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# CORRECTION OF MAIN RING QUADRUPOLE POSITIONS USING CLOSED ORBIT INFORMATION

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Abstract

The computer program OTRIM is described and results are presented for some simple cases. The program calculates corrections to be applied to the transverse positions of the Main Ring quadrupoles. The input data consists of a set of measurements of the transverse position of the closed orbit relative to the quadrupoles -- the beam sensor readings. The basic program assumes one beam sensor for each transverse direction at each Main Ring station i.e. $\sim$ one beam sensor for each transverse direction per quadrupole. An arbitrary beam sensor configuration may be introduced simply by specifying, in the input data, the numbers of the beam sensors of the basic configuration which are inoperative. The results of some simple misalignment cases indicate that, in principle, OTRIM can reduce the closed orbit deviation by factors $\sim 10^8$ when all beam sensors are operative. With 10% of the beam sensors inoperative, the maximum reduction factor ranges from $\sim$ 20-40 (inoperative beam sensors arbitrarily distributed) to <2 (inoperative beam sensors are consecutive). The case in which there are beam sensors at only the F quadrupoles in a given plane was also considered in order to obtain an indication of the loss of correction power in the event that it was desired to halve

the system cost. The results for this case indicate a maximum correction factor $\sim$ 20-40. While this factor is probably adequate, it must be noted that, in actual practice, it will be further reduced by inoperative beam sensors, by errors in the beam sensor readings and the position corrections applied to the quadrupoles, and perhaps by non-linear effects. Thus it is rather doubtful if the halved system would be adequate in actual practice.

## Introduction

The position of the closed orbit with respect to the beam elements of a synchrotron is determined by the magnetic fields and the positions of the beam elements. In this report we will show how errors in the positioning of the beam elements may be reduced by knowledge of the closed orbit position at a number of points around the accelerator -- the beam sensor readings. Since there are more beam elements than beam sensors in the present case, we cannot hope to uniquely remove a given position error.

The Main Ring of the NAL 200 GeV accelerator has six superperiods and uses a separated function lattice. The most critical alignment requirements are for the transverse (horizontal, i.e. radial; vertical) location of the quadrupoles. Thus the errors in the transverse location of the quadrupoles are the only beam element position errors that we consider. Laslett has calculated[1] that the quadrupoles must be located with an accuracy of .01 in. in these directions for a 75%

probability that the corresponding closed orbit deviation is contained within a width of 1 in.

The positioning of the quadrupoles may be considered to be achieved in three steps -

(a) Construction survey in which the quadrupoles are located using the station markers[2] which were placed to an accuracy ∿ 1/16 in. during construction.

(b) Refined survey using precise levels, tapes and an alignment laser. The wire alignment system described in the NAL Design Report has recently been discarded. The refined survey will probably be done one lattice cell at a time. Its accuracy should be such as to achieve a circulating beam.

(c) Final positioning of the quadrupoles using the beam sensor readings as described in this report. This same correction scheme may be used to relocate the quadrupoles if the closed orbit happened to move during the life of the accelerator.

## Theory

In Figure 1 is shown the arrangement for the first of the six superperiods of the Main Ring. There are 14 standard cells (C) a half cell (DF), a medium straight cell (CM) and a one half cell replacement containing the long straight section (FLD). The parameters for all of these cells are taken from the report by Garren[3]. Small changes to the FLD section were made by Bellendir and Teng[4] during the progress of this work. They have not been included.

The following notation is used in reference to the complete Main Ring - B(I), I = 1,210 - the beam sensors which measure the closed orbit displacement relative to the appropriate beam element. This information is then used to correct the quadrupole positions. It is assumed that a beam sensor is located at each station marker, including the one at the center of the long straight section.

Y(I), I = 1,210 - the spy station beam sensors. This information is not used to correct the quadrupole positions. Its sole use is to enable us to investigate the remaining closed orbit deviations at points intermediate to the B(I) after the quadrupole positions have been corrected. In the present case the spy stations are somewhat academic as we would not expect their readings to differ significantly from those of the beam sensors B(I) because of the small separation between B(I) and Y(I).

H(I), I = 1,222 - the transverse displacement of the Ith beam element. From Figure 1 we note that the I = 2,3,36,37, etc., elements are quadrupole doublets whereas the remaining elements are singlets. The main reason for this choice of beam elements is to obtain a program which does not exceed the available space in present large computers (CDC 6600, IBM 360/75). Because of the small separation between members of the above doublets, the assumption of no relative displacement of the members is probably quite accurate. Notice also from Figure 1 that there is no quadrupole associated with H(1), H(38) etc.

In this report we use the correction scheme which has been described by Laslett and Lambertson[5]. We define the matrix S with the equation

$$B = S\ H \qquad (1)$$

where B and H are given by

$$B = \begin{pmatrix} B(1) \\ B(2) \\ \vdots \\ B(NB) \end{pmatrix}$$ i.e. column matrix of beam sensor readings, NB = 210 here

$$H = \begin{pmatrix} H(1) \\ H(2) \\ \vdots \\ H(NH) \end{pmatrix}$$ i.e. column matrix of displacement of accelerator components. NH = 222 here

We assume that the horizontal plane motion is not coupled to that of the vertical plane. Thus the two directions are computed independently and each has its own S matrix. The S matrices are computed by the program SYNCH[6] and the details of this computation are given in Appendix 1. For the combined function machine that they considered, Laslett and Lambertson found it convenient to displace the beam elements in gangs rather than individually. Thus if G is the ganging matrix for the gang scheme chosen, we have

$$B = TH \qquad (2)$$

where T = SG. Although we only consider individual displacements (G = 1), our calculations are set up so that any desired G matrix may be introduced.

In practice we will be faced with the situation where the B matrix is given by a set of beam sensor readings and we wish to find the set of displacement corrections $H_c$ which are to be applied in order to give a zero B matrix. Ideally we would invert (2) and obtain

$$H_c = -H = -T^{-1} B \tag{3}$$

In general T is not square and so the above step is not possible. Furthermore, it is not possible even when T is a square matrix because B contains only relative beam displacements and thus a unique $H_c$ does not exist, i.e. T has a zero value determinant. Thus it is necessary to use a different approach from that of Eqn. (3). From Eqn. (2) we have

$$\overline{B}\, T = \overline{H}\, \overline{T}\, T \tag{4}$$

where $\overline{B}$ is the transpose of B, etc. Laslett and Lambertson show that Eqn. (4) also results if one fits the beam sensor readings by least squares. We define the matrix M by

$$M = \overline{T}\, T \tag{5}$$

Thus M is a symmetric matrix with either zero or positive eigenvalues. If the Kth eigenvalue and normalized eigenvector are respectively $\lambda^K$ and $V^K$ we have

$$MV^K = \lambda^K V^K \tag{6}$$

We then expand H in terms of the $V^L$

$$H = \Sigma_L A_L V^L \tag{7}$$

By multiplying Eqn. (4) by $V^K$ from the right and using Eqns. (5), (6), (7) we obtain

$$A_K = \frac{\bar{B}\, T\, V^K}{\lambda^K} \tag{8}$$

Next we define the Q matrix from

$$\bar{H}_c = -\bar{H} = \bar{B}\, Q \tag{9}$$

Thus from Eqns. (7), (8), (9) we have

$$Q = -\sum_L \frac{T\, V^L\, \bar{V}^L}{\lambda^L} \tag{10}$$

Laslett and Lambertson also note that the sum of squares of the beam sensor readings is given by

$$\sum_I B^2\,(I) = \bar{B}B = \sum_L \lambda^L\, A_L^2 \tag{11}$$

i.e. the component of this sum due to the Lth eigenvector is proportional to $\lambda^L$. Thus Eqn. (11) shows that the small eigenvalues and their eigenvectors can be dropped from the sum over L in Eqn. (10) (of course, the $\lambda=0$ ones must be dropped) and the result will not be greatly affected. This is the key point of the Laslett and Lambertson method. They found that dropping the small eigenvalue eigenvectors from the Q sum reduced the closed orbit deviation at points intermediate to the beam sensors in some cases. This effect should not be large in our case as we have $\sim$ 1 beam sensor per quadrupole. They also found that the total amount of corrective displacement was significantly reduced as small eigenvalue eigenvectors were omitted. In practice we calculate several Q matrices and investigate the behavior of $H_c$ as the number of omitted eigenvectors is varied.

Computer Program OTRIM

The computer program OTRIM performs the computation of the displacement corrections $H_c$ to be applied to the quadrupoles from a given set of beam sensor readings B. The program uses two magnetic tapes. Tape 3 is the input tape (read only) containing the SYNCH output and obtained as set out in Appendix 1. Tape 4 is a scratch tape (both read and write).

The input data sets required are as shown at the top of Figure 2 and will be described here in detail --

First Set (1 card only) Format (16I5). This is the program instruction card and contains, in order, the input values for the following variables:

ITEST = 0 - beam sensor readings to be input with punched cards

= 1 - run test case for which only non-zero displacement is H(2) = +1.0 and beam sensor readings are obtained by OTRIM from SYNCH output (Tape 3).

ISPY = 0 - skip spy stations

= 1 - calculate closed orbit displacement at spy stations after corrections $H_c$ have been applied to beam elements. It is run only for test case, ITEST = 1.

NHOR = 0 - skip horizontal plane

= 1 - run horizontal plane

NVERT = 0 - skip vertical plane

= 1 - run vertical plane

KAPUTH - the number of inoperative beam sensors for the horizontal plane.

KAPUTV - the number of inoperative beam sensors for the vertical plane.

NVCHEK = 0 - skip orthogonality test on eigenvectors of $\bar{T}T$.

= 1 - perform orthogonality test.

NHARM = 0 - skip the harmonic analysis of GV.

= 1 - perform the harmonic analysis of GV.

NNNQ - the number of Q matrices which are to be calculated. This number must be sufficiently small to allow them to be written on the scratch tape (Tape 4). Using NNNQ = 6 requires that Tape 4 be 2400 feet with density 800 b.p.i. This resulted in frequent parity errors so we have usually taken NNNQ = 5 and 556 b.p.i.

Second Set Format (16I5) KAP(J), J=1, KAPUTH. The identification numbers of the beam sensors which are inoperative (i.e. no available reading) in the horizontal plane.

Third Set Format (16I5) NQ(J), J=1, NNNQ. The numbers of small non-zero eigenvalue eigenvectors that are to be dropped from the horizontal plane Q matrices. These numbers must be arranged in descending order. For NB<NH (our case) the smallest NQ(J) may be NQ(NNNQ)=0. However, if NB=NH, the smallest allowable NQ(J) is NQ(NNNQ)=1 (see later).

<u>Fourth Set</u> Format (8F10.4) BS(J), J=1, NB. The beam sensor readings for the <u>horizontal</u> plane. The number fields of inoperative beam sensors remain blank. For test case runs (ITEST=1) no BS(J) data set is required as OTRIM obtains the appropriate data from the SYNCH output.

<u>Fifth Set</u> Format (16I5) KAP(J), J=1, KAPUTV
<u>Sixth Set</u> Format (16I5) NQ(J), J=1, NNNQ
<u>Seventh Set</u> Format (8F10.4) BS(J), J=1, NB

} vertical plane data

When both horizontal and vertical planes are to be computed, the order of the input data must be as above. If only one plane is to be computed, the input data consists of the instruction card and the three sets corresponding to that plane.

A listing of OTRIM is contained in Appendix 2 of this report. The operation of the program is outlined by the flow diagram in Figure 2. The program begins by reading into the columns of a matrix SY either the horizontal plane or the vertical plane SYNCH output, according to the instruction card. The SYNCH output gives the absolute displacements of the closed orbit for the sequential displacement of the first superperiod quadrupoles by 1 unit. The SYNCH displacement sequence is shown in Figure 1. We require the displacement of the closed orbit relative to the beam elements - i.e. the beam sensor readings. Also we must include matrix elements corresponding to the beam sensors at the center of the long straight sections. Thus OTRIM sets up the required S matrix as follows -

$$\left.\begin{array}{lll} S(1,1) = -1. & & \\ S(I,1) = 0. & I = 2,\ NB & \end{array}\right\} \quad (12)$$

$$\left.\begin{array}{lll} S(I,J) = SY(I,J-1) & I = 1,\ NB \text{ but } I \neq 2. & J = 2,3 \\ S(I,J) = SY(I,J-1) - 0.5 & I = 2. & J = 2,3 \end{array}\right\} \quad (13)$$

$$\left.\begin{array}{lll} S(I,J) = SY(I,J-1) & I = 1,\ NB \text{ but } I \neq J-1. & J = 4,35 \\ S(I,J) = SY(I,J-1) - 1.0 & I = J-1 & J = 4,35 \end{array}\right\} \quad (14)$$

$$\left.\begin{array}{lll} S(I,J) = SY(I,J-1) & I = 1,\ NB \text{ but } I \neq 35. & J = 36,37 \\ S(I,J) = SY(I,J-1) - 0.5 & I = 35. & J = 36,37 \end{array}\right\} \quad (15)$$

Eqn. (12) gives the elements corresponding to the beam sensor at the center of the long straight section. In Eqn. (13) it has been assumed that the displacement of beam sensor B(2) is given by 1/2 (H(2) + H(3)). A similar assumption is made for B(35) in Eqn. (15). The remaining beam sensors are assumed to be fixed to the adjacent quadrupoles - Eqn. (14). Eqns. (12), (13), (14), (15) set up 1/6 of the S matrix. The remaining 5/6 of this matrix is obtained by cyclic symmetry

$$S(I+\frac{N}{6} \times NB,\ J+\frac{N}{6} \times NH) = S(I,J) \qquad (16)$$

with N an integer. The indices I and J are modulo NB and NH respectively.

OTRIM next proceeds to calculate the matrix T=SG. We have simply used G=1 in this work, but any desired ganging matrix may be introduced at this point. Since we measure only relative displacement of the closed orbit, the displacement of all beam elements uniformly by 1 unit (horizontally or vertically) should produce no change in the closed orbit.

Thus the program checks for zero sum of the elements of each row of the T matrix. In fact the sums for the vertical direction may be $\sim 1 \times 10^{-2}$ due to slight vertical focussing at the entrance and exit planes of the bending magnets.

The inoperative beam sensors KAP(J) are allowed as the next step in the computation. The corresponding rows are dropped from the T matrix and the resulting reduced matrix is the R matrix. The matrix $M = \overline{R}R$ is then computed. KAP(I), T and M are all stored on the scratch tape for subsequent use.

The non-zero eigenvalues and their associated eigenvectors are computed and arranged in descending order by the subroutine EIGEN[7] for the matrix M. If NR is the number of rows in R and NR < NH (the present case) then the number of non-zero eigenvalues is NR. Thus the arguments of EIGEN are set up so as to compute for the NR largest eigenvalues. If we had a case where NR = NH, then the last of the NR eigenvalues computed would be zero (see below Eqn. 3). Thus for the NR = NH case the minimum value for NQ (NNNQ) is 1.

If NVCHEK = 1, then subroutine VCHEK checks that the eigenvectors of M are orthonormal using the equations

$$\begin{aligned} \overline{V}^I \, M \, V^J &= \lambda^J \qquad I = J \\ &= 0 \qquad I \neq J \end{aligned} \tag{17}$$

The subroutine assigns a limit of $(10^{-10} + 2 \times 10^{-8}\, \lambda^J)$ to the difference between left and right sides of Eqn. (17). Typically $\sim$ 10% of the smaller eigenvectors fall outside this limit by $\sim 10^{-9}$ and this is considered quite satisfactory.

If NHARM = 1, subroutine HAR will perform the harmonic analysis of $GV^L$. For each non-zero eigenvector, OTRIM lists the four largest amplitudes and their frequencies. In the present case the harmonic analysis is unreliable and should be omitted because it assumes equal argument increments whereas in fact the H(I) are spaced as shown in Figure 1. The harmonic behavior of the Main Ring is best understood by studying its harmonic response as shown in Appendix 3.

OTRIM is now ready to compute the NNNQ Q matrices which involve the omission or various numbers NQ of small eigenvalue eigenvectors. For reasons of computer economy the program calculates one column of <u>all</u> NNNQ Q matrices and stores these NNNQ columns as a record on Tape 4. It then proceeds to the next column of all Q matrices, and so on, until all NNNQ Q matrices are complete.

Each Q matrix is then read back from Tape 4 and the program computes the recommended displacement corrections HQ (equivalent to $H_c$) from Eqn. (9) and the total correction motion HQTOT.

$$HQTOT = \Sigma \ |HQ| \qquad (18)$$

For the test case the residual beam sensor readings BE are computed

$$BE = T \ (H+HQ) \qquad (19)$$

Also for the test case the subroutine NUMBER computes the maximum beam sensor reading, the r.m.s. beam sensor reading and the r.m.s. value of the local maxima of beam sensor reading$_1$. These parameters are computed both before and after application of HQ so that the improvement may be noted.

If ISPY = 1 and the test case is being run, OTRIM will proceed to compute, for each Q matrix, the residual displacement of the closed orbit relative to the beam elements at the spy stations, i.e. the residual beam sensor readings at the spy stations, BSP. Referring to Eqn. (1), we now have B replaced by Y and it is necessary to set up the corresponding spy S matrix. It is assumed that the spy station sensors are fixed to the adjacent quadrupoles. Thus the procedure is similar to that in Eqns. (12)-(15) except that in the present case there is no beam sensor whose motion is determined by the motion of two quadrupoles as in Eqns. (13) and (15). Also the spy S matrix is set up directly in the SY array into which the spy SYNCH data has been read (beginning in column 2) as the S array of OTRIM contains data which is required later. Thus the equations are

$$\left.\begin{array}{lll} SY(I,J) = 0. & I = 1, NB. & J = 1 \\ SY(I,J) = SY(I,J)-1. & I = 1, & J = 2 \\ SY(I,J) = SY(I,J)-1. & I = J-2, & J = 4,37 \end{array}\right\} (20)$$

All other elements in the first 1/6 of SY will be correct as read in. The remaining 5/6 of the matrix is obtained by cyclic symmetry and then the BSP values are computed.

OTRIM has now completed the computation for the plane in which it started. If this plane was the horizontal plane and NVERT = 1, then it returns to compute for the vertical plane. Otherwise, the computation is complete and the program ends.

Results

(a) No inoperative beam sensors, NKAPUT = 0

In Figure 3 we have plotted the displacement in both planes of the accelerator components before and after correction for the test case H(2) = +1.0. Only the components near H(2) are shown for two Q matrices (NQ=0 and NQ=108) which correspond to the omission of 0 and 108 non-zero small eigenvalue eigenvectors. As NQ is reduced, the maximum residual error is reduced and the accelerator components are placed on a smooth bump relative to the undisplaced positions. At most, the bump extends over $\sim$ one superperiod near the displaced element. The remainder of the accelerator is unchanged.

