Simulation of Conduction Cooled SRF Cavity

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Abstract—SRF cavities are used heavily in high energy physics research, but the operational complexities of SRF cavities prevent their widespread use in industry. One notable complexity of operating an SRF cavity is liquid helium coolant, and replacing liquid helium with cryocooler conduction cooling is an important research and development interest. As part of this effort, a system consisting of a 650 MHz single cell cavity, a commercial grade pulse tube refrigerator, and a thermal conduction link was simulated through finite element analysis. The electromagnetic fields inside the cavity, heat flow through the system, and temperature distribution of the thermal link and cavity were calculated. Based on the simulation, we found that a 2 W @ 4.2 K pulse tube refrigerator is capable of cooling a single cell Nb₃Sn coated cavity operating at up to 12 MV/m accelerating gradient. The maximum simulated temperature of the cavity was below 7 K, and the thermal conduction link was shown to significantly attenuate the cryocooler thermal oscillations at the location of the cavity.

I. INTRODUCTION

Radio frequency (RF) cavities are a key accelerator technology in particle physics research, and their use extends into the industrial sector. RF cavities show great potential in applications across a variety of fields, including medical imaging, sterilization, water treatment, and electron beam welding [1].

Although RF cavities can be constructed from normal conducting materials like copper, superconducting RF (SRF) cavities have substantially lower power losses. SRF cavities must be kept at cryogenic temperatures and are usually immersed in liquid helium. However, liquid helium systems are complicated, expensive, and large, so cryogenic systems which do not require liquid helium are an attractive alternative for cooling SRF cavities.

The Department of Energy and Fermilab have funded a Laboratory Directed Research and Development (LDRD) project to demonstrate conduction cooling of an SRF cavity. This project aims to design a system for keeping an SRF cavity cold using a cryocooler instead of liquid helium.

II. BACKGROUND

A. Basics of Superconductivity

Superconductivity is an effect observed in some materials below a critical temperature, T_c . Superconductive materials experience no electrical resistance to dc current and exhibit the Meissner effect, in which small external magnetic fields are expelled from the material. However, superconductivity breaks down if the material is subjected to temperatures higher than T_c or magnetic fields above a critical magnitude, B_c . The

Fig. 1. A 9 Cell 1.3 GHz Niobium Cavity [4].

maximum temperature, T_{max} , and maximum surface magnetic field, B_{pk} , of an SRF cavity must remain below these limits. SRF cavities are usually formed from niobium, which has the highest critical temperature of any element at 9.25 K and a critical field value of 206 mT at absolute zero temperature [2].

The Bardeen-Cooper-Schrieffer (BCS) Theory of Superconductivity describes superconductors as containing a mixture of normal electrons and Cooper pairs, pairs of electrons which can move freely through the metal lattice with no resistance [3]. When a superconductor is placed in a magnetic field which is weaker than B_c , a current of Cooper pairs is induced on the surface of the material. This current shields the normal electrons from the magnetic field, preventing resistive losses.

However, since Cooper pairs have mass, they cannot react instantaneously to changing magnetic fields. In ac fields, the Cooper pairs incompletely shield the normal electrons from the magnetic field, resulting in small resistive losses. The resistive losses of an SRF cavity can be quantified by the surface resistance R_s , which increases with temperature, frequency, and chemical impurities.

B. SRF Cavity Fundamentals

SRF cavities are hollow shells of metal which particles can pass through. Cavities are structured as elliptical cells with straight sections of beam pipe in between, as shown in Figure 1. A cavity can have one or more cells.

SRF cavities are designed with a unique shape which resonates at a particular frequency. When a source supplies electric power at this frequency to the cavity, the electromagnetic waves inside the cavity build up through positive interference. The electric field points along the axis of the cavity and oscillates at the resonant frequency. Charged particles which travel through a cell when the electric field is strongest will see a high voltage across the cell and gain energy.

