TM-271
0402

THIRD INTEGRAL RESONANT EXTRACTION FROM THE MAIN RING

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This was studied by K. Symon (FN-130, FN-134, FN-140, and FN-144). This type of analysis is applied here to derive some numerical results for the main ring.

Neglecting the y motion (vertical) and the curvature of the closed orbit the resonant x motion (horizontal) is given by (this is the same as that given by Symon except for minor changes of notation)

$$\begin{cases} \dfrac{d\phi}{d\theta} = \varepsilon - AR\cos(3\phi - \psi) - (B + 3C)R^2 \\ \dfrac{dR}{d\theta} = -AR^2 \sin(3\phi - \psi) \end{cases} \tag{1}$$

with the first integral

$$\varepsilon R^2 - \frac{2}{3} AR^3 \cos(3\phi - \psi) - \frac{1}{2}(B + 3C)R^4 = \text{const.} \tag{2}$$

where

θ = linear oscillation phase advance normalized to 2π per revolution,

R and ϕ are related to x and p_x by

$$\begin{cases} x = R\sqrt{\beta}\cos\left(\phi - \dfrac{61}{3}\theta\right) \\ p_x = \dfrac{R}{\sqrt{\beta}} \dfrac{\sin\left(\phi - \phi_o - \dfrac{61}{3}\theta\right)}{\cos\phi_o} \end{cases} \qquad \tan\phi_o \equiv \alpha \tag{3}$$

or

$$\begin{cases} R = (\beta p_x^2 + 2\alpha x p_x + \gamma x^2)^{1/2} \\ \phi = \frac{61}{3}\,\theta + \tan^{-1}\left(\alpha + \beta\,\frac{p_x}{x}\right) \end{cases} \quad (4)$$

and

$$\begin{cases} \varepsilon = \frac{61}{3} - \nu_x = \text{deviation of } \nu_x \text{ from resonant value } 20\tfrac{1}{3} \\ A \cos(61\,\theta - \psi) = \text{61st harmonic of } \left[\frac{\nu_x}{16}\,\beta^{5/2}\,\frac{B''}{(B\rho)}\right] \\ B = \text{0th harmonic (average) of } \left[\frac{\nu_x}{16}\,\beta^{3}\,\frac{B'''}{(B\rho)}\right] \\ C = \text{0th harmonic (average) of } \left[\frac{\nu_x}{16}\,\beta\gamma^{2}\right] \\ \alpha, \beta, \gamma = \text{conventional linear oscillation functions} \\ B'' = \frac{\partial^2 B_y}{\partial x^2} = \text{sextupole field} \\ B''' = \frac{\partial^3 B_y}{\partial x^3} = \text{octupole field} \\ (B\rho) = \text{rigidity of particle} = \frac{pc}{e} \end{cases}$$

First, we make some general statements about the error fields in the main ring bending and quadrupole magnets.

1. The sextupole error field in the ring bending magnets contains only harmonics 6, 12, 18 ...... 60, 66, ... (because there are 6 superperiods) and no 61st harmonic, and therefore, causes little or no harm.

2. The octupole error field in the quadrupoles will give

a nonvanishing 0th harmonic of $\beta^3 B'''$, hence a nonvanishing B. A tolerance on the octupole error field will be given. But it is expected that correcting octupole magnets will be installed to trim out the octupole error field. The design of the extraction system will be based on no octupole field, namely $B = 0$.

3. The kinematic term C is generally much smaller ($\sim 10^{-3}$) than the tolerance value of the octupole term B and can be neglected.