Figure 4 shows the variation in both planes of maximum |beam sensor reading| after correction, as NQ is changed. The initial displacements are also shown. The general trend is for a reduced residual beam sensor reading as NQ is reduced. For NQ = 0, the residual beam sensor readings are only $\sim 10^{-8}$. Even when NQ = 108 (i.e. slightly more than half the eigenvectors are omitted) it is still possible to reduce the closed orbit deviation by a factor $\sim$ 10. The variation of HQTOT with NQ is shown in Figure 5. It is seen that HQTOT may increase by a factor $\sim$ 2-3 as NQ is reduced over the range shown.

As will be discussed later (Appendix 3) it is important that OTRIM is able to correct 20th harmonic (integer closest to $\nu$) disturbances. Figures 6, 7, and 8 show the results when the 20th harmonic disturbance extended $\sim$ 1 wavelength. As NQ is reduced the beam elements are moved from their disturbed positions on to a small smooth bulge. Residual beam sensor readings $\sim 10^{-8}$ are theoretically possible in this case also. In Figure 8 we see that HQTOT tends to oscillate about the initial sum, first increasing and then decreasing as HQ is reduced.

Figures 9 and 10 present results for a disturbance of $\sim$ 1 wavelength of 40th harmonic. They indicate that smaller (factor 2-4) residual beam sensor readings are possible, but otherwise are similar to 20th harmonic case.

(b) 96 inoperative beam sensors, NKAPUT = 96

This case is interesting because it indicates whether we can hope to obtain sufficient information by placing beam sensors for a given plane at only the focussing quadrupoles for that plane, i.e. where the beam width function is a maximum. Such a scheme would represent a large financial saving. It is in contrast to (a) above where each beam sensor actually consists of a pair, one for the horizontal plane and one for the vertical plane. In the present case we have assumed that both horizontal plane and vertical plane beam sensors exist at B1, B2, B35, B36, B37, etc. Otherwise beam sensors exist only at QF quadrupoles in the plane of interest.

The results have been plotted in Figures 11 and 12 for the test case H(2) = +1.0. It is seen that the reduction in closed orbit deviation is now only ∿factor 20-40 in the most favorable case. Also the trend as NO is varied is not monatonic for small values of NQ. Thus it may be necessary to choose somewhat larger NQ values (and consequently a smaller reduction of closed orbit deviation) in an actual case. It is seen from Figure 12 that HQTOT tends to decrease slightly as NQ is reduced to small values ∿ 10. For further reduction of NQ HQTOT may increase slightly in some cases.

(c) 21 inoperative beam sensors, NKAPUT = 21

In this case we have returned to case (a) and imagined that 10% of the beam sensors are inoperative. This is an attempt to represent an actual situation since at any given time there will probably exist some inoperative beam sensors - say 10%. We chose 21 inoperative beam sensors in an arbitrary way and the results are plotted in Figures 13 and 14. It is seen that both the maximum residual beam sensor reading and HQTOT are rather slowly varying with NQ. The maximum reduction in closed orbit deviation is ∿ factor 40 now and so the correction is much poorer than in the case (a).

Also shown in Figures 13 and 14 is the case 21 consecutive inoperative beam sensors for the horizontal plane. As might be expected, this is a severe loss of information and OTRIM is only able to reduce the closed orbit deviation by ∿ 20%.

Discussion

Although we have considered only a few misalignment examples, these examples are important as Lambertson and Laslett found that the single misaligned quadrupole case was the most difficult to correct. Also the 20th harmonic is expected to be the most important misalignment harmonic (Appendix 3). In the present accelerator, these misalignments can be largely corrected by OTRIM even when a significant fraction of the beam sensors are inoperative. This difference in behavior is probably due to the fact that with the present separated function machine there is ∿ 1 beam sensor per focussing magnet whereas with the combined function machine of Lambertson and Laslett there was only ∿ 1 beam sensor per 7 focussing magnets. For the same reason, in the present case the spy stations do not show residual deviations that are much different from the residual deviations at the beam sensors. Also in our work there does not appear to be excessive total component displacement HQTOT as NQ is reduced. The residual beam sensor error is usually much smaller as NQ is reduced. Although it would be desirable to consider more misalignment examples, at present there appears to be no reason for not choosing NQ=0.

The question of whether it is sufficient to use beam sensors at only the QF quadrupoles of a given plane requires some further study before a definite answer may be given. The present work indicates that the closed orbit can still be corrected by a factor ∿ 20-40, which would probably be adequate. However, it must be noted that the above correction factor will be reduced when allowance is made for --

1. Inoperative beam sensors

2. Errors in beam sensor readings

3. Errors in position corrections applied to beam elements

4. Non-linear effects (if there are any?)

Thus it appears rather doubtful that it will be adequate to use beam sensors at only the QF quadrupoles. While such an arrangement may halve the cost of the beam sensor system, it reduces the maximum possible closed orbit correction by orders of magnitude.

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## References

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Applied Mathematics Division, Argonne National Laboratory, December, 1967; Revised, June 1968. The subroutine is set up for easy conversion to CDC 6600 use.

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Appendix 1 SYNCH Input Data

The lattice used is that described by Garren[3]. The input is set up for the system of beam sensors and spy stations shown in Figure 1. The elements of the first superperiod are displaced 1 unit in sequence. For each of these displacements SYNCH calculates the position of the closed orbit, in addition to other parameters, at all beam sensors and spy stations around the accelerator. The output which is written on the output Tape 5 under the format (A5, 2I3, 5F14.8) is as follows -

Name of Matrix (RING in our case)

Number of the beam sensor or spy staion, n. (1-420)

Total number of beam sensors and spy stations (420)

Distance from beginning to n

Closed orbit position, horizontal plane, at n

Closed orbit position, vertical plane, at n

Horizontal beta function at n

Vertical beta function at n

The desired SY matrix is then obtained by using appropriate read formats in OTRIM. In the case we have considered, Tape 5 was 2400 ft. long and 556 b.p.i.

# *Input Data for SYNCH*

```
FOR
 MIS.    RUN
         BMIS
         REM      DISPLACEMENTS OF QUADS. LATTICE IS 1969 REV. 2/14/69
         REM      B. SENSORS 1--35 ARE AT T. COLLINS STATIONS 0-34.TS-2
         REM      B. SENSORS 2,35 ARE 98.5FT FROM CENTRE LSS.A21+A23=A2
         REM      B.SENSOR34 IS (44IN.-QFE) DWNSTRM QFL. L1Q+L1B=L1
         REM      BEAM SPY STATIONS DNSTREAM QUAD ENTRANCE FROM SENSOR
         REM      AT LONG STR. (QFL,QFL),(QDL,QD),(QF,QFL),(QDL,QDL)
         REM                       ARE MOVED IN PAIRS SHOWN BY BRACKETS
         REM      UNITS ARE METERS, KILOGAUSS---FOR 400 GEV PROTONS
         REM      CELL QUADS QF2,QD2 = (QF,QD)/2=2.1336/2=1.0668
         REM      LONG STR.QUADS (QF,QD)DASH=QFL,QDL=(QF,QD)/2+(QFE,QDE)
         REM                                                 =1.31953
         REM      BENDING MAGNET (CURVED ORBIT)              =6.0706
         REM      MINI STRAIGHTS 0                           =0.3048
         REM      SHORT STRAIGHTS 00                         =2.1082
         REM      MEDIUM STRAIGHTS LM                        =14.859
         REM      LONG STRAIGHT LEFT L1                      =9.29767
         REM      LONG STRAIGHT CENTER L                     =5 .83429
         REM      LONG STRAIGHT RIGHT L2                     =2.92227
         REM      LONG STRAIGHT SEPARATION A1                =0.346202
         REM                               A2                =1.77292
         REM                               A3                =0.271018
 0       DRF      .3048
 00      DRF      2.1082
 LM      DRF      14.859
 L1      DRF      9.29767
 L2      DRF      2.92227
 LHA     DRF      25.417-45
 A1      DRF      0.346202
 A2      DRF      1.77292
 A3      DRF      0.271018
         REM      OSX IS SENSOR DIST. FROM QUAD, NORM CELL OSX+OOSX=OO
 OSX     DRF      0.1524
 OOSX    DRF      1.9558
 A23     DRF      0.881506
 A21     DRF      0.891414
 L1Q     DRF      0.96647
 L1B     DRF      8.3312
 LMX     DRF      14.7066
 GF      =        +25 .5344
 GD      =        -25 .5344
 BRHO    =        13373.8188
 BO      =        17.88392
 B       MAG      6.0706      .          BRHO        BO
 QFL     MAG      1.31953     GF         BRHO        0.
 QDL     MAG      1.31953     GD         BRHO        0.
 QF      MAG      2.1336      GF         BRHO        0.
 QD      MAG      2.1336      GD         BRHO        0.
 QF*     MAGS     QF          1.         1.          1.          1.
 QD*     MAGS     QD          1.         1.          1.          1.
 QFL*    MAGS     QFL         1.         1.          1.          1.
 QDL*    MAGS     QDL         1.         1.          1.          1.
 BL      MMM    8 B     0     B     0    B     0     B     0
         REM      T(I) ARE FIXED MATRICES, ALSO XL,FX,OX NORMAL CELLS
         REM      P(I) ARE MATRICES VARIED (FIRST SUPERPERIOD)
         REM      S(I) ARE SHIFT MATRICES
```

} *Misaligned Quadrupoles.*

```
XL       MMM     2     OOSX  BL
FX       MMM     2     QF    OSX
DX       MMM     2     QD    OSX
T1       EQU           LHA
T2       MMM     4     QF    A3    QFL   A23
T3       MMM     6     A21   QDL   A1    QDL   L2    BL
T4       MMM     2     QDL   OSX
T15      MMM     5     LMX   B     0     B     0
T66      MMM     2     QFL   L1Q
T67      MMM     7     L1B   B     0     B     0     B     0
T68      MMM     4     QFL   A1    QFL   A21
T69      MMM     4     A23   QDL   A3    QD
T70      EQU           LHA
S2       MMM     4     QF*   A3    QFL*  A23
S3       MMM     6     A21   QDL*  A1    QDL*  L2    BL
S4       MMM     2     QDL*  OSX
SF*      MMM     2     QF*   OSX
SD*      MMM     2     QD*   OSX
S66      MMM     2     QFL*  L1Q
S68      MMM     4     QFL*  A1    QFL*  A21
S69      MMM     4     A23   QDL*  A3    QD*
P1       EQU           T1
P3       EQU           T3
P4       EQU           T4
P6       EQU           FX
P10      EQU           FX
P14      EQU           FX
P18      EQU           FX
P22      EQU           FX
P26      EQU           FX
P30      EQU           FX
P34      EQU           FX
P38      EQU           FX
P42      EQU           FX
P46      EQU           FX
P50      EQU           FX
P54      EQU           FX
P58      EQU           FX
P62      EQU           FX
P66      EQU           T66
P68      EQU           T68
P69      EQU           T69
P8       EQU           DX
P12      EQU           DX
P16      EQU           DX
P20      EQU           DX
P24      EQU           DX
P28      EQU           DX
P32      EQU           DX
P36      EQU           DX
P40      EQU           DX
P44      EQU           DX
P48      EQU           DX
P52      EQU           DX
P56      EQU           DX
P60      EQU           DX
P64      EQU           DX
         PAGE
DO       SUB
KING     MMM     42    T1    S2    P3    P4    XL    P6    XL    P8    XL    P10   XL    P12
                       XL    P14   T15   P16   XL    P18   XL    P20   XL    P22   XL    P24
```

```
                        XL    P26   XL    P28   XL    P30   XL    P32   XL    P34   XL    P36
                        XL    P38   XL    P40   XL    P42   XL    P44   XL    P46   XL    P48
                        XL    P50   XL    P52   XL    P54   XL    P56   XL    P58   XL    P50
                        XL    P62   XL    P64   XL    P66   T67   P68   P69   T70   T1    T2
                        T3    T4    XL    FX    XL    DX    XL    FX    XL    DX    XL    FX
                        T15   DX    XL    FX    XL    DX    XL    FX    XL    DX    XL    FX
                        XL    DX    XL    FX    XL    DX    XL    FX    XL    DX    XL    FX
                        XL    DX    XL    FX    XL    DX    XL    FX    XL    DX    XL    FX
                        XL    DX    XL    FX    XL    DX    XL    FX    XL    DX    XL    FX
                        XL    DX    XL    T66   T67   T68   T69   T70   T1    T2    T3    T4
                        XL    FX    XL    DX    XL    FX    XL    DX    XL    FX    T15   DX
                        XL    FX    XL    DX    XL    FX    XL    DX    XL    FX    XL    DX
                        XL    FX    XL    DX    XL    FX    XL    DX    XL    FX    XL    DX
                        XL    FX    XL    DX    XL    FX    XL    DX    XL    FX    XL    DX
                        XL    FX    XL    DX    XL    FX    XL    DX    XL    FX    XL    DX
                        XL    T66   T67   T68   T69   T70   T1    T2    T3    T4    XL    FX
                        XL    DX    XL    FX    XL    DX    XL    FX    T15   DX    XL    FX
                        XL    DX    XL    FX    XL    DX    XL    FX    XL    DX    XL    FX
                        XL    DX    XL    FX    XL    DX    XL    FX    XL    DX    XL    FX
                        XL    DX    XL    FX    XL    DX    XL    FX    XL    DX    XL    FX
                        XL    DX    XL    FX    XL    DX    XL    FX    XL    DX    XL    T55
                        T67   T68   T69   T70   T1    T2    T3    T4    XL    FX    XL    DX
                        XL    FX    XL    DX    XL    FX    T15   DX    XL    FX    XL    DX
                        XL    FX    XL    DX    XL    FX    XL    DX    XL    FX    XL    DX
                        XL    FX    XL    DX    XL    FX    XL    DX    XL    FX    XL    DX
                        XL    FX    XL    DX    XL    FX    XL    DX    XL    FX    XL    DX
                        XL    FX    XL    DX    XL    FX    XL    DX    XL    T66   T57   T58
                        T69   T70   T1    T2    T3    T4    XL    FX    XL    DX    XL    FX
                        XL    DX    XL    FX    T15   DX    XL    FX    XL    DX    XL    FX
                        XL    DX    XL    FX    XL    DX    XL    FX    XL    DX    XL    FX
                        XL    DX    XL    FX    XL    DX    XL    FX    XL    DX    XL    FX
                        XL    DX    XL    FX    XL    DX    XL    FX    XL    DX    XL    FX
                        XL    DX    XL    FX    XL    DX    XL    T66   T67   T68   T59   T70
        CYB   42        RING              1  <-- Write tape
        END
        CALL            DO
S2      EQU             T2
P3      EQU             S3
        CALL            DO
P3      EQU             T3
P4      EQU             S4
        CALL            DO
P4      EQU             T4
P6      EQU             SF*
        CALL            DO
P6      EQU             FX
P8      EQU             SD*
        CALL            DO
P8      EQU             DX
P10     EQU             SF*
        CALL            DO
P10     EQU             FX
P12     EQU             SD*
        CALL            DO
P12     EQU             DX
P14     EQU             SF*
        CALL            DO
P14     EQU             FX
P16     EQU             SD*
        CALL            DO
P16     EQU             DX
```

```
P18     EQU         SF*
        CALL        DO
P18     EQU         FX
P20     EQU         SD*
        CALL        DO
P20     EQU         DX
P22     EQU         SF*
        CALL        DO
P22     EQU         FX
P24     EQU         SD*
        CALL        DO
P24     EQU         DX
P26     EQU         SF*
        CALL        DO
P26     EQU         FX
P28     EQU         SD*
        CALL        DO
P28     EQU         DX
P30     EQU         SF*
        CALL        DO
P30     EQU         FX
P32     EQU         SD*
        CALL        DO
P32     EQU         DX
P34     EQU         SF*
        CALL        DO
P34     EQU         FX
P36     EQU         SD*
        CALL        DO
P36     EQU         DX
P38     EQU         SF*
        CALL        DO
P38     EQU         FX
P40     EQU         SD*
        CALL        DO
P40     EQU         DX
P42     EQU         SF*
        CALL        DO
P42     EQU         FX
P44     EQU         SD*
        CALL        DO
P44     EQU         DX
P46     EQU         SF*
        CALL        DO
P46     EQU         FX
P48     EQU         SD*
        CALL        DO
P48     EQU         DX
P50     EQU         SF*
        CALL        DO
P50     EQU         FX
P52     EQU         SD*
        CALL        DO
P52     EQU         DX
P54     EQU         SF*
        CALL        DO
P54     EQU         FX
P56     EQU         SD*
        CALL        DO
P56     EQU         DX
P58     EQU         SF*
```

```
         CALL          DO
P58      EQU           FX
P60      EQU           SD*
         CALL          DO
P60      EQU           DX
P62      EQU           SF*
         CALL          DO
P62      EQU           FX
P64      EQU           SD*
         CALL          DO
P64      EQU           DX
P66      EQU           S65
         CALL          DO
P66      EQU           T66
P68      EQU           S68
         CALL          DO
P68      EQU           T68
P69      EQU           S69
         CALL          DO
         FIN
         STOP
EOF
```

Appendix 2 OTRIM Listing

The program was kept on the CIMS tape at N.Y.U. in order to reduce the probability of transmission error. The corresponding control cards are shown above the listing of OTRIM and it is also shown where any desired changes to the program would be inserted. The reader should consult the CIMS User's Manual for explanation of how changes are made. The data cards shown are for a test case run (ITEST=1) including spy stations (ISPY=1) in the horizontal plane (NHOR=1) and with 5 Q matrices to be calculated (NNNQ=5). The numbers of small eigenvalue eigenvectors to be omitted are 108, 68, 28, 8, 0 respectively.

In order to demonstrate how an arbitrary misalignment may be introduced, we have shown, at the appropriate place in the listing, the changes required to run the 20th harmonic misalignment.

