
200	K 0"-50"	1.0205	0.0271	1.0104	0.0268
165	π 0"-20"	1.0169	0.0321	0.9875	0.0311
165	π 20"-40"	0.9633	0.0327	0.9643	0.0327
165	K 0"-50"	0.9594	0.0362	0.9750	0.0368
140	π 0"-20"	0.9494	0.0250	0.9447	0.0249
140	π 20"-40"	0.9876	0.0258	0.9808	0.0256
140	K 0"-50"	1.0257	0.0355	1.0100	0.0349
100	π 0"-20"	0.9656	0.0345	0.9656	0.0345
100	π 20"-40"	1.0140	0.0323	1.0140	0.0323
100	K 0"-50"	1.0391	0.0571	1.0391	0.0323
-165	π 0"-20"	1.0185	0.0322	0.9832	0.0322
-165	π 20"-40"	0.9375	0.0307	0.9422	0.0307
-165	K 0"-50"	0.9447	0.0596	0.9603	0.0596

Table 5.15

Veto Deadtime Systematic Error		
Setting	$\Delta_{L.E.}$	$\Delta_{W.B.}$
+250	0.0002	0.0038
+200	0.0001	0.0017
+165	0.0046	0.0029
+140	0.0009	0.0004
+100	0.0041	0.0069
-165	0.0038	0.0020

Table 6.1

[illegible]

Table 6.1 (continued)

