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Beam Echoes in the Presence of Coupling

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Abstract

Transverse beam echoes could provide a new technique of measuring diffusion characteristics orders of magnitude faster than the current methods; however, their interaction with many accelerator parameters is poorly understood. Using a program written in C, we explored the relationship between coupling and echo strength. We found that echoes could be generated in both dimensions, even with a dipole kick in only one dimension. We found that the echo effects are not destroyed even when there is strong coupling, falling off only at extremely high coupling values. We found that at intermediate values of skew quadrupole strength, the decoherence time of the beam is greatly increased, causing a destruction of the echo effects. We found that this is caused by a narrowing of the tune width of the particles. Results from this study will help to provide recommendations to IOTA (Integrable Optics Test Accelerator) for their upcoming echo experiment.

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1 Introduction

One of the design challenges of high intensity accelerators is the management of diffusion effects, caused by space charge and other intensity-dependent effects. If left unmanaged, these effects can cause a large beam emittance growth rate, which can lead to beam loss over time. The first step to mitigating these effects is to accurately measure the diffusion rate. Unfortunately, the current method of achieving this (beam scraping) takes many hours to complete. Not only is this prohibitive for the development of more accurate correction methods, it is unfeasible for many accelerators, which cannot maintain a beam for that long.

The measurement of diffusion strength using transverse beam echoes could provide orders of magnitude improvement on measurement time. The strength of a beam echo is strongly correlated with the diffusion coefficient, and thus, measurements could be made in the time it takes to set up an echo effect, on the order of a few thousand turns. Currently, the theory of echoes is underdeveloped, and in order to accurately measure the diffusion, the delicate relationship between echo strength and many common accelerator parameters must be understood.

Building on previous research [1] which characterized the relationship between echo strength and many common accelerator parameters in 1 dimension, we characterize the effects of coupling between multiple dimensions on echo strength. We also attempt to understand the decoherence effect that we found occurs at certain skew quadrupole strengths. We hope to use the results of this research to develop a more robust theory of beam echoes, and to provide recommendations to the upcoming echo experiment at IOTA (Integrable Optics Test Accelerator).

2 Theory

2.1 What is a beam echo?

An analogue of spin echoes in NMR, beam echoes have been observed in both the transverse and longitudinal directions. In the transverse direction, particle motion is normally characterized in Floquet coordinates:

$$\xi = \frac{x}{\sqrt{\beta}} \qquad\qquad \eta = \sqrt{\beta}x + \frac{\alpha}{\sqrt{\beta}}x' \tag{1}$$

where x is the coordinate of transverse position, and α, β are the usual Courant Snyder parameters. Under the action of a linear lattice, used in accelerators to focus the beam, the particles will move in a circle in this coordinate system. Due to nonlinearities in the lattice, the rotation rate is dependent on the action J, defined by:

$$J = \frac{\xi^2 + \eta^2}{2} \tag{2}$$

This dependency is crucial to generating the echo effect. Suppose you have an initial beam distribution that is Gaussian in the action (See Figure 1). At t=0, we apply a one-turn dipole kick, shifting each particle by the same amount in the x' direction. The particles will now undergo oscillation around the equilibrium position, but at different rates. Because of this, the phase of the particles will become desynced, and the beam will undergo decoherence. Long after this decoherence, at t=τ, if we apply a one-turn quadrupole kick, we will see a temporary recoherence of the beam at t=2τ [2]. This phenomenon is known as a beam echo. Further echoes have been observed to occur at t=4τ,t=6τ, etc [4].

