POLARIZATION OF PROMPT MUONS

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ABSTRACT

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This paper presents measurements of the polarization of muons produced very near the point of proton - nucleon interaction. The experiment utilized a 400 GeV proton beam available in the Proton Central area of Fermilab. Muons were produced by the interaction of these protons with a variable density copper target. Extrapolation to infinite target density allowed elimination of contributions due to muons from meson decay. Measurements were made upon muons produced in the forward direction with energies near 185 GeV and upon muons produced with transverse momenta near 1.9 GeV/c and an energy of 54 GeV. In the first case only the longitudinal polarization was measured: $P = -0.01 \pm 0.14$. Under the second set of kinematic conditions both the longitudinal and transverse polarization were measured: $P_L = -0.06 \pm 0.16$, $P_T = -0.01 \pm 0.11$. These null measurements suggest that an electromagnetic process is the dominant mechanism for prompt muon production. The measurements also indicate an upper limit of $B_\mu(D^o)\sigma_{D^o} + B_\mu(D^+)\sigma_{D^+} < 6.7 \times 10^{-8}$ barns may be placed upon the production cross section for D particles.

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1. - INTRODUCTION

The first reports of prompt muon production[1 - 3] and the first attempt to measure their polarization[3] were made many years ago. These experiments had limited objectives and hence limited accomplishments. In particular the measurements did not indicate the large discrepancy between the actual rate of production of muons very near the point of hadron - nucleon interactions and the rate which could be calculated on the basis of known processes.

More recent measurements of the production of prompt muons either at large transverse momentum[4, 5] or at low p_t[6] show that the flux of these particles is much higher than can be accounted for by the decay of vector mesons or by Dalitz decay of etas.

It has been suggested that the anomalous portion of the prompt muon flux could be due to the weak decay of unknown particles[7, 8] or to some electromagnetic production mechanism other than those listed above.[9 - 11]

Since electromagnetic processes conserve parity, the muons produced by such a source have zero average polarization along their direction of flight. However, muons produced by weak decays of intermediate particles would be polarized. If there are no kinematic constraints, then the expected muon polarization in the parent rest frame would be near 1. In special cases such as the two body decay of a spin zero meson into a muon and neutrino the expected polarization of the muon is - 1 in the rest frame of the parent. If the mass of the parent is large compared with that of a muon and the production spectrum of the parent is sufficiently steep then the polarization in the lab frame would be nearly the same as the polarization in the rest frame of the parent.

The importance of this topic to particle physics will only be known when the production processes of prompt muons are fully understood. It has already been discovered that a small portion of the prompt muon flux is due to the leptonic decay of the J/ψ particle.[12, 13] At this time it is still not known whether the large portion of prompt muons whose source remained unexplained -- even after the recent flurry of new particle discoveries -- comes from a production mechanism as deeply interesting as the J/ψ.

The measurements discussed in this paper suggest that an electromagnetic process is responsible for the production of most prompt muons. We believe that these observations and other measurements of the properties of prompt muons give us insight into the character of interactions between the constituents (quarks) of colliding nucleons.

2. - Production and decay of polarized muons

2.1 - Some production mechanisms for polarized muons

It has been known for twenty years that various types of high energy interactions produce polarized muons. For example, the longitudinal polarization (component of the muon spin along its direction of flight) of muons produced by cosmic rays was measured in order to determine the energy spectrum of the parent pions.[14] Measurements have also been made at accelerators to observe the longitudinal[3, 15, 16] and transverse[17, 18] polarization (component of the muon spin perpendicular to its plane of production) of muons in efforts to determine properties of their parent particles or characteristics of the decays of these particles.

The production of longitudinally polarized muons via the decay of pions and kaons is well understood. A detailed discussion of this process is contained in Appendix 1. The decays of heavier mesons may also produce polarized muons. Let us consider a possible decay for a heavy spin zero meson, the D, into a kaon, muon, and neutrino. According to the helicity rule relativistic particles should be emitted left-handed (spin anti - aligned to their direction of motion). Thus for $D^+ \to \overline{K}^o \mu^+ \nu_\mu$ the neutrino will be produced left-handed and the μ^+ will be right-handed in the rest frame of the D. A Dalitz plot for this decay is shown in Fig. 1.

Point B and line A correspond to colinear emission of the neutrino and muon. The type of decay pictured for point B is suppressed since the spins of the muon and neutrino add to give a total angular momentum of 1. The conservation of angular momentum would be violated by such a decay.

However, for the type of decay pictured on line A, the spins of the muon and neutrino are in opposite directions so there is zero net angular momentum. For non-colinear decay configurations, the conservation of total angular momentum must be accomplished through a cancellation of spin angular momentum by orbital angular momentum.

Note that the neutrino is always emitted with a longitudinal polarization of - 1 since it has velocity c. The helicity of the muon is not so tightly constrained as its velocity is not the speed of light and in fact need not necessarily be relativistic. However, examination of the Dalitz plot as a function of T_{μ} reveals that the decay configurations are heavily weighted towards those in which the muon has a kinetic energy much larger than its rest mass. Hence the great majority of muons emitted by this decay mode will have a velocity near c and a polarization of +1 in the rest frame of the parent. Assuming that the D is produced with a fairly steep energy spectrum, the net polarization of muons with any given energy in the lab frame will be nearly the same as in the rest frame of the parent. A muon which is emitted in the rest frame of the D in a direction opposite the line of flight of the D may have a negative polarization in the laboratory. However, for any particular muon laboratory energy there will be many more muons produced by forward decays (from lower energy parents). Hence the average polarization observed in the lab for muons produced via this decay will be near +1.

2.2 - Characteristics of muon decays

For the decay $\mu^+ \rightarrow e^+ \bar{\nu}_\mu \nu_e$ the form of the positron spectrum is

$$\frac{d^2N}{d(\cos\theta)\,dx} \propto A(x)\,(1 + P\cos\theta \cdot B(x)) \text{ where } A(x) = 6x^2 - 4x^3,$$

P is the average muon polarization, θ is the angle between the positron momentum and the muon spin, x = (positron energy) / 53 MeV and $B(x) = (x - 1/2)/(3/2 - x)$.[19] Using this definition, x may range from 0 to 1. Both A (x) and B (x) take on their maximum values over this interval at $x = 1$. Note that the change in sign of B (x) at $x = 1/2$ reflects the fact that positrons with $1/2 < x < 1$ are more likely to be emitted near $\cos\theta = 1$ while muons with $0 < x < 1/2$ are more likely to be emitted near $\cos\theta = -1$. Thus high energy positrons are more likely to be emitted along the direction in which the muon spin is oriented.

Less information on muon polarization may be obtained from the decay $\mu^- \rightarrow e^- \nu_\mu \bar{\nu}_e$. These muons lose their polarization as they enter atomic orbits. Also, the majority of negative muons stopping within the aluminum polarimeter used in our experiments are captured by nuclei before they can decay. As noted in the "Data Analysis" section we use 18.5 for our ratio of (analyzing effectiveness for μ^+) / (analyzing effectiveness for μ^-).

3. - Apparatus

3.1 - Target

In order to measure the polarization of prompt muons, we must be able to distinguish between the polarization of muons from this source and the polarization of muons produced by meson decay. We accomplish this by using targets with different densities. Since relativistic mesons have long enough lifetimes to travel many interaction lengths, the probability that a meson will decay is inversely proportional to target density. Thus, an extrapolation to infinite target density allows us to eliminate contributions from meson decay.

Our most dense target consisted of 40 inches of solid copper. The second target contained 40 one inch thick blocks of copper with a one inch gap of air between each block. The least dense target was 40 one inch thick blocks of copper with a two inch air gap between each block. We label these targets according to their inverse densities: 1, 2, and 3. Targets 1 and 2 were backed by 2 meters and 1 meter of aluminum, respectively, so that any of the three targets could be inserted into a three meter long space in the beam line without having any open drift space behind any target. Each target had a surface perpendicular to the beam direction which was 1.625'' x 7.00''. The three targets were stacked (Fig. 2) so that a change in target density could easily be accomplished by a vertical movement of the target assembly which aligned the proper target with the beam. Nine feet of steel was placed directly behind the moveable target assembly to absorb any hadrons which leaked through the targets.

3.2 - Beam design

This experiment was done in the Proton Central area of Fermilab. Protons with an energy of 400 GeV were delivered onto our target. The muons produced there passed through 13 kg / cm^2 of material (mostly steel) in the area immediately downstream from the target.

In measuring the polarization of muons observed at 45 milliradians, we required that each muon pass through a counter placed 60 meters from the target at an angle of 45 mr. from the proton beam direction (Fig. 3). These muons then passed through 12 kg / cm^2 of earth and two trigger counters before reaching the polarimeter. The energy of muons stopping in the polarimeter was 54 ± 2 GeV. Although the mean transverse momentum of these muons was 2.4 GeV/c, their mean production transverse momentum was 1.9 GeV/c. The additional transverse momentum was imparted to the muons by multiple Coulomb scattering in the target and material immediately downstream from the target. The muon production angle in the center of momentum system of the colliding proton and nucleon was 58^{o}.

Measurement of the polarization of muons produced in the forward direction was accomplished in a somewhat different manner (Fig. 4). Extra precautions needed to be taken since muons produced by interaction of the proton beam with any material upstream from the target could simulate prompt muons produced in the forward direction. If these muons were allowed to pass down the beampipe and impinge on our target from the forward direction, then they would simulate promptly produced muons from our target since their intensity would change very little with target density. In order to eliminate these muons from the proton beam, a pitching magnet was placed 10 meters in front of the target. This standard Fermi Lab Vernier Dipole Magnet applied a 4 mr. vertical pitch to protons and an 8 mr. pitch to muons. This separated positive muons from the beam by 4 milliradians and negative muons from the beam by 12 milliradians. A test of this muon spoiler was made by inserting material into the beam upstream from the pitching magnet. The spoiler was determined to be effective in this test as well as in a test consisting of purposeful misalignment of the beam to cause scraping of the protons on the wall of the beampipe.

Muons produced in the target passed through the material immediately behind the target and into a steel filled bending magnet 6 meters long. This Fermi Lab Main Ring Magnet Type B-2 deflected muons by 22 mr. Positive muons then passed through 58 kg / cm^2 of earth and two trigger counters before reaching the polarimeter. Although the muons observed were nominally produced in the forward direction, multiple scattering in the target and material directly downstream from the target gave an R.M.S. momentum transfer of 640 MeV/c to the muons. Thus, the muons observed in the measurement had a transverse momentum distribution whose shape was approximately a Gaussian centered at zero. Since the width of this distribution was determined by the mean momentum transfer, the muons observed in this measurement were a sampling of muons whose production transverse momentum ranged from zero to beyond 1 GeV. Their energy was 185 ± 10 GeV.

3.3 - Polarimeter

The polarimeter consisted of a "sandwich" formed by 25 scintillation counters with aluminum slabs in front and in back of each counter (Fig. 5a). The counters were 24" x 36" x 0.25". Two counters were placed upstream from the polarimeter to detect muons entering it and an array of counters was positioned downstream from the polarimeter to detect muons leaving it.

A computer printout of a typical event is shown in Figure 5b. It consists of hits in a series of polarimeter counters beginning with the front counter. There are no hits in the rest of the polarimeter counters. The computer was instructed to examine the signals from the polarimeter whenever a trigger was received. This trigger consisted of hits in the two counters directly in front of the polarimeter, the first polarimeter counter, and no hits in the last polarimeter counter or in an array of counters behind the polarimeter. For the measurements at high transverse momentum, the trigger also required a hit in a counter placed 45 mr. off the beam line 60 meters from the target (see section 3.2). Hence a trigger was generated when a muon entered the front of the polarimeter and stopped before leaving the polarimeter.

A magnetic field was applied throughout the polarimeter by two 6'4'' x 11'8'' coils (Fig. 6). These coils were four feet apart. Each coil consisted of thirty turns of copper tube through which cooling water ran. The strength of the field was about thirty gauss. This caused the muons within the polarimeter to precess with a period of about 1.7 micro-seconds. The technique described here has been used by this group to measure the polarization of muons produced by cosmic rays[20] as well as muons produced by accelerators.[3, 15] Application of the magnetic field allowed us to avoid systematic errors by a method which is fully discussed in the ''Data'' section of this paper.

When a muon was observed to have stopped within the polarimeter, twenty - five clocks were started. Each clock was connected to a counter in the polarimeter. If a particle was subsequently detected in a particular counter, then the clock for that counter was stopped at the time this particle was observed. If a particle was observed in a counter adjacent to where the muon stopped and this particle was detected within 12.7 microseconds after the muon stopped, then this particle was assumed to be a decay positron (or electron). If the positron was detected in the counter immediately downstream from where the muon stopped, then it was counted as a ''forward'' decay. Positrons detected immediately upstream from the position at which the muon stopped were defined as ''backward'' decays. More discussion of the electronics is contained in Appendix 3.

3.4 - Possible sources for corrections to the data

The apparatus described in the previous sections comes very close to the ideal experiment in as much as all corrections are very minor. Examination of the data presented in the next section shows that there is little difference between the polarization one could calculate on the basis of the raw data points and that which may be calculated after considering corrections.

As noted in section 3.1, targets 1 and 2 are backed by aluminum. This extra material in the beam line means that we are looking at slightly different muon energies for different targets. The density 1 target has two meters (540 gm/cm^2) of aluminum behind it while target 2 has one meter of aluminum in back of it. The extra energy loss per meter of aluminum is about 0.67 GeV for 54 GeV muons and about 0.75 GeV for 185 GeV muons. The energy dependence for the combined production of muons via both meson decay and prompt sources goes roughly as $\frac{dI}{dx} = \frac{A\exp(-9x)}{x}$. This is equivalent to $\frac{dI}{I} = -(9 + \frac{1}{x})\,dx$. Thus one may compute corrections in intensity which are necessitated by the change in x for different targets. The correction is $dI/I = 0.027$ (for $dx = (1/\rho) = 1$ unit change in target density) at 55 GeV and $dI/I = 0.021$ (for $d(1/\rho) = 1$) for 185 GeV muons. The magnitude of this correction is readily apparent if it is made graphically. For instance, the data point for target 3 of the 185 GeV intensity data plotted in Fig. 7 is unmoved by this correction (since there is no extra material behind target 3) while the points for targets 2 and 1 are moved up by 0.021 and 0.042 units respectively. Clearly this is a minor correction.

In plotting the intensity, one must also consider a change in geometry which occurs when using different targets. Because of the difference in densities of the targets and the fact that the upstream faces of all the targets were positioned at the same point in the beam line, the mean point of production of muons is further downstream for the less dense targets. This means our detectors intercept a larger solid angle for less dense targets. However, since the detectors used to measure the intensity of muon production at 185 GeV (Fig. 7) were 1200' from the target while the change in the mean point of production is on the order of the sum of the interaction lengths for protons and mesons in copper (a total of about 1'), this correction is negligible. For our measurements at 54 GeV, a counter placed 200' from the target was used in defining intensities. Consequently, the correction for the increased solid angle at lower density is about 1°/o of the intensity.

As noted in section 3.2, any material in the beam line upstream from the target could cause the production of muons which simulated prompt muon production. This is because changes in target density would have very little effect on their intensity. Table 1 lists the various materials which were present in the beam line. These devices were used to monitor the beam profile and intensity. Altogether, they amounted to 0.753 gm / cm^2. Mesons created in this material could decay in 1.2 meters of air before reaching the target. If we use 175 gm / cm^2 as the mean free path for the interaction of pions and kaons in copper, then the distance traveled in the target by a typical meson during which it could decay to a muon is 0.2 meters. Thus an estimate for the ratio (muons produced by material upstream from the target) / (muons produced in the target) is $\frac{0.753\ (1.2 + 0.2)}{175\ (0.2)}$ = 0.030. This correction can be handled graphically by extrapolating to $1/\rho = -.03$ rather than to $1/\rho = 0$ in Fig. 7. Another source of muons which are not produced in the copper targets is production by hadrons which pass through the copper without interacting. These hadrons may produce muons either in the aluminum in back of targets 1 and 2 or in the steel which is behind the main target assembly (Fig. 2). The amount of muons produced by these hadrons is different for the various targets because there are somewhat different materials in back of each target. One may fairly easily estimate the percentage of the total muon flux due to hadrons escaping from the targets and correct the data for this effect.