OTRIM LISTING

```
EOF
G2D6002,T3400,CM337000,L4000.  NAL REMOTE BINGHAM
REWIND(OUTPUT)                                              C
REQUEST CIMST.                                              C
CIMS3.                                                      C
REQUEST,T312=TAPE3.  READ SYNCH                             C
REQUEST,T337=TAPE4.   WRITE,READ T,M,Q.                     C
REWIND(TAPE3)                                               C
REWIND(TAPE4)                                               C
MAP(PART)                                                   C
RUN(G,,,TAPEZ,EROUT)                                        C
*CXIT.
EREDIT.
*FIN.
REMOTE OUTPUT,252.
EOR
$GET      OTRIM               L  <------ changes if any.
$LAST                         R
$END                          Z
EOR
   1    1    1    0    0    0    0    0    5
  108   68   28    8    0
EOF
```

Control Cards For OTRIM Stored on CIMS. Tape at NYU.

Data

```
      PROGRAM OTRIM  (INPUT=223,OUTPUT=223,TAPE3,TAPE4)
C OTRIM READS IN SYNCH OUTPUT SY FROM TAPE3, GENERATES S,G,T,EM(=M).
C T=S*G.  B=T*H.  EM=T(TRANSPOSE)*T. GANGING MATRIX G=1 HERE.
C EIGEN DETERMINES E.VECTORS AND E.VALUES OF EM. Q MATRIX IS FOUND.
C HQ =B*Q ARE QUAD.POSITION CORRECTIONS,WHERE B ARE B.SENSOR READINGS.
C TAPE4 IS SCRATCH TAPE FOR T,M,AND AND Q/6 MATRICES
      COMMON/F202/ AMP,HQ,W2,KAP
      COMMON/F203/ INDEXX(223),TT(223,3),NQ(20)
      DIMENSION EM(223,223),B(223,223),VALU(223),W1(223),W2(223)
      DIMENSION AMP(223),AMPMX(4),IAMP(4),V(223,223),HQ(223),TEM(223)
      DIMENSION GG(223,223),BS(223),H(223),BE(223),VVJM(223)
      DIMENSION T(223,223),Q(223,223),SY(223,36),S(223,223)
      DIMENSION G(223,223),KAP(223),R(223,223),BSP(223)
      EQUIVALENCE (AMP,W1,BS),(S,T,EM,GG),(G,SY,R,B,V,Q),(HQ,TEM),(VALU,
     1H),(W2,BE,BSP,VVJM)
      LDIM=223
      NH=222
      NB=210
      NBS=NB/6
      NHS=NH/6
      NGC=NH
      NGR=NH
      NSR=NB
      NTR=NSR
      NTC=NGC
      NSC=NGR
      READ 3010,ITEST,ISPY,NHOR,NVERT,KAPUTH,KAPUTV,NVCHEK,NHARM,NNNQ
 3010 FORMAT (16I5)
      IF(NHOR.NE.1.AND.NVERT.NE.1) 3051,3052
 3051 PRINT 3050
 3050 FORMAT (34HOTRIM WAS NOT ASKED TO DO ANYTHING)
      CALL EXIT
 3052 PRINT 3011,ITEST,ISPY,NHOR,NVERT,KAPUTH,KAPUTV,NVCHEK,NHARM,NNNQ
```

```
3011 FORMAT (19HINSTRUCTION CARD***,6HITEST=,I1,2X5HISPY=,I1,2X5HNHOR=,
    1I1,2X  6HNVERT=,I1,2X7HKAPUTH=,I3,2X7HKAPUTV=,I3,2X7HNVCHEK=,I1,2X
    26HNHARM=,I1,2X5HNNNQ=,I1)
3020 CONTINUE
     REWIND 3
     REWIND 4
     IF(NHOR.EQ.1) GO TO 3030
     IF(NVERT.EQ.1) GO TO 3040
     CALL EXIT
3030 PRINT 3060
3060 FORMAT (1H1/,47H                    ORBIT ANALYSIS IN HORIZONTAL PLANE//)
     KAPUT=KAPUTH
     GO TO 3070
3040 PRINT 3080
3080 FORMAT (1H1/,45H                    ORBIT ANALYSIS IN VERTICAL PLANE//)
     KAPUT=KAPUTV
3070 CONTINUE
     DO 3090 J=1,36
     DO 3090 I=1,NB
     IF (NHOR.EQ.1) GO TO 3100
     READ (3,3120) MN,IPOS,KKK,EL,BETX5,SY(I,J),BETX7,BETY7
     GO TO 3090
3100 READ (3,3120) MN,IPOS,KKK,EL,SY(I,J),BETY5,BETX7,BETY7
3120 FORMAT (A5,2I3,5F14.8)
3090 READ (3,3120) MN,IPOS,KKK,EL,BETX5,BETY5,BETX7,BETY7
     READ (3,3120) EOF
     IF (EOF,3) 3130,3131
3131 PRINT 3140
3140 FORMAT (21HDO NOT FIND SYNCH EOF)
     CALL EXIT
3130 CONTINUE
6700 FORMAT (1X10(1XF9.5))
     NFLAG=1
3078 FORMAT(1X2I2)
C EXPAND SY MATRIX AND SUBTRACT OUT BEAM SENSOR DISPLACEMENT TO GET
C DISP. OF BEAM RELATIVE TO B.SENSORS.
     S(1,1)=-1.
     DO 1204 I=2,NSR
1204 S(I,1)=0.
     DO 1207 I=1,NSR
     DO 1207 J=2,3
     IF (I.EQ.2) GO TO 1208
     S(I,J)=SY(I,(J-1))
     GO TO 1207
1208 S(I,J)=SY(I,(J-1))-0.5
1207 CONTINUE
     DO 1211 I=1,NSR
     DO 1211 J=4,35
     IF (I.EQ.(J-1)) GO TO 1210
     S(I,J)=SY(I,(J-1))
     GO TO 1211
1210 S(I,J)=SY(I,(J-1))-1.
1211 CONTINUE
     DO 1215 I=1,NSR
     DO 1215 J=36,37
     IF (I.EQ.35) GO TO 1214
     S(I,J)=SY(I,(J-1))
     GO TO 1215
1214 S(I,J)=SY(I,(J-1))-0.5
1215 CONTINUE
C GENERATE REMAINING 5/6 OF S MATRIX
```

```
      NSCS=NSC/6
      NSRS=NSR/6
      DO 1021 J=1,NSCS
      DO 1021 I=1,NSR
      DO 1021 N=1,5
      IF (I+N*NSRS-NSR) 1023,1023,1024
 1023 S(I+N*NSRS,J+N*NSCS)=S(I,J)
      GO TO 1021
 1024 S(I+N*NSRS-NSR,J+N*NSCS)=S(I,J)
 1021 CONTINUE
 1029 FORMAT (3(2F14.8,5X))
C CONSTRUCT GANGING MATRIX G.EXAMPLE TRIVIAL CASE G=1.
      DO 1050 J=1,NGC
      DO 1050 I=1,NGR
      IF (J.EQ.I) GO TO 1051
      G(I,J)=0.
      GO TO 1050
 1051 G(I,J)=1.
 1050 CONTINUE
C CALCULATES T=S*G BY ROWS AND STORE IN T (EQUIV S) ROWS
      DO 1058 I=1,NSR
      DO 1057 J=1,NGC
      TEM(J)=0.
      DO 1057 K=1,NSC
 1057 TEM(J)=TEM(J)+S(I,K)*G(K,J)
      DO 1059 L=1,NGC
 1059 T(I,L)=TEM(L)
 1058 CONTINUE
C CHECK ZERO SUM OF ELEMENTS OF EACH ROW OF T MATRIX
      DO 1085 I=1,NTR
      TEM(I)=0.
      DO 1085 J=1,NTC
 1085 TEM(I)=TEM(I)+T(I,J)
      PRINT 6700,(TEM(I),I=1,NTR)
      NROW=0
      DO 1084 I=1,NTR
      IF(ABS(TEM(I)).GT.1.0E-6) NROW=1
 1084 CONTINUE
      IF(NROW.EQ.1.AND.NHOR.EQ.1) PRINT 1083
 1083 FORMAT (52H***ERROR      SUM OF ELEMENTS OF T ROW GREATER 1.0E-6)
C REDUCE T MATRIX BY B.SENSORS THAT ARE KAPUT. R IS REDUCED T MATRIX
      IF(KAPUT.EQ.0) GO TO 3162
      READ 3160,(KAP(I),I=1,KAPUT)
 3160 FORMAT (16I5)
      PRINT 3170
      PRINT 3160,(KAP(I),I=1,KAPUT)
 3170 FORMAT (36HB.SENSORS CRAPPED OUT ARE AS FOLLOWS/)
 3162 CONTINUE
      N=0
      DO 3180 I=1,NTR
      IF(KAPUT.EQ.0) GO TO 3200
      NNKAP=0
      DO 3190 L=1,KAPUT
      IF(KAP(L).EQ.I)NNKAP=1
 3190 CONTINUE
      IF(NNKAP.EQ.1) GO TO 3180
 3200 N=N+1
      DO 3210 J=1,NTC
 3210 R(N,J)=T(I,J)
 3180 CONTINUE
      NR=N
```

```
      WRITE (4) (KAP(I),I=1,KAPUT)
      DO 3191 I=1,NTR
 3191 WRITE (4) (T(I,J),J=1,NTC)
      END FILE 4
C CALCULATE M=EM=R(TRANSPOSE)*R
      DO 1066 I=1,NTC
      DO 1066 J=1,NTC
      EM(I,J)=0.
      DO 1066 K=1,NR
 1066 EM(I,J)=EM(I,J)+R(K,I)*R(K,J)
      DO 6210 I=1,NTC
 6210 WRITE (4) (EM(I,J),J=1,NTC)
      END FILE 4
C ONLY NR OF THE NTC E.VALUES WILL BE NON ZERO
      CALL TIME (9HSTART EIG)
      CALL EIGEN(EM,B,NTC,VALU,NR,SRNORM,LDIM)
      CALL TIME (7HEND EIG)
      PRINT 6070,SRNORM
 6070 FORMAT (1X,7HSRNORM=E30.14)
C MOVE E.VECTORS FROM COLUMNS OF EM TO COLUMNS OF V
      DO 6200 I=1,NTC
      DO 6200 J=1,NR
 6200 V(I,J)=EM(I,J)
C READ EM BACK IN FROM TAPE4 .STEP THROUGH T FIRST
      REWIND 4
      READ (4) (KAP(I),I=1,KAPUT)
      DO 6220 I=1,NTR
 6220 READ (4) EM(I,1)
      READ (4) EOF
      IF (EOF,4) 6230,6240
 6240 PRINT 6250
 6250 FORMAT (10HMISS T EOF)
      CALL EXIT
 6230 CONTINUE
      DO 6032 I=1,NTC
 6032 READ (4) (EM(I,J),J=1,NTC)
      READ (4) EOF
      IF (EOF,4) 6033,6006
 6006 PRINT 6007
 6007 FORMAT (1X20HCANNOT FIND EOF ON M)
      CALL EXIT
 6033 CONTINUE
      IF(NVCHEK.NE.1) GO TO 6300
      CALL VCHEK(LDIM,V,VALU,W1,W2,NTC,NR,EM)
 6300 CONTINUE
      IF(NHARM.NE.1) GO TO 6400
C FOURIER ANALYSIS OF G*V. RELOAD GANGING MATRIX G INTO GG.(G=1)
      CALL TIME (9HSTART HAR)
      DO 6310 I=1,NGR
      DO 6310 J=1,NGC
      IF(I.EQ.J) GO TO 6320
      GG(I,J)=0.
      GO TO 6310
 6320 GG(I,J)=1.
 6310 CONTINUE
      DO 6330 K=1,NTC
      DO 6340 I=1,NGR
      TEM(I)=0.
      DO 6340 J=1,NGC
 6340 TEM(I)=TEM(I)+GG(I,J)*V(J,K)
      CALL HAR(TEM,W2,NH)
```

```
      NGR2=NGR/2
      NGR1=NGR/2+1
      AMP(1)=W2(1)
      AMP(NGR1)=W2(NGR1)
      DO 6350 L=2,NGR2
      M=NGR2+L
 6350 AMP(L)=SQRT((W2(L))**2+(W2(M))**2)
      DO 6081 MM=1,4
      AMPMX(MM)=AMP(1)
      IAMP(MM)=0
      DO 6080 L=2,NGR1
      IF(AMP(L).LT.AMPMX(MM)) GO TO 6080
      AMPMX(MM)=AMP(L)
      IAMP(MM)=L-1
 6080 CONTINUE
      IM=IAMP(MM)+1
      AMP(IM)=0.
 6081 CONTINUE
      PRINT 6082,(AMPMX(I),I=1,4),(IAMP(I),I=1,4),VALU(K),K
 6082 FORMAT (1X4(1XE12.5),15X,4(2XI3),5XE12.6,5XI3)
 6330 CONTINUE
      CALL TIME (7HEND HAR)
 6400 CONTINUE
C GENERATE Q MATRIX.NQS,NQL ARE SUM LIMITS AT SMALL,LARGE E.VALUES RESP.
C E.VALUES =0 ARE EXCLUDED (NTC-NR).Q/6 IS GENERATED,SYMMETRY GIVES REM.
C READ IN T MATRIX IN PLACE OF GG
      REWIND 4
      READ (4) (KAP(I),I=1,KAPUT)
      DO 6087 I=1,NTR
 6087 READ (4) (T(I,J),J=1,NTC)
      READ (4) EOF
      IF(EOF,4) 6088,6089
 6089 NFLAG=8
      PRINT 3078,NFLAG
      CALL EXIT
 6088 CONTINUE
C  STORE NOS OF OPERATING B.SENSORS IN INDEXX.
      N=0
      DO 6095 I=1,NTR
      IF(KAPUT.EQ.0) GO TO 6096
      NNKAP=0
      DO 6097 L=1,KAPUT
      IF(KAP(L).EQ.I) NNKAP=1
 6097 CONTINUE
      IF(NNKAP.EQ.1) GO TO 6095
 6096 N=N+1
      INDEXX(N)=I
 6095 CONTINUE
      CALL TIME (7HSTART Q)
C NQ(I) MUST RUN FROM LARGEST TO SMALLEST NUMBER .Q/6 WRITTEN OVER T.
      READ 3160,(NQ(I),I=1,NNNQ)
      PRINT 6410,(NQ(I),I=1,NNNQ)
 6410 FORMAT (76HQ MATRICES OBTAINED FOR DELETION OF FOLLOWING NUMBERS O
     1F SMALL E.VECTORS****/,1X16I5)
      REWIND 4
      DO 6500 I=1,NNNQ
 6500 NQ(NNNQ+2-I)=NR-NQ(NNNQ+1-I)
      NQ(1)=0
 3077 FORMAT (4HNTC=,I3,5X3HNR=,I3,5X3HNB=,I3,5X5HNNNQ=,I3)
      DO 6094 M=1,NTC
      DO 6510 IJ=1,NB
```

```
 6510 TEM(IJ)=0.
      DO 6092 J=1,NTC
 6092 VVJM(J)=0.
      DO 6094 II=1,NNNQ
      NQS=NQ(II+1)
      NQL=NQ(II)+1
      DO 6093 J=1,NTC
      DO 6093 N=NQL,NQS
 6093 VVJM(J)=VVJM(J)+V(J,N)*V(M,N)/VALU(N)
      DO 6090 I=1,NR
      III=INDEXX(I)
      SUM=0.
      DO 6091 J=1,NTC
 6091 SUM=SUM-T(III,J)*VVJM(J)
 6090 TEM(I)=SUM
      WRITE (4) (TEM(I),I=1,NR)
 6094 CONTINUE
      PRINT 3077,NTC,NR,NB,NNNQ
      END FILE 4
      CALL TIME(5HEND Q)
C   READ Q BACK FROM TAPE4
      REWIND 4
      DO 220 KK=1,NNNQ
      PRINT 3078,KK
      IF (KK.EQ.1) GO TO 520
      KK1=KK-1
      DO 530 KKK=1,KK1
  530 READ (4) (TEM(M),M=1,NR)
  520 CONTINUE
      DO 540 M=1,NTC
      READ (4) (Q(I,M),I=1,NR)
      IF(M.EQ.NTC) GO TO 540
      NNNQ1=NNNQ-1
      DO 510 LL=1,NNNQ1
  510 READ (4) (TEM(I),I=1,NR)
  540 CONTINUE
  711 CONTINUE
      IF(KK.NE.1) GO TO 680
      IF(ITEST.EQ.1) GO TO 700
C READ IN B.SENSOR READINGS BS(I) AND OBTAIN CORRECTIONS OF QUADS HQ(I)
      READ 610,(TEM(I),I=1,NB)
  610 FORMAT (8F10.4)
      IF(KAPUT.EQ.0) GO TO 680
      DO 611 I=1,NR
      II=INDEX(I)
  611 BS(I)=TEM(II)
      GO TO 680
  700 CONTINUE
C FOR TEST ASSUME FIRST QUAD AT LONG STRAIGHT DISPLACED =+1
C THEN BS(I)=T(I,2)
      H(1)=0.
      H(2)=1.
      DO 660 I=3,NH
  660 H(I)=0.
      DO 662 I=1,NB
  662 TEM(I)=T(I,2)
      DO 640 I=1,NR
      II=INDEXX(I)
  640 BS(I)=TEM(II)
  680 CONTINUE
      IF(KK.EQ.1.AND.NHOR.EQ.1) GO TO 820
```