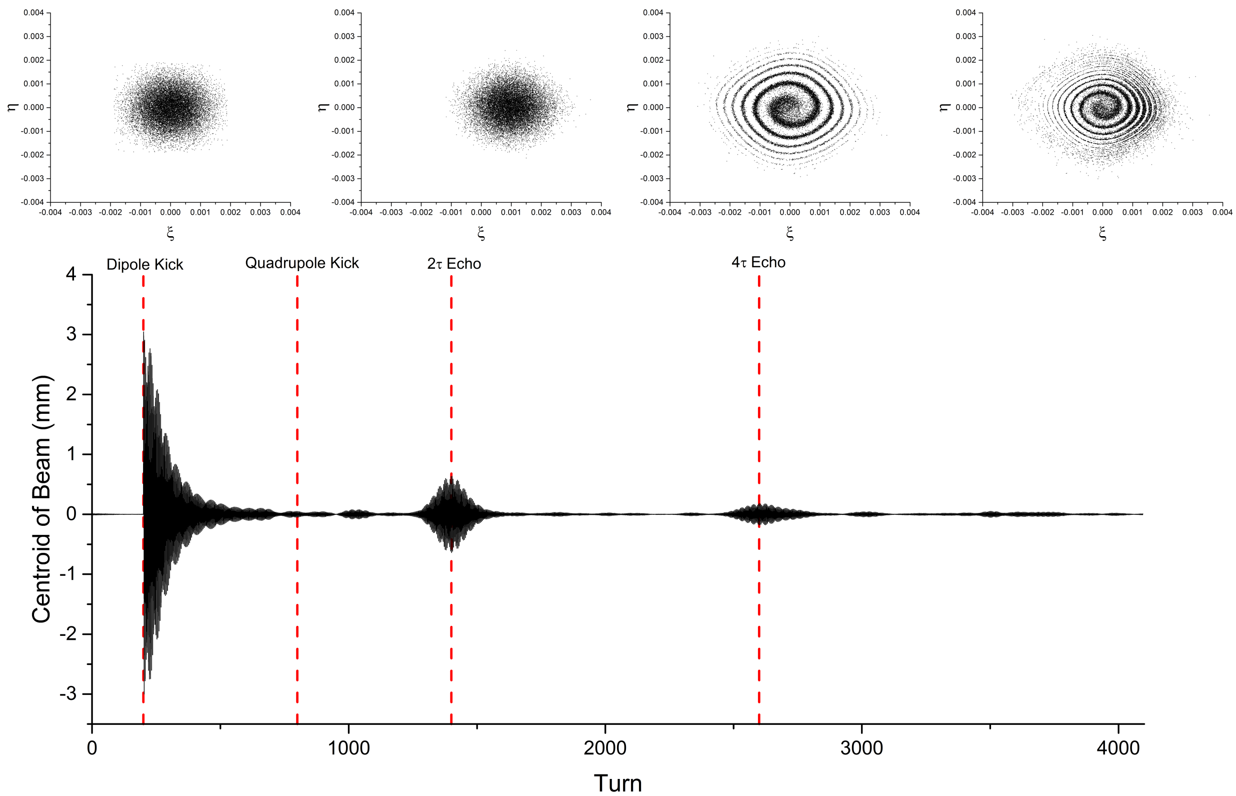

Figure 1: **Top:** Beam distribution profiles in Floquet coordinates, from left to right: Initial distribution, after dipole kick, after quadrupole kick, at 2τ echo.
Bottom: Centroid of the beam (average particle position) as a function of time.

2.2 Theory of Beam Echoes

The current established theory of beam echoes is first-order, and predicts the following expression for echo amplitude:

$$EchoAmpl_{linear} = \theta\beta Q, \qquad Q = q\omega'\epsilon\tau \tag{3}$$

where θ is the dipole kick strength, β is the betatron function at the dipole and quadrupole, q is the quadrupole kick strength, ω' is the slope of the angular betatron frequency, and ϵ is the initial emittance of the beam. This theory, while giving a relatively simple result, requires making a number of approximations, notably that the dipole and quadrupole kicks are small, and that τ is much greater than the decoherence time. We also define the the number of betatron oscillations per revolution as the tune of the beam.

As the approximations are generally good for the parameter space used in accelerators, the established theory does a decent job of establishing the relationship between echo amplitude and common accelerator parameters. However, it fails significantly at high quadrupole kick strengths (See Figure 2), and only predicts an echo at t=2τ. For this reason, using the results of previous research, a nonlinear theory of beam echoes was developed [3], which predicts the following for the echo amplitude:

$$EchoAmpl_{nonlinear} = \frac{\beta\theta Q}{[(1 + Q^2 - Q_2^2)^2 + 4Q_2^2]^{\frac{3}{4}}} \tag{4}$$

$$Q = q\omega'\epsilon\tau, \qquad Q_2 = \frac{q^2\omega'\epsilon\tau}{2} \tag{5}$$

where all variables are defined as before. While this theory is better, it is not yet ideal. While it better predicts the relationship between quadrupole kick strength and echo amplitude (See Figure 2), it still fails to predict the existence of echoes past t=2τ. This theory has recently been extended to include both nonlinear dipole and quadrupole kicks, which now predicts the existence of echoes at multiples of 2τ [3].