8.8	4.	4.	4.	4.	4.	4.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.
10.8	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.
12.8	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.
14.8	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.	3.
16.8	4.	4.	4.	4.	4.	4.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.	5.
18.8	4.	5.	5.	5.	5.	5.	6.	6.	6.	6.	6.	6.	6.	6.	6.	6.	6.	6.	6.
20.8	5.	6.	6.	6.	6.	6.	7.	7.	7.	7.	7.	7.	7.	7.	7.	7.	7.	7.	7.
22.8	7.	8.	9.	11.	12.	14.	16.	18.	20.	22.	24.	26.	28.	30.	32.	34.	36.	38.	40.
24.8	9.	11.	13.	15.	17.	19.	21.	23.	25.	27.	29.	31.	33.	35.	37.	39.	41.	43.	45.
27.8	12.	15.	17.	20.	24.	27.	31.	36.	41.	47.	53.	60.	67.	75.	84.	94.	104.	116.	128.
29.8	15.	18.	22.	26.	30.	35.	41.	47.	54.	61.	70.	79.	89.	101.	113.	127.	142.	158.	176.
32.8	18.	22.	26.	31.	37.	43.	50.	58.	66.	76.	87.	99.	113.	128.	144.	163.	183.	206.	231.
34.8	21.	25.	30.	36.	42.	50.	58.	67.	78.	90.	103.	118.	135.	154.	175.	199.	226.	256.	289.
37.8	23.	27.	33.	39.	46.	54.	64.	75.	87.	100.	116.	134.	154.	177.	202.	232.	265.	303.	347.
39.8	23.	28.	34.	41.	48.	57.	67.	79.	92.	107.	124.	144.	166.	192.	222.	257.	296.	342.	395.
42.8	23.	28.	34.	41.	48.	57.	67.	79.	92.	107.	124.	144.	166.	192.	222.	257.	296.	342.	395.
44.8	23.	28.	34.	41.	48.	57.	67.	79.	92.	107.	124.	144.	166.	192.	222.	257.	296.	342.	395.
47.8	22.	26.	31.	37.	43.	51.	60.	70.	82.	96.	113.	132.	154.	181.	212.	249.	294.	348.	413.
49.8	21.	25.	29.	34.	40.	47.	55.	65.	75.	88.	103.	120.	140.	164.	193.	226.	267.	316.	375.
52.8	21.	25.	29.	34.	39.	46.	53.	62.	72.	84.	97.	113.	132.	154.	181.	212.	250.	295.	350.
54.8	23.	27.	31.	36.	41.	48.	56.	65.	76.	88.	103.	121.	143.	168.	199.	234.	283.	341.	408.
57.8	27.	31.	36.	41.	48.	56.	65.	76.	88.	103.	121.	143.	168.	199.	234.	283.	341.	408.	484.
59.8	32.	38.	44.	51.	60.	70.	82.	97.	114.	134.	159.	189.	225.	270.	324.	393.	475.	575.	694.
62.8	40.	48.	56.	66.	77.	91.	107.	127.	151.	179.	214.	256.	308.	371.	449.	544.	658.	799.	928.
64.8	50.	60.	71.	84.	99.	117.	139.	166.	197.	236.	283.	340.	410.	495.	598.	722.	864.	1013.	1145.
67.8	62.	74.	88.	104.	124.	148.	177.	212.	253.	302.	361.	431.	516.	618.	739.	882.	1046.	1229.	1417.
69.8	74.	88.	106.	126.	151.	180.	214.	256.	306.	366.	439.	527.	633.	759.	906.	1071.	1243.	1395.	1483.
72.8	86.	103.	124.	148.	177.	212.	253.	302.	361.	431.	516.	618.	739.	882.	1046.	1229.	1417.	1566.	1694.
74.8	98.	117.	141.	169.	202.	242.	289.	345.	412.	492.	588.	702.	837.	997.	1181.	1387.	1607.	1819.	1988.
77.8	108.	130.	157.	188.	225.	269.	322.	384.	458.	547.	653.	779.	930.	1108.	1317.	1557.	1823.	2098.	2339.
79.8	117.	141.	170.	204.	245.	293.	350.	418.	499.	596.	712.	851.	1018.	1218.	1456.	1736.	2054.	2349.	2609.
82.8	124.	150.	181.	217.	261.	312.	373.	446.	533.	638.	763.	915.	1098.	1320.	1580.	1906.	2266.	2632.	2987.
84.8	129.	156.	188.	227.	273.	327.	391.	468.	561.	672.	806.	969.	1167.	1407.	1690.	2040.	2419.	2777.	3086.
87.8	131.	160.	193.	233.	280.	336.	403.	484.	580.	696.	836.	1007.	1215.	1467.	1768.	2116.	2466.	2808.	2952.
89.8	132.	161.	195.	235.	284.	341.	409.	491.	590.	708.	852.	1026.	1237.	1490.	1788.	2124.	2465.	2741.	2839.
92.8	131.	159.	194.	234.	282.	340.	408.	491.	589.	708.	850.	1023.	1229.	1474.	1758.	2086.	2373.	2608.	2684.
94.8	127.	156.	189.	229.	277.	333.	401.	481.	578.	693.	831.	997.	1193.	1422.	1683.	1962.	2231.	2437.	2510.
97.8	122.	150.	182.	221.	267.	322.	387.	464.	556.	666.	796.	950.	1131.	1340.	1574.	1822.	2059.	2244.	2327.
99.8	116.	142.	173.	210.	254.	305.	367.	439.	525.	627.	747.	888.	1051.	1237.	1443.	1661.	1870.	2043.	2139.
