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EFFECTS OF LOSING ONE QUADRUPOLE IN THE MAIN RING

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SUMMARY

The purpose of this note is to point out that, if focusing and defocusing quadrupoles of the main ring are of the same strength (which is not quite true in the actual case), either radial or vertical betatron oscillation (or both) becomes unstable when one quadrupole is inactive. Only exceptions are three short defocusing quadrupoles between stations 11 and 12 (new numbering system) of each superperiod. It is difficult and for many cases impossible to regain the stability by simply changing the strength of all quadrupoles by the same amount. On the other hand, if focusing and defocusing quadrupoles could be adjusted separately, betatron oscillations would become stable again. The amount of change needed is of the order of 1/3% ~1/2%. However, it must be remembered that at the injection (7 GeV) the momentum spread of the beam is ~±0.1% so that the focusing system could be near a stopband[1] for some particles, causing the beam size to increase by an intolerably large factor.

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QUALITATIVE DISCUSSION

The following treatment is taken from Courant-Snyder.[2] Equation of betatron oscillation is written in the form

$$\xi''(s) + [K_o(s) + k(s)]\xi(s) = 0$$

where $\xi(s)$ = horizontal or vertical displacement of a particle from the closed orbit. For a perfect system, $k(s) = 0$. When one quadrupole is inactive in the main ring,

$$k(s) \equiv k = + \text{ or } -0.0187 \text{ m}^{-2}$$

within that quadrupole. The change in tune is

$$\Delta\nu = (1/4\pi)\ \beta\cdot k\cdot L$$

where L is the quadrupole length and β is the average value of the betatron oscillation parameter in the quadrupole. With $\beta \simeq 100$ m and $L \simeq 2.1$ m, one gets

$$\Delta\nu \simeq + \text{ or } -0.31.$$

Existence of one inactive quadrupole changes the basic period of the system to one turn from 1/6 turn for the perfect system and $\nu = (2n+1)/2$, $n = 0,1,2,...$ are all at the boundary of a stopband. Since the design value is $\nu = 20.25 - 20.3$, the tune shift of + or -0.31 is large enough to drive the system to stopbands $\nu = 20$ or $\nu = 20.5$. Another way of seeing this is to estimate the change in β. When a quadrupole at $s = s_2$ is inactive, the value of $\beta(s)$ at $s = s_1$ changes from β_1 to $\beta_1 + \Delta\beta$:

$$\Delta\beta = (\beta_1\beta_2 Lk/2 \sin \mu_o) \cos (\mu_o+2\Delta\psi)$$

where $\mu_o \equiv$ phase advance per turn and $\Delta\psi \equiv$ phase advance from s_1 to s_2. With $\mu_o \equiv 2\pi\nu \simeq 40.5\ \pi$, one gets

$$\max |\Delta\beta| \simeq 200 \text{ m}$$

since it is possible to choose s_1 and s_2 such that

$$\beta_1, \beta_2 \simeq 100 \text{ m}$$

$$\cos (\pi_o + 2\Delta\psi) \simeq +1 \text{ or } -1.$$

The change in β is thus twice the unperturbed value causing β to vanish. This is not a proof but indicates strongly that the system may become unstable if one quadrupole is inactive.

NUMERICAL RESULTS

A straightforward numerical calculation has been done to confirm the conjecture made above. Focusing action of dipoles in the vertical plane is included in addition to quadrupoles. It is perhaps instructive to see the change in ν when the strength of one quadrupole is reduced from the design value while strengths of all other quadrupoles are kept at the design value. The upper diagram shows the change of tunes when the strength of one "focusing" quadrupole (7-foot long) in a normal cell or in a medium straight cell is reduced. The lower diagram is for one "defocusing" quadrupole. The system becomes unstable in one direction at $B'_F \simeq 0.5\ B'_o$ (design value) and at $B'_D \simeq 0.35\ B'_o$ with ν_x or $\nu_y = 20$. When the strength is reduced further, the opposite direction approaches $\nu = 20.5$ and, beyond that, there is no stable area in either plane. Notice that there is a fundamental difference between these situations and one in which strengths of all quadrupoles are increased or reduced by the same amount.[3] In the latter case, the lattice structure of the focusing system is intact and one finds many stable areas surrounded by stopbands. When the strength of one quadrupole is changed, one is dealing with essentially different focusing systems. The operating point of the system moves from the stability diagram of one system to that of another system. Strictly speaking, the statement that there is no stable area beyond a certain value of B_F' or B_D' means that the operating point always moves from a stopband of one system to a stopband of another system.

