TM-1738

Beam-beam interaction effects

...Leo Michelotti

Most of the calculations to be discussed were done within a scenario of 36×36 bunches circulating in a 25 cm (or, sometimes, 50 cm) low beta lattice whose bare tunes are $\nu_1 = 20.57837$ and $\nu_2 = 20.58987$. The proton transverse invariant emittance is assumed to be 30π mm-mr; the antiproton emittance is 22π. There are 33×10^{10} protons and 6×10^{10} antiprotons per bunch.

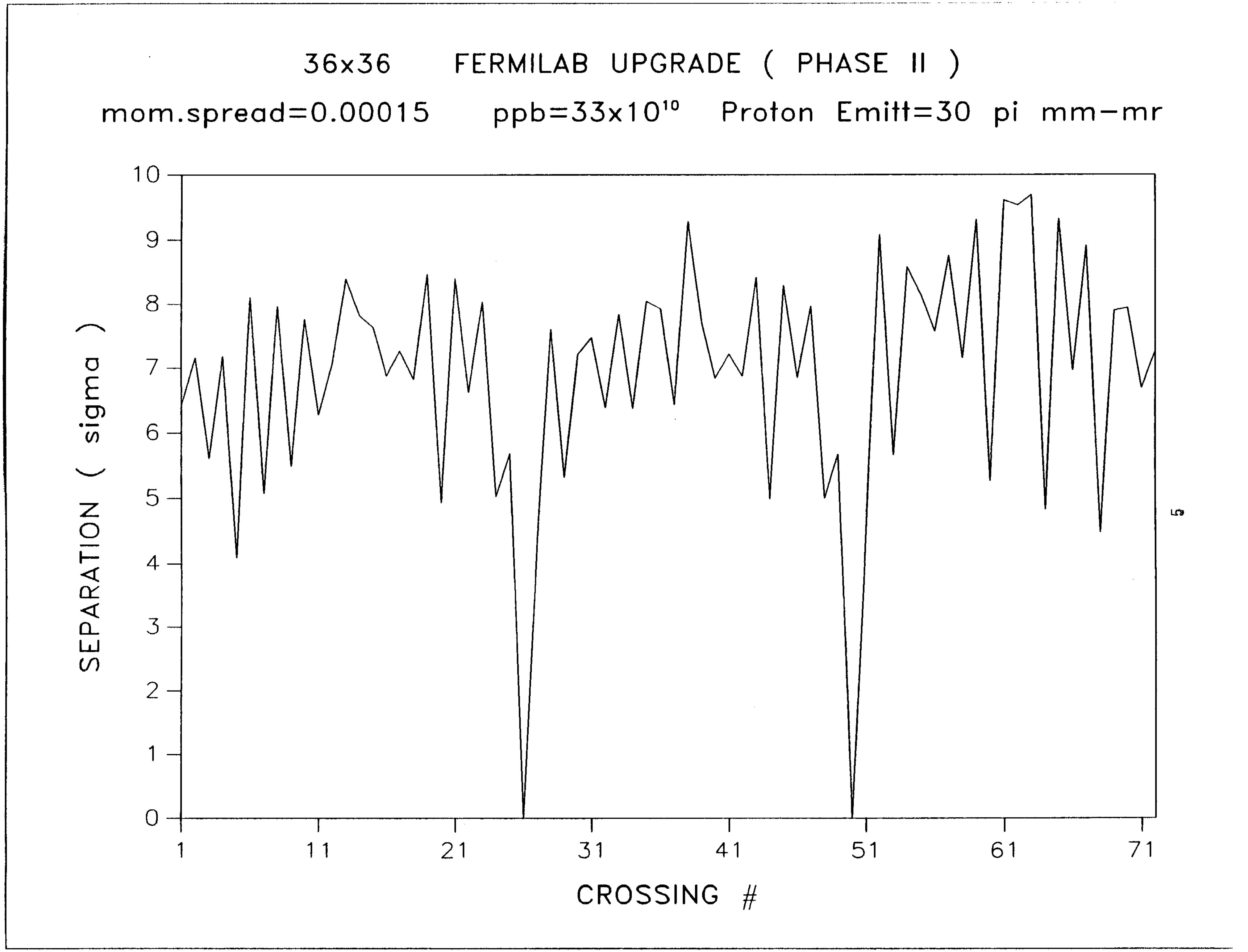
The separator voltages and kicks shown in this table are what we shall later refer to as "full separator strength." A few graphs will show quantities as functions of "normalized separator strength," which simply means we multiply all these voltages by some number smaller than one. The kicks shown here produce the closed orbit ...

Separator strengths

In subsequent discussion, "full excitation" of separators refers to the following values.

Label	Kick [μrad]	Voltage [kV]	Modules
.S1H	12.0000	100.00	2
.S2H	16.2920	135.77	2
.S3H	7.6341	63.62	1
.S4H	12.0000	100.00	2
.S5H	-3.2312	-26.93	1
.S6H	-7.6263	-63.55	1
.S1V	-8.0000	-66.67	1
.S2V	28.1301	234.42	4
.S3V	-12.1513	-101.26	2
.S4V	8.0000	66.67	1
.S5V	-6.4504	-53.75	1
.S6V	-14.6532	-122.11	2

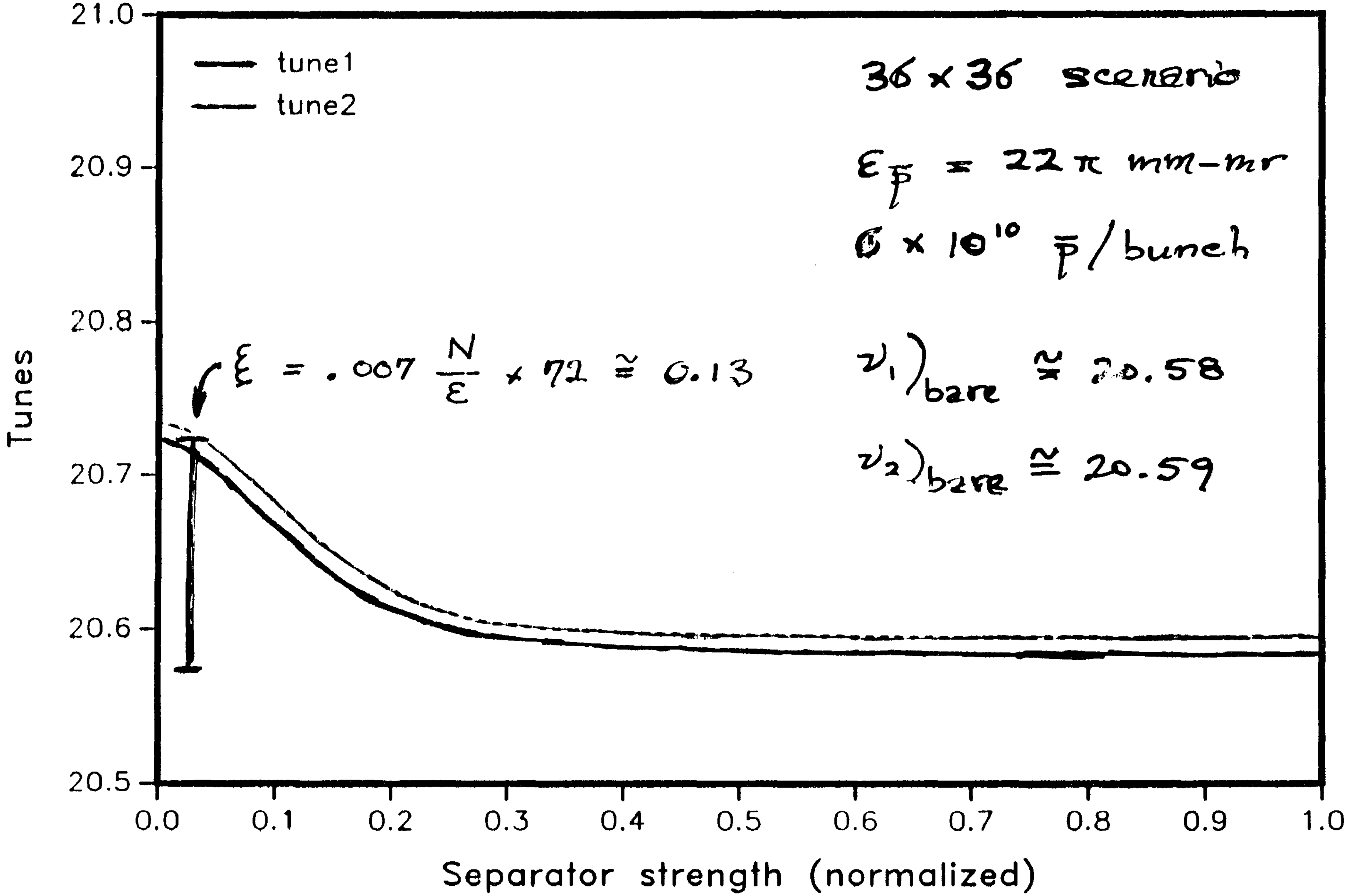
...displayed on this viewgraph: the 72 beam-beam hit sites are indexed on the abscissa, while the ordinate traces the separation in standard deviations, based on a Gaussian proton bunch. B0 and D0 are clearly recognizable as the two points in the ring corresponding to head-on beam-beam collisions.


36x36 FERMILAB UPGRADE (PHASE II)
mom.spread=0.00015 ppb=33x10^10 Proton Emitt=30 pi mm-mr
SEPARATION (sigma)
CROSSING #
0
1
2
3
4
5
6
7
8
9
10
1
11
21
31
41
51
61
71

The proton tunes *at zero transverse amplitude* are shown here as a function of normalized separator strength. When separators are off, tunes shift by the expected amount, ≈ 0.13, using $\xi = 0.007N[10^{10}]/\epsilon_{\text{inv}}[\pi\,\text{mm} - \text{mr}]$ as the beam-beam tune shift per collision. As separators achieve their full voltage, these tunes drop quickly to those expected from the two head-on collisions plus a little extra from the long-range beam-beam interactions. (We shall calculate their contribution later.)

Proton tunes @ zero amplitude

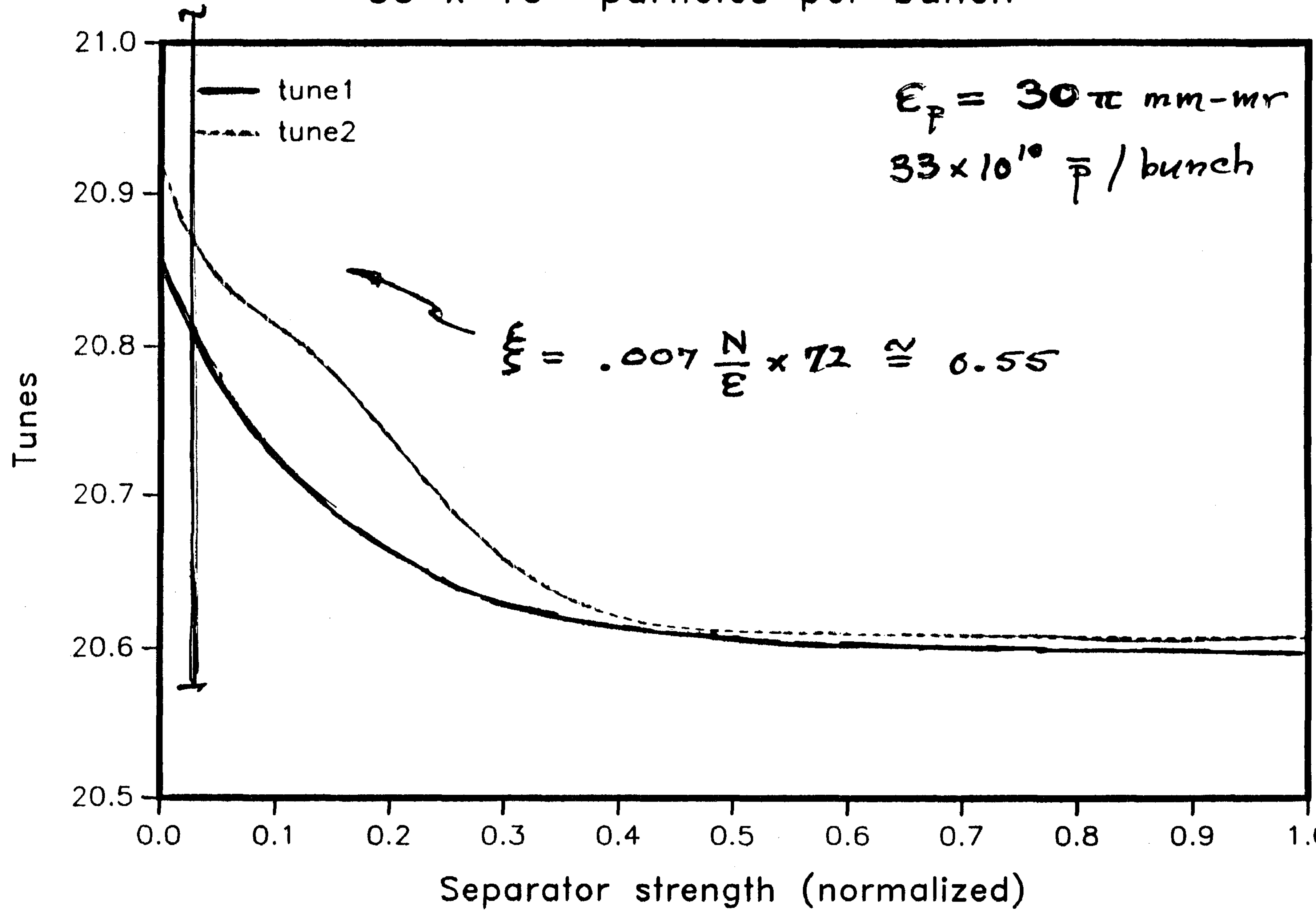
6 x 10^{10} antiprotons per bunch



The zero-amplitude antiproton tunes behave in a similar manner. However, the tune shift with separators off is lower than expected. The resolution of this discrepancy involves some physics of no relevance to the upgrade proposal, as nobody (yet) plans to run 36×36 without separators, but very charming for its own sake. To understand what is going on we must look at the clothed orbits.