The percentage of the incident proton flux which does not interact in the copper is the same for each target since each target contains 890 gm / cm^2 of copper. Using a proton interaction length of 140 gm / cm^2 in copper gives the result that $\exp(-890/140) = 0.0017$ is the fraction of the incident flux which does not interact.

One may also calculate the fraction of pions and kaons produced in the target which escape from the copper without interacting. Let N be the number of pions and kaons produced in the density one target and N' the number which do not interact within the target.

$$N = I_p \sigma \int_0^1 \exp(-6.36z)\, dz$$

$$N' = I_p \sigma \int_0^1 \exp(-6.36z)\exp(-5.09(1-z))\, dz$$

where I_p is the intensity of protons incident on the target, σ is the total cross section for all proton-nucleon interactions which produce a pion or kaon, 6.36 is the number of proton interaction lengths in 1 meter of copper, 5.09 is the number of meson interaction lengths in 1 meter of copper and the integration is performed over the length of the target. The integrations may easily be performed.

$$N = I_p \sigma \left[- \frac{\exp(-6.36z)}{6.36} \right] \Big|_0^1 = 0.16\, I_p \sigma$$

$$N' = I_p \sigma \left[\exp(-5.09) \right] \left[- \frac{\exp(-1.27z)}{1.27} \right] \Big|_0^1 = 0.0035\, I_p \sigma$$

Thus the fraction of pions and kaons which leak through the density one target is $N'/N = 0.022$. This is much larger than the fraction of protons which escape from the target and is the major source of muons produced down-stream from the target.

Mesons leaving target 1 will interact in the two meters of aluminum which are directly behind it. These mesons will contribute more muons than would have been produced in an infinitely long copper target since the mean free path for interaction in aluminum is longer. This allows the mesons a longer time in which to decay than if they were in copper. The fraction of muons contributed by these mesons is (N'x mean free path in Al) / (Nx mean free path in Cu) = $0.022\ (27/63)^{\frac{1}{3}}\ (890/280) = 0.05$ where the mean free path for mesons in a material is assumed to be proportional to (Atomic Mass $)^{\frac{1}{3}}$ / Density. The 460 gm / cm^2 of aluminum is 3.8 interaction lengths so the number of muons escaping from the aluminum behind the density one target is negligible.

Since there is only one meter of Al (1.9 interaction lengths) behind target 2, the fraction of mesons which leak through the copper and aluminum behind this target is $0.022 \exp(-1.9) = 0.0033$. These mesons will interact in the steel which is immediately behind the aluminum. The contribution of these mesons is $0.0033\ (56/63)^{\frac{1}{3}}\ (890/790) = 0.0036$ when expressed as a fraction of the total flux. This is more than an order of magnitude less than the correction previously calculated for the interaction of mesons in the aluminum. Hence it may be neglected and the correction for target 2 is the same as for target 1.

There is no aluminum behind target 3. Thus an estimate for the fraction of muons produced in material (steel) downstream from this target is $0.022\ (56/63)^{\frac{1}{3}} \times (890/790) = 0.024$.

The correction for the fraction of muons produced in material downstream from the copper targets may be made by shifting the data points 0.025 units to the right for target 3 and 0.05 units to the right for targets 1 and 2. This means that our targets have effective inverse densities which are 1.05, 2.05, and 3.025. As in the previous cases, this correction is quite minor.

4. - Data

4.1 - General considerations

Measurements were made at two different kinematic points since it was conceivable that there could be two important sources of prompt muon production. It was possible that electromagnetic processes could produce most of the prompt muons observed in the forward direction while the muons with large transverse momentum were predominately produced via weak decay of some intermediate particle. A further discussion of this possibility is contained in Appendix 2.

Since high energy positrons are emitted preferentially along the direction of muon polarization, a measurement of the decay asymmetry $A = \left(\frac{\text{forwards} - \text{backwards}}{\text{forwards} + \text{backwards}} \right)$ allowed us to determine the average muon polarization. Applying a magnetic field throughout the polarimeter enabled us to avoid making an error due to a systematic bias in the decay asymmetry measurement. As an example, if counter number 10 was the final counter hit by a muon, then it was assumed that this muon stopped in the aluminum slab between counters no. 10 and 11. Note that if the muon had in fact stopped in scintillation counter no. 10 then the decay positron would be defined as "backwards." Applying a magnetic field throughout the polarimeter allowed us to avoid the systematic bias which could have been introduced into the polarization measurement by this process.

The decay asymmetry as a function of time was shaped like a sine wave. The frequency of this sine wave was proportional to the strength of the magnetic field; the amplitude of the sine wave slowly decreased due to inhomogeneity of the magnetic field (Fig. 7). Decays of muons which stop in the scintillator caused a shift of the sine wave away from the $\frac{F - B}{F + B} = 0$ axis.

For a perfectly homogeneous magnetic field, all muons would precess with the same frequency which may be calculated from Larmor's formula $\nu_L = \frac{\mu B}{I\hbar}$ (μ is magnetic moment, B is field strength, $I\hbar$ is angular momentum). In this case, two muons which arrived with a particular angle between their magnetic moments would continue to have this angle between their directions of alignment even after many precession periods in the polarimeter. The decay asymmetry as a function of time would be a sine wave whose frequency was the Larmor precession frequency and whose amplitude was proportional to the polarization of the muons, $A = eP$. Since muons in different parts of the polarimeter were subject to slightly different magnetic fields, the actual decay asymmetry signal was the superposition of many sine waves each of which had a slightly different frequency. All the sine waves begin at $t = 0$ with the same amplitude (since muons arrive in all parts of the polarimeter with the same average polarization) but the amplitude of the sum of these sine waves decreases with time as they become out of phase with each other. Thus the decay asymmetry we observe looks like a sine wave whose frequency is equal to the average frequency of precession of the muons and whose amplitude slowly decreases. However, it is still true that the amplitude at $t = 0$

of the sine wave we observe is proportional to the average polarization of the muons. This is because at $t = 0$ the difference in precession frequencies has not yet had any effect.

4.2 - Data taken at 185 GeV; low transverse momentum

Analysis of the data taken at 185 GeV (Fig. 7) is simplified by the fact that only positive muons reached the polarimeter. One may rewrite the decay positron spectrum given in section 2.2 as $\frac{d^2N}{d(\cos\theta)\,dx} \propto 6x^2 - 4x^3 + (4x^3 - 2x^2)\,P\cos\theta$. Let $P_F(x, \cos\theta, z)$ be the probability that a positron emitted in an aluminum slab at depth z, angle θ and energy x will be detected by the scintillation counter which is in front of the aluminum. Then the intensity for forward decays detected by the polarimeter may be written as follows:

$$F(t) = I \exp(-t/\tau_+) \int (6x^2 - 4x^3 + (4x^3 - 2x^2)\,P\cos\theta)\,P_F(x, \cos\theta, z)\,dx\,dz\,d(\cos\theta)$$

where I is the constant of proportionality for the decay positron spectrum given above (I depends upon the intensity of muons stopping in the polarimeter), t is the time elapsed after stopping, and τ_+ is the lifetime of free muons (2.2 microseconds). The magnetic field applied throughout the polarimeter causes the muons to precess. Hence the polarization of positive muons may be expressed as $P = P_+ \exp(-\gamma t) \cos\omega t$ where $\exp(-\gamma t)$ reflects decreasing polarization due to magnetic field inhomogeneity, ω is the precession frequency, and the subscript is used to denote the sign of the muon. Using this expression for P, one may write:

$$F(t) = C(1 + e_+ P_+ \cos(\omega t)\exp(-\gamma t))\exp(-t/\tau_+) \qquad (1)$$

where

$$C = I \int (6x^2 - 4x^3)\, P_F(x, \cos\theta, z)\, dxdzd(\cos\theta)$$

$$e_+ = \frac{I}{C} \int (4x^3 - 2x^2)\, P_F(x, \cos\theta, z)\, dxdzd(\cos\theta)$$

One may derive a similar formula for the intensity of backwards decays as a function of time.

$$B(t) = C(1 - e_+ P_+ \cos(\omega t)\exp(-\gamma t))\exp(-t/\tau_+) + (s)\exp(-t/\tau_+)$$

This formula contains one extra term since, as explained in section 4.1, the decays of muons which stop in the scintillation counters are classified as backwards decays. Here s is a measure of the probability that muons will stop in the counters rather than in the aluminum.

This parameterization yields the following expression for the decay asymmetry:

$$A = \frac{F - B}{F + B} = \frac{2e_+ P_+ \cos(\omega t)\exp(-\gamma t) - s}{2 + s} \qquad (2)$$

Only cursory observation of the data is necessary to conclude that the polarization of prompt muons is smaller -- or possibly of the opposite sign -- compared with the muons from meson decay. Using the density 1/3 target the ratio (prompt muons) / (muons from mesons) is about 1/2 while for the density 1 target, this ratio is approximately 2. It is clear that the increase in the fraction of muons from prompt sources is associated with a decrease in the amplitude of the sine wave which represents the decay asymmetry. The two amplitudes derived by a least squares fit of equation (2) to the data are A (1) = -0.0555 ± 0.013 and A (3) = -0.0951 ± 0.013. Values derived from these fits for the other parameters are $\gamma = 0.242 \pm 0.066$, $\omega = 3.68 \times 10^6$ / sec, and $s = 0.124 \pm 0.007$.

A quantitative analysis of the decay asymmetry for prompt muons may be made by extrapolating $I_\mu A$ to $1/\rho = 0$. Here I_μ is the intensity we have previously measured for muon production. Both I_μ and $I_\mu A$ should vary linearly with $1/\rho$. This may be easily derived on the basis of two facts. Production of muons via meson decay is proportional to $1/\rho$ since the decay time for mesons allows them to traverse many mean free paths of interaction. Prompt muon production should be insensitive to target density and hence a constant for this variable. Thus the total intensity of muon production may be expressed as $I_\mu = I_m + I_p = C_1 (1/\rho) + C_2$, where I_m and I_p are the intensities of muons from meson decay and prompt production. One may derive the linear dependence of $I_\mu A$ on $1/\rho$ by considering the average polarization of the total muon flux, $P = \frac{I_m P_m + I_p P_p}{I_m + I_p}$.

Since the decay asymmetry amplitude is proportional to the polarization, $A = C_3 P = C_3 \left(\frac{I_m P_m + I_p P_p}{I_m + I_p} \right)$ and therefore $I_\mu A = C_3 (I_m P_m + I_p P_p)$. Thus the linear dependence is expressed by $I_\mu A = C_3 P_m C_1 (1/\rho) + C_3 P_p C_2$.

As shown in Fig. 8, our measurements indicate a decay asymmetry amplitude for prompt muons of $A_p = -0.001 \pm 0.025$. Since the amplitude of the decay asymmetry is proportional to the polarization and since the decay asymmetry is consistent with zero, this shows immediately that our result is consistent with zero longitudinal polarization for prompt muons. A more quantitative analysis of the polarization of prompt muons may be made if the polarization of muons from meson decay is known. A discussion of the calculation of the polarization of muons from meson decay is contained in Appendix 1. We calculate the average polarization of these muons to be -0.70 at this kinematic point. This allows us to calculate $e_+ = 0.185 \pm 0.01$ and to conclude that the average longitudinal polarization of prompt muons produced under these kinematic conditions is $P = -0.01 \pm 0.14$.

4.3 - Data taken at 54 GeV; general considerations

Since the contributions of negative muons must also be considered in the analysis of this data, extra terms must be added to equation (1). The intensity of forwards decays may be written as:

$$F(t) = C(1 + e_+P_+ \cos(\omega t)\exp(-\gamma t))\exp(-t/\tau_+) + \tag{3a}$$
$$+ R(1 + e_-P_- \cos(\omega t)\exp(-\gamma t))\exp(-t/\tau_-)$$

The first half of this expression is the contribution from positive muons as derived in section 4.2. The second half is the contribution from negative muons and is weighted by the ratio of negative to positive muons, R. We use $\tau_- = 0.86$ microseconds for the lifetime of negative muons in aluminum, e_- is our effective analyzing power for negative muons and other terms retain the definitions given with equation (1). The intensity of backwards decays may be similarly expanded to include contributions from negative muons:

$$B(t) = C(1 - e_+P_+ \cos(\omega t)\exp(-\gamma t))\exp(-t/\tau_+) + R(1 - e_-P_- \cos(\omega t)\exp(-\gamma t))\exp(-t/\tau_-) + s(1+R)\exp(-t/\tau_+) \qquad (3b)$$

where we have used the lifetime of free muons for both positive and negative muons in scintillator.

The value of R depends on the ratio (prompt muons) / (muons from meson decay) and hence varies with target density. One can express R in terms of the intensities of muons from meson decay and from prompt sources by $R = \frac{Q I_m + I_p}{I_m + I_p}$ where Q = (negative muons from meson decay) / (positive muons from meson decay) and we use a charge ratio of one for prompt production. One can estimate the value of Q from the known production spectra for mesons. Q may also be determined from the data by fitting $\frac{dN}{dt} = N_1 \exp(-t/\tau_+) + N_2 \exp(-t/\tau_-) + N_3$ where $\frac{dN}{dt}$ is the rate of decays observed. τ_- (0.86 microseconds) was used as the lifetime of negative muons in aluminum, τ_+ (2.2 microseconds) was used as the lifetime of muons in scintillator and of positive muons in aluminum. Q may be determined from the above fit since $N_1 = 0.5 I_p + (1-Q) I_m + s(I_m + I_p)$; $N_2 = 0.5 I_p + Q I_m$. The values for all these quantities except Q have been measured in our experiment. The fit of the data is consistent with our estimate of $Q = 0.8$ at this kinematic point.

4.4 - Transverse polarization measurement at 54 GeV

Data runs were taken with the magnetic field pointed eastward through the polarimeter and with the magnetic field pointed westward. Let ($\pm$) indicate the two possible directions of the magnetic field and φ be the angle between the plane of production of the muon and its polarization vector (Fig. 9). The expressions for the forward and backward intensities of decay as a function of time as given by equations (3) may be slightly modified to incorporate this information.

$$F(\pm) = [1 + e_+ P_+ \cos(\omega t \pm \varphi) \exp(-\gamma t)] \exp(-t/\tau_+) + R[1 + e_- P_- \cos(\omega t \pm \varphi) \exp(-\gamma t)] \exp(-t/\tau_-) \qquad (4)$$

$$B(\pm) = [1 - e_+ P_+ \cos(\omega t \pm \varphi) \exp(-\gamma t)] \exp(-t/\tau_+) + R[1 - e_- P_- \cos(\omega t \pm \varphi) \exp(-\gamma t)] \exp(-t/\tau_-) + s(1+R) \exp(-t/\tau_+)$$

Since $P_{transverse} = P \sin\varphi$ is the polarization of the muons perpendicular to their plane of production, we can obtain a measurement of this transverse component by analyzing the data according to the formula below which may be derived from (4).

$$\frac{F(+)+B(-)-F(-)-B(+)}{F(+)+B(-)+F(-)+B(+)} = \frac{e_+ P_{+tran} \exp(-t/\tau_+) + Re_- P_{-tran} \exp(-t/\tau_-) \quad 2\exp(-\gamma t)\sin(\omega t)}{2\exp(-\gamma t)[\exp(-t/\tau_+) + R\exp(-t/\tau_-)] + s(1+R)\exp(-t/\tau_+)}$$

Least squares fits of the data to this form yield values for γ and s which are consistent with our measurements on 185 GeV muons. The muon precession frequency, ω, was measured to be 2.41×10^6 /sec where the difference from the previous measurement is due to a change in current through the polarimeter magnet coils. We use $e_+ = 0.185$ as determined from the 185 GeV data and $e_- = 0.01$ is used as our analyzing effectiveness for negative muons.