Handwritten annotation (20th HARMONIC) — FOR 20th HARMONIC REPLACE AS SHOWN:

```
      DO 659  I= 1,5
  659 H(I) = Ø.
      DO 66Ø  I = 6,15
  66Ø H(I) = SIN(Ø.594868*(I-5))
      DO 661  I = 16, NH
  661 H(I) = Ø.
      PRINT 67ØØ, (H(I),I= 1,NH)
      DO 662  I= 1, NB
      TEM(I) = Ø.
      DO 662  J = 1, NH
```

```
      IF(KK.EQ.1.AND.NVERT.EQ.1) GO TO 830
      GO TO 829
  820 PRINT 822,KAPUT
      GO TO 829
  830 PRINT 824,KAPUT
  822 FORMAT (43HHORIZONTAL PLANE.NUMBER OF B.SENSORS KAPUT=,I3//)
  824 FORMAT (41HVERTICAL PLANE.NUMBER OF B.SENSORS KAPUT=,I3//)
  829 CONTINUE
      DO 810 I=1,NH
      HQ(I)=0.
      DO 810 J=1,NR
  810 HQ(I)=HQ(I)+BS(J)*Q(J,I)
      N=NR-NQ(KK+1)
      PRINT 460,N,NR
  460 FORMAT (1XI3,6HOF THE,I3,59HNONZERO E.VECTORS OMITTED. RECOMMENDED
     1 POSITION CORRECTIONS)
      PRINT 500,(HQ(I),I=1,NH)
  500 FORMAT (1X10(F10.7))
      HQTOT=0.
      DO 200 I=1,NH
  200 HQTOT=HQTOT+ABS(HQ(I))
      PRINT 201,HQTOT
  201 FORMAT (29HTOTAL MAGNET DISP.       HQTOT=,E10.3)
      IF(ITEST.NE.1) GO TO 220
C OBTAIN RESIDUAL BEAM ERROR BE(I) AFTER QUAD.POSITIONS CORRECTED
      DO 90 I=1,NB
      BE(I)=0.
      DO 90 J=1,NH
   90 BE(I)=BE(I)+T(I,J)*(H(J)+HQ(J))
C DETERMINE MAX,RMS,RMSMAX,DISPLACEMENTS OF EQUILIB ORBIT BEFORE AND AFT
C ER CORRECTION.
      IF(KK.NE.1) GO TO 850
      CALL NUMBER (BS,BSMAX,BSRMS,BSMRMS,NR)
  850 CONTINUE
      CALL NUMBER (BE,BEMAX,BERMS,BEMRMS,NR)
      PRINT 440
      PRINT 441,(BE(I),I=1,NB)
  440 FORMAT (49HBEAM WRT DISP.STRUCTURE AFTER CORRECTIONS APPLIED/)
  441 FORMAT (1X10(1XE9.2))
      PRINT 235
  235 FORMAT (15HVECTORS OMITTED,2X,8HBMAXIMUM,12X,12HBROOT MEAN S ,8X,1
     16HBMAX ROOT MEAN S,4X,14HTOTAL MAG DISP)
      PRINT 240,BSMAX,BSRMS,BSMRMS
      PRINT 245,N,BEMAX,BERMS,BEMRMS,HQTOT
  240 FORMAT (4X,3(10XE10.3),14X17HINITIAL CONDITION)
  245 FORMAT (1XI3,4(10XE10.3)///)
      REWIND 4
      IF (ITEST.EQ.1.AND.ISPY.EQ.1) GO TO 860
      GO TO 220
  860 CONTINUE
C READ IN SYNCH RESULTS FOR SPY STATIONS
      REWIND 3
      DO 6590 J=2,37
      DO 6590 I=1,NB
      READ (3,3120) MN,IPOS,KKK,EL,BETX5,BETY5,BETX7,BETY7
      IF (NHOR.EQ.1) GO TO 6600
      READ(3,3120) MN,IPOS,KKK,EL,BETX5,SY(I,J),BETX7,BETY7
      GO TO 6590
 6600 READ(3,3120) MN,IPOS,KKK,EL,SY(I,J),BETY5,BETX7,BETY7
 6590 CONTINUE
C EXPAND SPY SY MATRIX FOR H(1........222) CONVERT TO RELATIVE DISPL.
```

```
  662 TEM(I)=TEM(I)+T(I,J)*H(J)
      PRINT 6700,(TEM(I),I=1,NB)
      DO 640 I=1,NR
      II=INDEXX(I)
  640 BS(I)=TEM(II)
```

-36-

```
C NO. SPY STATIONS =NB ,NO.ACCEL.DISP. STILL NH.
      DO 6610 I=1,NB
 6610 SY(I,1)=0.
      SY(1,2)=SY(1,2)-1.
      DO 6611 J=4,NHS
      I=J-2
 6611 SY(I,J)=SY(I,J)-1.
      DO 6620 J=1,NHS
      DO 6620 I=1,NB
      DO 6620 NN=1,5
      IF(I+NN*NBS-NB) 6630,6630,6640
 6630 SY(I+NN*NBS,J+NN*NHS)=SY(I,J)
      GO TO 6620
 6640 SY(I+NN*NBS-NB,J+NN*NHS)=SY(I,J)
 6620 CONTINUE
      DO 6650 I=1,NB
      BSP(I)=0.
      DO 6650 J=1,NH
 6650 BSP(I)=BSP(I)+SY(I,J)*(H(J)+HQ(J))
      PRINT 6660
 6660 FORMAT (76HRELATIVE POSITION OF BEAM AT SPY STATIONS AFTER CORRECT
     1ION OF QUAD POSITIONS/)
      PRINT 441,(BSP(I),I=1,NB)
  220 CONTINUE
      IF(NHOR.NE.1) GO TO 870
      NHOR=0
      IF(NVERT.EQ.1) GO TO 3020
  870 CONTINUE
      RETURN
      END
      SUBROUTINE NUMBER(CI,CMAX,CRMS,CMRMS,NB)
      DIMENSION CI(223)
      COMMON/F203/ INDEXX(223),C(224),VC(224)
      DO 50 I=1,NB
   50 C(I)=CI(I)
      IMAX=1
      CMAX=ABS(C(1))
      DO 30 I=2,NB
      IF (ABS(C(I)).GT.CMAX) GO TO 20
      GO TO 30
   20 CMAX=ABS(C(I))
      IMAX=I
   30 CONTINUE
      CMS=0.
      DO 40 I=1,NB
   40 CMS=CMS+C(I)**2/NB
      CRMS=SQRT(CMS)
      C(NB+1)=C(1)
      C(NB+2)=C(2)
      II=0
      DO 60 I=1,NB
      IF(ABS(C(I+2)).LE.ABS(C(I+1)).AND.ABS(C(I+1)).GT.ABS(C(I))) GO TO
     161
      GO TO 60
   61 II=II+1
      VC(II)=ABS(C(I+1))
   60 CONTINUE
  140 FORMAT (1X5(10XE9.2))
      CMMS=0.
      DO 100 I=1,II
  100 CMMS=CMMS+VC(I)**2/II
```

```
      CMRMS=SQRT(CMMS)
      RETURN
      END
      SUBROUTINE TIME(WORD)
      CALL SECOND(T)
      PRINT 10,T,WORD
   10 FORMAT(4H0***F10.4,5X,A10)
      RETURN
      END
      SUBROUTINE EIGEN (A,B,NSUB,VALU,MSUB,SRNORM,NMAX)
C     EIGENVALUES AND EIGENVECTORS OF A REAL SYMMETRIC MATRIX
C SUB EIGEN RETURNS VECTORS IN COLUMNS OF EM.EIGENVALUES RETURN IN VALU.
C B IS SPACE FOR STORAGE
      DIMENSION A(NMAX,NSUB), B(NMAX,NSUB), VALU(MSUB)
      DIMENSION    DIAG(223), SUPERD(223), WVEC(223), PVEC(223),
     1             QVEC(223), VALL(223), Q(223), U(223),
     2             INDEX(223), FACTOR(223), V(223), T(223,3)
      COMMON/F202/ DIAG,SUPERD,WVEC,PVEC
      COMMON/F203/ INDEX,T
      EQUIVALENCE (WVEC,VALL,      FACTOR,U), (PVEC,QVEC,Q,V), (I1,T1),
     1            (I2,T2,ITER), (TEMP,  T0), (SUM,MATCH), (I,P),
     2            (DIV,SCALAR,TAU),  (ANORM2,ANORM,SUPERD(223)),
     3            (VTEMP,VNORM2,VNORM,INDEX(223))
      DATA (E1=2.0E-14)
C     HOUSEHOLDER SIMILARITY TRANSFORMATION TO CO-DIAGONAL FORM
      N=NSUB
      M=MSUB
      IF (M) 50, 50, 10
C     GENERATE IDENTITY MATRIX
   10 DO 40 I=2,N
      DO 40 J=2,N
      B(J,I)=0.0
      IF (I-J) 40, 25
   25 B(J,I)=1.0
   40 CONTINUE
   50 DO 200 I=1,N
C     REDUCE COLUMN OF MATRIX
      I1=I+1
      I2=I1+1
      IF(I2.GT.N) GO TO 160
      SUM=0.0
      DO 70 J=I2,N
   70 SUM=SUM+A(J,I)**2
      IF (SUM) 75,160,75
   75 J=I1
      TEMP=A(J,I)
      SUM=SQRTF(SUM+ TEMP  **2)
      A(J,I )=-SIGNF(SUM, TEMP   )
      WVEC(J )=SQRTF(      1.0+ABSF( TEMP   )/SUM)
      DIV=SIGNF(     WVEC(J )*SUM, TEMP   )
      DO 85 J=I2,N
   85 WVEC(J)=A(J,I)/DIV
      SCALAR=0.0
      DO 95 J=I1,N
      PVEC(J)=0.0
      DO 90 K=I1,N
   90 PVEC(J)=PVEC(J)+A(K,J)*WVEC(K)
      SCALAR=SCALAR+PVEC(J)*WVEC(J)
   95 CONTINUE
      SCALAR=SCALAR/2.0
      DO 120 J=I1,N
```

```
      QVEC(J)=PVEC(J)-SCALAR*WVEC(J)
      DO 120 K=I1,J
      A(K,J)=A(K,J)-(WVEC(K)*QVEC(J)+WVEC(J)*QVEC(K))
      A(J,K)=A(K,J)
  120 CONTINUE
      IF (M) 160, 160, 130
C     SAVE ROTATION FOR LATER APPLICATION TO CO-DIAGONAL VECTORS
  130 DO 150 K=2,N
      TEMP=0.0
      DO 140 J=I1,N
  140 TEMP=TEMP+WVEC(J)*B(J,K)
      DO 150 J=I1,N
      B(J,K)=B(J,K)-WVEC(J)*TEMP
  150 CONTINUE
C     MOVE CO-DIAGONAL FORM ELEMENTS FOR ITERATIVE PROCEDURE
  160 J=I
      DIAG(I)=A(J,I)
      SUPERD(I)=A(J+1,I)
  200 CONTINUE
C     GIVENS EIGENVALUE ITERATION FROM STURM CHAIN OF CO-DIAGONAL MINORS
      N=XABSF(N)
      M=XABSF(M)
C     CALCULATE NORM OF MATRIX AND INITIALIZE EIGENVALUE BOUNDS
      ANORM2=DIAG(1)**2
      DO 230 L=2,N
      Q(L-1)=SUPERD(L-1)**2
      ANORM2=DIAG(L)**2+Q(L-1)+Q(L-1)+ANORM2
  230 CONTINUE
      ANORM=SQRTF(ANORM2)
      DO 240 L=1,M
      VALU(L)=ANORM
      VALL(L)=-ANORM
  240 CONTINUE
      EPS1=ANORM*E1
      IF (EPS1) 250, 1000
  250 DO 570 L=1,M
C     CHOOSE NEW TRIAL VALUE WHILE TESTING BOUNDS FOR CONVERGENCE
  260 TAU=(VALU(L)+VALL(L))/2.0
      IF (2.0*(TAU-VALL(L))-EPS1) 570, 570, 270
C     DETERMINE SIGNS OF PRINCIPAL MINORS
  270 MATCH=0
      T2=0.0
      T1=1.0
      DO 450 L1=1,N
      P=DIAG(L1)-TAU
      IF (T2) 330, 300
  300 T1=SIGNF(1.0,T1)
  330 IF (T1) 400, 370
  370 T0=-SIGNF(1.0,T2)
      T2=0.0
      IF (Q(L1-1)) 410, 300
  400 T0=P-Q(L1-1)*T2/T1
      T2=1.0
C     COUNT AGREEMENTS IN SIGN (ZERO CONSIDERED POSITIVE)
  410 IF (T0) 440, 420, 430
  420 T2=T1
      IF (T2) 440, 430, 430
  430 MATCH=MATCH+1
  440 T1=T0
  450 CONTINUE
C     ESTABLISH TIGHTER BOUNDS ON EIGENVALUES
```

```
      DO 530 L1=L,M
      IF (L1-MATCH) 500, 500, 470
  470 IF (VALU(L1)-TAU) 260, 260, 480
  480 VALU(L1)=TAU
      GO TO 530
  500 VALL(L1)=TAU
  530 CONTINUE
      GO TO 260
  570 CONTINUE
C     EIGENVECTORS OF CO-DIAGONAL SYMMETRIC MATRIX--INVERSE ITERATION
      M=MSUB
      DO 970 I=1,M
C     CHECK FOR REPEATED VALUE
      IF(I.EQ.1) GO TO 725
  720 IF(VALU(I-1)-VALU(I)-(1.0E4)*EPS1) 730,725,725
  725 I1=-1
  730 I1=I1+1
C     TRIANGULARIZE CO-DIAGONAL FORM AFTER EIGENVALUE SUBTRACTION
      DO 760 L=1,N
      V(L)=EPS1
      T(L,2)=DIAG(L)-VALU(I)
      IF (L-N) 740, 735
  735 T(L,3)=0.0
      GO TO 760
  740 T(L,3)=SUPERD(L)
      IF (T(L,3)) 750, 745
  745 T(L,3)=EPS1
  750 T(L+1,1)=T(L,3)
  760 CONTINUE
      DO 820 J=1,N
      T(J,1)=T(J,2)
      T(J,2)=T(J,3)
      T(J,3)=0.0
      VTEMP=ABSF(T(J,1))
      IF (J-N) 785, 770
  770 IF (VTEMP) 820, 780
  780 T(J,1)=EPS1
      GO TO 820
  785 INDEX(J)=0
      IF (ABSF(T(J+1,1))-VTEMP) 810, 810, 790
  790 INDEX(J)=1
      DO 800 K=1,3
      VTEMP=T(J,K)
      T(J,K)=T(J+1,K)
      T(J+1,K)=VTEMP
  800 CONTINUE
  810 VTEMP    =T(J+1,1)/T(J,1)
      FACTOR(J)=VTEMP
      T(J+1,2)=T(J+1,2)-  VTEMP  *T(J,2)
      T(J+1,3)=T(J+1,3)-  VTEMP  *T(J,3)
  820 CONTINUE
      ITER=1
      IF (I1) 920, 860
C     BACK SUBSTITUTE TO OBTAIN EIGENVECTOR
  860 DO 870 L1=1,N
      L=N+1-L1
      V( L )=(V( L )-T(L,2)*V( L+1 )-T(L,3)*V( L+2 ))/T(L,1)
  870 CONTINUE
      GO TO (875,920), ITER
C     PERFORM SECOND ITERATION
  875 ITER=2
```

```
  880 DO 910 L=2,N
      IF (INDEX(L-1))890,900
  890 VTEMP=V(L-1)
      V(L-1)=V(L)
      V(L)=VTEMP
  900 V(L)=V(L)-FACTOR(L-1)*V(L-1)
  910 CONTINUE
      GO TO 860
C     ORTHOGONALIZE VECTOR TO OTHERS ASSOCIATED WITH REPEATED VALUE
  920 IF(I1.EQ.0) GO TO 945
      DO 940 L1=1,I1
      K=I-L1
      VTEMP=0.0
      DO 930 J=1,N
  930 VTEMP=VTEMP+A(J,K)*V(J)
      DO 940 J=1,N
  940 V(J)=V(J)-A(J,K)*VTEMP
      GO TO (880,945), ITER
C     NORMALIZE VECTOR
  945 VNORM2=0.0
      DO 950 L=1,N
  950 VNORM2=VNORM2+V( L )**2
      VNORM=SQRTF(VNORM2)
      DO 960 J=1,N
  960 A(J,I)=V( J )/VNORM
  970 CONTINUE
C     ROTATION OF CO-DIAGONAL VECTORS INTO MATRIX EIGENVECTORS
      N=NSUB
      DO 990 I=1,M
      DO 980 K=2,N
      U(K)=0.0
      DO 980 J=2,N
  980 U(K)=U(K)+B(J,K)*A(J,I)
      DO 990 J=2,N
  990 A(J,I)=U(J)
 1000 SRNORM=ANORM
      RETURN
      END
      SUBROUTINE VCHEK(IRD,V,R,TEM,CC,NI,III,A)
      DIMENSION V(223,223),A(223,223),R(223),CC(223),TEM(223)
      M=NI
      DO 270  K=1,III
      DO 271  I=1,M
      SUM=0.
      DO 272  J=1,M
      SUM=SUM+A(I,J)*V(J,K)
  272 CONTINUE
      TEM(I)=SUM
  271 CONTINUE
      DO 273 I=1,III
      SUM=0.
      DO 274 J=1,M
      SUM=SUM+TEM(J)*V(J,I)
  274 CONTINUE
      CC(I)=SUM
  273 CONTINUE
      IL=K-1
      IF(K.EQ.1) IL=2
      DO 700 II=1,III
      IF(II.EQ.K) GO TO 700
      IF(ABS(CC(II)).GT. CC(IL)) IL=II
```

```
 700 CONTINUE
     XIL=.0000000001
     IF(R(K).GT.0) XIL=XIL+R(K)*.00000002
     IF(ABS(CC(IL)) .LT.XIL.AND.ABS(R(K)-CC(K)).LT.XIL) GO TO 709
     PRINT 705 , K,R(K),CC(K),IL,XIL,CC(IL)
 705 FORMAT(1H0 I3,2E25.14,I7,2E25.14,7H  ***** )
     GO TO 712
 709 PRINT 710 , K,R(K),CC(K),IL,XIL,CC(IL)
 710 FORMAT(1H0 I3,2E25.14,I7,2E25.14)
 712 CONTINUE
 270 CONTINUE
 280 RETURN
     END
     SUBROUTINE HAR(V,W,MIN)
     COMMON/F203/ TBL(223),A(112),B(112),C(112),D(112),NC
     DOUBLE PRECISION TBL
     DIMENSION V(1),W(1)
     CALL HASTBL(MIN)
     N=NC/2
     IMAX=N+1
     A(1)=V(1)
     B(1)=0.0
     DO 5 I=2,IMAX
     L=NC+2-I
     A(I)=V(I)+V(L)
     B(I)=V(I)-V(L)
5    CONTINUE
     A(IMAX)=V(IMAX)
     CALL HARSUM
     F=1.0/FLOAT(N)
     DO 9 I=1,IMAX
     W(I)=C(I)*F
     IF (I .GE. IMAX) GO TO 9
8    CONTINUE
     K=N+I
     W(K)=D(I)*F
9    CONTINUE
     W(1)=0.5*W(1)
     W(IMAX)=0.5*W(IMAX)
     RETURN
     END
     SUBROUTINE HASTBL(MIN)
     COMMON/F203/ TBL(223),A(112),B(112),C(112),D(112),NC
     DOUBLE PRECISION TBL
     DOUBLE PRECISION PI,FNC,SAVE,FI,DCI,DSI,DB
     EQUIVALENCE (PI,PO)
     DIMENSION PO(2)
     DATA PO / 1721 6220 7732 5042 0550 B, 1641 6043 2304 6146 1213 B /
     IF (MIN .EQ. NC) RETURN
     FNC=0.0D0
     FI=0.0D0
     NC=MIN
     FNC=FLOAT(NC)
     DB=PI/FNC
     IF (MOD(NC,2) .NE. 0) GO TO 99
     IF (NC.LE.222) GO TO 2
99   CONTINUE
     PRINT 1000, NC
1000 FORMAT ( 14H0 ILLEGAL N.    ,I20 )
     STOP
2    CONTINUE
```

```
      N=NC/2
      NQ2=N/2
      TBL(1)=1.0D0
      TBL(N+1)=0.0D0
      TBL(NC+1)=-1.0D0
      IEO=2-MOD(N,2)
      DO 9 I=IEO,NQ2,IEO
      FI=FLOAT(I)
      SAVE=FI*DB
      DCT=DCOS(SAVE)
      DST=DSIN(SAVE)
      TBL(I+1)=DCT
      K=N-I
      TBL(K+1)=DST
      K=K+N
      TBL(K+1)=-DCT
      K=N+I
      TBL(K+1)=-DST
    9 CONTINUE
      RETURN
      END
      SUBROUTINE HARSUM
      COMMON/F203/ TBL(223),A(112),B(112),C(112),D(112),NC
      DOUBLE PRECISION TBL
      DOUBLE PRECISION AJ,BJ,CI,DI
      MR(K)=MIN0((MOD(K,NX4)),NX4-(MOD(K,NX4)))
      AJ=0.0D0
      BJ=0.0D0
      NX3=(NC*3)/2
      NX4=NC*2
      IMAX=NC/2+1
      IJA=0
      DO 9 I=1,IMAX
      CI=0.0D0
      DI=0.0D0
      IJ=0
      DO 8 J=1,IMAX
      IJC=MR(IJ)
      IJS=MR(IJ+NX3)
      AJ=DBLE(A(J))
      BJ=DBLE(B(J))
      CI=CI+AJ*TBL(IJC+1)
      DI=DI+BJ*TBL(IJS+1)
      IJ=IJ+IJA
    8 CONTINUE
      C(I)=SNGL(CI)
      D(I)=SNGL(DI)
      IJA=IJA+2
    9 CONTINUE
      RETURN
      END
      BLOCK DATA
      COMMON/F203/ TBL(223),A(112),B(112),C(112),D(112),NC
      DOUBLE PRECISION TBL
      DATA NC / 0 /
      END
  EOF
```