Antiproton tunes @ zero amplitude

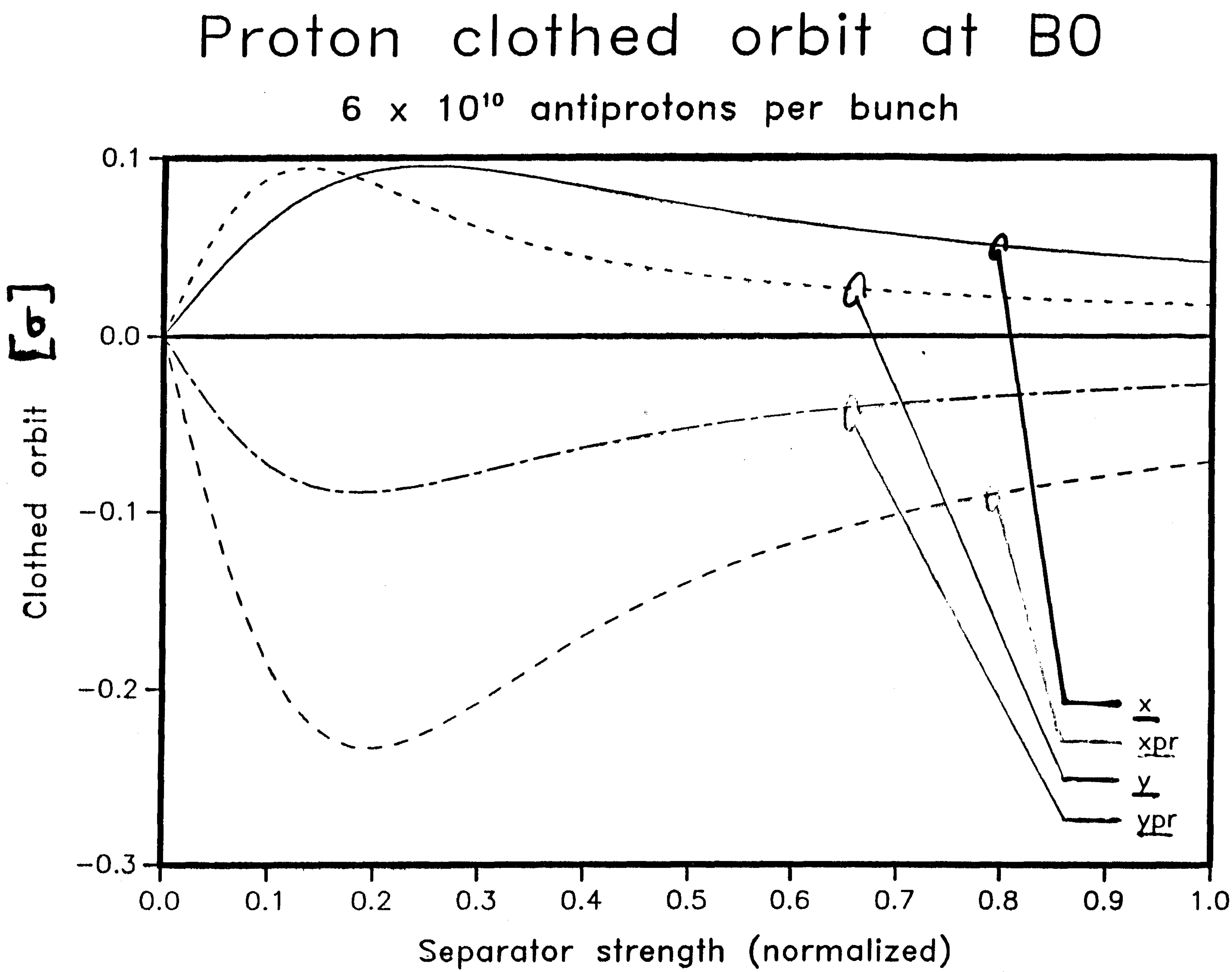
33×10^{10} particles per bunch



The closed orbit shown earlier was calculated assuming propagation by linear optics. The beam-beam interaction itself will modify this orbit, changing the "bare" closed orbit into a "clothed" orbit. Only at the interaction regions, B0 and D0, where the beams are brought into head-on collision, could such small shifts produce significant effects. If displacements become comparable to the transverse bunch width the result would be instability and/or loss of luminosity.

The next four figures show the proton and antiproton clothed orbits, in standard deviations, at B0 and D0. Curves labelled **xpr** and **ypr** refer to normalized x' and y' (i.e., $\alpha x + \beta x'$); thus, the angle corresponding to a limiting value of $x' \approx 0.1$, assuming a 50 μm bunch width, would be about 20 μrad. Some features of these figures are very sensitive to details of the scenario — bare tunes, separator configuration and strength, bunch shape — so that one should use caution in interpreting them: for example, there is no fundamental reason why the proton orbit moves positively at B0 and negatively at D0; these could become interchanged, or it may move in the same direction in both locations, depending on the scenario. On the other hand, the magnitudes of the displacements tend to vary little with small changes in the scenario and depend mostly on the strength of the beam-beam interaction. Indications are that this may produce a residual misalignment of $\leq 0.5\sigma$ even with separators at full strength. Assuming that bunch-to-bunch fluctuations would be much smaller, this could be compensated by fine tuning the separator voltages or otherwise adjusting the clothed orbits.

(These calculations, and the earlier ones of zero-amplitude tunes, were done using Newton's method on one species at a time rather than self-consistently. A self-consistent calculation will eventually be done, but results are not expected to change appreciably.)



Proton clothed orbit at B0
6 x 10^10 antiprotons per bunch
Clothed orbit [σ]
Separator strength (normalized)
0.1
0.0
−0.1
−0.2
−0.3
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
x
xpr
y
ypr

A few observations regarding angles. The (magnitude of the) limiting value of antiproton `ypr` at D0 is about 0.5. We can then solve for the limiting angle, y'; using $\alpha = 0$ at D0, we write

$$
\begin{aligned}
y' &\approx 0.5\,\frac{\sigma}{\beta^*} \\
&= 0.5\,\frac{\sigma}{l}\frac{l}{\beta^*}
\end{aligned}
$$

where l is the bunch length. Since $l/\beta^* \approx O(1)$ and $4\sigma/l = (2\sigma)/(l/2) \equiv \theta_{\text{bunch}}$ is the maximum possible angle subtended by a particle passing through the 1σ region of the bunch as it passes, we get

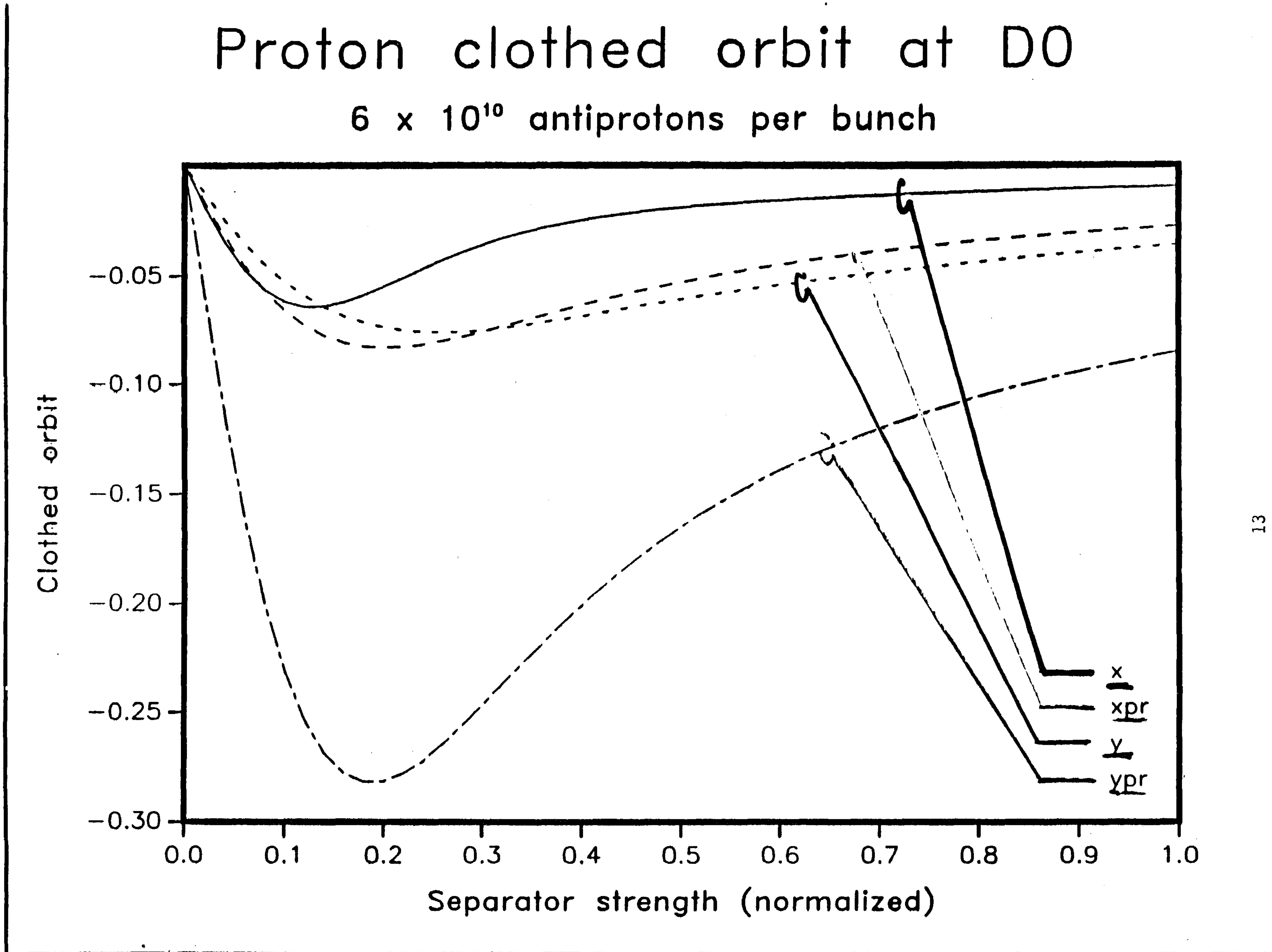
$$
y' \approx 0.1\,\theta_{\text{bunch}} \quad .
$$

The angle subtended by the closed orbit of the (probe) bunch is about 10% of the total available proton bunch angle.

Regrettably, if we follow a similar line of reasoning for individual probe particles in the bunch, the conclusion is not as pleasant. If the probe particles are assumed to lie on an invariant phase space distribution *before* taking the beam-beam kicks into account, then it must be that at D0

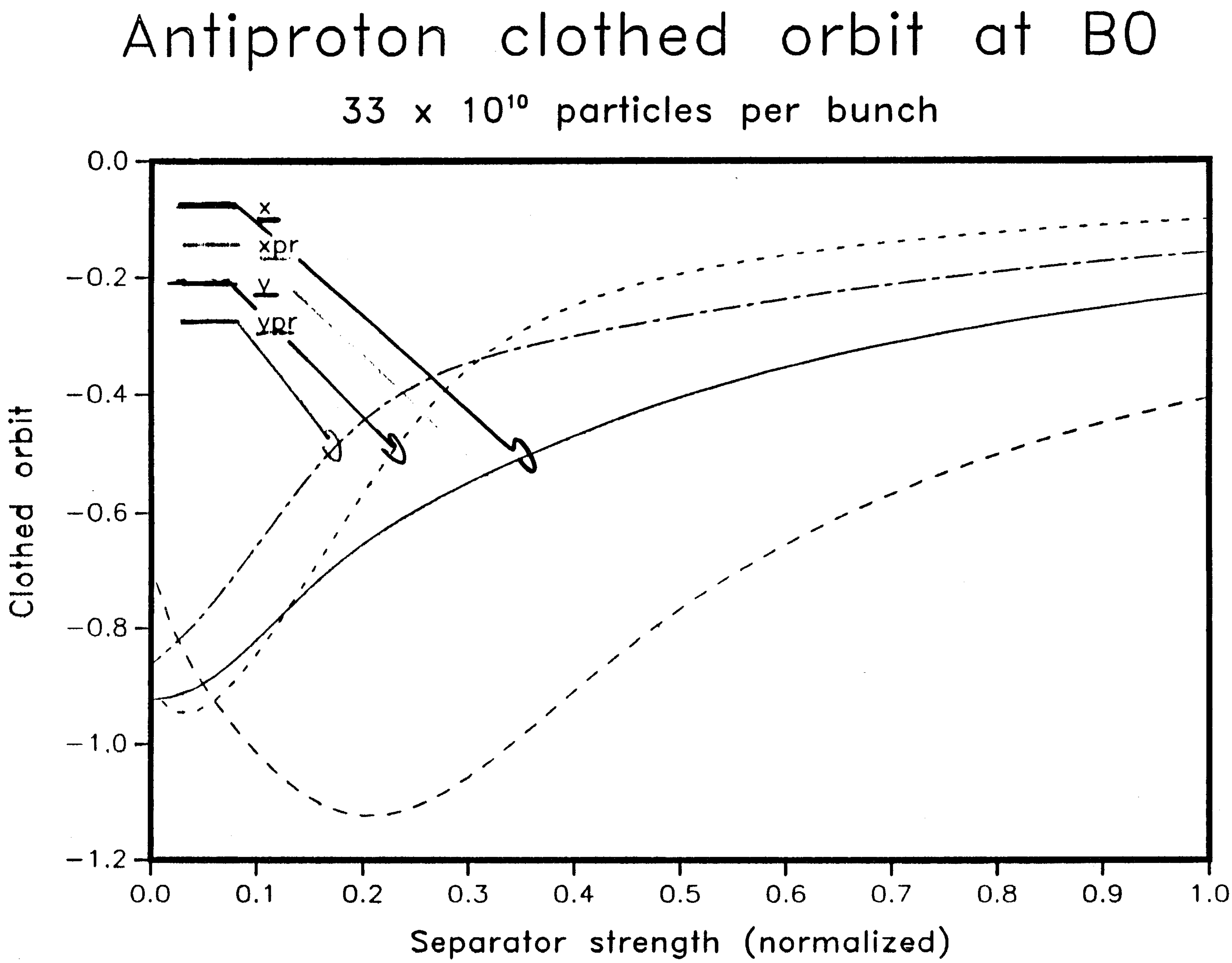
$$
\begin{aligned}
\sigma_{y'} &= \sigma_y/\beta^* \\
&= (\sigma_y/l)(l/\beta^*) \\
&\approx \frac{1}{4}\theta_{\text{bunch}} \quad .
\end{aligned}
$$

(Here, I'm taking $\sigma_p = \sigma_{\bar{p}}$ to make things simpler; otherwise, simply multiply by $\sigma_{\bar{p}}/\sigma_p$.) β^* is so small that particles within the probe bunch are traversing the source bunch at angles that are 25%, or so, of θ_{bunch}, in contradiction to the basic assumption of parallel passage required for the derivation of Montague's beam-beam kick. This raises the question, which


Proton clothed orbit at D0
6 x 10^10 antiprotons per bunch
Clothed orbit
-0.05
-0.10
-0.15
-0.20
-0.25
-0.30
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Separator strength (normalized)
x
xpr
y
ypr

we shall not address here, *of whether we need to modify the form of the beam-beam interaction for low beta collisions.*

