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FERMILAB ENERGY DOUBLER MAGNETS-MAGNETOSTATICS

S.C. Snowdon
Fermi National Accelerator Laboratory*
Batavia, Illinois 60510

Summary

Magnetic field computations have been performed for the three-inch circular aperture doubler magnets in the warm iron geometry. Locations for rectangular conductors along circular arcs have been found such that, in the absence of the construction errors, the sextupole and decapole terms have been removed from the dipole. For the quadrupole suitable locations for the conductors have been found that remove the duodecapole term. Field quality, longitudinally integrated fields, construction errors, forces, energy content, and eddy current heating under cycled conditions will be discussed for a 45 kG dipole and a 20 kG/inch quadrupole.

Design Considerations

The possibilities inherent in circular iron shields, elliptical iron shields, pancake coils, offset circular shell coils, and circular shell coils have been examined using complex variable methods.[1] Both the field in the transverse section and the longitudinally integrated field are calculated. For each geometry a search mode is employed to improve any one of several parameters specifying initial conductor locations. Thus, for circular shells, the radial position, azimuthal position, or azimuthal space between keystoned conductors may be adjusted. For pancake coils, the horizontal position or horizontal space between rectangular conductors may be varied. The end result of the search is a set of conductor locations that minimizes the energy content within the reference radius of all multipoles except the lowest. Although the search procedure may be incorporated into the longitudinally integrated fields, this was not done because, for relatively long magnets, the end effects are small and a few runs suffice to obtain the desired quality.

Additional calculations provide the field distribution and net flux entering the iron shield, eddy currents induced in various elements, and the electromagnetic force distribution in the conductors. Thus one may estimate field modifications induced by iron saturation, the iron cross section necessary to reduce return flux saturation, and the power loss in the bore tube liner, cryostat walls, and heat shield. From the force calculations realistic estimates are made of the banding tension necessary to restrict conductor movements and the spring constant[2] for the displacement of the coil package relative to the iron.

An iron shield in the shape of an upright ellipse has a reduced saturation effect on the field distribution and yields a coil arrangement for high quality fields that reflects the confocal nature of the coordinate system. Hence the usable aperture has even higher eccentricity than the shield although minimal net flux results. A horizontal aperture sufficiently small to provide an economic advantage over the circular case requires vertical injection and extraction. This imposes a considerable constraint on the elliptical shield which, therefore, was abandoned in favor of the circular shield. Having chosen a circular shield, the concentric nature of the coordinate system dictates that for high quality fields the conductor arrangement be circular. Field quality demands from orbit considerations, extraction requirements, and achievable construction tolerances set the aperture diameter at about three inches.

Considerations that lead to a choice of inner iron radius are as follow. In a typical cold iron design the shield is used directly to hold the coil package in place. Saturation effects on the magnetic field must be counterbalanced with additional correction windings. This design, however, makes maximum use of the iron in producing field. If, on the other hand, it is desired to have the dipoles and quadrupoles track with an accuracy sufficient to permit a single excitation current throughout the magnet system, then, for the coil package that yields a three inch aperture, the inner iron radius must be about four inches. For this radius there is sufficient space between the outside of the coil package and the iron to insert a thermally insulating support structure. Thus, the iron may remain at room temperature. In this warm iron design, although there is no significant saturation effect, more ampere-turns must be provided to offset the diminished utilization of the iron. We have opted for the warm iron design with the attendant possibility of simplifying the power distribution system.

For the most economical use of superconductor one may tailor the conductor size depending on its location in the magnetic field. Thus, in the dipole, two grades of multistrand cable were used. For the inner two shells a seven-strand cable of nominal size .150 in by .075 in was chosen[3] for which the effective JB product is 80% of short sample.[4] In the outer two shells an 11-strand cable of nominal size .150 in by .050 in was used for which the effective JB product is 70% of short sample.