It is convenient to define ($B + 3C < 0$ for main ring)

$$r \equiv -\frac{1}{2}\frac{A}{B + 3C} > 0 \text{ (ratio of sextupole to octupole fields)}$$

and rewrite Equ. (1) and (2) as

$$\left\{\begin{aligned} \frac{d\phi}{d\theta} &= \varepsilon - AR\cos(3\phi - \psi) + \frac{A}{2r}R^2 \\ \frac{dR}{d\theta} &= -AR^2 \sin(3\phi - \psi) \end{aligned}\right. \tag{5}$$

$$\varepsilon R^2 - \frac{2}{3}AR^3\cos(3\phi - \psi) + \frac{A}{4r}R^4 = \text{const.} \tag{6}$$

And we are only interested in the values of x and p_x for every third revolution at the electrostatic septum where

$$\left\{\begin{aligned} \beta &= \beta_s = 98 \text{ m} \\ \alpha &= \alpha_s = 0.45 \end{aligned}\right.$$

and

$$\theta = 0,\ 6\pi,\ 12\pi,\ 18\pi,\ \text{etc.}$$

or

$$\frac{61}{3}\theta = 0,\ 61(2\pi),\ 61(4\pi),\ 61(6\pi),\ \text{etc.}$$

At the electrostatic septum, therefore,

$$\begin{cases} x_s = R\sqrt{\beta_s} \cos \phi & \beta_s = 98 \text{ m} \\ p_{xs} = \dfrac{R}{\sqrt{\beta_s}} \dfrac{\sin (\phi - \phi_o)}{\cos \phi_o} & \tan \phi_o = \alpha_s = 0.45 \\ & \phi_o = 0.4229 \end{cases} \tag{7}$$

A. Fixed points

The 6 fixed points other than the origin R = 0 are given by $\frac{dR}{d\theta} = \frac{d\phi}{d\theta} = 0$. For $r > 0$.

3 stable fixed points:

$$\begin{cases} R_s = r + \sqrt{r^2 - \frac{2\varepsilon}{A} r} \approx 2r \\ \phi = \frac{\psi}{3}, \quad \frac{2\pi + \psi}{3}, \quad \frac{4\pi + \psi}{3}, \end{cases} \tag{8}$$

3 unstable fixed points:

$$\begin{cases} R_u = r - \sqrt{r^2 - \frac{2\varepsilon}{A} r} \approx \frac{\varepsilon}{A} \\ \phi = \frac{\psi}{3}, \quad \frac{2\pi + \psi}{3}, \quad \frac{4\pi + \psi}{3} \end{cases} \tag{9}$$

For no octupole field $r \to \infty$. The 3 stable fixed points recede to ∞ and the 3 unstable fixed points are exactly at $R_u = \frac{\varepsilon}{A}$.

B. Position and strength of the resonance exciting sextupole magnet

We would like the beam to pass the electrostatic septum at zero angle. This implies that at the septum $p_x = 0$ hence

$\phi = \phi_o$ (Equ. (7)) for the phase points streaming out along the separatrix. With no octupole field the separatrices in the (R, ϕ) plane are shown in Figure 1. We see from Figure 1 that for $\phi = \phi_o$ along a separatrix we must have

$$\frac{\psi}{3} = \phi_o + \frac{\pi}{6} = 0.9465 \qquad (10)$$

or

$$\psi = 3\ \phi_o + \frac{\pi}{2}$$

The 61st harmonics of the sextupole field is written as A cos $(61\ \theta - \psi)$. The sextupole magnet approximated as a δ-function should be placed at a location where

$$(\text{argument of cosine}) = n\pi$$

or at

$$\theta = \frac{n\pi + \psi}{61} = \frac{1}{61} \left(\frac{2n + 1}{2} \pi + 3\ \phi_o \right) \qquad (11)$$

$$n = 0, 1, 2 \ldots\ldots 120, 121.$$

In addition, to produce a large A the sextupole magnet should be placed where β is large. A convenient place is in medium straight MA with $n = 4$ and $\theta \equiv \theta_A = 0.0402(2\pi)$. With the electrostatic septum at $\theta = 0$ the medium straight MA has a "length" stretching from $\theta = 0.0404(2\pi)$ to $\theta = 0.0422(2\pi)$. Thus, the sextupole magnet should be placed at the beginning of MA.