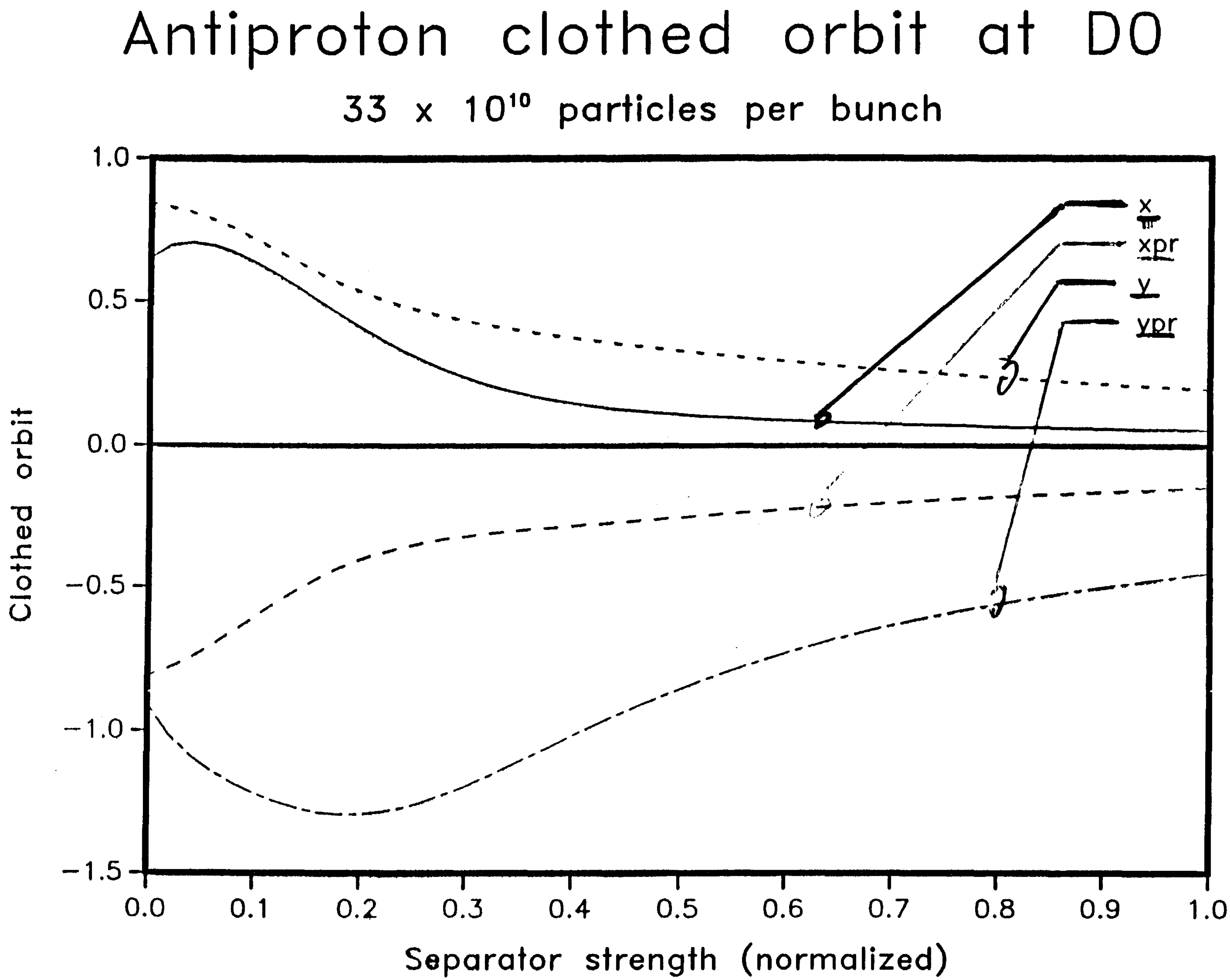
There is one more, somewhat perverse "point of interest" with regard to Montague's expression for the beam-beam kick. The beam-beam force is obtained by Lorentz transforming an electrostatic field computed in the rest frame of the source bunch. The computation is done assuming that in this frame the source looks like a very long cylinder; that is, the bunch length is increased by about 1000 via Lorentz transformation to its rest frame. However, consider viewing a beam-beam collision in the rest frame of a proton as it passes B0. What actually happens is a real, physical example of the classic "train in the tunnel" problem we all solved as undergraduates. We shall take the bunch length to be ≈ 50 cm and relativistic $\gamma \approx 1000$ in the (Fermi)lab frame. What we see is *not* a proton bunch of length 500 m: B0 itself is only ≈ 8 cm long in this frame, the anti-proton bunch is about 0.3 mm long, and the Tevatron itself is an ellipse with a 1 m semi-minor axis. Instead, the 500 m long string of protons at rest which appears in Montague's expression is actually laid out 8 cm at a time, much like a scroll being unrolled on one side and rolled up on the other. Orbits of individual protons look much like distorted cycloids, with an added delay that allows B0 to pass. However, the electromagnetic field from particles in the source bunch are also compressed into a transverse angular region of size that decreases inversely with γ. It is this final compression which makes Montague's expression still valid.


Antiproton clothed orbit at B0
33 x 10¹⁰ particles per bunch
x
xpr
y
ypr
Clothed orbit
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
−1.2
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Separator strength (normalized)

Notice that the clothed orbit of the antiprotons does not approach the central stable orbit as the separator voltage drops to zero. This observation is the key to understanding the "discrepancy" alluded to earlier between the observed and expected tune shift at zero separator strength. Here's what happens: Imagine increasing the beam-beam tune shift up from zero slowly while keeping separators turned off. As the zero-amplitude tune passes 21.0, a Hopf bifurcation occurs and two families of infinitely many closed orbits are spawned and begin to move outwards in phase space. These are, in fact, the stable resonant orbits in the islands of a $2\nu_x - 2\nu_y$ separatrix, *whose tunes remained pinned on the integer.* The zero orbit lies within the extremely small central stable region of this separatrix, and its neighboring orbits do indeed have the expected tuneshifts. However, as we continously reduce the normalized separator strength from 1 to 0 in these figures, the clothed orbit approaches one of the resonant orbits, not the central orbit.

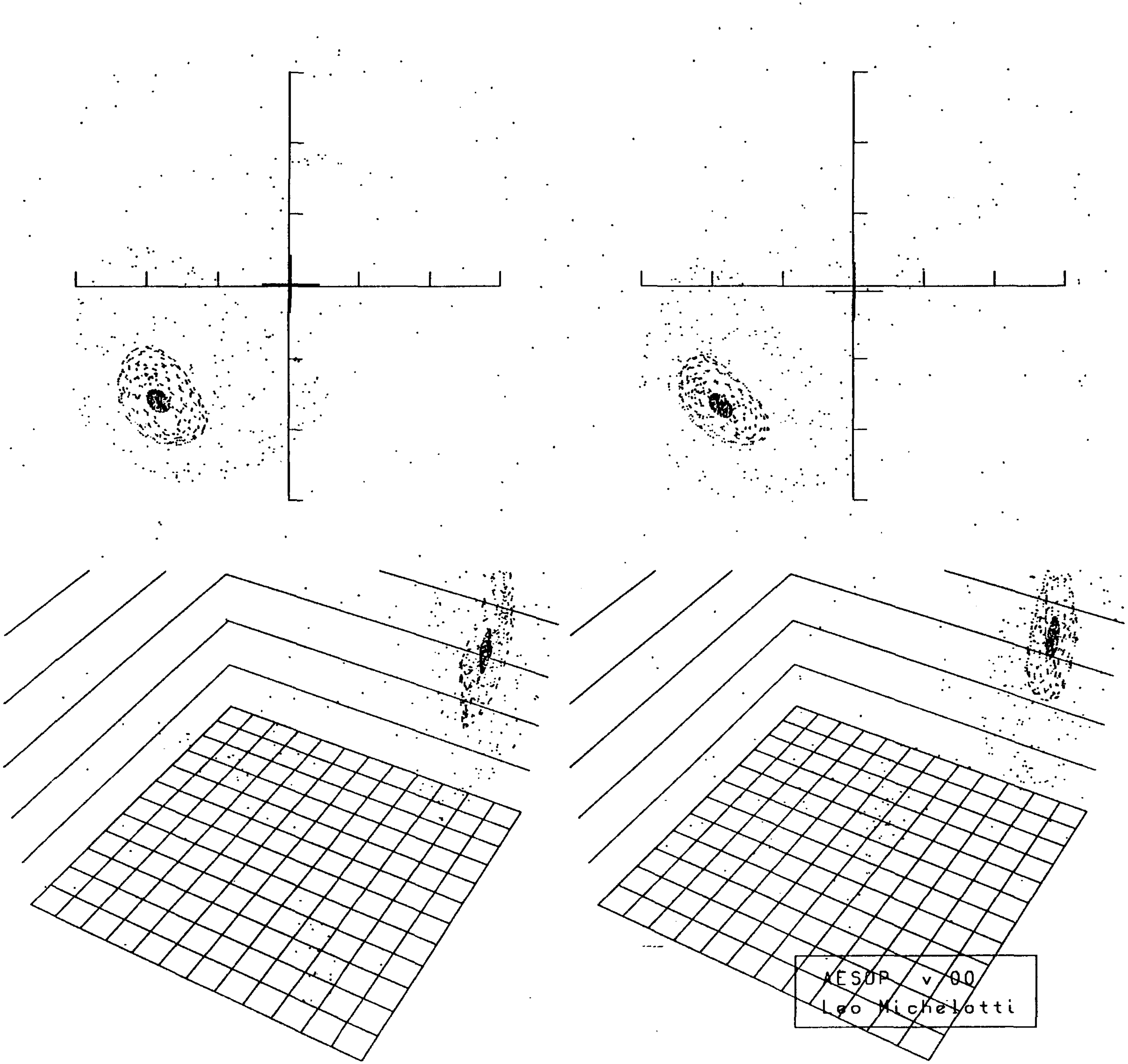
At least, that is what I think is happening: analysis is continuing, and I shall report the complete confirmation of this phenomenon, or if incorrect, the actual mechanism, in a future paper. (I should also mention that this pinning of the resonant orbits on integer tunes occurs in the round beam case; tunes get shifted by elliptic beams, whose orbits generally behave less regularly.)

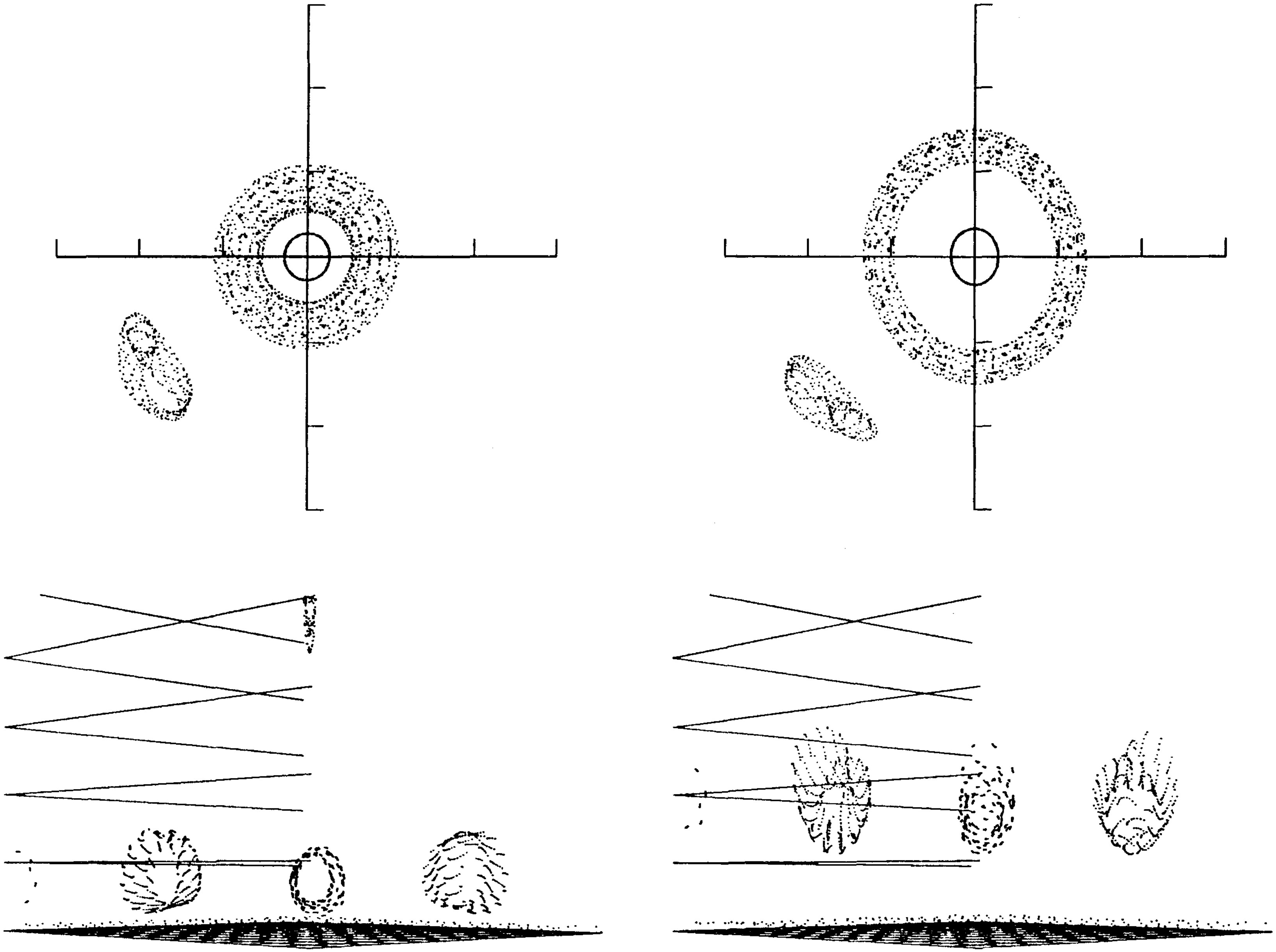
Direct observation of the results of a tracking program support the analytic (and semi-analytic) calculations, as seen ...



Antiproton clothed orbit at D0
33 x 10^10 particles per bunch
Clothed orbit
Separator strength (normalized)
1.0
0.5
0.0
−0.5
−1.0
−1.5
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
x
xpr
y
ypr

. . . here.