Several important parameters characterize the operation and efficiency of an SRF cavity, including accelerating gradient, dissipated power, quality factor, geometric factor, and normalized shunt impedance. In general, these parameters depend

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upon the cavity geometry and material, as well as on the driving source of RF power.

The accelerating gradient, E_{acc} , of an SRF cavity measures the maximum energy gain per unit length. A particle traveling along the axis of a cavity cell will see a voltage difference across the cell, V_{acc} . The accelerating gradient can be found by dividing by the length of the cell, d .

$$E_{acc} = \frac{V_{acc}}{d} \quad (1)$$

Since the RF field within the cavity varies with time, V_{acc} is dependent on the phase of the field when the particle enters the cavity and the speed of the particle moving through the cavity. To maximize the energy gain, particles must be carefully timed to reach the center of the cavity when the field is maximal.

Because a superconductor in an ac field has a nonzero surface resistance, an SRF cavity loses some energy in the form of heat. This dissipated power, P_d , is dependent on R_s and on the magnetic field, H . With S as the inner surface of the cavity,

$$P_d = \frac{1}{2} \int_S R_s |H|^2 dA \quad (2)$$

The quality factor, Q_0 , of a cavity is the ratio of energy gained to power dissipated in one period. Quality factor varies with accelerating gradient. For a cavity with angular frequency ω and a total energy U ,

$$Q_0 = \frac{\omega U}{P_d} \quad (3)$$

The geometric factor, G , of an SRF cavity can be determined from only the geometry of the cavity and the eigenmode of operation. With μ_0 as the permeability of free space,

$$G = \frac{\mu_0 \omega \int_V |H|^2 dV}{\int_S |H|^2 dA} \quad (4)$$

The geometric factor defines a relationship between the quality factor and the surface resistance:

$$G = Q_0 R_s \quad (5)$$

The normalized shunt impedance, R/Q , relates the voltage across the cavity to the stored energy.

$$R/Q = \frac{V_{acc}^2}{\omega U} \quad (6)$$

If R_s is assumed constant across the cavity, the dissipated power can be approximated by

$$P_d = \frac{V_{acc}^2 R_s}{G(R/Q)} = \frac{d^2 E_{acc}^2 R_s}{G(R/Q)} \quad (7)$$

C. Basics of Cryocoolers

An increasing demand for very low temperature, or cryogenic, applications has led to a growing market of commercial grade cryogenic coolers, or cryocoolers. Tabletop cryocoolers with high heat capacity are widely available and can cool applications to 4 K and below.

A pulse tube refrigerator (PTR) is a type of cryocooler with no moving parts at low temperature, an advantage which

Fig. 2. Parts of a Typical Pulse Tube Refrigerator [7].

reduces mechanical maintenance and vibrations within the cryocooler [5]. Figure 2 shows the typical components of a pulse tube refrigerator. Here the dotted line surrounds the thermally insulated section of the cryocooler. Thermal insulation can be achieved using vacuum and multi-layer insulation, which minimize both convection and radiation of heat [6].

The pulse tube refrigerator is composed of a piston, several heat exchangers (X_1 , X_L , and X_3), a regenerator, a pulse tube, an orifice, and a buffer volume. The piston moves back and forth, pushing the gas in and out of the insulated system and causing pressure variations in the pulse tube, a tube of fixed volume filled with gas. As the pressure in the pulse tube varies, the temperature rises and falls. During the low pressure stage of the cryocooler cycle, gas at room temperature T_a moves from the buffer volume, a relatively large volume of gas at essentially constant pressure, into the pulse tube. The flow of gas is limited by the orifice. At the same time, gas flows out of the left side of the pulse tube at a low temperature T_L , supplying the cooling power through the heat exchanger X_L . During the high pressure stage of the cycle, gas enters the pulse tube through the regenerator, a heat exchanger made from a solid porous material, and leaves through the orifice at a high temperature, removing heat from the system [7].

