Fermilab Proposal No. 350

Scientific Spokesman:

R. Kenney
California Inst. of Tech.
Pasasena, California 91109

(213)795-6841

A PROPOSAL TO STUDY π^o AND η INCLUSIVE PRODUCTION WITH INCIDENT π^- IN THE TRIPLE REGGE REGION

October 14, 1974

CALT-68-472

October 14, 1974

320

A PROPOSAL TO STUDY π^o AND η INCLUSIVE PRODUCTION WITH INCIDENT π^- IN THE TRIPLE REGGE REGION

ABSTRACT

We propose to measure the inclusive reactions

$$\pi^- p \to (\pi^o, \eta, \omega, \ldots) + X \quad (1)$$

$$\pi^- p \to (\pi^o, \eta, \omega, \ldots) + X^o \quad (2)$$

X^o = All neutral particles

in the essentially unexplored kinematic region $0 \le -t \le 3$ $(\text{GeV/c})^2$ and $0.8 \le x \le 1$.

The purposes of this study are: (1) To measure the ρ and A_2 trajectories out to larger $-t$ than can be reached in the exclusive charge exchange reactions due to vanishing counting rates; (2) To obtain data in the "triple Regge region" for comparison with theory for those reactions which are simplest from the theoretical point of view, and where the theory makes unambiguous predictions; and (3) To study the relation between high p_T and low p_T phenomena by measuring the above inclusive processes over a kinematic range extending from one region to the other. For example, we can test the prediction, suggested by the interchange model of high p_T processes, that $\alpha_\rho(t) \to -1$ for large $-t$.

The experiment will be run in the M2 beam at 100, 150, and 200 GeV/c with the equipment used in CEX E-111. The data will be reduced using the computing facilities available at BNL and LBL.

We request two running periods: first, a two week run to obtain a "first look" at 100 GeV/c and second, a three week run to finish the experiment. No additional equipment is necessary.

EXPERIMENTERS: CIT: A. Barnes, G. Fox, J. Mellema, A. Tollestrup, and R. Walker

LBL: O. Dahl, R. Johnson, R. Kenney, A. Ogawa, and S. Shannon

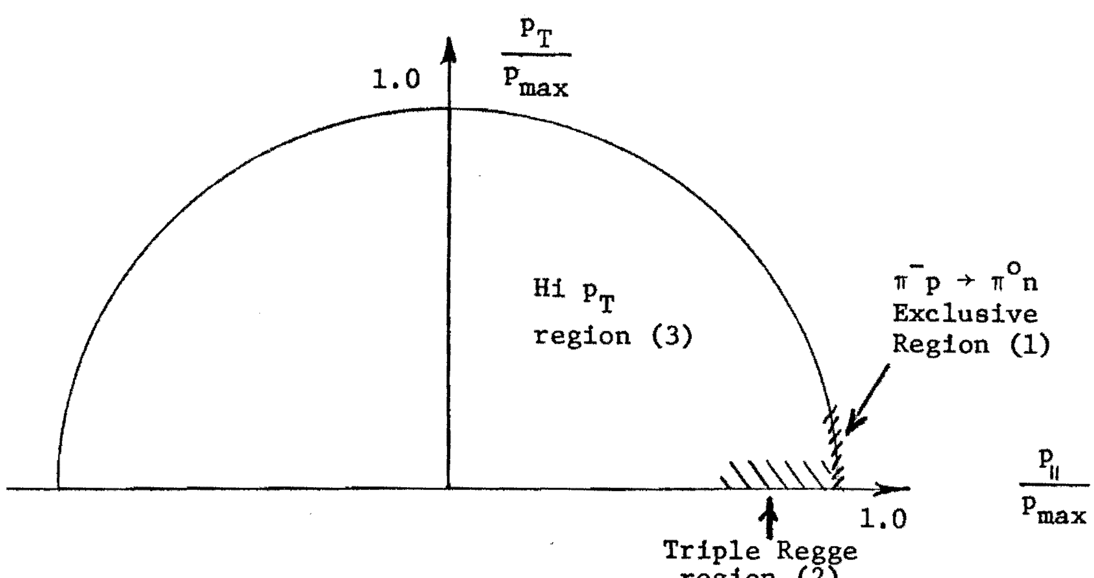
BNL: G. Donaldson, H. Gordon, and I. Stumer.

CO-SPOKESMEN: G. Fox and R. Kenney

I. Introduction.

We wish to measure inclusive meson production - mainly π^o and η with more limited information on ω and η' - through their photon decay detected in the apparatus used in the $\pi^- p \rightarrow (\pi^o, \eta)n$ experiment E-111. We are interested in the relatively unexplored "triple Regge" region where the meson (say π^o to be specific) carries off most of the momentum of the beam. We plan to study the region where $x \gtrsim 0.8$ (the Feynman x-value is $(p_{\parallel}/p_{max})_{CM} \simeq (E_{\pi^o}/E_{beam})_{lab}$) and $0 < -t < 3$ $(GeV/c)^2$ where the momentum transfer t is between the π^o and the beam π^-.

The motivation for this experiment came initially from the development of the triple Regge formalism[1)] which relates exclusive processes to inclusive processes. This formalism, which would be definitively tested by our proposed experiment, indicates how the ρ and A_2 trajectories could be obtained out to large values of -t from measurements of π^o and η inclusive production. The regions of the Peyrou plot where the triple Regge theory and the simple Regge theory have been applied are shown in the diagram below.



The triple Regge region has now been studied theoretically and the theory successfully compared with experiment for the case $pp \rightarrow pX$.

The inclusive reactions $\pi^- p \to (\pi^o, \eta)X$ are much simpler than almost any of the other reactions that can be studied, because at least in region 2 of the Peyrou plot they are dominated by the exchange of a single pole, namely, the ρ or A_2. Thus these reactions will continue to serve as a test case for theories because of their simplicity, much as their exclusive counterparts have in the past.

The exclusive charge exchange reactions of region 1 have been thoroughly investigated in Experiment 111 and other experiments at lower energies. The results for the charge exchange cross-section fit the simple Regge pole model prediction

$$\frac{d\sigma}{dt} = \beta(t)\ s^{2\alpha_\rho(t) - 2} \qquad (3)$$

surprisingly well. From the experimental measurements the ρ trajectory, $\alpha_\rho(t)$, has been determined as a function of t and is shown in figure 1. It is seen that the points fall close to a straight line thru the ρ and g mesons. It is also clear that little data is presently available for $-t > 1$ due to the small counting rate at these high t values.

A very impressive demonstration of how well the Regge formula (3) fits the data from Experiment 111 is shown in figure 2. Here we plot data from 6-200 GeV by scaling it with the factor $s^{2\alpha(t) - 2}$ to 100 GeV and then plotting the ratio of the scaled data to the 100 GeV result. If equation (3) is correct this should give a ratio of 1.0. It is seen that for $6 \le E_{LAB} \le 200$ GeV and $0 < -t < 1$ the prescription is an excellent fit to that data. Note that $d\sigma/dt$ varies by a factor of 2.10^4 within this kinematic region.