Appendix 3 Harmonic Response of Main Ring

The harmonic response of the Main Ring is obtained by the computer program HRESPON which is listed below. Starting from the SYNCH output tape HRESPON sets up the T matrix of Eqn. (2) by the same method used in OTRIM with the ganging matrix G=1. Then a set of beam element displacements is generated by

$$H(K) = \sin\,(0.001 \times I \times EL(K) + \phi(J)) \qquad (A1)$$

with $$\phi(J) = 2\pi\,(J-1)/NFI \qquad (A2)$$

where I = harmonic number

EL(K) = orbital distance to center of Kth beam element (metre)

$\phi(J)$ is the phase difference which may take NFI equally spaced values on the interval 0 - $2\pi$ as the index J is incremented.

For each harmonic I, HRESPON hunts on J to determine the phase $\phi(J)$ at which the maximum closed orbit deviation occurs. It also hunts on J to determine the phase at which the r.m.s. orbit deviation is a maximum. The output lists, for each harmonic,

I = harmonic number

NM = beam element where max. orbit deviation occurred

BMMAX = max. orbit deviation

PHM = phase $\phi(J)$ for max. orbit deviation

NRM = beam element where max. orbit deviation occurred for max. r.m.s. orbit deviation case.

BRMMAX = max. r.m.s. orbit deviation

PHRM = phase $\phi(J)$ for max. r.m.s. orbit deviation

The program requires only 1 data card (16I5) which specifies the variables -

NHOR = 0 skip horizontal plane

= 1 run horizontal plane

NVERT = 0 skip vertical plane

= 1 run vertical plane

NHAR - highest harmonic to be run (150 in listing below)

NFI - phase increments on interval 0 - $2\pi$ (10 in listing below)

In Figure 15 and Figure 16 we have plotted the Main Ring harmonic response for the horizontal and vertical planes. The curves have the same general form as that obtained by Laslett[8]. The positions of the maxima are in good agreement with the expected positions as given by $|6m\pm\nu_0|$ where m is an integer and $\nu_0 = 20$ is the integer closest to the $\nu$ value. The harmonic response of possible surveying schemes for the Main Ring are typically peaked towards low harmonics[8] due to short range correlations. Thus, although Figures 15 and 16 have large peaks at high harmonics, we would expect that the residual harmonics will peak at the integer (20) nearest the $\nu$ value when the harmonic response of the survey system is included.

## HRESPON LISTING

```
G3 6[illegible],CM222[illegible],T1[illegible],L2[illegible].NAL REMOTE BINGHAM
REQUEST,T322=TAPE3. READ SYNCH
REWIND(TAPE3)
MAP(PART)
RUN(G,,,,EROUT)
*CXIT.
EREDIT.
*FIN.
REMOTE OUTPUT,253.
EOR
      PROGRAM HRESPON(INPUT,OUTPUT,TAPE3)
C PROGRAM HRESPON DETERMINES THE HARMONIC RESPONSE OF THE CLOSED ORBIT.
      DIMENSION  EL(222),SY(222,36),S(222,222),TEM(222),H(222)
      DIMENSION B(210),BM(200),BRM(200),IB(200)
      TP=6.2831853
      TPL=0.001
      NH=222
      NB=210
      NBS=NB/6
      NHS=NH/6
      NSR=NB
      NSC=NH
      NTC=NH
      NTR=NB
      READ 3010,NHOR,NVERT,NHAR,NFI
 3010 FORMAT (16I5)
      IF(NHOR.NE.1.AND.NVERT.NE.1)3051,3052
 3051 PRINT 3050
 3050 FORMAT (36HHRESPON WAS NOT ASKED TO DO ANYTHING)
      CALL EXIT
 3052 PRINT 3010,NHOR,NVERT,NHAR,NFI
 3020 CONTINUE
      REWIND 3
      IF(NHOR.EQ.1)GO TO 3030
      IF(NVERT.EQ.1)GO TO 3040
      CALL EXIT
 3030 PRINT 3060
 3060 FORMAT (58H                          ,HARMONIC RESPONSE IN HORIZONTAL PL
     1ANE///)
      GO TO 3070
 3040 PRINT 3080
 3080 FORMAT (56H                          HARMONIC RESPONSE  IN VERTICAL PLAN
     1E///)
 3070 CONTINUE
      DO 3090 J=1,36
      DO 3095 I=1,NB
      IF (NHOR.EQ.1) GO TO 3100
      READ (3,3120)MN,IPOS,KK,EL(I),BETX5,SY(I,J),BETX7,BETY7
      GO TO 3095
 3100 READ (3,3120) MN,IPOS,KK,EL(I),SY(I,J),BETY5,BETX7,BETY7
 3120 FORMAT (A5,2I3,5F14.8)
 3090 READ (3,3120) MN,IPOS,KK,EEL,BETX5,BETY5,BETX7,BETY7
      READ (3,3120) EOF
      IF(EOF,3) 3130,3131
 3131 PRINT 3140
 3140 FORMAT(21HDO NOT FIND SYNCH EOF)
      CALL EXIT
 3130 CONTINUE
      PRINT 5700,(SY(I,1),I=1,NB)
```

```
 6700 FORMAT (1X10(1XF9.5))
 6701 FORMAT (1X10(1XF10.3))
C EXPAND SY MATRIX AND SUBTRACT OUT BEAM SENSOR DISPLACEMENT TO GET DISP
C OF BEAM RELATIVE TO B.SENSORS.
      S(1,1)=-1.
      DO 1204 I=2,NSR
 1204 S(I,1)=0.
      DO 1207 I=1,NSR
      DO 1207 J=2,3
      IF(I.EQ.2) GO TO 1208
      S(I,J)=SY(I,(J-1))
      GO TO 1207
 1208 S(I,J)=SY(I,(J-1))-0.5
 1207 CONTINUE
      DO 1211 I=1,NSR
      DO 1211 J=4,35
      IF(I.EQ.(J-1))GO TO 1210
      S(I,J)=SY(I,(J-1))
      GO TO 1211
 1210 S(I,J)=SY(I,(J-1))-1.
 1211 CONTINUE
      DO 1215 I=1,NSR
      DO 1215 J=36,37
      IF(I.EQ.35) GO TO 1214
      S(I,J)=SY(I,(J-1))
      GO TO 1215
 1214 S(I,J)=SY(I,(J-1))-0.5
 1215 CONTINUE
C GENERATE REMAINING 5/6 OF S MATRIX
      NSCS=NSC/6
      NSRS=NSR/6
      DO 1021 J=1,NSCS
      DO 1021 I=1,NSR
      DO 1021 N=1,5
      IF(I+N*NSRS-NSR) 1023,1023,1024
 1023 S(I+N*NSRS,J+N*NSCS)=S(I,J)
      GO TO 1021
 1024 S(I+N*NSRS-NSR,J+N*NSCS)=S(I,J)
 1021 CONTINUE
      PRINT 1029,S(176,2),S(1,39),S(1,2),S(36,39),S(141,2),S(1,75)
 1029 FORMAT (3(2F14.8,5X))
C CHECK ZERO SUM OF ELEMENTS OF EACH ROW OF S MATRIX
      DO 1085 I=1,NTR
      TEM(I)=0.
      DO 1085 J=1,NTC
 1085 TEM(I)=TEM(I)+S(I,J)
      PRINT 6700,(TEM(I),I=1,NTR)
      NROW=0
      DO 1084 I=1,NTR
      IF(ABS(TEM(I)).GT.1.E-6)NROW=1
 1084 CONTINUE
      IF(NROW.EQ.1) PRINT 1083
 1083 FORMAT (52H***ERROR.     SUM OF ELEMENTS OF T ROW GREATER 1.0E-6)
C CALCULATE NH H DIST.AROUND RING FROM NB B.SENSOR DISTANCES,EL(I).EL(1)
C   =0.
      EL(37)=EL(35)+2.74350
      EL(36)=EL(35)-2.384045
      EL(35)=EL(34)-1.62635
      DO 6720 J=1,33
      I=35-J
 6720 EL(I)=EL(I-1)-1.21920
```

```
      EL(4)=EL(3)-0.812165
      SM=EL(2)
      EL(3)=SM+2.384045
      EL(2)=SM-2.74358
      DO 6730 J=1,5
      DO 6730 I=1,37
 6730 EL(I+J*37)=6283.1853*FLOAT(J)/6.+EL(I)
      PRINT 6701,(EL(I),I=1,NH)
C  GENERATE HARMONIC RESPONSE OF CLOSED ORBIT
      DO 4000 I=1,NHAR
      DO 4030 J=1,NFI
      DO 4010 K=1,NH
 4010 H(K)=SIN(I*TPL*EL(K)+TP*(J-1)/NFI)
      DO 4020 L=1,NB
      B(L)=0.
      DO 4020 M=1,NH
 4020 B(L)=B(L)+S(L,M)*H(M)
      CALL NUMBER(B,BMAX,IBMAX,BRMS,NB)
      IB(J)=IBMAX
      BM(J)=BMAX
 4030 BRM(J)=BRMS
      CALL NJMBER (BM,BMMAX,IBMMAX,BMRMS,NFI)
      CALL NJMBER(BRM,BRMMAX,IBRMMAX,BRMRMS,NFI)
      PHM=360.*(IBMMAX-1)/NFI
      PHRM=360.*(IBRMMAX-1)/NFI
      NM=IB(IBMMAX)
      NRM=IB(IBRMMAX)
      IF(I.NE.1) GO TO 4060
      PRINT 4050
 4050 FORMAT (80H           HARMONIC  MAX.AMPLITUDE  PHASE           MAX.R
     1MS AMPLITUDE  PHASE        )
 4060 PRINT 4070,I,NM,BMMAX,PHM,NRM,BRMMAX,PHRM
 4070 FORMAT (12X,I3,1X,I3,2X,E12.4,2X,F6.1,9X,I3,2X,E12.4,2X,F6.1)
 4000 CONTINJE
      IF(NHOR.NE.1) GO TO 870
      NHOR=0
      IF(NVERT.EQ.1) GO TO 3020
  870 CONTINJE
      RETURN
      END
      SUBROUTINE NUMBER(C,CMAX,ICMAX,CRMS,NB)
      DIMENSION C(222)
      ICMAX=1
      CMAX=ABS(C(1))
      DO 30 I=2,NB
      IF(ABS(C(I)).GT.CMAX) GO TO 20
      GO TO 30
   20 CMAX=ABS(C(I))
      ICMAX=I
   30 CONTINUE
      CMS=0.
      DO 40 I=1,NB
   40 CMS=CMS+C(I)**2/NB
      CRMS=SQRT(CMS)
      RETURN
      END
EOR
        1  15.    10
EOF
```

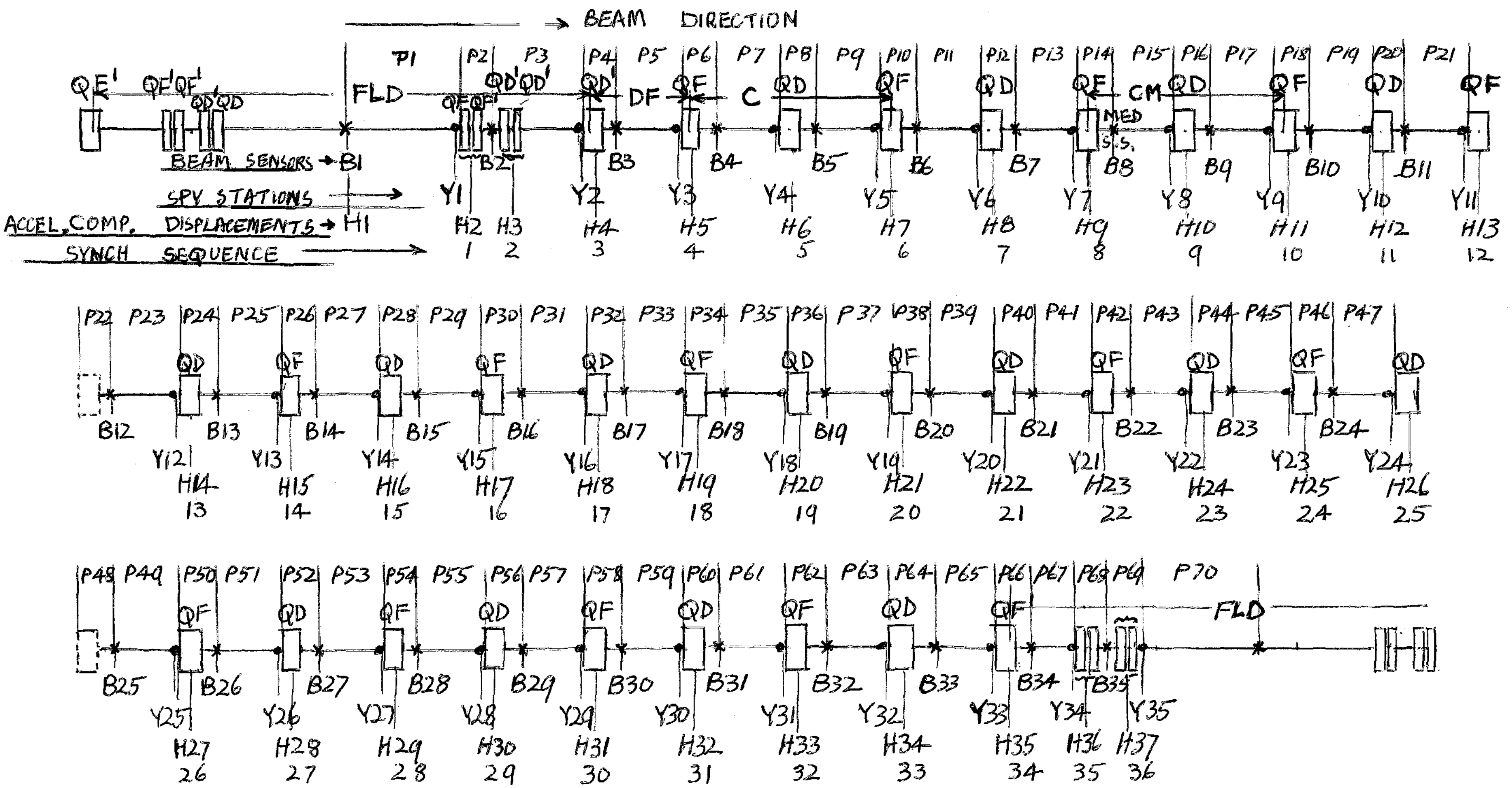
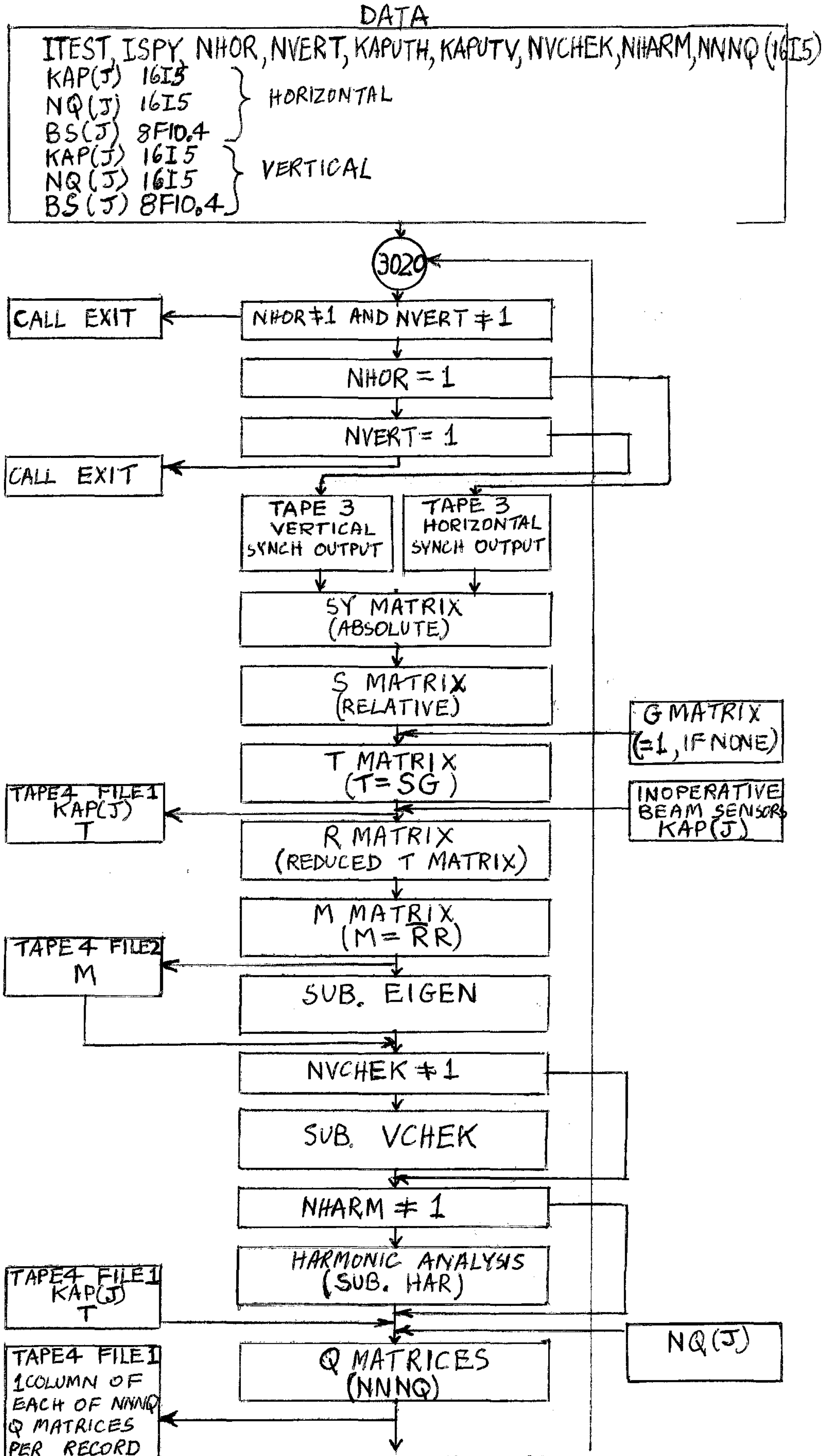



**FIG. 1** First superperiod of main ring showing locations of beam sensors, spy stations, accelerator component displacements. The displacement sequence used in SYNCH is also shown.