AESOP v 00
Leo Michelotti

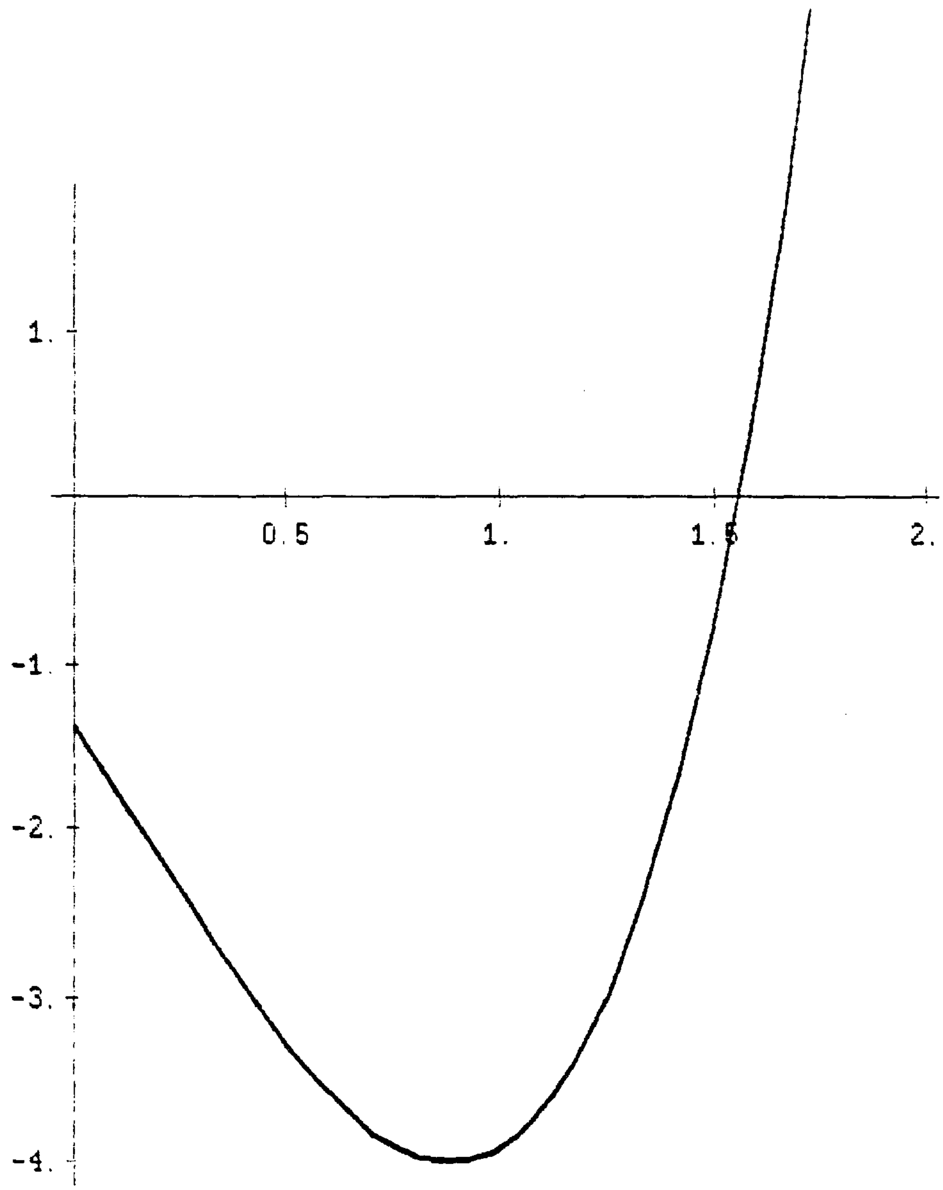


AESOP v.00
Leo Michelotti

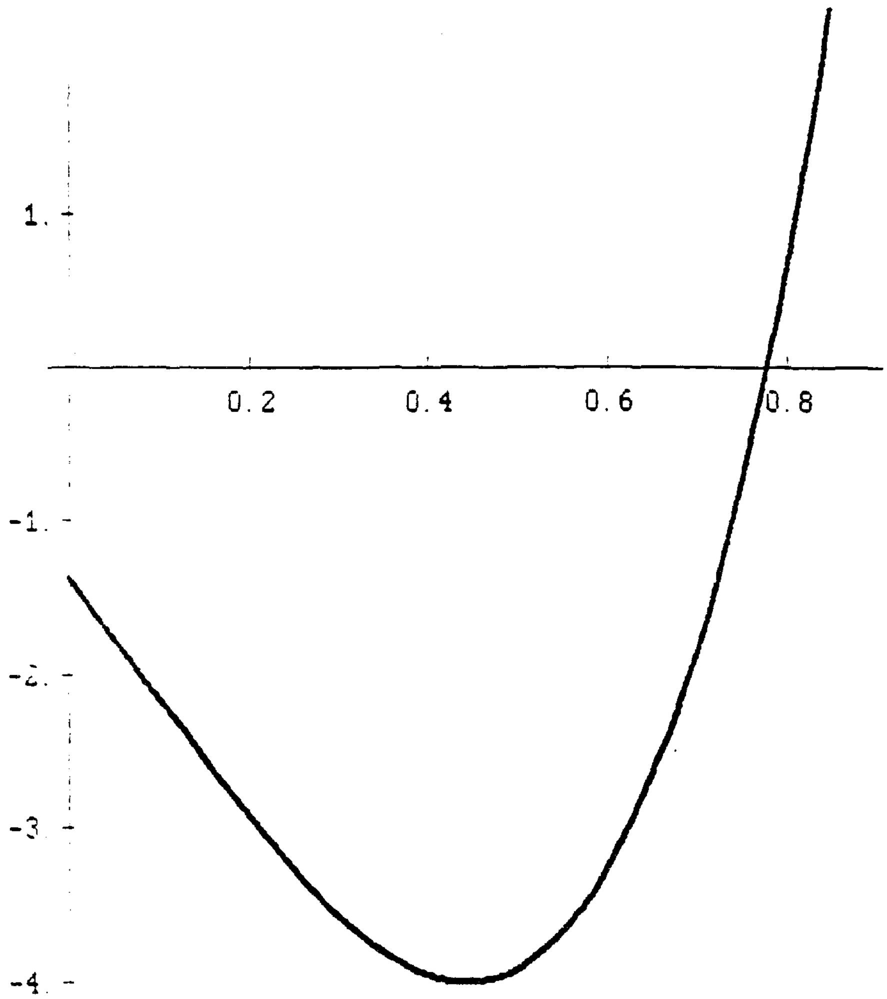
At what beam-beam strength would the closed orbit become unstable in the limit of zero long-range beam-beam interaction? In the weak-strong approximation, the discriminant function is

$$\Delta = [\cos\mu - \kappa\sin\mu + \frac{1}{2}\kappa^2 \sin\frac{\mu}{3}\sin\frac{2\mu}{3}]^2 - 1 \quad ,$$

where $\mu = 2\pi\nu$, and $\kappa = 4\pi\xi/\sqrt{2}$. The stability condition is $\Delta < 0$, and for $\nu \approx 20.6$ this occurs when $\kappa \leq 1.5+$, which makes the critical beam-beam tune shift (per collision) $\xi_{\rm crit} \approx 0.17$. The actual number is ≈ 0.008, so we are safe by about a factor of 20.


1.
0.5
1.
1.5
2.
-1.
-2.
-3.
-4.

Doing the same calculation in the strong-strong case (assuming equally strong p and $\bar{p}$ bunches) results in a discriminant that looks the same as before except that κ is replaced by 2κ. This reduces our safety margin by 2.


1.
0.2
0.4
0.6
0.8
-1.
-2.
-3
-4.

By averaging the Hamiltonian over angles, we project out its "shear" part whose gradients give us the amplitude dependence of the tune *to first order* in the beam-beam interaction. A partial wave analysis is applied to Montague's integral expression to separate the angle dependence: I_n represents a modified Bessel function of the first kind. With a change of variables and a little manipulation ...

Shearing tunes

Start from the transverse Hamiltonian with Montague kicks.

$$
\begin{aligned}
H(\underline{\phi}, \underline{I}; \theta) &= \underline{\nu} \cdot \underline{I} + \sum_{\theta_{bb}} U(\underline{x})\, \delta_{per}(\theta - \theta_{bb}) \\
x_k &= \sqrt{2\beta_k I_k} \sin(\psi_k(\theta) - \nu_k \theta + \phi_k), \qquad k = 1, 2 \\
U(\underline{x}) &= -\frac{1}{2} \frac{N r_p}{\gamma} (1 + 1/\beta^2) \int_0^\infty \frac{dt}{\sqrt{(t+\sigma_1^2)(t+\sigma_2^2)}} \\
&\quad \times \left(1 - \exp\left[-\frac{1}{2}\left(\frac{x_1^2}{t+\sigma_1^2} + \frac{x_2^2}{t+\sigma_2^2}\right)\right]\right)
\end{aligned}
$$

The beam-beam kick itself is

$$\Delta \underline{p} = \Delta \underline{x}' = -\partial U / \partial \underline{x} \ .$$

A partial wave analysis,

$$e^{z \cos \tau} = I_0(z) + 2 \sum_{n=0}^{\infty} I_n(z) \cos(n\tau) \ ,$$

plugged into the *first order* tuneshift relation,

$$\delta \underline{\nu} = \sum_{\theta_{bb}} \left\langle \frac{\partial U}{\partial \underline{I}}\, \delta_{per}(\theta - \theta_{bb}) \right\rangle_{\underline{\phi}, \theta} = \sum_{\theta_{bb}} \frac{1}{2\pi} \left\langle \left. \frac{\partial U}{\partial \underline{I}} \right|_{\theta_{bb}} \right\rangle_{\underline{\phi}} \ ,$$

produces the following expression,

GOTO NEXT TRANSPARENCY

...the tune shifts can be written as an integral which is amenable to numerical integration. Amazingly enough, the small amplitude limit, $\underline{I} \to \underline{0}$, can be worked out explicitly and actually takes on the correct value!

Shearing tunes (cont.)

$$\begin{aligned}
\delta\nu_1 &= \frac{1}{4\pi}\frac{Nr_p}{\gamma}\frac{\beta_1}{\sigma_1\sigma_2}\int_0^1 \frac{dw}{\sqrt{\xi_1^3\xi_2}}[\,Z_0(\zeta_1) - Z_1(\zeta_1)\,]\,Z_0(\zeta_2) \\
\xi_j &= 1 + (\sigma_j/\sigma_{\neg j} - 1)w \\
\zeta_j &= \frac{\beta_j I_j}{2\sigma_1\sigma_2}\frac{w}{\xi_j} \\
Z_n(\zeta) &= e^{-\zeta} I_n(\zeta) \;\; ,
\end{aligned}$$

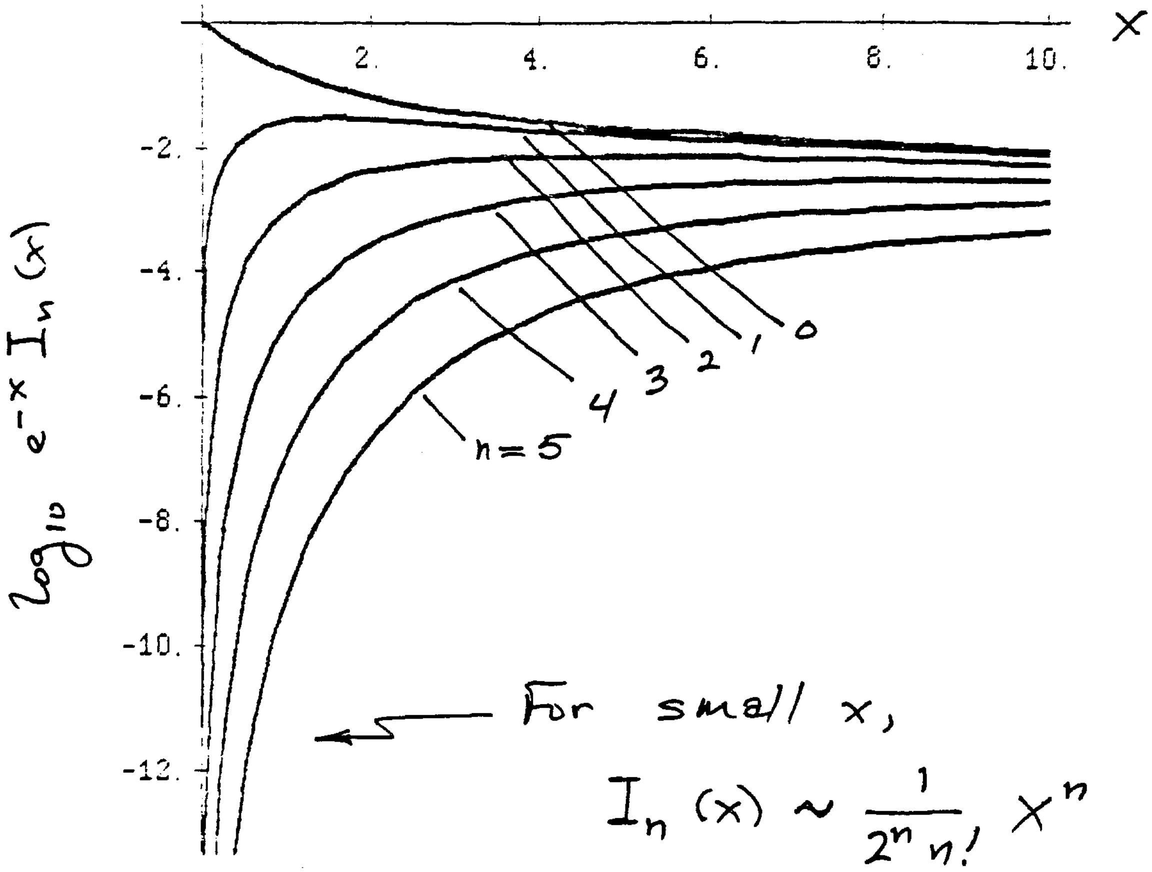
and similarly for $\delta\nu_2$. Interestingly enough, the limits work out correctly.

$$\begin{aligned}
\lim_{\underline{I}\to 0} \delta\nu_1 &= \frac{1}{4\pi}\frac{Nr_p}{\gamma}\frac{\beta_1}{\sigma_1\sigma_2}\int_0^1 \frac{dw}{\sqrt{\xi_1^3\xi_2}} \\
&= \frac{1}{2\pi}\frac{Nr_p}{\gamma}\frac{\beta_1}{\sigma_1(\sigma_1+\sigma_2)}
\end{aligned}$$

<u>For the record</u>:

$$\frac{1}{4\pi}\frac{Nr_p}{\gamma}\frac{\beta_1}{\sigma_1\sigma_2} \approx 0.007 \times \frac{N[\,10^{10}]}{\sqrt{\epsilon_1\epsilon_2}[\,\pi\,\mathrm{mm}-\mathrm{mr}]} \times \sqrt{\beta_1/\beta_2}$$

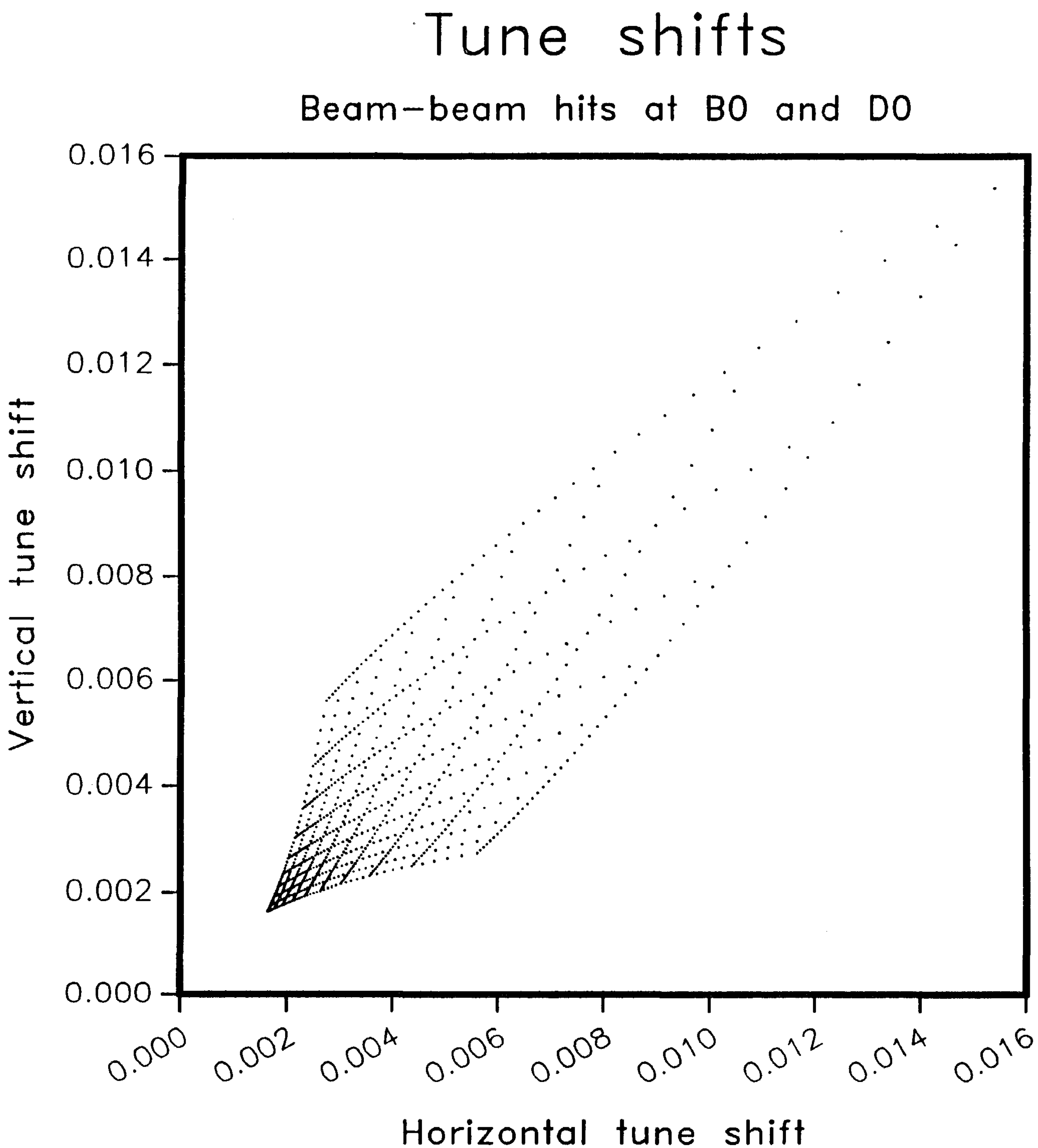
The function $Z_n(x) = e^{-x} I_n(x)$ is unfamiliar, so I show a few representative samples on a semi-log plot. Most of the integrand samples the small x part of this function. The important thing to note is exponential dependence on n. Resonance terms in the Hamiltonian have a form similar to the shear terms, but they involve Z_n's for $n \neq 0$. They therefore tend to decrease exponentially with n.