The necessary strength of the sextupole is given by the desired speed of streaming out of the phase points along the separatrix. With $\phi = \phi_o$ the second of Equ. (5) becomes

$$\frac{dR}{d\theta} = -AR^2 \sin (3\phi_o - \psi) = -AR^2 \sin (-\frac{\pi}{2}) = AR^2$$

The solution is

$$A\ (\Delta\theta) = \Delta\frac{1}{R} = \sqrt{\beta_s} \cos \phi_o \left(\Delta \frac{1}{x_s}\right) \quad (12)$$

We want the beam amplitude to grow from x_s = 30 mm = 0.03 m (position of septum) to x_s = 40 mm = 0.04 m in 3 revolutions ($\Delta\theta = 6\pi$). This gives

$$A = \frac{\sqrt{98\ \text{m}}}{6\pi} \cos (0.4229) \left(\frac{1}{0.03\ \text{m}} - \frac{1}{0.04\ \text{m}}\right) = 4.0\ \text{m}^{-1/2}$$

With a δ-function sextupole magnet at $\theta = \theta_A = \frac{4\pi + \psi}{61}$ where $\beta = \beta_A$ = 93 m we have

$$\text{61st harmonic of } \left[\frac{\nu_x}{16} \beta^{5/2} \frac{B''}{(B\rho)}\right]$$

$$= \text{61st harmonic of } \left[\frac{\nu_x}{16} \beta_A^{5/2} \frac{B''\ell}{(B\rho)} \frac{1}{\mathcal{R}} \delta(\theta - \theta_A)\right]$$

$$= \frac{\nu_x}{16\pi} \beta_A^{5/2} \frac{B''\ell}{(B\rho)} \frac{1}{\mathcal{R}} \cos 61 (\theta - \theta_A) \equiv A \cos (61\theta - \psi)$$

where ℓ = length of sextupole magnet, and $\mathcal{R}$ = radius of main ring. Therefore,

$$\frac{B''\ell}{(B\rho)} = \frac{16\pi}{\nu_x} \frac{\mathcal{R}}{\beta_A^{5/2}} A = \frac{48\pi}{61} \frac{10^3 \text{ m}}{(93 \text{ m})^{5/2}} 4.0 \text{ m}^{-1/2} \tag{13}$$

$$= 0.1186 \text{ m}^{-2}$$

At 200 BeV $(B\rho) = 0.67 \times 10^4$ kG m we get

$$\boxed{B''\ell = 794 \text{ kG/m}} \tag{14}$$

or

$$\begin{cases} \ell = 0.5 \text{ m, say} \\ B'' = 1588 \text{ kG/m}^2 \end{cases}$$

C. Onset of extraction

Extraction starts when the area of the stable triangle enclosed by the separatrices equals the emittance of the beam. Assuming the horizontal emittance of the beam to be $0.25\pi \times 10^{-6}$ m-rad this gives

$$\text{area} = \frac{3}{2} (R_u)^2 \sin \frac{\pi}{3} = \frac{3\sqrt{3}}{4} (R_u)^2 = 0.25\pi \times 10^{-6} \text{ m}$$

or

$$R_u = 0.778 \times 10^{-3} \text{ m}^{1/2}$$

and at the septum

$$\boxed{x_{su} = R_u \sqrt{\beta_s} \cos \frac{\psi}{3} = 0.0045 \text{ m} = 4.5 \text{ mm}} \tag{15}$$

The value of $\varepsilon = \frac{61}{3} - \nu_x$ at the onset of extraction is related to R_u by Equ. (8)

$$\varepsilon = \frac{A}{2} \frac{r^2 - (r - R_u)^2}{r} \qquad (16)$$

and depends on the amount of octupole error field present. For no octupole error field $r = \infty$ and

$$\boxed{\varepsilon = AR_u = 0.00311} \qquad (17)$$

During the beam spill of, say, 1 sec the value of ε must be controlled by the extraction trim quadrupoles to decrease smoothly from 0.00311 down to 0.