The present proposal is to study the π^o and η inclusive reactions in the triple Regge region (region 2). Specifically, for π^o production we will study the reactions:

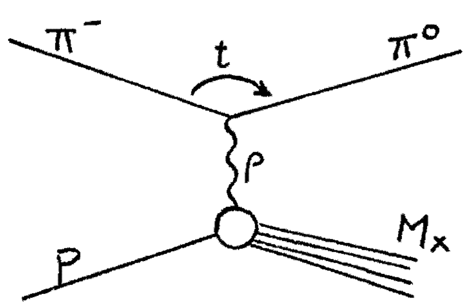
$$\pi^- p \rightarrow \pi^o X$$

$$\pi^- p \rightarrow \pi^o X^o$$

The first is the true inclusive π^o production and the second is π^o production where the final state is an all neutral one. The relevant variables in these reactions are shown in the diagram below. The theory, which is briefly described in Section II, predicts that the cross section for these reactions will be of the form

$$\frac{d^2\sigma}{dtdx} \sim (1 - x)^{1 - 2\alpha_\rho(t)} \qquad \text{reaction 1}$$

$$\frac{d^2\sigma}{dtdx} \sim \frac{1}{s}\,(1 - x)^{-2\alpha_\rho(t)} \qquad \text{reaction 2}$$

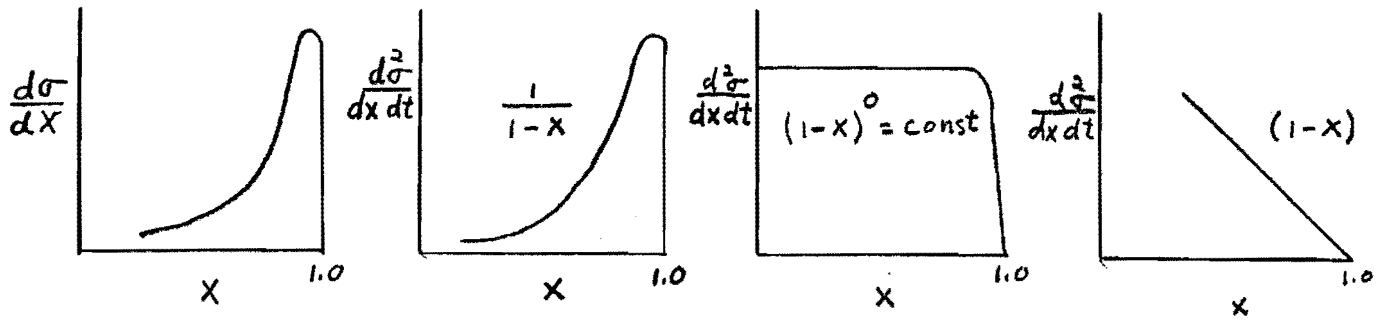


$$\frac{M_x^2}{s} = 1 - x$$

$$t = -p_T^2$$

For these equations to hold we must have s large and $(1 - x) \leq 0.2$ approximately.

To visualize the remarkable results predicted by these equations let's consider the x-dependence of the cross section for different t regions. The total cross section for reaction (2) as measured by the detector in our experiment would appear as shown in Fig. A below. Now assume for a moment that we look near $t = 0$ where the ρ-trajectory has an intercept of approximately 0.5. The cross section for reaction 2 would then be proportional to $1/(1 - x)$ and would appear as shown in Fig. B. If we look near t = -0.6 where $\alpha(t) = 0$, we would find a curve $(1 - x)^0$ such as in Fig. C. Finally as we go to a value of t such that α_ρ is equal to -0.5 the cross section would be proportional to (1 - x) as shown in Fig. D.



A B C D

These remarkable predictions of triple Regge theory have been verified by analysing some data from Experiment 111. The results corresponding to curves B, C, and D are shown in figures 3, 4, and 5. By fitting the experimental results at various values of t with the theoretical expressions, one obtains the trajectory $\alpha(t)$.

The same theory can of course be applied to η-production and the trajectory of the A_2 meson obtained. The trajectories obtained in this way from the sample of data from Experiment 111 are shown in figure 6. It should be emphasized that the data used had instrumental biases that preclude using it for a precision determination of the trajectories. The curves are shown to illustrate that the analysis has been tried and works beautifully.

The high p_T region 3 is to be studied in Experiment 268. However the measurements proposed here will extend to $p_T \approx 1.5 - 2.0\ (GeV/c)^2$ so that information will be obtained in the transition region between low and high p_T. This region is of some interest theoretically. For example, in an attempt to link the data in the triple Regge region with that in the high p_T region it has been postulated in the interchange model[2)] that the ρ-trajectory is not strictly linear but asymptotically approaches -1 as t becomes large and negative. Present measurements of the ρ-trajectory seem to be represented remarkably well by a straight line through the ρ and g mesons, extending in the negative t-region out to about t = -1.5. If the trajectory remains linear, $\alpha_\rho(-3)$ will assume a value less than -2, which is well below the postulated asymptotic value of -1. It is entirely possible, of course, that the ρ-trajectory continues to decrease linearly with increasing negative t, and that a completely new kind of physics takes over in the high p_T region. It will be interesting to obtain information on this question.

It should be emphasized that the major advantage of the proposed experiment for measuring the trajectories is that, rather than using the low counting rate exclusive cross sections for this purpose, one is using the high counting rate inclusive reactions as a source of data. It is this feature that will make it possible to collect an estimated 5,000 events with $x \geq 0.8$ and $2 < -t < 3$.

It should also be pointed out that the data will be sensitive to whether or not absorptive effects in the form of cuts are showing up. In the exclusive charge exchange reaction cuts only show up in the non-spin flip amplitude (which is small except for the region where $t \approx 0$.) That reaction is dominated by its spin flip amplitude which does not have important cut contributions. The spin flip amplitude is not expected to dominate the inclusive reaction and hence if cuts are important at high energies, the inclusive reaction should be a fruitful place to look for their effects.

II. Physics of the Triple Regge Region.

An important part of the high energy physics effort in strong interactions is devoted to the study of dynamics of simple exclusive processes such as $\pi^- p \rightarrow \pi^o n$ or $\pi^+ p \rightarrow \rho^o \Delta^{++}$. The Regge exchange picture has proved to be a surprisingly successful theory for such reactions. Since there are cuts in the j plane which could lie higher than the pole trajectory, and hence dominate at high energy, it was not expected that the simple Regge pole model should describe the high energy behavior of $\pi^- p \rightarrow (\pi^o, \eta)n$ so accurately. This success of the pole model may be fortuitous in that cuts are believed to be small in the spin flip amplitude which dominates $\pi^- p \rightarrow \pi^o n$. Thus it is interesting to study another reaction involving ρ-exchange but not expected to have a dominant spin flip amplitude to see whether simple Regge pole theory is still valid. Questions one must ask not only concern the energy dependence $s^{\alpha(t)}$ of the amplitudes, but also the t dependence: for instance does the ρ-trajectory remain linear, and is the dip at $t = -0.6$ present in all ρ-exchange processes? The recent interchange model[2)] for large p_T predicts that $\alpha_\rho(t)$ will break away from its simple form(s) past $t = -1.5$ $(\mathrm{GeV}/c)^2$ and in fact approach asymptotically to -1 as $-t \rightarrow \infty$. Both this prediction and cut modifications of ρ amplitudes are best examined at large −t where cross sections are small and thus are very hard to measure in exclusive processes at Fermilab energies.