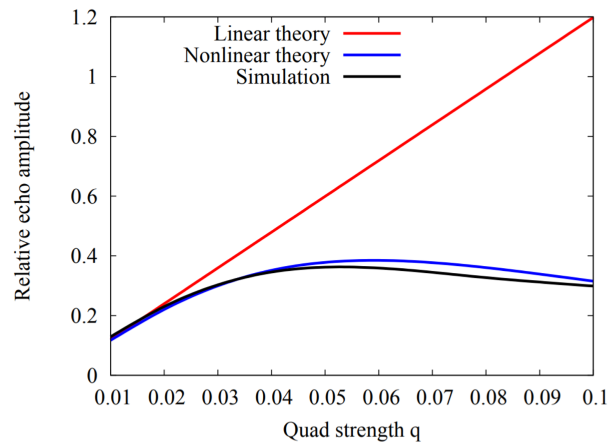

Figure 2: Comparison between linear and nonlinear predictions for echo amplitude as a function of quadrupole strength

3 Simulation

The simulation (See Figure 3) was modified from a previous simulation code in C. The program tracked the coordinates of individual particles as they propagated around a simulated ring. Parameters of the simulation were based the the previous beam echo experiment at RHIC (See Figure 4) [4]. The simulation was designed with three main components, which when alternated simulate all three effects occurring at once.

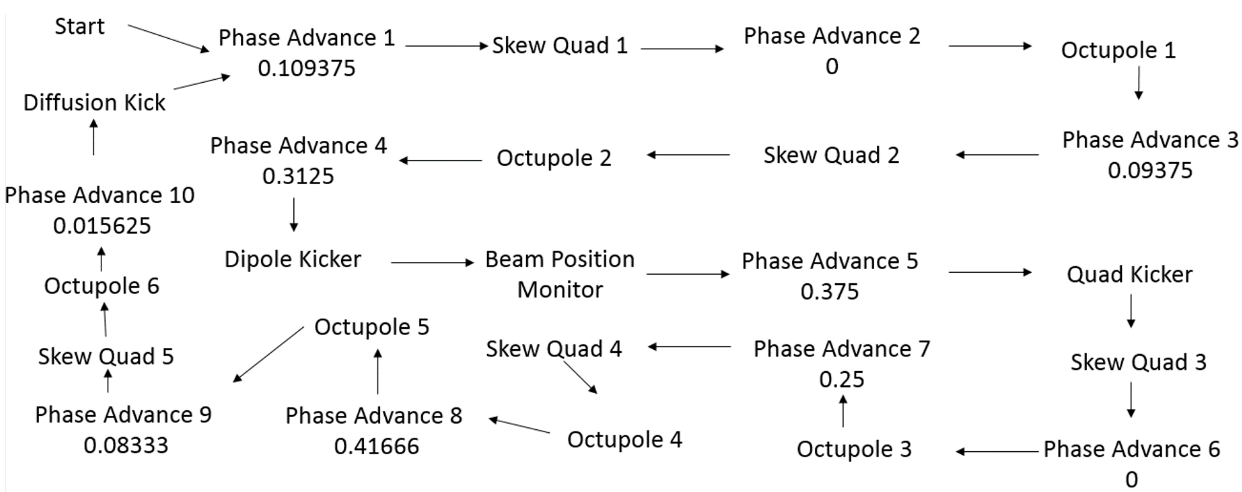

Figure 3: Simulation Setup. Numbers indicate fraction of total phase advance ($2\pi\nu$)

Number of Particles	20000
Initial Emittance	1.5E-7
Initial X Tune	0.235
Initial Y Tune	0.215-0.225
Betatron at BPM, Quadrupole, and Dipole (m)	10
Quadrupole Kick Strength (1/m)	0.007
Dipole Kick Strength	3.E-4
Time between Dipole and Quadrupole Kick	600-3000 Turns

Figure 4: Common Simulation Parameters based on 2005 RHIC Experiment

The components of the simulation are as follows:

1. Phase Advance: Simulates propagation through the FODO lattice. The transfer map is given by [5]:
$$\begin{bmatrix} \xi \\ \eta \end{bmatrix}_{new} = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \xi \\ \eta \end{bmatrix}_{old} \tag{6}$$
where ϕ is the phase advance in that segment.

