



## Magnetic Field Distribution in Doubler Type Dipoles

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April 2, 1974

Summary

A direct calculation is made of the two dimensional magnetic field from a conductor of wedge shaped cross section carrying a uniform current distribution. For sufficiently small conductors the wedge shape simulates the keystone conductor used in doubler type magnets. Field distributions are presented for the dual test dipoles, the doubler dipole, and the muon beam line dipole. A similar calculation representing the contribution of the magnet ends to the longitudinally integrated fields is also given for the same magnets.

Magnetic Field of Single Conductor in Doubler Geometry

Using complex notation the dipole magnetic field ( $H = H_x + iH_y$ ) for variable current density conductors and images is given by

$$H^* = -2i \iint J \left\{ \frac{1}{z-z_0-d} - \frac{1}{z+z_0+d} + \frac{1}{z-z_0^*-d} - \frac{1}{z+z_0^*+d} \right. \\ \left. + \frac{1}{\frac{R_1^2}{z-\frac{1}{z_0^*+d}}} - \frac{1}{\frac{R_1^2}{z+\frac{1}{z_0^*+d}}} + \frac{1}{\frac{R_1^2}{z-z_0^*-d}} - \frac{1}{\frac{R_1^2}{z+z_0^*+d}} \right\} r_0 dr_0 d\theta_0 \quad (1)$$

where  $z_0 = r_0 e^{i\theta}$  and integration is required only over the conductor located in the first quadrant. See. Fig. 1 for geometrical details.

If  $J$  is either constant as in the two-dimensional case or a function only of  $\theta_0$  as is convenient and reasonably realistic in the case of the end contributions which will be treated later, it is possible to integrate over the radial extent of the conductor. Individual integrals become

$$\begin{aligned} T_1(z) &= \int_{R_1}^{R_2} \frac{r_0 dr_0}{z-d-e^{i\theta_0}} \\ &= e^{-2i\theta_0} \left[ -(R_2-R_1)e^{i\theta_0} - (z-d) \ln \left( \frac{z-d-R_2e^{i\theta_0}}{z-d-R_1e^{i\theta_0}} \right) \right], \end{aligned} \quad (2)$$

$$\begin{aligned} T_2(z) &= \int_{R_1}^{R_2} \frac{r_0 dr_0}{z+d+r_0e^{i\theta_0}} \\ &= e^{-2i\theta_0} \left[ (R_2-R_1)e^{i\theta_0} - (z+d) \ln \left( \frac{z+d+R_2e^{i\theta_0}}{z+d+R_1e^{i\theta_0}} \right) \right], \end{aligned} \quad (3)$$

$$\begin{aligned} T_3(z) &= \int_{R_1}^{R_2} \frac{r_0 dr_0}{z-d-e^{-i\theta_0}} \\ &= e^{2i\theta_0} \left[ -(R_2-R_1)e^{-i\theta_0} - (z-d) \ln \left( \frac{z-d-R_2e^{-i\theta_0}}{z-d-R_1e^{-i\theta_0}} \right) \right], \end{aligned} \quad (4)$$

$$\begin{aligned} T_4(z) &= \int_{R_1}^{R_2} \frac{r_0 dr_0}{z+d+r_0e^{-i\theta_0}} \\ &= e^{2i\theta_0} \left[ (R_2-R_1)e^{-i\theta_0} - (z+d) \ln \left( \frac{z+d+R_2e^{-i\theta_0}}{z+d+R_1e^{-i\theta_0}} \right) \right], \end{aligned} \quad (5)$$

$$\begin{aligned}
 S_1(z) &= \int_{R_1}^{R_2} \frac{d+r_0 e^{-i\theta_0}}{z(d+r_0 e^{-i\theta_0})-R_1^2} \cdot r_0 dr_0 \\
 &= \frac{1}{2} \cdot \frac{R_2^2-R_1^2}{z} + (R_2-R_1) \cdot \frac{R_1^2}{z^2} e^{i\theta_0} \\
 &\quad + \frac{e^{2i\theta_0}}{z^3} \cdot R_1^2(R_1^2-zd) \ln \left[ \frac{z(d+R_2 e^{-i\theta_0})-R_1^2}{z(d+R_1 e^{-i\theta_0})-R_1^2} \right], \tag{6}
 \end{aligned}$$

$$\begin{aligned}
 S_2(z) &= \int_{R_1}^{R_2} \frac{d+r_0 e^{-i\theta_0}}{R_1 z(d+r_0 e^{-i\theta_0})+R_1^2} \cdot r_0 dr_0 \\
 &= \frac{1}{2} \cdot \frac{R_2^2-R_1^2}{z} - (R_2-R_1) \cdot \frac{R_1^2}{z^2} e^{i\theta_0} \\
 &\quad + \frac{e^{2i\theta_0}}{z^3} \cdot R_1^2(R_1^2+zd) \ln \left[ \frac{z(d+R_2 e^{-i\theta_0})+R_1^2}{z(d+R_1 e^{-i\theta_0})+R_1^2} \right], \tag{7}
 \end{aligned}$$

$$\begin{aligned}
 S_3(z) &= \int_{R_1}^{R_2} \frac{d+r_0 e^{i\theta_0}}{z(d+r_0 e^{i\theta_0})-R_1^2} \cdot r_0 dr_0 \\
 &= \frac{1}{2} \cdot \frac{R_2^2-R_1^2}{z} + (R_2-R_1) \cdot \frac{R_1^2}{z^2} e^{-i\theta_0} \\
 &\quad + \frac{e^{-2i\theta_0}}{z^3} \cdot R_1^2(R_1^2-zd) \ln \left[ \frac{z(d+R_2 e^{i\theta_0})-R_1^2}{z(d+R_1 e^{i\theta_0})-R_1^2} \right], \tag{8}
 \end{aligned}$$

$$S_4(z) = \int_{R_1}^{R_2} \frac{d+r_0 e^{i\theta_0}}{z(d+r_0 e^{i\theta_0})+R_1^2} \cdot r_0 dr_0$$

$$\begin{aligned}
&= \frac{1}{2} \cdot \frac{R_2^2 - R_1^2}{z} - (R_2 - R_1) \cdot \frac{R_1^2}{z^2} e^{-i\theta_0} \\
&+ \frac{e^{-2i\theta_0}}{z^3} \cdot R_1^2 (R_1^2 + zd) \ln \left[ \frac{z(d + R_2 e^{i\theta_0}) + R_1^2}{z(d + R_1 e^{i\theta_0}) + R_1^2} \right] \quad (9)
\end{aligned}$$