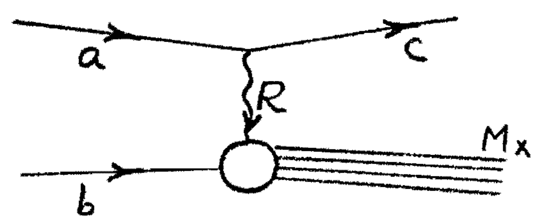
However it has recently been demonstrated that the same Regge phenomena may be studied in inclusive processes where the cross-sections are larger and so more amenable to study at large −t. The reactions studied so far are essentially[3)] the diffractive processes, mainly[4)]

$$pp \rightarrow p + \text{anything}$$

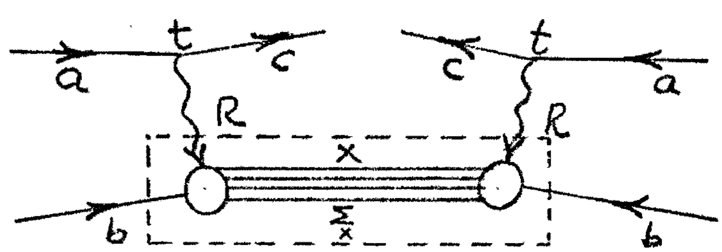
at both Fermilab[5)] and ISR energies[6)]. The general inclusive process $a + b \rightarrow c + \text{anything}$ can be described in terms of the two variables − the normal Feynman x (or the fraction of the maximum momentum p_{max} carried by c in the ab CM system) and the momentum transfer t. In the region of interest - high s, x near 1, and small t- two other useful variables - transverse momentum p_T and missing mass M - are given by

$$t \approx -p_T^{\,2}$$
$$1 - x \approx M^2/s.$$

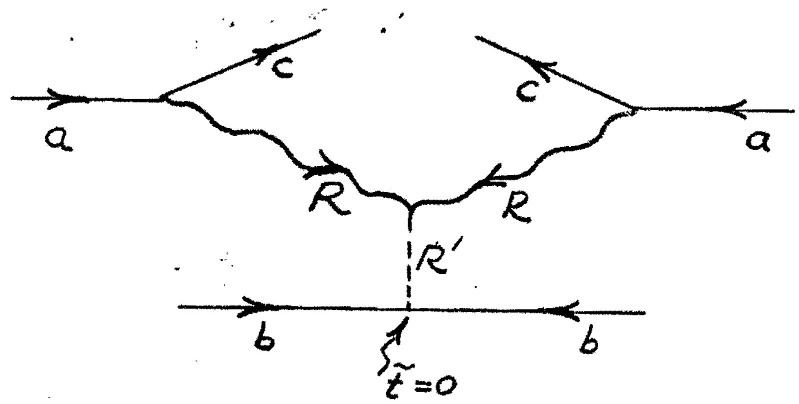
In this region, the amplitude for our inclusive process can be described by the diagram



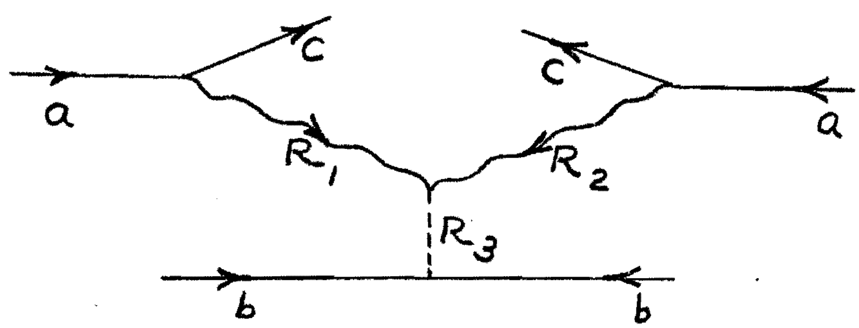
where the Regge pole R with trajectory $\alpha(t)$ is exchanged. Squaring and summing this amplitude leads to the following representation of the cross-section:



where the dotted box is formally proportional to the Reggeon-particle total cross-section and so can be equated by the generalized optical theorem to the imaginary part of the forward elastic Reggeon-particle amplitude. The latter is evaluated at the invariant energy M and so at large M can itself be approximated by the Regge form to give the final expression



which is summarized in figure 7 for the special case of $\pi^- p \to \pi^0$ + anything. Now the general case is exceedingly complicated because one must sum over all diagrams



allowed by conservation laws. Unfortunately the current data, which was predominately from the diffractive reaction $pp \to p + X$, allows a multitude of Regge exchanges. (Namely R_1 and R_2 can independently be Pomeron, ρ, ω, A_2, f, π, B ... exchange). One can nevertheless do very interesting analyses of the data, for each term has a different behavior, e.g.

$$\frac{d^2\sigma}{dtdx} \propto (1 - x)^{1 - \alpha_1(t) - \alpha_2(t)}$$

for R_3 = Pomeron, or

$$\frac{d^2\sigma}{dtdx} \propto \frac{1}{\sqrt{s}} (1 - x)^{1/2 - \alpha_1(t) - \alpha_2(t)}$$

for R_3 = f, ρ, etc.; and the data cover a range of x, s and t such that detailed fits can uncover the relative contributions of each term without unreasonable assumptions. The fits[8)] do show agreement between theory and experiment but the large number of pole terms - and the neccessity to neglect some of them (namely so-called interference terms when $R_1 \neq R_2$) - makes the fits neither transparent nor constrained. In particular it is impossible to ask about niceties of the theory and test for violations, so that there is no clear test for cut contributions. The tremendous advantage of the reactions

$$\pi^- p \rightarrow \pi^o + \text{anything}$$

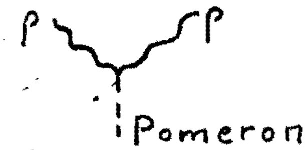
$$\pi^- p \rightarrow \eta + \text{anything}$$

lies in the fact that each allows only a single trajectory ($R_1 = R_2$) to be exchanged, ρ and A_2 exchange, respectively.