Having chosen the space allowed in the dipole for conductors, banding, cryostats, and supports, these general space allocations were incorporated into the quadrupole for maximum simplicity. These conditions and the desire to obtain the highest gradient possible in order to minimize longitudinal space allocation dictate an ungraded conductor design for the quadrupole. Thus the 11-strand conductor

*Operated by the Universities Research Association, Inc., under contract with the U.S. Energy Research and Development Administration.

was chosen which operates with a JB product at 63% of short sample.[4]

The length of the iron shield relative to the coil ends must be chosen. Calculations using many segments of linear current elements have been made[3] and indicate that for dipoles, in the absence of the iron shield, a field enhancement of some 20% is expected in the end region. Since the field enhancement by the iron in the transverse section is 18% it is desirable to terminate the iron somewhat before the conductors are turned around. In this manner there will be no significant field enhancement in the ends. For the quadrupole, since the maximum field is much less than 45 kG, the iron shield may be carried out over the coil ends.

Finally, it is to be noted that in construction both the dipoles and the quadrupoles the construction tolerance[5] on the location of conductor shell radii and azimuthal position of the shell is ±.002 in.

Tables 1 - 8 are self explanatory. Table 9 refers to the longitudinally integrated fields in which circular turn-around ends are used, the turn centers being separated by the length indicated. The entries T(N), S(N), and R(N) refer to coefficients in a multipole expansion of the longitudinally integrated magnetic field. Successive terms give ΔB at the reference radius for the dipole, sextupole, decapole, etc. The contribution due to the currents with no shield is T(N), the contribution from the iron shield is S(N), and R(N) is the ratio T(N) + S(N) divided by T(1) + S(1). Table 10 is a similar calculation in which the contribution due to the ends is omitted and the length set equal to one inch. The median plane field in the transverse section is given by BT. Columns BA, BS, and BN give respectively the contribution in the absence of the shield, the contribution due to the shield, and the total field normalized to unity at the origin. The entry DELR(N) is an estimate[6] of the magnitude of the change in R(N) induced by saturation effects in the iron shield. Tables 11 - 12 provide similar information relative to the quadrupole, T(N) etc. now stepping through quadrupole, duodecapole, etc.

References

1. W.W. Lee and S.C. Snowdon, IEEE Trans. on Nucl. Sci., NS-20, 726 (1973)

2. K. Halbach, Nucl. Instr. and Methods, 78, 185 (1970)

3. P.J. Reardon and B.P. Strauss, Ed., Fermilab Technical Note TM-421, May 1973

4. P. Price and R. Yamada, Fermilab Technical Note TM-538, Nov. 23, 1974

5. S. Ohnuma, Fermilab Technical Notes TM-502, June 14, 1974 and TM-510, July 17, 1972

6. M.A. Green, Kernforschungszentrum Karlsruhe External Report 3/71-7, June 1972

Table 1. Bending Magnets Performance Parameters

Field Strength	45 kG
Effective Field Length	240 in
Good Field Width	2.0 in
Field Quality (ΔB/B at 1 in Rad.)	±.05%

Table 2. Design Data for Bending Magnet

Conductor Current (45 kG)	2345 A
Conductor Size (no insulation)	
Inner 2 shells (7-strand)	.152 in by(.075/.0636)in
Outer 2 shells (11-strand)	.152 in by(.050/.0432)in
Effective Current Density	
(7-strand)	215 kA/in^2
(11-strand)	320 kA/in^2
Total Number of Turns	228
Insulation Thickness(spiral wrap)	.004 in
Inner Bore Tube Radius(304 SS)	1.125 in
Inner Bore Tube Wall Thickness	.050 in
Inner Cryostat Radius(304 SS)	2.50 in
Inner Cryostat Wall Thickness	.018 in
Outer Cryostat Radius(304 SS)	2.625 in
Outer Cryostat Wall Thickness	.018 in
Lamination Inner Radius(mild steel)	4.00 in
Lamination Thickness	.0625 in
Outside Dimension of Iron	16 in by 10 in
Total Length of Iron	234 in