2. Octupole Magnets: Simulates the nonlinearities that generate the phase decoherence. The transfer map is given by [5]:
$$\begin{aligned} \xi_{x_{new}} &= \xi_{x_{old}}, \qquad \xi_{y_{new}} = \xi_{y_{old}} \\ \eta_{x_{new}} &= \eta_{x_{old}} + k_{oct}(3\beta_x\beta_y\xi_{x_{old}}\xi_{y_{old}}^2 - \beta_x^2\xi_{x_{old}}^2) \\ \eta_{y_{new}} &= \eta_{y_{old}} + k_{oct}(3\beta_x\beta_y\xi_{x_{old}}^2\xi_{y_{old}} - \beta_y^2\xi_{y_{old}}^2) \end{aligned} \tag{7}$$
where k_{oct} is defined with respect to the magnetic field generated by the octupole as:
$$\begin{aligned} B_x &= \frac{(\beta\rho)k_{oct}}{l}(3x^2y - y^3) \\ B_y &= -\frac{(\beta\rho)k_{oct}}{l}(3xy^2 - x^3) \end{aligned} \tag{8}$$
where $(\beta\rho)$ is the beam rigidity and l is the octupole length.

3. Skew Quadrupole Magnets: Simulates the coupling between the dimensions. The transfer map is given by [5]:
$$\begin{aligned} \xi_{x_{new}} &= \xi_{x_{old}}, \qquad \xi_{y_{new}} = \xi_{y_{old}} \\ \eta_{x_{new}} &= \eta_{x_{old}} - k_{skew}\sqrt{\beta_y}\xi_{y_{old}} \\ \eta_{y_{new}} &= \eta_{y_{old}} - k_{skew}\sqrt{\beta_x}\xi_{x_{old}} \end{aligned} \tag{9}$$
where k_{skew} is defined with respect to the magnetic field generated by the skew quadrupole as:
$$\begin{aligned} B_x &= -\frac{(\beta\rho)k_{skew}x}{l} \\ B_y &= \frac{(\beta\rho)k_{skew}y}{l} \end{aligned} \tag{10}$$
where $(\beta\rho)$ is the beam rigidity and l is the skew quadrupole length.

The simulation was designed to both simulate 1D and 2D systems. In two dimensions, due to the extra coupling term present in the octupole magnet transfer maps, the relative octupole strength needed to be lower by a factor of 5. It was therefore important to check that the relationship between quadrupole strength and echo strength was independent of this change. As shown in Figure 5, this nearly identical for both the 1D ($k_{oct} = 1$) and 2D ($k_{oct} = 0.2$) systems.

