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BEAM INDUCED POWER LOSS TO BOOSTER LAMINATIONS

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PURPOSE

To execute an independent confirmation of the beam induced power loss to the booster laminations pointed out by A.G. Ruggiero.[1] The additional beam power that must be supplied by the booster radio frequency accelerating system to account for eddy current losses in the magnet laminations varies from about .2 kW at injection to 40 kW at transition.

GENERAL

Let the linear charge density per unit angle increment along the beam be $\lambda(\theta,t)$. If this is fourier analyzed assuming the charge is contained in stationary buckets

$$\lambda(\theta,t) = \sum_n \lambda_n e^{in(h\theta-\omega t)}, \qquad \text{(ab Coulomb)} \tag{1}$$

where h is the harmonic number and ω is the angular frequency of the radio frequency fields. The corresponding beam current is

$$I(\theta,t) = \frac{\omega}{h}\sum_n \lambda_n e^{in(h\theta-\omega t)}. \qquad \text{(ab Amp)} \tag{2}$$

A detailed electrodynamic calculation presented in Appendix I gives for the longitudinal electric field at the center of the beam:

$$E(\theta,t) = -4i\frac{c^2}{a^2}\frac{1}{h}\sum_n{}' \frac{\lambda_n}{n}[1+E(n\tfrac{h}{R},n\omega)]e^{in(h\theta-\omega t)}; \quad \text{(emu)} \tag{3}$$

and the prime indicates omission of the n = 0 term where a is the beam radius. If the wavelength of the beam modulation is long compared with the aperture b of the booster laminations

$$E(k,\omega) = -1 + \frac{1}{4}\mu^2 a^2(1+2\ \ln\frac{b}{a}) + i\frac{\omega a^2}{2cb} Z(\omega). \tag{4}$$

Here Z is the dimensionless measure of the lamination impedance[2] and

$$\mu^2 = k^2-\frac{\omega^2}{c^2}. \tag{5}$$

The power radiated by the beam to the walls is given by

$$P = R\int_{-\pi}^{\pi} E(\theta,t)I(\theta,t)d\theta, \qquad (6)$$

where R is the average orbital radius. Since $\lambda_{-n} = \lambda_n^*$ and $Z(-\omega) = Z^*(\omega)$,

$$P = \frac{8\pi Rc\omega^2}{h^2 b}\sum_{n=1}^{\infty} \lambda_n\lambda_n^* \mathcal{R}(n\omega), \quad \text{(emu)} \qquad (7)$$

where $\mathcal{R}(\omega)$ is the resistive part of the wall impedance[2] in dimensionless measure.

From Eq. (1), if

$$\phi = h\theta - \omega t, \qquad (8)$$

$$\lambda_n = \frac{h}{2\pi}\int_{-\pi}^{\pi} \lambda(\phi)e^{-in\phi}d\phi, \qquad (9)$$

and

$$\boxed{P = \frac{Rc\omega^2}{\pi b}\iint \lambda(\phi)\lambda(\phi')\sum_n{}' \mathcal{R}(n\omega)e^{in(\phi-\phi')}d\phi d\phi'.} \qquad (10)$$

Note that $\mathcal{R}(-\omega) = \mathcal{R}(\omega)$.

GENERAL DISTRIBUTION ASSUMING $\mathcal{R}(n\omega) = \mathcal{R}(\omega)$

For the booster laminations ω varies from about 30 MHz to 50 MHz. Reference (1) indicates that $\mathcal{R} \simeq .03$ and is sensibly constant for all higher frequencies. Hence (Eq. (10) may be evaluated assuming that $\mathcal{R}$ is frequency independent. Note that

$$\sum_n{}' e^{in(\phi-\phi')} = 2\pi\delta(\phi-\phi') - 1. \qquad (11)$$

$$I_0 = \langle I \rangle_{Av} = \frac{\omega}{h}\lambda_0 = \omega\langle\lambda\rangle_{Av} \tag{14}$$

Eq. (10) becomes

$$P = \frac{4\pi Rc\mathcal{R}}{b}\left(\frac{\langle\lambda^2\rangle_{Av}}{\langle\lambda\rangle^2_{Av}} - 1\right) I_0^2 \;. \quad \text{(emu)} \tag{15}$$