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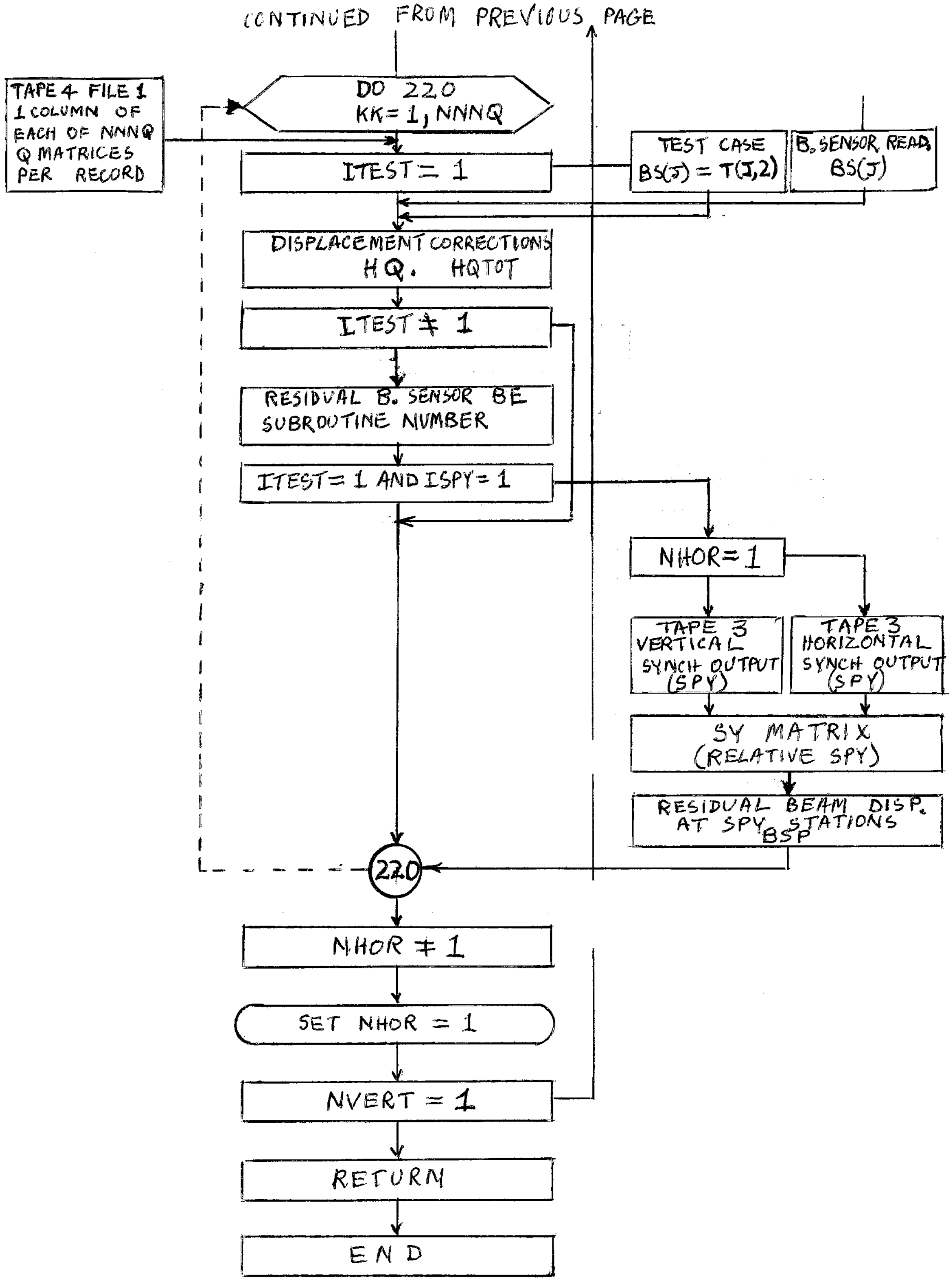
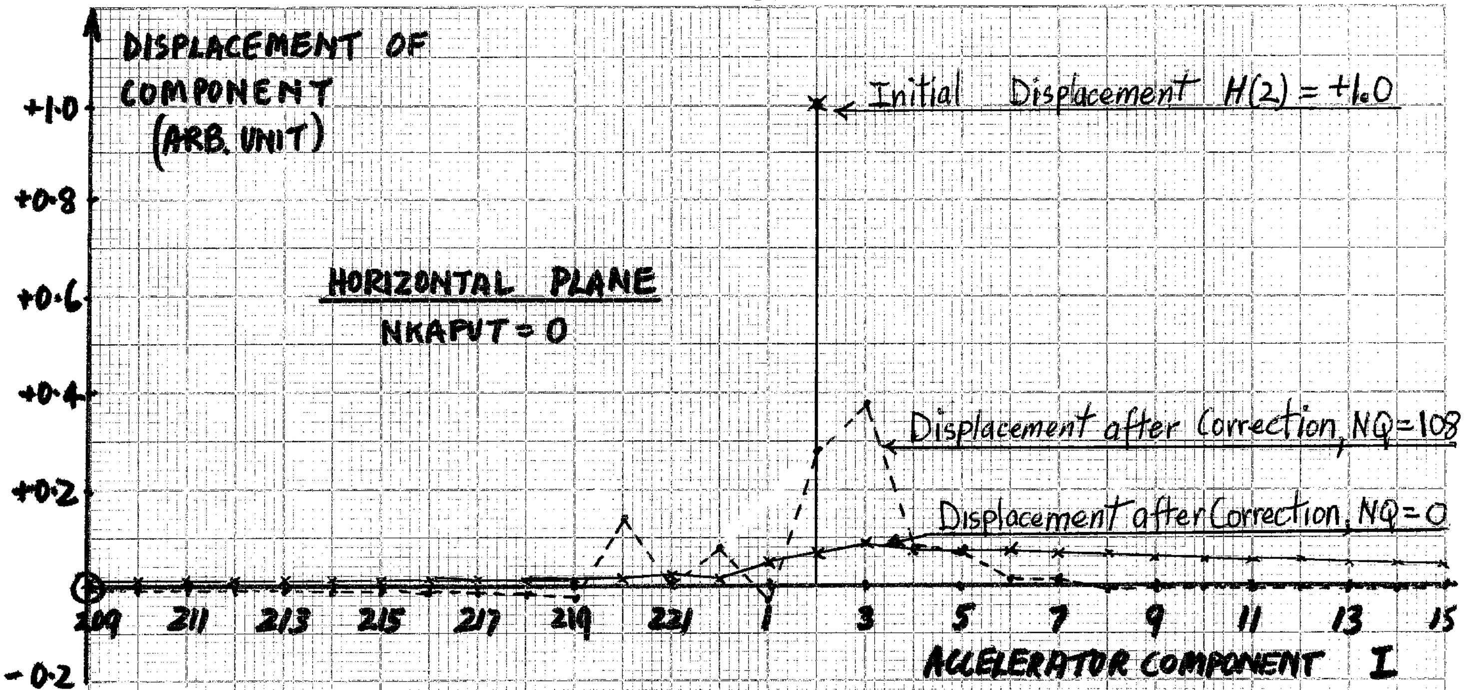



FIG. 2 Flow diagram for program OTRIM.



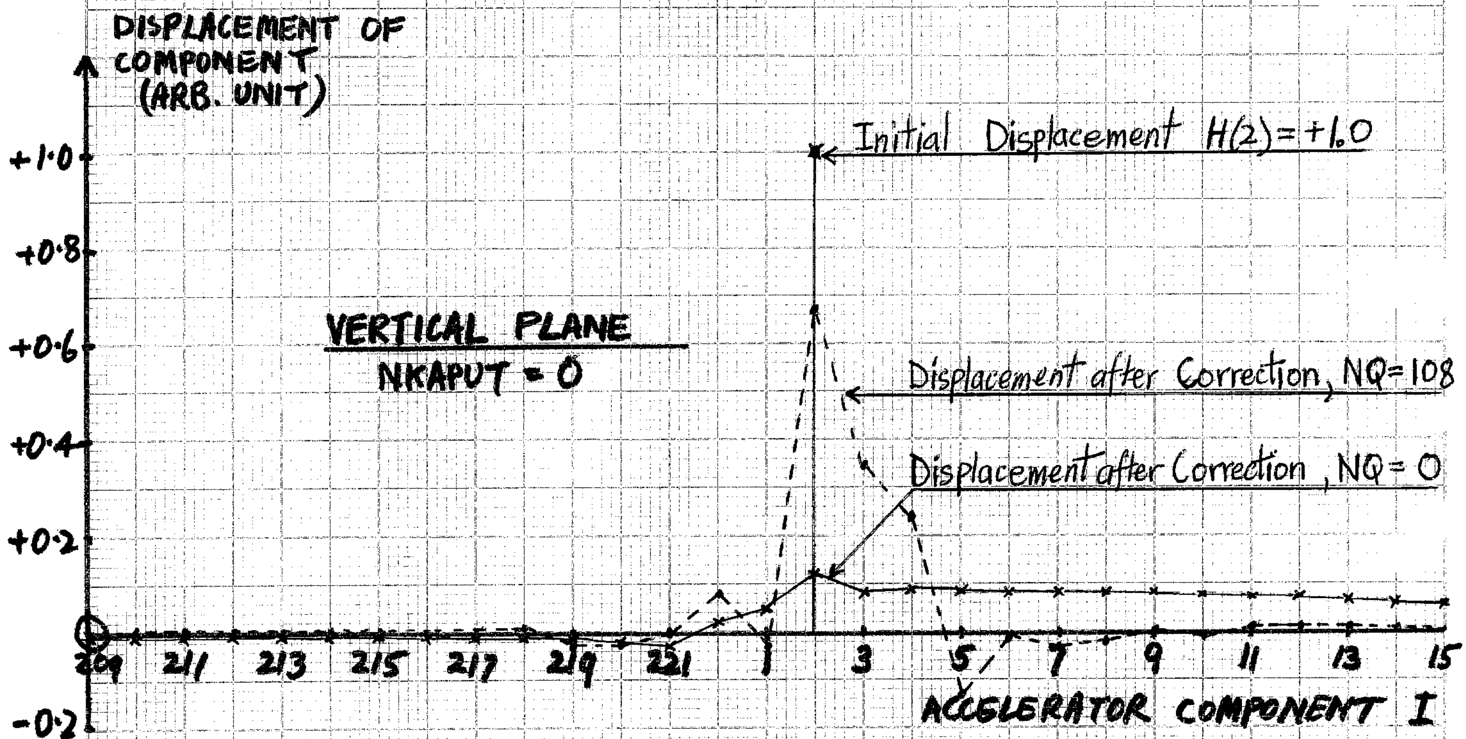
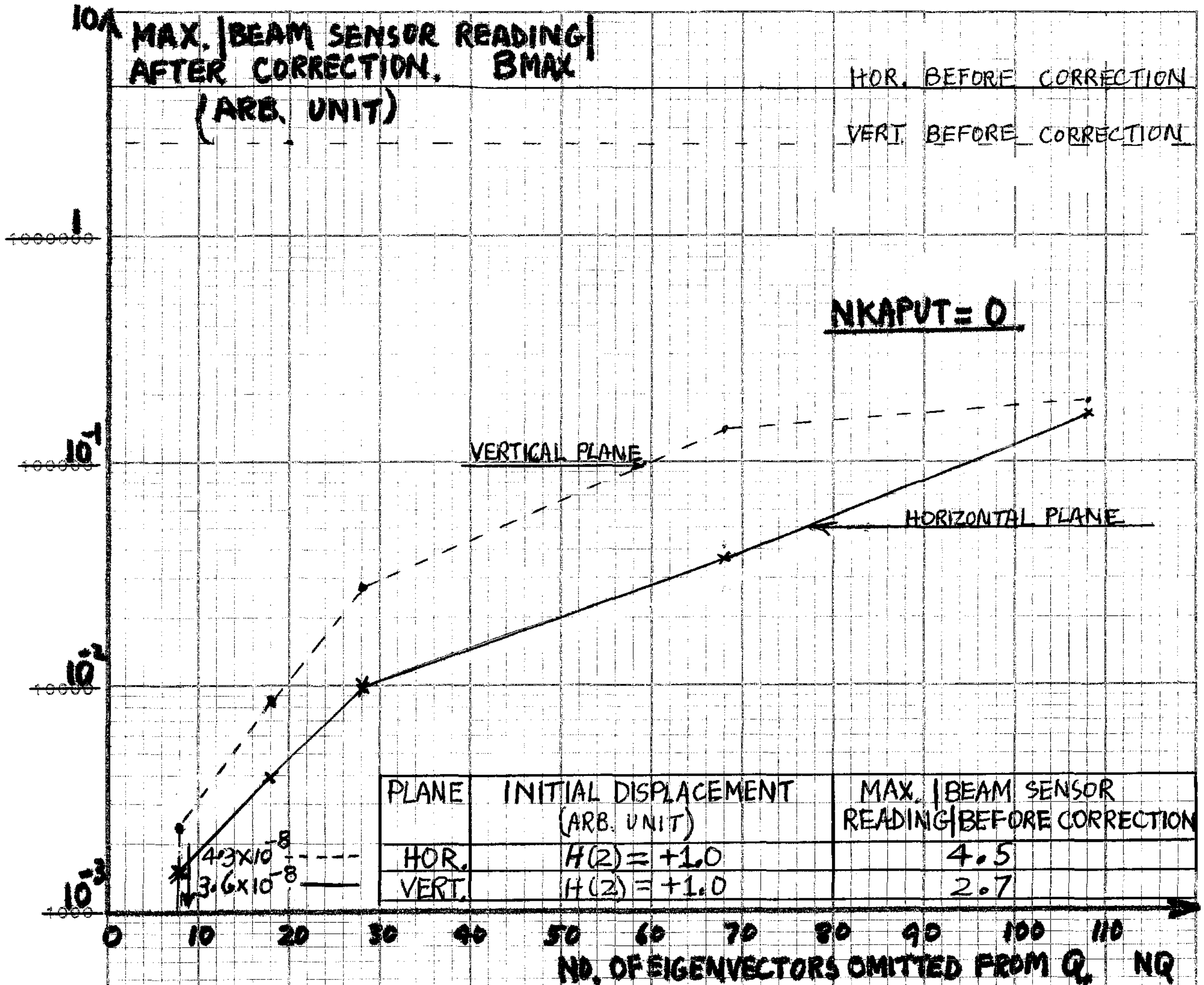




FIG.3 Displacement of accelerator components before and after correction for case where all beam sensors are operative

DATE




| PLANE | INITIAL DISPLACEMENT (ARB. UNIT) | MAX. \|BEAM SENSOR READING\| BEFORE CORRECTION |
|---|---|---|
| HOR. | H(2) = +1.0 | 4.5 |
| VERT. | H(2) = +1.0 | 2.7 |