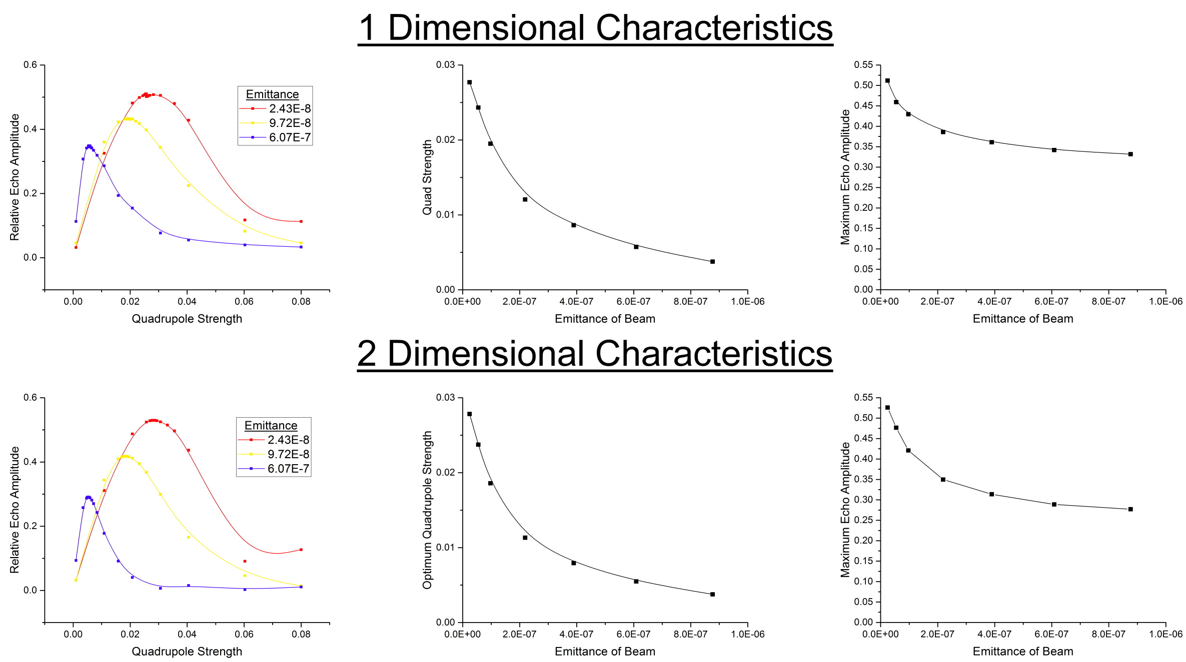

Figure 5: Echo Strength of 1D and 2D simulations as a function of quadrupole strength and emittance. The 2D Simulations were run with initial emittance in both dimensions equal, and skew quadrupole strength set to 0. Curves were generated from a spline of the data points.
Left: Echo amplitude as a function of quadrupole kick strength
Middle: Optimum quadrupole kick strength as a function of emittance
Right: Maximum relative echo amplitude (echo strength divided by dipole kick strength) achieved at the optimum quadrupole strength as a function of emittance

4 Results

4.1 Coupling

One of the major questions we sought to answer was: Does a dipole kick in the x direction generate an echo in the y direction through coupling? Not only can this happen, but the echo strength that can be generated is significant. As shown in Figure 6, as the skew quadrupole strength increases, the y echo strength increases. Looking at the centroid profiles, due to the coupling, a dipole kick in the x direction generates a dipole kick in the y direction of weaker strength. Since stronger coupling will generate a stronger dipole kick in the y direction, and echo strength is proportional to dipole strength, it is understandable why the y echo strength increases with echo strength. Interestingly, as the y echo strength increases, the x echo strength decreases. This is the cost of the coupling. Because the tune spread of particles is affected by motion in both the x and y dimensions, the coupling that leads to the y echo will cause weaker phase coherence in the x direction, lowering the echo strength.