The resultant field from one conductor suitably located in each quadrant to give a dipole field is

$$H^*(z) = -2i \int_{\theta_1}^{\theta_2} J(\theta_0) R(z, \theta_0) d\theta_0, \quad (10)$$

where

$$R(z, \theta_0) = T_1 - T_2 + T_3 - T_4 + S_1 - S_2 + S_3 - S_4. \quad (11)$$

Since  $z$  occurs in the denominator of the  $S$ -expressions one must evaluate these carefully for  $z = 0$ . Thus

$$\begin{aligned}
R(0, \theta_0) &= -4(R_2 - R_1) \cos \theta_0 + 2de^{-2i\theta_0} \ln \left( \frac{d + R_2 e^{i\theta_0}}{d + R_1 e^{i\theta_0}} \right) \\
&+ 2de^{2i\theta_0} \ln \left( \frac{d + R_2 e^{-i\theta_0}}{d + R_1 e^{-i\theta_0}} \right) \\
&- 2 \frac{d(R_2^2 - R_1^2)}{R_1^2} - \frac{4}{3} \frac{R_2^3 - R_1^3}{R_1^2} \cos \theta_0. \quad (12)
\end{aligned}$$

For the case of constant current density it would be possible to carry out the  $\theta_0$ -integration analytically. However, this is not done since a numerical integration technique must be developed for the variable case. Thus Eq. (10) is the final result for a single

conductor in the first quadrant. Summation of similar results for other conductors located in the first quadrant gives the general result.

### Longitudinally Integrated Magnetic Field

If a current distribution is localized in space and  $w$  is an axial coordinate orthogonal to the transverse coordinates  $x$  and  $y$ , Lambertson and others<sup>1</sup> have shown that by integrating the field equations along the  $w$ -axis from  $-\infty$  to  $+\infty$

$$\frac{\partial}{\partial x} \int H_y dw - \frac{\partial}{\partial y} \int H_x dw = 4\pi \int J_w dw , \quad (13)$$

and

$$\frac{\partial}{\partial x} \int H_x dw + \frac{\partial}{\partial y} \int H_y dw = 0. \quad (14)$$

These equations may be put into the complex notation for ease of calculation by defining

$$G_x = \int_{-\infty}^{\infty} H_x dw , \quad (15)$$

$$G_y = \int_{-\infty}^{\infty} H_y dw , \quad (16)$$

and

$$K = \int_{-\infty}^{\infty} J_w dw . \quad (17)$$

Then, Eqs. (13-14) become<sup>2</sup> using  $G = G_x + iG_y$ .

$$\frac{\partial G^*}{\partial z} = -2\pi i K \quad (18)$$

or, in integral form for the dipole symmetry considered here  
( $z$  is outside the current)

$$G^* = -2i \iint K \left\{ \frac{1}{z-z_o-d} - \frac{1}{z+z_o+d} + \frac{1}{z-z_o^*-d} - \frac{1}{z+z_o^*+d} \right. \\ \left. + \frac{1}{\frac{R_1^2}{z-\frac{1}{z_o^*+d}}} - \frac{1}{\frac{R_1^2}{z+\frac{1}{z_o^*+d}}} + \frac{1}{\frac{R_1^2}{z-\frac{1}{z_o+d}}} - \frac{1}{\frac{R_1^2}{z+\frac{1}{z_o+d}}} \right\} r_o dr_o d\theta_o \quad (19)$$

By comparing this with Eq. (5) one sees that the longitudinally integrated fields are distributed in the transverse coordinates as in a two-dimensional problem. The integrated fields replace the actual field and the integrated current density replaces the normal current density.

#### Magnet End Development

The conductor placement at the ends of the magnet is considered to be developed<sup>3</sup> from a flat curve in which connection between the two parallel sides is made by means of a semicircular arc. The details of the developed surface are presented in Fig. 2a. After the layer of conductors has been unwrapped, the parallel sides are described by

$$a = \frac{1}{2}(R_1+R_2)\left(\frac{\pi}{2} - \theta_o\right), \quad (20)$$

$$s = \frac{1}{2}(R_1+R_2)\left(\frac{\pi}{2} - \theta\right), \quad (21)$$

$$b = \frac{1}{2}(R_1+R_2)\left(\frac{\pi}{2} - \theta_1\right). \quad (22)$$

By assumption the unwrapped lager has a round end (Fig. 2b).  
Hence the current density in the axial direction is

$$J_w = J_o \cos \phi = J_o \frac{t}{\sqrt{w^2 + t^2}} . \quad (23)$$

Integrating this axially gives

$$K = \int J_w dw = J_o t \int_{\alpha}^{\beta} \frac{dw}{\sqrt{w^2 + t^2}} , \quad (24)$$

where

$$\alpha = \begin{cases} \sqrt{g^2 - t^2} & \text{for } t < g \\ 0 & \text{for } t > g \end{cases} , \quad (25)$$

and

$$\beta = \sqrt{h^2 - t^2} . \quad (26)$$

Hence the contribution of a single end to the integrated current is

$$K(\theta) = J_o t \left\{ \begin{array}{l} \ln \left( \frac{\sqrt{h^2 - t^2} + h}{\sqrt{g^2 - t^2} + g} \right) \\ \ln \left( \frac{\sqrt{h^2 - t^2} + h}{t} \right) \end{array} \right\} . \quad (27)$$

Note further, that any transverse straight section in the s-direction may be added without affecting Eq. (27) other than through the choice of origin of s.