Table 3. Stored Energy and Losses in Bending Magnet

Peak Stored Energy	.54 MJ
Inductance	.18 Hy
Repetition Period	60 sec
Eddy Current Losses	
Bore Tube	.13 W
Conductor Matrix	1.8 W
Inner Cryostat	.22 W
Outer Cryostat	.22 W
Heat Shield (20°K)	.2 W
Lamination (warm)	neg.
Hysteresis Losses	
Superconductor	3.3 W
Lamination (warm)	3.5 W

Table 4. Forces and Critical Fields in Bending Magnet

Central Field		45 kG
Maximum Field in Conductor		
Inner 2 shells (7-strand)		47 kG
Outer 2 shells (11-strand)		38 kG
Effective Radius of Conductor shells		1.896 in
Traction at Effective Radius		
Angle (Deg)	x-Traction (lb/in^2)	y-Traction (lb/in^2)
0	699	0
40	1521	-1174
50	1623	-985
90	0	0
Displacement Force		
(x-displ. = .010 in)		9.9lb/in
(y-displ. = .010 in)		9.9lb/in

Table 5. Focusing Magnet Performance Parameters

Gradient Strength	20.7 kG/in
Effective Gradient Length	62 in
Good Field Width	2.0 in
Gradient Quality	
($\Delta B/xB'_o$ at 1 in Rad.)	±.2%

Table 6. Design Data for Focusing Magnet

Conductor Current (20.7 kG/in)	2345 A
Conductor Size	
(no insulation)	.152 in by(.050/.0432)in
Effective Current Density	320 kA/in^2
Turns per Pole	51
Insulation Thickness	.004 in
(spiral wrap)	
Inner Bore Tube Radius(304 SS)	1.125 in
Inner Bore Tube Wall Thickness	.050 in
Inner Cryostat Radius(304 SS)	2.50 in
Inner Cryostat Wall Thickness	.018 in
Outer Cryostat Radius(304 SS)	2.625 in
Outer Cryostat Wall Thickness	.018 in
Lamination Inner Radius	4.00 in
(mild steel)	
Lamination Thickness	.0625 in
Outside Dimension of Iron	10.0 in Dia.
Total Length of Iron	66 in

Table 7. Stored Energy and Losses in Focusing Magnet

Peak Stored Energy	51 kJ
Inductance	.013 Hy
Repetition Period	60 sec
Eddy Current Losses	
Bore Tube	.002 W
Conductor Matrix	.20 W
Inner Cryostat	.004 W
Outer Cryostat	.004 W
Heat Shield (20°K)	.003 W
Lamination (warm)	neg.
Hysteresis Losses	
Superconductor	.56 W
Lamination (warm)	.25 W

Table 8. Forces and Critical Fields in Focusing Magnet

Central Gradient		20.7 kG/in
Maximum Field in Superconductor		31 kG
Effective Radius of Superconductor		1.838 in
Traction at Effective Radius		
Angle (Deg)	x-Traction (lb/in^2)	y-Traction (lb/in^2)
0	136	0
20	612	-1448
25	714	-1399
45	0	0
65	-1399	714
70	-1448	612
90	0	136
Displacement Force		
(x-displ. = .010 in)		2.7lb/in
(y-displ. = .010 in)		2.7lb/in

Table 9. INTEGRATED MULTIPOLE STRUCTURE OF C-SERIES GRADED DOUBLER DIPOLE

ORDER OF POLE	= 1	CALCULATIONAL MODE	= 1	NUMBER OF LAYERS	= 4
HIGHEST MULTIPOLE ORDER	= 19	CONDUCTOR CURRENT(A)	= 2345.0000	REFERENCE RADIUS(IN)	= 1.0000
INNER IRON RADIUS(IN)	= 4.0300	SIMPSONS RULE INTERVAL(DEG)	= 1.0000	HORIZONTAL INCREMENT(IN)	= .1000
INSULATION THICKNESS(IN)	= .0040				