D. Tolerance on octupole error field in the quadrupoles

The parameters given above are based on the assumption that all octupole error fields are trimmed out by correcting octupole magnets. The effect of the octupole field is to move the 3 stable fixed point inward from ∞ and to close the separatrices on the outside (dotted lines in Figure 1). The extreme tolerance condition is that the separatrix should not curve back before x_s has reached the desired maximum value x_{smax} (0.04 m for the main ring). But, in practice, the curvature of the separatrix caused by the octupole field becomes appreciable even at $R \approx R_s$. If the value of R corresponding to x_{smax} is larger than R_s the quality of the extracted beam will suffer. At $x_s = x_{smax} = 0.04$ m

$$R = R_{max} \cong \frac{x_{smax}}{\sqrt{\beta_s} \cos \frac{\psi}{3}} = \frac{0.04 \text{ m}}{\sqrt{98 \text{ m}} \cos 0.9465} = 6.91 \times 10^{-3} \text{ m}^{1/2}$$

The condition

$$R_s \approx 2r > R_{max} \approx 6.91 \times 10^{-3} \text{ m}^{1/2}$$

gives

$$r = -\frac{A}{2B} > 3.46 \times 10^{-3} \text{ m}^{1/2} \qquad (18)$$

or

$$-B < \frac{A}{2 \times 3.46 \times 10^{-3} \text{ m}^{1/2}} = 580 \text{ m}^{-1} \qquad (19)$$

The β values at the F and D quadrupoles in a normal cell are respectively 98 m and 27 m. Since each quadrupole is 7 ft long and the cell length is 195 ft, if all F quadrupoles have an octupole error field $-B_Q'''$ ($B_Q''' > 0$) and all D quadrupoles have octupole field $+B_Q'''$ we have

$$B = \text{average of } \left[\frac{\nu_x}{16} \beta^3 \frac{B'''}{(B\rho)} \right]$$

$$= \frac{61}{48} \left[(98 \text{ m})^3 - (27 \text{ m})^3 \right] \frac{-B_Q'''}{(B\rho)} \frac{7 \text{ ft}}{195 \text{ ft}}$$

$$= -(42 \times 10^3 \text{ m}^3) \frac{B_Q'''}{(B\rho)}$$

And the tolerance is given by

$$\frac{B_Q'''}{(B\rho)} = (0.024 \times 10^{-3} \text{ m}^{-3}) (-B) < 0.014 \text{ m}^{-4} \qquad (20)$$

At 200 BeV $(B\rho) = 0.67 \times 10^4$ kGm and the tolerance on B_Q''' is

$$\boxed{B_Q''' < 94 \text{ kG/m}^3} \qquad (21)$$

We note here that because of the difference in β values at the F and the D quadrupoles the contribution to B comes mainly from the F quadrupoles.

The measured B_Q''' is about 10 times this tolerance value. This indicates that correcting octupole magnets are necessary. Assuming for the correcting octupoles

$$\left\{ \begin{array}{l} \text{aperture} = 6 \text{ cm radius} \\ B''' = 10^5 \text{ kG/m}^3 \text{ (3.6 kG on pole tip)} \\ \text{located near } \beta = 98 \text{ m} \end{array} \right.$$

to compensate for $-B_Q''' = -1000$ kG/m^3 (say) in the sixteen 7 ft long F quadrupoles in one superperiod we have a length of

$$\frac{1000 \text{ kG/m}^3}{10^5 \text{ kG/m}^3} \times 16 \times 7 \text{ ft} = 1.12 \text{ ft}$$

Thus, to correct for the octupole error field at 200 BeV we need a 1.12 ft, 10^5 kG/m^3 octupole magnet per superperiod placed near β = 98 m. With these octupoles it should be easy to reduce the octupole field to less than 10% of the tolerance level and make the octupole field negligible in the operation of the beam extraction system.

