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A Longitudinal Emittance Measurement Program for the Fermilab Booster

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Abstract

We will describe a method of a longitudinal emittance measurement using TEK-DSA 602 scope and ACNET console program. The scope is used to measure Booster bunch length and a total peak to peak RF voltage on the gaps of the cavities. The signal for bunch length comes from 2 GHz bandwidth Resistive Wall Current Monitor. An extension of this method to measure greater than 100 values of the longitudinal emittance in one accelerator cycle is proposed. In addition this device can act as digital Mountain Range.

1 Introduction

The variables we will use for description of longitudinal motion[1] are

$$q = \phi - \phi_s \quad \text{and} \quad y = \frac{E - E_s}{\omega_{rf}}, \tag{1}$$

where q is dimensionless and y has the units of eV-sec. The ϕ is the phase and E the energy of a particle that passes the accelerating gap. We use the convention that $\phi = 0$ when the RF voltage is zero and rising. The ϕ_s and E_s

are the phase and energy of the synchronous particle and ω_{rf} is the angular radio frequency.

In terms of these variables, the longitudinal motion of the particles in circular accelerators with a time varying electric field at the accelerating gap is described by the Hamiltonian,

$$H(q,t) \equiv -\frac{1}{2}Ay^2 + B(q\Gamma + \cos(q+\phi_s)) = C, \tag{2}$$

where;

$$\Gamma = \sin(\phi_s) \quad , \quad \eta = \frac{1}{\gamma^2} - \frac{1}{\gamma_t^2}, \tag{3}$$

$$A = (\frac{hc}{R})^2 \frac{\eta}{E_s} \quad \text{and} \quad B = \frac{eV}{2\pi h}, \tag{4},$$

the radius of the Booster $R = 75.74$ meters, the harmonic number $h = 84$, V_0 is the peak RF voltage per turn, and the transition gamma $\gamma_t = 5.446$.

Each particle in the beam traces out a trajectory in phase space determined by the value of its Hamiltonian, C, and each particle has an emittance corresponding to the area enclosed by its trajectory. We will define the longitudinal beam emittance as the largest emittance of the particles in the beam.

The above discussion has been intended to clarify the terminology and introduce notation to be used here. In the next section we will present the formulae that are used to calculate beam parameters.

2 Calculational Method

The beam emittance ε_l as defined above is given by solving the Hamiltonian for y and integrating y as function of q

$$\varepsilon_l = \sqrt{\frac{2B}{A}} \int_{q_1}^{q_2} \sqrt{\cos(\phi_s + q) + \Gamma q + C dy}, \tag{5}$$

where,

$$C = \cos(\phi_s + q_1) + \Gamma q_1 \tag{6}$$

and $q_1 - q_2$ is the bunch length.

Although it is possible to find q_1 and q_2 from of the bunch length and synchronous phase in the existing program we follow the way the longitudinal emittance is calculated traditionally at Fermilab[2]:

$$\varepsilon_l = (Moving\_Bucket\_Area) * \left(\frac{Bunch\_Length}{Bucket\_Length}\right)^2 * C_A. \tag{7}$$

The C_A is the function displayed[3] in Figure 1. In the program the three curves are fitted to fifth order polynomials and for values of $\Gamma = \sin(\phi_s)$ in the indicated intervals linear interpolation is used[4]. The bucket length is defined as;

$$Bucket\_Length = \phi_L - \phi_R, \tag{8}$$

where $\phi_R = \pi - \phi_s$ and ϕ_L is the second point where the separatrix crosses the $y = 0$ line and can be found as a solution of the following equation

$$\Gamma\phi_L + \cos(\phi_L) = \Gamma\phi_R + \cos(\phi_R). \tag{9}$$

The ϕ_s can be calculated using the equation,

$$\phi_s = \frac{360}{2\pi} \arcsin(\frac{2\pi R}{V_0}\frac{dp}{dt}) \tag{10}$$

and the fact that we know $p(t)$, the linear momentum of the Booster beam, as a function of time. The V_0 is the total peak RF voltage supplied to the beam by all cavities in one Booster turn and it is a measured quantity. The moving bucket area can be found as an integral of the equation for the separatrix;

