PROTON DEUTERON ELASTIC SCATTERING
FROM 20 TO 220 GEV/C

by

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VITAE

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ABSTRACT

Proton deuteron elastic scattering was studied using facilities available at the Internal Target Area of the Fermi National Accelerator Laboratory. A deuteron gas jet was the target and the recoil deuterons were detected at large angles from the forward direction with a spectrometer employing a superconducting dipole and quadrupole doublet.

A total of 5×10^5 recoil deuterons were recorded for incident momenta, p_0 , from 20 to 220 GeV/c and with four-momentum transfer squared, t , in the range $.6 < -t < 3.0 (\text{GeV}/c)^2$. From these, approximately 150 differential cross sections were obtained covering our t and p_0 region as evenly as possible.

The results disagree with simple Glauber model predictions, and are typically significantly less than the predictions. The bulk of the data can be understood by adding corrections for the space-time development of the strong interaction to the Glauber model. In this context, we have measured the time constant of the strong interaction and its dependence on t . Explanations based upon the addition of intermediate inelastic states to the Glauber model do not reasonably explain the data either quantitatively or even qualitatively.

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CHAPTER 1

INTRODUCTION

Simple nuclei have received considerable attention in recent years both as a technique to study the space-time development of high energy processes¹ and as a way to measure interaction cross sections of unstable particles.² Of the various reactions and nuclear targets considered, high energy elastic proton deuteron scattering has the unique advantage of involving well understood single nucleon processes in a system where the nuclear physics can be reliably calculated. In an experiment at Fermi National Accelerator Laboratory, we studied this reaction as a function of both incident momentum, p_0 , and invariant four-momentum transfer, t . The incident momentum ranged from 20 to 220 GeV/c and t ranged from $-.6$ to -3.0 (GeV/c)². Deuterium gas was the target and the reaction was

$$p + d \rightarrow p + d \quad .$$

A multiple scattering theory for scattering from deuterium has been developed by Glauber.³ This model relates proton-deuteron scattering to proton-nucleon scattering by treating the deuteron as two separate nucleons and viewing the final reaction as the sum of several reactions. The incident proton hits either nucleon or both; the amplitude for each interaction is given from proton-nucleon scattering. When the proton hits one nucleon the interaction is referred to as single scattering (Fig. 1a). This process dominates at low t . When the incident proton hits both nucleons (Fig. 1b) the process is called double scattering. Then the total amplitude is the sum of the two amplitudes with the probability of the deuteron holding together folded in for elastic scattering. Double scattering dominates in the high t region studied here, $-t \geq .6(\text{gev}/c)^2$.

Recently, the possibility has been raised that the amplitude of the second interaction may be reduced because it occurs a very short time (10^{-24} sec) after the first interaction,^{4,5} a time comparable to the decay time of strongly interacting particles. Perhaps a particle which has interacted once is not immediately able to interact again, so that the space-time development of the interaction must be considered in reactions on nuclei. A previous proton deuteron elastic scattering experiment⁶ may have already seen this effect, if it exists, since it obtained cross sections which were lower than the Glauber predictions. The time between the first and second interaction is essentially the time required by the proton to cross the deuteron, a distance of 3 fermi. At relativistic incident

energies the relativistic length contraction effect shortens this time by effectively shrinking the size of the deuteron. At Fermilab energies the time becomes approximately inversely proportional to the incident energy. Therefore, information on the energy dependence of the cross section, as we have obtained in this experiment, may help lead to a better understanding of the space-time development of strong processes.

Other complications to Glauber theory whose importance has been considered recently are multiple exchanges and intermediate inelastic states⁷ (Fig. 1c). At least one source⁸ shows (using Regge phenomenology) that at Fermilab energies multiple exchanges are not important. Yet others⁹ (also using Regge phenomenology) have added these terms to the model and even fit data to obtain coefficients for the more important terms. Perhaps our data will help to clarify this question also.

The validity of the Glauber Model has been assumed in previous experiments to measure interaction cross sections of unstable particles.² With deuterons one such possibility is the measurement of $N-N^*$ elastic scattering, which would be accomplished via measurement of the $p+d \rightarrow N^*+d$ cross section in the double scattering region (Fig. 1d). In the Glauber picture, this final state interaction proceeds in two ways. Either the incident proton first scatters elastically off one nucleon and then produces the N^* by scattering off the second nucleon, or else the N^* is produced first and then scatters elastically off the second nucleon. The only unknown and not-directly-measurable amplitude is the N^* elastic one

amplitude, which can be inferred from the data. This work is directed towards testing the validity of extracting such cross sections by studying the elastic process where the cross sections of the constituent interactions are already known.

Fermilab's Internal Target Area, where this experiment was performed, is well suited for these measurements because the beam energy is continuously varied, making it ideal for energy dependence studies, and because the high incident flux enables the use of gas jet targets to obtain high luminosity without the ambiguities of multiple scattering present in a more dense target. Prior experiments at the site, studying the energy dependence of p-p elastic interactions^{10,11} and p-d elastic interactions¹² at low t , provided necessary associated measurements; for example, the low t p-d results provided a technique for normalizing the results, while the elastic proton-proton scattering results could be used with our p-d results to test the Glauber Model without problems of relative normalization. A large angle recoil spectrometer detected the deuterons recoiling from the target and independently measured the deuteron's production angle, θ , and momentum, p_r , with high precision, enabling the separation of elastic from inelastic scattering processes through kinematics. The t range of the spectrometer was well matched to the desired double scattering region of the proton-deuteron elastic interaction.

Chapter II describes the apparatus and the experimental technique. In Chapter III the reduction of the data to obtain cross sections is explained. In the last chapter we compare our results

with those of other elastic scattering experiments and relate them to various theoretical models.

CHAPTER 2

EXPERIMENTAL TECHNIQUE

2.1 APPARATUS OVERVIEW

The composite spectrometer system (Fig. 2) consists of the target, three independent detector systems, and the data acquisition system. The primary detector system, the spectrometer, detected the deuterons of interest and provided all the necessary information about them. The entire detector could be moved to be positioned at the appropriate angle for each t . The magnet currents could of course be scaled for the recoil deuteron momentum at that t . In Fig. 2 the spectrometer is the prominent portion containing the dipole (D), two quads (Q1,Q2) and five multi-wire proportional chambers (MWPC1-MWPC5).

The second detector system was the solid state spectrometers, which were used as monitors. These two independent arrays of detectors detected and measured the dE/dx and total energy of low t deuterons recoiling at nearly 90 degrees in the lab frame. Since the elastic cross section for low t proton deuteron scattering had already been accurately determined, the elastic peak in these stacks

could be used to infer the overall normalization.

The remaining detector system, the forward hodoscope, (downstream hodoscope in Fig. 2) detected the forward going scattered protons. It supplied positional information about these protons providing an additional constraint to identify elastic events when missing mass resolution was insufficient.

2.2 THE TARGET

The target was a pulsed deuterium gas jet^{13,14} injected downward transverse to the beam as shown in Figure 3. A series of high capacity diffusion pumps plus a buffer volume maintained the main ring vacuum pressure of $<10^{-6}$ torr away from the jet during operation. No cryogenics were used. The total jet-on time per main ring cycle (8-20 seconds) was limited to one second to prevent exceeding the pumping system capacity. Within this limit any assortment of pulses was possible. Typically, up to three pulses were used with durations from .1 to .8 seconds. A typical main ring cycle is shown in Fig. 4.

Setting the jet timing required two signals from the Fermilab operations center: t_0 , the signal marking the start of the cycle, and BDOT, a signal nominally one pulse for each beam energy increase of .01 GeV. The actual BDOT relation to beam energy was slightly different because of noise in the line and calibration problems, and was determined by comparing BDOT with better determined momentum values from the operations center. By clearing a scaler at each t_0

pulse and scaling BDOT pulses during the main ring cycle, we knew the momentum of the beam at all times after t_0 . The jets were timed relative to t_0 with BDOT as a guide. BDOT was also recorded for every event in both the spectrometer and monitor.

The principle of the jet¹⁵ is to force deuterium through a d'Laval nozzle¹⁶ at high enough injection pressure to create supersonic flow. This collimates the jet so that the gas goes directly across the beam and into the buffer volume and does not diffuse out into the main ring. This scheme maintained a (factor of ten) pressure differential between the buffer volume and the higher-vacuum beam chamber. The injection pressure could be varied from 70 psi to 280 psi which correspondingly varied the jet density around a value of 3×10^{-7} g/cm³. Any gas which would not clog or otherwise damage the nozzle could be used. The jet width as it crossed the beam was one centimeter. Assuming a beam of 10^{13} protons with a circulation frequency of 50,000 revolutions/sec, the luminosity is

$$L = 3 \times 10^{34} \text{ cm}^{-2} \text{ sec}^{-1} .$$

2.3 SPECTROMETER

The spectrometer detected and measured three quantities about each recoil deuteron: production angle, momentum, and time of flight (TOF) through the spectrometer. This enabled a determination both of the recoil particle mass and of the forward-going undetected mass. The front portion of the spectrometer, containing the quadrupole¹⁷

doublet and MWPC 1 measured the production angle while the rest of the spectrometer including the dipole¹⁷ and MWPC's 2-5 determined the momentum. The trigger scintillation counters doubled as the TOF detectors.

2.3.1 The Variables

The spectrometer reference frame is shown in Figure 5 along with its relation to the laboratory frame. The origin is at the target and the z-axis is along the spectrometer. The z-axis bends 12.5 degrees at the front of the dipole to follow the field axis and bends 12.5 degrees further at the dipole rear. The y-axis is vertical with positive upward. The x-axis is perpendicular to y and z with positive towards larger angles. For convenience the particles are often referred to with six coordinates: x, y, z, dx/dz and dy/dz (the derivatives with respect to z), and δ , where δ is

$$\delta = (p_r - p_c) / p_c \quad (2.1)$$

Units were centimeters for x, y, z and milliradians for dx/dz and dy/dz.

Separation of the elastic events could be accomplished by calculating the variable M_x , the mass of the undetected forward system. The equation for this is

$$M_x^2 = m_p^2 + 2p_0 (p_r \cos \theta - T_r \beta^*) \quad (2.2)$$

where T_r (p_r) is the kinetic energy (momentum) of the recoil particle and $\beta^* = p_0 / (E_0 + m_p)$ is the velocity of the cm system in the laboratory frame. For elastic scattering $M_x^2 = m_p^2$ so $\cos \theta = T_r / p_r \beta^*$. β^* ranges from

0.9 at an incident momentum of 20GeV/c to 1 at the higher momenta, thus the elastic kinematics (and the settings and acceptance of the spectrometer) change only slightly over the range of incident momenta considered in this experiment. θ is determined via the relation

$$\theta = \arccos(\cos(dy/dz) \cos(\theta_s + dx/dz)) , \quad (2.3)$$

where θ_s is the spectrometer angle shown in the figure. p_r is simply related to δ via

$$p_r = p_o (1 + \delta) , \quad (2.4)$$

where p_o is the central momentum the dipole is set for. p_o is supplied continuously by the Fermilab operations center in the form of BDOT nominally 100 times the momentum in GeV. The actual relation is

$$p_o = 6.14 + .01011 \text{ BDOT} . \quad (2.5)$$

t is determined via $t = -2m_d T_r$, which is the square of the difference between the initial and final deuteron 4-momenta since the initial 3-momentum of the deuteron is zero.

The error in determining the missing mass is determined from the three relations:

$$dM_x^2/d\theta = -2p_o p_r \sin\theta \quad (2.6)$$

$$dM_x^2/dp_r = 2p_o (\cos\theta - p_r/(E_r \beta^*)) \quad (2.7)$$

$$dM_x^2/dp_o = -2 (p_r \cos\theta - T_r p_o/E_o) . \quad (2.8)$$

Equation (2.8) shows that errors in p_o can cause systematic shifts of the peak, particularly at lower energies ($p_o < 50 \text{ GeV}_o$), but does not contribute to the resolution. The θ and p_r derivatives, however, are proportional to p_o so good resolution in θ and p_r is required,

especially at high energies.

2.3.2 The Chambers

Arrangements of the chamber sense planes are shown in Figure 6. MWPC's had active areas of 10 cm in x and 15 cm in y with 80 wires per x plane and 120 wires per y plane. Chambers 1, 2, and 4 (Fig. 6a) had two x planes, relatively offset by one-half wire spacing, and a vertical plane. Chamber 3 had only an x plane. Chamber 5 (Fig. 6b) had two wider and slanted (u,v) planes biased for x precision. The individual plane efficiencies were about 99% and the overall detection efficiency, aided by the redundant x planes, was 95%.

There were also four hodoscopes in the system. Their purpose was to aid in track reconstruction. However, in the final analysis these were unnecessary so they were only important as a source of multiple scattering.

Signals from the chambers were amplified and discriminated at the chamber and then transmitted to latch cards at the computer. Each trigger automatically cleared previous data from the latch, and caused the new data to be transferred to the computer to be stored on magnetic tape.

2.3.3 The Trigger And Time Of Flight

A fourfold coincidence of four scintillator counters grouped into two pairs composed the trigger. There was no hardware veto. The trigger efficiency was determined in test runs which included two extra trigger counters to be >99%. For the time of flight measurement, delays were arranged so a coincidence in the rear pair of trigger scintillators started the clock and the coincidence in the front pair stopped it. The distance between the detector pairs was 8 meters and the resolution always allowed complete separation of deuterons and protons. A sample TOF distribution is shown in Figure 7.

The hardware trigger admitted significant numbers of false triggers. Nearly all of these triggers were from particles which followed a direct line of sight from the beam pipe through the front and rear trigger counters, missing chambers 3 and 4 and the dipole completely. This background was allowed in order to keep the hardware trigger simple, and because it was easily removed with a software trigger.

2.3.4 Angle Measurement

Each superconducting quadrupole had an effective length of 63 cm with a 20 cm diameter circular aperture. The quadrupole doublet was set to focus horizontal parallel rays from the target to a point at a plane coincident with MWPC 1. In the ideal case this type of focus creates a one to one correspondence between dx/dz at the target and x

at chamber 1. Information from chamber 2 was used to correct dx/dz for chromatic aberrations (momentum dependence of focal plane) and for the effects of a non-ideal field; and to determine x , y , and dy/dz at the target. Wire orbit measurements were fit to develop the full relations.