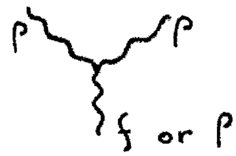
Returning to the exclusive processes, there is still no convincing Regge analysis of pp elastic scattering (simply because of the large number of spin states and exchanges just as in the inclusive case) whereas $\pi^- p \rightarrow (\pi^o,\eta) + n$ have formed the backbone of the theoretical analyses of exchange processes because of their extreme simplicity. The inclusive π^o and η production processes, discussed in this note, represent by far the cleanest test of the triple Regge formalism and the best hope to search for cuts and other interesting effects in strong interaction dynamics. Note that if cut contributions to the inclusive process do turn out experimentally to be small, then it will not be possible to use the standard explanation of a dominant spin flip amplitude as in $\pi^- p \rightarrow \pi^o n$. In fact a radical re-thinking of current absorption and cut theories will then be necessary. The measured cross-section is given by

$$\frac{d\sigma}{dtdx} = g_1 \; (1 - x)^{1 - 2\alpha(t)} + \frac{g_2}{\sqrt{s}} \; (1 - x)^{1/2 - 2\alpha(t)}$$

where g_1, g_2 represent

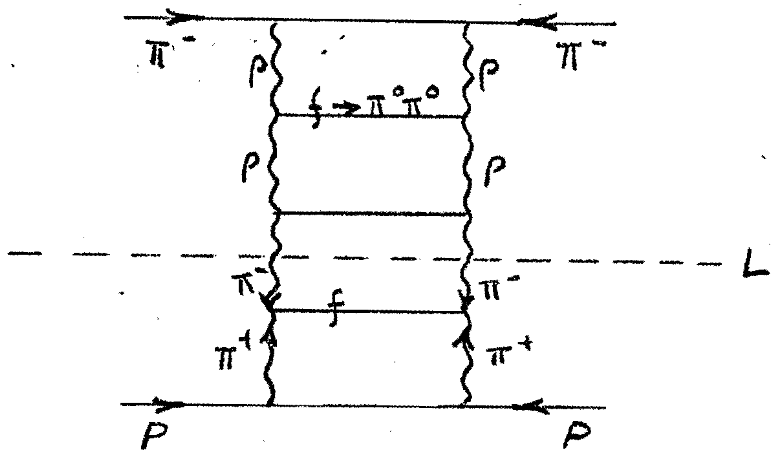


and



couplings. We can distinguish these terms by comparing the x dependence at two or more energies. Actually we expect the g_1 term to dominate at 100 GeV/c and above. This result, based on factorization and the $pp \to pX$ analysis, is not surprising as s ($\approx 2m\, P_{lab}$) is huge and the $1/\sqrt{s}$ coefficient suppresses the g_2 term. Thus we will have no problem in identifying $\alpha(t)$ and thus testing for cuts, linear trajectories etc. from our measured x dependence at fixed s. We can also extract the t dependence of the coupling constants g_1 for ρ and A_2 exchange. This will enable us to test for the dip at $t \approx -.6\ (\mathrm{GeV}/c)^2$ (familiar from $\pi^- p \to \pi^0 n$) expected for ρ exchange in naive Regge models. More generally we can examine the relation between the ρ and A_2 coupling constants. In exclusive channels this is given by exchange degeneracy, but this formalism has as yet not been extended to inclusive reactions.

We now consider the reaction $\pi^- p \to \pi^0 X^0$, X^0 all neutral. This extension was originally born of necessity as we had only a (biased) sample of all neutral data taken in conjunction with E-111 and not the full inclusive data for our feasibility studies. However the all neutral inclusive reaction has since turned out to be so intriguing that it would be nice to do a proper job of it. First we extract from the slightly doubtful if popular multiperipheral model the result[7)] that the s-dependence of reactions like $\pi^- p \to$ (all neutrals) can be described by poles in the j-plane and hence has a power law behavior. Perhaps the model is wrong but its prediction agrees with experiment and can be expressed in such general terms that it might also be true in more realistic theories. The multiperipheral model result follows at once from the fact that ladder diagrams like



split into two separate ladders having ths same form when cut by any line L. This implies we can sum them by a t-channel integral equation and obtain j-plane poles. The same technique will work for any cross-section of the form:

ab → any particle <u>not</u> satisfying some given property P.

In our case, the property P is being charged, while in another example $\bar{p}p$ annihilation, P is the property of being a baryon or antibaryon. Examination of experimental data confirms an approximate power law behavior for $\pi^- p \to$ all neutrals: in fact

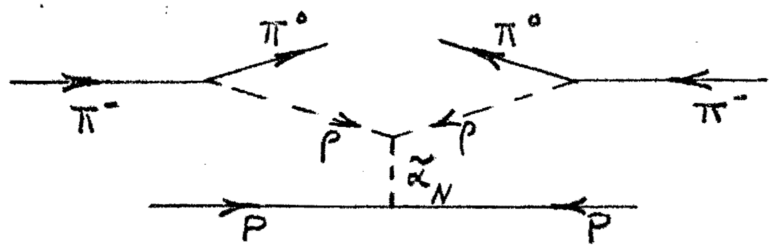
$$\sigma(\pi^- p \to \text{ all neutrals}) \sim s^{\tilde{\alpha}_N - 1}$$

where the (pseudo) trajectory has $\tilde{\alpha}_N \approx 0$. This contrasts with the total cross-section which is roughly constant with the trajectory $\alpha = 1$ (the Pomeron).

Now consider the process:

$$\pi^- p \to \pi^o + \text{all neutrals}$$

where the π^o is emitted in the triple Regge region, x near 1 and t small. As already illustrated in figure 1, one can clearly analyse this process in the triple Regge formalism to obtain the diagram



where in the bottom leg one has our new neutral pole with intercept $\tilde{\alpha}_N$ instead of the conventional Pomeron, f, and ρ trajectories. The corresponding cross-section is given by

$$\frac{d^2\sigma}{dtdx} \sim (1/M^2)^{1 - \tilde{\alpha}_N} \cdot (1 - x)^{1 - 2\alpha_\rho(t)}$$

or in the approximation that $\tilde{\alpha}_N = 0$,

$$\frac{d^2\sigma}{dtdx} \sim \frac{1}{s} (1 - x)^{-2\alpha_\rho(t)} .$$

We have used this formalism to analyse our present all neutral data and find the remarkable results displayed in figures 3 - 6. The formalism can be extended trivially to the general reaction $\pi^- p \to (\eta, \omega^o$ or indeed any neutral meson)+ all neutrals.

On a final theoretical note, we remark that the triple Regge formalism is valid in region of small t, large M^2 and large s/M^2. The latter condition implies x is near 1 [as $s/M^2 = 1/(1 - x)$]; experience with $pp \to pX$ has determined this region as $x > 0.8$ (or $s/M^2 > 5$). It is important to investigate whether the triple Regge behavior occurs over a larger or smaller x-region in reactions (2). Since only a single trajectory is exchanged in these reactions, the range of validity of the triple Regge formalism can be studied with greater precision.

III. Experimental Details.

The equipment needed for this experiment is part of that used in the charge exchange Experiment 111. Two different configurations of the detector will be used. In one, called Geometry I and shown in figure 8, the detector will be located to the side of the beam and will cover the higher t regions of the reaction. In the other arrangement (Geometry II), the detector will be directly in the beam as was the case for E-111 and will cover the low t values. The charged particle veto counters around the target for studying the all neutral final state will be the same as those used previously. The checkerboard counter has already been constructed and installed for E-268. The experimental equipment is essentially ready at present.