### Application

Many factors are involved in arriving at a suitable disposition of conductors. In general one desires the integrated fields to be as uniform as possible with a tolerance level in  $\Delta B/B$  definitely below 0.1% in the useful aperture. The total amount of superconductor used to achieve the desired central field must be kept to a minimum. This is accomplished in these designs by using two different sizes of superconductor, the large cross section being used in the highest field regions and the smaller cross section in the lower field regions. The criterion for the wire choice in each region is that some fraction of the critical current density must not be exceeded. Another general consideration in line with the critical current requirement stems from noting that the magnetic field at the conductors in the end region tends to be higher than the central field. This can be modified by choosing different axial lengths for each layer, by terminating the iron shielding short of the ends, and by "dogboning" the turns with the highest curvature.

There is no general method of optimizing the quality of the integrated field that incorporates all of these considerations. We chose to proceed as follows. Technical Services<sup>4</sup>, using a multipole expansion equivalent of the two-dimensional calculation reported here carried out the initial exploration of conductor locations in the body of the magnet and produced high field quality choices consistent with allowing fluid passages and a return bus conductor. These conductor locations were then inserted into LINDA, a magnetostatic iterative program, to determine the dividing line between the use of the two conductor types.



Technical Services again produced a high quality field utilizing the restriction introduced by LINDA. Practical magnet ends were then developed by Technical Services and modified after LINDA runs of the end configurations suggested a reasonable iron termination that would give no field rise over the central field. The net configuration was examined for field quality of the integrated fields. The difference between this quality and the previous high quality two dimensional adjustments were then used by Technical Services to obtain a field variation that could just be compensated by the application of magnet ends. This final configuration was tested by obtaining the integrated fields. All the results presented here follow from the above mentioned procedure.

### Results

Computer output records all the necessary input and giving median plane variation of the coil contribution, image or iron contribution, and resulting field. Two cases are considered for the doubler, Mark I and Mark II. For each case the distribution of the median plane field within the magnet body (Calculational Mode = 0) and the distribution of the median plane integrated field (Calculated Mode = 1) is given. The results for Mark II are definitely superior to those of Mark I and it is expected that further fine adjustments will provide even higher quality fields.

Similar results are given for the Mark I version of the large aperture muon beam line dipole.

References

1. Robert B. Meuser, "End Effect in Superconducting Beam Transport Magnets", IEEE Trans. Nucl. Sci. NS-18, 677 (1971). Integral theorem is attributed to G.R. Lambertson.
2. W.W. Lee and S.C. Snowdon, "Design of Dipole Magnet with Circular Iron Shield", IEEE Trans. Nucl. Sci. NS-20, 726 (1973).
3. M.A. Green, "The Elimination of High Multipoles in the Two Dimensional and Integrated Fields of Conductor-Dominated Dipole and Quadrupole Magnets with Iron Sheets", Lawrence Berkeley Laboratory UCID-3493 (Jan. 15, 1971).
4. Work carried out by G. Biallas.

Figure 1. Legend

O = Center

F = Right Hand Origin

F' = Left Hand Origin

$z_1$  = Image Position of  $z_0$

$R_1$  = Inner Radius of Wedge Conductor (F- $R_1$ )

$R_2$  = Outer Radius of Wedge Conductor (F- $R_2$ )

$R_i$  = Inner Radius of Iron (O-P)

d = Offset (F'-O or O-F)

$$H^* = 2iI \left( \frac{1}{z-z_0} + \frac{1}{z-z_1} \right)$$

$$z_0 = \begin{cases} d+r_0 e^{i\theta_0} & \text{Right Hand Wedge} \\ -d+r_0 e^{i\theta_0} & \text{Left Hand Wedge} \end{cases}$$

