Fermilab

COMPENSATION OF THE SOLENOID FIELD IN THE COLLIDING-BEAMS DETECTOR

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June 4, 1982

A. The Compensation Condition

We assume that 0 is the collision point, the detector solenoid is located at 0 and all compensating devices (solenoids and/or skew quadrupoles) are located between A and B on either side of 0. All these devices, including the detector solenoid, couple the horizontal and the vertical motions and are called couplers. The 4 x 4 transfer matrix from A to B is

$$M_{BA} = M_{Bn} N_n M_{nn-1} N_{n-1} \cdots\cdots N_2 M_{21} N_1 M_{1A} \qquad (1)$$

where

M_{mn} = uncoupled 4 x 4 transfer matrix from n to m (without couplers)

N_m = coupled transfer matrix of the m^{th} coupler with length taken out so that N_m acts at one point

If we project N_m to the collision point 0 by

$$P_m \equiv M_{m0}^{-1} N_m M_{m0}$$

we can write [For a more detailed discussion see S. Peggs, CERN/SPS/82-2(DI/MST)]

$$M_{BA} = M_{B0}P_nP_{n-1}\cdots P_2P_1M_{0A} \tag{2}$$

The condition for compensation is, then

$$\prod_{m=1}^{n} P_m = I = \text{unit matrix} \tag{3}$$

Generally, the couplers are weak and all P's are very close to the unit matrix. Writing $P_m \equiv I + K_m$ and keeping only linear terms in K_m we can rewrite Eq. (3) as

$$\sum_{m=1}^{n} K_m = 0 \tag{4}$$

Furthermore, for weak solenoids and skew quadrupoles K_m has the form

$$K_m = \left(\begin{array}{c|c} 0 & k_m \\ \hline -\bar{k}_m & 0 \end{array}\right) \tag{5}$$

where $\bar{k}_m$ = symplectic conjugate of k_m. Thus, we can finally write the compensation condition as

$$\sum_{m=1}^{n} k_m = 0 \tag{6}$$

We now give some explicit forms of k.

1) For weak solenoid

$$N = \left(\begin{array}{c|c} I & \theta I \\ \hline -\theta I & I \end{array}\right) \qquad (7)$$

where

$$\theta \equiv \frac{1}{2}\frac{B\ell}{(B\rho)} \qquad (B\rho) = \text{rigidity of beam}$$

and B and ℓ are the field and length of the solenoid. This gives

$$k = \theta M_x^{-1} M_y \qquad (8)$$

where M_x and M_y are the 2 x 2 horizontal and vertical transfer matrices from the collision point 0 to the middle of the solenoid.

(2) For weak skew quadrupole

$$N = \left(\begin{array}{cc|cc} \multicolumn{2}{c|}{I} & 0 & 0 \\ & & 1/f & 0 \\ \hline 0 & 0 & \multicolumn{2}{c}{I} \\ 1/f & 0 & & \end{array}\right) \qquad (9)$$

where

$$\frac{1}{f} = \frac{B'\ell}{(B\rho)} \qquad (B\rho) = \text{rigidity of beam}$$

and B' and ℓ are the gradient and the length of the skew quadrupole. This gives

$$k = \frac{1}{f} M_x^{-1} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} M_y \qquad (10)$$

In all cases the compensation is independent of the beam energy because (B) cancels out of the compensation condition (6).

B. Local Compensation

By local we mean that all couplers are located in the central drift space. In this case $M_x = M_y = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$ where L = distance from 0 to the coupler, and

$$k = \begin{cases} \theta \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} & \text{solenoid} \\ -\frac{L}{f} \begin{pmatrix} 1 & L \\ -\frac{1}{L} & -1 \end{pmatrix} & \text{skew quadrupole} \end{cases} \qquad (11)$$

(1) With solenoids the compensating condition is simply

$$\sum \theta_m = 0 \qquad (12)$$

This compensation is exact and straightforward, but the compensating solenoid would take up too much of the valuable drift spaces on either side of the detector.

(2) For skew quadrupoles, because the diagonal terms are equal and opposite in sign, compensation by skew quadrupoles alone is impossible.