However, in order to perform this experiment, it will be necessary to handle those cases where more than two gamma rays strike the detector or where additional charged particles accompany the π^o and cause pulses in the detector. In E-111 ($\pi^- p \rightarrow \pi^o n$), these cases were all automatically excluded; however since we are now studying inclusive reactions we must be able to reconstruct the π^o when it is accompanied by other particles from the same reaction.

We have studied this problem using data taken during the charge exchange experiment. For the geometry where the detector is located in the beam (Geometry II) we have found that only about 25% of the events have additional photons in the detector when $x > 0.8$. This results from requiring a very high x, in which case little energy is left over to make other gamma rays that fall within the relatively small angles subtended by the detector. Although we have not made measurements, calculations show that there will be significantly fewer events with additional photons for Geometry I.

Although the confusion of multi-gamma events is not expected to be serious we have nevertheless undertaken to solve the problem of reconstructing such events in the detector. The results of this study are described more fully in our accompanying report on parasitic running of experiment E-268 in the large p_T region where multigamma events present a more serious problem. An extensive program has been written for the event reconstruction and it has been run on the 7600 at BNL. From the success that we have had so far we do not believe that we will have any biases in this experiment from multi-gamma events. Adequate computer time will be available at LBL and BNL for the analysis of this data. Other possible problems are related to the energy resolution of the detector which limits the precision of the measurement of x and the missing mass. These problems have been studied in detail as discussed in the Feasibility Study submitted as a supplement to this proposal. It has been shown that they will not interfere with the goals of this experiment.

The cross sections used for estimating counting rates in this experiment are shown in figures 9 and 10. It is worth noting that the all inclusive cross-section shown in figure 9 is very much larger than the charge exchange cross section in figure 10. The $\pi^- p \rightarrow \pi^o X^o$ cross section is intermediate in value.

We plan to make measurements at 100, 150, and 200 GeV/c or greater for the incoming pion. This will allow for a complete check on the consistency of the s and M^2 dependence of the theoretical formulae. The cross-section $\pi^- p \rightarrow \pi^o X^o$ is small enough so that in some running the trigger must include the charged particle veto around the target. Information on this reaction will also be obtained during measurements of the all inclusive process by tagging those events in which none of the target veto counters was triggered. We can obtain a check on the value of the "pseudo" parameter $\tilde{\alpha}_N$ by measuring the all neutral cross section between 20 and 200 GeV for comparison of its energy dependence with the expected $1/p_{lab}$. This is a very simple measurement which will not require much time, and it will allow an independent verification of one term in the theoretical formula. The counting rates at high t, of course, are unknown as we do not know how the ρ-trajectory behaves. However, the sensitivity of the experiment is sufficient to insure that even if the trajectory continues to be linear, we will have a good determination of $\alpha_\rho(t)$ out to $t = -3$.

IV. Running Time Request.

Table I shows the estimated cross sections for the two reactions that we wish to study. The values at high t are, of course, very uncertain. Using these values we have estimated the running time required for these measurements.

We request two running periods. One two week period during which we would get a "first look" at the 100 GeV/c cross section. This would be followed later by a three week run to finish the experiment.

The number of events expected from such a run are shown in Table II. It is clear that the full inclusive reaction will yield an adequate number of events for an excellent determination of the ρ-trajectory out to at least t = -3. On the other hand the reaction $\pi^- p \rightarrow \pi^0 X^0$ will suffer from a lack of statistical accuracy past a t of -2.

Details of these estimates may be found in the Feasibility Study submitted as a supplement to this proposal.

Table I

Integrated cross sections in μb for $0.8 \leq x \leq 1$, and for the t intervals shown.

Reaction	p_{lab}	100 GeV/c	150 GeV/c	200 GeV/c
$\pi^- p \to \pi^o X$	$0 \leq -t \leq 1$	100	100	100
	$1 \leq -t \leq 2$	1-10	1-10	1-10
	$2 \leq -t \leq 3$	1/5-3	1/5-3	1/5-3
$\pi^- p \to \pi^o X^o$	$0 \leq -t \leq 1$	2	1.3	1.0
	$1 \leq -t \leq 2$	.07	.05	.035
	$2 \leq -t \leq 3$	.01	.006	.005

Table II

Division of Running Time and Expected Counts

		2 weeks		1 week		2 weeks	
		100 GeV/c		150 GeV/c		200 GeV/c	
		Geom. 1	Geom. 2	Geom. 1	Geom. 2	Geom. 1	Geom. 2
	Hours	100	60	50	30	100	60
	Δt						
$\pi^- p \to \pi^o X$	0 - 1	-	240K	-	120K	-	240K
	1 - 2	5K-50K	2.4K-24K	2.5K-25K	1.2K-12K	5K-50K	2.4K-24K
	2 - 3	1K-15K	-	500-7K	-	1K-15K	-
$\pi^- p \to \pi^o X^o$	0 - 1	-	14K	-	5K	-	7K
	1 - 2	320	500	110	160	160	250
	2 - 3	60	-	20	-	30	-

References

1. C.B. De Tar et al., Phys. Rev. Lett. 26, 675 (1971).
G.C. Fox, "Inclusive Structure of Diffraction Scattering," preprint (1973).

2. R. Blankenbecler and S.L. Brodsky, "Unified Description of Inclusive and Exclusive Reactions at all Momentum Transfers," SLAC-PUB-1430, preprint (1974).

3. Nondiffractive data in the triple Regge region is available for $pp \to nX$ from experiments at ISR and Fermilab (E-188A). Several exchanges (π, β, ρ, A_2...) are allowed in $pp \to nX$ and there is no data in the interesting region where t is large.

4. An analysis of all diffractive triple Regge data (except that of Fermilab E-186) is contained in R.D. Field and G.C. Fox, "Triple Regge and Finite Mass Sum Rule Analysis of the Inclusive Reaction $pp \to pX$," Nucl. Phys., (to be published).

5. Fermilab results on diffractive scattering in the triple Regge region from:
   a) FNAL πp and pp 30" bubble chamber data
   b) E-14A (Columbia, Stonybrook)
   c) E-188A (Imperial College, Rutgers, Upsala)
   d) E-186 (Dubna, Fermilab, Rockefeller, Rochester)

   Further diffractive triple Regge data can be expected in:
   e) E-221 (Columbia, Stonybrook)
   f) E-51A (Northwestern)
   g) E-96 (Focussing Spectrometer Facility)
   h) E-86A (Washington, Orsay)

6. ISR results on pp $\rightarrow$ pX in the triple Regge region come from CHLM collaboration, Nucl. Phys. B73, 40 (1974) and from earlier work quoted there.

7. P.W. Coulter and D.R. Snider, Phys. Rev. D8, 4055 (1973).

8. A.V. Stirling et al., Phys. Rev. Lett. 14, 736 (1965).
P. Sonderegger et al., Phys. Lett. 20, 75 (1966).
V.N. Bolotov et al., Nucl. Phys. B73, 365 (1974).