**FIG.4** EFFECT OF SMALL EIGENVALUE EIGENVECTOR OMISSION ON MAX. |BEAM SENSOR READING| AFTER CORRECTION.

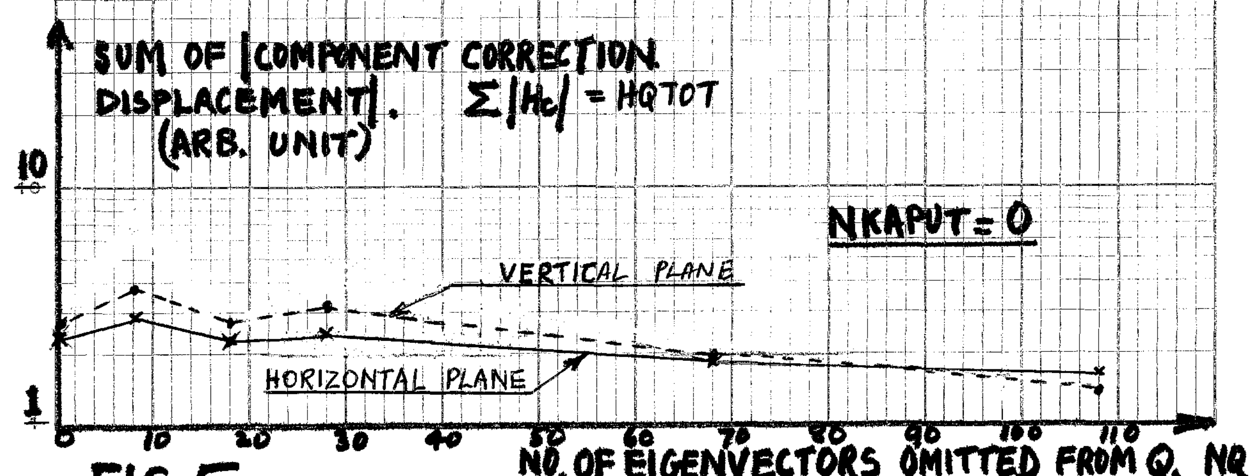



**FIG.5** EFFECT OF SMALL EIGENVALUE EIGENVECTOR OMISSION ON SUM OF |COMPONENT CORRECTION DISPLACEMENT|.

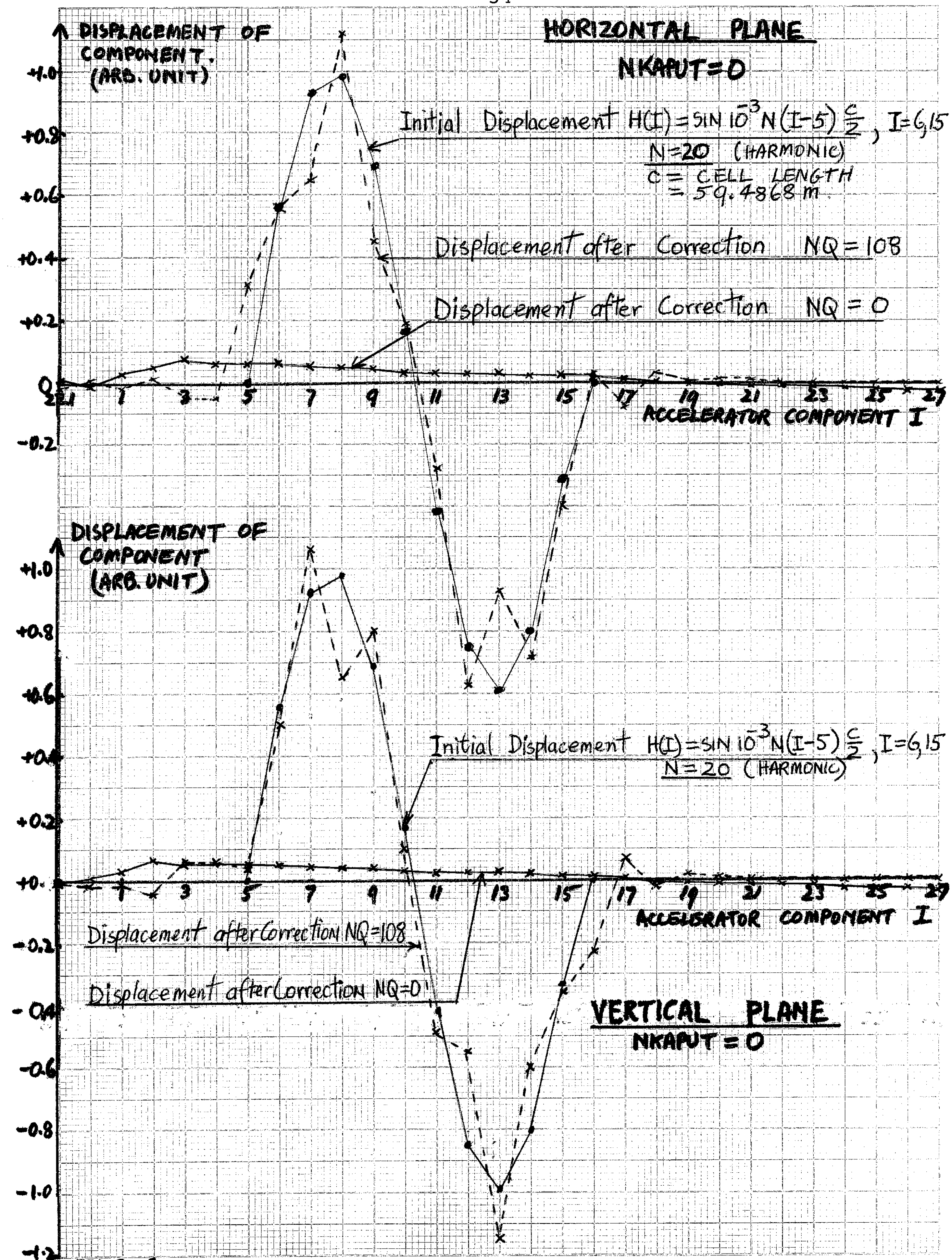



FIG.6 DISPLACEMENTS BEFORE AND AFTER CORRECTION. HARMONIC = 20

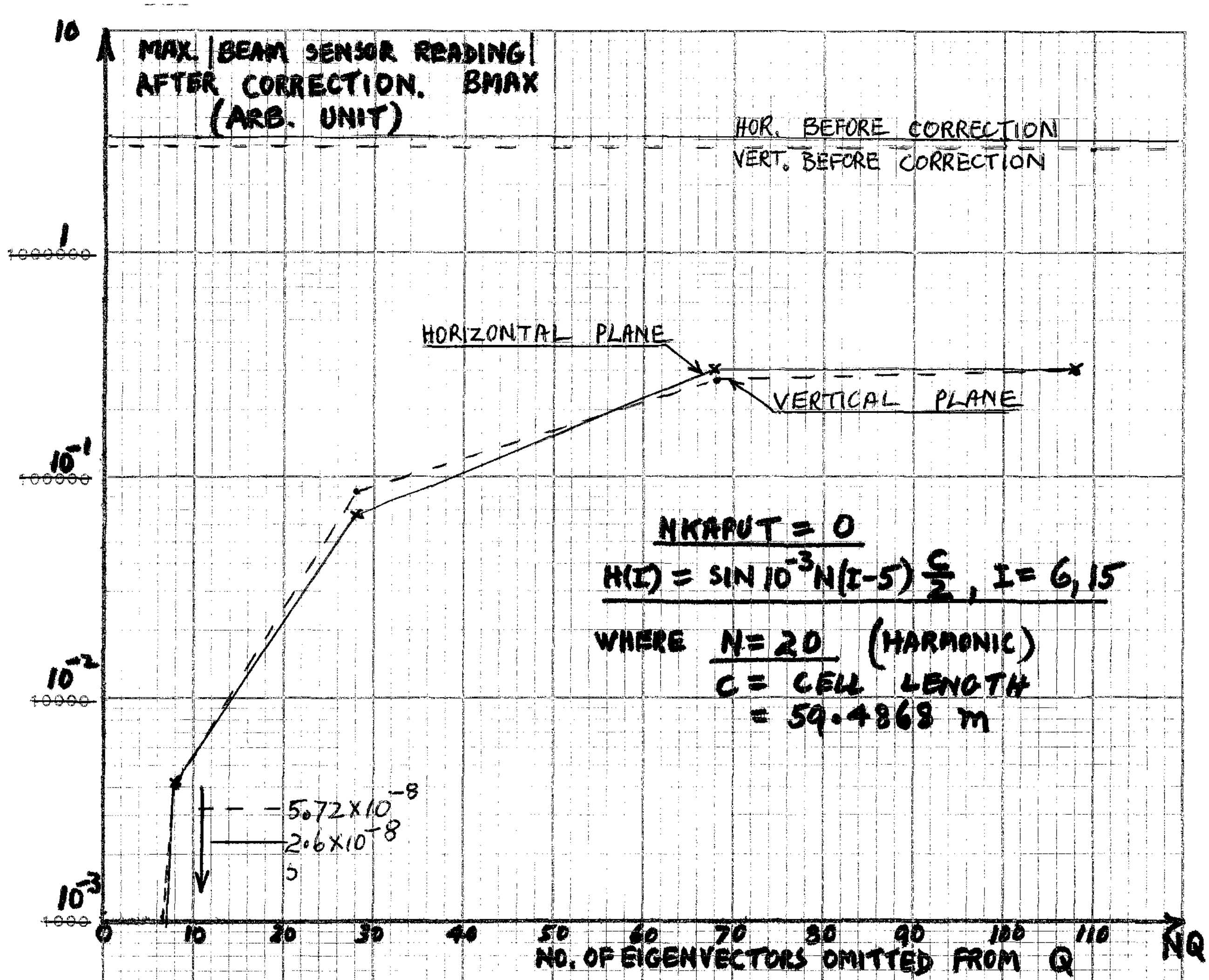



FIG. 7 EFFECT OF SMALL EIGENVALUE EIGENVECTOR OMISSION ON MAX. |BEAM SENSOR READING| AFTER CORRECTION

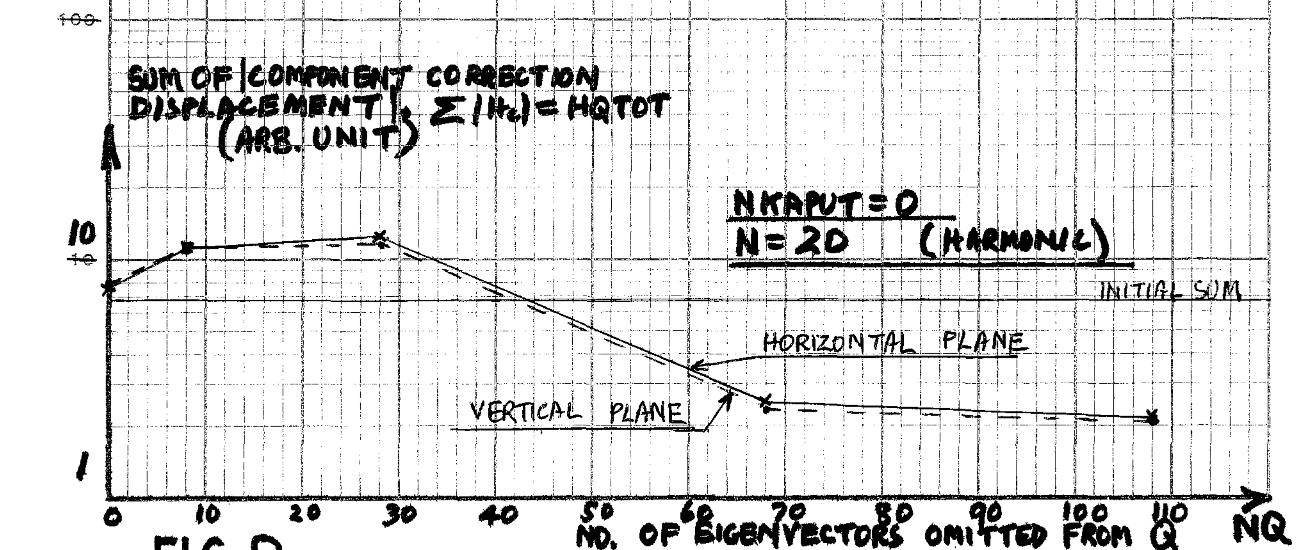



FIG. 8 EFFECT OF SMALL EIGENVALUE EIGENVECTOR OMISSION ON SUM OF COMPONENT CORRECTION DISPLACEMENT.

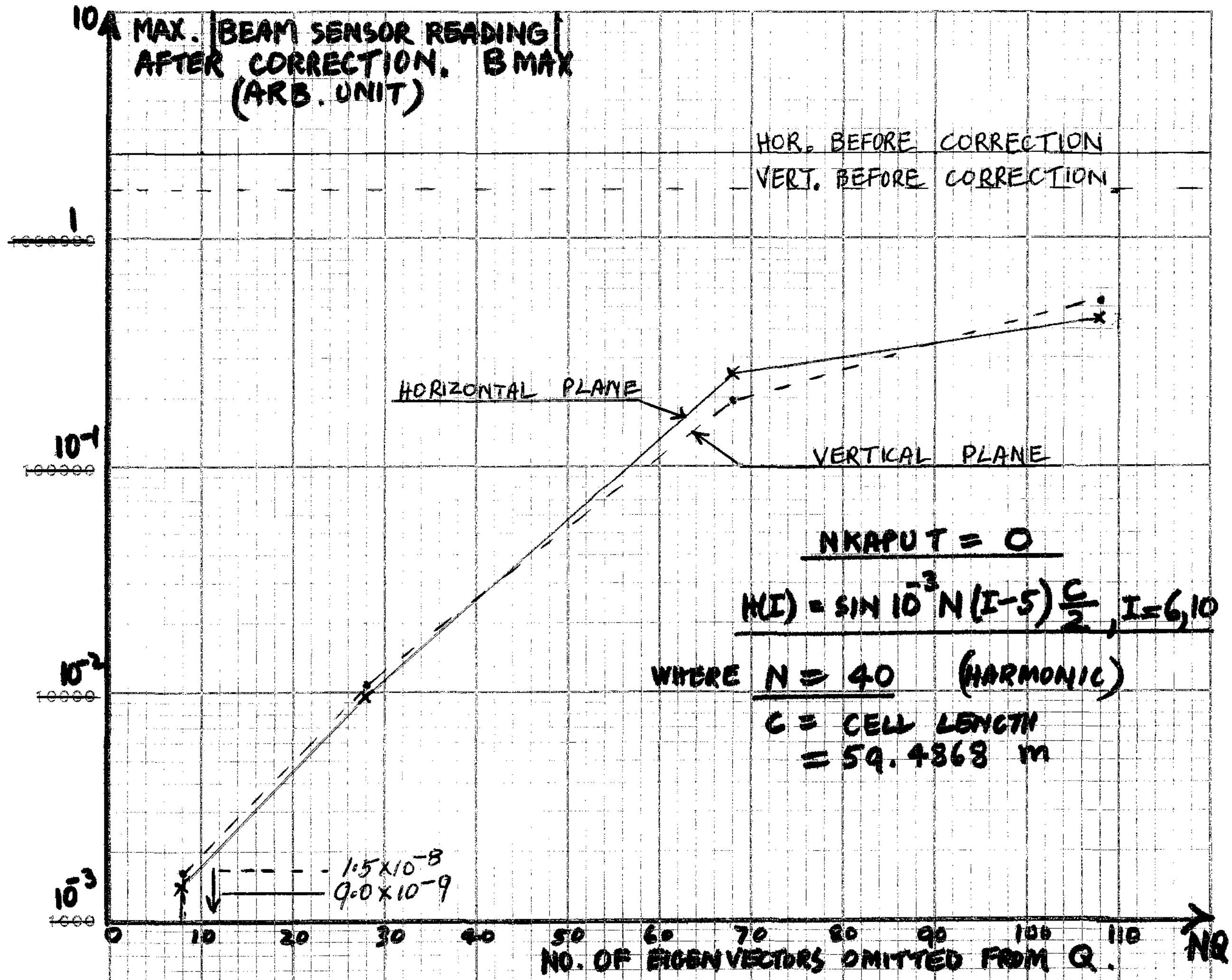



FIG. 9 EFFECT OF SMALL EIGENVALUE EIGENVECTOR OMISSION ON MAX. |BEAM SENSOR READING| AFTER CORRECTION.

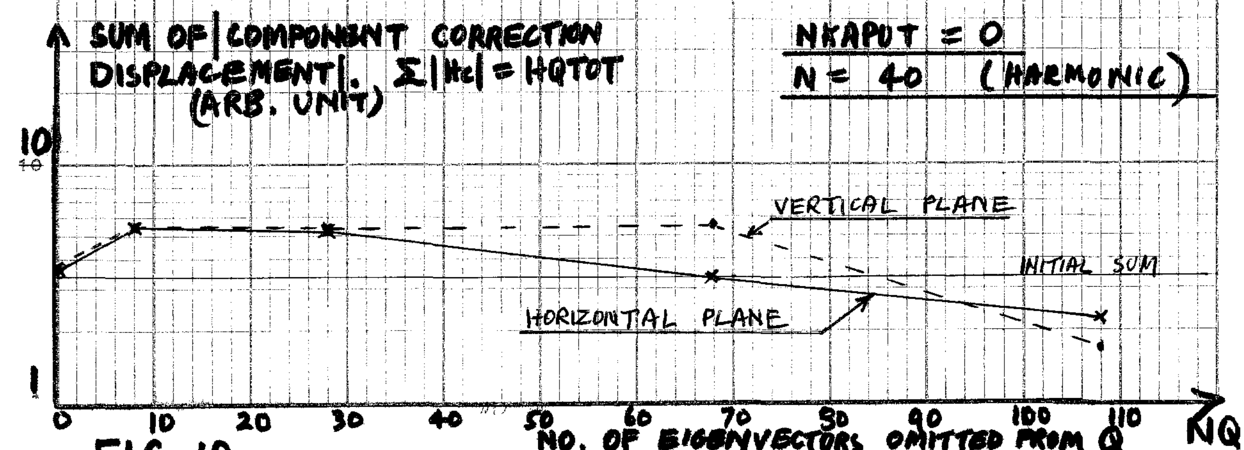



FIG. 10 EFFECT OF SMALL EIGENVALUE EIGENVECTOR OMISSION ON SUM OF |COMPONENT CORRECTION DISPLACEMENT|.