102.8	108.	133.	162.	196.	237.	285.	342.	408.	487.	579.	687.	813.	957.	1121.	1301.	1490.	1676.	1837.	1946.
112.8	69.	96.	125.	153.	183.	217.	257.	303.	356.	416.	484.	560.	645.	739.	841.	949.	1063.	1170.	1272.
122.8	33.	42.	52.	63.	76.	91.	108.	127.	149.	174.	202.	233.	268.	307.	351.	400.	453.	513.	578.
132.8	13.	17.	22.	27.	33.	41.	49.	58.	69.	81.	95.	111.	128.	149.	172.	198.	229.	264.	303.
142.8	10.	13.	17.	21.	26.	31.	37.	44.	52.	61.	71.	83.	97.	112.	131.	152.	177.	200.	245.
152.8	20.	24.	29.	34.	40.	46.	53.	61.	70.	79.	90.	102.	115.	130.	147.	167.	190.	217.	251.
162.8	36.	42.	48.	55.	63.	71.	80.	89.	99.	110.	122.	135.	149.	163.	179.	196.	215.	236.	259.
172.8	51.	58.	66.	75.	85.	95.	106.	117.	130.	144.	158.	173.	190.	207.	226.	246.	267.	289.	313.
182.8	60.	68.	78.	88.	100.	112.	126.	141.	157.	175.	196.	218.	243.	272.	305.	342.	385.	433.	482.
192.8	61.	71.	81.	93.	106.	121.	138.	158.	180.	206.	236.	271.	313.	363.	422.	491.	566.	637.	677.
202.8	60.	69.	81.	94.	109.	127.	148.	172.	202.	237.	280.	332.	395.	471.	561.	638.	755.	811.	785.
212.8	60.	70.	82.	96.	115.	136.	162.	193.	230.	276.	333.	402.	487.	589.	705.	825.	921.	946.	850.
222.8	65.	77.	92.	110.	131.	157.	188.	226.	272.	329.	399.	484.	588.	710.	847.	983.	1084.	1097.	773.
232.8	74.	89.	107.	129.	154.	185.	223.	268.	323.	389.	470.	568.	685.	822.	974.	1125.	1244.	1282.	1195.
242.8	82.	99.	120.	145.	174.	209.	251.	302.	362.	435.	522.	625.	748.	889.	1045.	1206.	1346.	1428.	1413.
252.8	84.	102.	124.	149.	179.	215.	257.	308.	367.	438.	522.	621.	736.	868.	1014.	1167.	1313.	1429.	1482.
262.8	79.	95.	115.	138.	165.	197.	235.	279.	331.	392.	463.	546.	641.	749.	870.	999.	1129.	1248.	1338.
272.8	67.	81.	97.	116.	138.	163.	193.	227.	267.	314.	367.	428.	490.	576.	664.	758.	857.	955.	1045.
282.8	55.	65.	77.	91.	107.	126.	147.	171.	199.	231.	267.	308.	354.	406.	463.	525.	591.	659.	726.
292.8	43.	51.	59.	69.	80.	93.	107.	123.	141.	162.	185.	211.	239.	271.	306.	344.	384.	427.	471.
302.8	34.	39.	45.	51.	59.	67.	76.	86.	98.	111.	125.	140.	158.	177.	197.	220.	244.	269.	296.
312.8	26.	29.	33.	37.	42.	47.	53.	59.	66.	74.	82.	91.	101.	112.	124.	137.	150.	165.	181.
322.8	19.	21.	23.	26.	28.	31.	35.	38.	42.	46.	51.	56.	62.	67.	74.	81.	88.	96.	105.
332.8	13.	14.	15.	16.	18.	19.	21.	23.	25.	27.	30.	32.	35.	38.	41.	45.	48.	52.	57.
342.8	9.	9.	10.	10.	11.	12.	13.	14.	15.	16.	17.	19.	20.	22.	23.	25.	27.	30.	32.
352.8	7.	7.	8.	8.	9.	10.	10.	11.	12.	13.	14.	16.	17.	18.	20.	22.	24.	26.	28.
362.8	8.	9.	9.	10.	11.	13.	14.	15.	17.	19.	20.	22.	24.	27.	29.	32.	35.	38.	41.
372.8	11.	12.	14.	16.	18.	20.	22.	24.	27.	30.	33.	36.	40.	44.	48.	52.	57.	62.	67.
382.8	15.	18.	20.	23.	26.	29.	33.	36.	40.	45.	49.	54.	60.	65.	71.	78.	84.	91.	99.
392.8	20.	23.	26.	30.	34.	38.	43.	48.	54.	60.	66.	72.	79.	87.	95.	103.	112.	121.	130.
402.8	23.	27.	31.	36.	41.	46.	52.	58.	64.	71.	79.	87.	95.	104.	113.	123.	133.	144.	155.
412.8	25.	29.	34.	39.	44.	50.	57.	63.	71.	78.	86.	95.	104.	114.	124.	135.	146.	158.	170.
422.8	25.	30.	34.	40.	45.	51.	58.	65.	72.	80.	89.	98.	107.	117.	128.	139.	151.	163.	176.
432.8	24.	28.	33.	38.	44.	50.	56.	63.	70.	78.	87.	96.	105.	115.	126.	137.	149.	161.	175.
442.8	22.	26.	30.	35.	40.	46.	52.	58.	65.	73.	81.	89.	99.	108.	119.	129.	141.	153.	166.
452.8	19.	22.	26.	30.	34.	39.	45.	51.	57.	64.	71.	79.	87.	96.	105.	115.	126.	137.	149.
462.8	15.	17.	20.	24.	27.	32.	36.	41.	46.	52.	58.	64.	71.	79.	87.	95.	105.	114.	125.
472.8	11.	13.	15.	17.	20.	23.	26.	29.	33.	38.	42.	47.	52.	58.	64.	71.	78.	86.	94.
482.8	7.	8.	9.	11.	12.	14.	16.	18.	21.	24.	26.	30.	33.	37.	41.	46.	50.	55.	61.
492.8	4.	5.	5.	6.	6.	7.	8.	9.											