The system becomes unstable in at least one direction when a quadrupole is totally missing (B_F' or $B_D' = 0$). Only exceptions are three short defocusing quadrupoles between stations 11 and 12 of each superperiod. Changes in ν and β are summarized in Table 1 when one of these is missing.

In general, it is difficult and for most cases impossible to regain the stability by simply changing the strength of all quadrupoles by the same amount. Even when the stability were obtained, the maximum value of β would be such that the beam size would increase by a large factor. This can be seen in Table 1. If focusing and defocusing quadrupoles are adjusted separately, one can obtain a stable operation. The amount of change needed is of the order of 1/3% ~1/2% of the original value. Although no systematic study has been made on the maximum value of β for such cases, it is expected that suppressing the beam size increase to below, say, 50% in both directions simultaneously is not an easy task. In this connection, it must be remembered that at the injection the momentum spread of the beam is ~±0.1% so that the operating point of the system could be near a stopband for some particles. One may, of course, be very lucky, the effect of an inactive quadrupole being compensated nicely by nonuniform remnant fields of quadrupoles. However, this fortuitous situation disappears quickly as the beam is accelerated to higher energies.

REFERENCES

1. S. Ohnuma, TM-314, July 7, 1971.
2. E.D. Courant and H. Snyder, Annals of Physics 3 (1958). See pp. 21-27.
3. The explanation that follows has been given by Lee Teng.

Table 1

Tunes and β when one Defocusing Quadrupole is Missing between Stations 11 and 12

	ν_x	ν_y	$(\beta_x)_{max}$	$(\beta_y)_{max}$
1.	20.449	20.134	763 m	344 m
2.	20.404	20.116	395 m	406 m
3.	20.302	20.049	172 m	928 m

1. quadrupole adjacent to station 11
2. middle quadrupole
3. quadrupole adjacent to station 12