The measured polarization of muons produced using the density one target was $P(1)_{tran} = -0.023 \pm 0.062$. The polarization measured for muons using the density 1/3 target was $P(3)_{tran} = 0.021 \pm 0.048$. A plot of this data is shown in Fig. 10. Assuming that the muons from meson decay have no transverse polarization, we can compute from these numbers the transverse polarization of promptly produced muons. $P(1\ \text{prompt})_{tran} = -0.047 \pm 0.127$ and $P(3\ \text{prompt})_{tran} = 0.081 \pm 0.185$. Using these two measurements, we find a best value for the transverse polarization of prompt muons in the direction perpendicular to the plane of production to be $P(\text{prompt})_{tran} = -0.01 \pm 0.11$ in the direction $(p_p \times p_\mu)$.

4.5 - Longitudinal polarization measurement at 54 GeV

Equations (4) yield the following formula for the decay asymmetry where $P_L = P\cos\varphi$ is the longitudinal component of the muon polarization.

$$\frac{F-B}{F+B} = \frac{[e_+ P_{L+} \exp(-t/\tau_+) + Re_- P_{L-} \exp(-t/\tau_-)]\cos(\omega t)\exp(-\gamma t) - (s/2)(1+R)\exp(-t/\tau_+)}{\exp(-t/\tau_+) + R\exp(-t/\tau_-) + (s/2)(1+R)\exp(-t/\tau_+)} \quad (6)$$

The values obtained for γ, ω, R and s in previous measurements agreed well with least squares fits to this data (Fig. 11). We also used $e_+ = 0.185$ and $e_- = 0.01$ as with the analysis of the transverse polarization data.

Fig. 12 shows the intensity of muons stopping in the polarimeter as a function of target density. For the solid copper target ($1/\rho = 1$), 51 °/o of the muons are from meson decays while 49 °/o come from prompt sources. The observed longitudinal polarization of these muons is $P_1 = -0.492 \pm 0.085$. Using our least dense target ($1/\rho = 3$), 74 °/o of the muons are from meson decays and 26 °/o are from prompt production. We observe a longitudinal polarization $P_3 = -0.640 \pm 0.109$. As before, we have calculated the expected polarization of muons from meson decay (see Appendix 1) for this kinematic point. Muons from this source have a polarization of -0.86. Using this information the polarization of positive prompt muons may be found from the measurements at either density. For target 1, P_1 (prompt) $= -0.08 \pm 0.17$ while the target 3, P_3 (prompt) $= 0.06 \pm 0.42$. The higher uncertainty in P_3 (prompt) is due to the smaller proportion of prompt muons observed using target 3 than using target 1. If both measurements are taken into consideration, we find the longitudinal polarization of prompt muons to be -0.06 ± 0.16 in this kinematic region.

5. - Conclusions

5.1 - Production mechanisms for prompt muons

The null values of our polarization measurements are strong evidence that prompt muons are produced predominately by electromagnetic processes. Most models for the production of prompt muons via the weak decay of some intermediate particle predict that the muons will have a longitudinal polarization of about +1 (such as in the case discussed in section 2.1). Our measurement of $P = -0.01 \pm 0.14$ indicate a limit of 13°/o of the prompt muons produced with $E = 185$ GeV in the forward direction by 400 GeV protons could come from such a source. Similarly, our observation $P = -0.06 \pm 0.16$ places a limit that only 10°/o of the muons produced with $E = 54$ GeV and a transverse momentum near 1.9 GeV could come from such a source.

Strong corroborative evidence for the conclusion that most prompt muons are produced electromagnetically has come from other experiments. Measurements of the rate of production of prompt muon pairs[11, 21] are consistent with the hypothesis that most prompt muons are produced in pairs. Although this is required by any electromagnetic production mechanism, it would be much more difficult to account for in any theory which postulates that a substantial number of prompt muons come from the decay of some intermediate particle. If we consider the creation of pairs of some heavy particle, each of which may decay by a semileptonic mode, then the ratio of single muons to muon pairs is limited by the branching ratio for this mode of decay. If for example, the branching ratio for a decay to a muon is 20°/o, then at least four times more single muons will be created than muon pairs. The creation of pairs of heavy leptons which decayed to muons could be a possible source for prompt muon pairs but recent results[22] indicate that at most only a small fraction of the total flux of prompt muons come from this source.

It should be noted that the results reported here differ by more than two standard deviations from the numbers reported for a polarization measurement at Serpukhov.[23] The polarization measurements were made for prompt muons produced by 70 GeV protons. The number reported as the result for the average polarization from measurements at transverse momenta of 2.0 and 2.8 GeV/c is -0.85 ± 0.36. We feel this is fairly difficult to reconcile with our measurement of $P = -0.06 \pm 0.16$ for 54 GeV muons produced at a transverse momentum of 1.9 GeV at Fermi Lab. These differences in measured polarizations cause a sharp divergence in the conclusions reached concerning the production mechanism for prompt muons. The theory advanced in reference 23 is that prompt muons are produced by the decay of some formerly unobserved particle which decays via a $V + A$ current.

As with our experiment, the Serpukov result depends on separating the polarization signal of promptly produced muons from that of muons produced by meson decay. This group at Serpukov has previously measured the intensity of prompt muon production and reported two different possible results depending upon different methods of evaluating their data.[24] These two different procedures for evaluating their data give much different results for the ratio (muons from prompt production) / (muons from meson decay). In the interpretation of their polarization data, the Russians have chosen to use the intensity results which give a higher flux of prompt muons compared with meson production and which also have a (prompt μ^+) / (prompt μ^-) ratio of 1.2. Their other method for evaluating the production of prompt muons gave a lower ratio for (prompt muons) / (meson production) and a charge ratio of 1. It should be noted that - 0.85 is very near the polarization which one can calculate for muons produced by meson decay under these kinematic conditions while the result predicted by most theories of prompt muon production through the decay of heavy intermediate particles is +1.

No paper has been published in any journal which gives the data for the Serpukhov measurement at $p_t = 2.0$ GeV/c. In reference 23, the data and formula used to compute the polarization of prompt muons produced at $p_t = 2.8$ GeV/c are given. This number may easily be computed and the result (which is not given in their text) is $P = -1.95 \pm 1.0$ where I have used the uncertainty indicated by the Russians in the graph on the last page of reference 23 (see Fig. 13). One must treat such results with a high degree of skepticism.

On the basis of available evidence, we conclude that the bulk of prompt muons are electromagnetically produced. Given this fact, further questions must be addressed concerning a more exact description of the production mechanism. Of course, a fraction of prompt muons come from well understood electromagnetic processes -- the decay of known vector mesons and Dalitz decay of etas. However, these sources fall far short of accounting for the total flux of prompt muons.[4-6] Two possibilities which must be considered are: (1) the muons may come from the internal conversion of photons created during the acceleration of quarks which are the constituents of colliding hadrons, or (2) they may be produced through the conversion of photons which come from quark annihilations taking place during the hadron collision. These two processes may be denoted as quark bremsstrahlung and quark annihilation.

Certainly each of these processes can be expected to contribute some portion of the flux of prompt muons. The invariant mass spectrum of muon pairs produced by quark bremsstrahlung would be expected to peak at a relatively small value while muon pairs produced through quark annihilations could easily have masses in the range 0.5 - 1.0 GeV/c^2. Examination of the available data indicates that the invariant mass spectrum of promptly produced muon pairs may be explained as the sum of three main contributions: (1) peaks due to the decays of known vector mesons, (2) a low mass continuum which is the result of Dalitz decay of etas, $\omega \to \gamma\mu\mu$ and quark bremstrahlung and (3) a high mass continuum due to quark annihilation.

5.2 - Upper limit on D production

Besides providing strong evidence that the bulk of prompt muons are produced electromagnetically, the polarization measurements discussed in this paper may be used to place an upper limit on $B_{\mu}(D^{o})\,\sigma_{D^{o}}$ + $B_{\mu}(D^{+})\,\sigma_{D^{+}}$. Here $\sigma_{D^{o}}$ and $\sigma_{D^{+}}$ are the production cross sections for D^{o} and D^{+} particles. $B_{\mu}(D^{o})$ and $B_{\mu}(D^{+})$ are the branching ratios of these particles to decays involving a muon. As discussed in section 2.1, the expected polarization of muons from a decay such as $D^{+} \rightarrow \bar{K}^{o}\mu^{+}\nu_{\mu}$ is near +1. However, our measurements indicate that only a small percentage of the prompt muon flux could come from a source which produces positive muons with this polarization. Since negative muons make only a small contribution to the polarization which we measure, we are insensitive to the production of the D^{-} and $\bar{D}^{o}$ whose decays are expected to produce negative muons. Monte Carlo calculations indicate that our measurements place an upper limit of $B_{\mu}(D^{o})\,\sigma_{D^{o}} + B_{\mu}(D^{+})\,\sigma_{D^{+}} <$ 6.7×10^{-8} barns. More details concerning this calculation are contained in Appendix 2.

Appendix - 1 The polarization of muons from meson decay

Let us consider the decay of pions in flight. According to the helicity rules, the neutrino produced in π^+ decay must have its spin aligned opposite to its direction of motion. Therefore, the μ^+ must also be emitted in the rest frame of the pion with negative helicity in order to conserve angular momentum. Similarly, in the decay of negative pions, both the anti - neutrino and μ^- must be produced with positive helicity in the rest frame of the pion. Although all the positive muons have a longitudinal polarization of -1 in the rest frame of their parent pions, an experimenter may expect to observe a much different polarization in the laboratory frame. Muons produced through pion decay have a velocity of 0.27c in the pion's rest frame. If the lab velocity of the pion is greater than this, the muon will travel in the forward direction in the lab regardless of the direction in which it is emitted in the rest frame of the pion. Thus, muons produced in the forward direction in the pion's rest frame will have negative helicity in the lab and those produced backwards in the rest frame of the pion will have positive helicity in the lab. In any particular range of muon energy, this will lead to an average muon polarization which is negative. This is because pions are produced with a steep energy dependence. Hence, for a particular muon energy, there are more muons produced by forward decays (of low energy pions) than by backward decays (of higher energy pions).

The average polarization of muons produced with a particular energy by pion decay may be computed if the energy spectrum of the parent pions is known. It has been shown[25] that the spin direction of a muon, as observed in the lab system, is given by the unit vector

$$s_{lab} = \frac{1}{m_\mu^2 + P_\pi \cdot P_\mu} \left(E_\pi \vec{P}_\mu - m_\mu \vec{P}_\pi - \frac{\vec{P}_\pi \cdot \vec{P}_\mu}{E_\mu + m_\mu} \vec{P}_\mu \right)$$

where $P_\pi = (E_\pi, \vec{P}_\pi)$, $P_\mu = (E_\mu, \vec{P}_\mu)$ are the four - vector momenta of the parent pion and muon. To find the expected longitudinal polarization of muons with a given energy produced by such a source, one need only find the weighted average of the longitudinal component of the spin vector defined above.

$$P = \int \frac{\gamma_\pi m_\mu^2 - E_\mu E'_\mu}{|\vec{P}| \cdot |\vec{P}'|} \cdot N(\gamma_\pi)\, d\gamma_\pi$$

where γ_π is the ratio of the pion's energy to its rest mass. $P'_\mu = (E'_\mu, P'_\mu)$ is the four - vector momentum of the muon in the rest frame of the pion and $N(\gamma_\pi)$ is a weighting function which takes into account both the pion energy spectrum and the probability that a pion with a particular energy will produce a muon with energy E_μ. The integration is performed over all pion energies which are allowed by kinematics to contribute to the flux of muons whose laboratory energy is E. One may convert the above integral into a discrete sum for calculation using a computer. For example, the expected longitudinal polarization of 185 GeV muons produced by pion decay is

$$P = \frac{\sum_{E_\pi} \frac{m_\mu^2 (E_\pi / m_\pi) - E_\mu E'_\mu}{|\vec{P}_\mu| \, |\vec{P}'_\mu|} (\exp(-8.6 x_\pi))(1/x_\pi^2)}{\sum_{E_\pi} \exp(-8.6 x_\pi)(1/x_\pi^2)}$$

where I have used $\frac{d\sigma_\pi}{dE} \propto \exp(-8.6 x_\pi)$ as the energy dependence for the production of pions and $x_\pi = E_\pi / 400$ GeV. The expected polarization of these muons is -0.61.

If one is interested in computing the longitudinal polarization of muons produced with a particular energy and a specified transverse momentum, then another factor must be added to the weighting function. In our experiment at Fermilab we observed muons at an angle of 45 mr from the beam center line. Clearly, the parent pions must have been produced at some angle near this. In fact, Monte Carlo calculation of multiple Coulomb scattering indicates that a typical muon which we detected at 45 mr was actually produced at about 35 mr. The equation used to express the transverse momentum dependence for pion production was $\frac{d\sigma_\pi}{dp_t} \propto p_t \exp(-p_t/250 \text{ MeV})$. With the addition of this factor, the equation for the expected polarization of muons from pion decays becomes

$$P = \frac{\sum_{E_\pi} \frac{m_\mu^2 (E_\pi / m_\pi) - E_\mu E'_\mu}{|\vec{P}_\mu| \, |\vec{P}'_\mu| \cdot x_\pi^2} \exp(-8.6 x_\pi) E_\pi \sin\theta \cdot \exp(-E_\pi \sin\theta / 250 \text{ MeV})}{\sum_{E_\pi} \exp(-8.6 x_\pi)(1/x_\pi)^2 E_\pi \sin\theta \cdot \exp(-E_\pi \sin\theta/250 \text{ MeV})}$$

where θ is the angle between the direction in which a pion is produced and the beam center line. Again the sum is over all pion energies which can possibly produce muons with the proper energy. The expected polarization in the laboratory of muons produced by pion decay with $E_{\mu} = 54$ GeV and $P_t = 1.9$ GeV/c is -0.80.

The formulas given above for the polarization of muons produced by pion decay may be applied to the case of two body K - meson decays into a muon and neutrino. One need only replace E_{π} by E_k, m_{π} by m_k and use the values for $P'_{\mu} = (E'_{\mu}, \vec{P}'_{\mu})$ which are correct for the rest frame of a parent kaon. Since a muon produced by kaon decay has a larger energy in the rest frame of its parent than a muon produced by pion decay, there is a corresponding greater difference in the laboratory energy of muons produced by forwards and backwards decays. For example, a 55 GeV muon may be produced by a forward decay from a 55 GeV pion or kaon and also by a backwards decay from a 96 GeV pion or a 2500 GeV kaon. As the production spectra of pions and kaons are fairly similar, this means that muons produced by two body kaon decay will be more polarized than those from pion decay. This is because in the case of kaon decay a larger percentage of muons with a particular energy will be produced by a forward decay (from a low energy parent) than by a backward decay (from a high energy parent). Indeed, calculation shows that for 185 GeV muons produced in the forward direction by two body kaon decay, the expected longitudinal polarization in the laboratory is -0.98; while for 54 GeV muons produced with $p_t = 1.9$ GeV/c by two body kaon decay, the expected polarization is -0.99.

Although the two body decay of a kaon to a muon and neutrino is the decay mode which is most important in this experiment, it is not the only mode which must be considered. Osborne[26] has estimated the average polarization which may be expected from muons produced by kaon decays. As previously noted, the production spectrum of kaons has a steep energy dependence. Hence, those decay modes in which the muon retains a large fraction of the kaon energy are more likely to contribute muons which will be observed in our experiment. Ranked according to their significance in our measurements, various modes in which muons may be produced by kaon decay are:

1) directly in two body decay
2) indirectly through the decays of kaons to pions
3) directly in three body decays.

In two body decays of kaons to muons and neutrinos, the average muon will retain 0.58 of the energy of the parent kaon; for the case of the decay of a kaon to two pions and the pions to muons, these muons will, on the average, have 0.39 of the parent kaon energy; while for three body decay modes (either to three bodies which include a muon or indirectly through three body decays which include a pion), the average muon will be produced with an even smaller fraction of the energy of the parent kaon.

The branching ratio for two body decays of kaons to muons and neutrino is $64^{o}/o$ for both K^{+} and K^{-}. Decays of kaons to particles which include pions and no muons account for nearly $100^{o}/o$ of K - short decays and $28^{o}/o$ of both K^{+} and K^{-} decays. Taking into account these large branching ratios and the different fraction of the parent kaon energy which is retained for various decay modes, it can be seen that the contribution from kaon decays to three particles which include a muon is small. This is fortunate as it is not easy to calculate the expected polarization of muons produced by $K_{\mu 3}$ decay.