Table 6.2

[illegible]

Table 6.2 (continued)

8.8	7.	7.	7.	7.	7.	7.	7.	7.	7.	8.	8.	8.	8.	8.	8.	8.	9.	9.	9.
10.8	7.	7.	7.	7.	7.	7.	7.	7.	7.	8.	8.	8.	8.	8.	8.	8.	9.	9.	9.
12.8	8.	9.	9.	9.	9.	9.	9.	9.	9.	10.	10.	10.	10.	10.	10.	10.	11.	11.	11.
14.8	11.	12.	13.	14.	15.	16.	17.	18.	19.	20.	21.	22.	23.	24.	25.	26.	27.	28.	29.
16.8	15.	17.	19.	21.	24.	27.	30.	34.	37.	41.	46.	50.	55.	61.	66.	72.	78.	85.	92.
18.8	21.	24.	28.	32.	36.	41.	46.	52.	58.	65.	72.	80.	88.	97.	106.	116.	126.	137.	149.
20.8	29.	34.	40.	46.	53.	60.	68.	77.	86.	97.	108.	120.	132.	145.	160.	175.	191.	208.	227.
22.8	39.	46.	55.	63.	73.	84.	96.	108.	122.	137.	153.	170.	189.	208.	229.	252.	276.	301.	329.
24.8	55.	65.	77.	90.	104.	120.	137.	156.	176.	199.	223.	248.	276.	306.	339.	373.	410.	450.	493.
27.8	72.	86.	102.	120.	140.	161.	185.	211.	240.	271.	305.	341.	381.	425.	471.	522.	576.	635.	698.
29.8	90.	108.	129.	151.	177.	205.	236.	271.	308.	350.	395.	445.	499.	550.	602.	656.	714.	776.	841.
32.8	107.	129.	154.	181.	213.	248.	286.	329.	377.	429.	487.	551.	621.	698.	782.	875.	977.	1089.	1211.
34.8	121.	146.	175.	207.	244.	285.	331.	382.	439.	502.	572.	650.	736.	832.	938.	1056.	1186.	1331.	1492.
37.8	138.	158.	190.	226.	267.	313.	365.	423.	488.	561.	642.	733.	834.	947.	1074.	1217.	1376.	1555.	1757.
39.8	155.	185.	219.	259.	303.	350.	402.	460.	525.	599.	688.	789.	902.	1030.	1174.	1337.	1522.	1732.	1972.
42.8	175.	215.	259.	309.	363.	423.	488.	558.	634.	721.	820.	934.	1071.	1226.	1403.	1605.	1838.	2106.	2417.
44.8	192.	235.	289.	349.	413.	483.	558.	638.	726.	826.	938.	1071.	1231.	1413.	1623.	1864.	2145.	2473.	2857.
47.8	212.	255.	319.	389.	463.	543.	628.	718.	818.	928.	1051.	1211.	1393.	1604.	1848.	2133.	2467.	2857.	3347.
49.8	233.	285.	359.	439.	523.	613.	708.	808.	918.	1041.	1191.	1373.	1584.	1828.	2113.	2447.	2837.	3327.	3917.
52.8	255.	319.	403.	493.	583.	683.	793.	903.	1033.	1183.	1363.	1574.	1818.	2103.	2437.	2827.	3317.	3907.	4597.
54.8	279.	353.	443.	543.	643.	753.	873.	993.	1133.	1293.	1483.	1703.	1953.	2243.	2577.	2967.	3417.	3967.	4657.
57.8	305.	389.	489.	599.	709.	829.	959.	1099.	1259.	1439.	1649.	1899.	2189.	2513.	2883.	3313.	3803.	4353.	4963.
59.8	333.	423.	533.	643.	753.	873.	993.	1133.	1293.	1473.	1673.	1913.	2193.	2517.	2887.	3317.	3807.	4357.	4967.
62.8	363.	463.	573.	683.	793.	913.	1043.	1193.	1353.	1533.	1733.	1973.	2253.	2577.	2947.	3377.	3867.	4417.	5027.
64.8	395.	505.	615.	725.	835.	955.	1085.	1235.	1405.	1585.	1785.	2025.	2305.	2629.	2999.	3429.	3919.	4469.	5079.
67.8	429.	549.	659.	769.	879.	999.	1129.	1279.	1459.	1649.	1859.	2099.	2379.	2703.	3073.	3493.	3973.	4523.	5133.
69.8	465.	585.	695.	805.	915.	1035.	1165.	1315.	1495.	1685.	1895.	2135.	2415.	2739.	3109.	3529.	3999.	4549.	5159.
72.8	503.	623.	733.	843.	953.	1073.	1203.	1353.	1533.	1723.	1933.	2173.	2453.	2777.	3147.	3567.	4037.	4587.	5197.
74.8	543.	663.	773.	883.	993.	1113.	1243.	1403.	1583.	1773.	1983.	2223.	2503.	2827.	3197.	3617.	4087.	4637.	5247.
77.8	585.	705.	815.	925.	1035.	1155.	1285.	1445.	1625.	1815.	2025.	2265.	2545.	2869.	3239.	3659.	4129.	4679.	5289.
79.8	629.	749.	859.	969.	1079.	1199.	1329.	1489.	1669.	1859.	2069.	2299.	2569.	2893.	3263.	3683.	4153.	4703.	5313.
82.8	675.	795.	905.	1015.	1125.	1245.	1375.	1535.	1715.	1905.	2115.	2345.	2605.	2929.	3299.	3719.	4189.	4739.	5349.
84.8	723.	843.	953.	1063.	1173.	1293.	1423.	1583.	1763.	1953.	2163.	2393.	2653.	2977.	3347.	3767.	4237.	4787.	5397.
87.8	773.	893.	1003.	1113.	1223.	1343.	1473.	1633.	1813.	2003.	2213.	2443.	2703.	3027.	3397.	3817.	4287.	4837.	5447.
89.8	825.	945.	1055.	1165.	1275.	1395.	1525.	1685.	1865.	2055.	2265.	2495.	2755.	3079.	3449.	3869.	4339.	4889.	5499.
92.8	879.	999.	1109.	1219.	1329.	1449.	1579.	1739.	1919.	2109.	2319.	2549.	2809.	3133.	3503.	3923.	4393.	4943.	5553.
94.8	935.	1055.	1165.	1275.	1385.	1505.	1635.	1795.	1975.	2165.	2375.	2605.	2865.	3189.	3559.	3979.	4449.	5009.	5619.