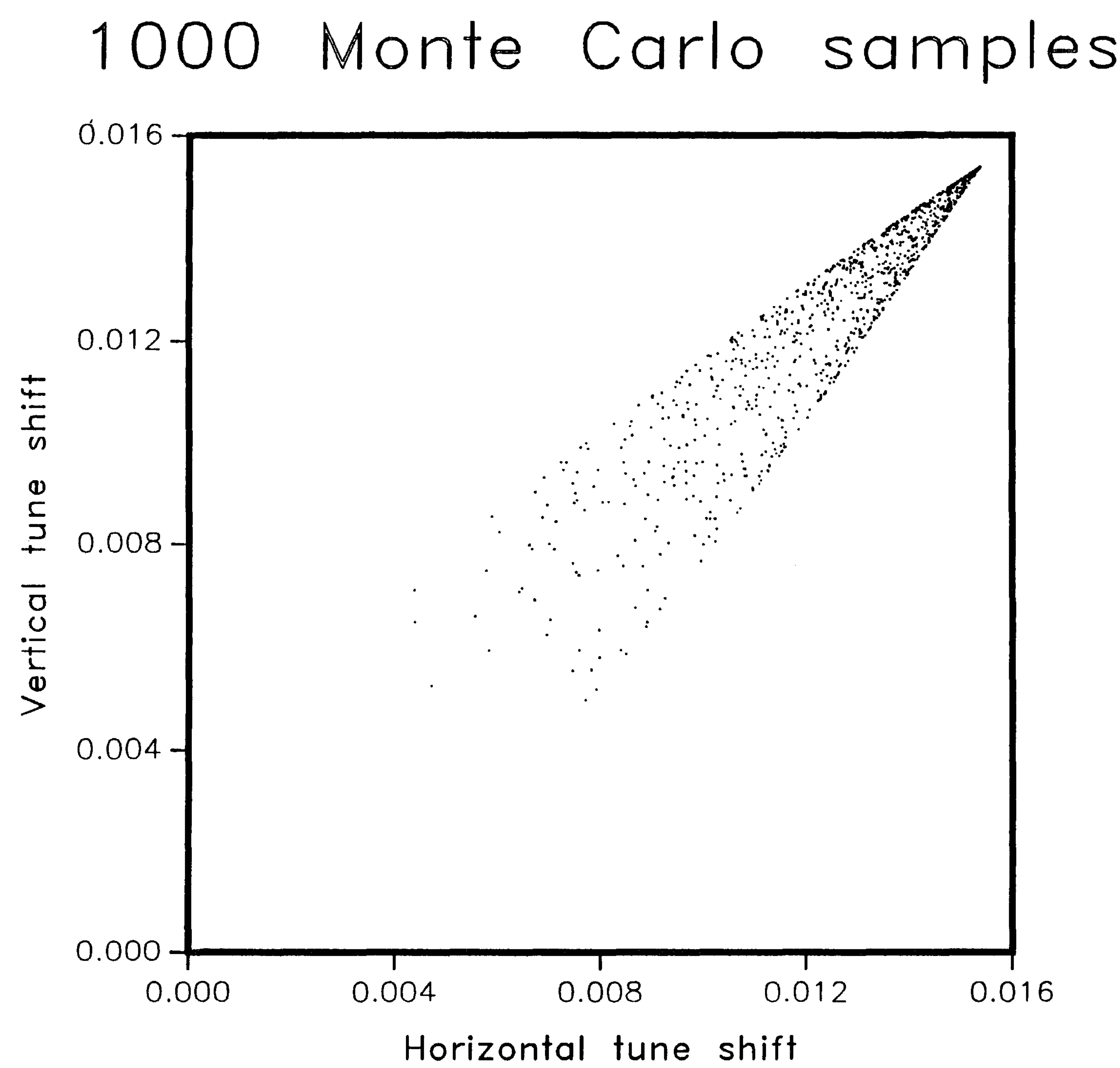
X
2.
4.
6.
8.
10.
-2.
-4.
-6.
-8.
-10.
-12.
$\log_{10} e^{-x} I_n(x)$
0
1
2
3
4
n = 5
For small x,
$I_n(x) \sim \frac{1}{2^n n!} x^n$

7/31/89

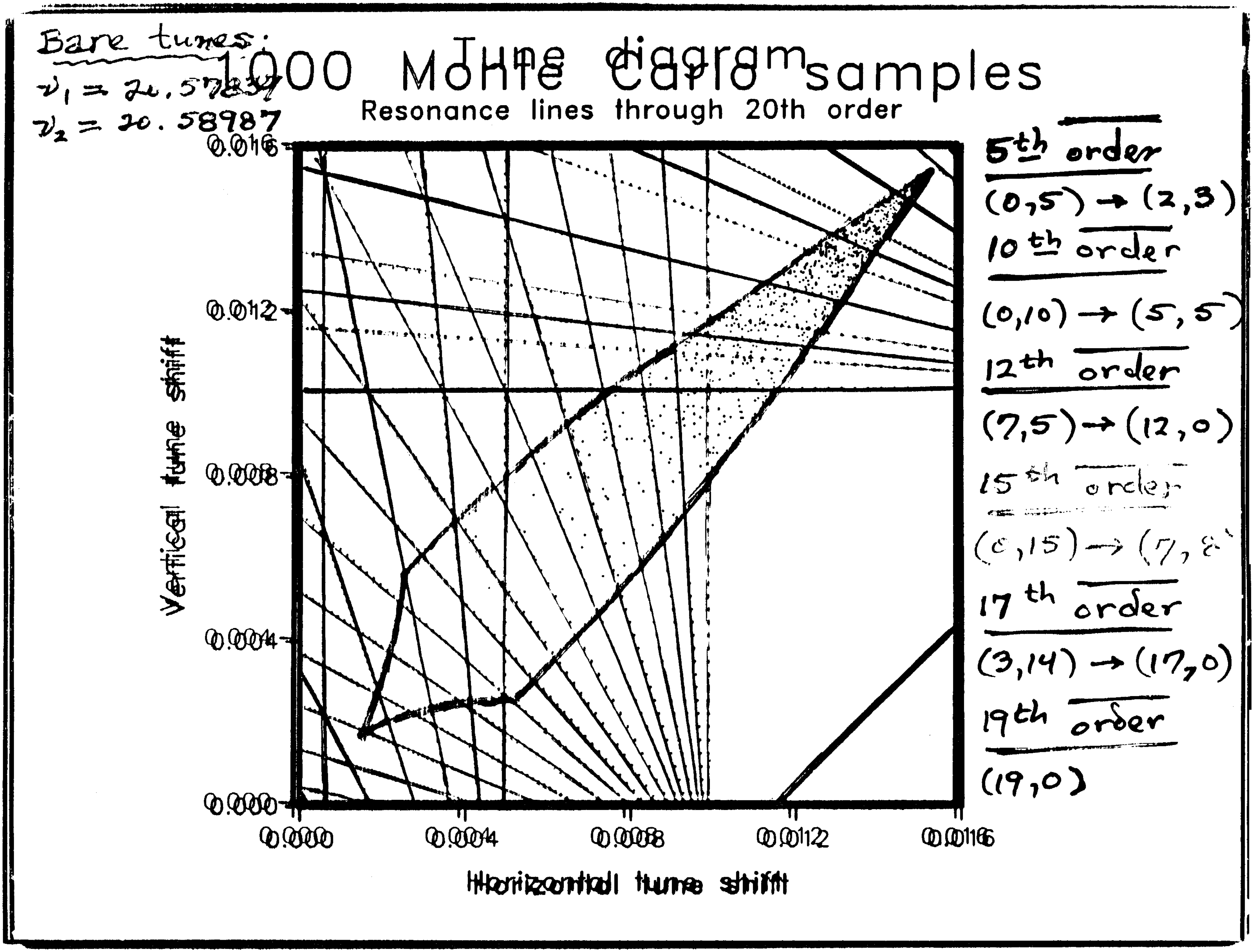
Here is the tune shift footprint of the antiprotons, calculated by numerically integrating the previous expressions. The lines are an (I_1, I_2) grid ranging from 0 to 4 $\sigma_1\sigma_2/\sqrt{\beta_1\beta_2}$, an amplitude range of about 4 standard deviations *of the proton (source) bunch.* To get a better idea of where the antiprotons are actually located ...


Tune shifts
Beam-beam hits at B0 and D0
Vertical tune shift
Horizontal tune shift
0.000
0.002
0.004
0.006
0.008
0.010
0.012
0.014
0.016

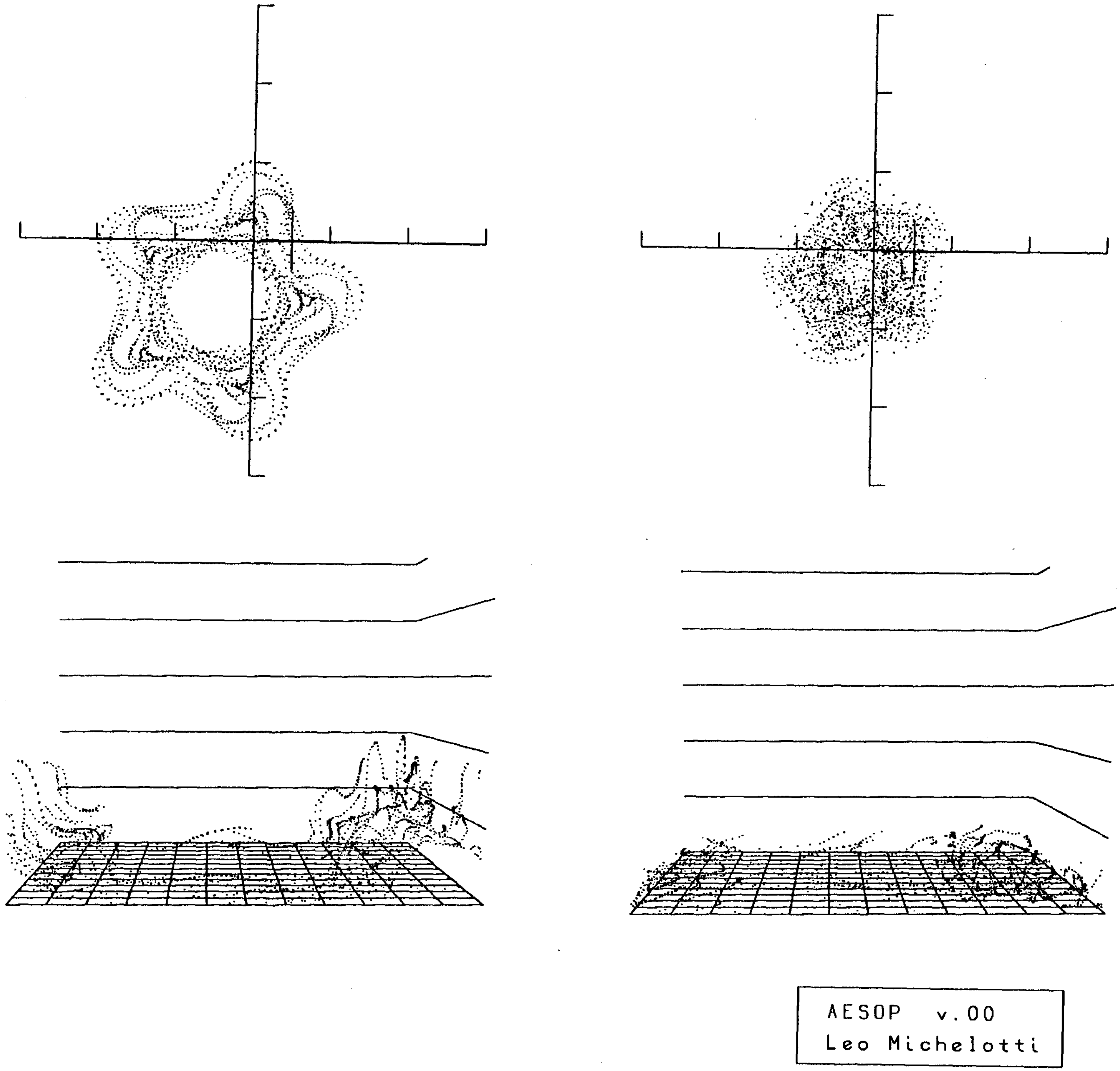
. . . we show a Monte Carlo bunch of 1000 antiprotons. We now overlay this with a tune diagram . . .


1000 Monte Carlo samples
0.016
0.012
0.008
0.004
0.000
Vertical tune shift
0.000
0.004
0.008
0.012
0.016
Horizontal tune shift

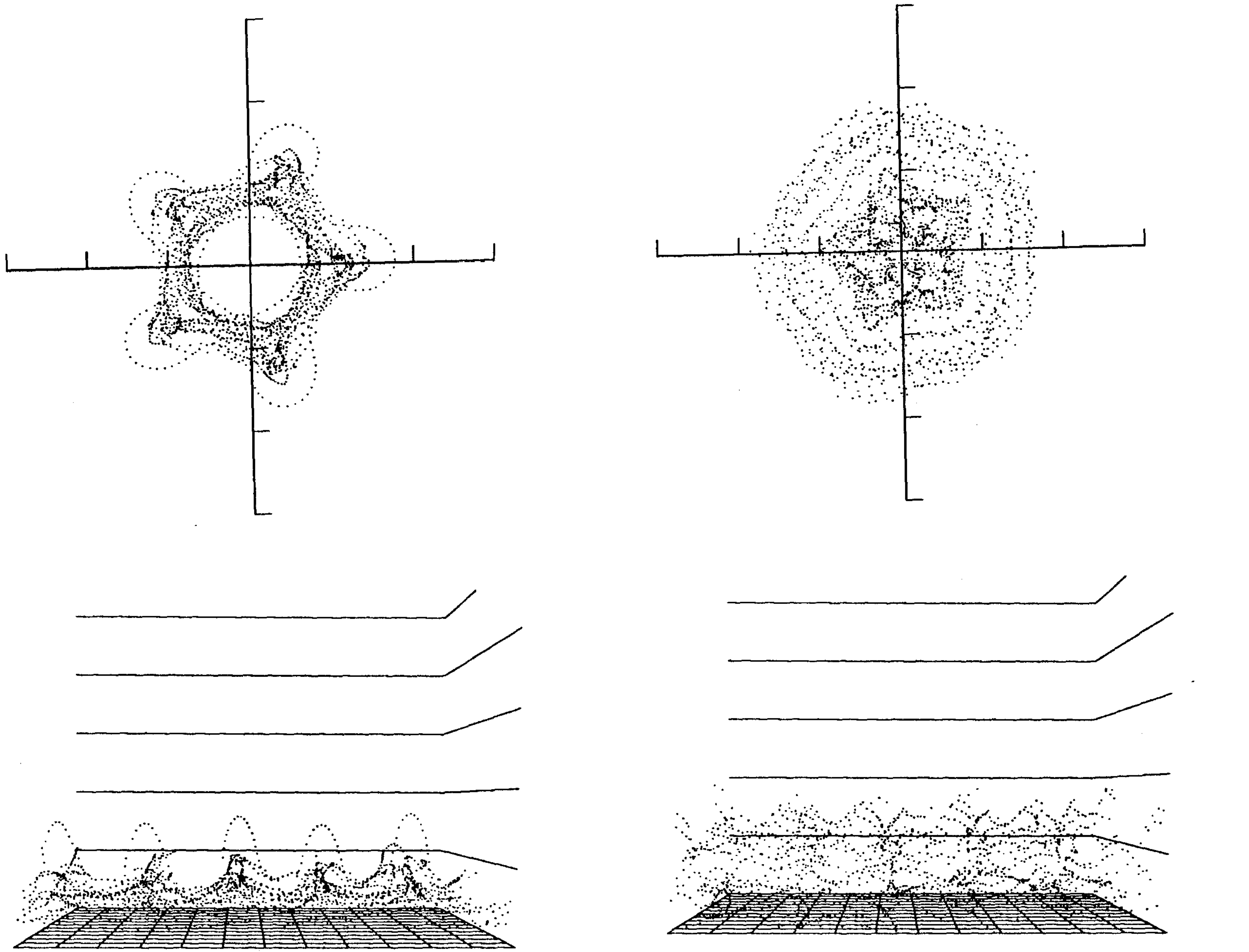
...showing resonance lines through 20^{th} order, based on a "model" low beta lattice whose bare tunes were given on the first page. The importance of those fifth order resonances is illustrated ...


