national accelerator laboratory

TM-483
0428

FORCES IN DOUBLER TYPE SHEET CURRENT DIPOLES

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March 22, 1974

Summary

The force distribution on a current sheet that simulates the current density distribution in the energy doubler dipole is calculated. For a perfectly centered dipole the magnetic forces result in a compressive force vertically and an elongative force horizontally. Net forces and torques on displaced current distribution are also calculated. Results are given for the dual test dipole, the doubler dipole, and the muon beam line dipole.

Magnetic Field

In complex variable notation, the magnetic field is given by

$$H^* = -2ii_o\, a \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left[\frac{1}{z-\delta-\Delta-ae^{i\theta}} + \frac{1}{z-\frac{b^2}{\delta^*+\Delta+ae^{-i\theta}}} \right] \cos\theta d\theta$$

$$-2ii_o\, a \int_{\frac{\pi}{2}}^{-\frac{\pi}{2}} \left[\frac{1}{z-\delta+\Delta-ae^{i\theta}} + \frac{1}{z-\frac{b^2}{\delta^*-\Delta+ae^{-i\theta}}} \right] \cos\theta d\theta \qquad (1)$$

where it is assumed that the sheet current density is $i_o\cos\theta$. Figure 1 should be consulted for geometrical quantities. If one designates $s = e^{i\theta}$ then

$$H^* = -i_o a \int_{C_1} \left[\frac{1}{z-\delta-\Delta-as} + \frac{a+(\delta^*+\Delta)s}{az-[b^2-(\delta^*+\Delta)z]} \right] (1+\frac{1}{s^2})ds$$

$$-i_o a \int_{C_2} \left[\frac{1}{z-\delta+\Delta-as} + \frac{a+(\delta^*-\Delta)s}{az-[b^2-(\delta^*-\Delta)z]} \right] (1+\frac{1}{s^2})ds. \qquad (2)$$

Integration gives

$$H^* = -i_o \Bigg\{ - 2\pi i - \ln\left(\frac{ia-z+\delta+\Delta}{ia+z-\delta-\Delta} \cdot \frac{ia+z-\delta+\Delta}{ia-z+\delta-\Delta}\right)$$

$$- \frac{a}{z-\delta-\Delta} \left[- 2i + \frac{a}{z-\delta-\Delta} \ln\left(\frac{ia-z+\delta+\Delta}{ia+z-\delta-\Delta}\right) \right]$$

$$- \frac{a}{z-\delta+\Delta} \left[2i + \frac{a}{z-\delta+\Delta} \ln\left(\frac{ia+z-\delta+\Delta}{ia-z+\delta-\Delta}\right) \right]$$

$$- \frac{2ia(\delta^*+\Delta)}{b^2-(\delta^*+\Delta)z} + \frac{2ia(\delta^*-\Delta)}{b^2-(\delta^*-\Delta)z}$$

$$- \frac{a^2b^2}{[b^2-(\delta^*+\Delta)z]^2} \left[i\pi + \ln\left(\frac{i[b^2-(\delta^*+\Delta)z]-az}{i[b^2-(\delta^*+\Delta)z]+az}\right) \right]$$

$$- \frac{a^2b^2}{[b^2-(\delta^*-\Delta)z]^2} \left[i\pi + \ln\left(\frac{i[b^2-(\delta^*-\Delta)z]+az}{i[b^2-(\delta^*-\Delta)z]-az}\right) \right]$$

$$- \frac{b^2}{z^2} \cdot \ln\left(\frac{i[b^2-(\delta^*+\Delta)z]-az}{i[b^2-(\delta^*+\Delta)z]+az} + \frac{i[b^2-(\delta^*-\Delta)z]+az}{i[b^2-(\delta^*-\Delta)z]-az}\right) \Bigg\}$$

where the arguments of the logarithmic terms have been arranged so that the solution is appropriate to the inside region.