$$z_1 = R_i^2 / z_0^*$$

$$I = \int r_0 dr_0 d\theta_0$$

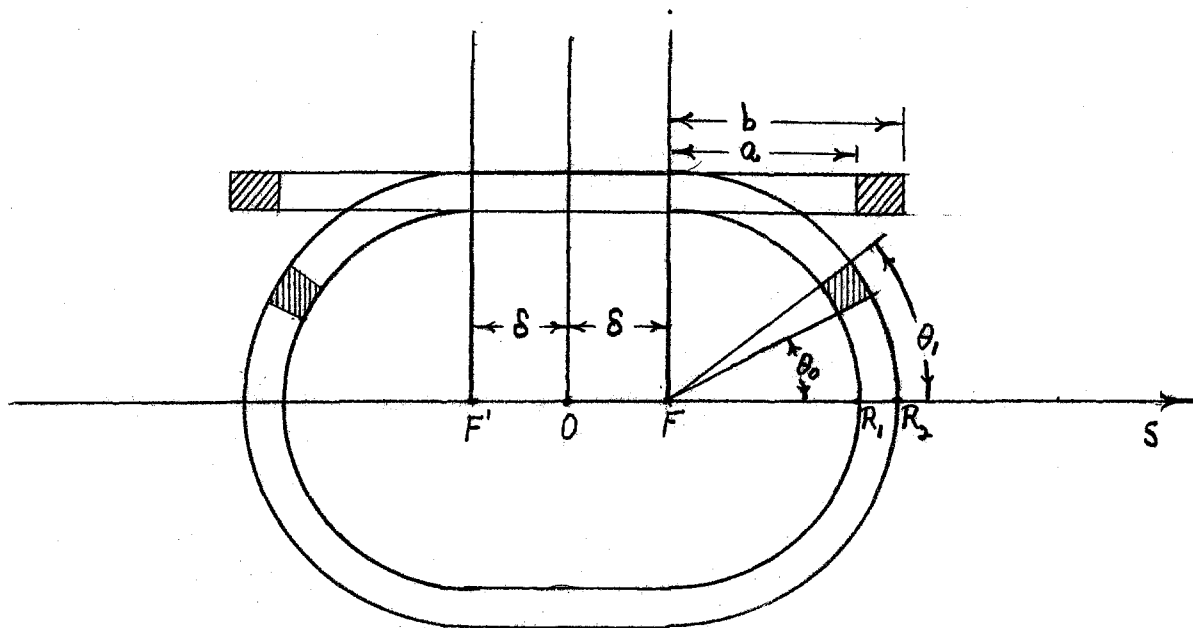


Figure 2a. Unwrapping of Conductor (See Legend)

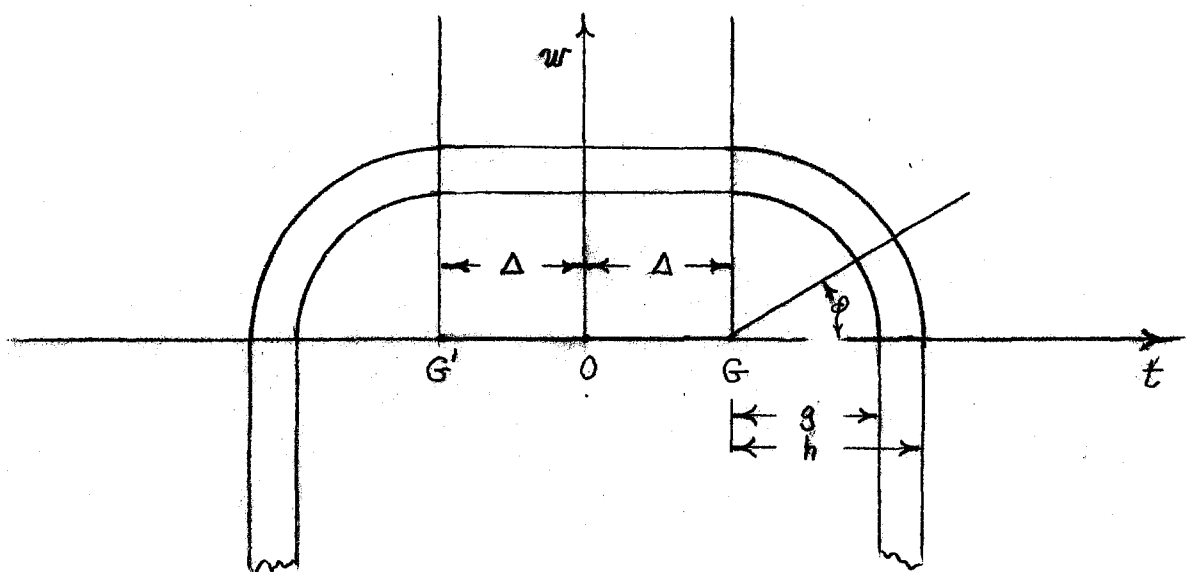


Figure 2b. Plan View of Unwrapped Conductor  
(See Legend)

# DIRECT INTEGRATION OF SHELL MODEL DOUBLER MAGNET Mark I

|                             |   |           |                          |   |        |                       |          |
|-----------------------------|---|-----------|--------------------------|---|--------|-----------------------|----------|
| CALCULATIONAL MODE          | = | 0         | NUMBER OF LAYERS         | = | 8      |                       |          |
| CONDUCTOR CURRENT(A)        | = | 2815.0000 | REFERENCE RADIUS(IN)     | = | 1.0000 | INNER IRON RADIUS(IN) | = 3.0000 |
| SIMPSONS RULE INTERVAL(DEG) | = | 1.0000    | HORIZONTAL INCREMENT(IN) | = | .1000  | HORIZONTAL OFFSET(IN) | = .3750  |
| FLAT BEND CENTER(IN)        | = | 0.0000    | INSULATION THICKNESS(IN) | = | .0015  |                       |          |

| LAYER | TURNS   | CURDEN<br>(KA/IN/IN) | THETAS<br>(DEG) | THETAF<br>(DEG) | SPACER<br>(IN) | RINNER<br>(IN) | ROUTER<br>(IN) | LENGTH<br>(IN) |
|-------|---------|----------------------|-----------------|-----------------|----------------|----------------|----------------|----------------|
| 1     | 18.0000 | 268.0200             | 0.0000          | 76.7019         | 0.0000         | .8750          | 1.0326         | 1.0000         |
| 2     | 22.0000 | 408.2161             | 0.0000          | 51.5719         | 0.0000         | 1.0954         | 1.2516         | 1.0000         |
| 3     | 16.0000 | 407.4277             | 0.0000          | 33.2774         | 0.0000         | 1.2530         | 1.4085         | 1.0000         |
| 4     | 4.0000  | 267.1165             | 53.0960         | 67.1163         | 0.0000         | 1.0954         | 1.2516         | 1.0000         |
| 5     | 2.0000  | 268.1132             | 82.4773         | 90.9972         | 0.0000         | .8750          | 1.0326         | 1.0000         |
| 6     | 3.0000  | 267.1543             | 68.9841         | 79.4979         | 0.0000         | 1.0954         | 1.2516         | 1.0000         |
| 7     | 1.0000  | 268.0644             | 91.1576         | 95.4032         | 0.0000         | .8750          | 1.0326         | 1.0000         |
| 8     | 3.0000  | 407.4216             | 41.5394         | 47.7790         | 0.0000         | 1.2530         | 1.4085         | 1.0000         |