Bare tunes:
$\nu_1 = 20.57837$
$\nu_2 = 20.58987$
Tune diagram
1000 Monte Carlo samples
Resonance lines through 20th order
Vertical tune shift
Horizontal tune shift
0.016
0.012
0.008
0.004
0.000
0.000
0.004
0.008
0.012
0.016
5th order
(0,5) → (2,3)
10th order
(0,10) → (5,5)
12th order
(7,5) → (12,0)
15th order
(0,15) → (7,8)
17th order
(3,14) → (17,0)
19th order
(19,0)

. . . on these AESOP plots. The top two viewports contain two-dimensional projections of four-dimensional transverse phase space: the upper left (right) shows the orbit projected along normalized horizontal (vertical) Cartesian coordinates; the axes extend over $\pm 1.5\sigma$. The offsets are precisely those predicted by our Newton's method calculation of the clothed orbit. By subtracting off the clothed orbit . . .

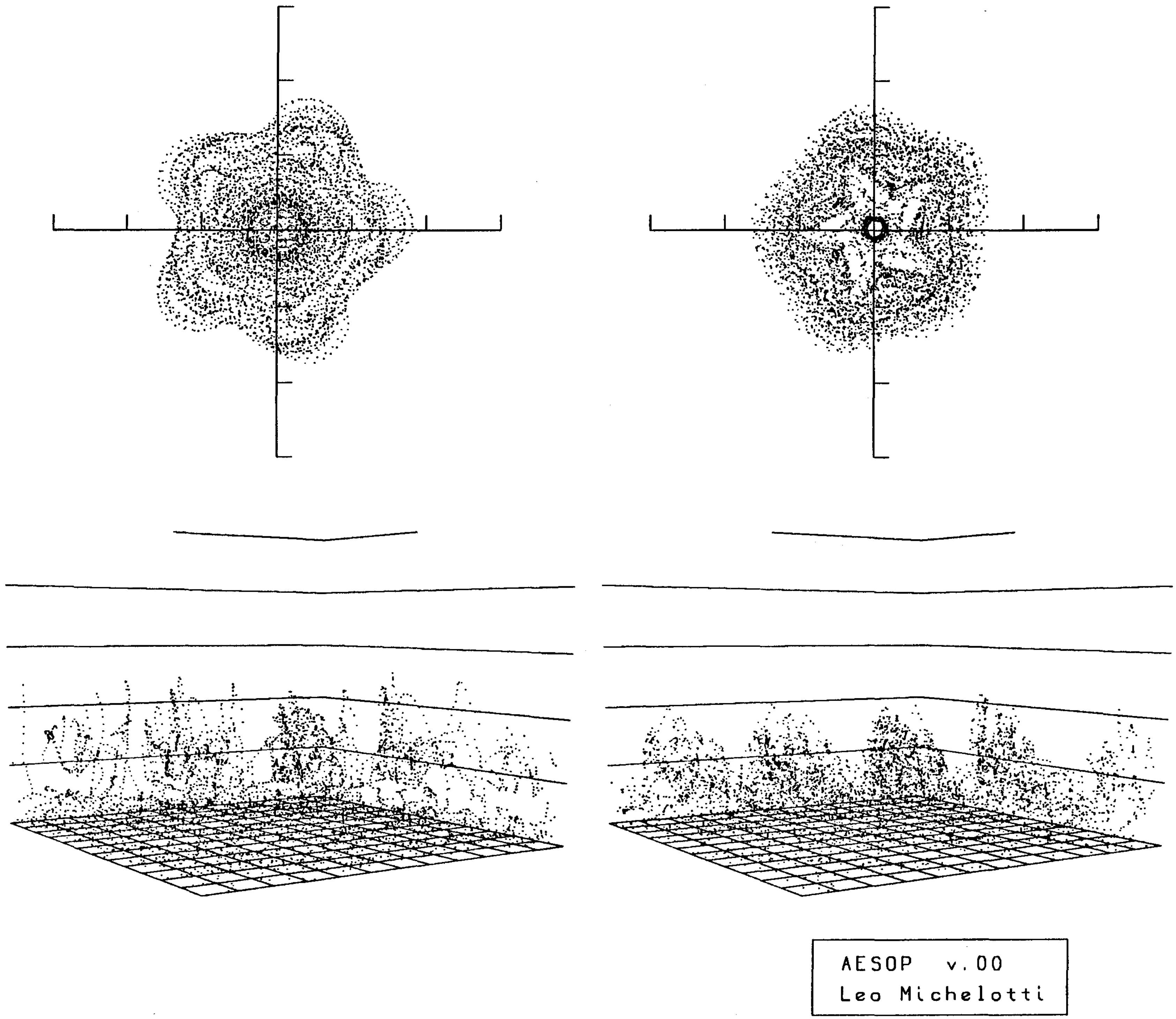

AESOP v.00
Leo Michelotti

. . . we recenter these orbits, a necessary step for interpreting the "angle-angle-action" plots in the lower two viewports. The third axis of the left (right) plot is respectively the horizontal (vertical) action, or amplitude squared. Viewed along the ν_x axis, we see the influence of the $5\nu_x$ resonance, but by rotating the view a little . . .



AESOP v.00
Leo Michelotti

...we can see some structure arising the $3\nu_x + 2\nu_y$ direction as well. By adjusting the bare tunes slightly, we avoid these aesthetic but destabilizing resonances but at the risk of running into ...


AESOP v.00
Leo Michelotti

...the $2\nu_x - 2\nu_y$ resonance along the diagonal. Being a difference resonance, however, this should be more stable.

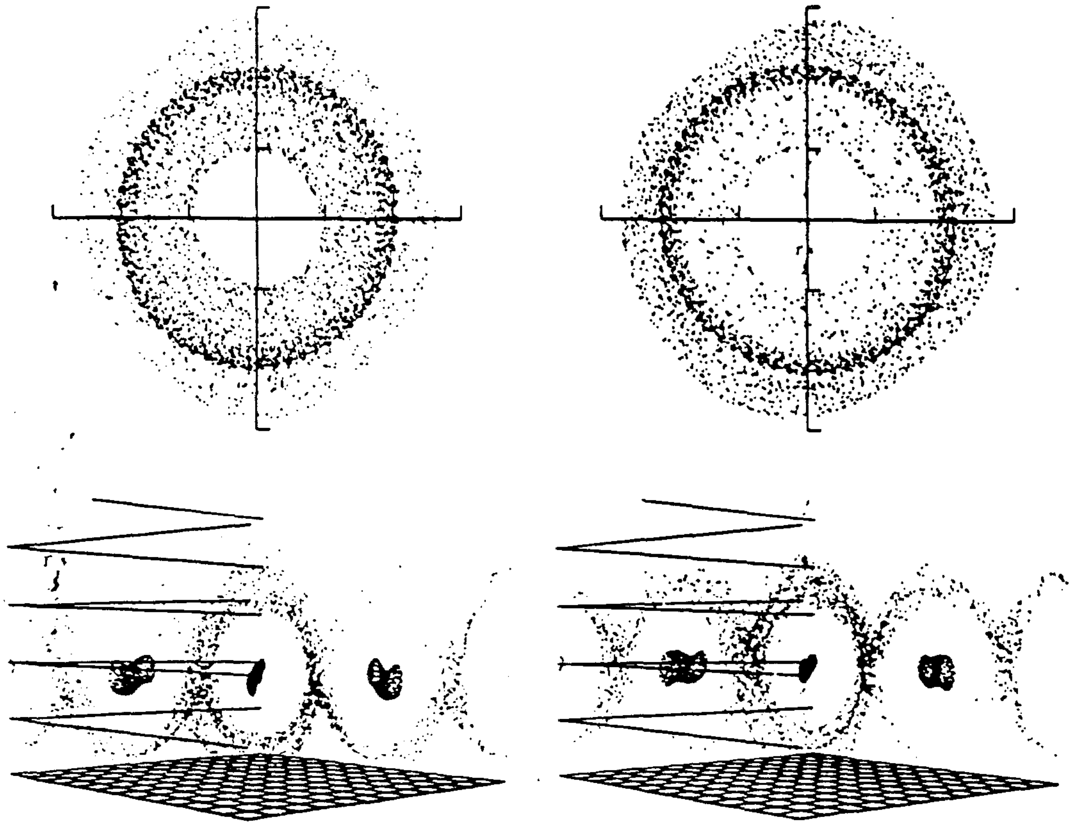
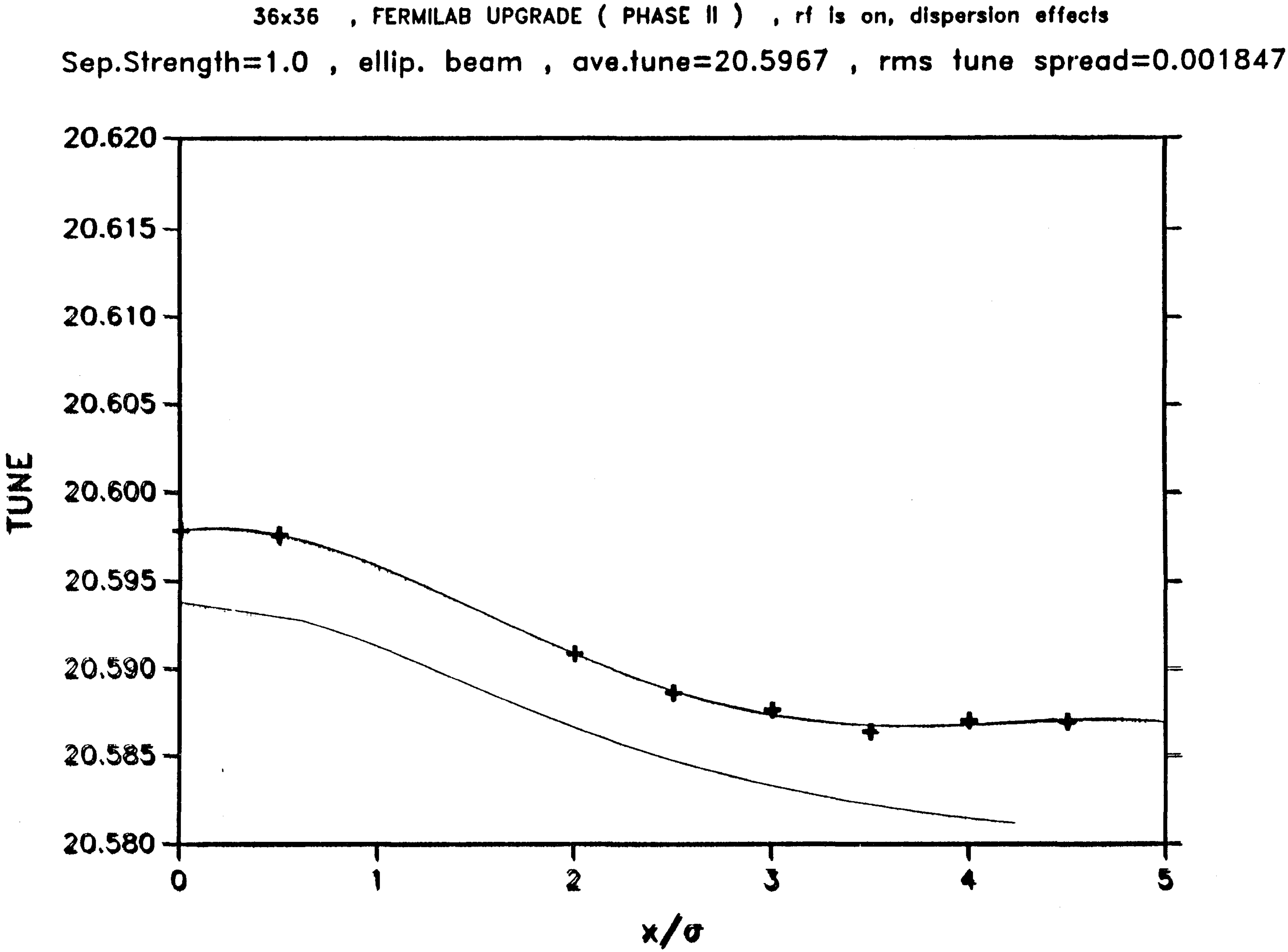


Figure 1: Separatrix of a $2\nu_x - 2\nu_y$ resonance and island orbits.

The tune footprint induced by the two head-on collisions does not yet include the contribution arising from the long-range beam-beam hits. We can estimate what is missing by comparing to tracking. The lower curve was generated by moving on the tune-space footprint along the curve $I_2 = 0$; the upper was obtained by Fourier transforming horizontal position samples obtained by a tracking program with normalized separator strength set to 1 and taking a range of initial conditions along the horizontal direction. (Note: the σ of the abscissa refers to the source bunch, in this case, protons.) There are two observations to be made: (1) the tune spread is about the same in both cases, while (2) tunes obtained by tracking are systematically larger by about 0.004. This difference is explainable as the contribution coming from the long range part of the beam-beam interaction, as we shall see next.


36x36 , FERMILAB UPGRADE (PHASE II) , rf is on, dispersion effects
Sep.Strength=1.0 , ellip. beam , ave.tune=20.5967 , rms tune spread=0.001847
TUNE
20.620
20.615
20.610
20.605
20.600
20.595
20.590
20.585
20.580
0
1
2
3
4
5
x/σ

The long range part of the beam-beam interaction, obtained by omitting the exponential piece, looks like a magnetic field — and why shouldn't it? — with multipole content given by the very simple formula on this viewgraph. Note well that d in this formula is the complex number $x + iy$, where x and y are the components of the separation from the source bunch. Plugging in the numbers, $N = 33$, $E = 1$, and setting $n = 1$ (quadrupole), we see that the normal quad effective strength is

$$\left.\frac{B'l}{B\rho}\right|_{effective} [\mathrm{m}^{-1}] = -9.6 \times 10^{-4} \frac{x^2 - y^2}{(x^2 + y^2)^2} [\mathrm{mm}^{-2}] \ .$$

We can can do a "back of the envelope" calculation to get a crude estimate of the tune shift: let us take $\beta_x \approx 50\mathrm{m}$, $|d| \approx 5\mathrm{mm}$, and sum over the seventy long range beam-beam hits.

$$|\delta\nu|_{\mathrm{longrange}} = \frac{1}{4\pi}\left|\sum \beta_x \frac{B'l}{B\rho}\right| \leq 70 \times \frac{50}{4\pi} \times \frac{0.00096}{5^2} \approx 0.012$$

In fact, this overestimates the effect by a factor of 3. The following sums were calculated in detail using the closed orbit.

$$\begin{aligned} s_x &= \sum \beta_x \frac{x^2 - y^2}{(x^2 + y^2)^2} &= -58[\mathrm{m/mm}^2] \\ s_y &= \sum \beta_y \frac{x^2 - y^2}{(x^2 + y^2)^2} &= +18[\mathrm{m/mm}^2] \end{aligned}$$

These yield the tuneshifts

$$\begin{aligned} \delta\nu_x &= -7.6 \times 10^{-5}\, s_x[\mathrm{m/mm}^2] &= +0.0044 \\ \delta\nu_y &= -7.6 \times 10^{-5}\, s_y[\mathrm{m/mm}^2] &= -0.0014 \end{aligned}$$

The 0.004 shift that is observed between the analytic tunes arising from the two head-on collisions and those obtained from a tracking program is therefore just what one would expect from the long-range contributions.