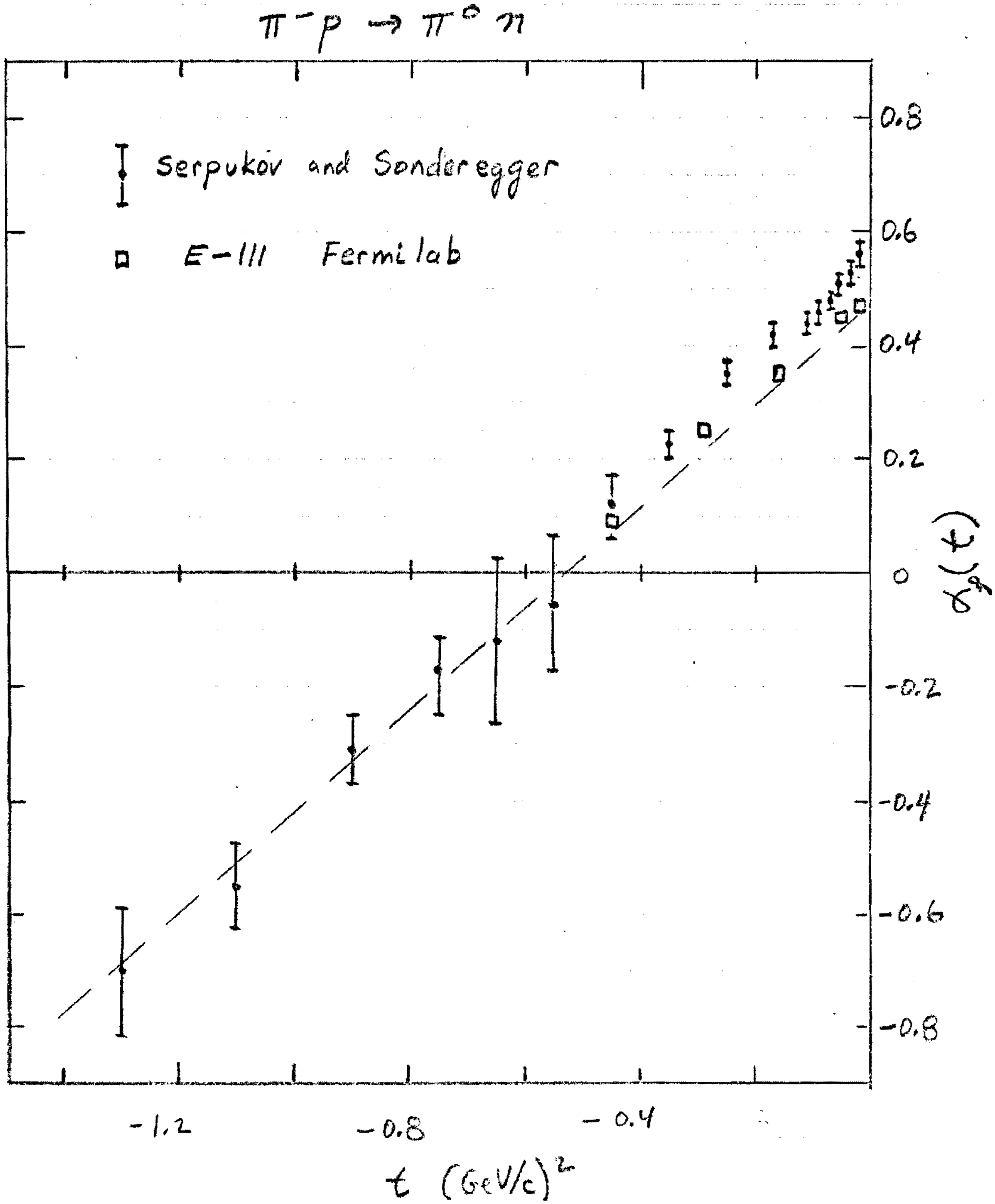


Fig. 1: The ρ trajectory determined from several measurements in the range from 6 GeV to 100 GeV.[8)]