| X(IN)   | BT(KG-IN) | BA(KG-IN) | BS(KG-IN) | BN      |
|---------|-----------|-----------|-----------|---------|
| 0.00000 | -44.42070 | -35.32326 | -8.59144  | 1.00000 |
| .10000  | -44.41655 | -35.32419 | -8.59236  | .99991  |
| .20000  | -44.40436 | -35.30924 | -8.59511  | .99963  |
| .30000  | -44.38493 | -35.28522 | -8.59971  | .99919  |
| .40000  | -44.35962 | -35.25348 | -8.60614  | .99862  |
| .50000  | -44.33018 | -35.21576 | -8.61442  | .99796  |
| .60000  | -44.29649 | -35.17395 | -8.62455  | .99725  |
| .70000  | -44.26019 | -35.12966 | -8.63654  | .99652  |
| .80000  | -44.22128 | -35.08390 | -8.65039  | .99580  |
| .90000  | -44.18064 | -35.03652 | -8.66612  | .99509  |
| 1.00000 | -44.13931 | -35.48558 | -8.68373  | .99434  |
| 1.10000 | -44.12059 | -35.42635 | -8.70324  | .99345  |
| 1.20000 | -44.07445 | -35.34979 | -8.72466  | .99221  |
| 1.30000 | -37.98874 | -29.24074 | -8.74800  | .85520  |

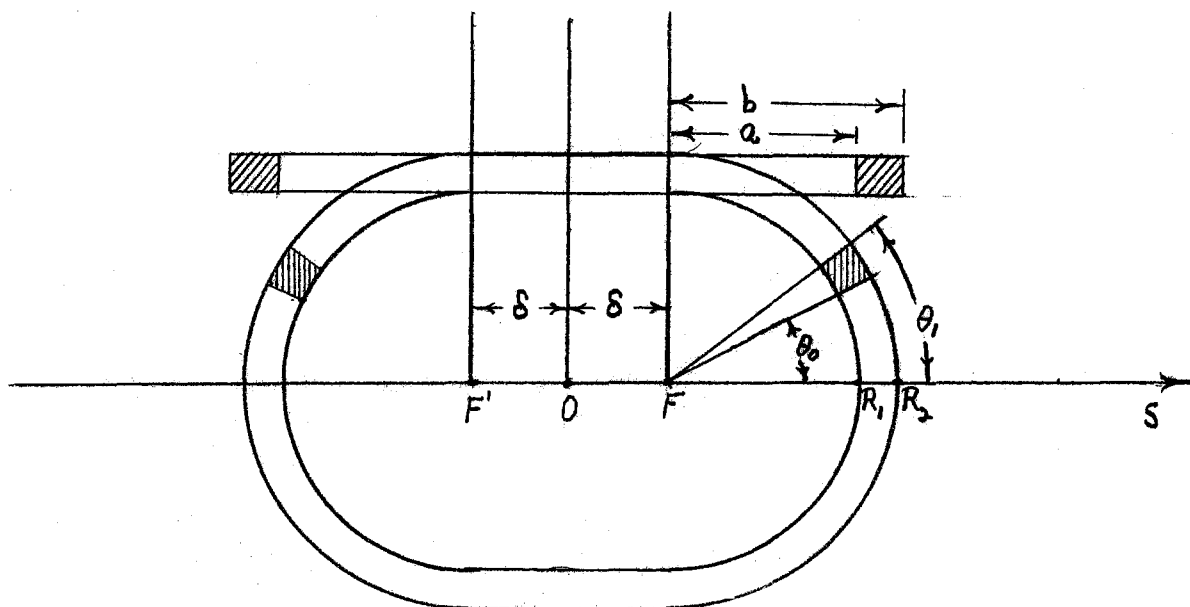


Figure 2a. Unwrapping of Conductor (See Legend)

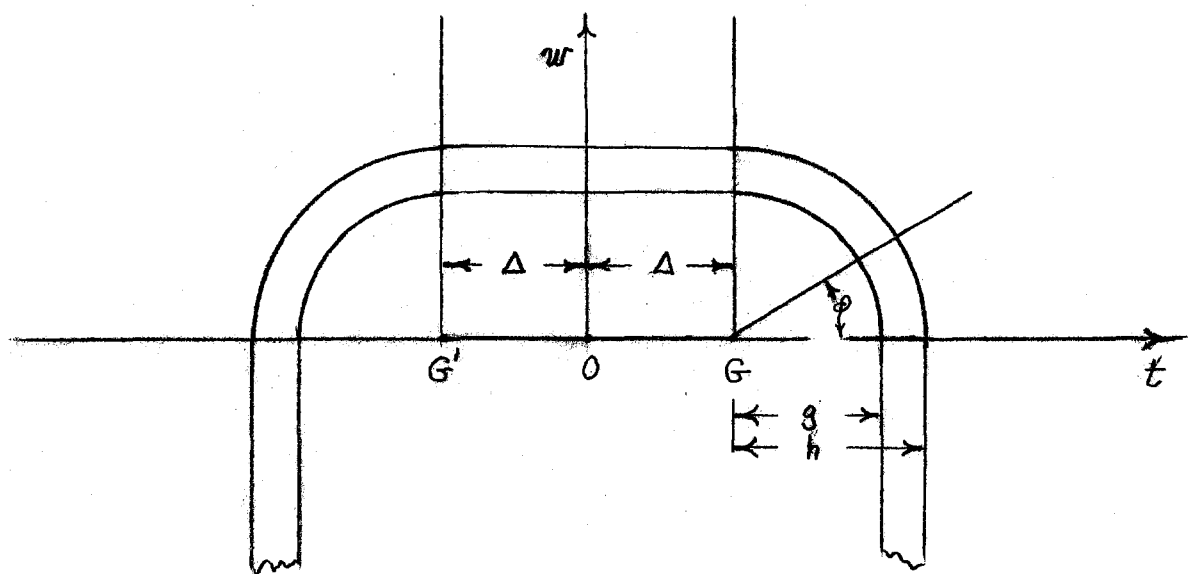


Figure 2b. Plan View of Unwrapped Conductor  
(See Legend)