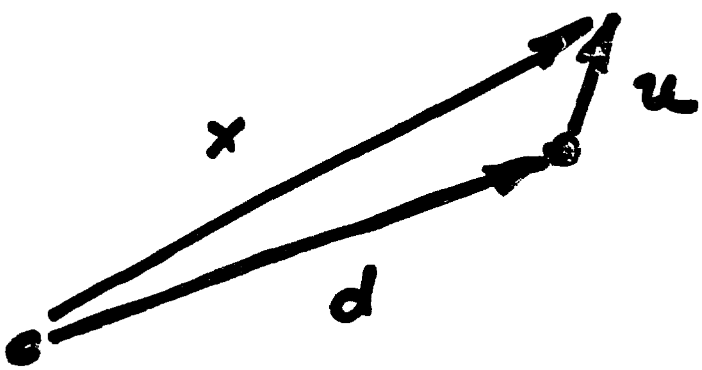
$\boxed{\text{Beam-beam kick}}$

A particle passing a round Gaussian beam experiences a transverse kick.

$$\begin{aligned}
\Delta \underline{x}' &= -Nr_p \frac{1}{\gamma}(1+\frac{1}{\beta^2})\,(1-\exp[-r^2/2\sigma^2]) \times \underline{x}/|\underline{x}|^2 \\
&\approx -Nr_p \frac{1}{\gamma}(1+\frac{1}{\beta^2}) \times \underline{x}/|\underline{x}|^2 \\
\Delta \underline{x}'[10^{-8}] &= -2.9\,\left(N[10^{10}]/E[\mathrm{TeV}]\right)\,\left(\underline{x}/|\underline{x}|^2\right)[mm^{-1}]
\end{aligned}$$

The long range beam-beam kick effectively looks like a magnetic field and can be expanded in multipoles. Expand $x = d + u$, where d is the new central closed orbit, and u is small.

$$\frac{B_o R}{B\rho}\int d\theta\,(b_n + i a_n) = 2.9 \times 10^{-11}\mathrm{m}\,\frac{N[10^{10}]}{E[\mathrm{TeV}]}\,(-1)^n\,(1/d)^{n+1}$$



$d = d_1 + i d_2$, ... etc.

The fact that the tune spread is not appreciably influenced by the long-range collisions is not surprising, as their contribution should be down by two orders of magnitude in σ/d compared to the tune shift.

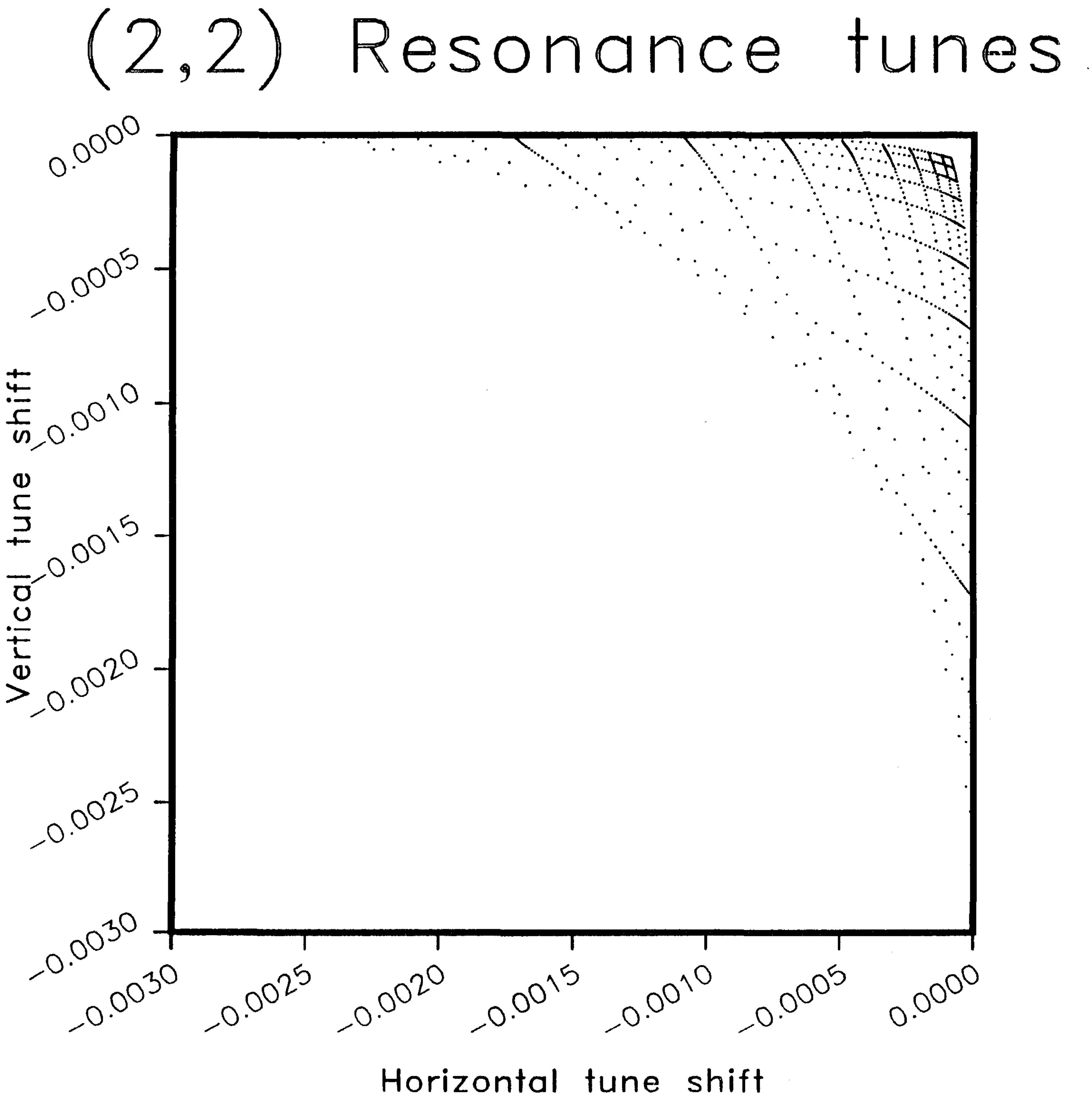
Head-on beam-beam collisions give rise to even order resonances of the form $2\underline{m} \cdot \underline{\nu} + 2n = 0$. Projecting the resonant orbits onto the I_1, I_2 plane traces out a locus described by

$$\frac{[\,\underline{m} \cdot (\underline{\nu}_{\rm bare} + \delta\underline{\nu}_0) + n\,] + \underline{m} \cdot \underline{\nu}_{\rm shear}(\underline{I})}{\underline{m} \cdot \underline{\nu}_{\rm res}(\underline{I})} = \pm\frac{1}{2} \quad ,$$

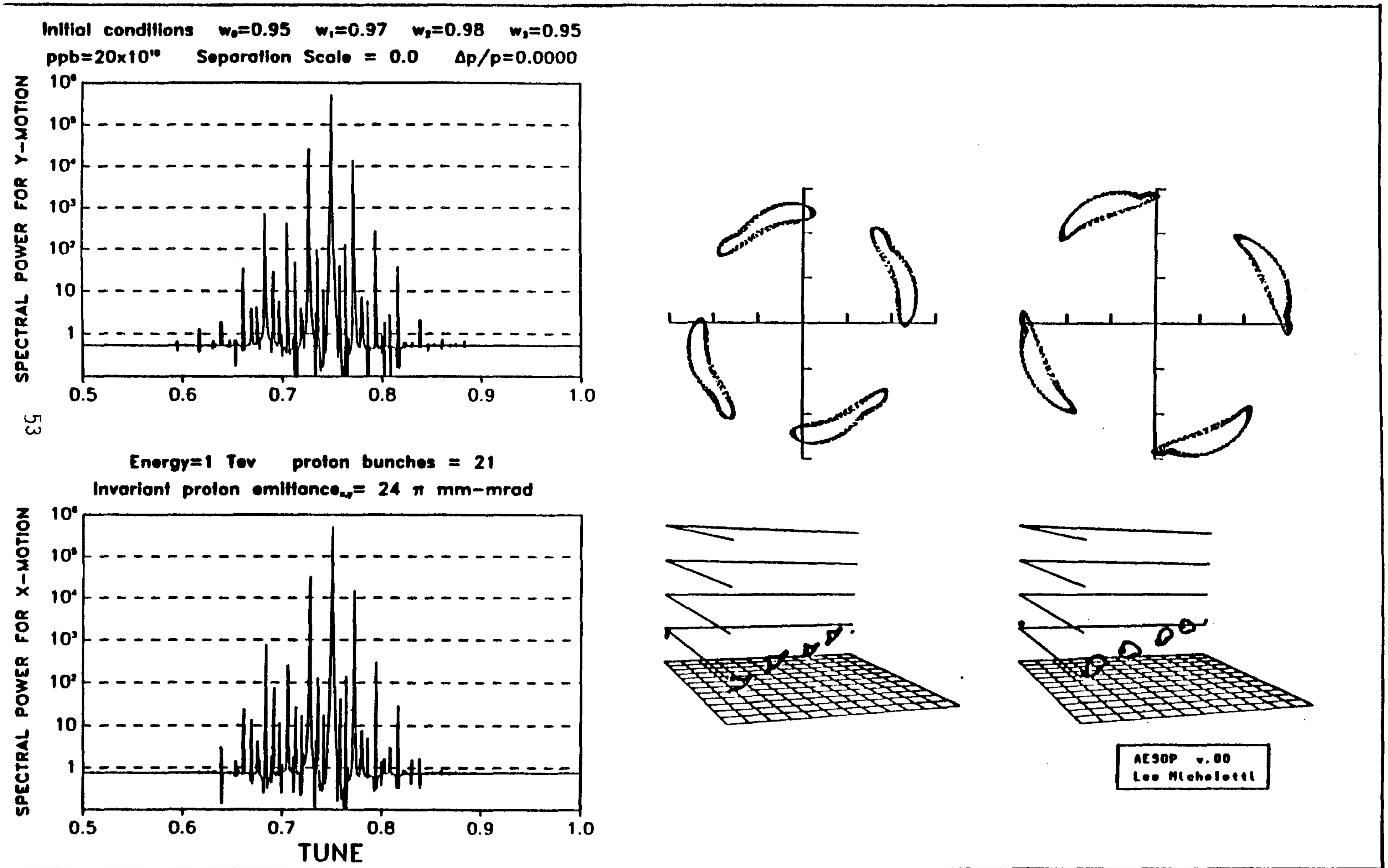
where $\underline{\nu}_{\rm shear/res}(\underline{I}) = \partial H_{\rm shear/res} \partial \underline{I}$. The integral expression for the resonating tunes is similar to that for the shearing tunes. First define $\alpha \equiv 1/(m_1 + m_2)$. (I'm assuming sum resonances, otherwise substitute $|m_1|$, etc.) Then the horizontal resonating tune is

$$\begin{aligned} \nu_{\rm res,\ 1} &= -\frac{1}{4\pi}\frac{Nr_p}{\gamma}\frac{\beta_1}{\sigma_1\sigma_2} \times (2|\cos 2\pi n/3|) \\ &\times \alpha \int_0^1 \frac{dw}{w^{1-\alpha}\sqrt{\xi_1^3 \xi_2}} [\, Z_{m_1-1}(\zeta_1) - 2Z_{m_1}(\zeta_1) + Z_{m_1+1}(\zeta_1)\,]\, Z_{m_2}(\zeta_2) \quad , \end{aligned}$$

with a similar expression for the vertical component. The quantity in parentheses arises from coherently summing the contributions from B0 and D0. The appearance of higher order Z functions makes these tunes exponentially decreasingly small relative to the shear tunes.


(2,2) Resonance tunes
Vertical tune shift
0.0000
-0.0005
-0.0010
-0.0015
-0.0020
-0.0025
-0.0030
-0.0030
-0.0025
-0.0020
-0.0015
-0.0010
-0.0005
0.0000
Horizontal tune shift

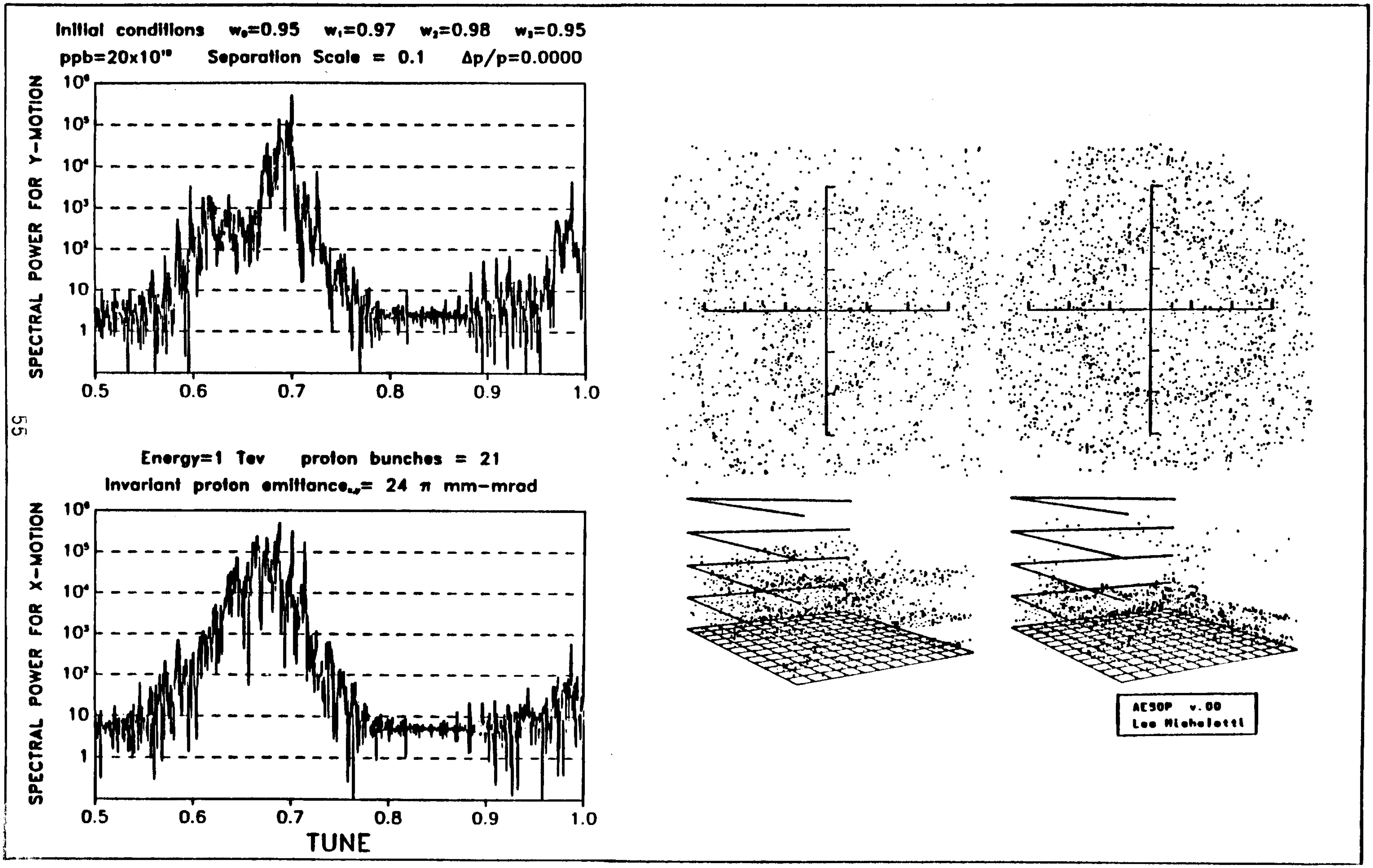
This is not the full story, of course. It is hard to summarize the sort of anecdotal information obtained by staring at tracking "data" from a variety of lattices and separators. Much of what we saw was expected: provided one avoided overlapping, low order resonances, orbits of "moderate" amplitude would start out on KAM tori ...