For z on the current sheet the field is discontinuous such that

$$H_t^+ - H_t^- = 4\pi i_o \cos\theta \quad , \quad H_n^+ = H_n^- \tag{4}$$

which yields

$$H^*(+) - H^*(-) = -4\pi i i_o \cos\theta \, e^{-i\theta}. \tag{5}$$

Hence the average field in the current sheet is

$$\langle H^* \rangle_{Av} = H^*(-) - i\pi i_o(1 + e^{-2i\theta}). \tag{6}$$

Force

If S_x and S_y designate stresses on a unit area of the current sheet, then

$$S^* = S_x - iS_y = - i\langle H^* \rangle_{Av} i_o \cos\theta. \tag{7}$$

The net force per unit length of current sheet is then

$$F^* = \oint_c S^* \, a d\theta. \tag{8}$$

Torque

The torque per unit length of the current sheet is

$$T_z = \text{Real} \left[i \oint_c z \, S^* \, a d\theta \right]. \tag{9}$$

There can be no torque about the central axis since the iron shield in symmetrical about this axis. However, if $z = \delta + z'$, the origin of z' being center of the displaced bore tube then,

$$T_{z'} = \text{Real}\left[i\oint_c z' \, S^* \, a d\theta \right] . \tag{10}$$

But

$$T_z = 0 = \text{Real}\left[i\oint_c (\delta + z') S^* \, a d\theta \right] . \tag{11}$$

Hence

$$T_{z'} = - \text{Real}\left[i\delta\oint_c S^* \, a d\theta \right] = - \text{Real}\,(i\delta F^*). \tag{12}$$

Results

Application of the previous formulas using input data appropriate for the dual test dipole, the doubler dipole, and the muon beam line dipole gives the results shown in Tables (1-3).