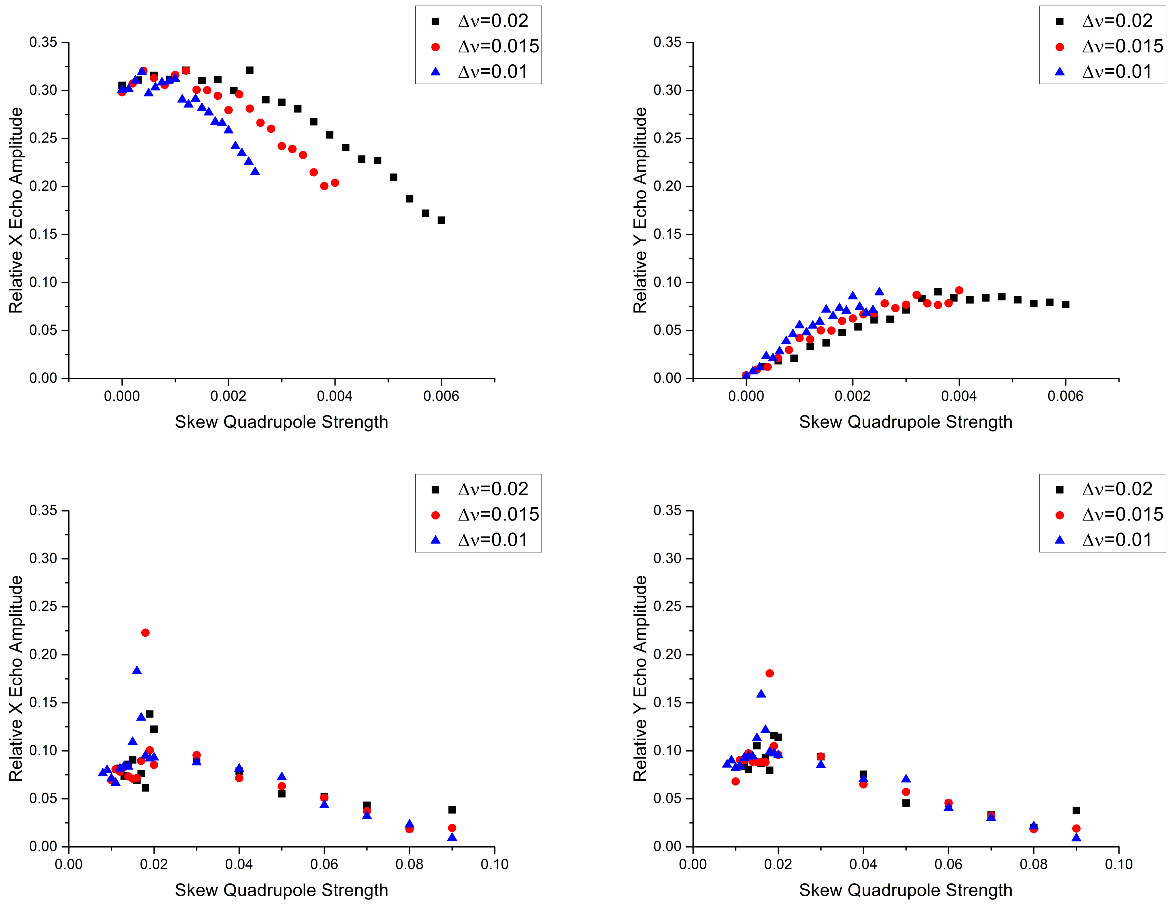

Figure 6: Echo Strength as a function of coupling (skew quadrupole) strength for various initial tune separations.
Top: X Echo strength **(left)**, Y Echo strength **(right)** as a function of low skew quadrupole strength.
Bottom: X Echo strength **(left)**, Y Echo strength **(right)** as a function of high skew quadrupole strength.

The results at very strong skew quadrupole strengths are surprising.As echos are generated from delicate phase interactions, it is not unreasonable to think that strong coupling effects would affect this and destroy the echoes. However this does not happen until extremely strong coupling. At skew quadrupole strengths that are merely strong, the echo is significant, including an interesting spike in echo amplitude around $k_{skew} = 0.02$ that occurs for all initial tune separations, although at different magnitudes. This "anomaly" is completely unexpected, and warrants further investigation. Given the strengths of the coupling considered, it is not surprising to see that the x and y echo strengths are nearly identical.

4.2 Decoherence

An interesting effect occurs at medium skew quadrupole strengths. For some reason, as skew quadrupole strength increases, the decoherence time of the beam increases massively to a maximum, followed by a decrease back to low decoherence time (See Figure 7). This effect is correlated the initial tune separation of the particles, defined as the difference between the initial x tune and the initial y tune. As initial tune separation increases, so does the skew quadrupole strength at which the maximum decoherence time is observed.