The entire cooling cycle takes about one second to complete. During each cycle, the temperature within the pulse tube fluctuates, causing periodic temperature variations in the cooled application. In practice, most PTRs are designed in a U-shape so that the low temperature section, or cold head, of the cryocooler is positioned at the bottom of the cooler rather than in the middle. Two-stage pulse tube refrigerators, in which the first stage pre-cools the second stage, can reach temperatures as low as 2.1 K [7].

D. Conduction Cooling a Cavity

Advances in technology and the development of materials with favorable properties have paved the way for cooling an SRF cavity using a cryocooler rather than liquid helium. The high critical temperature of niobium allows cavities to operate above liquid helium temperatures. Furthermore, coating SRF cavities with Nb_3Sn has been shown to substantially reduce the surface resistance (and thus, the heat dissipation of the cavity). The less stringent cooling requirements of modern SRF cavities, in combination with the growing availability of high capacity cryocoolers, have made conduction cooling SRF cavities possible. A thermal link was constructed to connect an SRF cavity to a cryocooler and allow heat to flow. The link

was designed to conduct heat well at cryogenic temperatures and in vacuum, maintain good thermal contact with both the cryocooler and cavity, and prevent the mechanical vibrations of the cryocooler from affecting the cavity.

E. Finite Element Analysis

Finite element analysis (FEA) is a technique for numerically approximating the solutions to partial differential equations. Systems of partial differential equations with known boundary conditions are common in physics calculations, including heat transfer and electromagnetic field problems. Finite element analysis finds numerical values for unknown fields by dividing a continuous geometry into small discrete volumes. This process is called meshing, and the divided geometry is a mesh. Numerical values for the unknown fields are approximated for each element in the mesh, and the continuous solution is found by combining the results from all of the elements.

COMSOL is a software which implements FEA to solve problems across a variety of physics domains. COMSOL physics interfaces can be combined to perform multiphysics simulations, which model interactions from more than one branch of physics. To model the conduction cooled cavity, both electromagnetic and thermodynamic effects must be considered.

III. PROJECT GOALS AND PROCEDURE

To demonstrate conduction cooling of an SRF cavity, a Cryomech PT420 pulse tube refrigerator (shown in Figure 3) will be used to cool a single cell 650 MHz SRF cavity with Nb₃Sn coating. The system was simulated to calculate RF parameters such as ω , P_d , and B_{pk} , to determine the temperature distribution across the cavity and cooling link, and to examine the effect of cryocooler temperature oscillations on the cavity temperature.

A. Geometry

The simulated geometry includes the SRF cavity, vacuum space inside the cavity, cryocooler plate, and thermal link, as shown in Figure 4. Figure 5 shows half of the total geometry with the parts color coded.

Heat is generated on the inner surface of the cavity, and it flows through the thermal link to the cryocooler plate. To establish good thermal contact between the cavity and the cooling link, niobium rings are welded around the equator of the cavity. These rings are then bolted to the cooling link, providing a path for heat to flow out of the cavity. The thermal link is attached to the cryocooler plate with a U-shaped bend, which provides flexibility to isolate the mechanical vibrations of the cryocooler from the cavity. The entire system is kept under vacuum and is insulated with multi-layer insulation.

B. Material Properties

Several material-specific properties are needed to simulate heat transfer across the cavity and cooling link, including thermal conductivity, specific heat capacity, and thermal contact resistance of bolted connections. The cavity and cooling link

Fig. 3. A Cryomech PT420 Two-Stage Pulse Tube Refrigerator [8].

Fig. 4. CAD Model of a 650 MHz SRF Cavity, Cooling Link, and Cryocooler Plate.

are composed of three metals: aluminum, copper, and niobium. In Figure 6, aluminum is shown in blue, copper in orange, and niobium in red.