As noted previously, muons produced directly via two body decays of kaons will have longitudinal polarizations in the laboratory of - 0.98 and - 0.99 for our measurements at 185 and 54 GeV respectively. Muons produced by the decay of kaons to pions will have an expected polarization near that which was computed in the first part of this Appendix since they are produced from the decay of pions (i.e. - 0.61 and - 0.80 for our 185 and 54 GeV measurements). Osborne estimates that muons produced via $K_{\mu 3}$ decay may have a polarization whose sign is opposite to that of muons produced through the other modes and may have a magnitude near 0.27. The energy dependence for the production of K mesons in our experiment is much steeper than the spectrum used to make this estimate. Consequently, 0.27 is an overestimate of the polarization of muons from $K_{\mu}3$ decay produced in our experiment.

The expected polarization of muons from meson decay may be computed by averaging the calculated polarizations for each decay mode. Each term in the average must be weighted according to the number of muons produced by that mode and the likelihood that the muon would have the correct energy and transverse momentum to reach our apparatus. We find the expected polarization of muons from meson decay to be -0.70 and -0.86 for our 185 and 54 GeV measurements described in the main text. Due to possible small errors in meson energy spectra and in the contribution from $K_{\mu 3}$ decay, each of these magnitudes may be in error, though probably not by more than 5°/o.

Appendix - 2 Calculation of the upper limit on D production

Several searches for the production of charmed mesons by hadronic interactions have been conducted.[27-30] In no case has any positive indication been found. The measurements discussed in this paper may be used to set a much lower limit on the production of D mesons in proton nucleon collisions than the 2×10^{-6} barns limit set by the most recent publication.[30] It should be noted that the mass of the D was not known when these experiments were performed and that, in general, they were not sensitive to the production of particles whose masses were less than 2 GeV.

Calculation of an upper limit on $B_\mu(D^o)\,\sigma_{D^o} + B_\mu(D^+)\,\sigma_{D^+}$ may be done on the basis of our polarization measurements and a few reasonable assumptions. One must ascertain the level at which muons could be produced by the decays of the D^o and D^+ under the requirement that this level must be consistent with our observations. The D^+ and D^o were assumed to have masses of 1.876 and 1.865 GeV/c^2 respectively and decay via a V - A current to $K\mu\nu$ (see Fig. 14). Although the details of the production spectrum of the D are not known at this time, it seems likely that the x and p_t dependence of D production should be roughly similar to that of the J. For the purpose of this calculation, we have assumed $\frac{d^2\sigma_D}{dx\,dp_t} \propto \exp(-10x)\exp(-1.5\,p_t)$ in the center of momentum frame of the two colliding nucleons.

The production and decay characteristics given in the previous paragraph provide a well - defined shape for the muon spectrum generated by D decay. This shape is probably fairly accurate; however, it should be noted that if the true production spectrum is substantially different from that given above, or if the decay of the D to leptons takes place predominately via some other mode, then the shape of the muon spectrum generated by D decay will be altered. As examples, if the true production spectrum of the D goes as exp (- 11.5x), then fewer muons from D decay would reach our apparatus, or if the true production spectrum of the D is proportional to exp (- 1.0 p_t), then more muons from D decay would reach our apparatus. Later in this Appendix our upper limit on $B_\mu (D^o) \sigma_{D^o} + B_\mu (D^+) \sigma_{D^+}$ will be given in conjunction with the effects of different production spectra on this calculation.

It seems likely that little error is made in our calculation of the shape of the muon spectrum from D decay by assuming that the muons are predominately produced via $D \to K\mu\nu$. Written in terms of quarks this is $c\bar{d} \to s\bar{d} + \mu\nu$ for $D^+ \to \bar{K}^o \mu^+ \nu_\mu$ and $c\bar{u} \to s\bar{u} + \mu\nu$ for $D^o \to K^- \mu^+ \nu_\mu$. These decays satisfy the rule $\Delta Q = \Delta S = \Delta C$, $\Delta I = 0$ and, hence, are heavily favored compared with other possible modes.[31] Two body decay, $D^+ \to \mu^+ \nu_\mu$, would give muons with a much higher kinetic energy in the rest frame of the D; however, it is unlikely that this mode has a high branching ratio. Lee and Gaillard[31] estimate $(D^+ \to \mu^+ \nu) / (D^+ \to \bar{K}^o l^+ \nu) = (m_D / m_K) \times 10^8 / [(m_D / 1 \text{ GeV})^5 \times 10^{10}] = 4 \times 10^{-4}$. The decay $D \to K^* \mu\nu$ is also possible, but is must certainly have a smaller branching ratio than $D \to K\mu\nu$, as there is less phase space available for the decay to K^*. It should also be noted that $D \to K \ldots$ is a case of $0^- \to 0^-$ so that the leptons produced in these decays may form an S - wave state while $D \to K^* \ldots$ is a $0^- \to 1^-$ and, therefore, the leptons are induced to form a P - wave. However, these factors do not necessarily dictate that the muons produced by $D \to K^* \mu\nu$ will have very different kinetic energies in the rest frame of the D than those produced by $D \to K\mu\nu$. A comparison of Dalitz plots (Fig. 1 and Fig. 15) shows that in each case the bulk of the muons are relativistic in the rest frame of the D, though a typical muon does receive a somewhat lower energy in $D \to K^* \mu\nu$ than in $D \to K\mu\nu$. For instance, the upper limit on the kinetic energy which the muon may receive in K^* production is 623 MeV, while 769 MeV is the maximum possible kinetic energy received by the muon in $D \to K\mu\nu$. Thus, the contribution from muons produced by $D \to K^* \mu\nu$ will probably not make a large difference in the shape of the muon spectrum expected from D decay.

We may now place constraints on the heights of various points within this spectrum. Muons created by D decay will have a polarization near 1 as discussed in section 2 of this paper. However, muons created by any electromagnetic process must necessarily have 0 longitudinal polarization. This is because electromagnetic processes must conserve parity. We have measured $P = -0.01 \pm 0.14$ for 185 GeV prompt muons produced in the forward direction and $P = -0.06 \pm 0.16$ for 54 GeV prompt muons with p_t near 1.9 GeV/c. Therefore, 13°/o of the total flux of prompt muons is the upper limit which may be set for the production of muons via D decay at the first point while a 10°/o limit may be set at the second.

We have also measured the production characteristics of prompt muons.[6, 11] It certainly seems a good approximation to suggest that the production spectrum for electromagnetically produced muons is close to that which we measured for prompt muons as a whole. Consequently, our measurement at p_t of 1.9 GeV places a more severe restriction on the production of muons by D decay than our measurement at low p_t. This is due to the steeper dependence upon p_t of muons produced electromagnetically compared with those from D decay.

Monte Carlo calculation indicates that if equal numbers of muons were created by D decay and by the conversion of photons, then in the kinematic area corresponding to our measurement at high p_t, the ratio (muons from D decay) / (muons from photons) would be 10. The calculation was based on electromagnetic creation of muon pairs with a production spectrum $\frac{d^2\sigma}{dx\, dp_t} \propto \exp(-3.5\, p_t) \exp(-10.4x)$ and the mean invariant mass of these pairs was assumed to be about 0.8 GeV, which is consistent with our measurements.[11] The result of this calculation may be combined with our measurement of the total cross section for prompt muon production (about 6.7 microbarns) and our null measurement of polarization (an upper limit of 10°/o of the total prompt muon flux at $E = 54$ GeV and $P_t = 1.9$ could come from a source with $P = 1$) to give an upper limit on $B_\mu(D^0)\,\sigma_{D^0} + B_\mu(D^+)\,\sigma_{D^+}$.

The null polarization measurement indicates:

$$\frac{\text{(number of muons from } D^{o} \text{ and } D^{+} \text{ decay in this kinematic area)}}{\text{(number of muons from electromagnetic processes in kinematic area)}} \leq 0.1$$

The Monte Carlo calculation mentioned at the beginning of this paragraph indicates:

$$\frac{10 \cdot \text{total number of muons from } D^{o} \text{ and } D^{+} \text{ decay}}{\text{total number of muons from em. processes}} = \frac{\text{number of muons from } D^{o} \text{ and } D^{+} \text{ Decay in this kinematic area}}{\text{number of muons from em. processes in this kinematic area}}$$

Combining this with the inequality above gives:

$$\frac{10 \cdot B_{\mu}(D^{o})\,\sigma_{D^{o}} + B_{\mu}(D^{+})\,\sigma_{D^{+}}}{6.7 \times 10^{-6} \text{ barns}} \leq 0.1$$

Therefore, $B_{\mu}(D^{o})\,\sigma_{D^{o}} + B_{\mu}(D^{+})\,\sigma_{D^{+}} \leq 6.7 \times 10^{-8}$ barns.

As previously explained, this calculation depends upon the form used for the production spectrum of the D. We have assumed that the production spectrum is proportional to exp $(-10x)$. It is possible that the true production spectrum is somewhat different than this. Detailed Monte Carlo calculation shows that if the true production spectrum is proportional to exp $(-11.5x)$, then the sensitivity of our experiment to the production of muons by D decay is changed by less than 10 % (i.e. the change in our upper limit would be less than 10 %). However, we are quite heavily dependent upon the exponent used for the transverse momentum dependence of the production spectrum. If one believes that the production spectrum of the D is proportional to exp ($-p_t$) rather than exp ($-1.5\ p_t$), then one would be entitled to change the upper limit set by our measurement by a factor of about 3 (the upper limit would be about 2.2×10^{-8} barns).

Appendix 3 - Polarimeter Interface

1. Design Considerations

The function of the polarimeter interface was to monitor the scintillation counters in the polarimeter during the time immediately after a muon stopped within the polarimeter. This was done in order to gather information about the decay electrons from these muons. Signals from all the counters which detected a particle within 12.7 microseconds after a muon stopped were noted by the interface. The position of each counter and the time at which it was hit were recorded by the interface, and this information was sent to the computer. This time interval was deemed sufficiently long (about six decay lifetimes for a free muon) to detect decay electrons and, in fact, was long enough to give us a good measure of the rate of "accidental" decays which were due to a polarimeter counter being fired by some source other than a decay electron. These accidentals were constant as a function of time.

For the purposes of discussion, the interface may be considered to consist of three parts, each of which is considered in a separate section of this Appendix. In reality, each part of the interface was connected to the other two parts by many lines, and few of the operations performed within the interface were completely limited to a single part.

2. Emitter coupled logic

ECL was used for those functions within the interface where the greatest speed was desired. A typical time delay encountered between arrival of input signals to an ECL chip and the beginning of the appropriate output from that chip is 2 nanoseconds for series 10, 000 Motorola ECL chips. Motorola chips from different series and chips of other manufactures are within about a factor of two from this speed.

One operation handled by ECL was the treatment of the "Trigger" signal which indicated that a muon had stopped within the polarimeter. This Trigger was ignored if an earlier event was still in the process of being recorded by the interface or being read by the computer. It was also ignored if for any reason an "Inhibit" signal was sent to the interface through the Camac dataway system. This feature could be used to reject triggers which did not occur during the beam spill or to reject triggers which occured during a spike in the beam spill. If a Trigger was accepted then it was converted from ECL to TTL logic levels and sent to the TTL part of the interface.

A second operation performed by ECL was the initial handling of signals from each of the polarimeter counters. When a Trigger was accepted, 25 "latches" were opened -- a separate latch for each counter. During the next 12.7 microseconds, any signal which indicated that a counter had detected a particle caused the latch for that counter to be closed. The purpose of each of these latches was to open when a muon stopped within the polarimeter and to close if a decay electron was detected in the counter connected to that particular latch. Provision was made so that we could avoid having latches closed by a second particle entering the polarimeter while the latches were open. Signals from each of the latches were converted from ECL to TTL logic levels and sent to the TTL part of the interface.

3. Transistor - Transistor logic

TTL was used for those functions of the interface where time delays and pulse widths on the order of 20 nsec. were acceptable. Included in this part of the interface was a 20 megahertz clock signal which was generated using a quartz crystal. The frequency of this signal was cut in half by using a flip flop resulting in a clock signal whose period was 100 nsec. This was used to determine the time elapsed between the arrival of a Trigger (indicating a stopped muon) and the closing of a latch (indicating the possible detection of a decay electron).

When a Trigger was sent to TTL, 25 separate "gates" were opened. Each gate then allowed clock pulses to pass through it and be counted by an eight bit binary counter. If a latch was closed at some subsequent time, then the gate corresponding to that particular latch was closed and no more clock pulses were counted in that particular eight bit counter. There was no latch connected to one of these gates. When the number of counts in the eight bit counter attached to this gate reached 128, then the rest of the gates were closed. All counters which had counted up to 128 were ignored. The only counters which had less than 128 counts in them were those whose gates had been shut when a latch had closed. Thus, if any of the polarimeter counters detected a particle in less than 12.8 μ sec. after a muon stopped within the polarimeter, then an eight bit binary counter retained the information as to when this particular detector had been hit. This part of the interface may be thought of as 25 very fast digital stop-watches all of which were started at the same time by a muon stopping within the polarimeter, with the "stop" control of each stopwatch connected to a different scintillation counter.

The number (position) of the polarimeter counter and the elapsed time recorded by the stopwatch was sent to the Camac part of the interface for each stopwatch which had been stopped in less than 12.8 μ sec. The data was sent one word at a time with each word containing the information from one stopwatch. Priority encoders were used to select the order in which the words of data were sent. The information from the eight bit binary counters was sent to multiplexers and the output of the priority encoders was used to select the proper information from the multiplexers.

Another operation performed by this section of the interface was to determine the number of latches which had been closed. A logic chain was set up to determine whether ≥ 3, ≥ 2 or exactly 1 latch(es) had been closed before 12.8 μ sec. had elapsed. In this way, provision was made so that it was easily possible to ignore events in which several polarimeter counters were hit by some shower of particles. The information as to the number of clocks which needed to be read was sent to the Camac part of the interface as part of each word of data.

4. Camac

The specification of the logic levels used in Camac necessitated that TTL chips be used for this part of the interface. It was built in a single width module which plugged into a Camac crate and was connected to the other parts of the interface using a cable which consisted of twisted pairs of wires. One wire in each pair was held at ground. In this way, cross talk between the lines was minimized.

This part of the interface decoded signals from the Camac dataway to determine whether the interface was being addressed by the computer. If the interface was being addressed, the desired function (Clear, Initialize, Abort, Inhibit, or Read) was also decoded, and the corresponding signals were sent to the other parts of the interface. This part of the interface handled the individual words of data which were sent to the computer. It sent the signal to the TTL part which cleared out a word of data from one stopwatch and caused a new word of data from another stopwatch to be ready for the computer to read.