# DIRECT INTEGRATION OF SHELL MODEL DOUBLER MAGNET Mark 1

|                             |   |           |                          |   |        |                       |          |
|-----------------------------|---|-----------|--------------------------|---|--------|-----------------------|----------|
| CALCULATIONAL MODE          | = | 0         | NUMBER OF LAYERS         | = | 8      |                       |          |
| CONDUCTOR CURRENT(A)        | = | 2815.0000 | REFERENCE RADIUS(IN)     | = | 1.0000 | INNER IRON RADIUS(IN) | = 3.0000 |
| SIMPSONS RULE INTERVAL(DEG) | = | 1.0000    | HORIZONTAL INCREMENT(IN) | = | .1000  | HORIZONTAL OFFSET(IN) | = .3750  |
| FLAT BEND CENTER(IN)        | = | 0.0000    | INSULATION THICKNESS(IN) | = | .0015  |                       |          |

| LAYER | TURNS   | CURDEN<br>(KA/IN/IN) | THETAS<br>(DEG) | THETAF<br>(DEG) | SPAGER<br>(IN) | RINNER<br>(IN) | ROUTER<br>(IN) | LENGTH<br>(IN) |
|-------|---------|----------------------|-----------------|-----------------|----------------|----------------|----------------|----------------|
| 1     | 18.0000 | 268.0200             | 0.0000          | 76.7019         | 0.0000         | .8750          | 1.0326         | 1.0000         |
| 2     | 22.0000 | 408.2161             | 0.0000          | 51.5719         | 0.0000         | 1.0954         | 1.2516         | 1.0000         |
| 3     | 16.0000 | 407.4277             | 0.0000          | 33.2774         | 0.0000         | 1.2530         | 1.4085         | 1.0000         |
| 4     | 4.0000  | 267.1165             | 53.0960         | 67.1163         | 0.0000         | 1.0954         | 1.2516         | 1.0000         |
| 5     | 2.0000  | 268.1132             | 82.4773         | 90.9972         | 0.0000         | .8750          | 1.0326         | 1.0000         |
| 6     | 3.0000  | 267.1543             | 64.9441         | 74.4979         | 0.0000         | 1.0954         | 1.2516         | 1.0000         |
| 7     | 1.0000  | 268.0644             | 91.1576         | 95.4032         | 0.0000         | .8750          | 1.0365         | 1.0000         |
| 8     | 3.0000  | 407.4216             | 41.5394         | 47.7790         | 0.0000         | 1.2530         | 1.4085         | 1.0000         |

| X(IN)   | BT(KG-IN) | BA(KG-IN) | BS(KG-IN) | BN      |
|---------|-----------|-----------|-----------|---------|
| 0.00000 | -44.42070 | -35.32926 | -8.59144  | 1.00000 |
| .10000  | -44.41655 | -35.32419 | -8.59236  | .99991  |
| .20000  | -44.40436 | -35.30924 | -8.59351  | .99963  |
| .30000  | -44.38493 | -35.28522 | -8.59971  | .99919  |
| .40000  | -44.35962 | -35.25344 | -8.60614  | .99862  |
| .50000  | -44.33019 | -35.21576 | -8.61442  | .99796  |
| .60000  | -44.29649 | -35.17395 | -8.62455  | .99725  |
| .70000  | -44.26019 | -35.12966 | -8.63654  | .99652  |
| .80000  | -44.23428 | -35.08390 | -8.65039  | .99580  |
| .90000  | -44.20264 | -35.03652 | -8.66612  | .99509  |
| 1.00000 | -44.16431 | -35.08558 | -8.68373  | .99434  |
| 1.10000 | -44.12059 | -35.42635 | -8.70324  | .99345  |
| 1.20000 | -44.07445 | -35.34979 | -8.72466  | .99221  |
| 1.30000 | -37.98374 | -29.24974 | -8.74800  | .85520  |

DIRECT INTEGRATION OF SHELL MODEL DOUBLER MAGNET Mark II

|                             |   |           |                          |   |        |                       |          |
|-----------------------------|---|-----------|--------------------------|---|--------|-----------------------|----------|
| CALCULATIONAL MODE          | = | 0         | NUMBER OF LAYERS         | = | 8      |                       |          |
| CONDUCTOR CURRENT(A)        | = | 2815.0000 | REFERENCE RADIUS(IN)     | = | 1.0000 | INNER IRON RADIUS(IN) | = 3.0625 |
| SIMPSONS RULE INTERVAL(DEG) | = | 1.0000    | HORIZONTAL INCREMENT(IN) | = | .1000  | HORIZONTAL OFFSET(IN) | = .3750  |
| FLAT BEND CENTER(IN)        | = | 0.0000    | INSULATION THICKNESS(IN) | = | .0015  |                       |          |