LAYER	TURNS	CURDEN (KA/IN/IN)	THETAS (DEG)	THETAF (DEG)	SPACER (IN)	RINNER (IN)	ROUTER (IN)	LENGTH (IN)
1	29.00	215.531	.1267	81.7456	.00045	1.5000	1.6650	234.0000
2	27.00	215.531	.1136	68.1899	.00027	1.6800	1.8450	236.0000
3	30.00	320.522	.0939	45.8796	-.00062	1.9450	2.1100	239.0000
4	28.00	320.522	.0908	39.4065	-.00052	2.1250	2.2900	240.0000

MULTIPOLE COEFFICIENTS

T(N) =	8.818E+03	-9.544E+00	-2.327E+00	-1.054E+01	-1.577E-01	5.659E-01	-2.392E-01	7.678E-02	-5.920E-02	2.781E-02
S(N) =	2.012E+03	6.692E+00	-5.665E-02	-1.269E-03	2.295E-06	2.709E-07	6.746E-10	-6.172E-11	-6.610E-13	9.761E-15
R(N) =	1.000E+00	-2.633E-04	-2.201E-04	-9.733F-04	-1.456E-05	5.225E-05	-2.209E-05	7.089E-06	-5.466E-06	2.568E-06

X(IN)	BT(KG-IN)	BA(KG-IN)	BS(KG-IN)	BN
.00000	10833.13097	8818.04655	2012.08442	1.00000
.10000	10833.10220	8817.35386	2012.15134	1.00000
.20000	10830.01240	8817.66038	2012.35202	.99999
.30000	10829.84728	8817.16102	2012.68627	.99997
.40000	10829.57039	8816.41666	2013.15373	.99995
.50000	10829.10414	8815.35021	2013.75393	.99991
.60000	10828.30377	8813.81753	2014.48624	.99983
.70000	10826.92438	8811.57499	2015.34938	.99970
.80000	10824.58596	8808.24202	2016.34394	.99949
.90000	10820.72729	8803.25997	2017.46732	.99913
1.00000	10814.56807	8795.84930	2018.71877	.99856
1.10000	10805.66826	8784.97138	2020.09688	.99769
1.20000	10790.92907	8769.32903	2021.60004	.99638
1.30000	10770.77413	8747.54767	2023.22646	.99452
1.40000	10744.00066	8719.02652	2024.97414	.99205
1.50000	10713.75622	8686.91535	2026.84087	.98925

Table 10. INTEGRATED MULTIPOLE STRUCTURE OF C-SERIES GRADED DOUBLER DIPOLE

ORDER OF POLE =	1	CALCULATIONAL MODE =	0	NUMBER OF LAYERS =	4
HIGHEST MULTIPOLE ORDER =	19	CONDUCTOR CURRENT(A) =	2345.0000	REFERENCE RADIUS(IN) =	1.0000
INNER IRON RADIUS(IN) =	4.0000	SIMPSONS RULE INTERVAL(DEG)=	1.0000	HORIZONTAL INCREMENT(IN) =	.1000
INSULATION THICKNESS(IN) =	.0040				

LAYER	TURNS	CURDEN (KA/IN/IN)	THETAS (DEG)	THETAF (DEG)	SPACER (IN)	RINNER (IN)	ROUTER (IN)	LENGTH (IN)
1	29.00	215.531	.1267	81.7456	.00045	1.5000	1.6650	1.0000
2	27.00	215.531	.1138	68.1849	.00027	1.6800	1.8453	1.0000
3	30.00	320.522	.0989	49.8796	-.00062	1.9450	2.1100	1.0000
4	28.00	320.522	.0908	39.4065	-.00052	2.1250	2.2900	1.0000