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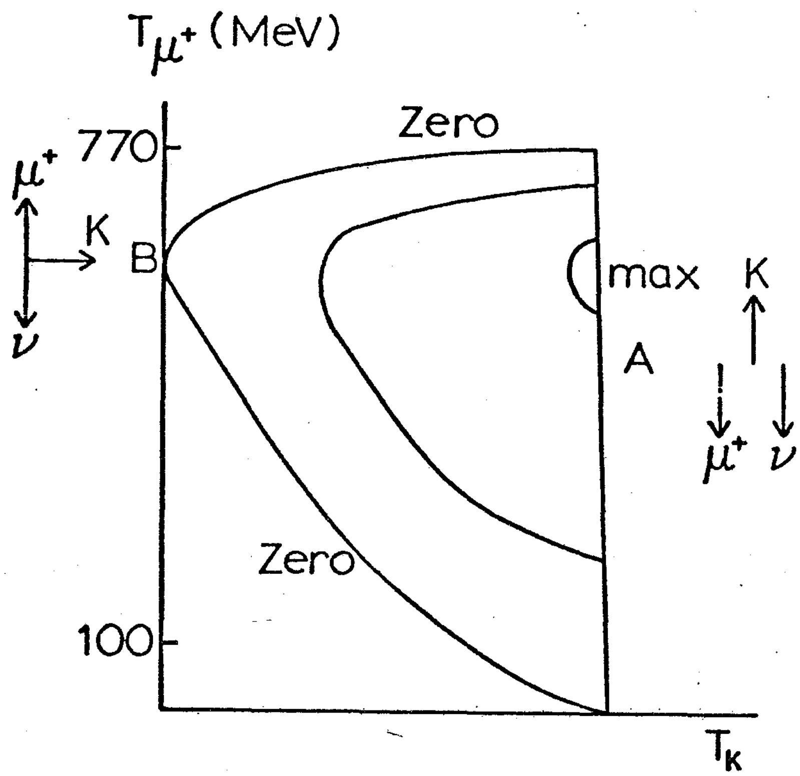
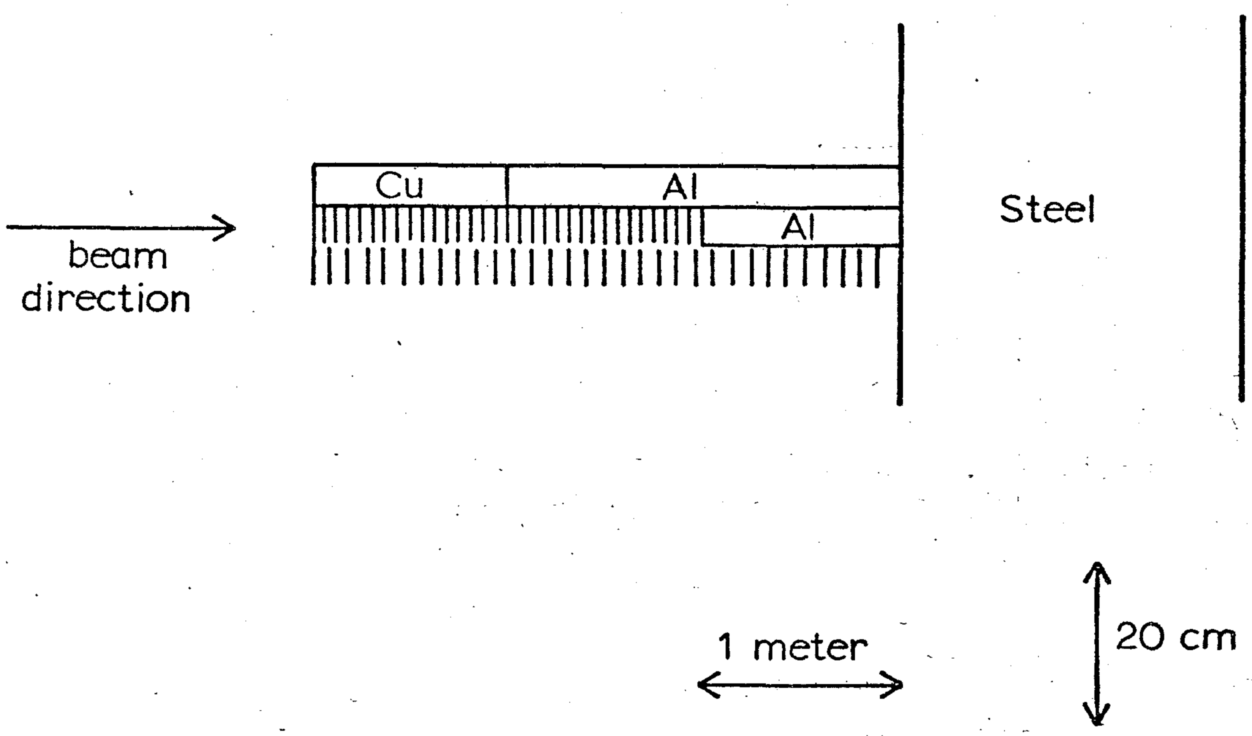


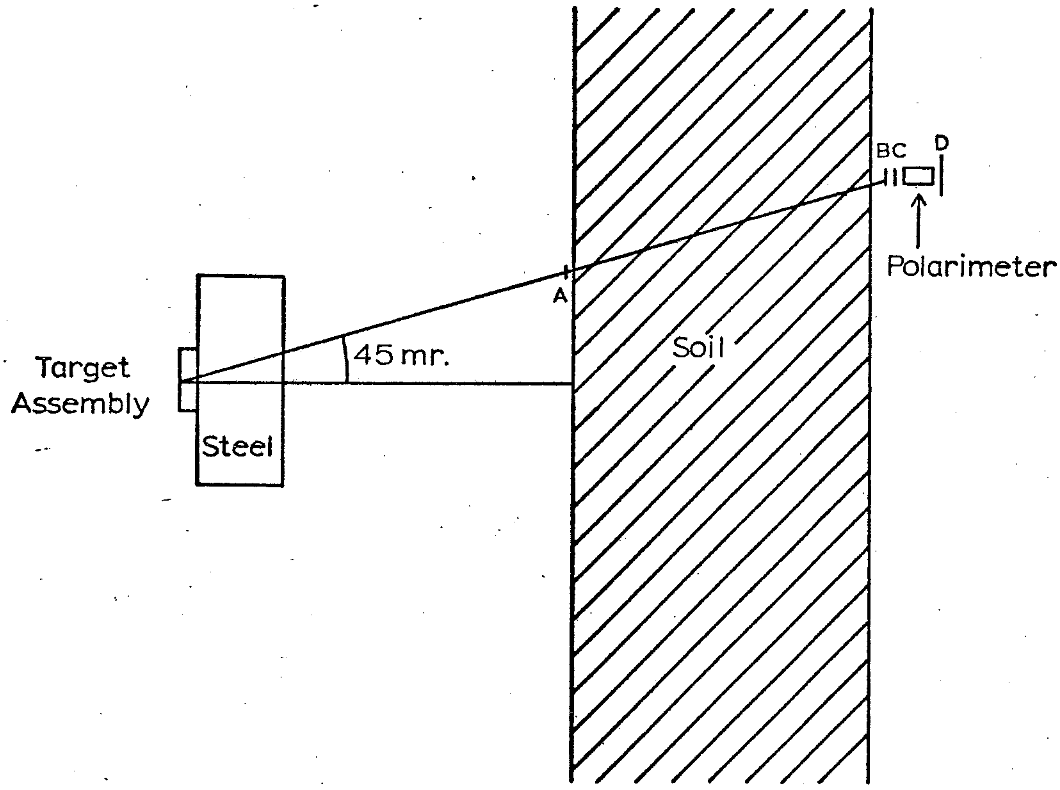
Dalitz plot for the decay $D^+ \rightarrow \bar{K}^o \mu^+ \nu_\mu$. Arrows indicate directions of particles momenta in the rest frame of the D for decay configurations corresponding to point B and line A. Those areas of the Dalitz plot which would have the highest density and lowest density of events are indicated. Note that most decays will have $T_{\mu^+} > 100$ MeV.

Fig. 1



The main target assembly and steel backing. Target 1 consisted of forty inches of solid copper backed by 2 meters of aluminum. Target 2 was constructed from 40 one inch thick copper blocks with one inch of air between each block and was backed by one meter of aluminum. Target 3 contained 40 one inch thick blocks of copper with 2 inches of air between each block.

Fig. 2



A view of the placement of the apparatus for the measurements at 54 GeV. The signals from the counters in the polarimeter were examined to look for a stopping muon when a particle hit counters A, B, and C but did not hit any counter in array D.

Fig. 3

Fig. 4

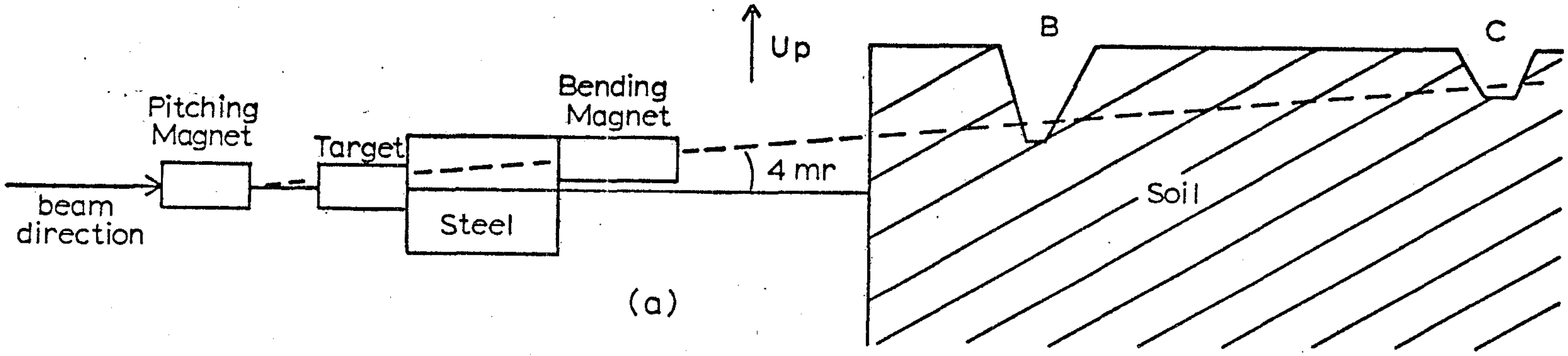


Figure (a) is a block diagram of the experiment showing the effect of the pitching magnet. The 185 GeV measurement was done in pit C, then the polarimeter was moved to pit B for the 54 GeV measurement.

Figure (b) is a block diagram of the experiment showing the effect of the bending magnet. Positive muons produced in the forward direction were directed towards the polarimeter in the C pit.

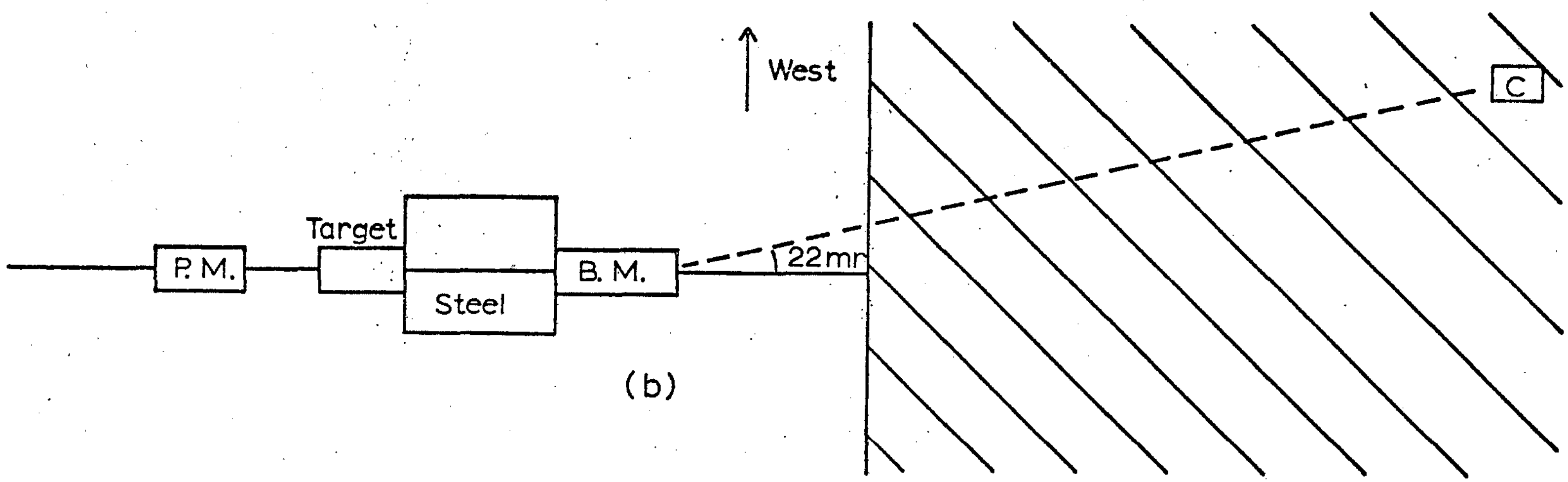
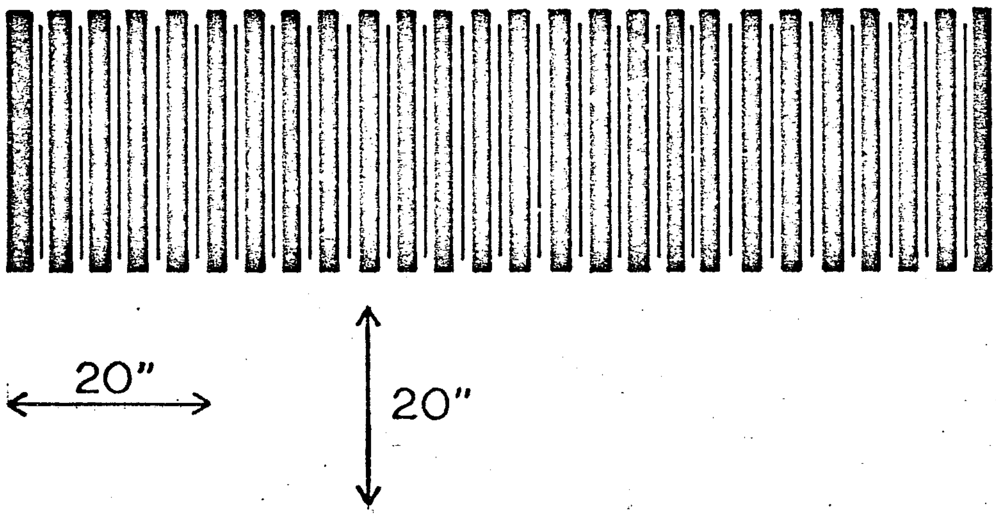


Fig. 5 (a)

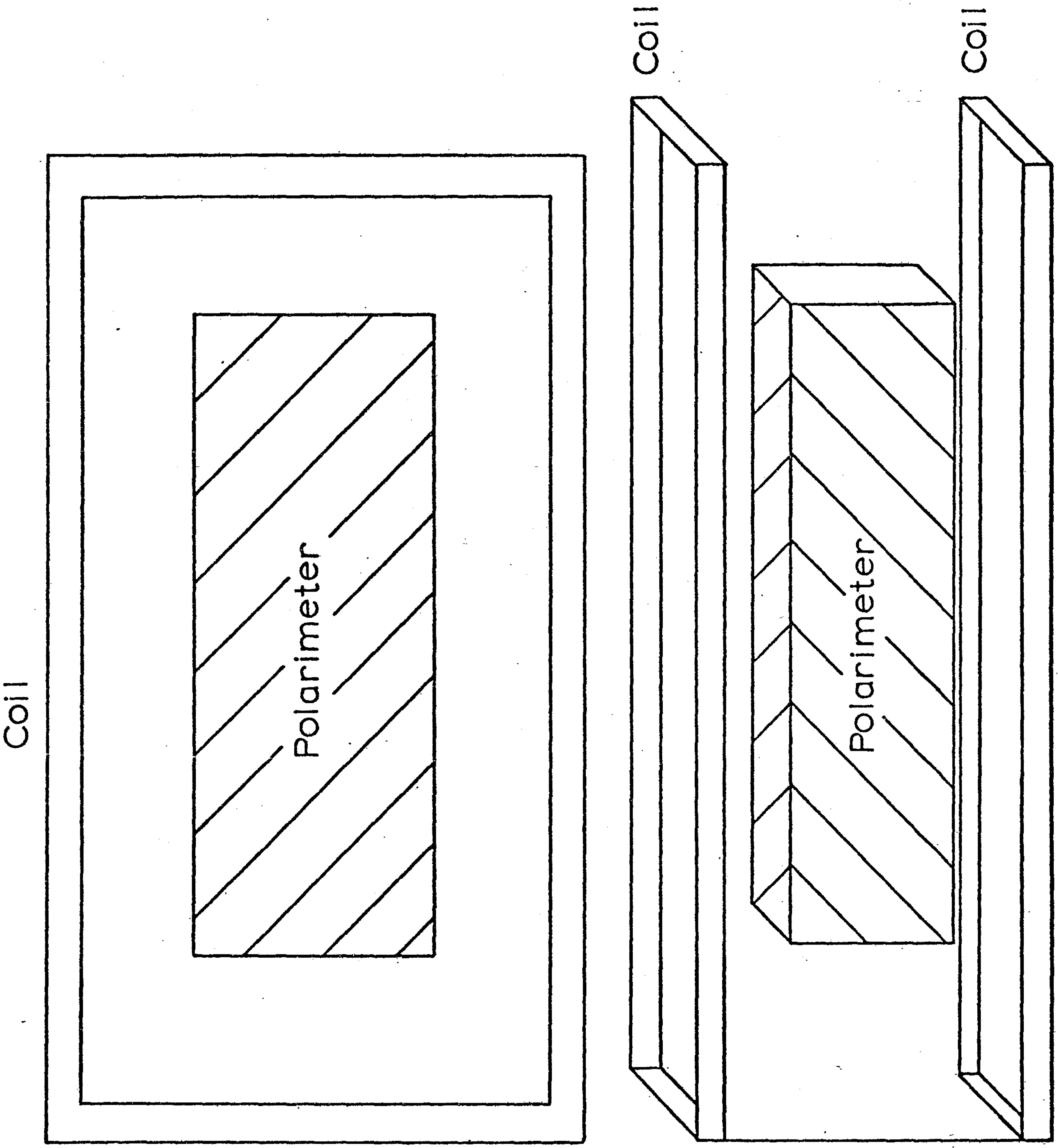


The polarimeter consisted of 26 slabs of aluminum which were 26" by 38" by 2" and 25 scintillation counters which were 24" by 36" by 0.25".

```
 1   3   5   7   9   11   13   15   17   19   21   23   25
 + + + + + + + + + + + + + + + + + +
                                      7
```