$$Moving\_Bucket\_Area = \sqrt{\frac{2B}{A}} \int_{\phi_L}^{\phi_R} \sqrt{\cos(\phi) + \Gamma q + C_1}\, d\phi, \tag{11}$$

where,

$$C_1 = \cos(\phi_s + q_R) + \Gamma q_R \tag{12}$$

The $q_R = \pi - \phi_s$ and q_L is found from;

$$\cos(\phi_s + q_L) + \Gamma q_L = \cos(\phi_s + q_R) + \Gamma q_R \tag{13}$$

After integration

$$Moving\_Bucket\_Area = \alpha(\Gamma)\frac{16R}{h^2 c}\sqrt{\frac{heV_0E_s}{2\pi\eta}}. \tag{14}$$

The program uses lookup table for $\alpha(\Gamma)$ which is indentical to the table in Bouvet at al[5].

3 Measurement of the bunch length and RF Voltage

As we have said above, the program uses the TEK DSA602 scope to measure the RF voltage, V_0, and the *Bunch_Length*. The RF voltage is the sum of the voltages coming from upstream gap monitors located in each RF cavity. These signals are properly phased by adjusting lengths of connecting cables and the summed signal is brought to the scope. The program uses a linear fit to compensate for the changes in the voltage as function of the frequency and at the same time to compansete for the loss due to the lossy cables and reduction factor of the pickup monitors. RF voltage and a bunch length are measured every 3 ms during the Booster cycle starting at 3 ms after injection. The signal for beam analyses comes from 2 GHz bandwidth Resistive Wall Current Monitor. For each point total of $2.56\mu s$ of the beam is analyzed. This portion of the beam is digitized in 5120 points and stored in the scope's memory. The stored waveform is then transferred trough ACNET and numerically analyzed. The bunch parameters are derived based on the sample of 55 consecutive bunches. The baseline of the signal is defined as follows. First we find the voltage distribution, $N(V)$. The digitized values of the voltages are in an interval between -127 and +128 in some arbitrary units. Next we take cut at zero value and for purpose of defining baseline of the signal consider only region of negative voltage. We define the baseline of the signal, V_b as

$$V_b = \overline{V} + \sigma \tag{15}$$

where

$$\overline{V} = \frac{\sum_{V=-127}^{0} V N(V)}{\sum_{V=-127}^{0} N(V)} \tag{16}$$

and

$$\sigma^2 = \frac{\sum_{V=-127}^{0} (V - \overline{V})^2 N(V)}{\sum_{V=-127}^{0} N(V)} \tag{17}$$

Figures 2, 3 and 4 are voltage distributions at 3, 9 and 33 ms.