The wire orbit technique¹⁸ uses wires to mimic particle trajectories through the quadrupoles. Unlike the particles however, the wire trajectories could be measured at both the target and chamber 1. Then polynomial relations (one for each target coordinate) were created with the chamber 1 trajectories as input, and the coefficients were adjusted to give the target trajectories of the wires as output. The same relation is also valid for real particles.

The main ring vacuum was extended to MWPC 1, making multiple scattering up to that point negligible. After chamber 1, two trigger counters and three hodoscopes between chambers 1 and 2 caused significant multiple scattering and made the accuracy of the corrections t dependent. At $t=-.6$ these errors caused 60% of the resolution width, as determined by tests without the hodoscopes. The no-multiple-scattering-limit errors on the target information were .2 mr and 5 mr for dx/dz and dy/dz , respectively, and several millimeters for x and y . At $-t=1.0$, the chromatic aberrations plus multiple scattering increased the errors to .5mr and 10mr for dx/dz and dy/dz . There was a systematic correction of 2% for the chromatic aberration determined by minimizing the width of the elastic peak.

2.3.5 Momentum Measurement

The superconducting dipole had a one meter effective length and a 20 cm diameter circular bore. The bend angle was 25 degrees and the dipole axis was aligned at 12.5 degrees with respect to the front of the spectrometer to maximize the acceptance. Helium replaced air along the particle trajectory to reduce scattering. The coordinates at chambers 2 thru 5 (equivalently $x, y, dx/dz$, and dy/dz both before and after the magnet) were used to define the momentum; because of the fringe fields associated with a superconducting magnet, a detailed study was required to determine the relation. First the field was measured. Then a fit was made keeping all terms up to fifth order in an expansion and requiring the appropriate symmetries and consistency with Maxwell's Equations.

A computer simulation then created pseudo-particles, or rays, and tracked them through the dipole obtaining some 4000 sets of 8 coordinates ($x, y, dx/dz, dy/dz$ both before and after the dipole) and an associated δ , related to the momentum via Equation (2.1). These rays were used to develop a relation between the coordinates and δ for real particles. The simulation was also used to test the sensitivity of the tracks to the field fit. It showed that keeping only first order terms for the x and z fields, and keeping out to only third order in the y field changed the coordinates of the output particle by less than a micron. The final work was done with the simpler fit.

The equation for obtaining δ was

$$\delta = \sum_{i \geq j \geq k}^7 c_{ijk} x_i x_j x_k, \quad (2.9)$$

where x_i $i=1,6$ are some subset of 6 of the above 8 coordinates and $x_7=1$. The use of all 8 coordinates would overconstrain the equation. The c_{ijk} are determined by an analytic chi squared type minimization using the 4000 pseudo-particles. (See the Appendix A for details.) In the minimization the errors due to multiple scattering and finite wire spacing were included. Four different sets of six coordinates were used and the errors were added in five different amounts obtaining 20 different sets of coefficients. The quality of the fits was determined by the chi square of the 4000 rays with multiple scattering added. The average of the best two fits gives the best results. Another fit was found whose answer, minus the best answer, varies rapidly as a function of multiple scattering. Comparing the two answers allowed a check on the consistency of the momentum measurement, and helped determine the amount of multiple scattering a particle had undergone. The error in calculating δ from the fit alone is $<5 \times 10^{-4}$. Including finite wirespacing the error in δ is 1×10^{-3} . Including multiple scattering this correction is $<.003$, i.e., the error in the momentum measured at the dipole is .15-.3% depending on t .

Since the number of quantities measured was greater than the number minimally necessary to calculate the momentum, the above procedure could be repeated replacing δ in Equation (2.9) with any

other of the nine quantities recorded for the pseudo-particles. This was done with dx/dz (before the dipole) as the replacement, providing a way to calculate a quantity measured by the spectrometer. Comparing the measured and calculated values gave a second check on multiple scattering.

2.4 MONITORS

The two stacks of fully depleted silicon surface barrier solid state detectors are shown in Figure 8. They are located within the main ring vacuum 200 cm from the target with stack two directly above stack one. They accept events from an area larger than the target to insure that all real events are detected. Their angles with respect to the beam are

$$\theta(\text{stack 1}) = 84.50 \pm .07 \text{ degrees}$$

$$\theta(\text{stack 2}) = 84.57 \pm .07 \text{ degrees} \quad .$$

These angles are determined from both survey measurements, and, more accurately, using the range of the deuteron in stack one, where the elastic deuterons barely penetrate the back the rear detector. The details are in Chapter 3. The angles of the two stacks differ because of the their relative vertical displacement.

The stacks have slightly different configurations. The E counters are approximately 1500 and 2000 microns thick for stacks one and two respectively. The dE/dx detectors are about 200 microns thick. The distance between the front and rear detectors in a stack is less than two cm. Stack one has one collimator, and stack two has

two, to define the angular acceptance of the stacks. The angular acceptances are

$$\Omega \text{ (stack 1)} = 4.16 \times 10^{-6} \text{ steradians}$$

$$\Omega \text{ (stack 2)} = 4.70 \times 10^{-6} \text{ steradians}$$

There is a relative error in Ω of 2% between stacks. Uncertainties, such as exactly how well a collimator defines the acceptance, cause the absolute errors to be slightly larger.

The detection efficiency of a silicon detector is essentially 100%. Therefore, the only corrections are from multiple scattering losses and background. Background corrections are discussed in the next chapter. For stack 1 the multiple scattering losses are negligible. In stack two the presence of two collimators causes a 3% correction. The multiple scattering losses are determined by a Monte Carlo calculation.

Simultaneous pulses from both detectors of a single stack defined a monitor event. Scalers recorded the number of monitors in each stack. For a fraction of the monitors the pulse heights of both detectors were digitized and recorded in a CAMAC buffer memory. For these monitors BDOT was also recorded. The fraction (the monitor divide) which had their pulse heights recorded could be varied from run to run to maintain an approximate one to one ratio between the number of monitors and the number of spectrometer events. The fraction recorded ranged from 1/20 to 1 over the course of the experiment. The monitor information was transferred from the buffer memory to the computer whenever a trigger occurred in the main spectrometer.

2.5 FORWARD HODOSCOPE

The hodoscope (Figure 9) was placed approximately 23 meters downstream of the target and approximately 30 cm from the main ring beam to detect the forward proton of elastic events. It could be moved ± 15 cm horizontally and perpendicular to the beamline, and be rotated to maintain perpendicularity to the target. This served to center the hodoscope at the expected position of the elastic protons. This position, in terms of distance from the main ring beam pipe, varied approximately as $\sqrt{-t}/p$.

The active area of the hodoscope was 1.9 cm horizontally by 6.4 cm vertically. It consisted of 60 scintillator elements each .64cmX.64cmX1.27cm with the square face to the target. They were arranged in three rows of twenty elements. In each row, elements were overlapped slightly more than 50% so that particles emanating from the target would trigger two adjacent elements. This coincidence style configuration was required because background radiation near the beam pipe resulted in a multi-megacycle counting rate per element.

The overall detection efficiency (with coincidence required) was 75-80%. This was determined using data from this experiment which did not require but had available the hodoscope, and also, and with more statistics and rigor using p-p data (Ref. 10). This relatively low efficiency for scintillator resulted from the intense radiation (damage) and from attempts to reduce the background via threshold adjustment and tight timing.

The hodoscope information was used by relating it to information about the recoil particle. From kinematics, assuming the recoil particle to be elastic, the position of the elastic forward proton in the hodoscope could be predicted. A distribution of the difference between the predicted position and the actual position of any detected particles, summed over many events, showed a peak around zero corresponding to elastic events. Use of the hodoscope was limited to the 200 Gev data for $1. < -t < 3.0$ and was only used in the analysis for $-t > 2.0$. Even then it was only used as a check on the results obtained fitting the missing mass distribution. In all cases the two results agreed.

2.6 DATA ACQUISITION

A PDP11/45 controlled the experiment. It handled data accumulation and magnetic tape writing, and did a partial data reduction to monitor the performance of the apparatus. Separate electronics (see Fig. 10) determined the presence of an event and interrupted the computer to initiate data transfer. Coincidences from a fast OR (10ns) of T1 and T2 were fanned out to a fast OR with T3 and T4, and to a delay for use in the time of flight (TOF) measurement. The trigger, a full fourfold coincidence of T1-T4 caused a computer interrupt, the latching of data and the start of the TOF time-to-digital converter circuit. The delayed T1-T2 coincidence arrived shortly after to stop the TOF counter. Meanwhile, the computer inhibited the experiment from accepting further triggers and the latching of data was completed. Then the

data readout began, serially reading and clearing scalers, TDC's, ADC's, BDOT, a buffer memory containing monitor events (see prior sections), and finally the latched MWPC and hodoscope data. To save time the data was transferred to core rather than directly to tape, or, if core was already full, to disk. Upon completion of the readout, the system was reset and the inhibit removed to accept more monitors and await a new trigger. Data was transferred to magnetic tape between jet pulses.

On a low priority basis the computer partially reduced the events to provide immediate feedback on the system. Some of the information available was chamber and hodoscope distributions and efficiencies, TOF and incident momentum distributions, monitor data, and information about consistency checks made on the data. The analysis did not affect the contents of the data tapes in any way.

The maximum collection rate of this system was 800 events per ramp. At that rate the computer had no time to analyze data and under some circumstances had trouble transferring all the data to tape. Typical collection rates were much lower, ranging from about 300 triggers per ramp at $t = -.6$ to 60 triggers per ramp at $t = -3.0$. The detection rates for elastic scattering events for the same t values were 50 events per ramp (15,000 per hour) and < 5 per hour. Data from 100,000 triggers fit onto one tape.

Approximately 50 data tapes containing elastic proton deuteron events were taken. On these tapes were just over 300,000 recoil deuterons. The remainder of the triggers were protons, pions, and

background, primarily false triggers. At each t value at least two runs were taken, one with a single jet pulsed from 20 to 80 GeV/c with the spectrometer set according to the recoil kinematics for an incident momentum of 40 GeV/c, and one with two jets from 70 to 130 and 190-220 GeV/c with the spectrometer set for 100 GeV/c. Filling the gap from 130 to 190 GeV/c would have required a separate run because of target-pumping-system-capacity considerations and therefore would have increased the experiment run time by 50%. Care was taken to accumulate the events as evenly in t as possible. At $t = -.6$ 50,000 recoil deuterons were recorded and at $t = -3.0$, where the elastic cross section is down four orders of magnitude, 7500 recoil deuterons were taken.

CHAPTER 3

ANALYSIS

The analysis of the data was a two stage process. First the undesired triggers were removed and the deuteron tracks reconstructed. The results of this were then written to a summary tape in a condensed format. All the monitor events were written to the summary tape. The condensed data were then analyzed and reanalyzed improving resolution, setting cuts, binning the data, etc., to obtain the number of elastic events in bins of t and p_0 . Then the normalization was set for each data point using the monitor for luminosity and a Monte Carlo for the spectrometer acceptance.

3.1 TRACK RECONSTRUCTION

Track reconstruction has been done using two methods. The first method¹⁹ used the hodoscopes to clarify multiple track confusion. That technique reconstructed about 65% of the data. The final technique used expected properties of the tracks rather than the hodoscopes to sort out multiple hit wires in chambers. The technique properly reconstructed 99% of the events, except for chamber

inefficiencies.

Before attempting to reconstruct an event, a software trigger was applied, requiring at least one hit wire in each of chambers 1, 2, 4, and 5. This removed false triggers. The few real triggers cut here were accounted for by the acceptance correction. The fraction of the triggers cut here was very t dependent varying from 20% to 95% from $t=-.6$ to $t=-3.0$, respectively. The remaining events were assumed to be real. The deuterons, isolated using the time of flight, were then tested for having sufficient planes with wires fired to do the reconstruction. 5% of the events failed here. 99% of the remaining events were properly reconstructed.

The reconstruction started at chamber 5 and worked forward, first finding x coordinates, then going back and doing y coordinates. At each chamber 90% of the events were easily reconstructed since there was only one wire fired in each plane in the chamber. At chamber 5 x and y were determined from the crossed u,v planes and were tested to be within the physical chamber. Then in chambers 4, 2, and 1, pairs, or two fired wires were found that were near each other in x , one in each x plane. This eliminated confusion from noisy wires and other background sources. Pairs at chamber 4 and 2 (or all hits if no pairs were found) plus all fired wires in chamber 3 were tested in combinations for a track whose coordinates at the dipole center, projected from both front and rear, matched within empirical limits (Fig. 11). If no combination gave a result within the limits, the best result was used. The y coordinates were then done similarly except that there were no redundant planes, and

chamber 1 was used instead of chamber 3. The x coordinate at chamber 1 was found by projecting forward from chambers 2 and 3 and finding the nearest pair in chamber 1 (or single hit if no pair was found.)

3.2 THE ELASTIC SIGNAL

The two independent variables of interest were the invariant four-momentum squared, t , and the incident momentum, p_0 . All the data were divided into bins in t and p_0 in such a way as to have approximately 200 elastic events in each bin where possible. The variables required to be measured to distinguish elastic events were the recoil angle, θ , and recoil momentum, p_r . These allowed the calculation of M_x and an equivalent variable $\Delta p_r/p_c$.

To calculate $\Delta p_r/p_c$ each event was subjected to an elastic hypothesis, i.e., the measured recoil momentum was compared to that predicted from the production angle and elastic kinematics. A typical distribution in the difference between the measured and inferred momentum, divided by p_c , for an incident momentum of 120 GeV/c, is shown in Fig. 12. A clean peak is evident near $\Delta p_r/p_c = 0.0$, with a broad distribution of counts for $\Delta p_r/p_c < 0.0$, corresponding to inelastic processes. The absence of events at $\Delta p_r/p_c > 0$, a region where events are kinematically forbidden, indicates the absence of backgrounds.