For a pure, normal conducting metal at cryogenic temperatures, the thermal conductivity k as a function of temperature T can be calculated using the Wiedemann-Franz Law, which relates thermal conductivity to electrical conductivity σ [9].

$$\frac{k}{\sigma} = L_0 T \quad (8)$$

The free electron Lorenz number, L_0 , is independent of temperature and material and is calculated from the Boltzmann constant k_B and the charge of the electron e .

$$L_0 = \frac{\pi^2}{3} \left(\frac{k_B}{e} \right)^2 = 2.45 \times 10^{-8} \text{ W } \Omega \text{ K}^{-2} \quad (9)$$

Fig. 5. CAD Model of Cavity (Green), Niobium Rings (Pink), Aluminum Cooling Link (Dark Grey), Cryocooler Plate (Gold), End Caps (Purple), and Vacuum Space Inside Cavity (Light Grey).

Fig. 6. CAD Model of Geometry Composed of Niobium (red), Aluminum (blue), Copper (orange), and Vacuum (Light Grey).

The residual resistivity ratio (RRR) of a material relates its electrical resistivity at room temperature, $\rho(273\text{ K})$, to its resistivity at 4.2 K, ρ_0 .

$$\text{RRR} = \frac{\rho(273\text{ K})}{\rho_0} \quad (10)$$

Values of $\rho(273\text{ K})$ are commonly tabulated, and the conductivity of a material at cryogenic temperatures can be approximated as ρ_0^{-1} . Combining Equations 8 and 10, the thermal conductivity of a metal at low temperature is

$$k = L_0 T \frac{\text{RRR}}{\rho(273\text{ K})} \quad (11)$$

Fig. 7. Thermal Conductivity of 5N Aluminum and 5N Copper [10].

Fig. 8. Thermal Conductivity of Superconducting Niobium [11].

The thermal link is constructed from 5N aluminum, aluminum which is at least 99.999% pure. Values of $\text{RRR} = 500$ and $\rho(273\text{ K}) = 2.7 \times 10^{-8} \Omega\text{ m}$ were used to find the thermal conductivity of the aluminum. The cryocooler plate is 5N copper, which can be modeled with $\text{RRR} = 250$ and $\rho(273\text{ K}) = 1.7 \times 10^{-8} \Omega\text{ m}$ [10]. The thermal conductivity of the aluminum and copper are shown in Figure 7.

The SRF cavity and the rings welded around the equator are made of niobium, which is a superconductor with a much more complicated thermal conductivity. The calculations explained by Koechlin and Bonin [11] were used to determine the thermal conductivity of niobium, which is plotted in Figure 8.

To model heat flow through time, the heat capacity of each metal must be known. The NIST Cryogenic Technologies Group Resources maintains an Index of Material Properties [12]. The index provides polynomial curve fits for the specific heat capacity of alloys. The curves for 3003F Aluminum and Copper (OFHC) were used to model 5N aluminum and 5N

Fig. 9. Heat Capacity of Aluminum, Copper, and Niobium [12], [13].

Fig. 11. Surface Resistance of SRF Cavity with Nb₃Sn Coating (courtesy: Sam Posen).

Fig. 10. Thermal Contact Resistance of Bolted Al-Al and Nb-Al Connections [14].

copper. To model the specific heat capacity of niobium, a polynomial fit published by Andrew Davies for niobium at temperatures less than 9 K was used [13]. The heat capacities of all three metals are shown in Figure 9.

The thermal contact resistance between bolted connections must also be considered. Using experimental data, the contact resistances between aluminum and niobium and between aluminum and aluminum were found [14]. These contact resistances are shown in Figure 10.

The surface resistance of a Nb₃Sn coated cavity is another temperature-dependent property and is shown in Figure 11 [15].

To model electromagnetic fields inside the cavity, the space was assumed to be perfect vacuum with the permittivity of free space ϵ_0 and the permeability of free space μ_0 .

C. Cryocooler Heat Capacity Measurements

The heat capacity of the cryocooler describes the amount of dissipated power it can absorb from the cavity at a given temperature. Accurate measurements of the cryocooler's heat capacity are crucial for simulating heat transfer through the system.