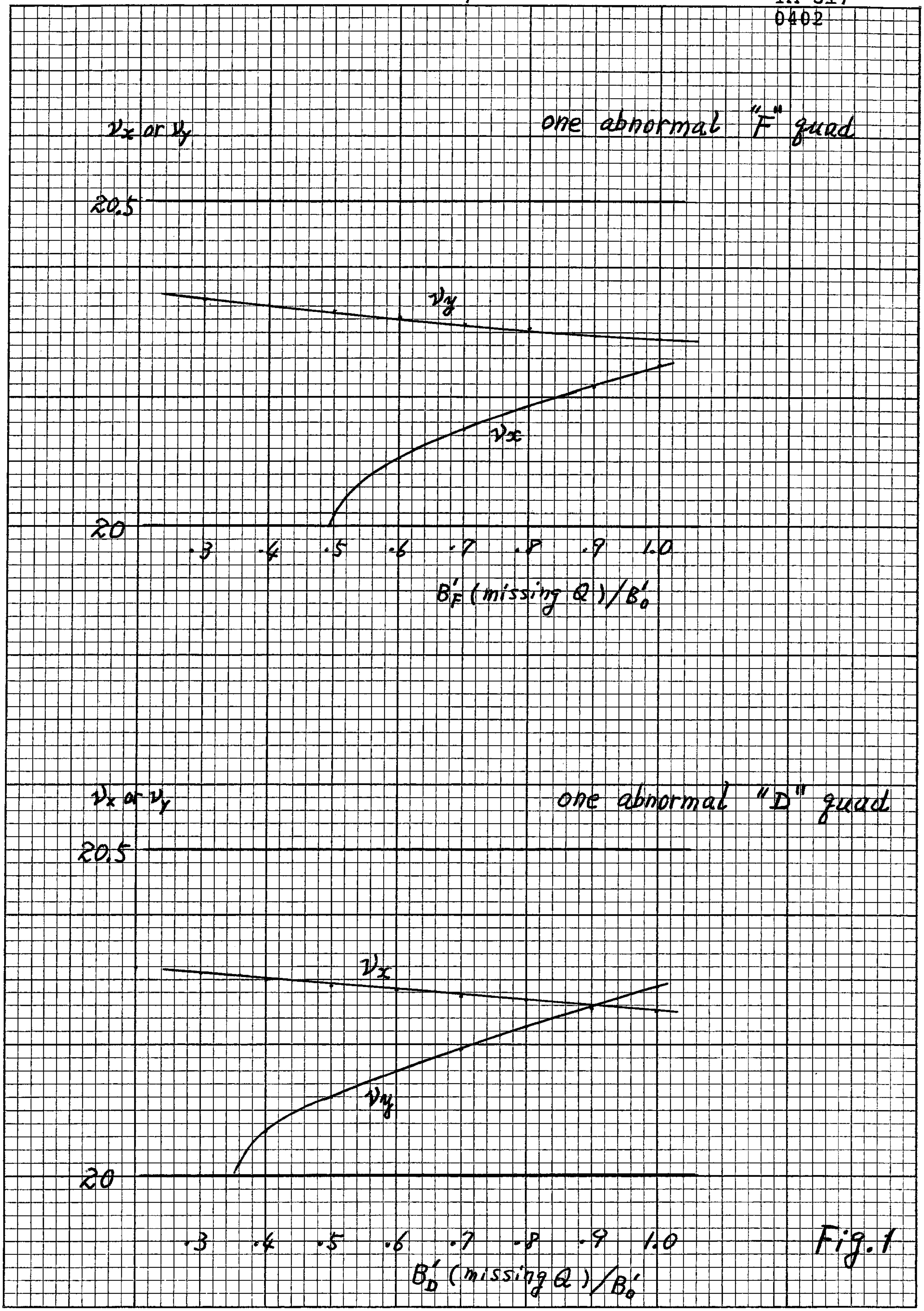
ν_x or ν_y
one abnormal "F" quad
20.5
ν_y
ν_x
20
.3 .4 .5 .6 .7 .8 .9 1.0
B'_F (missing Q)/B'_0
ν_x or ν_y
one abnormal "D" quad
20.5
ν_x
ν_y
20
.3 .4 .5 .6 .7 .8 .9 1.0
B'_D (missing Q)/B'_0

Fig. 1

ADDENDUM

Lee Teng investigated the same problem applying the formalism of Courant-Snyder (TM-313, TM-313-A). He explains the failure of the analytical approach in detail using an invariant quantity of the betatron motion.

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ADDENDUM TO TM-317

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D. Edwards pointed out that when one quadrupole is missing one can usually regain the stable betatron oscillation in both horizontal and vertical directions by simply making the next (upstream or downstream) quadrupole inactive. This has been confirmed by numerical calculations. When two normal quadrupoles are missing together, tunes are 20.17 (horizontal) and 20.24 (vertical); modified values of β are 400~420 (horizontal) and 380~400 (vertical). This would be better than changing the strength of all quadrupoles by the same amount when one cannot control B_F' and B_D' separately.

It should be emphasized here that, whatever remedies are employed, the condition at the injection point is quite different from the normal case so that the transport system has to be retuned, not an easy task since there are six parameters to be adjusted (β_x, α_x, β_y, α_y, and two dispersion parameters). For example, if the transport system is not readjusted, the mismatching will double the beam size and the entire effect (mismatching and the increase of β) is to increase the beam size four times.

I am grateful to D. Edwards for his suggestions.