4 Operational Experience and Future Development

A present the system described above will measure a value of the longitudinal emittance in approximately 2 minutes. An example of a set of measurements is given in Figures 5 and 6 where the longitudinal emittance is measured every three milliseconds in the acceleration cycle. We expect to improve the software such that we can take an arbitary number of emittance values, but the limitations of our present system are obvious, ie. we cannot take multiple values of the emittance in one Booster beam cycle and the system cannot be made to readout any faster that the one point every 2 minutes. These limitations come from the nature of the TEK scope (only 32k samples of memory and 60 Hz maximum trigger rate) and the ACNET system that is used to readout and analyze the data. In a more powerful system we would propose to replace the hardware so that these limitations disappear. A possible system is shown in Figure 7. The heart of such a system would be a 2 Gs/s digitizer, with good analog bandwidth and dynamic range (eg. 1 Ghz and 8 bits) and a modest fast memory depth (8k samples). The digitizers should be able to operate on an external clock and we would like to be able to read out the fast memory into a large, regular, dual-ported memory in one millisecond. The system shown can then be operated in two distinct ways. If the 4 digitizers are run in series then one can get 4 microseconds worth of data at 2 Gs/s into the regular memory every 1/4 millisecond. This information can then be analyzed to get a value of the longitudinal emittance and we will be able to get greater than 100 values on the longitudinal emittance in one Booster acceleration cycle. If the 4 digitizers are run in parallel then one can get an effective sampling rate of 8 Gs/s and by using the external clock feature of the digitizer one can subdivide the 8k samples of each digitizer arbitarily in the cycle. It is easy to see that one can use this method to generate a digital mountain range (8k samples = 128 * 64 samples, 64 samples @ 2 Gs/s = 32 ns). This mountain range scheme offers considerable advantages over the analog system now being used in that one has the option to analyze the longitudinal bunch data. In this system we are talking about a large amount of data generation and we propose using powerful microprocessors to do data reduction so that only a small amount of useful data is sent to the user consoles.

5 References

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5. C. Bouvet et.al., "A Selection of Formulae and Data Useful for the Design of A.G. Synchrotrons", CERN/MPS-SI/Int. DL/70/4, April 1970.