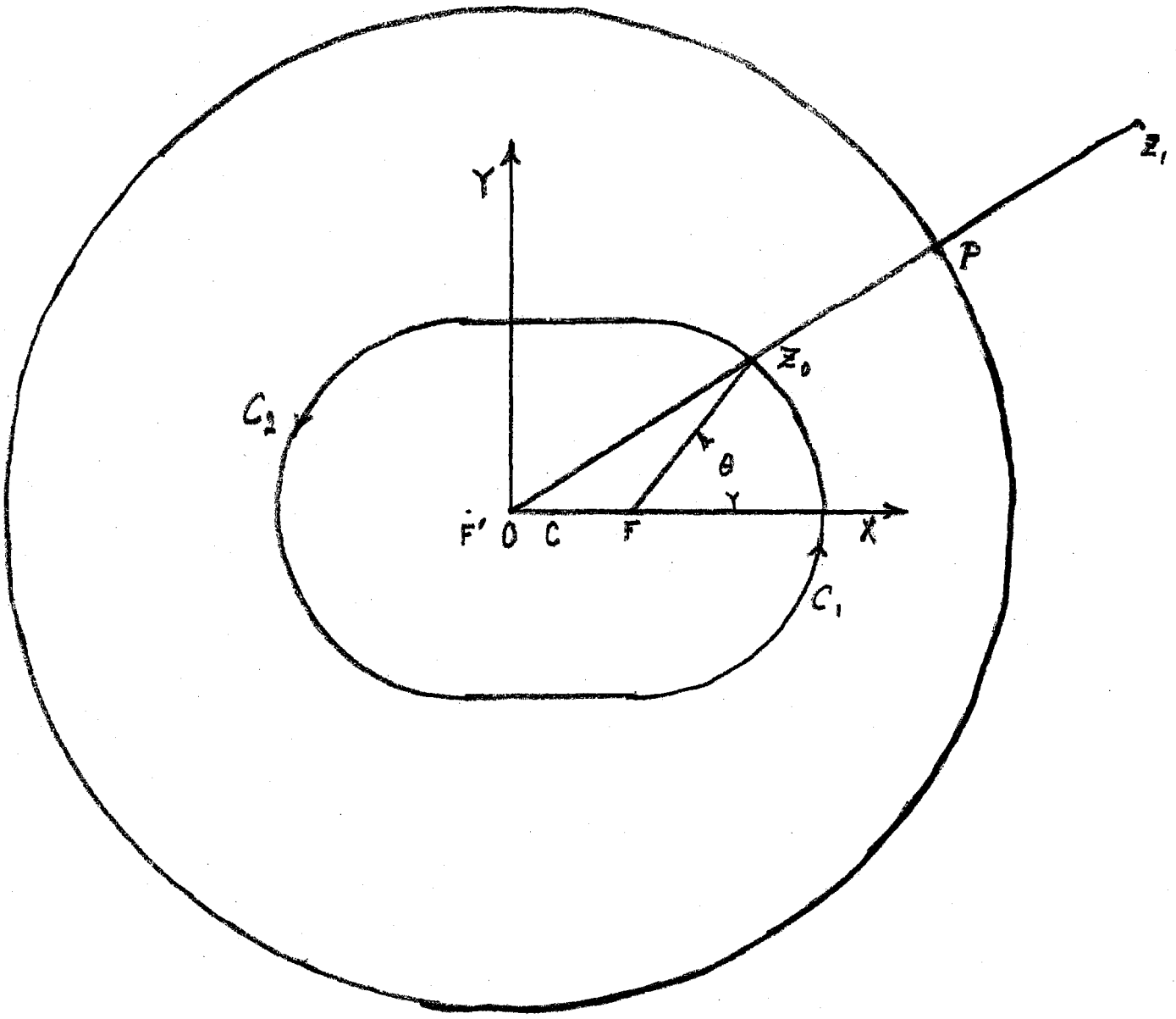

Figure 1. Sheet Current Dipole (See Legend)

Figure 1. Legend

0 - Equilibrium Center

C - Displaced Center

F - Origin of Right Hand Circular Sheet Current

F' - Origin of Left Hand Circular Sheet Current

z_1 = Image Position of z_o

a = Radius at Current Sheet $(F-z_o)$

b = Inner Radius of Iron (0-P)

Δ = Offset (F'-C or C-F)

δ = Displacement of Bore Tube Center (0-C)

$$H^* = -2iI\left(\frac{1}{z-z_o} + \frac{1}{z-z_1}\right)$$

$$z_o = \left\{ \begin{array}{ll} \delta+\Delta+ae^{i\theta} & \text{Right Hand Sheet} \\ \delta-\Delta+ae^{i\theta} & \text{Left Hand Sheet} \end{array} \right\}$$

$$z_1 = \frac{b^2}{z_o^*}$$

$$I = i_o a \cos\theta d\theta$$

Note that the origin of θ is F for $-\frac{\pi}{2}<\theta<\frac{\pi}{2}$ and F' for $\frac{\pi}{2}<\theta<\frac{3\pi}{2}$

Table 1. Magnetic Stresses in Dual Test Dipole

Conductor Current	2625A
Number of Conductors	130
Current Sheet Radius	1.340 in
Current Sheet Offset	.000 in
Inner Radius of Iron	3.050 in
Central Field	37.6 kG

Stress:

Quadrant	Angle (Deg)	X-Stress (P/in/in)	Y-Stress (P/in/in)
1	0	441	0
	10	503	-385
	20	666	-691
	30	877	-858
	40	1062	-863
	50	1146	-724
	60	1078	-495
	70	842	-251
	80	462	-68
	90	0	0
2	90	0	0
	100	-462	-68
	110	-842	-251
	120	-1078	-495
	130	-1146	-724
	140	-1062	-863
	150	-877	-858
	160	-666	-691
	170	-503	-385
	180	-441	0

Table 1. (Cont.)