To measure the heat capacity, a thermometer and a cartridge heater, which uses a resistive load to produce Joule heating, were connected to each stage of the pulse tube refrigerator. A Lakeshore TC 355 temperature controller with Labview was used to set the output of the first stage heater, H1, and read the temperature of both stages. Because the temperature controller can set the heater output with only limited precision, the heater on the second stage, H2, was powered more precisely with a Keithley 2230G-30-1 current source. As the current was varied, the voltage across the heater was measured using a Keysight 33461A digital multimeter, and the input power to the heater was found by Equation 12.

$$P_{in} = IV \quad (12)$$

The power supplied to H1 was adjusted to five values between 0-75 W, and the power supplied to H2 was varied between 0-5 W. For each case, the temperature of stage 2 (T2) was recorded, along with the amplitude of the cyclic temperature oscillations. Figure 12 shows the temperature of stage 2 as a function of the input power to H1 and H2. Note that increasing the heat load on stage 2 (H2) clearly increases the temperature, while the heat load on stage 1 (H1) has only a limited impact on the temperature. Therefore, the main factor influencing the cryocooler temperature is the dissipated power from the SRF cavity, not the temperature of stage 1. Figure 13 shows the amplitude of the cyclic temperature oscillations at stage 2 for various input power. The period of oscillation is about one second, and the amplitude of these oscillations also increases with the input power.

Fig. 12. Temperature at Stage 2 for Input Powers H1 and H2.

Fig. 13. Amplitude of Temperature Oscillations at Stage 2 for Input Powers H1 and H2.

D. Simulation Procedure

The final simulation goals were accomplished by splitting the work into discrete steps, which provided several advantages. The simulations were simpler to set up and faster to run, and the results were much easier to validate. Dividing the work into smaller parts also required less computing power, less memory, and fewer software licenses.

1) *Heat Transfer Through Cooling Link:* First, heat transfer through the cryocooler plate, cooling link, and niobium rings was modeled. The geometry used is shown in Figure 14. The dissipated power of the cavity, P_d , was represented as a heat flux entering the cooling link through the weld faces of the niobium rings. The temperature at the top of the cryocooler plate, T_{cryo} , was determined for the given heat flux from Figure 12. Using these two boundary conditions, a heat transfer simulation was performed to find the equilibrium temperature across the geometry. The input parameter P_d was varied, and the average temperature of the weld faces, T_{weld} ,

Fig. 14. Geometry Used for First Heat Transfer Simulation with Weld Faces (red) and Top of Cryocooler Plate (blue).

was recorded for each case.

2) *Cryocooler Oscillations:* The previous simulation focused on finding the steady state thermodynamic solution of the cooling link. However, the cryocooler temperature fluctuates periodically, so assuming a constant temperature at the top of the cryocooler plate is unrealistic. To model the cyclic variation in temperature, the temperature at the cryocooler plate was modified to a periodic function.

$$T(t) = T_0 + A \sin(2\pi ft) \quad (13)$$

Here T_0 is the cryocooler's average temperature, A is the amplitude of oscillation, and f is the frequency of oscillation. For the simulation, values of $T_0 = 4.2$ K, $A = 0.25$ K, and $f = 1$ Hz, which model the cryocooler operation with a dissipated power of about 1.5 W (see Figures 12, 13), were used.

Due to time constraints, the time-dependent simulation was performed only on the partial geometry in Figure 14, not on the full geometry. The oscillations in temperature at the weld faces of the rings were compared to the temperature oscillations at the cryocooler plate.

3) *Multiphysics Simulation of Cavity:* In the second set of simulations, the cavity was simulated without the niobium rings and cooling link. An eigenfrequency study was performed on the vacuum space within the cavity to find the resonant frequency ω , along with the H field, G , U , R/Q , and B_{pk} . The simulation returns relative values for the energy levels within the cavity, but without information about the driving source, it cannot calculate the absolute magnitude of the electric and magnetic fields. To scale the results, at least one value must be assumed. For this simulation, E_{acc} was provided as a parameter, and the electric and magnetic field values were multiplied by the ratio of E_{acc} to the arbitrary accelerating gradient found in the simulation to produce meaningful values.