MULTIPOLE COEFFICIENTS

T(N) =	3.664E+01	-2.861E-02	1.709E-04	-4.335E-02	-4.386E-04	2.444E-03	-1.001E-03	3.364E-04	-2.518E-04	1.195E-04
S(N) =	8.336E+00	2.818E-02	-2.287E-04	-9.220E-06	9.864E-09	1.133E-09	2.939E-12	-2.544E-13	-2.721E-15	4.000E-17
R(N) =	1.000E+00	-9.641E-06	-1.284E-06	-9.639E-04	-9.750E-06	5.434E-05	-2.226E-05	7.479E-06	-5.598E-06	2.656E-06
DELR(N)=	7.574E-04	1.578E-05	5.917E-07	2.642E-08	1.284E-09	6.567E-11	3.473E-12	1.881E-13	1.037E-14	5.801E-16

X(IN)	BT(KG-IN)	BA(KG-IN)	BS(KG-IN)	BN
.00000	44.98035	36.64467	8.33569	1.00000
.10000	44.98035	36.64438	8.33597	1.00000
.20000	44.98033	36.64352	8.33682	1.00000
.30000	44.98028	36.64206	8.33822	1.00000
.40000	44.98011	36.63991	8.34019	.99999
.50000	44.97957	36.63685	8.34272	.99998
.60000	44.97817	36.63237	8.34580	.99995
.70000	44.97506	36.62562	8.34944	.99988
.80000	44.96882	36.61519	8.35363	.99974
.90000	44.95735	36.59899	8.35836	.99949
1.00000	44.93771	36.57408	8.36364	.99905
1.10000	44.90598	36.53653	8.36944	.99835
1.20000	44.85731	36.48153	8.37578	.99726
1.30000	44.78689	36.40425	8.38264	.99570
1.40000	44.69381	36.30381	8.39000	.99363
1.50000	44.59437	36.19649	8.39788	.99142

MAX. FIELD ON IRON(KG) = 15.6944 IRON PERMEABILITY AT BMAXFE= 233.6375 FLUX IN IRON(KG-IN) = 67.7712

Table 11. INTEGRATED MULTIPOLE STRUCTURE OF C-SERIES UNGRADED DOUBLER QUADRUPOLE

ORDER OF POLE =	2	CALCULATIONAL MODE =	1	NUMBER OF LAYERS =	4
HIGHEST MULTIPOLE ORDER =	14	CONDUCTOR CURRENT(A) =	2345.0000	REFERENCE RADIUS(IN) =	1.0000
INNER IRON RADIUS(IN) =	4.0000	SIMPSONS RULE INTERVAL(DEG)=	1.0000	HORIZONTAL INCREMENT(IN) =	.1000
INSULATION THICKNESS(IN) =	.0040				

LAYER	TURNS	CURDEN (KA/IN/IN)	THETAS (DEG)	THETAF (DEG)	SPACER (IN)	RINNER (IN)	ROUTER (IN)	LENGTH (IN)
1	17.00	320.522	1.4482	34.1045	-.00164	1.5000	1.6650	59.0000
2	14.00	320.522	1.3003	25.6932	-.00108	1.6800	1.8450	60.0000
3	11.00	320.522	1.1304	18.0238	-.00028	1.9450	2.1100	61.0000
4	9.00	320.522	1.0382	13.7738	-.00009	2.1250	2.2900	62.0000

MULTIPOLE COEFFICIENTS

T(N) =	1.223E+03	-5.575E-01	-2.927E+00	1.270E-01	-5.742E-02
S(N) =	5.538E+01	5.764E-03	2.541E-07	-2.162E-10	-1.063E-13
R(N) =	1.000E+00	-4.313E-04	-2.258E-03	9.931E-05	-4.488E-05