$\Delta p_r/p_c$ has several advantages over other variables. The width and position of the elastic peak in this variable is independent of p_0 . The width of the elastic peak in the variable M_x is dependent on

energy, and for a finite incident energy bin the peak in M_X^2 is not Gaussian. $\Delta p_r/p_0$ also has an advantage over another variable, Feynman x of the forward going system, which is approximately $1-(M_X^2-m_p^2)/s$. Since $\Delta p_r/p_0$ has a more direct relation to measured quantities, the source of anomalous shifts, tails, wide peaks, etc., is more transparent. Several cuts were made to improve the quality of the peak. The momentum was calculated in two ways (see Section 2.3.5 and Fig. 13) and events were cut if the answers disagreed by $>.7\%$. dx/dz at the front of the magnet was calculated (see Section 2.3.5 and Fig. 14) and compared with the measured value, and events were cut if the two answers disagreed by $>1\text{mr}$. Tracks were projected to the dipole center from the front and rear, and the events were cut if the x and y projections disagreed by more than $.5\text{cm}$ and 5cm , respectively (see Fig. 15). These four cuts removed 5% of the events except at $-t < 1.0$ where the increased multiple scattering increased the percentage cut to a high of 22% at $t = -.6$.

Another cut was added to help define the acceptance of the spectrometer through the quadrupoles. A study was done tracing pseudo-particles from the target through an equivalent ideal quadrupole doublet to test which particles would hit the magnet walls. From this a well defined region in dx/dz and dy/dz at the target was found within which particles would rarely hit the walls. That region was:

$$|dy/dz| \leq 52 \text{ mr}$$

$$|dx/dz + 2.5x| \leq 15 \text{ mr}$$

$$|8| \leq .1$$

Particles outside that region were cut.

Two corrections were necessary because deuterons traversing the spectrometer interacted with matter in their path²⁰ (scintillators, chambers, etc.). Ionization losses caused the dipole measured deuteron momentum to be less than the actual recoil deuteron momentum, and had to be corrected for in order to center the elastic peak and improve the chromatic aberration corrections at the quadrupoles. Nuclear scattering losses caused deuterons to break up in flight reducing the number of deuterons detected. The sizes of both corrections are inversely related to t and were calculated from the amount and type of matter in the recoil deuteron path (approximately equivalent to 1g/cm^2 of carbon). The dominant contributions were from the first two trigger counters and three hodoscopes. The energy lost to ionization varied from 4 to 13 MeV, and nuclear scattering removed 1.5% to 3.% of the deuterons from the sample. Straggling, or fluctuations in the ionization losses, were comparable in size to the error in the dipole measurement of the momentum. At $t = -.6, -1.3,$ and -3.0 straggling contributed twice as much, as much, and one-third as much, respectively, to the total momentum error, as did the dipole measurement error.

The number of elastic events was extracted by fitting the $\Delta p_r/p_c$ distribution, with a Gaussian for the elastic peak, two resonance shapes for N^* 's at $M_x = 1.52$ and 1.69 GeV and a third order polynomial for the background, constrained to be zero near $M_x = m_p + m_\pi$. The exact cutoff value was set from the width of the elastic peak. Chi squares

for the fit were typically one per degree of freedom or less (due to zeros at $\Delta p_r/p_0 > 0.0$.) The difference in results with and without N^* peaks was negligible except for $-t \geq 2.5$. The fits also returned the error including background and N^* uncertainties.

The fitting routine rebinned the data in t and p_0 and added runs together to get approximately 200 elastic events per bin, where possible. It was also used to check the acceptance of the spectrometer and select data from regions with good acceptance. In this way 200 data points were obtained whose overall errors were 10% or so.

3.3 ACCEPTANCE

The spectrometer acceptance for each data point was determined using a Monte Carlo calculation. In the Monte Carlo 1000 elastic events were randomly generated at the target with distributions mimicking the actual target distributions and traced through the spectrometer. The results of this were fed to the same analysis routine that analyzed the summary tapes.

The range of generated dx/dz values was set from the desired t bin. The dy/dz range was ± 75 mr, larger than the accepted ± 52 mr to insure full coverage of the spectrometer. At $-t \geq 2.0$ in good acceptance regions the acceptance (69%) was determined by this difference. A transport relation derived from the wire orbit data transported the event to chamber 1. An ideal quad approximation was used to test that the event did not hit the magnet walls. Transport

through the dipole was accomplished by the same technique used to generate the pseudo-particles in Section 2.3.5. Multiple scattering was included at every step, including the effects of the proportional chamber sense wires. Ionization losses were lumped into the first two trigger counters and three hodoscopes. The physical dimensions of all the chambers and the dipole were considered. The trigger counters and hodoscopes were deliberately oversized so their dimensions were not considered. When passing these generated rays through the analysis routine, the same target cuts and momentum corrections were used as for the data. The other four cuts, which tested the amount of scattering an event had suffered, were removed, however, since these cut mainly long multiple scattering tails, and tails were not considered in the Monte Carlo. The Monte Carlo also did not account for possibility of the recoil deuterons undergoing nuclear scattering in the spectrometer.

Several checks were made on the Monte Carlo by comparing its results with data. Comparisons for three of the quantities calculated are shown in Figures 13, 14, and 15. The resolutions from the Monte Carlo and data were also compared and showed good agreement. Figure 16 shows another check comparing the shapes of the acceptance as determined by the data and Monte Carlo. Figure 17 shows the range of acceptance results obtained, depending upon the width of the t bin and the incident momentum.

A full acceptance distribution in t for a sample Monte Carlo run is shown in Figure 18. The width of the good acceptance region was approximately $\pm 10\%$ of the central t value. The acceptance shifted to

lower t at energies below 40 GeV as expected from kinematics. At $-t > 1.2$ this shift made it impossible to obtain data points at precisely the same t for all p_0 , given the data we had. At $-t > 1.8$ and $p_0 = 200 \text{ GeV}_c$ the t acceptance shifted to higher t creating a similar problem there. The Monte Carlo did not automatically reproduce the t dependence of the cross section. To correct for this dependence, the Monte Carlo was often run for several small t bins instead of the single large one used with the data. Then a weighted average was taken. Because the data points were usually generated from regions with good (flat) acceptance, this effect was often negligible and always $< 10\%$. The overall uncertainty in the Monte Carlo calculation is estimated to be $\pm 8\%$.

3.4 LUMINOSITY

For each p_0 bin, for each run, the luminosity was determined from the monitor stacks, particularly the thinner, better understood stack. Recoil measurement techniques with solid state detectors are described in Reference 12 and references cited therein. The deuterons were isolated via a E vs. dE/dx scatter plot (Figure 19), and were then plotted vs. dE/dx to isolate the elastic signal. dE/dx rather than E was used since a portion of the deuterons penetrated the stack. All events within 3σ of the peak were considered possible elastic candidates. The number of inelastics among the candidates (2%) was estimated from a convolution of a formula for inelastics (from previous experiments)²¹ with the width of the elastic peak. The lack of events outside kinematically

allowed regions indicates that there are no other backgrounds. The position of the elastic peak determined corrections for variations in the angle caused by motion of the beam and subsequent movement of the target to follow the beam. The angle and the average p_0 were used together in a formula from previous data to calculate $d\sigma/d\Omega$ for the elastic peak. The number of events in the peak was scaled up to correct for the monitor divide. All this put together gives the luminosity as

$$L = \frac{\text{no. of monitors (scaled)}}{d\sigma/d\Omega \cdot \Omega}$$

Most of the above used techniques and even computer programs taken directly from References 21 and 22.

The second stack was used to check the primary stack for time variations and degradation or other problems. The ratio between the number of elastic events from stacks one and two was .87, varying only a few percent from run to run. Since the answer from the second stack was not as precisely obtained as for the primary stack, this variation is not important. The ratio of the apertures of the two stacks was $.89 \pm .01$. The difference in angle contributed a factor of .94, and the multiple scattering losses in stack two contributed a factor of 1.03. the expected ratio then was $.86 \pm .01$, in good agreement with the actual value.

The absolute angle of stack one was determined most accurately via a range technique. Fortuitously, the stack thickness ($.4088\text{g/cm}^2$) corresponded almost exactly with the deuteron range at

the stack angle for $p_0=150$ GeV/c; 50% of the deuterons penetrated the stack. At $p_0=20$ GeV/c the penetration effect on the distribution was noticeable but small. The angle corresponding to these effects is 4.50 degrees from perpendicular to the beam. At 4.40 degrees the effect would not have been visible for $p_0=20$, and for $p_0=150$ there would have been less than 20% penetration. At 4.60 degrees there would be 50% penetration at $p_0=20$. Therefore, the error in the angle from this source could not exceed .1 degrees.

Uncertainties in the stack thickness were another, smaller source of error in the stack angle. Ortec, the manufacturer of the detectors, gives the detector thickness to the nearest micron and the density as 1 micron=.2325 mg/cm², implying a <1% error in all values. However, even if a 5% error is assumed this corresponds to an error of only .05 degrees. We estimate the overall error to be .07 degrees corresponding to a 6% error in the normalization. Note that this error is not subject to the usual 1σ , 2σ , 3σ , idea of Gaussian distributed errors. An error of .14 degrees is virtually impossible by both quantitative and qualitative arguments; it is more like a 4 or 5σ effect.

3.5 NORMALIZATION

Setting the normalization entailed collecting the effects of all aforementioned cuts and corrections, carefully avoiding double counting, and checking in as many ways as possible that other corrections were not necessary. First the number of elastic events

had to be scaled up for cuts made to the data but not the Monte Carlo. These included the 5% chamber inefficiency cut, the four cuts removing the large multiple scattering events (5% to 22%), and nuclear scattering losses (1.5% to 3%). Then the events were scaled up by the Monte Carloed spectrometer acceptance, MC. Here, the Monte Carlo dy/dz generation of events over only .150 radians of a possible 2π radians was folded in, plus a $\sin\theta$ term for the transformation from the spectrometer frame to the lab frame. Then, by dividing by the luminosity, L , and the t bin, Δt , the invariant cross section, $d\sigma/dt$, in millibarns/(GeV/c)² is obtained as

$$\frac{d\sigma}{dt} = \frac{\text{no. events}}{\text{corrections}} \frac{1}{MC} \frac{.150/\sin}{2\pi} \frac{1}{L} \frac{1}{\Delta t} .$$

The process was checked in several ways. Some elastic proton proton data was taken in a region where the cross section had been previously measured,^{23,24} and compared to those results as shown in Figures 20, 21, and 22. The differences are within quoted errors, showing that the overall procedure is probably correct. Also, two (very different) track reconstruction techniques gave the same results (within 2%), indicating that our understanding of those techniques is probably correct. Third, small amounts of data taken in a separate run six months before the main data, and using different momentum calculations and different analysis routines, give the same results as the final data and procedures.

One very important check that has been made is that the monitor stacks and the spectrometer see the whole/same target region. The target x and y distributions are reconstructed from spectrometer

information and show the x distribution to be approximately Gaussian with $\sigma = .5\text{cm}$, and the y distribution to fall more sharply than Gaussian with a full width full maximum of 1cm . Both distributions are centered around zero to within a σ of their widths. The Monte Carlo predicts a good acceptance region of $>\pm 2\text{cm}$ in x and y implying the spectrometer sees the whole target. The target x distribution is also known from the width of the elastic peak in the monitor stacks to have a σ of $.4$ to $.45\text{ cm}$, slightly narrower than the spectrometer obtained σ reflecting the better resolution of the monitor stacks. These tests have been done for all runs and always give good results, so we have concluded that there is no correction necessary for this effect.

The overall uncertainty in the normalization is estimated to be 10%, primarily due to uncertainty in the stack angle and collimator size. Errors in corrections to the spectrometer results add only small amounts to the overall uncertainty. $d\sigma/dt$ varied mostly as a function of t , from a high of $20\text{ microbarns}/(\text{GeV}/c)^2$ to a low of $.5\text{ nanobarns}/(\text{GeV}/c)^2$. Variations in $d\sigma/dt$ with p_0 were much smaller, always less than a factor of three over the entire p_0 range measured.

CHAPTER 4

RESULTS AND DISCUSSION

A list of cross sections obtained from the data is given in the Table of Cross Sections at the end of the chapter. Portions of these data are shown in Figures 23 and 24. In Figure 23 results of other experiments are shown for comparison. Our results are below the lower energy results of Bradamante et al.²⁵ and are above or equal to the higher energy results of Goggi et al.,²⁶ consistent with the energy dependence trends of our data. Our results however are in direct conflict with results of Allaby et al.²⁷ at 25 GeV/c; their results are nearly a factor of two higher than ours. No explanation for this has been found, but our results have been checked in many ways previously described to be sure our normalization is correct. The curves shown in the figures are the simple Glauber model predictions.

4.1 GLAUBER MODEL

The Glauber model^{3,9,28-30} assumes that nucleons in a nucleus interact with incident hadrons exactly as they do when they are not in a nucleus. The total interaction amplitude is merely a coherent sum of the individual amplitudes, and a phase shift analysis is typically the technique used to facilitate the summation over amplitudes. To simplify the calculation, the assumption is made for elastic scattering that the initial and final state wavefunctions are identical, i.e., that the nucleons of the deuteron do not move during the interaction time.

The amplitude for a deuteron interaction may be written²⁸

$$F_{fi}(\underline{q}) = \langle f | F(\underline{q}, \underline{s}) | i \rangle \quad , \quad (4.1)$$

$$F(\underline{q}, \underline{s}) = F_S(\underline{q}, \underline{s}) + F_D(\underline{q}, \underline{s})$$

$$F_S(\underline{q}, \underline{s}) = \exp(\frac{1}{2}i\underline{q} \cdot \underline{s}) f_n(\underline{q}) + \exp(-\frac{1}{2}i\underline{q} \cdot \underline{s}) f_p(\underline{q})$$

$$F_D(\underline{q}, \underline{s}) = \frac{i}{2\pi k} \int \exp(i\underline{q} \cdot \underline{s}) f_n(\underline{q}' + \underline{q}/2) f_p(-\underline{q}' + \underline{q}/2) d^2 \underline{q}' \quad ,$$

where $\underline{q}$ is the 4-momentum transfer, $\underline{s}$ are the components of the coordinates of the nucleons in the deuteron in a plane perpendicular to the direction of the incident proton, $|f\rangle$ and $|i\rangle$ are final and initial states, and $F(\underline{q}, \underline{s})$ is the result of the phase shift analysis; F_S (F_D) is the single (double) scattering part. Setting $|f\rangle = |i\rangle = \phi(r)$, the ground state deuteron wave function, simplifies

$F_{ii}(\underline{q})$ to

$$F_{ii}(\underline{q}) = S(\underline{q}/2)f_n(\underline{q}) + S(-\underline{q}/2)f_p(\underline{q}) + (i/2\pi k) \int S(\underline{q}')f_n(\underline{q}/2+\underline{q}')f_p(\underline{q}/2-\underline{q}') d^{(2)}\underline{q}' \quad (4.2)$$

where f_n , f_p are the neutron and proton elastic scattering amplitudes, and, approximating $\underline{q} \cdot \underline{s} = \underline{q} \cdot \underline{r}$ (following Glauber) since $\underline{s}$ is nearly parallel to the plane containing the momentum transfers $\underline{q}$,

$$S(\underline{q}) = \int \exp(i\underline{q} \cdot \underline{r}) |\phi(\underline{r})|^2 d^3\underline{r} \quad (4.3)$$

is the deuteron form factor, and contains all the information on the deuteron state.