The magnetic field H on the inner surface of the cavity was then used as input to a heat transfer simulation through

Fig. 15. Temperature Distribution Through Cooling Link with $P_d = 2$ W and $T_{cryo} = 4.5$ K.

the cavity. The dissipated power in each mesh element, dA , was calculated using the known H field and the temperature-dependent R_s . The heat flows out through the notches in the cavity where the niobium rings are welded on. These notches were approximated as having a spatially uniform, constant temperature T_n , which was found from the accelerating gradient and the results of the thermal link simulation. The cavity simulation was repeated for different values of E_{acc} , and temperature difference across the cavity ΔT_{cavity} was recorded for each case.

Additionally, the effects of R_s on ΔT_{cavity} were examined. The R_s values shown in Figure 11 assume an even coating of Nb_3Sn across the cavity. The simulation was repeated for cases where the coating was properly applied on only 90% and on only 80% of the cavity. To model these cases, the surface resistance of a "bad" coating, $R_{s,bad}$ was calculated as three times the ideal surface resistance, $R_{s,ideal}$, which is shown in Figure 11. The total surface resistance of a cavity with a coating properly applied on pct percent of the surface was then found as

$$R_s = pct R_{s,ideal} + (1 - pct) R_{s,bad} \quad (14)$$

In addition, a pure niobium cavity with no Nb_3Sn coating was simulated, and the limitations of this cavity were examined.

4) *Multiphysics Simulation of Entire Geometry*: Finally, the entire geometry was simulated. The electromagnetic fields within the cavity were calculated, and the heat flow of the dissipated power from the inner surface of the cavity to the cryocooler plate was modeled. The results from this simulation support the previous simulations.

IV. SIMULATION RESULTS

A. Heat Transfer Through Cooling Link

The temperature distribution of the cooling link was found for eighteen values of the dissipated power P_d . Shown in Figure 15 is the temperature map for $P_d = 2$ W. Figure 16 shows the average temperature of the niobium weld faces as a function of dissipated power.

Fig. 16. Average Temperature of Cryocooler Plate and Niobium Ring Weld Faces For Various Total Dissipated Power.

Fig. 17. Temperature Oscillations with Time.

B. Cryocooler Oscillations

The periodic cryocooler temperature, approximated as a sine wave, is shown in blue in Figure 17, and the spatial average of the weld face temperature is shown in red. Although the temperature at the weld faces does oscillate with time, the amplitude of the oscillations is about one-fourth of the amplitude of the cryocooler oscillations. Since niobium is a relatively poor thermal conductor, these oscillations would likely attenuate even further across the cavity surface.

C. Multiphysics Simulation of Cavity

The magnitude of the electric and magnetic fields inside the cavity were found in the second simulation and are shown, scaled for $E_{acc} = 10$ MV/m, in Figures 18 and 19. The electric field is strongest in the center of the cavity and points along the cavity axis. The magnetic flux is strongest around the cavity equator. The calculated resonant frequency of the cavity is 647

Fig. 18. Electric Field Magnitude Inside the Cavity.

Fig. 19. Magnetic Flux Magnitude Inside the Cavity.

MHz. Using the frequency and fields, we can find the other RF parameters and compare with the theoretical values for this shape cavity. The RF parameters found with an assumed accelerating gradient of 10 MV/m are shown in Table I.