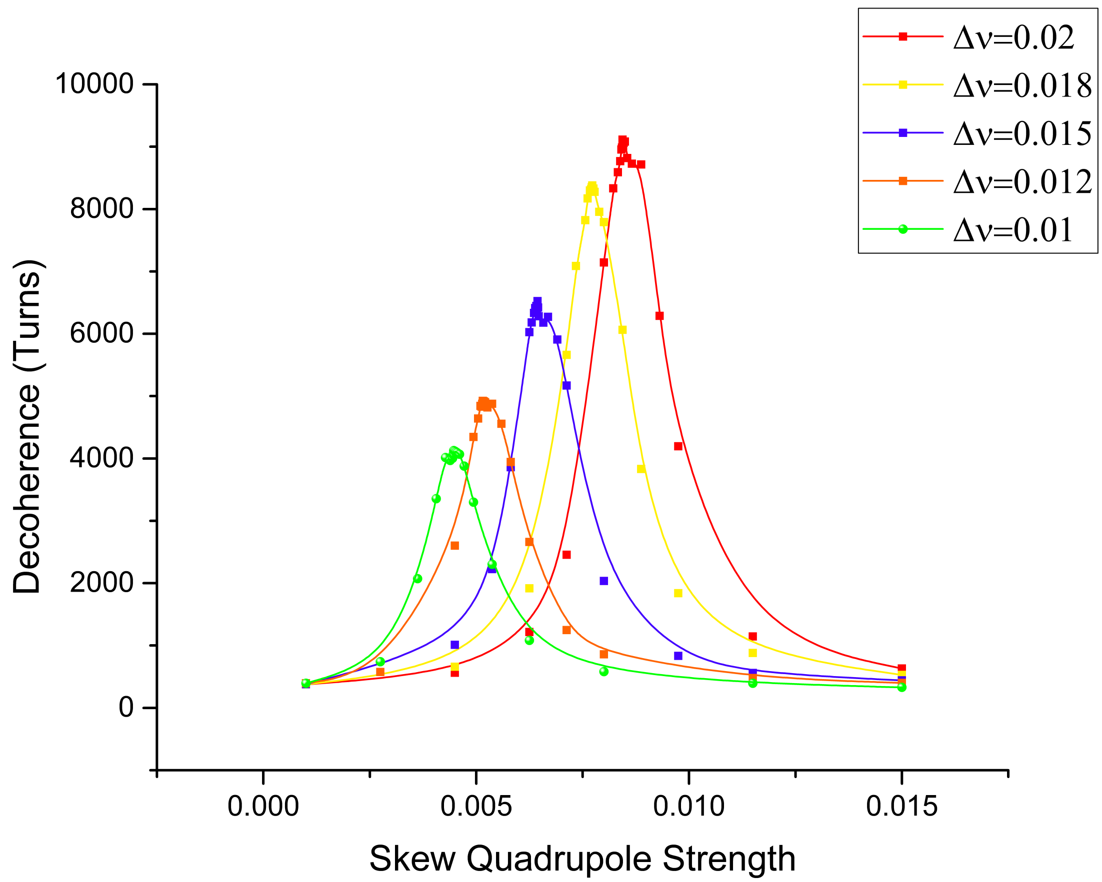

Figure 7: Decoherence Time (defined as the time for the centroid amplitude to decay to $\frac{1}{e}$ of the original dipole kick amplitude) as a function of skew quadrupole strength, at various initial tune separations

This decoherence effect is correlated with particles shifting their tunes to the difference resonance (See Figure 8). As the coupling increases, particles begin to shift their y tune up to be equal to the x tune, or visa versa, with a higher fraction of particles undergoing this effect at skew quadrupole strengths. At both low skew quadrupole strengths, where the number of particles that move is small, and high skew quadrupole strengths, where most of the particles move, the decoherence time is low. But in the middle, where the number of particles that move their tune is approximately half, the decoherence time is high. This explains the relationship between initial tune separation and location of maximum decoherence, as a larger initial tune separation implies a stronger coupling needed to shift particles to the difference resonance.

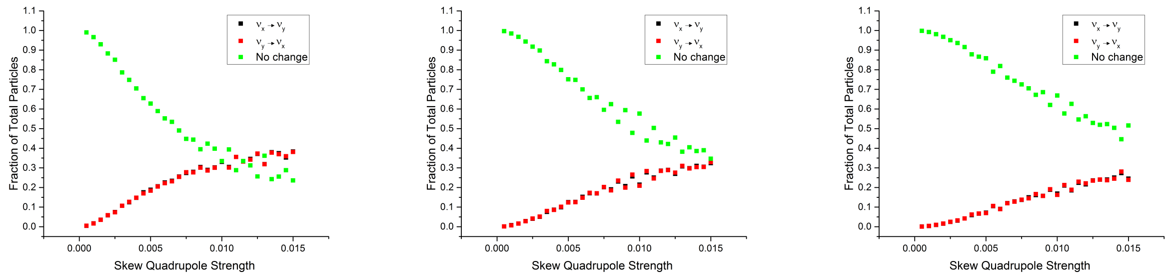

Figure 8: Distribution of Particles that move their tune as a function of skew quadrupole strength, at different initial tune separations, (**left to right**): 0.01, 0.015, 0.02.

When dealing with an initial Gaussian beam distribution, even though all the particles are given the same initial tune, they will undergo a tune shift proportional to their initial action. There will therefore be a Gaussian distribution in the tune of the particles, which gives the decoherence effect that generates the echo. The decoherence time is determined by the inverse of the width of the tune distribution. For some reason, as the coupling increases, this tune width narrows around the maximum decoherence, then widens again past the maximum (See Figure 9). This explains the decoherence effect; however, the narrowing of the tune width is still unexplained.