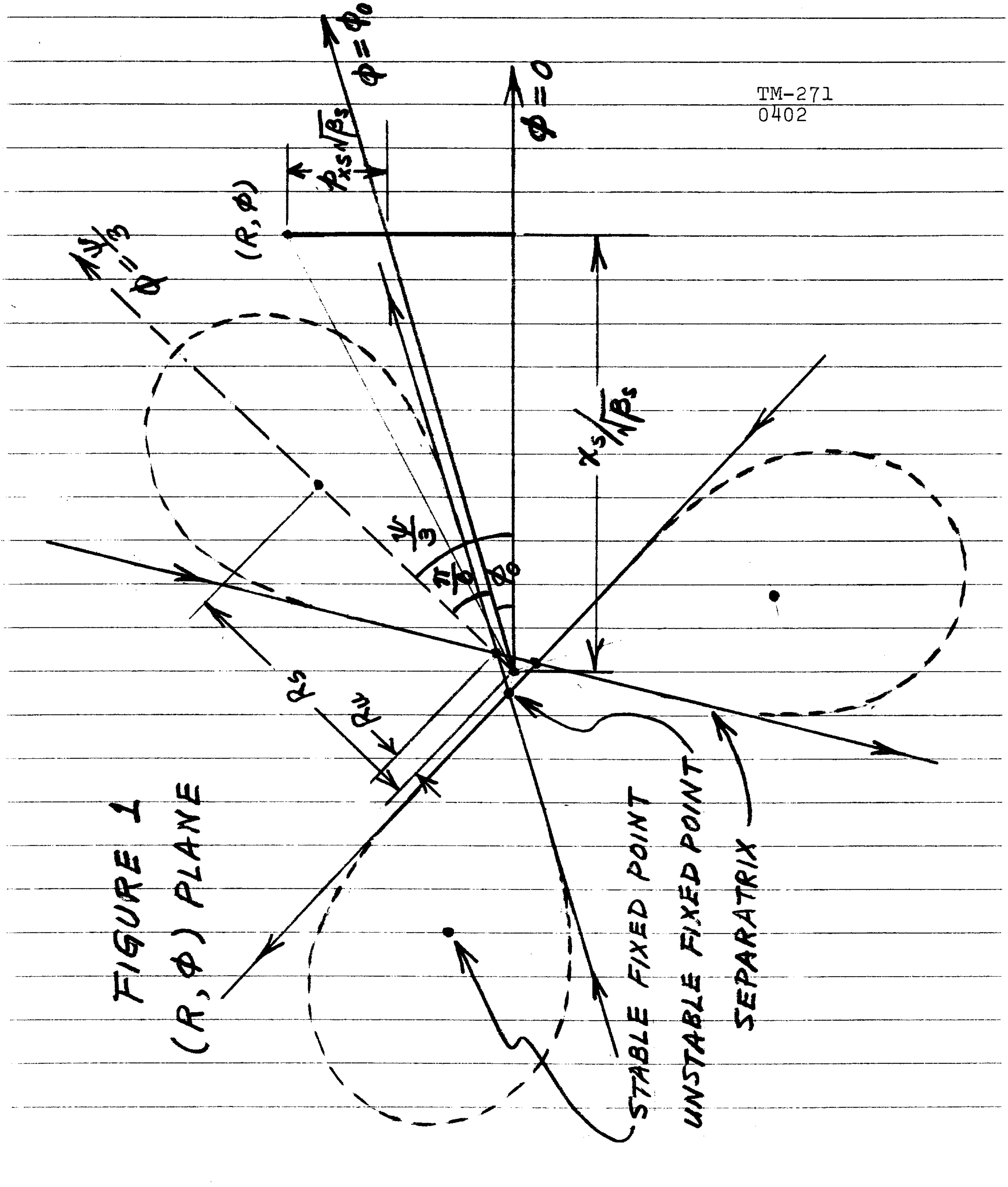

FIGURE 1
(R, Φ) PLANE