X(IN)	GT(KG-IN/IN)	GA(KG-IN/IN)	GS(KG-IN/IN)	GN
.00000	1279.25813	1223.35889	55.89924	1.00000
.10000	1279.25807	1223.35883	55.89924	1.00000
.20000	1279.25724	1223.35799	55.89925	1.00000
.30000	1279.25347	1223.35418	55.89929	1.00000
.40000	1279.24209	1223.34270	55.89939	.99999
.50000	1279.21224	1223.31264	55.89960	.99996
.60000	1279.13772	1223.23774	55.89999	.99991
.70000	1278.95849	1223.05786	55.90062	.99977
.80000	1278.54817	1222.64657	55.90160	.99945
.90000	1277.65836	1221.75534	55.90302	.99875
1.00000	1275.84891	1219.94391	55.90500	.99734
1.10000	1272.31069	1216.40301	55.90768	.99457
1.20000	1265.59915	1209.68796	55.91119	.98932
1.30000	1252.94411	1197.02841	55.91570	.97943
1.40000	1228.63782	1172.71643	55.92139	.96043
1.50000	1180.21556	1124.28713	55.92843	.92258

Table 12. INTEGRATED MULTIPOLE STRUCTURE OF C-SERIES UNGRADED DOUBLER QUADRUPOLE

ORDER OF POLE =	2	CALCULATIONAL MODE =	0	NUMBER OF LAYERS =	4
HIGHEST MULTIPOLE ORDER =	18	CONDUCTOR CURRENT(A) =	2345.0000	REFERENCE RADIUS(IN) =	1.0000
INNER IRON RADIUS(IN) =	4.0000	SIMPSONS RULE INTERVAL(DEG)=	1.0000	HORIZONTAL INCREMENT(IN) =	.1000
INSULATION THICKNESS(IN) =	.0040				

LAYER	TURNS	CURDEN (KA/IN/IN)	THETAS (DEG)	THETAF (DEG)	SPACER (IN)	RINNER (IN)	ROUTER (IN)	LENGTH (IN)
1	17.00	320.522	1.4482	34.1045	-.00164	1.5000	1.6650	1.0000
2	14.00	320.522	1.3003	25.6932	-.00108	1.6800	1.8450	1.0000
3	11.00	320.522	1.1304	18.0238	-.00028	1.9450	2.1100	1.0000
4	9.00	320.522	1.0382	13.7738	-.00009	2.1250	2.2900	1.0000

MULTIPOLE COEFFICIENTS

T(N) =	1.979E+01	4.129E-06	-4.792E-02	2.240E-03	-9.584E-04
S(N) =	8.933E-01	9.372E-05	4.487E-09	-3.348E-12	-1.677E-15
R(N) =	1.000E+00	4.730E-06	-2.317E-03	1.083E-04	-4.634E-05
DELR(N)=	6.176E-05	6.044E-08	1.585E-10	5.260E-13	1.598E-15

X(IN)	GT(KG-IN/IN)	GA(KG-IN/IN)	GS(KG-IN/IN)	GN
.00000	20.68322	19.78988	.89335	1.00000
.10000	20.68322	19.78988	.89335	1.00000
.20000	20.68322	19.78988	.89335	1.00000
.30000	20.68322	19.78987	.89335	1.00000
.40000	20.68320	19.78985	.89335	1.00000
.50000	20.68304	19.78969	.89335	.99999
.60000	20.68244	19.78908	.89336	.99996
.70000	20.68051	19.78714	.89337	.99987
.80000	20.67535	19.78197	.89339	.99962
.90000	20.66312	19.76971	.89341	.99903
1.00000	20.63668	19.74324	.89344	.99775
1.10000	20.58327	19.68979	.89348	.99517
1.20000	20.47963	19.58609	.89354	.99016
1.30000	20.28103	19.38741	.89361	.98055
1.40000	19.89468	19.00097	.89371	.96187
1.50000	19.11673	18.22291	.89382	.92426

MAX. FIELD ON IRON(KG) = 6.9576 IRON PERMEABILITY AT BMAXFE= 667.6402 FLUX IN IRON(KG-IN) = 14.4223