The first two terms are the single scattering terms, and are negligible at $-t > .6(\text{GeV}/c)^2$. The last term is the double scattering term important in the t region studied here. $S(\underline{q})$ has been measured by studying low t pd elastic scattering¹² and is given by

$$S(\underline{q}) = .34e^{141.5t} + .58e^{26.1t} + .08e^{15.5t} \quad (4.4)$$

Quadrupole contributions to $S(\underline{q})$ are expected to be small³¹ and have been neglected. The steep t dependence of $S(\underline{q})$ facilitates expanding F_n and f_p in the integral of Equation (4.2) to give

$$\frac{d\sigma}{dt} = (I_G^2/\pi) \frac{d\sigma}{dt} pp(t/4) \frac{d\sigma}{dt} np(t/4) \quad , \quad (4.5)$$

where the Glauber integral, I_G , is

$$I_G = 1/2 \int S(t) \exp(b_{pp}t/2) \exp(b_{np}t/2) dt \quad (4.6)$$

and b_{pp} and b_{np} are the elastic pp and np slope parameters.

Equation (4.2) with (4.5) shows that the Glauber picture of double scattering is equivalent to the situation in Fig. 25b.³² The amplitude is the probability $S(q)$ of having the initial state shown, times the product of the amplitudes for the two interactions, all integrated over all possible initial states. This in turn is related to the case where the initial deuteron has zero internal momentum. For the single scattering case (Fig. 25a) the amplitude is equivalent to simply the product of the probability of the spectator nucleon initially having the proper final momentum times the amplitude for the proper momentum being transferred to the hit nucleon.

We have calculated I_G using Equations (4.4) in (4.6) and also using form factors from Reference 28 and find a value of $I_G = .028 \text{mb}^{-1} \pm 3\%$ over our range of energies and t . I_G has also been measured¹² and gives the same value, but with larger errors, for our energy region.

For f_n and f_p we use the fairly standard approximation³³ $f_n = f_p$ and use a recent parameterization³⁴ for $(d\sigma/dt)_{pp}$

$$\frac{d\sigma}{dt} = 74.9 \exp(10.8t + 1.9t^2) (s/s_0)^{2(.98+2t)-2} \quad (4.7)$$

where $s_0 = 188(\text{GeV}/c)^2$. This parameterization, based on Regge phenomenology, and fit to the results of the high precision experiment of the Fermilab Single Arm Spectrometer Group, agrees well with results of other experiments^{23,24} and with our own pp data. Another parameterization considered was Lorentz contracted geometric scaling;³⁵ however, this parameterization, while accounting for gross features of the cross section over a wider range of energies than the chosen parameterization, is accurate in our energy range only to within $\pm 15\%$. Since the nucleon-nucleon cross section enters quadratically into the pd elastic scattering prediction, this would contribute a 30% error into the Glauber model. With the chosen parameterization the estimated uncertainty in the prediction is $\pm 10\%$.

It is clear from Figure 24 that, except at our highest t and energy values, the results are below or equal to the Glauber predictions. The results of Bradamante at 10 GeV/c are also below Glauber. At ISR energies²⁶ the results are consistent with Glauber within large errors and over a somewhat restricted t range. We have attempted to explain the discrepancy between the data and the model both by considering the validity of approximations made in the original derivation and by including in the model physical effects not considered in the simple picture.

4.2 RECOIL EFFECTS

One approximation made in the derivation is that the incident particle does not change momentum after interacting with the first nucleon in the deuteron, i.e., the recoil of the incident particle is ignored. Faldt³⁶ has studied this attempting to explain Bradamante's data and has determined that including recoil effects reduces the Glauber prediction for double scattering by an approximate multiplicative factor $1/(1+|t|/16m_p^2)$. At $-t=1(\text{GeV}/c)^2$ this reduces the Glauber prediction 7%, hardly sufficient to explain our data or even Bradamante's data. Also, the t dependence of the factor is not consistent with the data. Moreover, as often occurs (although coincidentally) with such factors, this factor is almost exactly cancelled, including its t dependence, by another factor obtained when eliminating another approximation as shown later. We conclude therefore, that recoil effects do not explain our data or Bradamante's data.

4.3 INELASTIC EFFECTS

The physical effect most often included in the Glauber model to modify its predictions is the contribution of inelastic intermediate states (IIS). Contributions from IIS to the Glauber model are expected to be reasonably in phase with the double scattering elastic amplitude,^{26,37} and so, increase the cross section. Also, since the relevant inelastic cross sections ($N^*(1520, 1690, 2190)$ production and non-resonant background) have a slower falloff in t than the elastic

cross sections, as shown from both previous measurements³⁸⁻⁴⁰ and from trends in our pp and pd distributions; their relative contribution to the cross sections should increase with increasing $-t$.

The energy dependence of the contribution is related to several effects, one being the energy dependence of the cross sections as a function of M_x^2 relative to the elastic cross sections and another being that a fixed M_x^2 corresponds to a smaller $1-x$ at higher incident energy. The latter effect is important since it is expected that IIS contributions are limited to states produced with $1-x$ below some small fixed value, so at higher incident energies more states lie below the limiting value of $1-x$. Overall, the effects of IIS are expected to become more important with increasing incident energy.

Goggi et al., explain their results using IIS by adding a Regge parameterization for that contribution to the Glauber model. The resulting predictions are in excellent agreement with the data. This is reasonable, since their results in the double scattering region are above the simple Glauber predictions, and since the incident energy for their data is extremely high. Our results and the results of Bradamante, however, are below the Glauber predictions implying that IIS contributions would have to be approximately 90 degrees out of phase with the elastic amplitudes (the amplitudes are squared in double scattering) to explain the data. Also, the discrepancy has a t and p_0 dependence exactly opposite to the expected dependence of IIS contributions. Hence, it appears that in the region covered by our data another effect is more important than IIS, although IIS may

contribute significantly to some parts of the data.

4.4 MODIFIED GLAUBER MODEL

More realistic versions of the Glauber model should account for the space-time development⁴ of the intermediate system as well as include intermediate states with inelastic components. Space-time development effects assume that strong interactions require a finite amount of time, τ_0 , to occur, a time comparable to the time available, τ , i.e., the time to cross the deuteron. This implies that incident protons may not be able to interact twice in the available time, naturally predicting a reduction in the cross section in the double scattering region. Any inelastic components would then be added in phase and perhaps explain the higher cross sections found for the highest t and incident energies measured. We introduce the time dependence phenomenologically by redefining the double scattering portion of the Glauber scattering amplitude as

$$F_D^{\text{new}}(\underline{q}, \underline{s}) = F_D^{\text{old}}(\underline{q}, \underline{s}) [(1-A) + A(1 - \exp(-\tau/\tau_0))] , \quad (4.8)$$

allowing for a portion of the interaction with no transient effect and a portion with an exponential relation to the time; the choice of an exponential form is made by analogy to other, well understood, processes. Equation (4.8) can be written more simply as

$$F_D^{\text{new}}(\underline{q}, \underline{s}) = F_D^{\text{old}}(\underline{q}, \underline{s}) \times [1 - A \exp(-\tau/\tau_0)] . \quad (4.9)$$

Now, when performing the integral over d^3r , while the first term still factors out a term $S(q')$, the r dependence of τ destroys this for the second term. The time between interactions is proportional to the distance between the nucleons in the deuteron so, parameterizing $\tau/\tau_0 = gr$, where $g = g(q, p_0)$, for the second term the integral becomes

$$S_2(q) = \int \exp(i\mathbf{q} \cdot \mathbf{s}) |\phi(r)|^2 e^{-gr} d^3r . \quad (4.10)$$

Note that the approximation $\mathbf{q} \cdot \mathbf{s} = \mathbf{q} \cdot \mathbf{r}$ is not made here, in contrast to Equation (4.3). A simple wavefunction⁴¹ is used

$$|\phi(r)|^2 = (4R)^{-3/2} \exp(-r^2/4R^2) \quad (4.11)$$

$$R^2 = 33 \text{ GeV}^{-2} .$$

The details of the integral are shown in the Appendix B, and the result is

$$S_2(q) = S(q \sin \theta) (.44 + .56 \exp(-2Rg)) , \quad (4.12)$$

and the $S(q \sin \theta)$ part is exact while the form of the remainder is approximate. The presence of the $\sin$ factor in Equation (4.12) results from not making the approximation $\mathbf{q} \cdot \mathbf{s} = \mathbf{q} \cdot \mathbf{r}$. Hence, the factor should be present when integrating both terms in Equation (4.9), not just the second term, and also in Equation (4.3).

When integrating over d^2q , the $\sin\theta$ factor increases the value of the Glauber integral by a factor

$$I_G^{\text{new}} = I_G \times (1/(1-|t|/16m_p^2)) \quad (4.13)$$

Recoil effects³⁶ described earlier, however, almost exactly cancel this effect, leaving $I_G = .028 \text{mb}^{-1}$ as originally obtained. The independence of g and q' so that the exponential term is preserved on integration over q' is discussed in the Appendix B.

At this point the new Glauber prediction is expressed as

$$\frac{d\sigma}{dt} = \frac{d\sigma}{dt} \text{ Glauber} [1 - A(.44 + .56e^{-2Rg})]^2 \quad (4.14)$$

To further parameterize $g(q, p_0)$ requires consideration of the effects determining τ and τ_0 . τ is affected by time dilation, so $\tau = rm_p/p$. Equations (4.9) through (4.12) effectively average over r giving $\langle \tau \rangle = \langle r \rangle m_p/p_0 = 2Rm_p/p_0$. $\langle \tau \rangle = (.70/p_0) \times 10^{-23} \text{sec}$. By analogy to other systems τ_0 is expected to be inversely related to t , so τ_0 is parameterized as $b_1/(-t)^c$. For example, the rate for dipole radiation of photons goes as the energy of the photon cubed. Thus, if the scattering proceeded by the exchange of massless vector particles, we would expect $c=3/2$; however, any reasonably small positive value of c would be consistent with the model. The order of magnitude for b_1 can be estimated from N^* decays, assuming the decay mechanism there is related. N^* (1470, 1520) have widths of 100 MeV

corresponding to lifetimes of 1.4×10^{-24} sec. The pion emitted has an energy of .5Gev, and, since the transfer per interaction in double scattering is $t/4$, these two effects cancel giving $b_1 = 1.4 \times 10^{-24}$. Combining the two effects gives an estimate for $\tau/\tau_0 = 5t^c/p_0$. Putting the parameterization into Equation (4.14) gives

$$\frac{d\sigma}{dt} = \frac{d\sigma}{dt} \text{ Glauber} [1 - A(.44 + .56e^{-b(-t)^c/p_0})]^2 \quad (4.15)$$

where A, b, and c are fit from the data, and b is expected to be about 5.

The fit was made after eliminating half the $t = -.6$ and $-.8$ data points to even out the distribution of data points, and eliminating data with $-t \geq 2.3$ to eliminate points which require other effects to explain them since they already lie above the Glauber prediction.

With I_G fixed at $.028 \text{ mb}^{-1}$ the results are

Chisq/DF = 1.16 for 106 data points

$A = .22 \pm .04$

$b = 6.0 \pm 4.0$

$c = 4.2 \pm 1.3$,

and the errors in b and c are due to uncertainties of the fit while the error in A is due to uncertainties in normalization. Allowing

I_G to be fit also, gives the results:

Chisq/DF = 1.15 for 106 data points

$I_G = .031 \pm .002$

$A = .29 \pm .07$

$$b=5.1 \pm 3.$$

$$c=3.3 \pm 1.$$

b agrees surprising well with the estimated value of 5, c is not unreasonable although it does not match the photon exchange analogy of $c=3/2$, and the fit value of the Glauber integral is in reasonable agreement with the calculated value. The fit is shown graphically in Figure 26.

Adding the points with $-t \geq 2.3$ causes a rapid drop in b and an increase in c and in the Chisq/DF. Also the contributions of these points to the Chisq/DF are abnormally large, being 1.5/DF for 3 points at $t=-2.3$, and 7/DF (5 points) and 10/DF (4 points) at $t=-2.5$ and -3.0 , respectively. Since these points lie above the predictions of the unmodified Glauber model this seems to imply that inelastic states (or other effects) are becoming important and does not imply that the model is wrong. Further consideration of this requires a more detailed model.

Other variations of this fit which have been tried are:

replacing $(-t)^c$ with $(-t+m^2)^c$

$$\frac{d\sigma}{dt} = \frac{d\sigma}{dt} \text{ Glauber}^x [(1-A)+A(1-\exp(-\tau/\tau_0))] \quad , \quad (4.16)$$

and combinations of the two.

The first variation is suggested if one believes that the exponential terms may be related to hadron form factors (which have recently also been parameterized as exponentials)⁴² and gives a Chisq/DF=1.14 for $0 < m < 1\text{GeV}$ and increases the Chisq/DF outside that region. $m=4$ is approximately the upper limit of acceptable values with

Chisq/DF=1.31. While this does not rule out the possibility that this relates to hadron form factors, neither is it compelling evidence of any such relation or of the need for a $-t+m^2$ type parameterization. The second variation assumes incoherence between the time dependent and time independent terms, and gives Chisq/DF=1.14, $A=.39 \pm .07$, and b and c are the same as for the original fit. Combinations of the two give the same results.