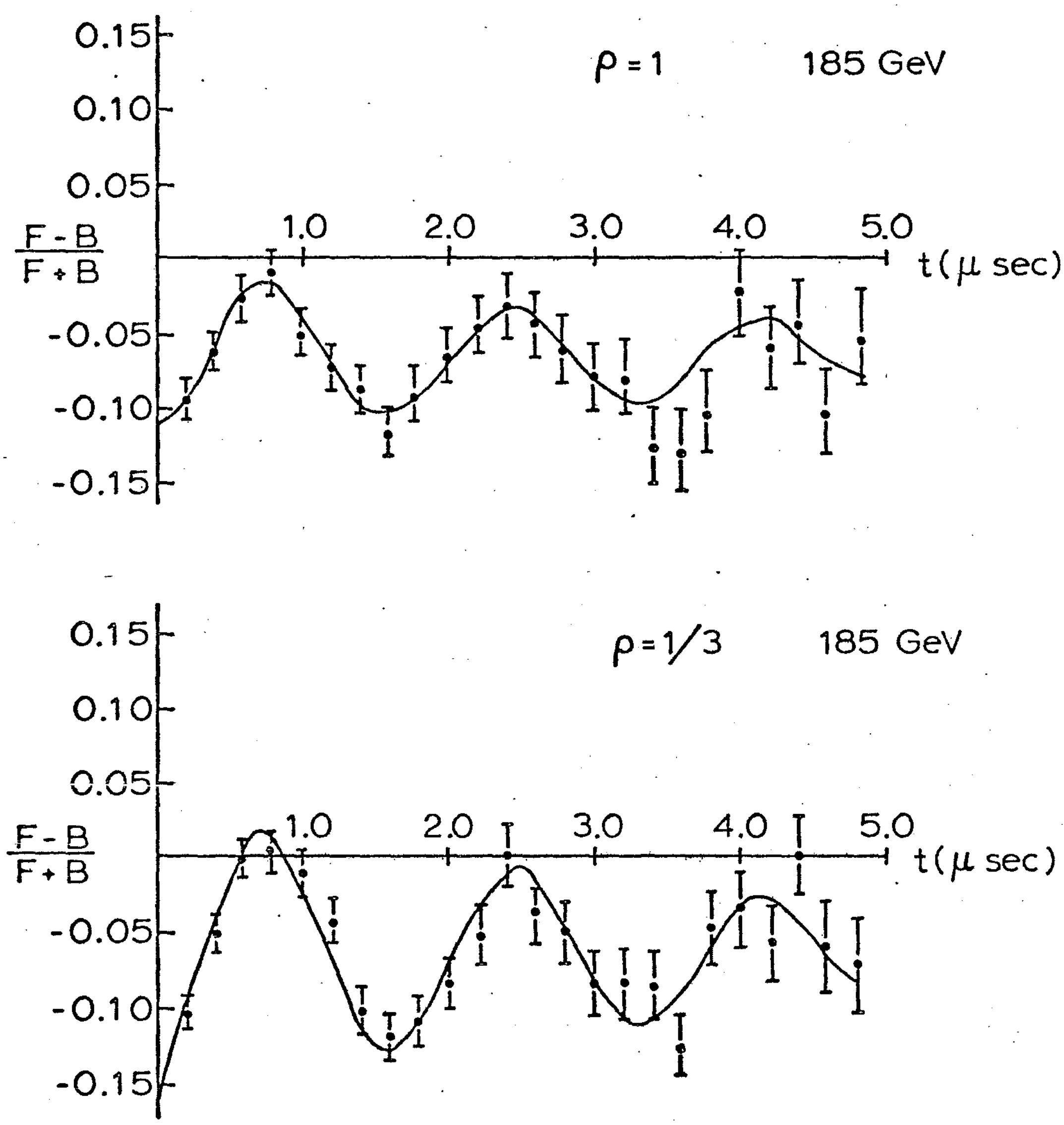
Fig. 5 (b)

This computer output of a typical event shows that a muon was detected in counters 1 through 18. The decay positron was detected in counter 19 at a time 1.4 microseconds after the muon stopped. This event would be defined as a "forward" decay.



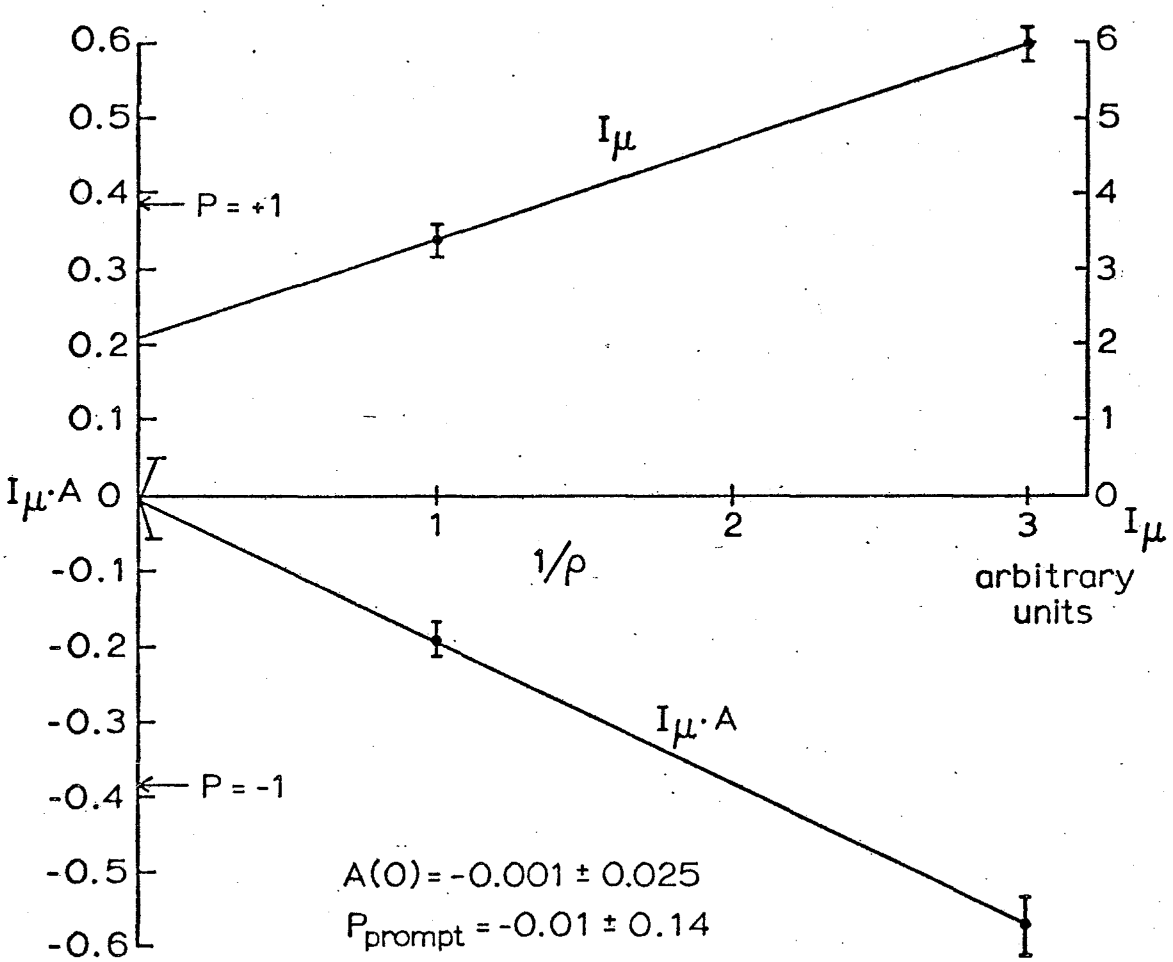
An overhead view and a side view of the polarimeter and polarimeter magnet coils.

Fig. 6



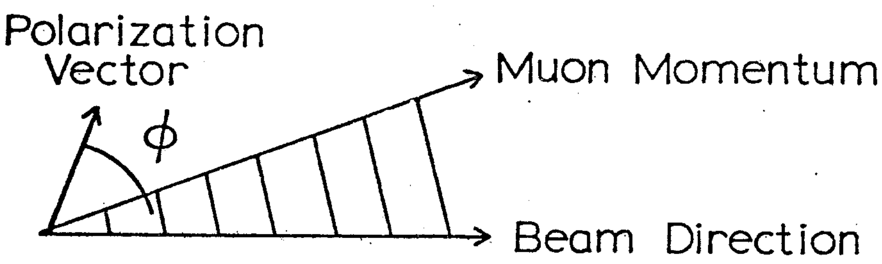
Plots of the decay asymmetry as a function of precession time using targets 1 and 3. The shift away from the $\frac{F-B}{F+B} = 0$ axis is due to muons stopping in the scintillator. Each graph also displays the functional values derived by a least squares fit of the data to equation (2).

Fig. 7



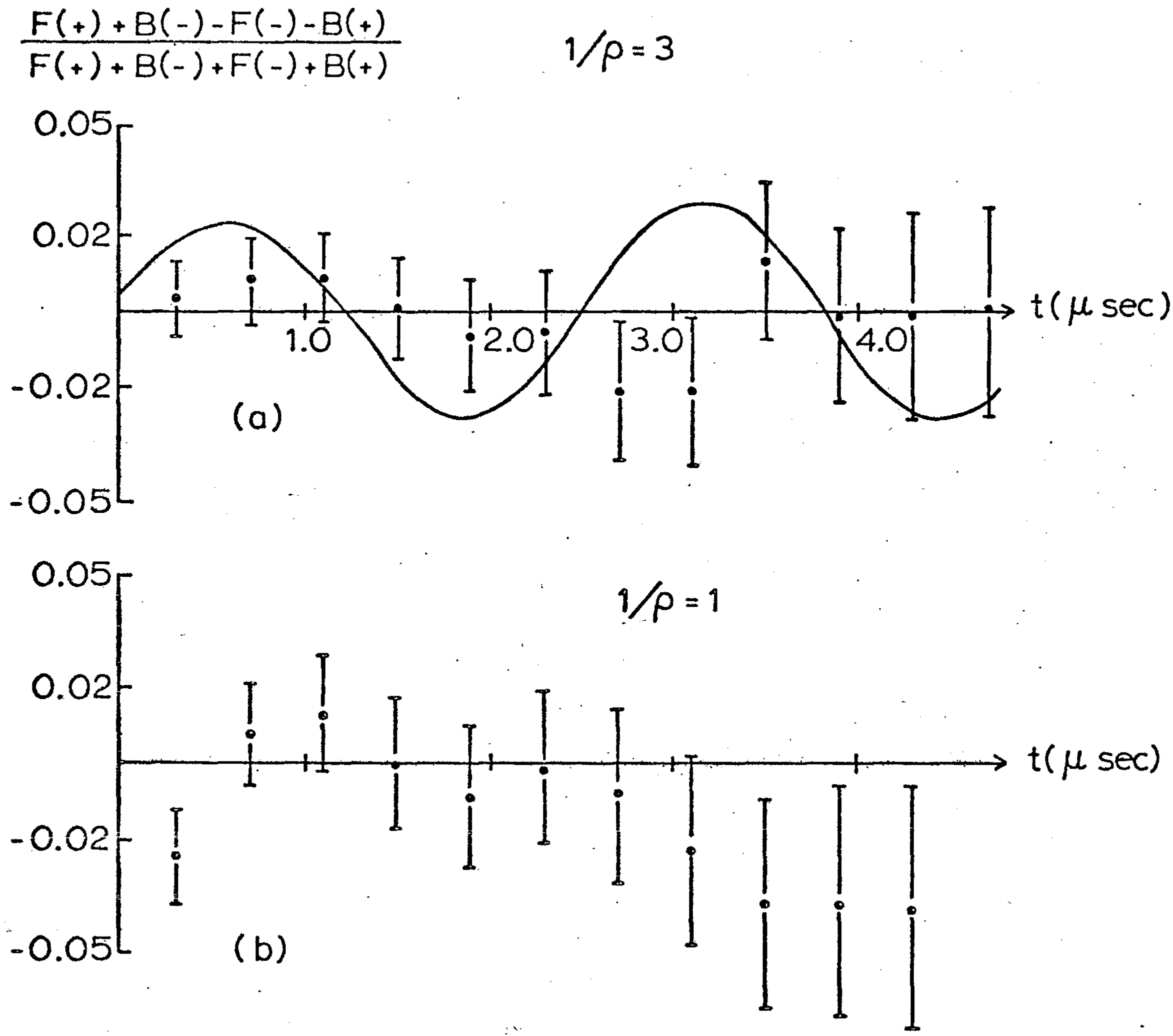
By extrapolating $I_\mu \cdot A$ to $1/\rho = 0$ we determine the decay asymmetry for muons from prompt production. If prompt muons had polarizations near $+1$ or -1, then the intercept of $I_\mu \cdot A$ would be near $+0.39$ or -0.39.

Fig. 8



The plane of production of a muon is defined by the beam direction and the direction of flight of the muon. For a positive muon, the polarization vector is oriented in the direction of its spin. Φ is the angle between the plane of production and the polarization vector.

Fig. 9



Transverse polarization data taken using targets 3 and 1. The line in figure (a) gives the functional values of equation (5) using parameters determined by a least squares fit of the data. Known values for γ, ω and $\bar{s}$ were used.