C. Remote Compensation

This can be accomplished with either solenoids or skew quadrupoles. The skew quadrupoles are much smaller and hence, more attractive. In general, four skew quadrupoles are necessary to satisfy the four compensation conditions

$$-\sum_{m=1}^{4} (B'\ell)_m \, M_{mx}^{-1} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} M_{my} = \tfrac{1}{2}(B\ell) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{13}$$

where, now, (B) is that of the detector solenoid. This is best solved by computer. But for an order of magnitude estimate we can take the special simple case for which $\alpha_x^* = \alpha_y^* = 0$ at the collision point 0. Then

$$M_x^{-1} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} M_y = -\sqrt{\beta_x \beta_y} \begin{pmatrix} \sqrt{\dfrac{\beta_x^*}{\beta_y^*}} \sin\phi_x \cos\phi_y & \sqrt{\beta_x^* \beta_y^*} \sin\phi_x \sin\phi_y \\ -\dfrac{1}{\sqrt{\beta_x^* \beta_y^*}} \cos\phi_x \cos\phi_y & -\sqrt{\dfrac{\beta_y^*}{\beta_x^*}} \cos\phi_x \sin\phi_y \end{pmatrix} \tag{14}$$

where β_x^*, β_y^* are values at 0; β_x, β_y are values at the skew quadrupole; and ϕ_x, ϕ_y are the phase advances from 0 to the skew quadrupole. For D. Johnson's Type E low-β insertion, the parameters at the four locations A48, A49, B11, B12 (taken at the edges of the lattice quadrupoles) are

Location	ϕ_x (rad)	ϕ_y (rad)	β_x (m)	β_y (m)
A48	-1.533914	-3.833340	137.28201	23.21214
A49	-1.457322	-1.627188	953.48737	61.75198
B0(0)	0	0	0.80825	0.89524
B11	1.730012	1.565236	68.99403	861.99964
B12	4.010746	1.666772	24.85311	106.07594

(Note: α_x^* and α_y^* at B0 are not exactly zero but they are very small.)

With these values and $B\ell$ = 15 kG x 5 m for the detector solenoid we get the following strengths for the four skew quadrupoles.

Location	$B'\ell$ (kG)
A48	2.8794
A49	-6.6124
B11	6.8285
B12	-1.3086

The sign convention on $B'\ell$ is the following:

+ for horizontally defocusing } quadrupole
- for horizontally focussing } quadrupole

rotated clockwise 45° when viewed along beam direction.

These are rather modest skew quadrupoles. In general, one notes that the strengths $B'\ell$ of the skew quadrupoles required are of the order of

$$\frac{(B\ell)_{solenoid}}{2\sqrt{\beta_x\beta_y}} \text{ which in this case is } \frac{15 \times 5}{2 \times 50} = 0.75 \text{ kG.}$$

The values of $B'\ell$ given above are that large only because of unfavorable cancellations of the effects of the four skew quadrupoles.

These compensating skew quads and the detector solenoid are presumably turned on at the same time when the low-β insertion is turned on. If it is desirable to leave the detector solenoid on all the time, before the low-β is turned on, the compensating skew quads should be set to a different set of strengths appropriate for the normal-β.

It would be convenient to have a computer program which gives the compensation as well as the low-β insertion parameters.

ADDENDUM

COMPARISON OF DETECTOR SOLENOID FIELD TO ERROR SKEW -
QUAD FIELD IN DIPOLES OF THE TEVATRON COLLIDER

(1) In a solenoid with field B_s and length ℓ_s the orbit equation is

$$\begin{cases} x'' = \frac{B_s}{B\rho} y' \\ y'' = -\frac{B_s}{B\rho} x' \end{cases}$$

which gives on going through the solenoid

$$\Delta y' = -\frac{B_s \ell_s}{B\rho} x' \qquad \text{(A-1)}$$

(2) In a dipole with length ℓ_D, field B_D and error skew-quad coefficient

$$a_1 = \frac{1}{B_D} \frac{\partial B_x}{\partial x}$$

The horizontal field is

$$B_x = a_1 B_D x$$

which gives, in going through the dipole

$$\Delta y' = \frac{B_x \ell_D}{B\rho} = a_1 \frac{B_D \ell_D}{B\rho} x \sim a_1 \frac{B_D \ell_D}{B\rho} \beta_x x' \qquad \text{(A-2)}$$

From Eqs. A-1 and A-2 we see that we must compare

$$a_1 (B\ell)_D \beta_x \qquad \text{to} \qquad (B\ell)_s$$

(3) For Tevatron dipole $a_1 \sim 10^{-4}\ \text{in}^{-1} = 4 \times 10^{-3}\ \text{m}^{-1}$ and at a fairly high value of $\beta_x \sim 80$ m we have

$$a_1 (B\ell)_D \beta_x \sim (4 \times 10^{-3}\ \text{m}^{-1})\ (40\ \text{kG} \times 6\ \text{m}) \times 80\ \text{m} \sim 77\ \text{kGm}.$$

This is roughly equal to

$$(B\ell)_s \sim 15\ \text{kG} \times 5\ \text{m} \sim 75\ \text{kGm}$$

of the Tevatron detector solenoid.

The conclusion is, therefore, that there is no need to provide individual compensation for the detector solenoid. The overall correction skew quads which correct for the skew quad errors in dipoles should be able to easily take care of also the detector solenoid. Indeed they could probably also take care of the correction at the injection energy of 150 GeV when the effect of the solenoid field is ~6.7 times that of the error skew quad field in a dipole.