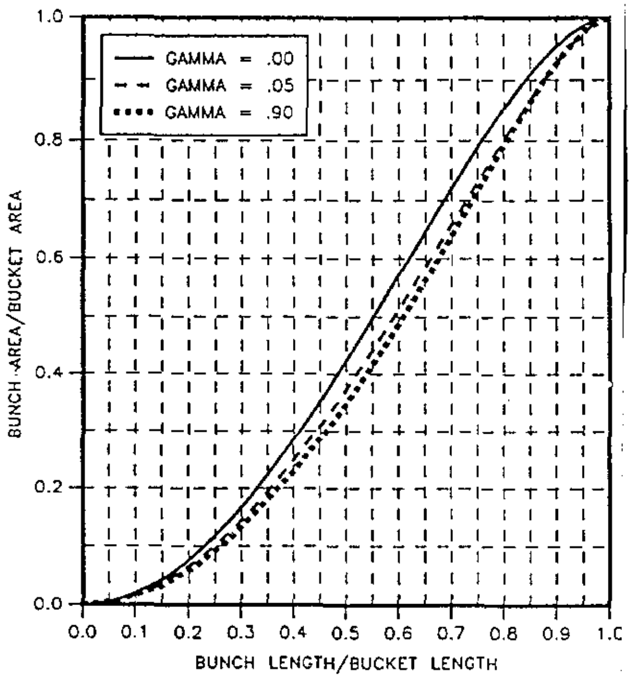

Figure 1

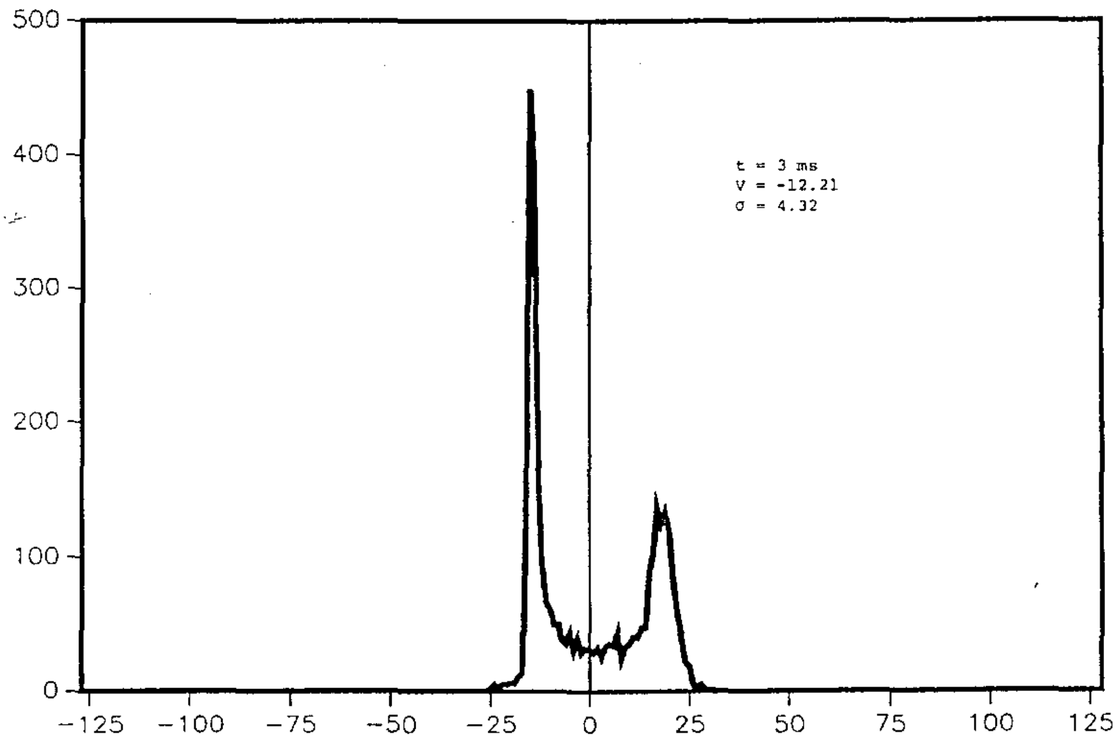

Figure 2