Using the magnetic field calculated in the RF eigenfrequency study, the temperature distribution across the cavity was found. The heat flow depends on the choice of E_{acc} and R_s . Figure 20 shows an example of the temperature across the cavity with $E_{acc} = 10$ MV/m, $T_n = 4.52$ K, and 100% ideal R_s . The temperature difference across the elliptical surface of the cavity is 130 mK, which is small compared to the temperature difference across the thermal link. Moreover, the average temperature and maximum temperature of the cavity elliptical surface are within 2% of each other. These trends were also apparent for the other simulated values of

Fig. 20. Temperature Distribution in Cavity.

the accelerating gradient and surface resistance. Therefore, the spatial temperature variations across the cavity are largely inconsequential, and only the average temperature of the cavity elliptical surface is reported for the remainder of the simulation results.

The accelerating gradient was adjusted to four different values. For each value of E_{acc} , the dissipated power was approximated using Equation 7, using the simulated values for the geometric factor and normalized shunt impedance shown in Table I and assuming a surface resistance of 10 nΩ. The notch temperature T_n was then determined from Figure 16. Using this notch temperature and the known H field on the cavity surface, heat flow through the cavity was modeled. The accelerating gradient and surface resistance were varied, and the average temperature of the cavity was recorded. These results are shown in Figure 21. It can be seen that small changes in the quality of the Nb₃Sn coating make very little impact on the cavity temperature.

To investigate the importance of the Nb₃Sn coating, a cavity without the coating was simulated. A notch temperature $T_n = 5$ K and a dissipated power $P_d = 2$ W were assumed. At 5 K, pure, uncoated niobium has an approximate surface resistance of 680 nΩ [16]. Using Equation 7, the accelerating gradient for these conditions was found to be 1.4 MV/m. The simulated temperature distribution for this case was nearly identical to that shown in Figure 20 with a maximum temperature of 5.12 K. However, the surface resistance of pure niobium rises with temperature, and the heat capacity of the cryocooler is quickly exceeded at accelerating gradients above 1.4 MV/m. The Nb₃Sn coating is a key factor in enabling conduction cooling for cavities at mid-range accelerating gradients.

D. Multiphysics Simulation of Entire Geometry

The previous simulations can be used together to approximate the temperature across the cavity and cooling link for different values of the accelerating gradient. To verify these

TABLE I
SIMULATED RF PARAMETERS AND THEORETICAL VALUES.

Parameter	Symbol	Simulated Value	Theoretical Value
Peak Surface Magnetic Flux	B_{pk}	38.2 mT	36.0 mT
Peak Surface Electric Field	E_{pk}	17.5 MV/m	16.0 MV/m
Geometric Factor	G	265	266
Normalized Shunt Impedance	R/Q	138	138
Dissipated Power	P_d	1.50 W	
Stored Energy	U	9.49 J	
Quality Factor	Q_0	2.6×10^{10}

Fig. 21. Average Cavity Temperature for Varying Accelerating Gradient and Surface Resistance.

Fig. 22. Average Cavity Temperature for Varying Dissipated Power.

approximations, a multiphysics simulation of the entire geometry was performed. The maximum temperature of the niobium, T_{max} , was found as a function of dissipated power, P_d . The results are shown in Figure 22. These results match very well with the combined results of the previous simulations.

V. CONCLUSIONS

Conduction cooling of a Nb₃Sn coated single cell 650 MHz SRF cavity was simulated using the geometry of the cavity, measured heat capacity of a Cryomech PT420 cryocooler, and thermodynamic properties of a designed cooling link. For accelerating gradients between 6 and 12 MV/m, the average equilibrium temperature of the cavity was below 6 K, within the superconducting regime of niobium. Moreover, the temperature of the cavity elliptical surface was found to be nearly spatially uniform. The Nb₃Sn coating was shown to be of critical importance for operating the cavity at an accelerating gradient above 1.5 MV/m. Additionally, the temperature oscillations of the cryocooler were modeled, and the amplitude of temperature fluctuations significantly dampened at the location of the cavity. Based on the simulation results, a commercial cryocooler and the designed thermal link are a viable alternative to liquid helium for SRF cavities with Nb₃Sn coating in applications which require an accelerating gradient of 12 MV/m or below.

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