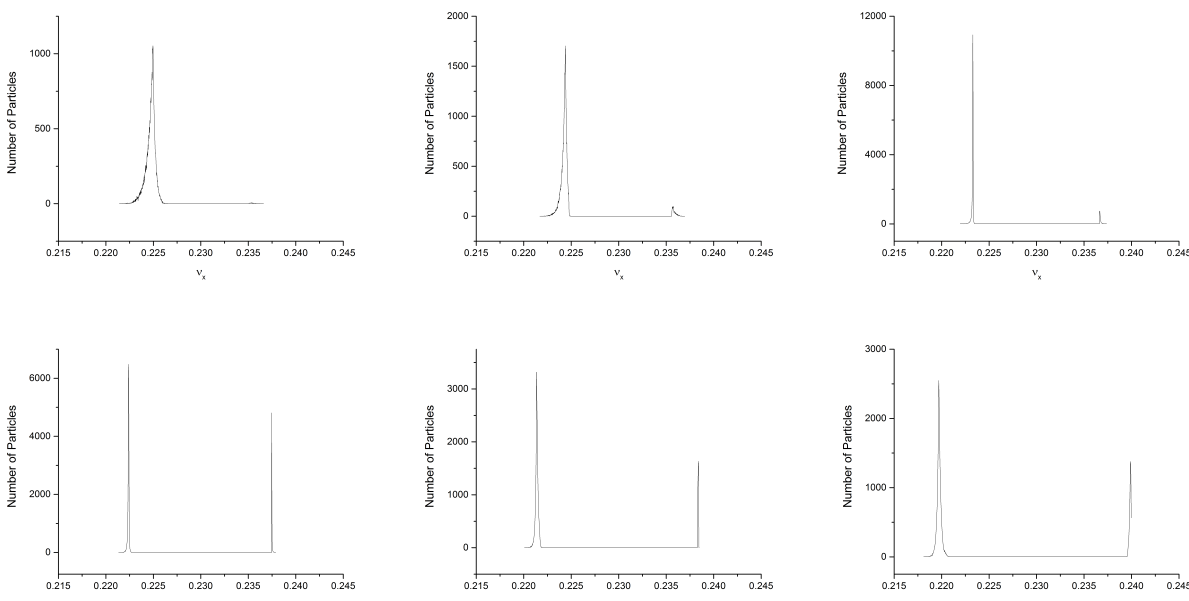

Figure 9: Tune Distribution in Y direction for initial tune separation $\Delta\nu = 0.01$ at different skew quadrupole strengths. Note the width of the peaks is minimum at the location of the maximum decoherence, corresponding to the skew quadrupole strength $k_{skew} = 0.0045$. Skew quadrupole strengths in the plots are(**left to right, top to bottom**): 0.0005,0.002,0.0035,0.0045,0.0055,0.007.

5 Conclusions

In conclusion, we have found that coupling between the dimensions can generate an echo both in the x and y direction from a dipole kick solely in the x direction. This makes sense, as the coupling generates a dipole kick in the y direction from the x dipole kick. As the coupling increases, so does the y echo amplitude, at the cost of the x echo amplitude decreasing. It is surprising that the echo effect is not destroyed until the coupling is extremely strong. There are still significant echoes at strong coupling, including an apparent spike in echo strength that merits investigation. This is an important result for the feasibility of measuring diffusion using beam echoes, as many accelerators have non-negligible coupling.

At skew quadrupole strengths in the range 0.003-0.01, the decoherence time is massively increased. The location of the the maximum decoherence time and the location of this maximum in skew quadrupole strength are both proportional to the initial tune separation. This appears to be related to particles moving their tune from the initial values to the difference resonance. This effect is correlated with a narrowing in the tune width of the particles, which is inversely proportional to the decoherence time, however, the cause of the narrowing of this tune width is unknown. In the future, we hope to explain the source of the narrowing of the tune width, as well as explore the echo effects of coupled systems with diffusion. We hope to use these and future results to develop a theoretical model of beam echoes in 2 dimensions, including coupling and diffusion effects.

6 Acknowledgements

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7 References

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