Fig. 10

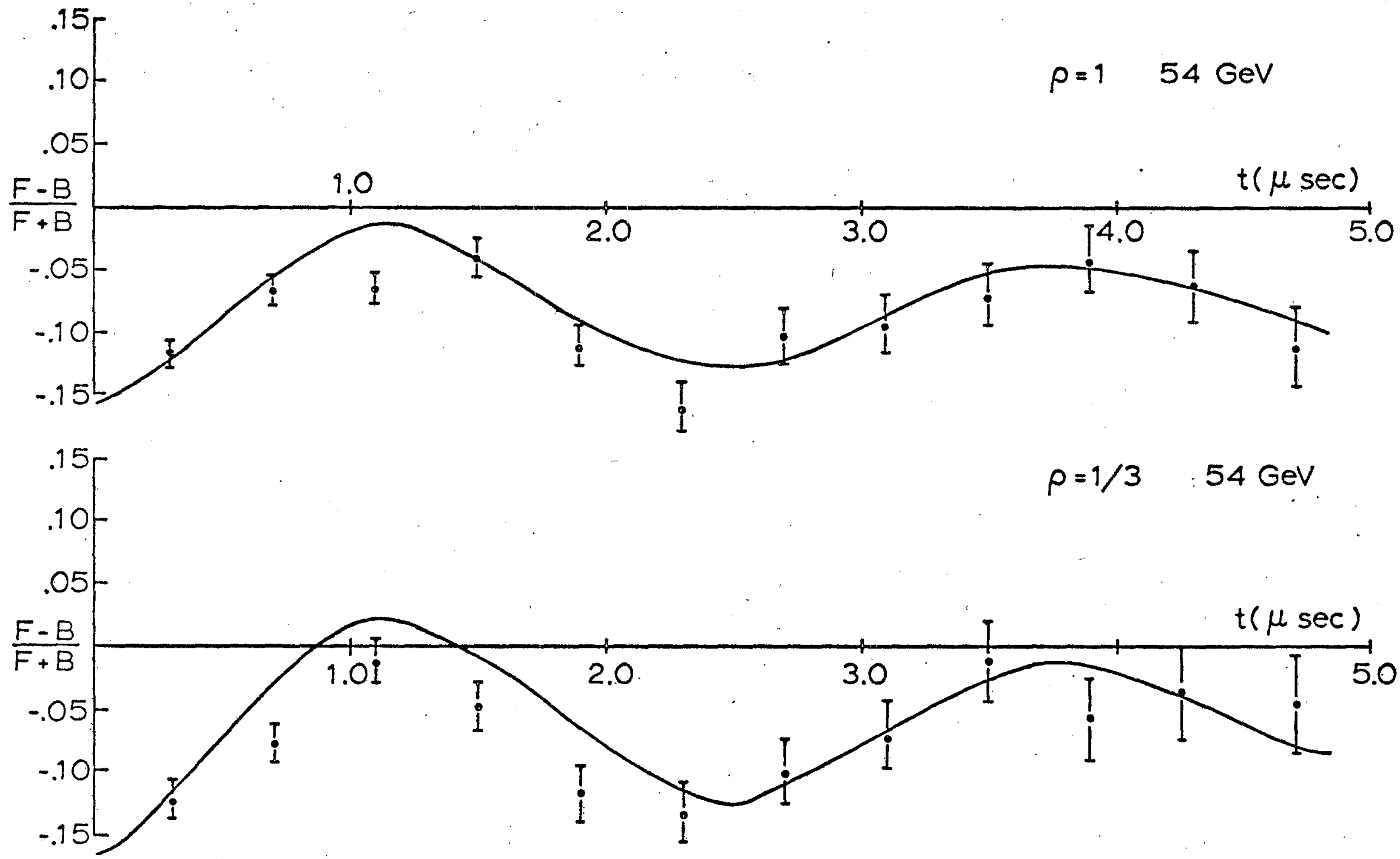
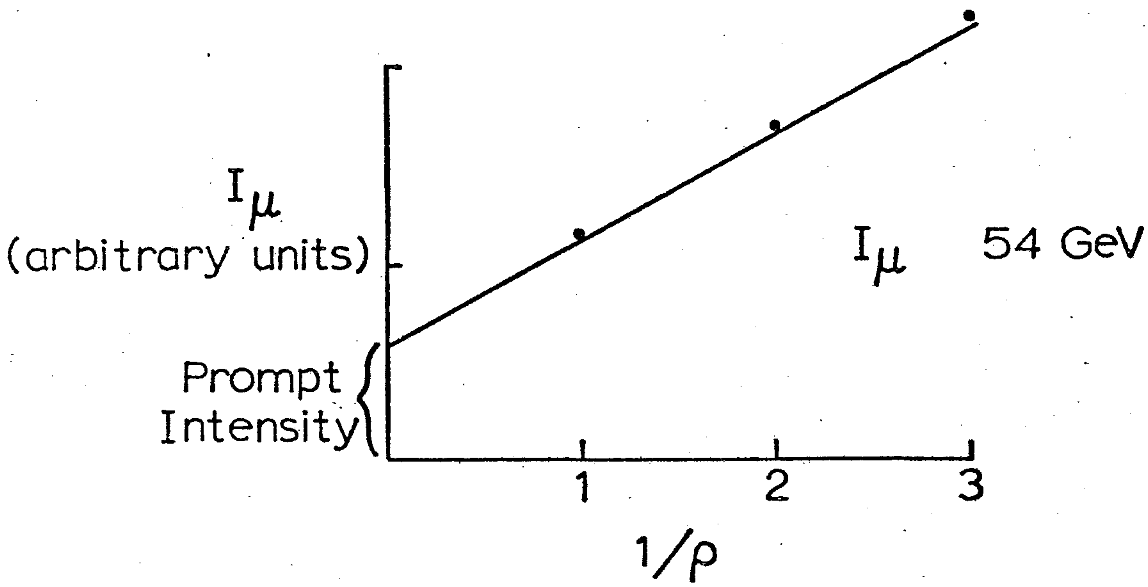


Fig. II

Plots of the decay asymmetry as a function of precession time for targets 1 and 3 at 54 GeV. Also shown are the functional values of equation (6) using parameter values determined by a least squares fit of the data.



Intensity of muons stopping in the polarimeter as a function of 1/(target density). The points show the raw data while the line takes into account the corrections discussed in section 3.4

Fig. 12

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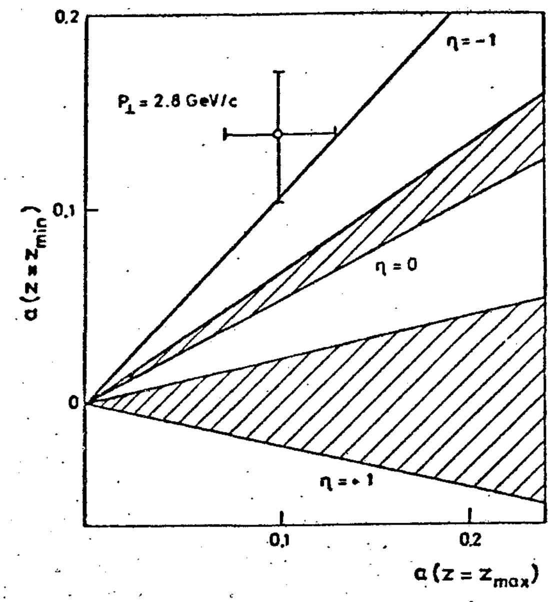


A graph from the last page of reference 23. I have added the line indicating $\eta = -2$. This parameter is used by the Russians to indicate their measured polarization for prompt muons. Physically possible values for η lie in the area on the graph between the lines $\eta = -1$ and $\eta = +1$.

Fig. 13

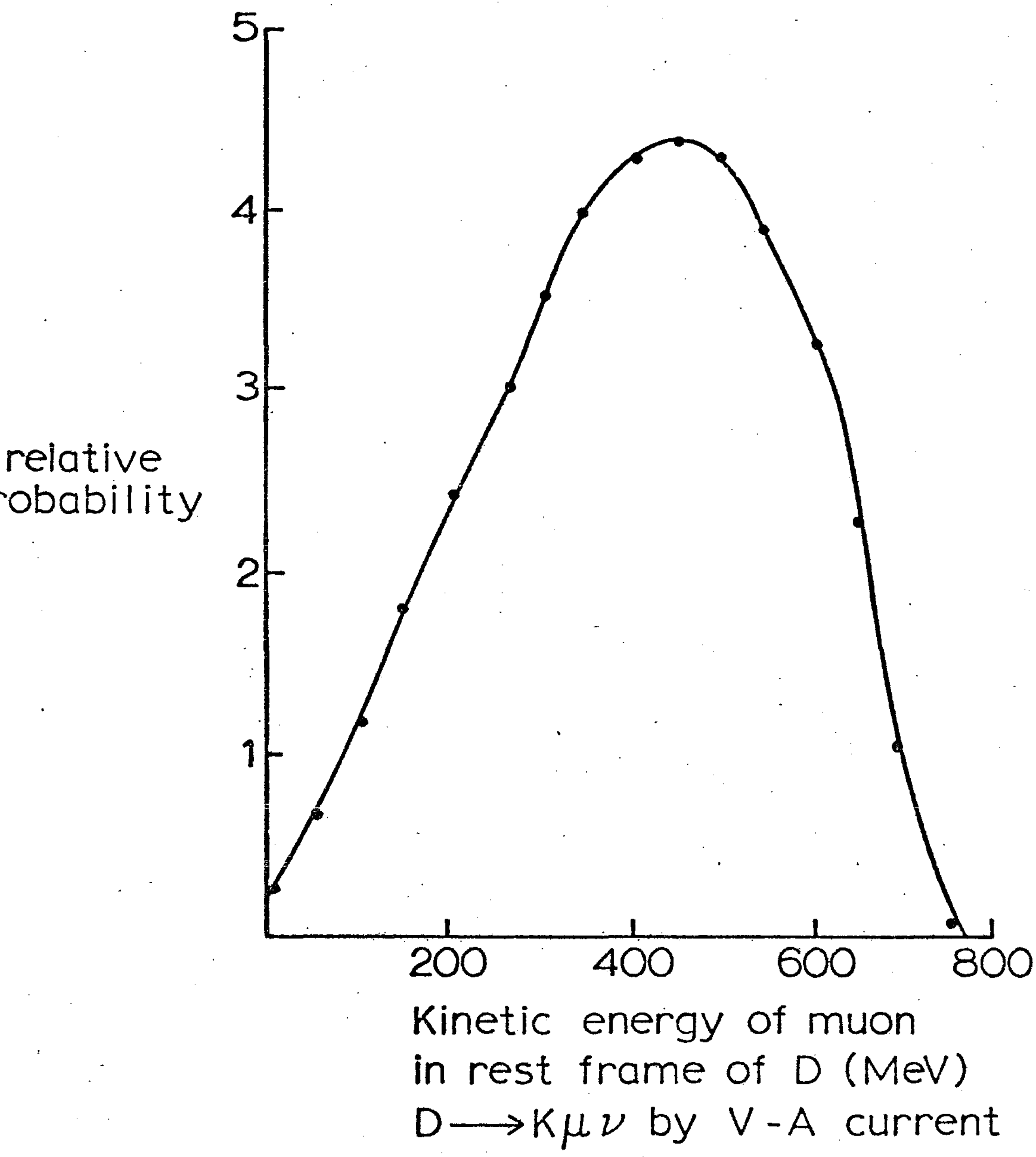
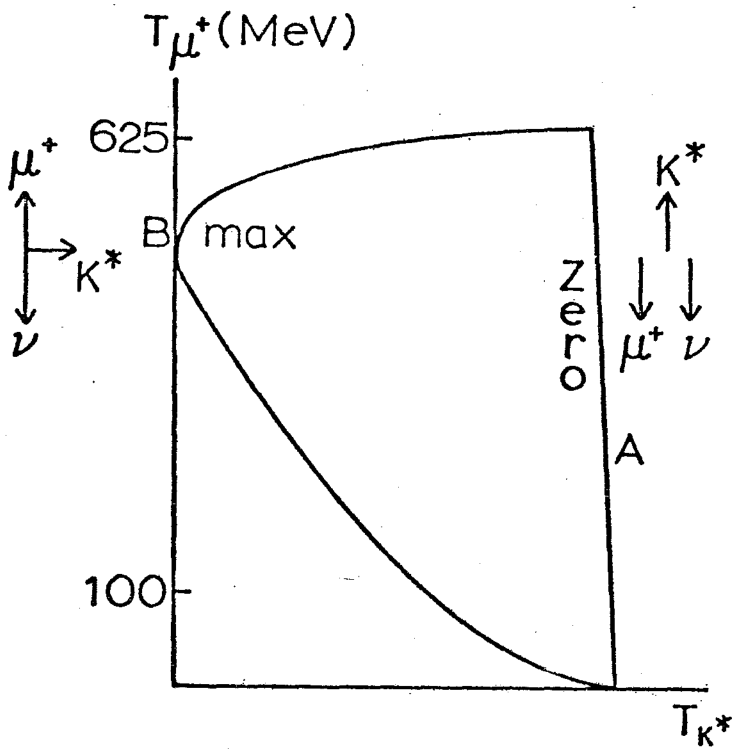

5
4
3
2
1
relative
robability
200
400
600
800
Kinetic energy of muon
in rest frame of D (MeV)
D⟶Kμν by V-A current

Fig. 14



Dalitz plot for the decay $D^+ \to K^* \mu^+ \nu_\mu$. Arrows indicate directions of particles momenta for decay configurations corresponding to point B and line A. Those areas of the Dalitz plot which would have the highest density and lowest density of events are indicated. Note that most decays will have $T_{\mu^+} > 100$ MeV.

Fig. 15

Table 1

Function	Material	gm/cm^2
Vertical and	2 plates Ti, 0.002''	0.046
Horizontal SWIC's	2 plates mylar, 0.003''	0.017
Secondary Emission	21 foils Al, 0.002''	0.288
Monitor	2 windows Ti, 0.002''	0.046
Ion Chamber	2 foils brass, 0.005''	0.203
	3 foils Al, 0.001''	0.020
Window on Vacuum Chamber	Ti, 0.002''	0.023
	1.7 meter air	0.110 (eff)
	Total	0.753

COMMENTS

Polarization of Prompt Muons Produced at $P_t = 2.15$ GeV/c by 400-GeV Proton Interactions*

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The polarization of prompt muons produced at a center-of-mass angle of 61° and a transverse momentum of 2.15 GeV/c by the interaction of 400-GeV protons was measured to be -0.135 ± 0.20. This value, consistent with zero, differs from the large value reported from similar measurements at 70 GeV and is inconsistent with the proposal that the prompt leptons observed at large transverse momenta are derived from weak decays of intermediate particles.

It has been proposed[1] that the anomalously large production of prompt leptons at large values of p_t by nucleon-nucleon interactions might be derived from the weak decays of intermediate particles. Most plausible descriptions of such decays would lead to a large polarization of leptons so produced along their direction of flight. Recent communications by Anisimova *et al.*[2] and Abramov *et al.*[3] report measurements of the polarization of prompt muons produced by the interaction of 70-GeV protons with nuclei. They find a value of -0.85 ± 0.37 for the polarization of positive muons produced at an angle of 90° in the center-of-mass system with transverse momenta of 2.0 and 2.8 GeV/c. Such a result, indeed any value of the longitudinal polarization other than zero, indicates that the production must be mediated by a parity-nonconcerving interaction.

We have now made similar measurements at Fermilab of the longitudinal polarization of prompt muons produced with transverse momenta of 2.15 GeV/c by the interaction of 400-GeV protons with nuclei. Muons, produced through the interaction of protons with a variable-density copper target, passed through the target and through steel shielding near the target with trajectories defined by counters set 60 m from the target at an angle of 45 mr from the proton beam direction. The muons then passed through 60 m of earth to stop in a polarimeter designed to measure the polarization of the muons in the direction of their flight and the component of polarization perpendicular to the plane of production. The characteristics of the target and a more complete description of the beam have been presented previously.[4]

The energy of the muons was defined by their range as 54 ± 2 GeV. While the mean transverse momentum of the muons emerging from the target assembly was then about 2.40 GeV/c, the mean production transverse momenta was calculated to be 2.15 ± 0.10 GeV/c when the effects of the multiple scattering of the muons by the material of the target assembly were considered; the muon angle of production in the center-of-mass system of the nucleon-nucleon interaction was then 61°.