97.8	993.	1113.	1223.	1333.	1443.	1563.	1693.	1853.	2033.	2223.	2433.	2663.	2923.	3247.	3617.	4037.	4507.	5057.	5667.
100.8	1053.	1173.	1283.	1393.	1503.	1623.	1753.	1913.	2093.	2283.	2493.	2723.	2983.	3307.	3677.	4097.	4567.	5117.	5727.
112.8	1223.	1343.	1453.	1563.	1673.	1793.	1923.	2083.	2263.	2453.	2663.	2893.	3153.	3477.	3847.	4267.	4737.	5287.	5897.
114.8	1265.	1385.	1495.	1605.	1715.	1835.	1965.	2125.	2305.	2495.	2705.	2935.	3195.	3519.	3889.	4309.	4779.	5329.	5939.
116.8	1309.	1429.	1539.	1649.	1759.	1879.	2009.	2169.	2349.	2539.	2749.	2979.	3239.	3563.	3933.	4353.	4823.	5373.	5983.
118.8	1355.	1475.	1585.	1695.	1805.	1925.	2055.	2215.	2395.	2585.	2795.	3025.	3285.	3609.	3979.	4399.	4869.	5419.	6029.
120.8	1403.	1523.	1633.	1743.	1853.	1973.	2103.	2263.	2443.	2633.	2843.	3073.	3333.	3657.	4027.	4447.	4917.	5467.	6077.
122.8	1453.	1573.	1683.	1793.	1903.	2023.	2153.	2313.	2493.	2683.	2893.	3123.	3383.	3707.	4077.	4497.	4967.	5517.	6127.
124.8	1505.	1625.	1735.	1845.	1955.	2075.	2205.	2365.	2545.	2735.	2945.	3175.	3435.	3759.	4129.	4549.	5019.	5569.	6179.
126.8	1559.	1679.	1789.	1899.	2009.	2129.	2259.	2419.	2609.	2799.	3009.	3239.	3499.	3823.	4193.	4613.	5083.	5633.	6243.
128.8	1615.	1735.	1845.	1955.	2065.	2185.	2315.	2475.	2655.	2845.	3055.	3285.	3545.	3869.	4239.	4659.	5129.	5679.	6289.
130.8	1673.	1793.	1903.	2013.	2123.	2243.	2373.	2533.	2713.	2903.	3113.	3343.	3593.	3917.	4287.	4707.	5177.	5727.	6337.
132.8	1733.	1853.	1963.	2073.	2183.	2303.	2433.	2593.	2773.	2963.	3173.	3403.	3653.	3977.	4347.	4767.	5237.	5787.	6397.
134.8	1795.	1915.	2025.	2135.	2245.	2365.	2495.	2655.	2835.	3025.	3235.	3465.	3715.	4039.	4409.	4829.	5299.	5849.	6459.
136.8	1859.	1979.	2089.	2199.	2309.	2429.	2559.	2719.	2909.	3099.	3309.	3539.	3789.	4113.	4483.	4903.	5373.	5923.	6533.
138.8	1925.	2045.	2155.	2265.	2375.	2495.	2625.	2785.	2965.	3155.	3365.	3595.	3845.	4169.	4539.	4959.	5429.	5979.	6589.
140.8	1993.	2113.	2223.	2333.	2443.	2563.	2693.	2853.	3033.	3223.	3433.	3663.	3913.	4237.	4607.	5027.	5497.	6047.	6657.
142.8	2063.	2183.	2293.	2403.	2513.	2633.	2763.	2923.	3103.	3293.	3503.	3733.	3983.	4307.	4677.	5097.	5567.	6117.	6727.
144.8	2135.	2255.	2365.	2475.	2585.	2705.	2835.	2995.	3175.	3365.	3575.	3805.	4055.	4379.	4749.	5169.	5639.	6189.	6799.
146.8	2209.	2329.	2439.	2549.	2659.	2779.	2909.	3069.	3249.	3439.	3649.	3879.	4129.	4453.	4823.	5243.	5713.	6263.	6873.
148.8	2285.	2405.	2515.	2625.	2735.	2855.	2985.	3145.	3325.	3515.	3725.	3955.	4205.	4529.	4899.	5319.	5789.	6339.	6949.
150.8	2363.	2483.	2593.	2703.	2813.	2933.	3063.	3223.	3403.	3593.	3803.	4033.	4283.	4607.	4977.	5397.	5867.	6417.	7027.
152.8	2443.	2563.	2673.	2783.	2893.	3013.	3143.	3303.	3483.	3673.	3883.	4113.	4363.	4687.	5057.	5477.	5947.	6497.	7107.
154.8	2525.	2645.	2755.	2865.	2975.	3095.	3225.	3385.	3565.	3755.	3965.	4195.	4445.	4769.	5139.	5559.	6029.	6579.	7189.
156.8	2609.	2729.	2839.	2949.	3059.	3179.	3309.	3469.	3649.	3839.	4049.	4279.	4529.	4853.	5223.	5643.	6113.	6663.	7273.
158.8	2695.	2815.	2925.	3035.	3145.	3265.	3395.	3555.	3735.	3925.	4135.	4365.	4615.	4939.	5309.	5729.	6199.	6749.	7359.
160.8	2783.	2903.	3013.	3123.	3233.	3353.	3483.	3643.	3823.	4013.	4223.	4453.	4693.	5017.	5387.	5807.	6277.	6827.	7437.
162.8	2873.	2993.	3103.	3213.	3323.	3443.	3573.	3733.	3913.	4103.	4313.	4543.	4783.	5107.	5477.	5897.	6367.	6917.	7527.
164.8	2965.	3085.	3195.	3305.	3415.	3535.	3665.	3825.	4005.	4195.	4405.	4635.	4875.	5199.	5569.	5989.	6459.	7009.	7619.
166.8	3059.	3179.	3289.	3399.	3509.	3629.	3759.	3919.	4099.	4289.	4499.	4729.	4969.	5293.	5663.	6083.	6553.	7103.	7713.
168.8	3155.	3275.	3385.	3495.	3605.	3725.	3855.	4015.	4195.	4385.	4595.	4815.	5055.	5379.	5749.	6169.	6639.	7189.	7799.
170.8	3253.	3373.	3483.	3593.	3703.	3823.	3953.	4113.	4293.	4483.	4693.	4923.	5163.	5487.	5857.	6277.	6747.	7297.	7907.
172.8	3353.	3473.	3583.	3693.	3803.	3923.													