| LAYER | TURNS   | CURDEN<br>(KA/IN/IN) | THETAS<br>(DEG) | THETAF<br>(DEG) | SPACER<br>(IN) | RINNER<br>(IN) | ROUTER<br>(IN) | LENGTH<br>(IN) |
|-------|---------|----------------------|-----------------|-----------------|----------------|----------------|----------------|----------------|
| 1     | 16.0000 | 267.4469             | .1582           | 68.4856         | 0.0000         | .8750          | 1.0326         | 1.0000         |
| 2     | 3.0000  | 267.4161             | 77.3493         | 90.1606         | 0.0000         | .8750          | 1.0326         | 1.0000         |
| 3     | 1.0000  | 267.5583             | 94.5380         | 98.7493         | 0.0000         | .8892          | 1.0466         | 1.0000         |
| 4     | 22.0000 | 407.5915             | .1318           | 51.7954         | 0.0000         | 1.0950         | 1.2512         | 1.0000         |
| 5     | 4.0000  | 266.7569             | 53.0960         | 67.1392         | 0.0000         | 1.0950         | 1.2512         | 1.0000         |
| 6     | 3.0000  | 266.7939             | 75.6304         | 86.1614         | 0.0000         | 1.0950         | 1.2512         | 1.0000         |
| 7     | 16.0000 | 406.9258             | .1146           | 33.3805         | 0.0000         | 1.2550         | 1.4105         | 1.0000         |
| 8     | 5.0000  | 407.0132             | 39.3049         | 49.6984         | 0.0000         | 1.2550         | 1.4105         | 1.0000         |

| X(IN)   | BT(KG-IN) | BA(KG-IN) | BS(KG-IN) | BN      |
|---------|-----------|-----------|-----------|---------|
| 0.00000 | -44.37571 | -35.99964 | -8.37607  | 1.00000 |
| .10000  | -44.37435 | -35.99747 | -8.37688  | .99997  |
| .20000  | -44.37389 | -35.99160 | -8.37929  | .99989  |
| .30000  | -44.36704 | -35.98373 | -8.38331  | .99980  |
| .40000  | -44.36477 | -35.97583 | -8.38895  | .99975  |
| .50000  | -44.36515 | -35.96896 | -8.39619  | .99976  |
| .60000  | -44.36704 | -35.96199 | -8.40505  | .99980  |
| .70000  | -44.36624 | -35.95071 | -8.41553  | .99979  |
| .80000  | -44.35551 | -35.92789 | -8.42762  | .99954  |
| .90000  | -44.32518 | -35.88385 | -8.44133  | .99886  |
| 1.00000 | -44.26335 | -35.80667 | -8.45667  | .99747  |
| 1.10000 | -44.15287 | -35.67923 | -8.47364  | .99498  |
| 1.20000 | -43.94330 | -35.45106 | -8.49224  | .99026  |
| 1.30000 | -37.80176 | -29.28929 | -8.51247  | .85186  |



# DIRECT INTEGRATION OF SHELL MODEL DOUBLER MAGNET Mark II

|                             |   |           |                          |   |        |                       |          |
|-----------------------------|---|-----------|--------------------------|---|--------|-----------------------|----------|
| CALCULATIONAL MODE          | = | 1         | NUMBER OF LAYERS         | = | 8      |                       |          |
| CONDUCTOR CURRENT(A)        | = | 2815.0000 | REFERENCE RADIUS(IN)     | = | 1.0000 | INNER IRON RADIUS(IN) | = 3.0625 |
| SIMPSONS RULE INTERVAL(DEG) | = | 1.0000    | HORIZONTAL INCREMENT(IN) | = | .1000  | HORIZONTAL OFFSET(IN) | = .3750  |
| FLAT BEND CENTER(IN)        | = | 0.0000    | INSULATION THICKNESS(IN) | = | .0015  |                       |          |

| LAYER | TURNS   | CURDEN<br>(KA/IN/IN) | THETAS<br>(DEG) | THETAF<br>(DEG) | SPACER<br>(IN) | RINNER<br>(IN) | ROUTER<br>(IN) | LENGTH<br>(IN) |
|-------|---------|----------------------|-----------------|-----------------|----------------|----------------|----------------|----------------|
| 1     | 16.0000 | 267.4469             | .1662           | 68.4856         | 0.0000         | .8750          | 1.0326         | 239.5460       |
| 2     | 3.0000  | 267.4161             | 77.3493         | 90.1606         | 0.0000         | .8750          | 1.0326         | 239.5460       |
| 3     | 1.0000  | 267.5583             | 94.5380         | 98.7493         | 0.0000         | .8892          | 1.0466         | 239.5460       |
| 4     | 22.0000 | 407.5915             | .1318           | 51.7954         | 0.0000         | 1.0950         | 1.2512         | 241.1230       |
| 5     | 4.0000  | 266.7569             | 53.0960         | 67.1392         | 0.0000         | 1.0950         | 1.2512         | 241.1230       |
| 6     | 3.0000  | 266.7939             | 75.6304         | 86.1614         | 0.0000         | 1.0950         | 1.2512         | 241.1230       |
| 7     | 16.0000 | 406.9258             | .1146           | 33.3805         | 0.0000         | 1.2550         | 1.4105         | 240.8060       |
| 8     | 5.0000  | 407.0132             | 33.3049         | 49.6984         | 0.0000         | 1.2550         | 1.4105         | 240.8060       |

| X(IN)   | BT(KG-IN)    | BA(KG-IN)   | BS(KG-IN)   | BN      |
|---------|--------------|-------------|-------------|---------|
| 0.00000 | -10798.72067 | -8759.60174 | -2039.11894 | 1.00000 |
| .10000  | -10799.26402 | -8758.95109 | -2039.31293 | .99996  |
| .20000  | -10797.04788 | -8757.15291 | -2039.89497 | .99985  |
| .30000  | -10795.47259 | -8754.60740 | -2040.96519 | .99970  |
| .40000  | -10794.00004 | -8751.77619 | -2042.22384 | .99956  |
| .50000  | -10792.86361 | -8748.89235 | -2043.37125 | .99946  |
| .60000  | -10791.75391 | -8745.64607 | -2046.10784 | .99935  |
| .70000  | -10789.61133 | -8740.97724 | -2048.63409 | .99916  |
| .80000  | -10784.61675 | -8733.06613 | -2051.55057 | .99869  |
| .90000  | -10774.32625 | -8719.46837 | -2054.15789 | .99774  |
| 1.00000 | -10755.69237 | -8697.13569 | -2058.55668 | .99602  |
| 1.10000 | -10724.26145 | -8661.61382 | -2062.64763 | .99310  |
| 1.20000 | -10667.14596 | -8600.01455 | -2067.13141 | .98782  |
| 1.30000 | -9131.96758  | -7109.95890 | -2072.00868 | .85028  |