The polarimeter consisted, basically, of 24 layers of 5-cm-thick aluminum plates backed by 6-mm sheets of scintillator, each viewed by a phototube. The plates and scintillator sheets were 60-cm high and 90-cm wide, aligned normal to the muon beam direction. A 30-G magnetic field, directed in the plane of production of the detected muons and perpendicular to the beam direction, served to precess the muons. The polarization was then measured by determining the direction of the positive muon decay (forwards or backwards) as a function of the precession time. The polarimeter and the techniques used to determine muon polarizations have been described elsewhere.[5] These measurements have served to calibrate the polarimeter; and in the configuration used here, the asymmetry amplitude for the decay of positive muons which stop in the aluminum can be described by the relation

$$A_+ = (F-B)/(F+B) = e_+ P, \tag{1}$$

where F and B represent the intensity of muons decaying forwards and backwards, P is the longitudinal polarization of the muons, and e_+ is the calibration factor which has been determined to be 0.185 ± 0.01 for this polarimeter through analyses of other experiments.[5]

For the analysis of this experiment, where both positive and negative muons stop in the polarimeter and the muons precess about the direction of the magnetic field, the amplitude $A(t)$ will be a function of time and F and B will vary with time in a complex, but well defined manner:

$$F = [1 + e_+P_+\cos(\omega t)\exp(-\gamma t)]\exp(-t/\tau_+) + R[1 + e_-P_-\cos(\omega t)\exp(-\gamma t)]\exp(-t/\tau_-),$$
$$B = [1 - e_+P_+\cos(\omega t)\exp(-\gamma t)]\exp(-t/\tau_+) + R[1 - e_-P_-\cos(\omega t)\exp(-\gamma t)]\exp(-t/\tau_-) + s(1+R)\exp(-t/\tau_+). \tag{2}$$

Here we take the value of e_- as 0.01 for the analyzing effectiveness of negative muons; s, the proportion of decays from muons which stop in the scintillator, was measured to be 0.13 in previous work[5]; γ, the measure of the decoherence induced by field inhomogeneities, has been measured previously and is equal to 0.3; ω, the precession frequency, is equal to 2.43×10^6/sec; τ_+, the lifetime of free muons, positive muons in aluminum, and all muons in scintillator, is 2.2 μm sec; and τ_-, the lifetime of negative muons in aluminum, is 0.86 μm sec. The value of R, the ratio of negative muons to positive muons, varies with the target density even as the ratio of muons from meson decays to prompt muons so varies. We use for the ration, $R = (0.80 I_m + I_p)/(I_m + I_p)$, where I_m and I_p are the intensities of muons from meson decay and from prompt production. Measurements of the total intensity $I = F + B$, as a function of time fitted to the forms defined in Eq. (2) are in accord with the choice of $R_m = 0.80$.

The results of the measurements which define the polarization of the prompt muons are shown in Fig. 1. From an analysis of the variation of the intensity of muons stopping in the polarimeter with respect to the target density, ρ, shown in the upper panel, we find that 51% of the muons generated by proton interactions with the solid copper target ($\rho = 1$) are derived from meson decays and that 49% are prompt muons. Only 26% of the muons from the density-$\frac{1}{3}$ target ($\rho = \frac{1}{3}$) are

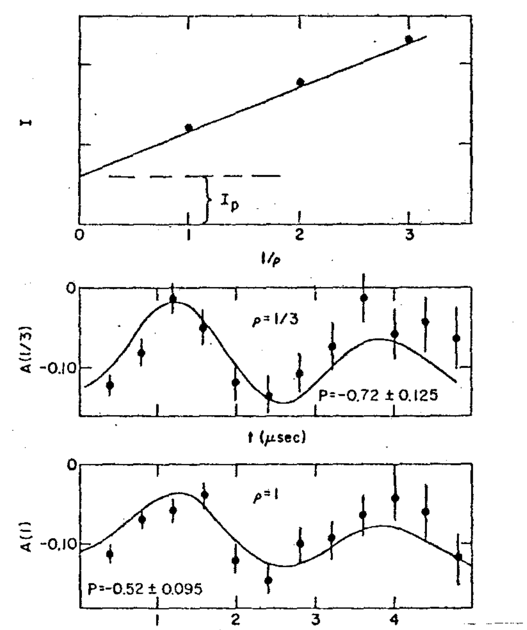

FIG. 1. The upper panel shows the variation of intensity of muons stopping in the polarimeter as a function of a target density. The solid points show the raw data; the line shows the data corrected for the effects of material before the target. The intensity is presented in arbitrary units. The points on the two lower panels show the measured muon-decay asymmetry, $A = (F-B)/(F+B)$, for two different target densities, while the solid curves show least-squares fits to Eq. (2) defining the value of the muon polarizations P.

prompt muons while 74% come from meson decays. The lower two panels show the results of the measurements of the asymmetry as a function of time for the two different target densities together with least-squares fits to the forms given in Eq. (2) which define the values of the polarizations. The polarization of the muons from the density-1 (solid copper) target is found to be $P(\rho=1)=-0.52\pm0.095$ while $P(\rho=\frac{1}{3})=-0.72\pm0.125$.

For any meson-production spectrum which falls off rapidly with increasing energy, the polarization of the muons from the meson decays is nearly equal to the polarization in the meson decay system. Explicit calculations of the meson spectrum, using parametrizations which fit the known production spectra adequately, give a value of -0.90 ± 0.04 for the polarization of the muons; using these numbers, we deduce a value of the polarization of the prompt muons from the density-1 measurement of -0.123 ± 0.20 and a value of -0.205 ± 0.48 for the density-$\frac{1}{3}$ measurements. Considering both measurements, we find a best value of -0.135 ± 0.20 for the polarization of the prompt muons. This value is consistent with zero, consistent with the null value of polarization found for prompt muons in the forward direction,[5] and consistent with the hypothesis that the bulk of the prompt muon flux originates in electromagnetic processes. The result is quite different from a value near $+1.0$ expected from the more conventional models of prompt-muon production through the weak decays of heavy intermediate particles, and places a limit of about 10% on the portion of the muon flux derived from such sources. We also believe that our result is difficult to reconcile with the large negative values of polarization reported at 70 GeV.[2,3]

The transverse polarization of the muons in a direction normal to the plane of production was measured to be 0.108 ± 0.079 in the direction $\vec{p}_p\times\vec{p}_\mu$. Presuming no contribution from muons from meson decay, transverse polarization of the prompt muons was 0.22 ± 0.16, consistent with zero.

We would like to thank the personnel of Fermilab, and especially those of the Proton Laboratory, for their invaluable assistance in setting up and running of the experiment.

---

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[5]L. B. Leipuner *et al.*, Phys. Rev. Lett. 36, 1007 (1976).

Polarization of Prompt Muons

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The polarization of 185-GeV prompt muons produced in the forward direction by the interaction of 400-GeV protons has been measured to be $P=0.00\pm 0.10$ along their direction of flight. The null value for the polarization suggests that the muons are produced through electromagnetic interactions.

It seems most probable that the anomalously large direct lepton flux observed in many experiments[1] is either derived from electromagnetic production mechanisms which are not now thoroughly understood or from the weak-interaction decays of particles[2] which have not been previously identified. If the production process is indeed electromagnetic, we will expect that the polarization of the leptons in the direction of their momentum will be zero, inasmuch as the electromagnetic interactions are known to conserve parity. However, if the leptons are produced through the weak decays of intermediate particles, the leptons will probably be polarized. For interactions mediated by the charged weak currents, this polarization will be $+1$ for positive leptons produced with velocities near c in the system of the parent particle if there is no kinematic constraint on the spin direction or -1 when the spin direction is constrained as for the two-body decays of spin-zero mesons to neutrinos and leptons. The polarization of leptons from weak decays mediated by neutral currents is not yet known. In view of the importance of measurements which will better define the character of the production of prompt leptons, we have measured the polarization of 185-GeV prompt positive muons produced by the nuclear interactions of 400-GeV protons from the Fermilab accelerator.

The polarization of the muons was determined in a manner similar to that used previously in measurements of the polarization of muons produced through the interactions of protons from accelerators[3] and in the measurement of the polarization of cosmic-ray muons.[4] The muons were stopped in a "polarimeter" constructed of 25 3-ft× 2-ft× 2-in. slabs of aluminum backed by a covering of scintillation counters. Coils were energized which produced a magnetic field of about 30 G through the polarimeter in a direction perpendicular to the direction of incidence of the incoming muons. The position of the stopped muon was determined by the registration of the scintillation counters in the muon path and clocks were started at the time of the stop. The passage of the decay electron through the counter upstream (back) or downstream (front) of the stop position then stopped the clock assigned to that counter, defining the decay time and the direction of decay. Since a positive muon decays such that the high-energy electrons are emitted preferentially in the direction of the muon polarization, the recorded direction defines the direction of polarization in a statistically defined way. In particular, the ratio of decays (front − back)/(front + back) will vary sinusoidally with the precession frequency and the amplitude of the sine wave will be proportional to the polarization. Such a measurement of the Fourier component of the variation of the decay asymmetry with time is insensitive to the systematic sources of error which tend to plague more straightforward measurements of front-back asymmetries.

A brief description of the experimental arrangement has been published previously.[5] The polarimeter was housed in a building placed in a pit dug in the soil about 1200 ft from the target area of the proton central beam area. The center of the polarimeter lay on a line to the target which made an angle of 22 mrad with the extension of the proton beam line. Only muons with an energy of about 185 GeV will pass through the 13 kg/cm² of material (mostly steel) in the target area and the 57.6 kg/cm² of earth in the region between the end of the target hall and the polarimeter and then stop in the polarimeter. In the course of the measurements, the muons from the target were bent through the requisite angle of 22 mrad into the polarimeter by means of a bending magnet downstream of the target. Therefore the measurements concerned positive muons produced in the forward direction upon production. However, since the rms momentum transfer due to multiple Coulomb scattering in the target and the material directly downstream of the target is about 630 MeV/c, the measurements can be considered as sampling the production of muons up to transverse momenta near 1.0 GeV/c.

Since the muons produced by the interaction of the protons in the copper target are derived from the decays of mesons produced in the target as well as from the direct interactions, it is necessary to measure the polarization of the direct, or prompt, component in a manner which allows the exclusion of the muons from meson decay. We do this in a manner similar to that used to construct the intensity of direct muons from a flux of direct muons and muons from meson decay; we measured the polarization as a function of target density. Since the probability of a meson decaying before it is effectively removed from the beam through interactions is inversely proportional to the target density, an appropriate extrapolation of the polarization measured as a function of the inverse density to the value at zero inverse target density (or infinite density) will define the polarization of the prompt muons.

The graph of Fig. 1 shows the measured decay-asymmetry amplitudes plotted as a function of (precession) time from the time when the muon stopped. Measurements were made of the polarization of the muons produced from a copper-air target with an effective density of one-third that of copper and a solid copper target. Muon flux measurements were made at the same densities and with a target which had an effective density of one-half that of copper. The results of these intensity measurements, interpreted as described previously,[5] show that about 65% of the flux at density one-third was generated by meson decays and about 35% was from the direct processes. With the solid copper target, about 37% of the muons were from meson decays and 63% were

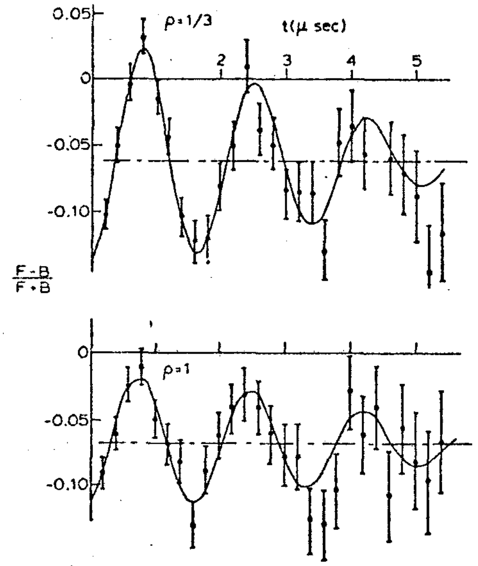

FIG. 1. Muon decay asymmetries as a function of precession time for target densities of one-third [$A(3) = 0.0782 \pm 0.0062$] and one [$A(1) = 0.0490 \pm 0.0073$] times the density of copper.

from the direct processes. It is obvious upon inspection that the polarization of the muons from the target of solid copper is appreciably smaller than the polarization of the muons from the target of density one-third, indicating that the polarization of the prompt muons is different from, and much smaller or of opposite sign than, that of the muons from meson decays. Since the magnetic field varied somewhat over the dimensions of the apparatus, the precession frequency varied correspondingly and the mean asymmetry then diminished with time. It is then an adequate approximation for our purpose to consider that the asymmetry varies with time as

$$(F-B)/(F+B) = A\cos(\omega t)\sin(Dt)/Dt,$$

where A is the amplitude at $t=0$, ω is the mean precession frequency, and D is a measure of the spread of frequencies. The values of A derived from a least-squares fit[6] of the data by this form are $A(3) = -0.0782 \pm 0.0062$ and $A(1) = -0.0490 \pm 0.0073$, where $A = (F-B)/F+B)$ with F the intensity of decays forward and B the intensity of decays backward, and the argument of A is the inverse density in units of the density of copper.

The amplitude of the prompt or direct portion of the flux can be determined simply by extrapolating the values of I_μ, the intensity, and $I_\mu A$ to

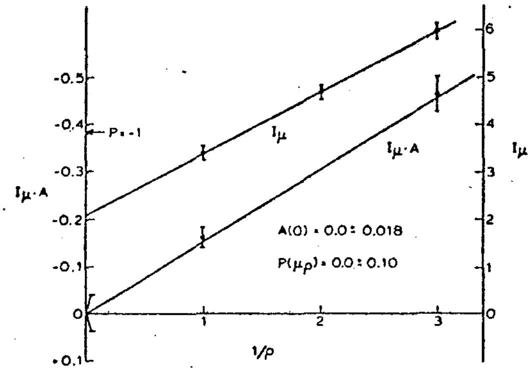

FIG. 2. Intensity versus inverse density of the target and intensity times the muon decay-asymmetry amplitude versus inverse target density. The intensity is measured in arbitrary units. The position of the intercept expected for a prompt muon polarization of −1.0 is shown; the corresponding intercept for a polarization of +1.0 would lie at a value of $I_\mu A$ of +0.38. The points show the raw data while the solid line represents a best fit after small corrections are made for the effects of proton interactions upstream from the target.

the values at $1/\rho = 0$ as both of these quantities should vary linearly with $1/\rho$. The graph of Fig. 2 shows the measured intensity, I_μ, and the product $I_\mu A$ as functions of $1/\rho$. The points show the uncorrected data and the solid line gives the values of $I_\mu P$ after a small correction for the presence of muons produced by the interaction of the proton beam with material upstream of the target. The extrapolation of the (corrected) values of $I_\mu P$ to zero inverse density gives the amplitude for the prompt muons of 0.00 ± 0.018. This can be converted to the polarization, if the polarization of the muons from the meson decay is known. This polarization depends upon the spectrum of meson production as, roughly speaking, muons which are produced from pion and K-meson decays in the forward direction in the meson center-of-mass system retain a negative polarization in the laboratory system, while those which decay backwards in the meson center-of-mass system will have a positive polarization in the laboratory system. Since the meson production spectrum has a steep energy dependence, there are many more relatively low-energy mesons which decay forwards in their system to produce 185-GeV muons than there are higher-energy mesons which decay backwards to give muons with a laboratory energy of 185 GeV and, hence, there is a net negative polarization.

As a consequence of kinematic effects, for muons emitted from a parent particle with velocities near c in the parent-particle system, the mean polarization in the laboratory system will be nearly equal to the center-of-mass polarization if the production spectrum of the parent is sufficiently steep (as for pions or K mesons). Detailed calculations using meson spectra from 24-GeV protons on copper[7] scaled to 400 GeV indicate that the polarization of muons from pion decays is about −0.62 and polarization of muons from K-meson decays is about −0.98. These values are not especially sensitive to the production spectrum. With about 22% of the meson-derived muon flux from K^+-meson decay, the polarization of the muons from all meson decays will be −0.70. Using this value for the polarization of the muons from meson decays, we find that the asymmetry amplitude A equals $0.172P$, where P is the polarization, and the polarization of the prompt muons is $P = 0.00 \pm 0.10$. The muons produced by the direct processes are then not polarized in the direction of flight in the laboratory system. This lack of polarization suggests that the muons are produced electromagnetically.

While the conduction of this measurement was made possible by the existence of the Fermi National Accelerator Laboratory, it was made easy and pleasant by the exemplary cooperation we have received by everyone at Fermilab. We feel indebted particularly, however, to Dr. Roy Rubinstein who provided solutions for the real parts of the complex problems we posed him and taught us to ignore the imaginary parts.

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[6]We are indebted to Dr. G. Takhtamyshev for adapting the Dubna fitting program FUMILI for use on the PDP-11 computer.

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