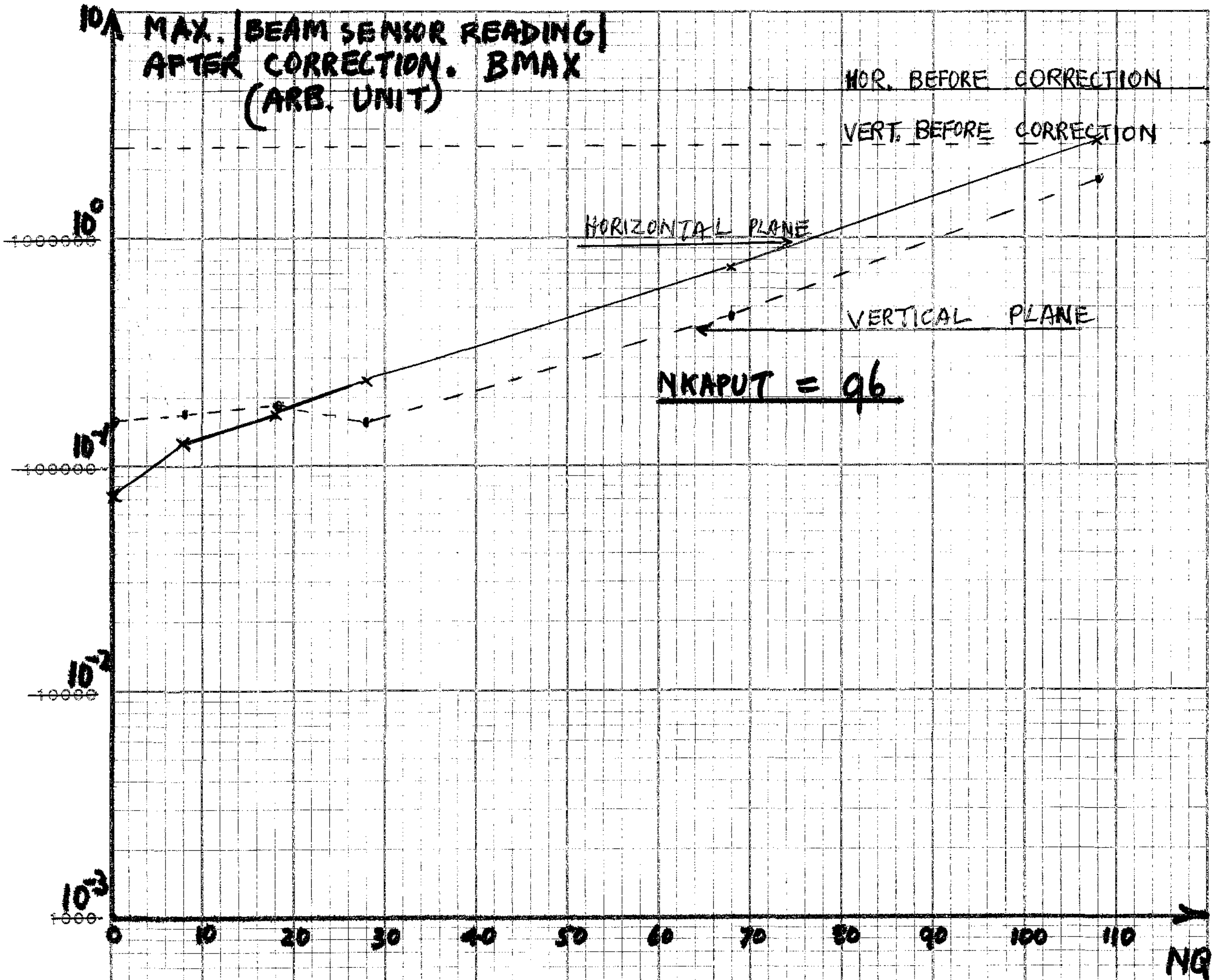



FIG. 11 EFFECT OF SMALL EIGENVALUE EIGENVECTOR OMISSION ON MAX. |BEAM SENSOR READING| AFTER CORRECTION. NKAPUT=96

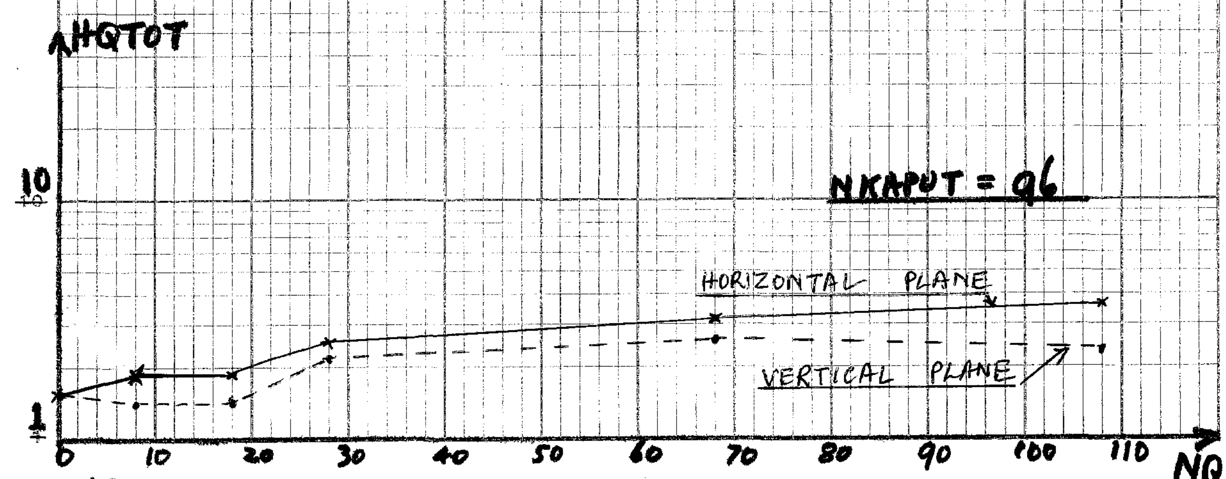



FIG. 12 EFFECT OF SMALL EIGENVALUE EIGENVECTOR OMISSION ON SUM OF |COMPONENT CORRECTION DISPLACEMENT|. NKAPUT=96

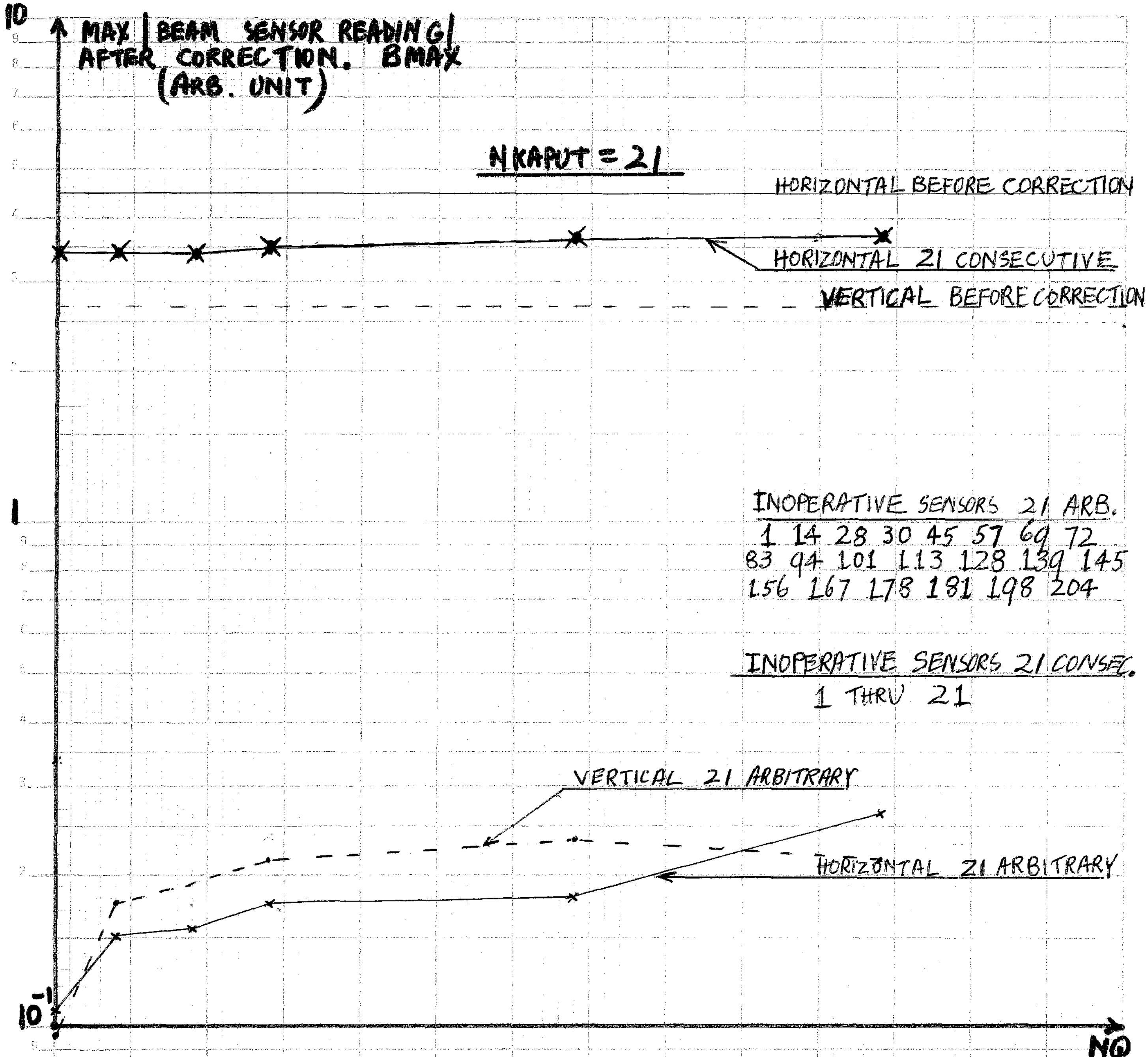



FIG 13 EFFECT OF SMALL EIGENVALUE EIGENVECTOR OMISSION ON BMAX FOR NKAPUT = 21 CASES SHOWN.

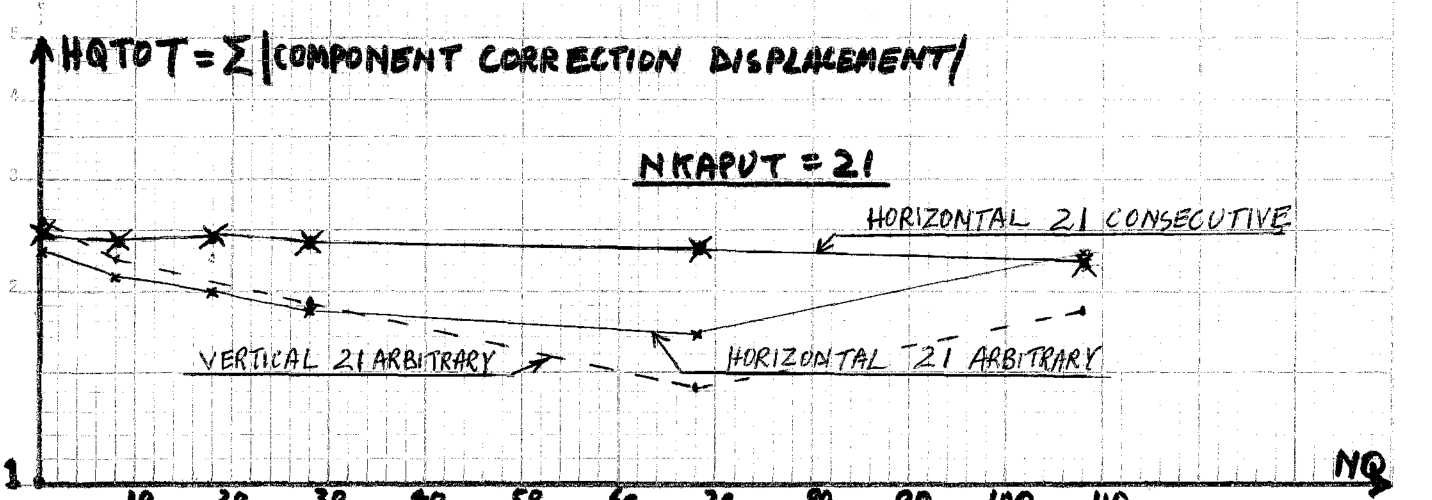



FIG.14 EFFECT OF NQ ON HQTOT FOR NKAPUT=21 CASES SHOWN

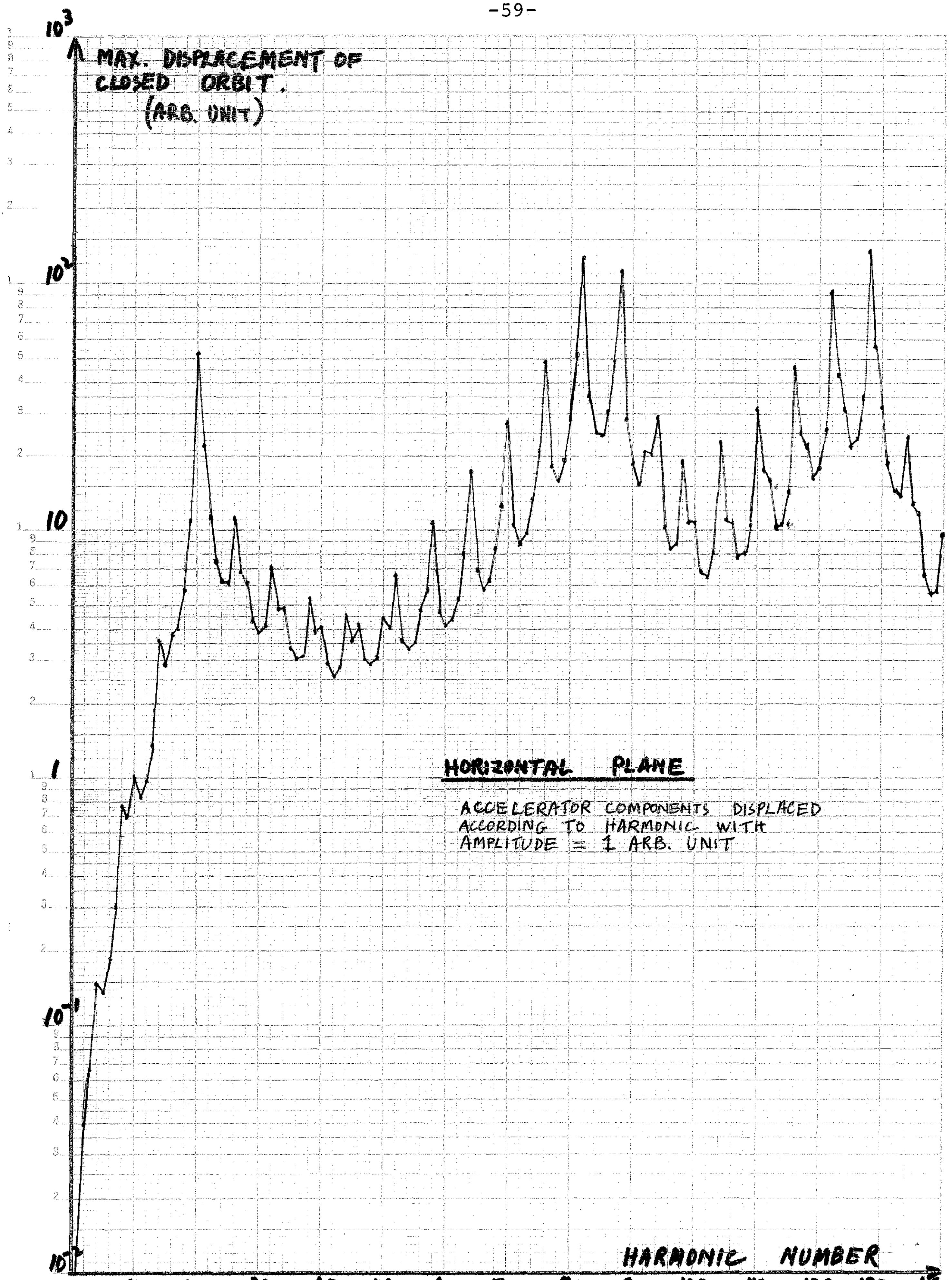



FIG. 15 HARMONIC RESPONSE IN HORIZONTAL PLANE.

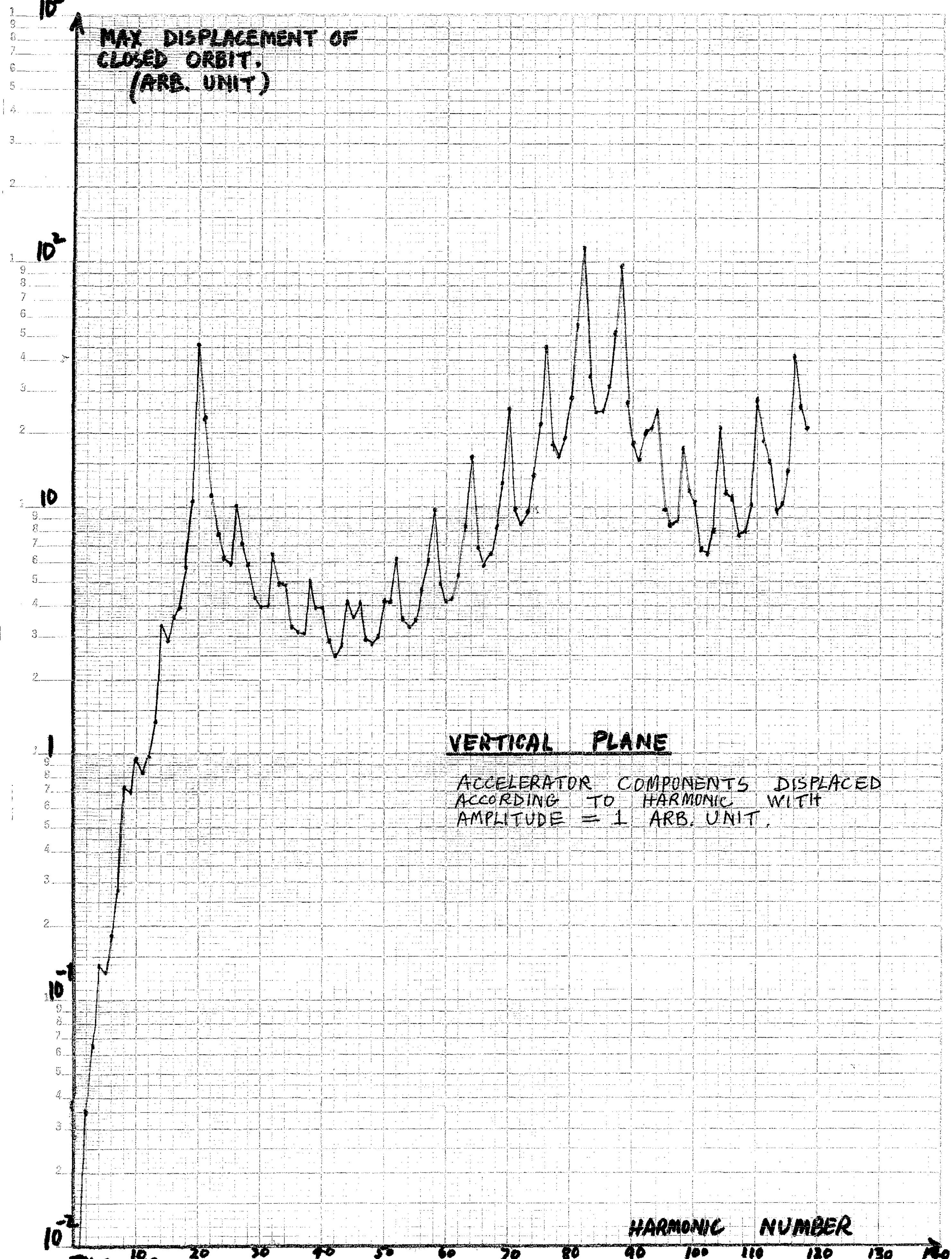



FIG. 16 HARMONIC RESPONSE IN VERTICAL PLANE.