Quadrant	Angle (Deg)	X-Stress (P/in/in)	Y-Stress (P/in/in)
3	180	-441	0
	190	-503	385
	200	-666	691
	210	-877	858
	220	-1062	863
	230	-1146	724
	240	-1078	495
	250	-842	251
	260	-462	68
	270	0	0
4	270	0	0
	280	462	68
	290	842	251
	300	1078	495
	310	1146	724
	320	1062	863
	330	877	858
	340	666	691
	350	503	385
	360	441	0

Resultant Forces and Torque (about bore tube center)

X-Displ. (in)	Y-Displ. (in)	X-Force (P/in)	Y-Force (P/in)	Z-Torque (in·P/in)
.010	.000	5.35	.00	.00
.000	.010	.00	5.35	.00
.010	.010	5.35	5.35	.00

Table 2. Magnetic Stresses in Doubler Dipole

Conductor Current	2815A
Number of Conductors	140
Current Sheet Radius	1.1325 in
Current Sheet Offset	.375 in
Inner Radius of Iron	3.0625 in
Central Field	45.5 kG

Stress:

Quadrant	Angle (Deg)	X-Stress (P/in/in)	Y-Stress (P/in/in)
1	0	534	0
	10	651	-759
	20	965	-1367
	30	1374	-1710
	40	1740	-1743
	50	1925	-1497
	60	1832	-1069
	70	1431	-596
	80	775	-210
	90	0	0
2	90	0	0
	100	-775	-210
	110	-1431	-596
	120	-1832	-1069
	130	-1925	-1497
	140	-1740	-1743
	150	-1374	-1710
	160	-965	-1367
	170	-651	-759
	180	-534	0

Table 2. (Cont.)

Quadrant	Angle (Deg)	X-Stress (P/in/in)	Y-Stress (P/in/in)
3	180	-534	0
	190	-651	759
	200	-965	1367
	210	-1374	1710
	220	-1740	1743
	230	-1925	1497
	240	-1832	1069
	250	-1431	596
	260	-775	210
	270	0	0
4	270	0	0
	280	775	210
	290	1431	596
	300	1832	1069
	310	1925	1497
	320	1740	1743
	330	1374	1710
	340	965	1367
	350	651	759
	360	534	0

Resultant Force and Torque (about bore tube center)

X-Displ. (in)	Y-Discpl. (in)	X-Force (P/in)	Y-Force (P/in)	Z-Torque (in·P/in)
.010	.000	11.04	.00	.00
.000	.010	.00	9.48	.00
.010	.010	11.04	9.48	.0156

Table 3. Magnetic Stresses in Muon Line Dipole

Conductor Current	2380A
Number of Conductors	306
Current Sheet Radius	2.489 in
Current Sheet Offset	1.000 in
Inner Radius of Iron	5.750 in
Central Field	40.5 kG

Stress:

Quadrant	Angle (Deg)	X-Stress (P/in/in)	Y-Stress (P/in/in)
1	0	642	0
	10	719	-548
	20	926	-988
	30	1191	-1237
	40	1419	-1264
	50	1514	-1089
	60	1410	-782
	70	1088	-440
	80	585	-160
	90	0	0
2	90	0	0
	100	-585	-160
	110	-1088	-440
	120	-1410	-782
	130	-1514	-1089
	140	-1419	-1264
	150	-1191	-1237
	160	-926	-988
	170	-719	-548
	180	-642	0

Table 3. (Cont.)

Quadrant	Angle (Deg)	X-Stress (P/in/in)	Y-Stress (P/in/in)
3	180	-642	0
	190	-719	548
	200	-926	988
	210	-1191	1237
	220	-1419	1264
	230	-1514	1089
	240	-1410	782
	250	-1088	440
	260	-585	160
	270	0	0
4	270	0	0
	280	585	160
	290	1088	440
	300	1410	782
	310	1514	1089
	320	1419	1264
	330	1191	1237
	340	926	988
	350	719	548
	360	642	0

Resultant Force and Torque (about bore tube center)

X-Displ. (in)	Y-Displ. (in)	X-Force (P/in)	Y-Force (P/in)	Z-Torque (in·P/in)
.010	.000	17.82	.00	.00
.000	.010	.00	13.70	.00
.010	.010	17.82	13.70	.0412