Another fit has been tried which has no theoretical background and is only an attempt to check our estimate of our errors by testing how small a Chisq/DF can be obtained from a fit to the data. If a fit with Chisq/DF<.8 or so can be obtained from a relatively simple parameterization then we have probably overestimated our errors. The parameterization tried was a combination of the above two parameterizations times another factor, F, defined as

$$F=I (1+dp_0+et+fp_0^2+gt^2+hp_0t)$$

where I,d,e,f,g, and h are free to vary. The Chisq for this fit was 1.0/DF indicating that our errors are probably approximately correct.

4.5 CONSTITUENT QUARK MODEL

The existence of both a time dependent and time independent term in the fit suggest that nucleons are composed of pieces, and, therefore, that the simple space-time development arguments made for the fit are related to the constituent quark model.⁴³ This model assumes the nucleons are made of n (typically 3) independent quark clusters⁴⁴ composed each of a valence quark and a share of the gluons

and sea quarks. Typical nucleon nucleon interactions involve one cluster from each nucleon, and after interacting, independent of the final state, the clusters are excited and require a finite time to be able to interact again; or, alternatively, the reaction requires a finite amount of time to occur and a nucleon does not interact with two nucleons simultaneously. In the picture, in deuteron interactions, viewing the deuteron as two groups of three quarks rather than a bound state of six quarks;⁴⁵ after the incident nucleon interacts with one nucleon, only $n-1$ quark clusters are available to interact fully with the second nucleon while the one cluster which has already interacted is not fully available; it contributes to the time dependent term. This implies that A is given by $A=1/n$; $A=.29\pm.07$ from the fit in which the Glauber integral was allowed to vary, which matches well with $n=3$ or 4 , and is not inconsistent with $n=5$.

The time dependence of the interaction is not well specified in this model, however, it does require the assumption that the lifetime of the excited cluster is inversely related to t which is the same assumption used by the space-time arguments.

This model has the additional feature that it shows how to add inelastic state contributions to the model. In order for an inelastic state to contribute, the same quark cluster must interact with both nucleons, once to excite the cluster, and then the second time because, since it is the only excited part of the incident nucleon, an inelastic state is involved only if that cluster partakes in the second interaction. Hence, inelastic amplitudes can only be

added in to the time dependent term of the fit, implying that inelastic terms contribute less than otherwise expected at high energies, except at correspondingly high t . This type of argument may help explain our $-t \geq 2.3$ data.

4.6 SUMMARY

We have measured differential proton deuteron cross sections by performing a recoil experiment at the Internal Target area at Fermilab, using a large angle single arm recoil spectrometer to detect the deuterons recoiling from a gas jet target and identify the elastic process. Using simultaneous measurements from a solid state spectrometer at low t to set the normalization, we find that our results are significantly different from the Glauber model, generally lower, and with a slower fall off in t . The discrepancy between our data and the Glauber picture has led us to conclude that the basic Glauber model is incomplete. Some effect, probably the finite time required for strong interactions, reduces the elastic cross sections. The cross sections drop with increasing energy more rapidly than predicted by Glauber, and above some medium energy dependent on t , become constant.

We find that the data can be understood by assuming that after a nucleon interacts once it contains both time dependent and time independent amplitudes for a second interaction, implying the nucleon is composed of pieces. We measure a characteristic strong interaction time, τ_0 , and find a t dependence (here t is for a single

scatter);

$$\tau_0 = 1.4 \pm .8 \times 10^{-24} / (4t)^{3.5 \pm 1} .$$

Comparison with the quark model yields a prediction of 3-4 quarks per nucleon consistent with the standard three.

Inelastic intermediate states apparently do not contribute significantly, except at very high t and p_0 . How they contribute is unresolved, although our data (with $-t > 2.2(\text{GeV}/c)^2$ cross sections greater than the simple Glauber predictions and with $p_0 = 210 \text{ GeV}/c$ results greater than the corrected Glauber predictions) and the higher energy ISR results indicate that they should be in phase with the elastic amplitudes to increase the cross sections. Also, the constituent quark model gives some indication on how to relate the inelastics to the space-time effects, although it will probably require more data than we have available to investigate this.

TABLE OF CROSS SECTIONS

$t \pm \Delta t$ (GeV/c) ²	$p_0 \pm \Delta p_0$ GeV/c	$d\sigma/dt \pm \Delta d\sigma/dt$ microbarns / GeV ²
.60 .02	22.5 2.5	.180 x 10 ² .21 x 10 ¹
.60	27.5	.175 x 10 ² .20 x 10 ¹
.60	32.5	.159 x 10 ² .18 x 10 ¹
.60	37.5	.176 x 10 ² .20 x 10 ¹
.60	47.5	.161 x 10 ² .19 x 10 ¹
.60	52.5	.139 x 10 ² .17 x 10 ¹
.60	57.5	.147 x 10 ² .18 x 10 ¹
.60	62.5	.124 x 10 ² .15 x 10 ¹
.60	67.5	.152 x 10 ² .19 x 10 ¹
.60	72.5	.164 x 10 ² .20 x 10 ¹
.60	77.5	.136 x 10 ² .18 x 10 ¹
.60	82.5	.178 x 10 ² .22 x 10 ¹
.60	87.5	.157 x 10 ² .20 x 10 ¹
.60	92.5	.142 x 10 ² .18 x 10 ¹
.60	97.5	.154 x 10 ² .20 x 10 ¹
.60	102.5	.128 x 10 ² .16 x 10 ¹
.60	107.5	.169 x 10 ² .21 x 10 ¹
.60	115.0 5.0	.155 x 10 ² .17 x 10 ¹
.60	125.0	.131 x 10 ² .15 x 10 ¹
.60	197.5 7.5	.114 x 10 ² .12 x 10 ¹
.60	212.5	.111 x 10 ² .13 x 10 ¹

$t \pm \Delta t$ (GeV/c) ²	$p_0 \pm \Delta p_0$ GeV/c	$d\sigma/dt \pm \Delta d\sigma/dt$ microbarns / GeV ²
.80 .025	22.5 2.5	.768 x 10 ¹ .85 x 10 ⁰
.80	27.5	.791 x 10 ¹ .86 x 10 ⁰
.80	32.5	.717 x 10 ¹ .79 x 10 ⁰
.80	37.5	.742 x 10 ¹ .83 x 10 ⁰
.80	42.5	.924 x 10 ¹ .10 x 10 ¹
.80	47.5	.636 x 10 ¹ .73 x 10 ⁰
.80	52.5	.623 x 10 ¹ .72 x 10 ⁰
.80	57.5	.498 x 10 ¹ .60 x 10 ⁰
.80	62.5	.530 x 10 ¹ .63 x 10 ⁰
.80	67.5	.599 x 10 ¹ .70 x 10 ⁰
.80	72.5	.495 x 10 ¹ .64 x 10 ⁰
.80	77.5	.519 x 10 ¹ .64 x 10 ⁰
.80	82.5	.652 x 10 ¹ .81 x 10 ⁰
.80	87.5	.533 x 10 ¹ .66 x 10 ⁰
.80	92.5	.595 x 10 ¹ .73 x 10 ⁰
.80	97.5	.663 x 10 ¹ .81 x 10 ⁰
.80	102.5	.521 x 10 ¹ .66 x 10 ⁰
.80	107.5	.540 x 10 ¹ .70 x 10 ⁰
.80	115.0 5.0	.589 x 10 ¹ .64 x 10 ⁰
.80	125.0	.531 x 10 ¹ .57 x 10 ⁰
.80	197.5 7.5	.526 x 10 ¹ .55 x 10 ⁰
.80	212.5	.558 x 10 ¹ .63 x 10 ⁰

$t \pm \Delta t$ (GeV/c) ²	$p_0 \pm \Delta p_0$ GeV/c	$d\sigma/dt \pm \Delta d\sigma/dt$ microbarns / GeV ²
1.00 .03	25.0 5.0	.308 x 10 ¹ .35 x 10 ⁰
1.00	35.0	.280 x 10 ¹ .32 x 10 ⁰
1.00	45.0	.268 x 10 ¹ .33 x 10 ⁰
1.00 .06	55.0	.219 x 10 ¹ .24 x 10 ⁰
1.00	65.0	.242 x 10 ¹ .27 x 10 ⁰
1.00	72.5 2.5	.234 x 10 ¹ .30 x 10 ⁰
1.00	77.5	.200 x 10 ¹ .25 x 10 ⁰
1.00	82.5	.253 x 10 ¹ .31 x 10 ⁰
1.00	87.5	.211 x 10 ¹ .25 x 10 ⁰
1.00	92.5	.205 x 10 ¹ .26 x 10 ⁰
1.00	97.5	.170 x 10 ¹ .21 x 10 ⁰
1.00	102.5	.156 x 10 ¹ .20 x 10 ⁰
1.00	107.5	.233 x 10 ¹ .29 x 10 ⁰
1.00	115.0 5.0	.194 x 10 ¹ .21 x 10 ⁰
1.00	125.0	.164 x 10 ¹ .18 x 10 ⁰
1.00	197.5 7.5	.165 x 10 ¹ .18 x 10 ⁰
1.00	212.5	.184 x 10 ¹ .20 x 10 ⁰
1.12 .04	22.5 2.5	.272 x 10 ¹ .29 x 10 ⁰
1.12	27.5	.178 x 10 ¹ .20 x 10 ⁰
1.20	22.5	.198 x 10 ¹ .22 x 10 ⁰
1.20	27.5	.143 x 10 ¹ .16 x 10 ⁰
1.20	32.5	.114 x 10 ¹ .14 x 10 ⁰
1.20	37.5	.884 x 10 ⁰ .11 x 10 ⁰

$t \pm \Delta t$ (GeV/c) ²	$p_0 \pm \Delta p_0$ GeV/c	$d\sigma/dt \pm \Delta d\sigma/dt$ microbarns / GeV ²
1.20 .04	42.5 2.5	.115 x 10 ¹ .15 x 10 ⁰
1.20	50.0 5.0	.817 x 10 ⁰ .91 x 10 ⁻¹
1.20	60.0	.884 x 10 ⁰ .10 x 10 ⁰
1.20	70.0	.693 x 10 ⁰ .84 x 10 ⁻¹
1.20	75.0	.817 x 10 ⁰ .98 x 10 ⁻¹
1.20	85.0	.760 x 10 ⁰ .88 x 10 ⁻¹
1.20	95.0	.760 x 10 ⁰ .88 x 10 ⁻¹
1.20	105.0	.798 x 10 ⁰ .97 x 10 ⁻¹
1.20	115.0	.703 x 10 ⁰ .84 x 10 ⁻¹
1.20	125.0	.684 x 10 ⁰ .83 x 10 ⁻¹
1.20	197.5 7.5	.561 x 10 ⁰ .69 x 10 ⁻¹
1.20	212.5	.770 x 10 ⁰ .95 x 10 ⁻¹
1.22	27.5 2.5	.126 x 10 ¹ .10 x 10 ⁰
1.22	32.5	.126 x 10 ¹ .10 x 10 ⁰
1.31	45.0 5.0	.57 x 10 ⁰ .6 x 10 ⁻¹
1.31	55.0	.56 x 10 ⁰ .6 x 10 ⁻¹
1.31	65.0	.46 x 10 ⁰ .5 x 10 ⁻¹
1.31	75.0	.56 x 10 ⁰ .6 x 10 ⁻¹
1.31	95.0	.41 x 10 ⁰ .4 x 10 ⁻¹
1.31	105.0	.48 x 10 ⁰ .4 x 10 ⁻¹
1.31	115.0	.42 x 10 ⁰ .4 x 10 ⁻¹
1.31	125.0	.39 x 10 ⁰ .4 x 10 ⁻¹
1.31	135.0	.44 x 10 ⁰ .4 x 10 ⁻¹
1.31	210.0	.34 x 10 ⁰ .3 x 10 ⁻¹

$t \pm \Delta t$ (GeV/c) ²	$p_0 \pm \Delta p_0$ GeV/c	$d\sigma/dt \pm \Delta d\sigma/dt$ microbarns / GeV ²
1.27 .045	22.5 2.5	.109 x 10 ¹ .12 x 10 ⁰
1.27	27.5	.104 x 10 ¹ .12 x 10 ⁰
1.36	22.5	.828 x 10 ⁰ .10 x 10 ⁰
1.36	27.5	.688 x 10 ⁰ .84 x 10 ⁻¹
1.40	32.5	.437 x 10 ⁰ .55 x 10 ⁻¹
1.40	37.5	.409 x 10 ⁰ .53 x 10 ⁻¹
1.40 .09	42.5	.484 x 10 ⁰ .58 x 10 ⁻¹
1.40	47.5	.372 x 10 ⁰ .44 x 10 ⁻¹
1.40	52.5	.372 x 10 ⁰ .44 x 10 ⁻¹
1.40	57.5	.326 x 10 ⁰ .41 x 10 ⁻¹
1.40	62.5	.326 x 10 ⁰ .41 x 10 ⁻¹
1.40	67.5	.381 x 10 ⁰ .52 x 10 ⁻¹
1.40	72.5	.474 x 10 ⁰ .73 x 10 ⁻¹
1.40	75.0 5.0	.298 x 10 ⁰ .39 x 10 ⁻¹
1.40	85.0	.298 x 10 ⁰ .39 x 10 ⁻¹
1.40	95.0	.307 x 10 ⁰ .40 x 10 ⁻¹
1.40	105.0	.344 x 10 ⁰ .42 x 10 ⁻¹
1.40	120.0 10.	.260 x 10 ⁰ .30 x 10 ⁻¹
1.40	205.0 15.	.260 x 10 ⁰ .30 x 10 ⁻¹