Table 6.3

[illegible]

Table 6.3 (continued)

8.0	10.	10.	10.	10.	10.	10.	10.	10.	10.	11.	11.	11.	11.	11.	12.	12.	12.	12.	13.	13.
10.0	10.	10.	11.	11.	11.	12.	12.	13.	13.	13.	14.	15.	15.	16.	17.	17.	18.	19.	20.	21.
12.0	12.	12.	13.	14.	15.	15.	16.	18.	19.	20.	21.	23.	24.	26.	27.	29.	31.	33.	35.	37.
14.0	15.	16.	17.	19.	21.	22.	24.	27.	29.	31.	34.	37.	40.	43.	46.	50.	53.	57.	61.	65.
16.0	20.	22.	24.	27.	30.	33.	37.	40.	45.	49.	53.	58.	64.	69.	75.	81.	88.	95.	102.	109.
18.0	27.	30.	34.	39.	43.	49.	54.	60.	67.	74.	81.	89.	98.	107.	117.	127.	137.	149.	161.	173.
20.0	36.	41.	47.	54.	61.	69.	77.	86.	96.	107.	118.	131.	144.	158.	172.	188.	205.	222.	241.	260.
22.0	47.	55.	63.	72.	83.	94.	106.	119.	134.	149.	165.	183.	202.	222.	244.	267.	291.	317.	345.	374.
24.0	64.	75.	87.	100.	115.	132.	149.	169.	190.	213.	237.	264.	292.	323.	356.	391.	429.	469.	512.	558.
26.0	82.	97.	114.	132.	152.	175.	199.	226.	255.	287.	322.	359.	400.	444.	491.	542.	597.	657.	721.	789.
29.0	101.	120.	141.	165.	191.	220.	252.	287.	326.	368.	414.	465.	520.	580.	645.	716.	793.	876.	967.	1066.
32.0	119.	142.	168.	197.	229.	265.	304.	348.	397.	450.	509.	573.	644.	722.	808.	901.	1004.	1117.	1241.	1377.
34.0	134.	161.	190.	224.	262.	304.	351.	403.	461.	525.	596.	675.	763.	859.	967.	1085.	1217.	1363.	1525.	1705.
37.0	145.	174.	207.	244.	286.	333.	386.	446.	512.	586.	668.	760.	863.	977.	1106.	1249.	1410.	1590.	1793.	2022.
39.0	151.	182.	216.	256.	301.	351.	408.	473.	545.	626.	716.	818.	933.	1062.	1208.	1372.	1559.	1770.	2012.	2269.
42.0	152.	183.	219.	259.	305.	357.	416.	482.	557.	642.	738.	845.	967.	1106.	1262.	1441.	1645.	1878.	2148.	2461.
44.0	149.	180.	215.	255.	300.	352.	411.	477.	553.	638.	734.	844.	968.	1109.	1270.	1454.	1665.	1908.	2190.	2520.
47.0	145.	174.	208.	247.	291.	342.	399.	464.	538.	622.	717.	826.	949.	1090.	1250.	1435.	1648.	1894.	2181.	2517.
49.0	141.	170.	203.	241.	284.	333.	390.	454.	527.	610.	705.	813.	938.	1080.	1244.	1434.	1654.	1911.	2213.	2570.
52.0	141.	170.	203.	241.	284.	334.	392.	458.	533.	619.	719.	833.	966.	1120.	1298.	1507.	1750.	2033.	2360.	2732.
54.0	147.	177.	212.	253.	299.	353.	415.	487.	570.	667.	779.	910.	1063.	1243.	1454.	1700.	1984.	2303.	2646.	2987.
57.0	162.	195.	234.	280.	333.	394.	466.	549.	648.	763.	899.	1059.	1249.	1474.	1739.	2046.	2392.	2762.	3166.	3393.
59.0	186.	225.	270.	324.	386.	460.	546.	648.	769.	912.	1082.	1285.	1527.	1816.	2155.	2527.	2980.	3417.	3786.	3903.
62.0	219.	265.	319.	383.	459.	548.	654.	780.	929.	1107.	1320.	1575.	1881.	2245.	2672.	3159.	3680.	4184.	4552.	4649.
64.0	259.	314.	379.	456.	547.	656.	784.	936.	1118.	1336.	1596.	1908.	2279.	2719.	3229.	3800.	4395.	4930.	5278.	5267.
67.0	304.	369.	447.	538.	646.	775.	927.	1108.	1324.	1582.	1889.	2255.	2689.	3197.	3778.	4414.	5055.	5606.	5916.	5932.
69.0	352.	428.	518.	624.	749.	898.	1075.	1284.	1533.	1829.	2181.	2598.	3087.	3654.	4295.	4984.	5665.	6227.	6510.	6343.
72.0	401.	487.	589.	709.	851.	1020.	1219.	1455.	1734.	2065.	2457.	2918.	3456.	4074.	4764.	5496.	6204.	6764.	7004.	6749.
74.0	447.	543.	656.	790.	947.	1133.	1353.	1612.	1918.	2280.	2705.	3203.	3780.	4437.	5168.	5915.	6623.	7156.	7342.	7021.
77.0	490.	594.	718.	863.	1034.	1235.	1472.	1751.	2079.	2465.	2916.	3442.	4045.	4724.	5461.	6214.	6992.	7397.	7546.	7231.
79.0	528.	640.	771.	926.	1108.	1322.	1572.	1867.	2212.	2616.	3085.	3627.	4245.	4931.	5667.	6469.	7353.	7677.	7394.	7394.
82.0	565.	678.	816.	979.	1170.	1393.	1654.	1960.	2317.	2732.	3213.	3764.	4386.	5072.	5799.	6523.	7168.	7629.	7866.	7547.
84.0	596.	709.	853.	1021.	1219.	1449.	1718.	2031.	2396.	2819.	3305.	3861.	4485.	5168.	5901.	6680.	7251.	7721.	7903.	7733.
87.0	627.	734.	881.	1054.	1256.	1492.	1766.	2095.	2488.	2952.	3473.	4057.	4703.	5423.	6207.	7049.	7543.	7952.	8039.	7956.
89.0	652.	752.	903.	1079.	1284.	1523.	1801.	2124.	2498.	2929.	3424.	3986.	4617.	5369.	6242.	7279.	7451.	7953.	8176.	7973.