UNIFORM PHASE SPACE DENSITY IN STATIONARY BUCKET

In this case

$$\lambda(\phi) = \lambda_{max} \cos\frac{\phi}{2} \;. \tag{16}$$

Hence,

$$\langle\lambda\rangle_{Av} = \frac{2}{\pi}\lambda_{max}, \tag{17}$$

and

$$\langle\lambda^2\rangle_{Av} = \frac{1}{2}\lambda_{max}. \tag{18}$$

For Eq. (15) one has

$$P = 4\pi\left(\frac{\pi^2}{8} - 1\right)\frac{Rc}{b}\mathcal{R}\, I_0^2 \;. \quad \text{(emu)} \tag{19}$$

In the booster R = 7547.17 cm, $\mathcal{R}$ = .03, b ≃ 2.5 cm, I_0 = .022 ab Amp at injection. Thus

$$P = 3.8 \times 10^9 \frac{\text{ergs}}{\text{sec}} = .38 \text{ kW} \tag{20}$$

COSINE DISTRIBUTION

In this case, let

$$\lambda(\phi) = \langle\lambda\rangle_{Av}(1 + \cos\phi). \tag{21}$$

Then

$$\langle\lambda^2\rangle_{Av} = \frac{3}{2}\langle\lambda\rangle^2_{Av} , \tag{22}$$

and for Eq. (15) one has

$$P = 2\pi \frac{Rc}{b} \mathcal{R} I_0^2 . \qquad \text{(emu)} \tag{23}$$

For the booster at injection this gives

$$P = .81 \text{ kW}. \tag{24}$$

RECTANGULAR DISTRIBUTION

Let

$$\lambda(\phi) = \begin{cases} 0 & -\pi < \phi < \alpha \\ \lambda_{max} & -\alpha < \phi < \alpha \\ 0 & \alpha < \phi < \pi . \end{cases} \tag{25}$$

Then

$$\langle\lambda\rangle_{Av} = \frac{\alpha}{\pi} \lambda_{max} , \tag{26}$$

and

$$\langle\lambda^2\rangle_{Av} = \frac{\alpha}{\pi} \lambda^2_{max} . \tag{27}$$

For Eq. (15) this yields

$$P = 4\pi \left(\frac{\pi}{\alpha} - 1\right) \frac{Rc}{b} \mathcal{R} I_0^2 . \text{ (emu)} \tag{28}$$

In the booster, if one chooses α to be one-half of the angular bunch length at transition,[3] $\alpha = .2$ rad. Then at transition

$$P = 70 \text{ kW}. \tag{29}$$

UNIFORM PHASE SPACE DISTRIBUTION IN ACCELERATING BUCKET

Gumowski and Reich[4] have given the linear charge density distribution to be expected for a uniformly filled bucket. A program BUCKET has been written to determine the radio frequency regime for the booster that will maintain constant bucket size. In particular, one may calculate $\langle\lambda\rangle_{Av}$ and $\langle\lambda^2\rangle_{Av}$ throughout the accelerating regime. The extra radio frequency power given by Eq. (15) has been evaluated and is tabulated in the attached computer output. Since the bunch length near transition is estimated incorrectly in this theory, the estimate of Eq. (29) may be substituted.

CIRCUMFERENCE FACTOR

The previous calculations have been made assuming that the magnet laminations fill the entire ring. More realistic estimates of the power may be obtained by dividing the previous calculations by the circumference factor. For the booster this factor is 1.72. Thus

Distribution	Power (kW)
Stationary Bucket (Inj.)	.22
Cosine (Inj.)	.47
Rectangular (Trans.)	40
Accelerating Bucket	Divide table by 1.72

APPENDIX: BEAM ELECTRODYNAMICS

Reference (1) gives the method of calculating the wall impedance $Z(\omega)$ required in Eq. (4). Here we are interested in finding the fields in cylindrical geometry of a beam of radius a moving along the axis of a cylinder of radius b.