$t \pm \Delta t$ (GeV/c) ²	$p_0 \pm \Delta p_0$ GeV/c	$d\sigma/dt \pm \Delta d\sigma/dt$ microbarns / GeV ²
1.45 .05	25.0 10.	.420 x 10 ⁰ .48 x 10 ⁻¹
1.55	25.0	.313 x 10 ⁰ .34 x 10 ⁻¹
1.55	35.0	.215 x 10 ⁰ .27 x 10 ⁻¹
1.60 .10	45.0	.172 x 10 ⁰ .20 x 10 ⁻¹
1.60	55.0	.154 x 10 ⁰ .18 x 10 ⁻¹
1.60	65.0	.176 x 10 ⁰ .21 x 10 ⁻¹
1.60	80.0 10.	.141 x 10 ⁰ .18 x 10 ⁻¹
1.60	100.0	.126 x 10 ⁰ .17 x 10 ⁻¹
1.60	120.0	.976 x 10 ⁻¹ .14 x 10 ⁻¹
1.60 .075	205.0 15.	.118 x 10 ⁰ .18 x 10 ⁻¹
1.78 .08	25.0 5.0	.134 x 10 ⁰ .15 x 10 ⁻¹
1.78	35.0	.998 x 10 ⁻¹ .12 x 10 ⁻¹
1.80	45.0	.855 x 10 ⁻¹ .11 x 10 ⁻¹
1.80 .11	55.0	.542 x 10 ⁻¹ .68 x 10 ⁻²
1.83 .08	70.0 10.	.606 x 10 ⁻¹ .69 x 10 ⁻²
1.80 .11	80.0	.624 x 10 ⁻¹ .78 x 10 ⁻²
1.80	100.0	.532 x 10 ⁻¹ .73 x 10 ⁻²
1.80	120.0	.492 x 10 ⁻¹ .72 x 10 ⁻²
1.80	205.0 15.	.484 x 10 ⁻¹ .71 x 10 ⁻²

$t \pm \Delta t$ (GeV/c) ²	$p_0 \pm \Delta p_0$ GeV/c	$d\sigma/dt \pm \Delta d\sigma/dt$ microbarns / GeV ²
1.94 .065	25.0 5.0	.717 x 10 ⁻¹ .83 x 10 ⁻²
1.97 .10	35.0	.465 x 10 ⁻¹ .54 x 10 ⁻²
2.00 .13	45.0	.414 x 10 ⁻¹ .44 x 10 ⁻²
2.00	60.0 10.	.293 x 10 ⁻¹ .38 x 10 ⁻²
2.00	85.0 15.	.236 x 10 ⁻¹ .34 x 10 ⁻²
2.00	115.0	.294 x 10 ⁻¹ .69 x 10 ⁻²
2.00	205.0	.185 x 10 ⁻¹ .41 x 10 ⁻¹
2.06 .14	25.0 5.0	.340 x 10 ⁻¹ .39 x 10 ⁻²
2.16 .11	40.0 10.	.215 x 10 ⁻¹ .26 x 10 ⁻²
2.20 .14	60.0	.122 x 10 ⁻¹ .20 x 10 ⁻²
2.30	80.0	.848 x 10 ⁻² .97 x 10 ⁻³
2.30	110.0 20.	.697 x 10 ⁻² .82 x 10 ⁻³
2.30	205.0 15.	.657 x 10 ⁻² .17 x 10 ⁻²
2.38 .12	30.0 10.	.119 x 10 ⁻¹ .14 x 10 ⁻²
2.58 .16	55.0 15.	.515 x 10 ⁻² .67 x 10 ⁻³
2.50	80.0 10.	.475 x 10 ⁻² .71 x 10 ⁻³
2.50	110.0 20.	.374 x 10 ⁻² .58 x 10 ⁻³
2.50	205.0 15.	.414 x 10 ⁻² .11 x 10 ⁻²
2.87 .14	30.0 10.	.192 x 10 ⁻² .32 x 10 ⁻³
3.00 .18	60.0 20.	.111 x 10 ⁻² .17 x 10 ⁻³
3.00	100.0 30.	.808 x 10 ⁻³ .12 x 10 ⁻³
3.00	205.0 15.	.606 x 10 ⁻³ .21 x 10 ⁻³

INTERACTION MECHANISMS

pd $\rightarrow$ pd
Single
Scattering

(a) Dominant at $-t \leq .5 (\text{GeV}/c)^2$

pd $\rightarrow$ pd
Double
Scattering

(b) Dominant at $-t \geq .5 (\text{GeV}/c)^2$ Low S

pd $\rightarrow$ pd
Double
Scattering

(c) Increasing Contribution with
Increasing S

pd $\rightarrow$ N^{*}d
Double
Scattering

(d) N^{*} Production to Measure N^{*} N
Elastic Amplitude

Figure 1

Figure 2

4.2 RECOIL EFFECTS

One approximation made in the derivation is that the incident particle does not change momentum after interacting with the first nucleon in the deuteron, i.e., the recoil of the incident particle is ignored. Faldt³⁶ has studied this attempting to explain Bradamante's data and has determined that including recoil effects reduces the Glauber prediction for double scattering by an approximate multiplicative factor $1/(1+|t|/16m_p^2)$. At $-t=1(\text{GeV}/c)^2$ this reduces the Glauber prediction 7%, hardly sufficient to explain our data or even Bradamante's data. Also, the t dependence of the factor is not consistent with the data. Moreover, as often occurs (although coincidentally) with such factors, this factor is almost exactly cancelled, including its t dependence, by another factor obtained when eliminating another approximation as shown later. We conclude therefore, that recoil effects do not explain our data or Bradamante's data.

4.3 INELASTIC EFFECTS

The physical effect most often included in the Glauber model to modify its predictions is the contribution of inelastic intermediate states (IIS). Contributions from IIS to the Glauber model are expected to be reasonably in phase with the double scattering elastic amplitude,^{26,37} and so, increase the cross section. Also, since the relevant inelastic cross sections ($N^*(1520, 1690, 2190)$ production and non-resonant background) have a slower falloff in t than the elastic

cross sections, as shown from both previous measurements³⁸⁻⁴⁰ and from trends in our pp and pd distributions; their relative contribution to the cross sections should increase with increasing $-t$.

The energy dependence of the contribution is related to several effects, one being the energy dependence of the cross sections as a function of M_X^2 relative to the elastic cross sections and another being that a fixed M_X^2 corresponds to a smaller $1-x$ at higher incident energy. The latter effect is important since it is expected that IIS contributions are limited to states produced with $1-x$ below some small fixed value, so at higher incident energies more states lie below the limiting value of $1-x$. Overall, the effects of IIS are expected to become more important with increasing incident energy.

Goggi et al., explain their results using IIS by adding a Regge parameterization for that contribution to the Glauber model. The resulting predictions are in excellent agreement with the data. This is reasonable, since their results in the double scattering region are above the simple Glauber predictions, and since the incident energy for their data is extremely high. Our results and the results of Bradamante, however, are below the Glauber predictions implying that IIS contributions would have to be approximately 90 degrees out of phase with the elastic amplitudes (the amplitudes are squared in double scattering) to explain the data. Also, the discrepancy has a t and p_0 dependence exactly opposite to the expected dependence of IIS contributions. Hence, it appears that in the region covered by our data another effect is more important than IIS, although IIS may

contribute significantly to some parts of the data.

4.4 MODIFIED GLAUBER MODEL

More realistic versions of the Glauber model should account for the space-time development⁴ of the intermediate system as well as include intermediate states with inelastic components. Space-time development effects assume that strong interactions require a finite amount of time, τ_0 , to occur, a time comparable to the time available, τ , i.e., the time to cross the deuteron. This implies that incident protons may not be able to interact twice in the available time, naturally predicting a reduction in the cross section in the double scattering region. Any inelastic components would then be added in phase and perhaps explain the higher cross sections found for the highest t and incident energies measured. We introduce the time dependence phenomenologically by redefining the double scattering portion of the Glauber scattering amplitude as

$$F_D^{\text{new}}(\underline{q}, \underline{s}) = F_D^{\text{old}}(\underline{q}, \underline{s}) [(1-A) + A(1 - \exp(-\tau/\tau_0))] , \quad (4.8)$$

allowing for a portion of the interaction with no transient effect and a portion with an exponential relation to the time; the choice of an exponential form is made by analogy to other, well understood, processes. Equation (4.8) can be written more simply as

$$F_D^{\text{new}}(\underline{q}, \underline{s}) = F_D^{\text{old}}(\underline{q}, \underline{s}) \times [1 - A \exp(-\tau/\tau_0)] . \quad (4.9)$$

Now, when performing the integral over d^3r , while the first term still factors out a term $S(q')$, the r dependence of τ destroys this for the second term. The time between interactions is proportional to the distance between the nucleons in the deuteron so, parameterizing $\tau/\tau_0 = gr$, where $g = g(q, p_0)$, for the second term the integral becomes

$$S_2(q) = \int \exp(i\mathbf{q} \cdot \mathbf{s}) |\phi(r)|^2 e^{-gr} d^3r \quad (4.10)$$

Note that the approximation $\mathbf{q} \cdot \mathbf{s} = \mathbf{q} \cdot \mathbf{r}$ is not made here, in contrast to Equation (4.3). A simple wavefunction⁴¹ is used

$$|\phi(r)|^2 = (4R)^{-3/2} \exp(-r^2/4R^2) \quad (4.11)$$

$$R^2 = 33 \text{ GeV}^{-2}$$

The details of the integral are shown in the Appendix B, and the result is

$$S_2(q) = S(q \sin \theta) (.44 + .56 \exp(-2Rg)) \quad , \quad (4.12)$$

and the $S(q \sin \theta)$ part is exact while the form of the remainder is approximate. The presence of the $\sin$ factor in Equation (4.12) results from not making the approximation $\mathbf{q} \cdot \mathbf{s} = \mathbf{q} \cdot \mathbf{r}$. Hence, the factor should be present when integrating both terms in Equation (4.9), not just the second term, and also in Equation (4.3).

When integrating over d^2q , the $\sin\theta$ factor increases the value of the Glauber integral by a factor

$$I_G^{\text{new}} = I_G \times (1/(1-|t|/16m_p^2)) \quad . \quad (4.13)$$

Recoil effects³⁶ described earlier, however, almost exactly cancel this effect, leaving $I_G = .028 \text{mb}^{-1}$ as originally obtained. The independence of g and q' so that the exponential term is preserved on integration over q' is discussed in the Appendix B.

At this point the new Glauber prediction is expressed as

$$\frac{d\sigma}{dt} = \frac{d\sigma}{dt} \text{ Glauber} [1 - A(.44 + .56e^{-2Rg})]^2 \quad . \quad (4.14)$$

To further parameterize $g(q, p_0)$ requires consideration of the effects determining τ and τ_0 . τ is affected by time dilation, so $\tau = rm_p/p$. Equations (4.9) through (4.12) effectively average over r giving $\langle \tau \rangle = \langle r \rangle m_p/p_0 = 2Rm_p/p_0$. $\langle \tau \rangle = (.70/p_0) \times 10^{-23} \text{sec}$. By analogy to other systems τ_0 is expected to be inversely related to t , so τ_0 is parameterized as $b_1/(-t)^c$. For example, the rate for dipole radiation of photons goes as the energy of the photon cubed. Thus, if the scattering proceeded by the exchange of massless vector particles, we would expect $c=3/2$; however, any reasonably small positive value of c would be consistent with the model. The order of magnitude for b_1 can be estimated from N^* decays, assuming the decay mechanism there is related. N^* (1470, 1520) have widths of 100MeV

corresponding to lifetimes of 1.4×10^{-24} sec. The pion emitted has an energy of .5Gev, and, since the transfer per interaction in double scattering is $t/4$, these two effects cancel giving $b_1 = 1.4 \times 10^{-24}$. Combining the two effects gives an estimate for $\tau/\tau_0 = 5t^c/p_0$. Putting the parameterization into Equation (4.14) gives

$$\frac{d\sigma}{dt} = \frac{d\sigma}{dt} \text{ Glauber} [1 - A(.44 + .56e^{-b(-t)^c/p_0})]^2 \quad (4.15)$$

where A, b, and c are fit from the data, and b is expected to be about 5.

The fit was made after eliminating half the $t = -.6$ and $-.8$ data points to even out the distribution of data points, and eliminating data with $-t \geq 2.3$ to eliminate points which require other effects to explain them since they already lie above the Glauber prediction.

With I_G fixed at $.028 \text{ mb}^{-1}$ the results are

Chisq/DF = 1.16 for 106 data points

$A = .22 \pm .04$

$b = 6.0 \pm 4.0$

$c = 4.2 \pm 1.3$,

and the errors in b and c are due to uncertainties of the fit while the error in A is due to uncertainties in normalization. Allowing I_G to be fit also, gives the results:

Chisq/DF = 1.15 for 106 data points

$I_G = .031 \pm .002$

$A = .29 \pm .07$

$$b=5.1 \pm 3.$$

$$c=3.3 \pm 1.$$

b agrees surprising well with the estimated value of 5, c is not unreasonable although it does not match the photon exchange analogy of $c=3/2$, and the fit value of the Glauber integral is in reasonable agreement with the calculated value. The fit is shown graphically in Figure 26.

Adding the points with $-t \geq 2.3$ causes a rapid drop in b and an increase in c and in the Chisq/DF. Also the contributions of these points to the Chisq/DF are abnormally large, being 1.5/DF for 3 points at $t=-2.3$, and 7/DF (5 points) and 10/DF (4 points) at $t=-2.5$ and -3.0 , respectively. Since these points lie above the predictions of the unmodified Glauber model this seems to imply that inelastic states (or other effects) are becoming important and does not imply that the model is wrong. Further consideration of this requires a more detailed model.

Other variations of this fit which have been tried are:

replacing $(-t)^c$ with $(-t+m^2)^c$

$$\frac{d\sigma}{dt} = \frac{d\sigma}{dt} \text{ Glauber}^x [(1-A)+A(1-\exp(-\tau/\tau_0))] \quad , \quad (4.16)$$

and combinations of the two.

The first variation is suggested if one believes that the exponential terms may be related to hadron form factors (which have recently also been parameterized as exponentials)⁴² and gives a Chisq/DF=1.14 for $0 < m < 1\text{GeV}$ and increases the Chisq/DF outside that region. $m=4$ is approximately the upper limit of acceptable values with

Chisq/DF=1.31. While this does not rule out the possibility that this relates to hadron form factors, neither is it compelling evidence of any such relation or of the need for a $-t+m^2$ type parameterization. The second variation assumes incoherence between the time dependent and time independent terms, and gives Chisq/DF=1.14, $A=.39\pm.07$, and b and c are the same as for the original fit. Combinations of the two give the same results.