92.0	635.	766.	919.	1097.	1305.	1547.	1827.	2153.	2530.	2965.	3464.	4032.	4669.	5370.	6117.	6968.	7556.	8073.	8201.	8047.
94.0	643.	776.	930.	1110.	1319.	1563.	1846.	2173.	2553.	2991.	3494.	4068.	4712.	5422.	6178.	6940.	7563.	8141.	8323.	8044.
97.0	649.	782.	938.	1119.	1329.	1574.	1858.	2187.	2569.	3009.	3515.	4092.	4740.	5454.	6212.	6971.	7652.	8139.	8289.	7977.
99.0	653.	786.	942.	1124.	1335.	1580.	1864.	2194.	2576.	3017.	3523.	4099.	4746.	5455.	6205.	6949.	7608.	8066.	8186.	7861.
102.0	654.	788.	944.	1125.	1336.	1581.	1865.	2194.	2576.	3014.	3516.	4087.	4726.	5423.	6155.	6875.	7505.	7933.	8035.	7718.
112.0	642.	773.	925.	1101.	1305.	1548.	1810.	2122.	2479.	2906.	3346.	3863.	4432.	5044.	5677.	6291.	6722.	7204.	7335.	7158.
122.0	602.	724.	866.	1029.	1217.	1433.	1681.	1964.	2287.	2653.	3066.	3528.	4037.	4585.	5157.	5722.	6233.	6626.	6832.	6797.
132.0	539.	650.	778.	926.	1097.	1292.	1516.	1772.	2064.	2395.	2769.	3187.	3647.	4143.	4660.	5173.	5642.	6021.	6260.	6322.
142.0	468.	567.	681.	813.	965.	1140.	1340.	1569.	1829.	2124.	2457.	2827.	3234.	3671.	4128.	4584.	5010.	5370.	5622.	5729.
152.0	402.	489.	598.	707.	841.	995.	1171.	1373.	1602.	1862.	2154.	2479.	2838.	3226.	3636.	4052.	4451.	4801.	5064.	5208.
162.0	347.	423.	511.	613.	731.	866.	1021.	1197.	1390.	1626.	1884.	2172.	2493.	2843.	3219.	3607.	3991.	4339.	4617.	4791.
172.0	301.	368.	445.	533.	636.	753.	888.	1042.	1218.	1419.	1647.	1904.	2192.	2511.	2858.	3224.	3599.	3924.	4189.	4355.
182.0	265.	323.	389.	467.	555.	657.	774.	909.	1062.	1238.	1438.	1666.	1923.	2209.	2522.	2852.	3180.	3476.	3710.	3868.
192.0	236.	286.	344.	412.	489.	578.	680.	797.	932.	1085.	1261.	1460.	1685.	1936.	2209.	2497.	2783.	3048.	3276.	3464.
202.0	211.	256.	307.	366.	435.	513.	604.	707.	826.	962.	1118.	1295.	1494.	1716.	1959.	2217.	2481.	2739.	2978.	3191.
212.0	192.	231.	278.	331.	393.	464.	547.	642.	751.	877.	1021.	1186.	1373.	1584.	1816.	2067.	2320.	2586.	2824.	3028.
222.0	181.	218.	261.	312.	371.	440.	519.	612.	720.	846.	991.	1160.	1353.	1573.	1817.	2080.	2349.	2602.	2813.	2957.
232.0	182.	219.	263.	315.	376.	447.	531.	630.	746.	883.	1044.	1232.	1450.	1699.	1974.	2265.	2547.	2783.	2938.	2952.
242.0	195.	235.	284.	341.	408.	488.	582.	695.	828.	986.	1173.	1393.	1648.	1939.	2257.	2583.	2880.	3092.	3156.	3030.
252.0	216.	262.	317.	382.	458.	551.	661.	791.	946.	1131.	1349.	1607.	1905.	2242.	2606.	2991.	3278.	3484.	3483.	3233.
262.0	241.	293.	356.	430.	519.	624.	749.	898.	1076.	1288.	1538.	1831.	2170.	2552.	2961.	3367.	3713.	3913.	3868.	3522.
272.0	264.	323.	393.	476.	575.	692.	832.	999.	1197.	1432.	1710.	2034.	2400.	2820.	3270.	3724.	4098.	4302.	4222.	3794.
282.0	284.	348.	424.	515.	622.	750.	902.	1083.	1298.	1552.	1851.	2201.	2603.	3054.	3535.	4009.	4402.	4605.	4500.	4022.
292.0	300.	369.	449.	545.	656.	786.	950.	1150.	1378.	1647.	1964.	2334.	2759.	3235.	3742.	4239.	4651.	4866.	4764.	4274.
302.0	312.	383.	468.	569.	689.	831.	1001.	1202.	1441.	1724.	2057.	2446.	2894.	3396.	3934.	4466.	4916.	5168.	5093.	4600.
312.0	320.	393.	481.	586.	711.	859.	1035.	1245.	1494.	1790.	2140.	2550.	3024.	3560.	4141.	4725.	5231.	5533.	5475.	4951.
322.0	325.	401.	491.	599.	727.	881.	1063.	1281.	1541.	1850.	2218.	2651.	3155.	3729.	4358.	4997.	5558.	5886.	5814.	5225.
332.0	328.	405.	498.	608.	740.	899.	1086.	1312.	1582.	1904.	2289.	2742.	3274.	3880.	4549.	5225.	5808.	6137.	6029.	5379.
342.0	329.	407.	501.	614.	748.	910.	1103.	1335.	1612.	1945.	2342.	2812.	3363.	3991.	4677.	5366.	5949.	6266.	6132.	5439.
352.0	328.	407.	501.	615.	751.	915.	1111.	1346.	1628.	1966.	2370.	2848.	3406.	4041.	4730.	5419.	6000.	6314.	6173.	5447.
362.0	324.	403.	498.	611.	748.	911.	1108.	1343.	1626.	1965.	2368.	2845.	3401.	4032.	4718.	5405.	5992.	6314.	6171.	5413.
372.0	319.	397.	490.	603.	737.	899.	1093.	1326.	1605.	1939.	2336.	2805.	3351.	3973.	4651.	5336.	5927.	6255.	6103.	5323.
382.0	312.	388.	4																	