# DIRECT INTEGRATION OF SHELL MODEL MUON MAGNET MARK I

|                             |   |           |                          |   |        |                       |          |
|-----------------------------|---|-----------|--------------------------|---|--------|-----------------------|----------|
| CALCULATIONAL MODE          | = | 1         | NUMBER OF LAYERS         | = | 5      |                       |          |
| CONDUCTOR CURRENT(A)        | = | 2380.0000 | REFERENCE RADIUS(IN)     | = | 2.2500 | INNER IRON RADIUS(IN) | = 5.7500 |
| SIMPSONS RULE INTERVAL(DEG) | = | 1.0000    | HORIZONTAL INCREMENT(IN) | = | .1000  | HORIZONTAL OFFSET(IN) | = 1.0000 |
| FLAT BEND CENTER(IN)        | = | 0.0000    | INSULATION THICKNESS(IN) | = | .0015  |                       |          |

| LAYER | TURNS   | CURDEN<br>(KA/IN/IN) | THETAS<br>(DEG) | THETA <sup>2</sup><br>(DEG) | SPACER<br>(IN) | RINNER<br>(IN) | ROUTER<br>(IN) | LENGTH<br>(IN) |
|-------|---------|----------------------|-----------------|-----------------------------|----------------|----------------|----------------|----------------|
| 1     | 48.0000 | 224.5244             | .0630           | 87.3895                     | 0.0000         | 2.2500         | 2.4331         | 120.0000       |
| 2     | 1.0000  | 224.6759             | .0645           | 95.2199                     | 0.0000         | 2.2500         | 2.4121         | 120.0000       |
| 3     | 50.0000 | 342.5520             | .0573           | 55.6858                     | 0.0000         | 2.4650         | 2.6179         | 122.9980       |
| 4     | 16.0000 | 224.3136             | .0568           | 82.2997                     | 0.0000         | 2.4650         | 2.6179         | 124.4060       |
| 5     | 39.0000 | 342.5236             | .0516           | 39.9237                     | 0.0000         | 2.6220         | 2.7747         | 122.9980       |

| X(IN)   | BT(KG-IN)   | BA(KG-IN)   | BS(KG-IN)   | BN      |
|---------|-------------|-------------|-------------|---------|
| 0.00000 | -5078.21509 | -3787.52242 | -1290.59458 | 1.00000 |
| .10000  | -5078.27463 | -3787.53130 | -1290.54313 | 1.00001 |
| .20000  | -5079.44350 | -3787.55474 | -1290.78876 | 1.00004 |
| .30000  | -5079.71484 | -3787.58331 | -1291.93153 | 1.00010 |
| .40000  | -5079.07350 | -3787.70203 | -1291.37148 | 1.00017 |
| .50000  | -5079.50051 | -3787.69133 | -1291.30869 | 1.00025 |
| .60000  | -5079.97167 | -3787.62843 | -1292.34324 | 1.00035 |
| .70000  | -5080.46402 | -3787.48974 | -1292.37528 | 1.00044 |
| .80000  | -5080.95239 | -3787.24747 | -1293.70492 | 1.00054 |
| .90000  | -5081.41335 | -3786.18102 | -1294.53232 | 1.00063 |
| 1.00000 | -5081.82546 | -3786.36818 | -1295.45768 | 1.00071 |
| 1.10000 | -5082.17207 | -3785.69090 | -1296.48118 | 1.00078 |
| 1.20000 | -5082.43752 | -3784.33448 | -1297.50304 | 1.00083 |
| 1.30000 | -5082.61068 | -3783.78719 | -1298.32350 | 1.00087 |
| 1.40000 | -5082.68179 | -3782.53899 | -1300.14281 | 1.00088 |
| 1.50000 | -5082.64093 | -3781.17959 | -1301.56125 | 1.00097 |
| 1.60000 | -5082.47477 | -3779.39567 | -1303.97910 | 1.00084 |
| 1.70000 | -5082.16410 | -3777.46742 | -1304.59668 | 1.00079 |
| 1.80000 | -5081.67371 | -3775.26441 | -1306.41430 | 1.00068 |
| 1.90000 | -5080.97334 | -3772.74105 | -1308.23229 | 1.00054 |
| 2.00000 | -5079.98269 | -3769.33163 | -1310.15101 | 1.00035 |
| 2.10000 | -5079.61650 | -3766.44569 | -1312.17081 | 1.00008 |
| 2.20000 | -5078.75486 | -3762.46260 | -1314.29206 | .99971  |
| 2.30000 | -5077.24267 | -3757.72753 | -1316.51514 | .99922  |
| 2.40000 | -5075.88729 | -3752.04687 | -1318.84042 | .99856  |
| 2.50000 | -5065.45222 | -3745.18392 | -1321.26830 | .99768  |
| 2.60000 | -5060.65269 | -3736.95353 | -1323.79915 | .99654  |
| 2.70000 | -5053.14626 | -3726.71287 | -1326.43339 | .99506  |
| 2.80000 | -5043.51326 | -3714.34189 | -1329.17137 | .99317  |
| 2.90000 | -5031.20918 | -3599.19570 | -1332.01348 | .99074  |
| 3.00000 | -5015.43274 | -3580.47266 | -1334.96009 | .98764  |
| 3.10000 | -4994.66195 | -3556.65032 | -1338.01154 | .98355  |
| 3.20000 | -4963.51933 | -3522.35117 | -1341.16815 | .97741  |
| 3.30000 | -4937.58157 | -2973.15129 | -1344.43028 | .85022  |