Another fit has been tried which has no theoretical background and is only an attempt to check our estimate of our errors by testing how small a Chisq/DF can be obtained from a fit to the data. If a fit with Chisq/DF<.8 or so can be obtained from a relatively simple parameterization then we have probably overestimated our errors. The parameterization tried was a combination of the above two parameterizations times another factor, F, defined as

$$F=I (1+dp_0+et+fp_0^2+gt^2+hp_0t)$$

where I,d,e,f,g, and h are free to vary. The Chisq for this fit was 1.0/DF indicating that our errors are probably approximately correct.

4.5 CONSTITUENT QUARK MODEL

The existence of both a time dependent and time independent term in the fit suggest that nucleons are composed of pieces, and, therefore, that the simple space-time development arguments made for the fit are related to the constituent quark model.⁴³ This model assumes the nucleons are made of n (typically 3) independent quark clusters⁴⁴ composed each of a valence quark and a share of the gluons

and sea quarks. Typical nucleon nucleon interactions involve one cluster from each nucleon, and after interacting, independent of the final state, the clusters are excited and require a finite time to be able to interact again; or, alternatively, the reaction requires a finite amount of time to occur and a nucleon does not interact with two nucleons simultaneously. In the picture, in deuteron interactions, viewing the deuteron as two groups of three quarks rather than a bound state of six quarks;⁴⁵ after the incident nucleon interacts with one nucleon, only $n-1$ quark clusters are available to interact fully with the second nucleon while the one cluster which has already interacted is not fully available; it contributes to the time dependent term. This implies that A is given by $A=1/n$; $A=.29\pm.07$ from the fit in which the Glauber integral was allowed to vary, which matches well with $n=3$ or 4 , and is not inconsistent with $n=5$.

The time dependence of the interaction is not well specified in this model, however, it does require the assumption that the lifetime of the excited cluster is inversely related to t which is the same assumption used by the space-time arguments.

This model has the additional feature that it shows how to add inelastic state contributions to the model. In order for an inelastic state to contribute, the same quark cluster must interact with both nucleons, once to excite the cluster, and then the second time because, since it is the only excited part of the incident nucleon, an inelastic state is involved only if that cluster partakes in the second interaction. Hence, inelastic amplitudes can only be

added in to the time dependent term of the fit, implying that inelastic terms contribute less than otherwise expected at high energies, except at correspondingly high t . This type of argument may help explain our $-t \geq 2.3$ data.

4.6 SUMMARY

We have measured differential proton deuteron cross sections by performing a recoil experiment at the Internal Target area at Fermilab, using a large angle single arm recoil spectrometer to detect the deuterons recoiling from a gas jet target and identify the elastic process. Using simultaneous measurements from a solid state spectrometer at low t to set the normalization, we find that our results are significantly different from the Glauber model, generally lower, and with a slower fall off in t . The discrepancy between our data and the Glauber picture has led us to conclude that the basic Glauber model is incomplete. Some effect, probably the finite time required for strong interactions, reduces the elastic cross sections. The cross sections drop with increasing energy more rapidly than predicted by Glauber, and above some medium energy dependent on t , become constant.

We find that the data can be understood by assuming that after a nucleon interacts once it contains both time dependent and time independent amplitudes for a second interaction, implying the nucleon is composed of pieces. We measure a characteristic strong interaction time, τ_0 , and find a t dependence (here t is for a single

scatter);

$$\tau_0 = 1.4 \pm .8 \times 10^{-24} / (4t)^{3.5 \pm 1} .$$

Comparison with the quark model yields a prediction of 3-4 quarks per nucleon consistent with the standard three.

Inelastic intermediate states apparently do not contribute significantly, except at very high t and p_0 . How they contribute is unresolved, although our data (with $-t > 2.2(\text{GeV}/c)^2$ cross sections greater than the simple Glauber predictions and with $p_0 = 210 \text{ GeV}/c$ results greater than the corrected Glauber predictions) and the higher energy ISR results indicate that they should be in phase with the elastic amplitudes to increase the cross sections. Also, the constituent quark model gives some indication on how to relate the inelastics to the space-time effects, although it will probably require more data than we have available to investigate this.

TABLE OF CROSS SECTIONS

$t \pm \Delta t$ (GeV/c) ²	$p_0 \pm \Delta p_0$ GeV/c	$d\sigma/dt \pm \Delta d\sigma/dt$ microbarns / GeV ²
.60 .02	22.5 2.5	.180 x 10 ² .21 x 10 ¹
.60	27.5	.175 x 10 ² .20 x 10 ¹
.60	32.5	.159 x 10 ² .18 x 10 ¹
.60	37.5	.176 x 10 ² .20 x 10 ¹
.60	47.5	.161 x 10 ² .19 x 10 ¹
.60	52.5	.139 x 10 ² .17 x 10 ¹
.60	57.5	.147 x 10 ² .18 x 10 ¹
.60	62.5	.124 x 10 ² .15 x 10 ¹
.60	67.5	.152 x 10 ² .19 x 10 ¹
.60	72.5	.164 x 10 ² .20 x 10 ¹
.60	77.5	.136 x 10 ² .18 x 10 ¹
.60	82.5	.178 x 10 ² .22 x 10 ¹
.60	87.5	.157 x 10 ² .20 x 10 ¹
.60	92.5	.142 x 10 ² .18 x 10 ¹
.60	97.5	.154 x 10 ² .20 x 10 ¹
.60	102.5	.128 x 10 ² .16 x 10 ¹
.60	107.5	.169 x 10 ² .21 x 10 ¹
.60	115.0 5.0	.155 x 10 ² .17 x 10 ¹
.60	125.0	.131 x 10 ² .15 x 10 ¹
.60	197.5 7.5	.114 x 10 ² .12 x 10 ¹
.60	212.5	.111 x 10 ² .13 x 10 ¹

$t \pm \Delta t$ (GeV/c) ²	$p_0 \pm \Delta p_0$ GeV/c	$d\sigma/dt \pm \Delta d\sigma/dt$ microbarns / GeV ²
.80 .025	22.5 2.5	.768 x 10 ¹ .85 x 10 ⁰
.80	27.5	.791 x 10 ¹ .86 x 10 ⁰
.80	32.5	.717 x 10 ¹ .79 x 10 ⁰
.80	37.5	.742 x 10 ¹ .83 x 10 ⁰
.80	42.5	.924 x 10 ¹ .10 x 10 ¹
.80	47.5	.636 x 10 ¹ .73 x 10 ⁰
.80	52.5	.623 x 10 ¹ .72 x 10 ⁰
.80	57.5	.498 x 10 ¹ .60 x 10 ⁰
.80	62.5	.530 x 10 ¹ .63 x 10 ⁰
.80	67.5	.599 x 10 ¹ .70 x 10 ⁰
.80	72.5	.495 x 10 ¹ .64 x 10 ⁰
.80	77.5	.519 x 10 ¹ .64 x 10 ⁰
.80	82.5	.652 x 10 ¹ .81 x 10 ⁰
.80	87.5	.533 x 10 ¹ .66 x 10 ⁰
.80	92.5	.595 x 10 ¹ .73 x 10 ⁰
.80	97.5	.663 x 10 ¹ .81 x 10 ⁰
.80	102.5	.521 x 10 ¹ .66 x 10 ⁰
.80	107.5	.540 x 10 ¹ .70 x 10 ⁰
.80	115.0 5.0	.589 x 10 ¹ .64 x 10 ⁰
.80	125.0	.531 x 10 ¹ .57 x 10 ⁰
.80	197.5 7.5	.526 x 10 ¹ .55 x 10 ⁰
.80	212.5	.558 x 10 ¹ .63 x 10 ⁰

$t \pm \Delta t$ (GeV/c) ²	$p_0 \pm \Delta p_0$ GeV/c	$d\sigma/dt \pm \Delta d\sigma/dt$ microbarns / GeV ²
1.00 .03	25.0 5.0	.308 x 10 ¹ .35 x 10 ⁰
1.00	35.0	.280 x 10 ¹ .32 x 10 ⁰
1.00	45.0	.268 x 10 ¹ .33 x 10 ⁰
1.00 .06	55.0	.219 x 10 ¹ .24 x 10 ⁰
1.00	65.0	.242 x 10 ¹ .27 x 10 ⁰
1.00	72.5 2.5	.234 x 10 ¹ .30 x 10 ⁰
1.00	77.5	.200 x 10 ¹ .25 x 10 ⁰
1.00	82.5	.253 x 10 ¹ .31 x 10 ⁰
1.00	87.5	.211 x 10 ¹ .25 x 10 ⁰
1.00	92.5	.205 x 10 ¹ .26 x 10 ⁰
1.00	97.5	.170 x 10 ¹ .21 x 10 ⁰
1.00	102.5	.156 x 10 ¹ .20 x 10 ⁰
1.00	107.5	.233 x 10 ¹ .29 x 10 ⁰
1.00	115.0 5.0	.194 x 10 ¹ .21 x 10 ⁰
1.00	125.0	.164 x 10 ¹ .18 x 10 ⁰
1.00	197.5 7.5	.165 x 10 ¹ .18 x 10 ⁰
1.00	212.5	.184 x 10 ¹ .20 x 10 ⁰
1.12 .04	22.5 2.5	.272 x 10 ¹ .29 x 10 ⁰
1.12	27.5	.178 x 10 ¹ .20 x 10 ⁰
1.20	22.5	.198 x 10 ¹ .22 x 10 ⁰
1.20	27.5	.143 x 10 ¹ .16 x 10 ⁰
1.20	32.5	.114 x 10 ¹ .14 x 10 ⁰
1.20	37.5	.884 x 10 ⁰ .11 x 10 ⁰

$t \pm \Delta t$ (GeV/c) ²	$p_0 \pm \Delta p_0$ GeV/c	$d\sigma/dt \pm \Delta d\sigma/dt$ microbarns / GeV ²
1.20 .04	42.5 2.5	.115 x 10 ¹ .15 x 10 ⁰
1.20	50.0 5.0	.817 x 10 ⁰ .91 x 10 ⁻¹
1.20	60.0	.884 x 10 ⁰ .10 x 10 ⁰
1.20	70.0	.693 x 10 ⁰ .84 x 10 ⁻¹
1.20	75.0	.817 x 10 ⁰ .98 x 10 ⁻¹
1.20	85.0	.760 x 10 ⁰ .88 x 10 ⁻¹
1.20	95.0	.760 x 10 ⁰ .88 x 10 ⁻¹
1.20	105.0	.798 x 10 ⁰ .97 x 10 ⁻¹
1.20	115.0	.703 x 10 ⁰ .84 x 10 ⁻¹
1.20	125.0	.684 x 10 ⁰ .83 x 10 ⁻¹
1.20	197.5 7.5	.561 x 10 ⁰ .69 x 10 ⁻¹
1.20	212.5	.770 x 10 ⁰ .95 x 10 ⁻¹
1.22 x .08	27.5 2.5	.126 x 10 ¹ .10 x 10 ⁰
1.22 x .08	32.5	.126 x 10 ¹ .10 x 10 ⁰
1.31 x .08	45.0 5.0	.57 x 10 ⁰ .6 x 10 ⁻¹
1.31 x .08	55.0	.56 x 10 ⁰ .6 x 10 ⁻¹
1.31 x .08	65.0	.46 x 10 ⁰ .5 x 10 ⁻¹
1.31 x .08	75.0	.56 x 10 ⁰ .6 x 10 ⁻¹
1.31	95.0	.41 x 10 ⁰ .4 x 10 ⁻¹
1.31	105.0	.48 x 10 ⁰ .4 x 10 ⁻¹
1.31	115.0	.42 x 10 ⁰ .4 x 10 ⁻¹
1.31	125.0	.39 x 10 ⁰ .4 x 10 ⁻¹
1.31	135.0	.44 x 10 ⁰ .4 x 10 ⁻¹
1.31	210.0	.34 x 10 ⁰ .3 x 10 ⁻¹

$t \pm \Delta t$ (GeV/c) ²	$p_0 \pm \Delta p_0$ GeV/c	$d\sigma/dt \pm \Delta d\sigma/dt$ microbarns / GeV ²
1.27 .045	22.5 2.5	.109 x 10 ¹ .12 x 10 ⁰
1.27	27.5	.104 x 10 ¹ .12 x 10 ⁰
1.36	22.5	.828 x 10 ⁰ .10 x 10 ⁰
1.36	27.5	.688 x 10 ⁰ .84 x 10 ⁻¹
1.40	32.5	.437 x 10 ⁰ .55 x 10 ⁻¹
1.40	37.5	.409 x 10 ⁰ .53 x 10 ⁻¹
1.40 .09	42.5	.484 x 10 ⁰ .58 x 10 ⁻¹
1.40	47.5	.372 x 10 ⁰ .44 x 10 ⁻¹
1.40	52.5	.372 x 10 ⁰ .44 x 10 ⁻¹
1.40	57.5	.326 x 10 ⁰ .41 x 10 ⁻¹
1.40	62.5	.326 x 10 ⁰ .41 x 10 ⁻¹
1.40	67.5	.381 x 10 ⁰ .52 x 10 ⁻¹
1.40	72.5	.474 x 10 ⁰ .73 x 10 ⁻¹
1.40	75.0 5.0	.298 x 10 ⁰ .39 x 10 ⁻¹
1.40	85.0	.298 x 10 ⁰ .39 x 10 ⁻¹
1.40	95.0	.307 x 10 ⁰ .40 x 10 ⁻¹
1.40	105.0	.344 x 10 ⁰ .42 x 10 ⁻¹
1.40	120.0 10.	.260 x 10 ⁰ .30 x 10 ⁻¹
1.40	205.0 15.	.260 x 10 ⁰ .30 x 10 ⁻¹