SOURCE

All field and source quantities are assumed to vary harmonically with time as $e^{-i\omega t}$. Then, using a vector decomposition previously employed,[5] the current density becomes

$$J = -i\frac{\omega}{c}\,\nabla W + \vec{L}\sigma - i\frac{\omega}{c}\,\nabla \times \vec{L}T, \text{ (gaussian)} \tag{1}$$

where the operator $\vec{L}$ is taken to be

$$\vec{L} = \vec{k}\times\nabla, \tag{2}$$

$\vec{k}$ being a unit vector along the z-axis. For the transverse magnetic mode let

$$\nabla^2 W = -\rho \tag{3}$$

$$\nabla^2 T = -\tau \tag{4}$$

$$\sigma = 0 \tag{5}$$

$$W = \frac{\partial T}{\partial z} \tag{6}$$

then

$$J = i\omega\tau\vec{k}. \tag{7}$$

If ρ is fourier analyzed in the z-variable

$$\rho = \int \rho_0(k)e^{ikz}dk, \tag{8}$$

τ may be written as

$$\tau = -i \int \frac{\rho_0(k)}{k}\, e^{ikz}dk. \tag{9}$$

POTENTIALS

Since σ is taken to be zero, the transverse electric potential, $U_{TE} = 0$. The transverse magnetic potential U_{TM} will be taken as U and fourier analyzed in z to give

$$U = \int u(k,r)e^{ikz}dk. \tag{10}$$

Since the charge ρ is contained within the cylinder r = a and

$$\nabla^2 U + \frac{\omega^2}{c^2} U = -4\ \pi\tau, \tag{11}$$

one has

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) - \mu^2 u = \begin{cases} \frac{4\pi i}{k}\ \rho_0(k) & r<a \\ 0 & r>a\ , \end{cases} \tag{12}$$

where

$$\mu^2 = k^2 - \frac{\omega^2}{c^2}\ . \tag{13}$$

Then

$$u = -\frac{4\pi i\rho_0(k)}{k\mu^2} \begin{cases} 1 + E\ I_0(\mu r) & r<a \\ C\ I_0(\mu r) + DK_0(\mu r) & r>a\ . \end{cases} \tag{14}$$

FIELDS

$$E = 4\pi\tau\vec{k} + \nabla \times \vec{L}U \qquad \text{(gaussian)} \tag{15}$$

$$\vec{H} = -i\frac{\omega}{c}\ \vec{L}U \qquad \text{(gaussian)} \tag{16}$$

Thus

$$E_r = -\int \frac{4\pi\rho_0(k)}{\mu} \begin{Bmatrix} E\ I_1(\mu r) \\ CI_0(\mu r) - DK_1(\mu r) \end{Bmatrix} e^{ikz}dk \tag{17}$$

$$E_\theta = 0$$

$$E_z = -\int \frac{4\pi i\rho_0(k)}{k} \begin{Bmatrix} 1 + E\, I_0(\mu r) \\ C\, I_0(\mu r) + DK_0(\mu r) \end{Bmatrix} e^{ikz} dk \tag{19}$$

$$H_r = H_z = 0 \tag{20}$$

$$H_\theta = -\int \frac{4\pi\omega\rho_0(k)}{\mu kc} \begin{Bmatrix} E\, I_1(\mu r) \\ C\, I_1(\mu r) - DK_1(\mu r) \end{Bmatrix} e^{ikz} dk \tag{21}$$

where the upper entry in the brackets pertains to $0 < r < a$ and the lower entry to $a < r < b$.

BOUNDARY CONDITIONS AT EDGE OF BEAM

Matching the tangential components of $\vec{E}$ and $\vec{H}$ at $r = a$ gives

$$C\, I_0(\mu a) + DK_0(\mu a) = 1 + E\, I_0(\mu a) \tag{22}$$

$$C\, I_1(\mu a) - DK_1(\mu a) = E\, I_1(\mu a). \tag{23}$$

Thus

$$C - E = \mu a\, K_1(\mu a) \tag{24}$$

$$D = \mu a\, I_1(\mu a).$$

BOUNDARY CONDITIONS AT WALL

If Z is the impedance of the wall at $r = b$

$$E_z = -ZH_\theta, \qquad (r = b) \tag{26}$$

where Z is given in Reference (1). Using Eqs. (19), (21), and (26) for $r = b$ gives

$$\left[i\mu c I_0(\mu b) + \omega Z I_1(\mu b)\right]C + \left[i\mu c K_0(\mu b) - \omega Z K_1(\mu b)\right]D = 0. \tag{27}$$