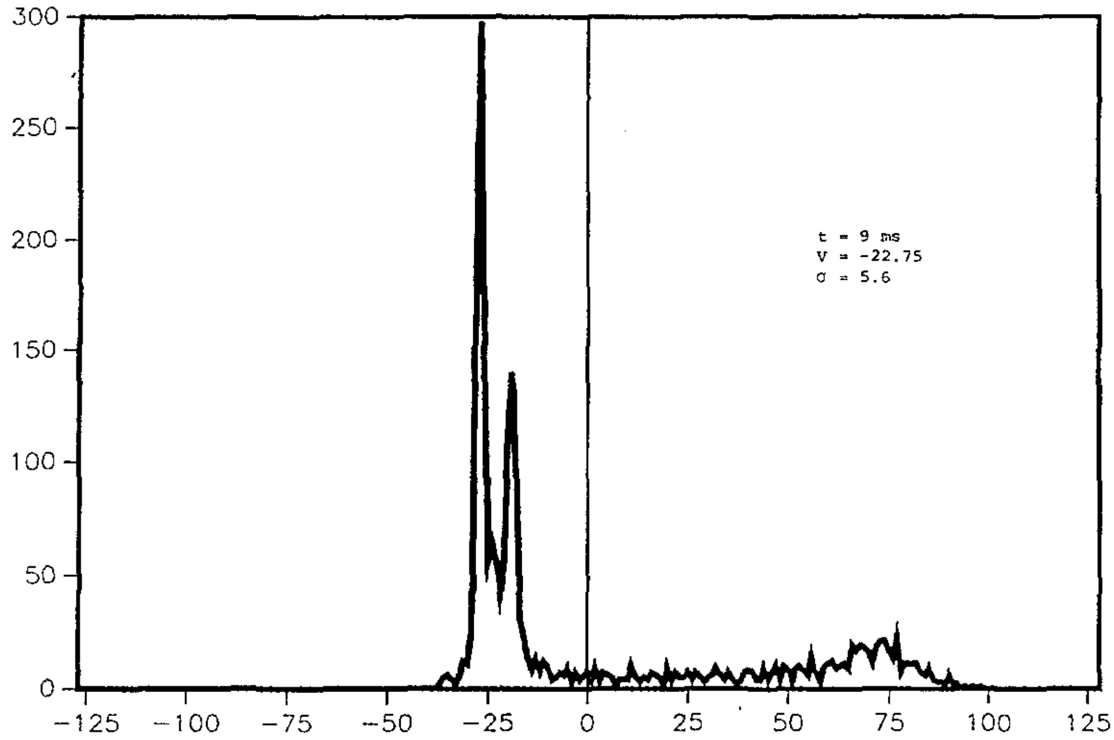

Figure 3

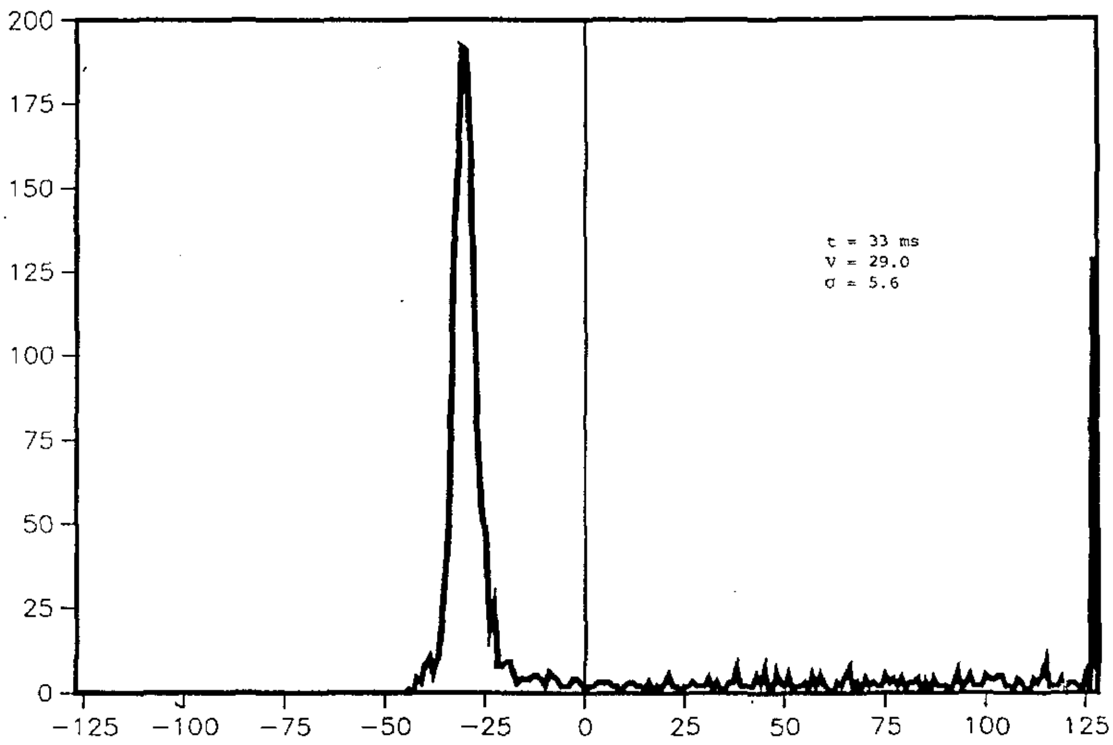

Figure 4

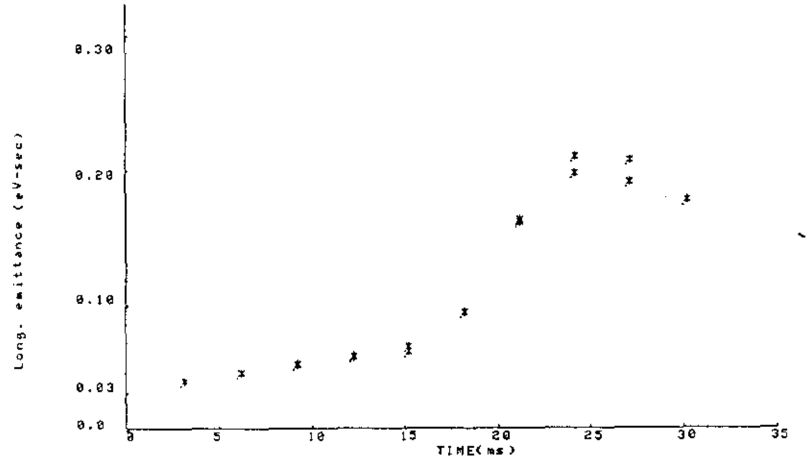