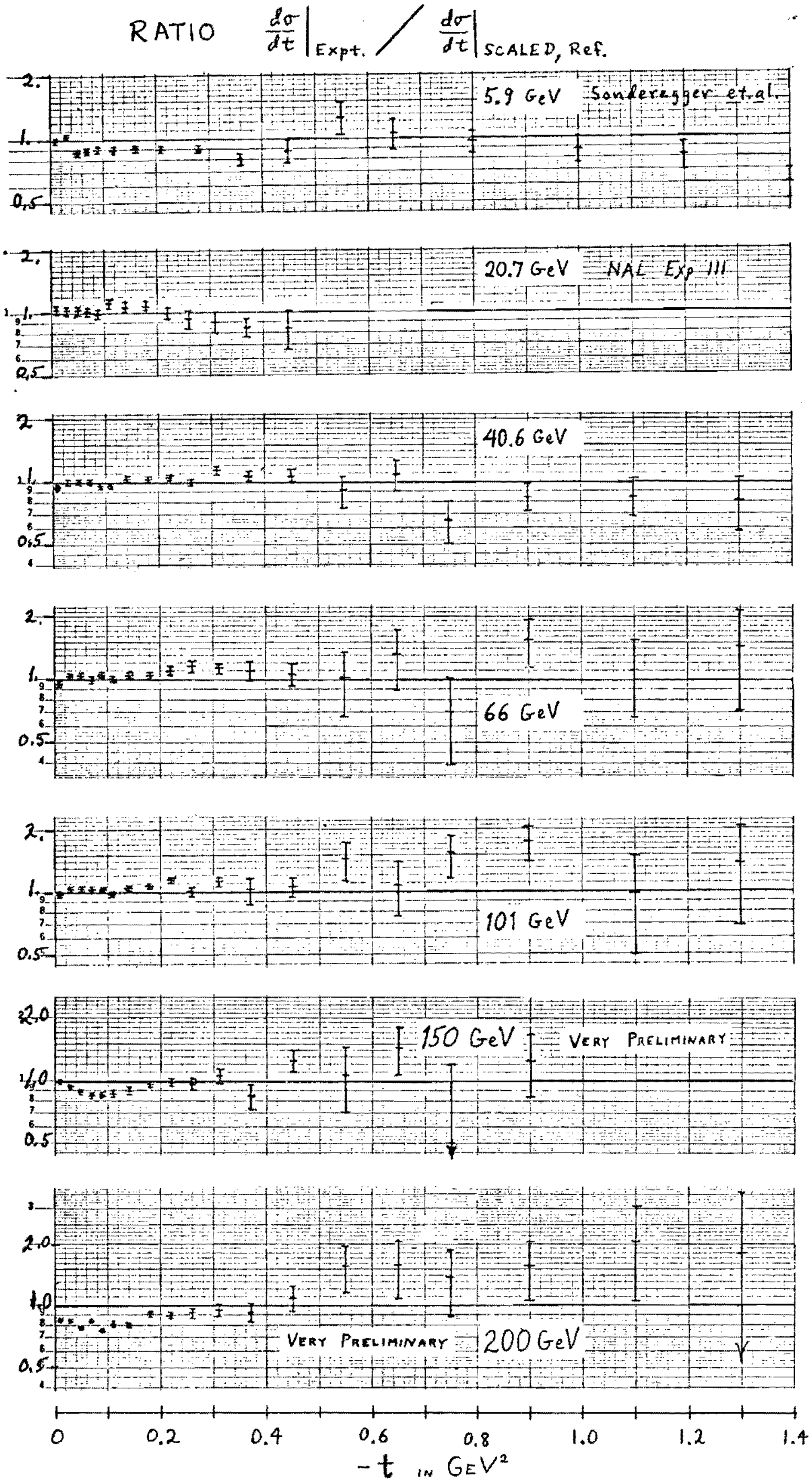


Fig. 2: Comparison of data on $\pi^- p \rightarrow \pi^0 n$ in the 6 GeV - 200 GeV interval with data at 100 GeV by scaling the former with the Regge formula (3). This ratio of scaled data to the 100 GeV result should be equal to 1.0 if equation (3) is correct.

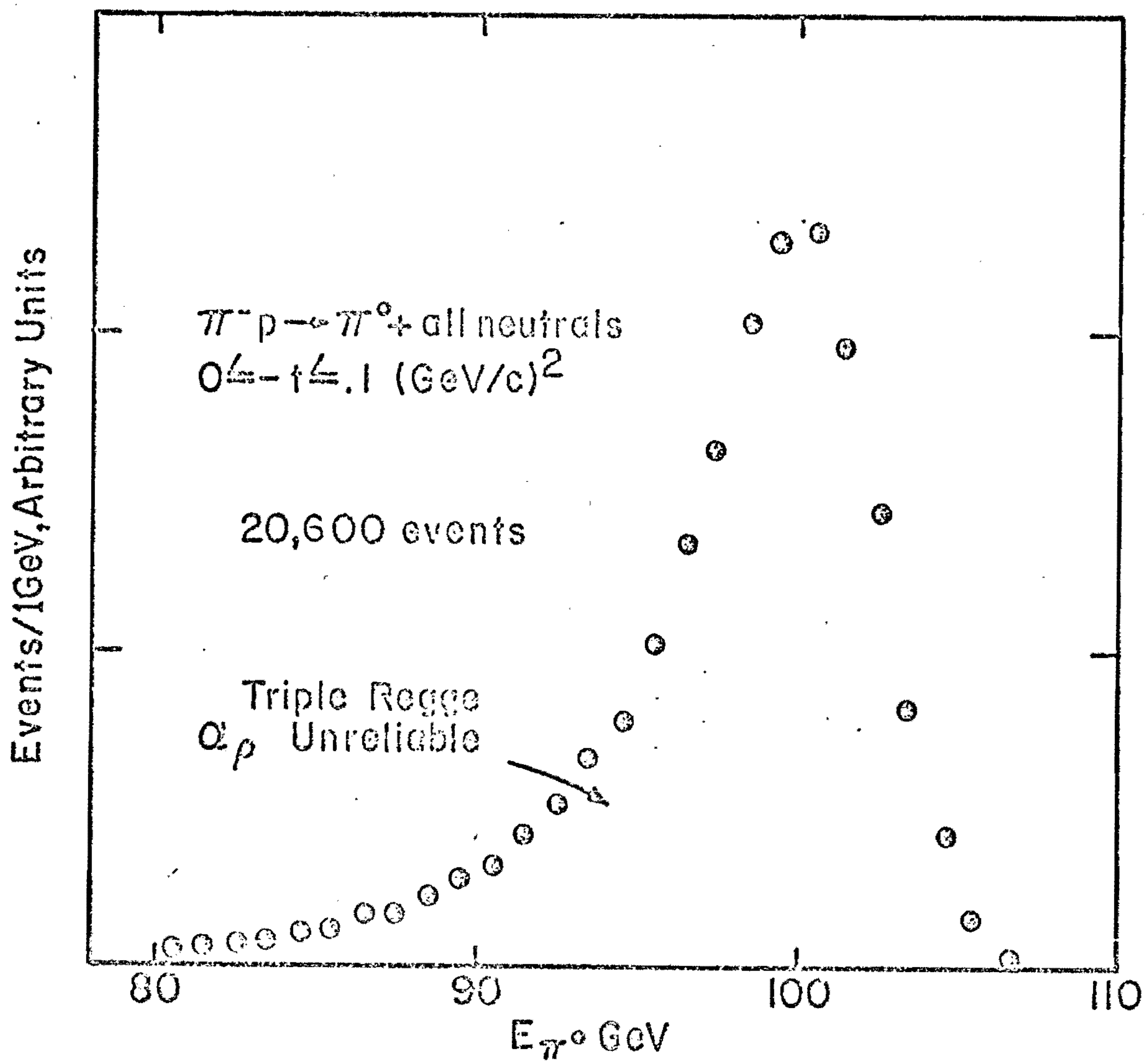


Fig. 3: Energy distribution of π^o from preliminary analysis of the 101 GeV/c data on $\pi^- p \rightarrow \pi^o X^o$ (all neutrals) in the range $0 \leq -t \leq 0.1$ $(GeV/c)^2$. The peak has a large contribution from π^o Δ^o final states where "Δ" stands for the nucleon resonances (N*) excited in the scattering. Extraction of the triple Regge α_ρ is unrealiable for the small t values in this distribution because of the relatively large yield of Δ^o compared with higher masses of the inclusive final state. (The triple Regge analysis derives α_ρ from these high mass states.) This distribution can be compared with Fig. B in the text.