Equations (25) and (27) give

$$C = -\mu a I_1(\mu a) \cdot \frac{K_0(\mu b) + \frac{i\omega Z}{\mu c} K_1(\mu b)}{I_0(\mu b) - \frac{i\omega Z}{\mu c} I_1(\mu b)} , \tag{28}$$

Equation (24) then gives

$$E = -\mu a \cdot \frac{I_0(\mu b)K_1(\mu a) + K_0(\mu b)I_1(\mu a) - \frac{i\omega Z}{\mu c}\left[I_1(\mu b)K_1(\mu a) - K_1(\mu b)I_1(\mu a)\right]}{I_0(\mu b) - \frac{i\omega Z}{\mu c} I_1(\mu b)} . \tag{29}$$

For small argument v

$$I_0(v) \simeq 1 \qquad I_1(v) \simeq \frac{1}{2} v$$

$$K_0(v) \simeq -\ln\alpha v \qquad K_1(v) \simeq \frac{1}{v} + \frac{v}{2}\ln\alpha v - \frac{v}{4} , \tag{30}$$

where $\ln\alpha = \gamma - \ln 2 = .5772157 - \ln 2$.

Hence for $\mu b << 1$ and small $|Z|$

$$C(\mu,\omega) = \frac{1}{2}\mu^2 a^2 \ln\alpha\mu b + i\frac{\omega a^2}{2cb} Z \tag{31}$$

$$D(\mu,\omega) = \frac{1}{2}\mu^2 a^2 \tag{32}$$

$$E(\mu,\omega) = -1 + \frac{\mu^2 a^2}{4}\left(1 + 2\ln\frac{b}{a}\right) + i\frac{\omega a^2}{2cb} Z . \tag{33}$$

ELECTRIC FIELD AT ORIGIN

The longitudinal electric field for r = 0 from Eq. (19) is

$$E_z = -4\pi i \int \frac{\rho_0(k)}{k}(1+E)\, e^{ikz} dk . \quad \text{(gaussian)} \tag{34}$$

TRANSITION TO TOROIDAL GEOMETRY

For large radius one may replace k and z by

$$k = n\frac{h}{R} \qquad z \to R\theta . \tag{35}$$

But

$$\lambda(\phi) = \frac{\pi a^2 R}{h} \rho, \tag{36}$$

and from Eq. (8) and Eq. (9) in the main text

$$\pi a^2 h \rho_0(k) \rightarrow \lambda_n. \tag{37}$$

Thus Eq. (34) becomes

$$E_\theta = -\frac{4i}{h} \frac{c^2}{a^2} \sum_n{}' \frac{\lambda_n}{n} \left[1+E(n\frac{h}{R}, n\omega)\right] e^{in(h\theta-\omega t)}, \quad \text{(emu)}$$

where c^2 is required to change λ_n and E_θ from gaussian to emu units as are used in the main text.

REFERENCES

1. A.G. Ruggiero, Energy Loss due to the Resistive Magnet Lamination in the NAL Booster, NAL Physics Note FN-230, May 15, 1971.

2. S.C. Snowdon, Wave Propagation between Booster Laminations Induced by Longitudinal Motion of Beam, NAL Technical Memorandum TM-277, Nov. 3, 1970; for equivalent circuit solution see: A.G. Ruggiero, Longitudinal Space Charge Forces within a Bunched Beam in the Presence of Magnetic Lamination, NAL Physics Note FN-220, January 4, 1970.

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4. I. Gumowski and K.H. Reich, Synchrotron Motion in the Presence of Space Charge, CERN/SI/Int. DL/70-6, July 30, 1970.

5. S.C. Snowdon, Journ. Math. Phys. 2, 719 (1961).

CONSTANT AREA BUCKETS FOR BOOSTER 8 GE'

RADIUS(M)=	75.47170	DELP/P =	.00080	KETRAN(GEV)=	4.17150	AVBINJ(KG)=	.28482	AVBMAX(KG)=	3.92859
FREQ(HZ) =	15.00000	HARM. NO.=	84.00000	NO. POINTS=	40.00000	FACTOR =	1.00000	GAP(IN) =	2.05100
HBMDIA(IN)=	3.00000	VBMDIA(IN)=	1.25000	PART(E+12)=	3.85000				