Figure 5

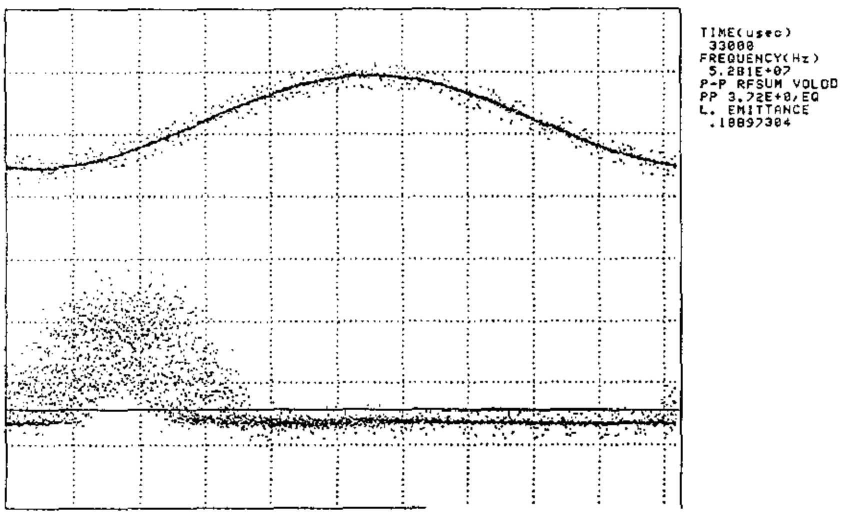

Figure 6

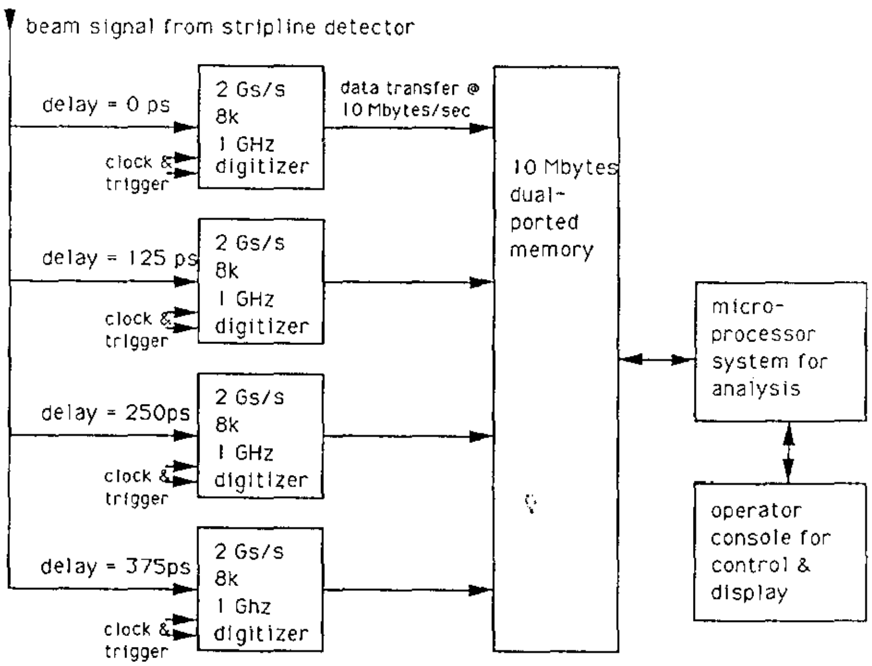

Figure 7