Initial conditions w_0=0.95 w_1=0.97 w_2=0.98 w_3=0.95
ppb=20x10^{10} Separation Scale = 0.0 Δp/p=0.0000
SPECTRAL POWER FOR Y-MOTION
10^6
10^5
10^4
10^3
10^2
10
1
0.5
0.6
0.7
0.8
0.9
1.0
Energy=1 Tev proton bunches = 21
Invariant proton emittance$_{x,y}$= 24 π mm-mrad
SPECTRAL POWER FOR X-MOTION
TUNE
AESOP v.00
Lee Michelotti

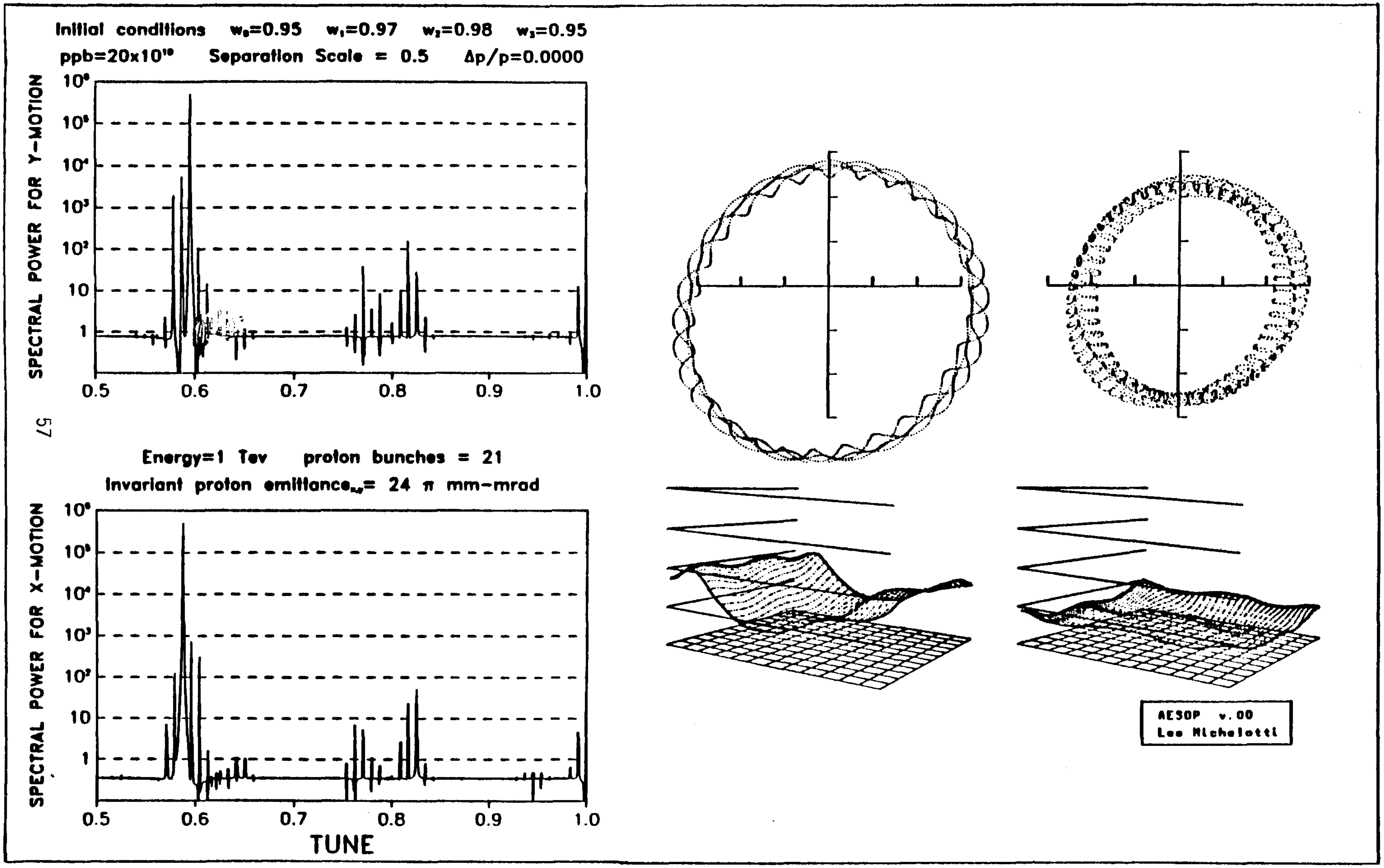
Fig. 1

...become chaotic when the separation reached some critical value, and



Initial conditions w_0=0.95 w_1=0.97 w_2=0.98 w_3=0.95
ppb=20×10^{10} Separation Scale = 0.1 $\Delta p/p$=0.0000
SPECTRAL POWER FOR Y-MOTION
10^6 10^5 10^4 10^3 10^2 10 1
0.5 0.6 0.7 0.8 0.9 1.0
Energy=1 Tev proton bunches = 21
Invariant proton emittance$_{x,y}$= 24 π mm-mrad
SPECTRAL POWER FOR X-MOTION
10^6 10^5 10^4 10^3 10^2 10 1
0.5 0.6 0.7 0.8 0.9 1.0
TUNE
AESOP v.00
Leo Michelotti

...quiet down again after the separation became large enough. At larger amplitudes, we ran into such things as ...


Initial conditions w0=0.95 w1=0.97 w2=0.98 w3=0.95
ppb=20x10^10 Separation Scale = 0.5 Δp/p=0.0000
SPECTRAL POWER FOR Y-MOTION
10^6
10^5
10^4
10^3
10^2
10
1
0.5 0.6 0.7 0.8 0.9 1.0
Energy=1 Tev proton bunches = 21
Invariant proton emittance x,y= 24 π mm-mrad
SPECTRAL POWER FOR X-MOTION
10^6
10^5
10^4
10^3
10^2
10
1
0.5 0.6 0.7 0.8 0.9 1.0
TUNE
AESOP v.00
Leo Michelotti

...phase locking of orbits influenced by the $2\nu_x - 2\nu_y$ resonance (which is understandable in terms of perturbation theory), or ...

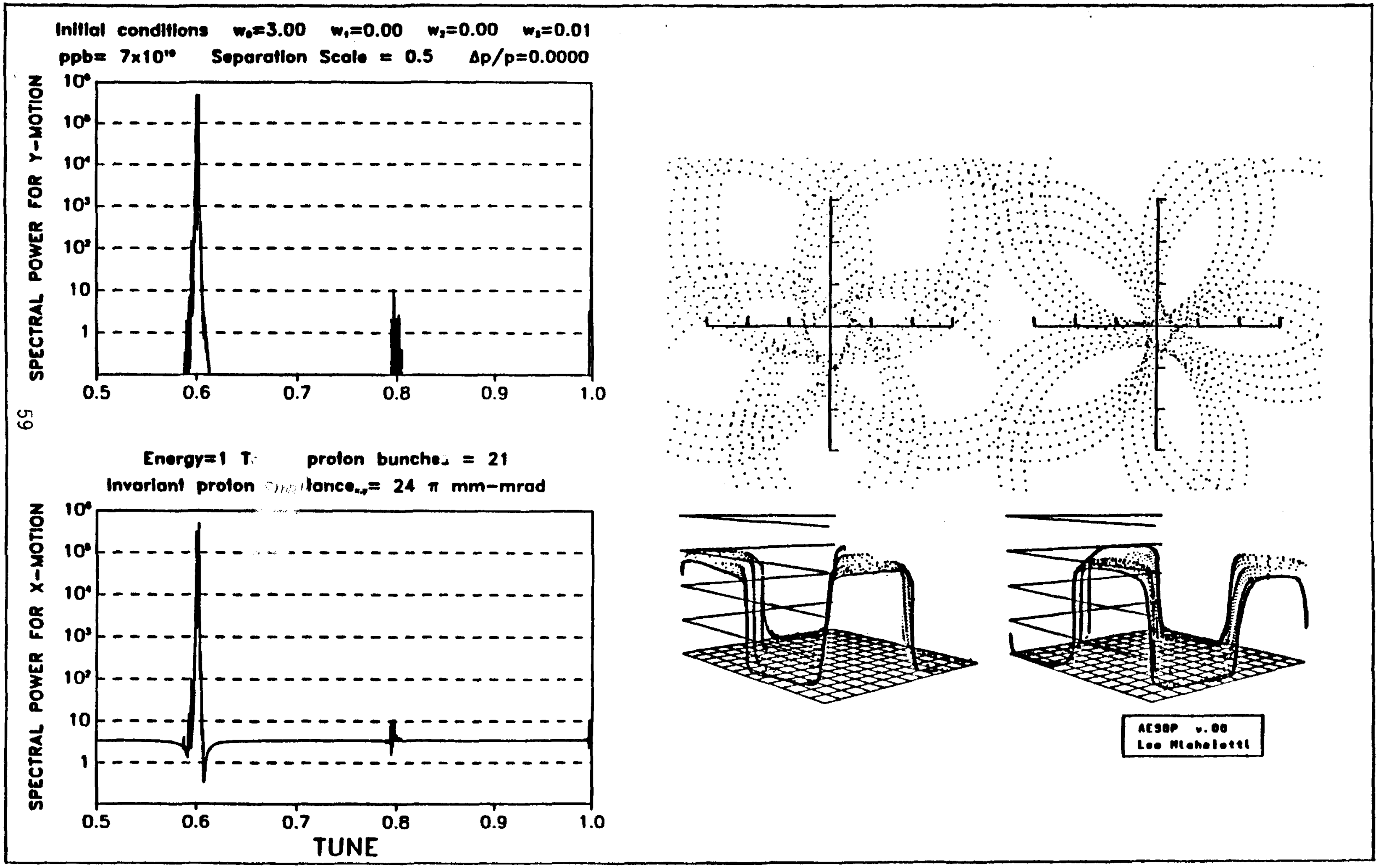
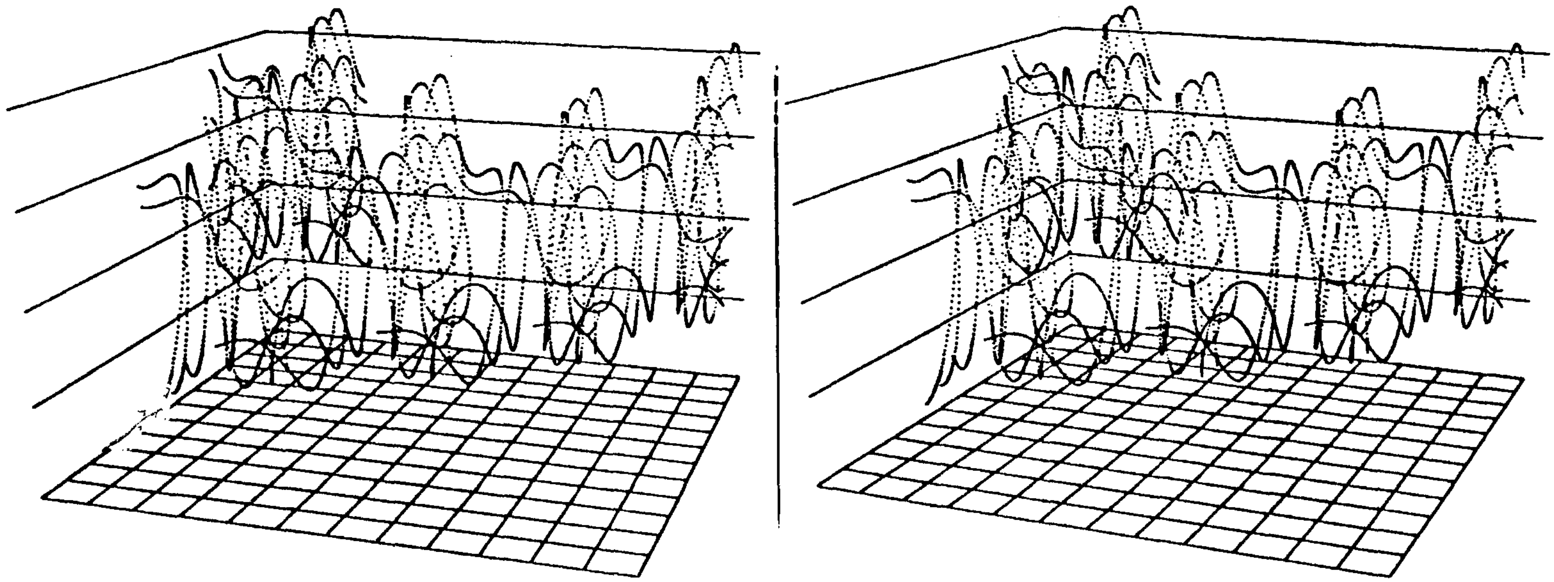

Initial conditions w_0=3.00 w_1=0.00 w_2=0.00 w_3=0.01
ppb= 7×10^{10} Separation Scale = 0.5 $\Delta p/p$=0.0000
SPECTRAL POWER FOR Y-MOTION
10^6
10^5
10^4
10^3
10^2
10
1
0.5
0.6
0.7
0.8
0.9
1.0
Energy=1 T proton bunches = 21
Invariant proton emittance = 24 π mm-mrad
SPECTRAL POWER FOR X-MOTION
TUNE
AESOP v.00
Leo Michelotti

Fig. 5

... "tangled" orbits. These were orbits which seemed to exist in an intermediate region between regular and chaotic. Most likely, they represent particles which will occasionally receive a strong kick from a long-range bunch, inducing transitions between tori. This could, in principle, lead to phase space diffusion, but this has not yet been observed; these orbits twist in unpredictable ways, but generally remain bounded; the particular one shown here was followed for over 100,000 iterations without its exhibiting appreciable increase in mean amplitude. Even if we could prove it would eventually diverge, there remains the statistical problem of determining how much of phase space is filled with these "anomolies."

We have not yet proved that helical separation can not be made to work, but we continue to hunt for problem areas. For example, we recently have been looking at dispersion effects, which makes bunches elliptical and oscillates the mean crossing point. Elliptical beams do produce dirtier orbits than round beams, but disastrous behavior seems to be associated with failure to avoid low order resonances, especially the 5^{th}s. More needs to be done to make sure that tune modulation does not produce phase space diffusion, but we are encouraged by the first of our ...



Stereo view; tangled orbit 11/29/88 LH

Fig 16

Conclusions (or strong indications):

(1) The effect of the long-range beam-beam encounters on particles in the (transverse) core of a bunch will be to shift the tune by about 1/2 the amount experienced in one head-on collision. Their contribution to the tune spread will be small (about $0.2^2 = 4\%$, or less, of their contribution to the tune shift for 5 σ minimum separation.)

(2) Particles in the tails of the distribution exhibit fascinating phenomena which indicate that they still feel the effect of those long-range encounters, but whose influence on beam lifetimes is not (yet) understood.

(3) Most of the physics will thus come from the two head-on collisions. It *may* be necessary to correct the clothed orbit at B0 and D0 to keep the beams centered, avoid instability, and keep the luminosity high.

(4) In any case, we are probably being conservative and overestimating effects by using Montague's form for the beam-beam kick. At low beta, beta is comparable to the bunch length, which means that individual antiprotons traverse the proton bunches at non-negligible angles. This means they will sample a beam-beam kick "averaged" over some transverse distance, which will tend to make the beam-beam force profile flatter, reducing both the linear tune shifts and the strength of the nonlinearities.

(5) Nonetheless, there remains an order of magnitude or more safety margin in the criterion for linear stability, while an uncompensated $\Delta\beta/\beta$ could be on the order of 10-15%.