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Calculation of Integrated Luminosity for Beams Stored in the TEVATRON Collider

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Introduction

A model for calculating the integrated luminosity of beams stored in the Tevatron collider will be presented. The model determines the instantaneous luminosity by calculating the overlap integral of bunched beams passing through the interaction region. The calculation accounts for the variation in beam size due to the beta functions and also for effects due to finite longitudinal emittance and non-zero dispersion in the interaction region. The integrated luminosity is calculated for the beams as they evolve due to processes including collisions and intrabeam scattering. The model has been applied to both the extant and upgraded Tevatron collider, but is not limited to them.

The original motivation for developing the computer model was to determine the reduction in luminosity due to beams with non-zero longitudinal emittances. There are two effects: 1) The transverse beam size is increased where the dispersion is non-zero; 2) The finite length of the beam bunch combined with an increasing β function results in an increased transverse beam size at the ends of the bunch. The derivation of a sufficiently useful analytic expression for the luminosity proved to be intractable. Instead, a numerical integration computer program was developed to calculate the luminosity in the presence of a finite longitudinal emittance. The program was then expanded into a model which allows the luminosity to vary due to changes in emittances and reduction in bunch intensities. At that point, it was not difficult to calculate the integrated luminosity.

Luminosity

Instantaneous Luminosity

A general expression for instantaneous luminosity has been given by M. Month [MM]:

$$c \; [\; |\beta_1 - \beta_2|^2 - |\beta_1 \times \beta_2|^2 \;]^{1/2} \int \rho_1 \; \rho_2 \; dV,$$

in which ρ is the volume particle density, βc is the particle velocity, c is the speed of light and dV is dx dy dz. For the Tevatron it is reasonable to assume $\beta_1 = -\beta_2$ which gives:

$$2 \; \beta c \int \rho_1 \; \rho_2 \; dV.$$

The particle density can be written as [MM]:

$$\rho_i(x,y,z,t) = N_i \; S_i(x,y,z) \; T_i(z - \delta_i).$$

N_i is the total number of particles in bunch i. S_i is the spatial density function. T_i is a dimensionless function which accounts for the displacement of the bunch from the origin. δ_i is given by $\pm \beta$ c $(t-t_i)$, where t_i is the time when the center of bunch i reaches z=0. The ± sign ambiguity is resolved by choosing the protons to move in the +z direction. The origin of time is chosen to be the instant at which the center of the proton bunch crosses z=0.

Bunch Luminosity

The bunch luminosity is the integrated luminosity obtained from the passage of one bunch through the other [MM]. This represents integrals over space as well as time. For the Tevatron, a realistic time interval for the bunch integration is < 20 nsec.

$$L_{bunch} = 2 \; \beta c \; N_1 \; N_2 \int dt \; T_1 \; T_2 \int dV \; S_1 \; S_2 \; .$$

Luminosity per Turn

If there are B identical bunches in each beam around the accelerator circumference, and f is the revolution frequency, the luminosity per turn can be written as

$$L(t) = B \; L_{bunch} \; f.$$

For the Tevatron, f = 47713 Hz. The model assumes that there are no changes in the beam parameters on the time scale of one turn (about 21 μsec). L(t) has units of instantaneous luminosity.

Integrated Luminosity

If L(t) is the luminosity per turn at a time t and T is the store duration, then the integrated luminosity in the store is given by

$$\int_0^T L \; (t) \; dt.$$

As an example, suppose L(t) had a time dependence characterized by an exponential decay with a constant lifetime τ. In this case the integrated luminosity can be derived analytically:

$$\text{for } L(t) = L_o \; e^{-t/\tau},$$

$$\int L dt = L_o \; \tau \; [\; 1 - e^{-T/\tau} \;].$$

A numerical example is given by $L_o = 1 \times 10^{30}$ cm^{-2} sec^{-1}, τ = 27 hours and T = 27 hours (along with 10^{24} barn = 1 cm^2) which yield $\int$Ldt = 60 inverse nanobarns or 60/nb.

Measured Luminosity

However, a simple exponential decay does not adequately describe the behavior of the luminosity in the Tevatron as Figure 1 clearly demonstrates. Here, on this semilog plot, the CDF monitor of the luminosity does not have a single slope in time. Indeed, the luminosity lifetime increases with time. (The curves will be discussed in a later section.)

Emittances and Beam Sizes

The spatial density function S depends on the beam size in all three dimensions. At locations in the lattice where the dispersion is zero, the 95%

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normalized transverse emittance ϵ is defined to be related to the rms beam size (σ_o) by:

$$\epsilon = \frac{6\ p/m}{\beta_L} \sigma_o^2 \pi .$$

where p is the beam momentum and m is the beam particle rest mass. The lattice amplitude function β_L is given by:

$$\beta_L = \beta_o - 2\ \alpha_o\ z + \frac{1 + \alpha_o^2}{\beta_o}\ z^2$$

Table 1 lists the values of β_o and α_o for various Tevatron lattices at the center of the straight section in which CDF is located.