NIT	TIME (MSEC)	KE (GEV)	RADFREQ (MHZ)	PHIS (DEGREES)	VPT (MV)	BUCKET ALPHA	S	POWER (KW)	RES
5	.00083	.207242	30.465166	16.5667	.169100	.5335	.0316	.9644	.02260
4	.00167	.229483	31.615038	22.7443	.248653	.4270	.0270	1.3505	.02234
4	.00250	.268143	33.381552	27.0853	.315078	.3606	.0253	1.8361	.02445
4	.00333	.325220	35.566339	30.7787	.371101	.3091	.0246	2.4912	.02576
4	.00417	.402832	37.944195	34.2241	.418134	.2652	.0245	3.3465	.02705
3	.00500	.502778	40.306320	37.5704	.457563	.2263	.0246	4.4022	.02814
3	.00583	.626208	42.494780	40.8531	.490876	.1916	.0247	5.6346	.02894
3	.00667	.773505	44.416922	44.0546	.519475	.1611	.0249	7.0119	.02944
3	.00750	.944313	46.039536	47.1380	.544489	.1346	.0250	8.5076	.02973
3	.00833	1.137682	47.371537	50.0685	.566685	.1120	.0251	10.1084	.02986
3	.00917	1.352223	48.445104	52.8229	.586485	.0929	.0253	11.8157	.02990
3	.01000	1.585254	49.301112	55.3923	.604048	.0770	.0256	13.6470	.02989
3	.01083	1.837904	49.980213	57.7802	.619350	.0637	.0260	15.6421	.02986
3	.01167	2.105189	50.518488	60.0002	.632259	.0525	.0267	17.8776	.02982
3	.01250	2.386050	50.945999	62.0736	.642583	.0431	.0278	20.5032	.02978
3	.01333	2.678389	51.286892	64.0287	.650108	.0352	.0294	23.8386	.02975
3	.01417	2.980080	51.560138	65.9019	.654604	.0282	.0319	28.6690	.02972
3	.01500	3.288984	51.780468	67.7444	.655824	.0220	.0360	37.4068	.02963
3	.01583	3.602958	51.959253	69.6394	.653467	.0162	.0437	60.9252	.02966
4	.01667	3.919865	52.105256	71.7693	.647010	.0099	.0641	217.6944	.02964
4	.01750	4.237578	52.225243	92.0850	.613045	.0048	.1252	3749.3960	.02963
4	.01833	4.553993	52.324432	102.2293	.621061	.0107	.0503	141.0841	.02961
3	.01917	4.867036	52.406899	104.4431	.617055	.0135	.0365	70.1513	.02960
3	.02000	5.174670	52.475814	105.6665	.607006	.0154	.0299	52.1526	.02959
3	.02083	5.474910	52.533669	106.4889	.592105	.0168	.0260	44.5012	.02958
3	.02167	5.765829	52.582426	107.1024	.572886	.0178	.0234	40.3446	.02957
3	.02250	6.045567	52.623646	107.5987	.549703	.0188	.0215	37.7170	.02956
3	.02333	6.312343	52.658570	108.0306	.522844	.0197	.0203	35.8649	.02956
3	.02417	6.564465	52.688193	108.4306	.492560	.0206	.0194	34.4282	.02955
2	.02500	6.800338	52.713313	108.8248	.459098	.0215	.0189	33.2286	.02955
2	.02583	7.018473	52.734575	109.2334	.422701	.0226	.0186	32.1534	.02955
3	.02667	7.217494	52.752498	109.6760	.383612	.0238	.0187	31.1297	.02954
3	.02750	7.396150	52.767502	110.1738	.342079	.0253	.0190	30.0993	.02954
3	.02833	7.557318	52.779922	110.7549	.298354	.0271	.0196	29.0161	.02954
3	.02917	7.688011	52.790030	111.4592	.252688	.0295	.0208	27.8283	.02954
3	.03000	7.799385	52.798037	112.3517	.205328	.0327	.0225	26.4682	.02954
3	.03083	7.886740	52.804108	113.5555	.156501	.0375	.0253	24.8252	.02954
4	.03167	7.949531	52.808367	115.3568	.106383	.0455	.0304	22.6826	.02953
4	.03250	7.987363	52.810908	118.7226	.054981	.0632	.0419	19.4077	.02953

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