Table 6.4

Parameters of Oscillation Fits					
Data Set	$\chi^2(\sin^2 2\theta = 0)$	χ^2_{min}	Δm^2	$\sin^2 2\theta'$	Comments
ν	11.0465	9.0400	1252	0.55	nominal M.C.
ν	9.4536	6.3767	42	0.06	stretched M.C.
ν^c	4.7155	0.2247	132	0.16	nominal M.C.
ν^c	4.9032	0.2595	132	0.16	stretched M.C.
$\nu + \nu^c$	15.7620	12.5431	1252	0.61	nominal M.C.
$\nu + \nu^c$	14.3595	8.7544	10	0.43	stretched M.C.

Table 6.5a

λ Distribution 90% Points for 3 D.F.				
$\Delta m^2/\sin^2 2\theta$	0	0.04	0.10	0.26
90	5.0	5.7	5.1	5.0
162	5.0	5.0	5.2	4.6
302	5.0	4.6	4.5	4.3
902	5.0	5.0	5.0	4.7

Table 6.5b

λ Distribution 90% Points for 15 D.F.				
$\Delta m^2/\sin^2 2\theta$	0	0.04	0.10	0.26
90	6.6	6.3	4.9	5.0
162	6.6	6.3	4.5	3.5
302	6.6	6.4	5.3	4.9
902	6.6	6.5	6.5	5.9

Table 6.5c

λ Distribution 90% Points for 18 D.F.				
$\Delta m^2/\sin^2 2\theta$	0	0.04	0.10	0.26
90	6.3	5.9	4.9	4.9
162	6.3	6.3	4.3	2.9
302	6.3	6.1	4.9	4.3
902	6.3	6.3	6.1	5.9

Dirac neutrino $m_\nu \neq 0$
 $\eta \neq 0$ L_{TOT} conserved
 Figure 1.1a

Majorana neutrino $m_\nu \neq 0$
 $\eta \neq 0$ L_{TOT} not conserved
 Figure 1.1b

Majorana $m_\nu = 0$ $\eta \neq 0$
 Figure 1.1c

Figure 1.2

Figure 1.3a

Figure 1.3b

Figure 1.4b

Figure 1.4a

Figure 1.5

Figure 1.6a

Figure 1.6b

Figure 1.7

Figure 1.8

Figure 1.9

Figure 1.10a

Figure 1.10b

Figure 1.11

Figure 2.1

Figure 2.2

Figure 2.3

Figure 2.4

Figure 2.5

Figure 2.6a

Figure 2.6b

Figure 2.6c

Figure 2.7

Figure 2.8

Figure 2.9

Figure 2.10a

Figure 2.10b

Figure 2.10c

Figure 2.11a

PION ν FLUX Z=1116 meters KAON ν FLUX Z=1116 meters

Figure 2.11b

PION ν FLUX INTEGRATED OVER Z KAON ν FLUX INTEGRATED OVER Z

Figure 2.12a

Figure 2.12b

Figure 2.13a

Figure 2.13b

Figure 2.14

Lab E

Figure 2.15a

Figure 2.15b

Figure 2.16
BEAMLINE
MONITORING
SYSTEM

Figure 2.17

Figure 2.18

Figure 2.19a

Figure 2.19b

Figure 2.20a

Figure 2.20b

Figure 2.21a

Figure 2.21b

Figure 2.22a

Figure 2.22b

Figure 2.22c

Figure 2.22d

Figure 2.22e

Figure 2.22f

Figure 2.23a

Figure 2.23b

Figure 2.24a

Figure 2.24b

Figure 3.1

Figure 3.2

Figure 3.3

Figure 3.4

Figure 3.5

Figure 3.6a

Figure 3.6b

Figure 3.7

Figure 3.8

WONDER BUILDING FRONT END

Figure 3.9a

Figure 3.9b

Efficiency
Trigger:

6-JAN-82 19:41:21
 RUN 29 PLACE 51
 EVNT 129 CEXIT 1

Figure 4.1

30-JAN-82 12:55:46
 RUN 97 PLACE 32
 EVNT 6734 CEXIT 1

Figure 4.2

Figure 4.3a

Figure 4.3b

Figure 4.4

Figure 4.5

Figure 4.6

Figure 4.7

Figure 4.8

Figure 4.9

Figure 5.1

Figure 5.2a

Lab E

Figure 5.2b
Wonder Bldg.

Figure 5.3

Figure 5.4

Figure 5.5

Figure 5.6

Figure 5.7

RELATIONSHIP BETWEEN VERTEX + RADIUS CUTS

Figure 5.8

Figure 5.9a

Figure 5.9b

Figure 5.10

Figure 5.11

Figure 5.12

Figure 5.13a

Figure 5.13b

Figure 5.14a

Figure 5.14b

Figure 5.15a

Figure 5.15b

Figure 5.16a

ν DATA CORRECTED FOR NORMALIZATION and NOMINAL MC

Figure 5.16b

ν DATA CORRECTED FOR NORMALIZATION and "STRETCHED" MC

Figure 5.17a

ν DATA CORRECTED FOR NORMALIZATION and NOMINAL MC

Figure 5.17b

ν DATA CORRECTED FOR NORMALIZATION and "STRETCHED" MC

Figure 6.1a

ν DATA CORRECTED FOR NORMALIZATION and NOMINAL MC

Figure 6.1b

ν DATA CORRECTED for NORMALIZATION and NOMINAL MC

Figure 6.2a

Figure 6.2b

Figure 6.2c

Figure 6.3

Figure 6.4

Figure 6.5

Figure 6.6

Figure 6.7

Figure 6.8a
 ν DATA ONLY

Figure 6.8b
 ν^e DATA ONLY

Figure 6.8c
COMBINED $\nu + \nu^e$

Figure 6.9

Figure 6.10

Figure 6.11

Figure 6.12

Figure B1.1a

Figure B1.1b