$t \pm \Delta t$ (GeV/c) ²	$p_0 \pm \Delta p_0$ GeV/c	$d\sigma/dt \pm \Delta d\sigma/dt$ microbarns / GeV ²
1.45 .05	25.0 10.	.420 x 10 ⁰ .48 x 10 ⁻¹
1.55	25.0	.313 x 10 ⁰ .34 x 10 ⁻¹
1.55	35.0	.215 x 10 ⁰ .27 x 10 ⁻¹
1.60 .10	45.0	.172 x 10 ⁰ .20 x 10 ⁻¹
1.60	55.0	.154 x 10 ⁰ .18 x 10 ⁻¹
1.60	65.0	.176 x 10 ⁰ .21 x 10 ⁻¹
1.60	80.0 10.	.141 x 10 ⁰ .18 x 10 ⁻¹
1.60	100.0	.126 x 10 ⁰ .17 x 10 ⁻¹
1.60	120.0	.976 x 10 ⁻¹ .14 x 10 ⁻¹
1.60 .075	205.0 15.	.118 x 10 ⁰ .18 x 10 ⁻¹
1.78 .08	25.0 5.0	.134 x 10 ⁰ .15 x 10 ⁻¹
1.78	35.0	.998 x 10 ⁻¹ .12 x 10 ⁻¹
1.80	45.0	.855 x 10 ⁻¹ .11 x 10 ⁻¹
1.80 .11	55.0	.542 x 10 ⁻¹ .68 x 10 ⁻²
1.83 .08	70.0 10.	.606 x 10 ⁻¹ .69 x 10 ⁻²
1.80 .11	80.0	.624 x 10 ⁻¹ .78 x 10 ⁻²
1.80	100.0	.532 x 10 ⁻¹ .73 x 10 ⁻²
1.80	120.0	.492 x 10 ⁻¹ .72 x 10 ⁻²
1.80	205.0 15.	.484 x 10 ⁻¹ .71 x 10 ⁻²

$t \pm \Delta t$ (GeV/c) ²	$p_0 \pm \Delta p_0$ GeV/c	$d\sigma/dt \pm \Delta d\sigma/dt$ microbarns / GeV ²
1.94 .065	25.0 5.0	.717 x 10 ⁻¹ .83 x 10 ⁻²
1.97 .10	35.0	.465 x 10 ⁻¹ .54 x 10 ⁻²
2.00 .13	45.0	.414 x 10 ⁻¹ .44 x 10 ⁻²
2.00	60.0 10.	.293 x 10 ⁻¹ .38 x 10 ⁻²
2.00	85.0 15.	.236 x 10 ⁻¹ .34 x 10 ⁻²
2.00	115.0	.294 x 10 ⁻¹ .69 x 10 ⁻²
2.00	205.0	.185 x 10 ⁻¹ .41 x 10 ⁻¹
2.06 .14	25.0 5.0	.340 x 10 ⁻¹ .39 x 10 ⁻²
2.16 .11	40.0 10.	.215 x 10 ⁻¹ .26 x 10 ⁻²
2.20 .14	60.0	.122 x 10 ⁻¹ .20 x 10 ⁻²
2.30	80.0	.848 x 10 ⁻² .97 x 10 ⁻³
2.30	110.0 20.	.697 x 10 ⁻² .82 x 10 ⁻³
2.30	205.0 15.	.657 x 10 ⁻² .17 x 10 ⁻²
2.38 .12	30.0 10.	.119 x 10 ⁻¹ .14 x 10 ⁻²
2.58 .16	55.0 15.	.515 x 10 ⁻² .67 x 10 ⁻³
2.50	80.0 10.	.475 x 10 ⁻² .71 x 10 ⁻³
2.50	110.0 20.	.374 x 10 ⁻² .58 x 10 ⁻³
2.50	205.0 15.	.414 x 10 ⁻² .11 x 10 ⁻²
2.87 .14	30.0 10.	.192 x 10 ⁻² .32 x 10 ⁻³
3.00 .18	60.0 20.	.111 x 10 ⁻² .17 x 10 ⁻³
3.00	100.0 30.	.808 x 10 ⁻³ .12 x 10 ⁻³
3.00	205.0 15.	.606 x 10 ⁻³ .21 x 10 ⁻³

INTERACTION MECHANISMS

pd $\rightarrow$ pd
Single
Scattering

(a) Dominant at $-t \leq .5 (\text{GeV}/c)^2$

pd $\rightarrow$ pd
Double
Scattering

(b) Dominant at $-t \geq .5 (\text{GeV}/c)^2$ Low S

pd $\rightarrow$ pd
Double
Scattering

(c) Increasing Contribution with
Increasing S

pd $\rightarrow$ N^{*}d
Double
Scattering

(d) N^{*} Production to Measure N^{*} N
Elastic Amplitude

Figure 1

Figure 2

TARGET SYSTEM

Figure 3

MAIN RING CYCLE

Beam Intensity $\approx 1-2 \times 10^{13}$ Protons
 Acceleration $\approx 120 \text{ GeV/c/sec}$

Jet Pulses (as Measured from Target Chamber Vacuum)
 $\approx 70-130 \text{ GeV/c}$ and $190-210 \text{ GeV/c}$

Figure 4

REFERENCE FRAMES

Figure 5

CHAMBER LAYOUTS

wirespacing is $2\text{mm} \times \cos 15^\circ$
 X projection wirespacing is 2 mm

(b) Chamber 5

Figure 6

TIME OF FLIGHT

$$t = -1.3 (\text{GeV}/c)^2$$

Distance Travelled = 8 meters (26 ns)

Time Zero Arbitrary

Figure 7

(a) STACK ANGLE

(b) DETECTOR LAYOUT

Figure 8

HODOSCOPE

(a) Sideview (one Column of 3 Elements)

(b) Top View

Figure 9

Figure 10

'MIDMAG' SCHEME

Figure 11

Figure 12

MOMENTUM CHECK

$$\delta = (p_r - p_c) / p_c$$

 $\delta_A = \text{BEST VALUE}$
 $\delta_B = \text{OTHER VALUE}$
 $\tau = -2.0 \text{ MONTE CARLO}$

Figure 13

SCATTERING CHECK

dx/dz EVALUATED
AT DIPOLE FRONT

I $t = -2.0$ MONTE CARLO

Figure 14

'MIDMAG' CHECK

$X_{F(B)}$ = Projection of X Coordinate
to Dipole Center from Front(Back)

I t = -2.0 MONTE CARLO

Figure 15

MONTE CARLO ACCEPTANCE

Spectrometer Set at:

$$t_0 = -.6 (\text{GeV}/c)^2$$

$$p_0 = 40 \text{ GeV}/c$$

== MONTE CARLO RESULTS

▲ RAW DATA SCALED

● DATA $\times$ EXP. $(4t/t_0)$ SCALED

Figure 17

MONTE CARLO ACCEPTANCE

Spectrometer set at:
 $t_0 = -2.0 \text{ (GeV/c)}^2$
 $p_0 = 40 \text{ GeV/c}$

$p_0 = 25 \text{ GeV/c}$
$p_0 = 100 \text{ GeV/c}$

MONITOR SPECTRUM

Figure 19

Figure 20

Figure 21

Figure 22

Figure 23

Figure 24

GLAUBER MODEL MECHANISMS

(a) Single Scattering

(b) Double Scattering

Figure 26

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APPENDIX A MOMENTUM EQUATION

The momentum is related to the coordinates via

$$\delta^c = c_{ijk} x_i x_j x_k, \text{ (summation implied)} \quad (\text{A.1})$$

where δ^c is the calculated value of δ , different from δ^d the actual value, and the summation is over $i=j, 7$, $j=k, 7$, and $k=1, 7$. Defining d_1 by the assignment

$$d_1 = x_i x_j x_k \quad (\text{A.2})$$

$$c_1 = c_{ijk}$$

gives

$$\delta^c = c_1 d_1 \quad (\text{A.3})$$

The error in δ^c due to errors in the coordinates is.

$$\begin{aligned} \Delta \delta^c &= c_1 \Delta d_1 \\ &= c_{ijk} (\Delta x_i x_j x_k + x_i \Delta x_j x_k + x_i x_j \Delta x_k) \end{aligned} \quad (\text{A.4})$$

x_i is defined from the wirespacing of the planes, with a scaling parameter a to vary the amount of error included,

$$\Delta x_i = a/2 \times \text{wirespacing}_i \quad (\text{A.5})$$

The variable chosen for minimization is

$$x^2 = (\delta_n^d - \delta_n^c)(\delta_n^d - \delta_n^c) + (\Delta \delta_n^c)(\Delta \delta_n^c) \quad (\text{A.6})$$

Then require

$$dX^2/dc_1 = 0 \quad , \quad (A.7)$$

$$0 = 2d_1(c_j d_{jn} - \delta_n^d) + 2\Delta_{ln}(c_j \Delta_{jn}^d) \quad (A.8)$$

$$d_{ln} \delta_n^d = c_j (d_{jn} d_{ln} + \Delta_{jn}^d \Delta_{ln}^d) \quad (A.9)$$

Define

$$\begin{aligned} q_1 &= d_{ln} \delta_n^d \\ o_{jl} &= d_{jn} d_{ln} \\ a^2 e_{jl} &= \Delta_{jn}^d \Delta_{ln}^d \\ v_{jl} &= o_{jl} + a^2 e_{jl} \end{aligned} \quad (A.10)$$

Then from Equation (A.9)

$$q_1 = c_j v_{jl} \quad , \quad (A.11)$$

which is just a simple matrix equation. Inverting (in general v will have a nonzero determinant) gives

$$c_j = (v^{-1})_{jl} q_1 \quad , \quad (A.12)$$

so the c_j are uniquely defined for any $\underline{a}$, and the procedure for obtaining them is as above.

The final choice of $\underline{a}$ comes from studying the results for various $\underline{a}$. With $a=0$ and coordinates with six digit accuracy, the equation is virtually errorless, but when coordinates with reasonable errors are used, the equation gives meaningless answers. $a>0$ reduces the effect errors have on the result, but also reduces the accuracy at the no error limit. For $\underline{a}$ large such that o_{ij} is negligible, the accuracy with and without errors becomes comparable to ignoring the fringe fields altogether. The best compromise depends upon the errors in the real data.

The details of the assignment (A.2) are constrained only by programming algorithm considerations. The most significant terms in the series are the linear terms and most terms including a dx/dz coordinate. Terms including a simple x coordinate are also important. Other terms add a little, but are mostly retained for speed in the computing algorithm.

APPENDIX B

MODIFIED GLAUBER DERIVATION

When performing the integral over d^3r , the modified model does not generate a factor which is directly $S(q)$, the deuteron form factor. Hence, the approximation $\underline{q} \cdot \underline{s} = \underline{q} \cdot \underline{r}$ is not useful and, in fact, only complicates the integral. The integral over d^2q' to generate I_G is also more complicated and must be considered in some detail.

B.1 MODIFIED FORM FACTOR

Starting from Equation (4.10) and doing the integral in Cartesian coordinates

$$I = (4\pi R^2)^{-3/2} \int \exp(i\underline{q} \cdot \underline{s}) \exp(-r^2/4R^2) \exp(-g|z|) dx dy dz \quad . \quad (B.1)$$

Here, since the incident proton does not change direction appreciably during the interaction, the approximation has been made, for purposes of the time factor, $\exp(-gr)$, that $r=z$. The approximation is expected to produce negligible error, and is in the same spirit as

many approximations made in the standard model to get to this point in the derivation. The absolute value of z is taken since the incident proton must hit the first nucleon first, i.e., only the distance between the nucleons is relevant, not the order of the nucleons.

The integral factors into a product of integrals

$$I = I_x I_y I_z \quad (B.2)$$

$$I_x = (4\pi R^2)^{-1/2} \int \exp(iq_x x) \exp(-x^2/4R^2) dx, \quad (B.3)$$

$$I_z = (4\pi R^2)^{-1/2} \int \exp(-z^2/4R^2) \exp(-g|z|) dz, \quad (B.4)$$

and I_y is the same as I_x after replacing $x \rightarrow y$.

I_x is a common fourier transform whose result is

$$I_x = \exp(-R^2 q_x^2) \quad (B.5)$$

Similarly,

$$I_y = \exp(-R^2 q_y^2) \quad (B.6)$$

Using $q_x^2 + q_y^2 = q^2 \sin^2 \theta$ gives

$$I_x I_y = S(q \sin \theta), \quad (B.7)$$

where S is the deuteron form factor.

I_z is evaluated by using the symmetry of the integrand to convert the integral to an integral from 0 to ∞ with the absolute value of z removed. Then $\exp(-gz)$ is expanded as a power series so the integral becomes a sum of integrals of the type

$$\int z^k \exp(-a^2 z^2) dz = \frac{1}{2} a^{-(k+1)} \Gamma((k+1)/2) . \quad (B.8)$$

Hence,

$$I_z = \pi^{-1/2} \sum_{k=0}^{\infty} \frac{(-2gR)^k}{k!} \Gamma((k+1)/2) . \quad (B.9)$$

Since $\Gamma(1/2) = \pi^{1/2}$, this result has the proper limit for $g \rightarrow 0$.

To approximate this result note that

$$\Gamma((k+1)/2) = 1 + 10\% , \quad k=1,4 \quad (B.10)$$

and that in our region higher terms will be unimportant, except that they may tend to steepen the drop in the cross section at the low energy end. Then, removing the excess of the $k=0$ term from the summation, I_z is fairly accurately given in our range as

$$I_z = (\pi^{1/2} - 1) / \pi^{1/2} + 1/\pi^{1/2} e^{-2gR} . \quad (B.11)$$

B.2 PARTICLE EXCITATION

In the course of deriving the Glauber integral with space-time effects included, a question on the nature of the excitation of the nucleon is raised--do the nucleons become excited as they approach one another or does the excitation occur only after the interaction, i.e., in the following sketches, with the vertical axis as the excitation level, and time going to the right--which reflects the actual occurrence?

Assuming $g(q, p_0) = f(p_0)q^n$, then, since $(q/2 \pm q')$ is transferred to N_1 (N_2), if case (a) is true, then g is a function also of q' ; while if case (b) is correct, then the q' dependence drops out to first order. In case (b)

$$g = f(p_0) \left(\left(\frac{q}{2} - q' \right)^n + \left(\frac{q}{2} + q' \right)^n \right) \quad (B.12)$$

Since $q' \ll q$, Equation (B.12) can be expanded

$$\begin{aligned} g &= f(p_0) \left(1 - \frac{2nq'}{q} + 1 + \frac{2nq'}{q} + O((q')^2) \right) \\ g &= f(p_0) 2 \left(\frac{q}{2} \right)^n + O((q')^2) \end{aligned} \quad (B.13)$$

There are arguments for both cases, and, while we have not been able to find any reason to conclusively favor either of them, since we also do not expect to be able to discern between the two cases experimentally, it is expected that either choice is sufficiently

close to reality to be useful. Therefore, for ease of calculation,
we proceed using case (b).