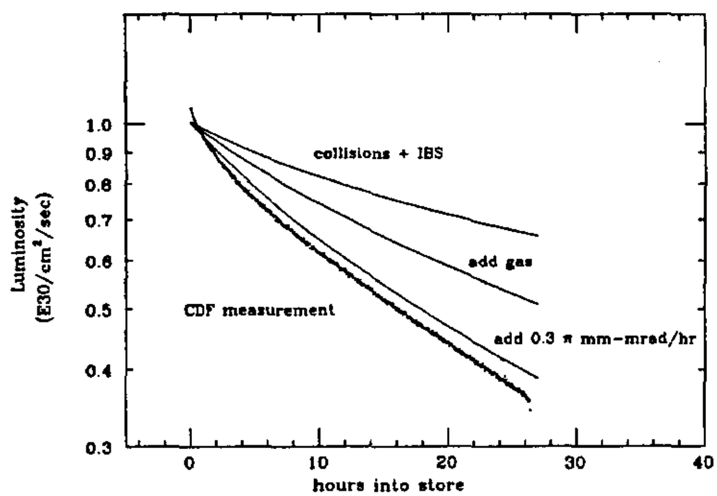

Figure 1. Time dependence of luminosity. The solid lines represent results of the model starting at t=0.

Table 1. Lattice Parameters.

	1989 Run		Upgrade	
			50 cm	25 cm
	Injection	Low β	Low β	Low β
β_{ox} (m)	72.3	0.427	0.495	0.241
β_{oy} (m)	72.6	0.402	0.500	0.248
α_{ox}	-.469	0.163	0.001	0.022
α_{oy}	0.467	-.056	0.016	0.108
η_o (m)	2.41	0.194	0.015	0.054
η'_o	0.021	-.146	0.084	0.178
γ_t	18.74	17.88	18.60	18.60
tune	19.4	19.4	20.6	20.6
Energy	150 GeV	900 GeV	1 TeV	1 TeV
RF	---	1.1 MV	1.1 MV	1.1 MV

From Table 1 one can see that the dispersion function

$$\eta = \eta_o + \eta'_o\ z.$$

is not zero through the interaction region. In this case, the momentum spread due to a finite longitudinal emittance increases the transverse beam size. The model assumes the resulting rms beam size (σ) is:

$$\sigma^2 = \sigma_o^2 + (\ \eta\ \sigma_p\ )^2$$

where σ_o is the rms beam size due to transverse emittance alone and σ_p is the rms momentum spread.

Evolution of Luminosity

Bunch Intensities

The bunch intensities are reduced using two effects. The first is due to proton antiproton collisions at 1.8 TeV (which are the whole point of the collider). The loss rate is proportional to the luminosity and the cross section:

$$dN/dt = \sigma_{in}\ L.$$

The model assumes σ_{in} = 44 mbarn. The other modelled source of intensity loss is due to nuclear scattering off the gas molecules in the beam tube [MOR]. The gas is assumed to be represented by the following:

Warm sections	1×10^{-8} torr	60% H_2	40% CO
Cold sections	5×10^{-11} torr	75% H_2	25% He

This gas composition yields a bunch intensity lifetime of 260 hours which is consistent with observations on very long stores.

Bunch Emittances

The transverse emittances are evolved using three effects: intrabeam scattering [AP], multiple Coulomb scattering off gas molecules within the beam tube [MOR], and an arbitrary source of emittance growth. The gas gives an emittance growth time of 212 hours. The longitudinal emittance evolves with intrabeam scattering only. The motions in the two transverse planes are assumed to be fully coupled.

The particular formulation of intrabeam scattering used in the model has been published by L. Evans [LE] and his full formulation is used in the model. However, sometimes it is convenient to have a quick estimate of the emittance growth rates due to intrabeam scattering. The following empirical expressions are adequate for such estimates if one has a Tevatron bunch of 6E10 intensity at 1 TeV with 1 MV/turn RF voltage.

$$\frac{d\epsilon}{dt} = C\ \epsilon_x^{-1.24}\ \epsilon_p^{-0.68}$$

The constant C depends on which emittance one is evaluating:

$$\text{for } \epsilon = \epsilon_x\ ,\ C = 36.4\ \pi \text{ mm-mrad / hour}$$

$$\text{and for } \epsilon = \epsilon_p\ ,\ C = 9.70 \text{ eV-sec / hour.}$$

ϵ_x is the horizontal 95% normalized transverse emittance and ϵ_p is the longitudinal emittance. The growth rate scales with bunch intensity. Full transverse coupling causes equal sharing of the horizontal emittance growth with the vertical; the sum of the two emittance growths is still given by the above expression.

