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RAISING THE SPACE-CHARGE LIMIT OF A SYNCHROTRON BY MEANS OF RF QUADRUPOLES

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ABSTRACT

A method is suggested for containing a high-intensity beam in a synchrotron by applying a transverse electromagnetic field which varies in space and time (by means of rf quadrupoles) in a manner to neutralize the electric and magnetic self-force of the beam. The same method may be applied to diminish the beam-beam interaction in colliding-beam storage rings.

INTRODUCTION

At a sufficiently high intensity, the size of the beam in a synchrotron grows because of a transverse self-force due to the electric space charge and the magnetic pinch effect. The resultant self-force causes a shift, $\Delta\nu$, in the number of betatron oscillations per turn. When the intensity is such that ν approaches a resonance condition, then the beam size increases until the self-force is diminished because of the reduced density of the beam.

A convenient formula[1] for the maximum current is $I_{amp} = 3 \times 10^7 \nu\Delta\nu \; (a/R)^2 \; (\beta\gamma)^3 \; F$, where a is the radius of the beam, R is the radius of the accelerator, and F is the bunch factor. The rapid variation with energy, $(\beta\gamma)^3$, arises because at relativistic velocities the electric and magnetic pinch-force of the beam tends exactly to cancel the space-charge force.

The formula has been unfortunate because it gives rise to a temptation to insure an intense beam simply by increasing the height of the gap--a very expensive measure. Furthermore, allowing the beam to grow in size at injection corrupts the beam henceforth, makes it difficult to extract, and would lower the luminosity for colliding-beam experiments. The formula also assumes that no neutralization of the space charge occurs due to low-energy electrons. This ion neutralization does occur to some extent,[2] and can cause the magnetic force to dominate and hence to vitiate the formula--reducing the limit. Image effects also complicate the analysis.

The proper antidote to the self-force is not to increase the aperture but rather to apply an electromagnetic field which varies in space and time in a manner to produce a force that should counter the self-force and thereby keep $\Delta\nu$ (and hence the beam size) as small as possible. Indeed it should be feasible to measure the electric and magnetic field of the beam (or better, to measure ν itself) and then to keep $\Delta\nu$ small by a feedback system.

Let us consider a few examples, the first being a proton beam of uniform density produced by H^- ions being injected and stripped in a synchrotron magnet excited at a constant current, multiple scattering being neglected. As the beam increases with time, the self-force increases and hence an increasingly strong compensating quadrupole focusing (or defocusing) force should be applied. In the case of a separated-function lattice, the quadrupole strengths of all the focusing magnets could be changed--but since the beam grows slowly by being in a state of resonance, special compensating quads might be located at a few positions rather than many.

In principle, exact compensation of the self-force should allow for any beam current, but other kinds of instabilities can be expected to impose a limitation as the intensity is increased.

There are a number of methods for adjusting the quadrupole strengths. The simplest would be to do so empirically, by programming the corrections, relying on the excellent pulse-to-pulse constancy of the linac beam for reproducibility. Another method would be to measure the actual electric and magnetic field of the beam and use this electronically to calculate and then set the strengths. The most direct method would be to measure ν in a non-destructive manner and then to keep ν constant by feeding that signal back to the compensating quads. One method of measuring ν would be to have transverse deflecting plates followed at some distance by a beam-position monitor. This signal would be fed back to the deflecting plates through a high-gain, but amplitude-limited, amplifier and would lead to an oscillation related to the frequency ν . Of course, in all this discussion both horizontal and vertical focusing forces must be considered separately.

In general the beam will not have a uniform density as has been assumed in the above discussion. In principle a multipole compensating field could be applied to match the distribution, i.e., octupole lens, etc., in addition to the quadrupoles. However, it should be pointed out that a non-uniform density distribution, such as one that is gausian in shape, will automatically change to a uniform distribution as beam blow-up starts to occur. This is because the dense inner part of the beam will approach a resonant blow-up first and so will grow into the less dense part of the beam which will not yet have started to grow. Thus if a particular radius which includes most of the beam is chosen and a correction appropriate to a uniform distribution out to that radius is applied, then as the space-charge limit is approached, the beam within the radius will become uniform and will not grow further. The beam outside the chosen radius will be lost. I suppose there would be a slow leakage at the surface of the beam insofar as the surface does not represent a step-function. This may be a method of cleaning up the outside tails of a beam.

Now let us apply an rf bunching field. Then the longitudinal density of the beam will change as bunches form, and a rapid variation in the compensating quadrupole strength must be made to match the varying density in the beam. As the bunches form, the self-force will become stronger in proportion to the bunching factor.

The varying compensation could be done in principle by a feedback system if the system of detectors and quads would respond very rapidly, i.e., to high harmonics of the applied rf voltage. That is too much to expect. In actuality the density variation in the bunch can be approximated by a few harmonic terms of the applied rf chosen to match the bunch structure. These could be found by feeding the detector signals to resonance circuits tuned to harmonics of the rf applied voltage. The slowly-varying amplitudes of these components could then be fed back to corresponding quads which are excited by harmonics of the rf--with appropriate phases, of course. Alternatively, the amplitudes and phases could be programmed empirically depending upon the pulse-to-pulse constancy of modern accelerators for reproducibility.

The new piece of equipment required at this stage would be the rf harmonically excited quads.[3] These magnetic (or electrostatic) quads would resemble the resonant rf accelerating cavity, except that for low harmonics they would be more complex. A structure which comes to mind for the first harmonic would be four mutually parallel quarter-wave lines (or, better, half-wave lines) which are arranged symmetrically about the beam and parallel to its longitudinal direction. The structure for higher harmonics would be simple cavities excited in a mode to give a quadrupole field.

It would appear that the above discussion of a bunched beam can easily be extended to actual acceleration, and that only practical considerations of building and powering the rf compensating magnets would limit the intensity that can be attained--apart from other kinds of instabilities. Of course, even a factor of two increase is worthwhile.

The excitation of the compensating force will appear as a slow pulse at injection time, a few milliseconds in length, rising as protons are injected, and then rising further as they become bunched. Finally, as the increasing energy of the particles implies a greater cancellation of the electric and magnetic self-force, the compensating force will diminish, but in any case it will soon become negligible compared to the increasingly stronger focusing forces of the magnetic lattice. Hopefully, even one set of rf quads operating at the fundamental rf frequency will be enough. The rf quads must be tuned to resonance, but the frequency change is only about ten percent because the pulse is so short. It would also appear that rf quadrupoles might also be used to compensate for beam-beam interactions which produce changes in ν in colliding-beam storage rings where a bunched beam is used. In that case the strengths of the quadrupoles would have to be greater than they would be at injection energy. The compensating quads would not be located at the interaction region, but in each of the beams separately, at appropriate places in the lattices.

The above work was done in China at the Institute of High Energy Physics at Peking. It grew out of discussion with Dr. Fang Zuo-Shieh about the space-charge limit of accelerators, for which I am indebted to him.

References

[1] F. T. Cole et al. MURA Report 462, June 1959.

[2] I understand that this kind of ion neutralization has been done in a controlled manner in the USSR by admitting just enough gas into the donut so that, on the average, the electric and magnetic self-force just cancel. Of course that will not work for a bunched beam. See also J. Herrera and B. Zotter, BNL Report 50980, UC-28, Dec. 15, 1978.

[3] Quadrupole lenses excited at very high frequencies have been used in the USSR to provide transverse focusing in linacs.