Comparison to Present Tevatron

Table 2 gives the observed values of intensities and emittances for a store from the present run. These are used in the model to predict the evolution of the luminosity and the curves in Figure 1 represent the predictions of the model as the various luminosity degradation mechanisms are added. At t=0 the value of the luminosity calculated by the overlap integral model agrees to 5% with the value measured by CDF. However, starting at t = 0 hours, the collisions, intrabeam scattering (IBS) and the interactions with the gas are not sufficient to describe the time behavior of the luminosity. It is necessary to postulate another ad hoc source of transverse emittance growth. Possible sources of this additional growth are discussed elsewhere [DH].

Table 2. Parameters for Store 2030
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	Time into Store 0 hour	10 hours
Bunch Intensities		
protons	65	61×10^9
antiprotons	11	10×10^9
Transverse emittances		
protons	17	22 π m-mrad
antiprotons	11	14 π mm-mrad
Longitudinal emittances		
protons	3.2	3.9 eV-sec
antiprotons	2.8	3.6 eV-sec
Luminosity	1.06	0.62×10^{30} cm^{-2} sec^{-1}

6 bunches per beam; 900 GeV per beam

Figure 2 shows the results of restarting the model at t=10 hours into the store. This gives slightly better agreement in the initial luminosity, and the luminosity behavior predicted by the model is not qualitatively different from starting it at t=0.

For the first 10 hours of this store, the model predicts ∫Ldt to be 33/nb from collisions and intrabeam scattering alone. Including gas scattering reduces it to 31/nb. The ad hoc transverse emittance growth rate of 0.3 π m-mrad/hr reduces it to 29/nb, which is very close to the measured value of 28/nb. Restarting the model at t=10 hours yields 35, 32 and 30/nb which again compares closely to the measured value which happens to be 28/nb again.

Application to Collider Upgrade

Table 3 gives beam parameters for a possible version of the collider upgrade. The upgrade lattice incorporating two low β inserts of 25 cm β^* each is used. Table 4 shows the integrated luminosity for the various causes of luminosity degradation considered in the model. The store length is taken to be 10 hours.

Conclusions

In the present Tevatron, the integrated luminosity is determined by more than collisions losses, intrabeam scattering and gas scattering. Postulating an additional source of transverse emittance growth of about 0.3 π mm-mrad/hour brings the model closer to the observations.

In the upgraded collider, the integrated luminosity per day could approach 1/pb. This may be compared to the 6.6/pb delivered so far in the present run.

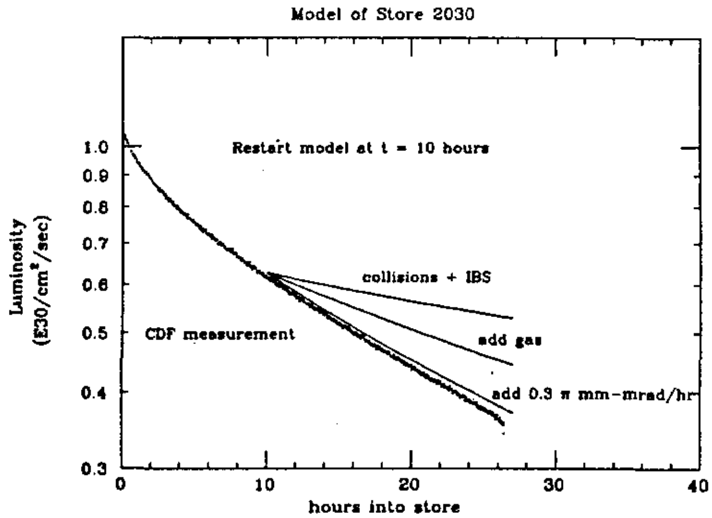

Figure 2. Same store as Figure 1 but the solid lines represent results of the model starting at t=10 hours.

Table 3. Upgrade Example Parameters

	protons	antiprotons
Intensities	60	50 $\times 10^{10}$
Emittances		
transverse	12	12 π mm-mrad
longitudinal	3	3 eV-sec

22 bunches per beam; 1 TeV per beam
3×10^{31} cm^{-2} sec^{-1} Initial luminosity

Table 4. ∫Ldt for Upgrade Example

collisions and intrabeam scattering	0.87 /pb
add gas scattering	0.83 /pb
add 0.30 π mm-mrad/hour	0.77 /pb

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[LE] L. Evans, Physics of High-Energy Particle Accelerators (BNL/SUNY Summer School, 1983), AIP Conference Proceedings No. 127, 1985, p. 243; L. R. Evans, Proceedings of the 12th International Conference on High-Energy Accelerators (Fermilab, 1983), p. 229; L. Evans, Physics of Particle Accelerators (SLAC Summer School 1985 and Fermilab Summer School 1984), AIP Conference Proceedings No. 153, 1987, p. 1723.

[MM] Mel Month, Collider Performance with Ideal Collisions, June 1, 1985 (Accel. Div. Report 85-1; DO Note 201).

[MOR] Vacuum and Beam Lifetime in the Doubler, H. Mizuno et al., Fermilab Internal Report UPC #119 (1979).