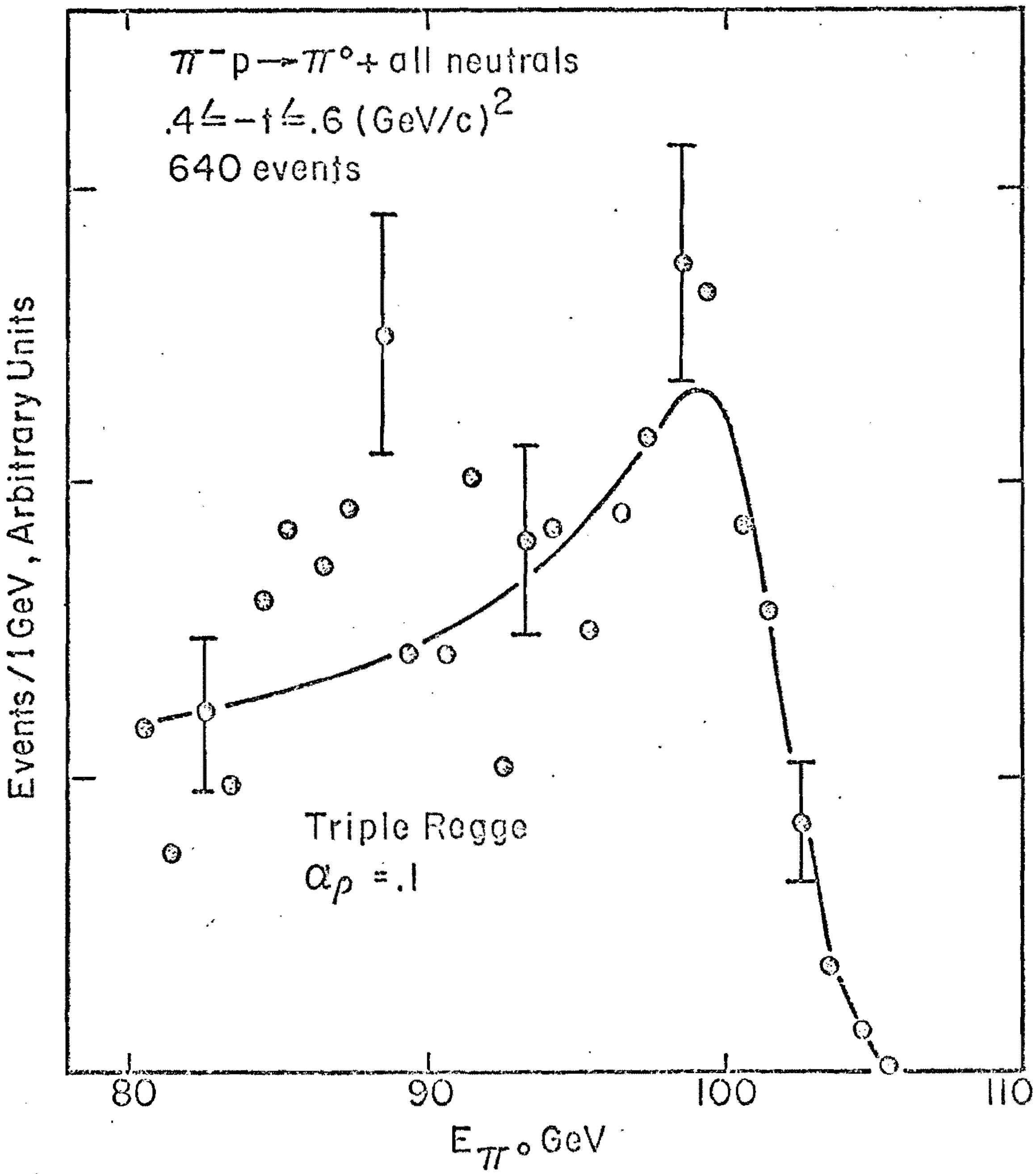


Fig. 4: The distribution in this figure is like that in Fig. 3, except here the t range is $0.4 \leq -t \leq 0.6\ (GeV/c)^2$. In this region of t, the number of high mass final states dominates by a factor of 4 and permits the triple Regge analysis to extract the value of $\alpha_\rho = 0.1$, averaged over the internal in t. This distribution can be compared with Fig. C in the text.

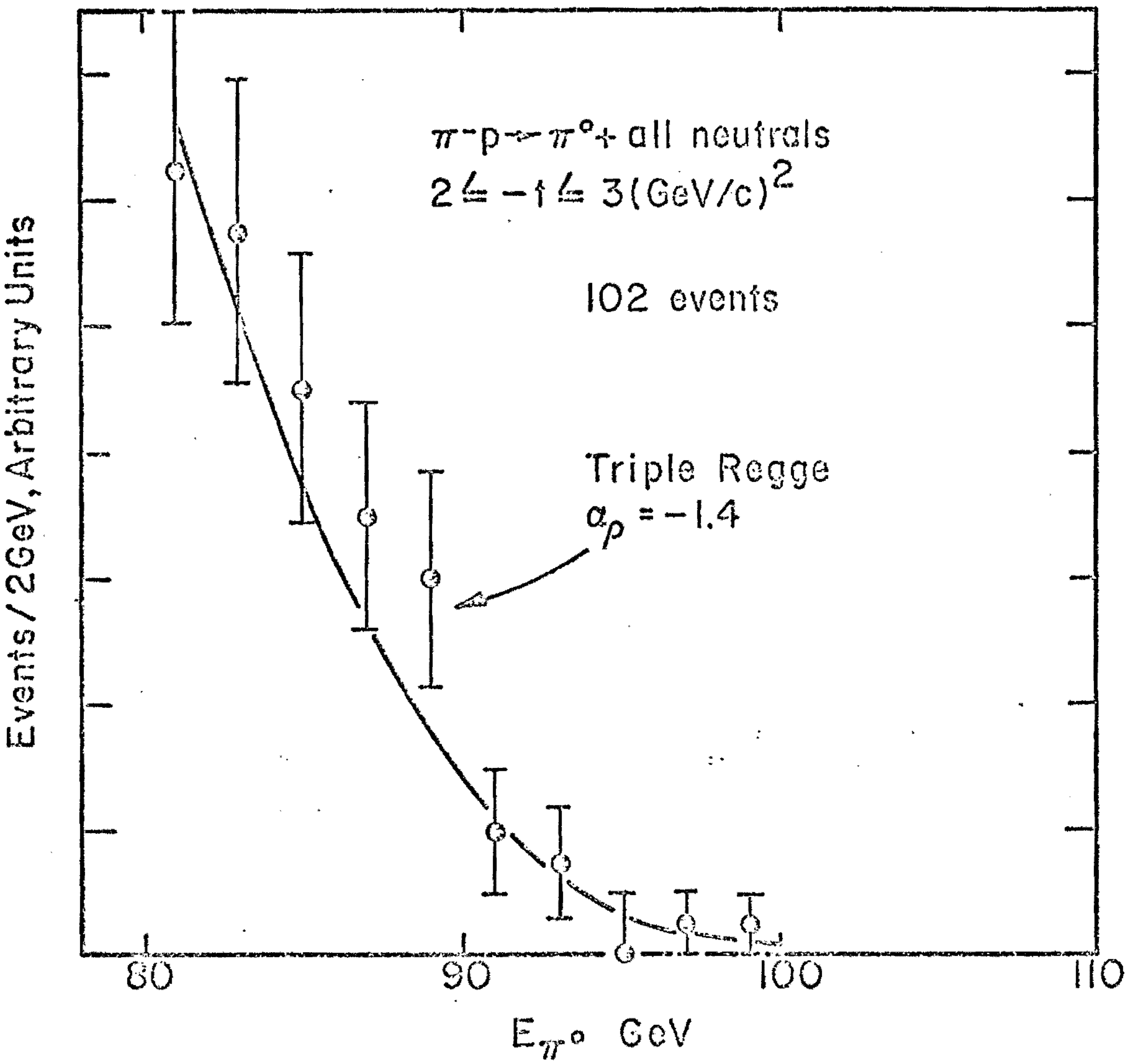


Fig. 5: The distribution in this figure is like that in Figs. 3 and 4, except for the range in t, namely, $2 \leq -t \leq 3$, in which Δ^0 production is negligible. Triple Regge analysis yields $\alpha_\rho = -1.4$ averaged over this interval in t. This distribution may be compared with Fig. D in the text.

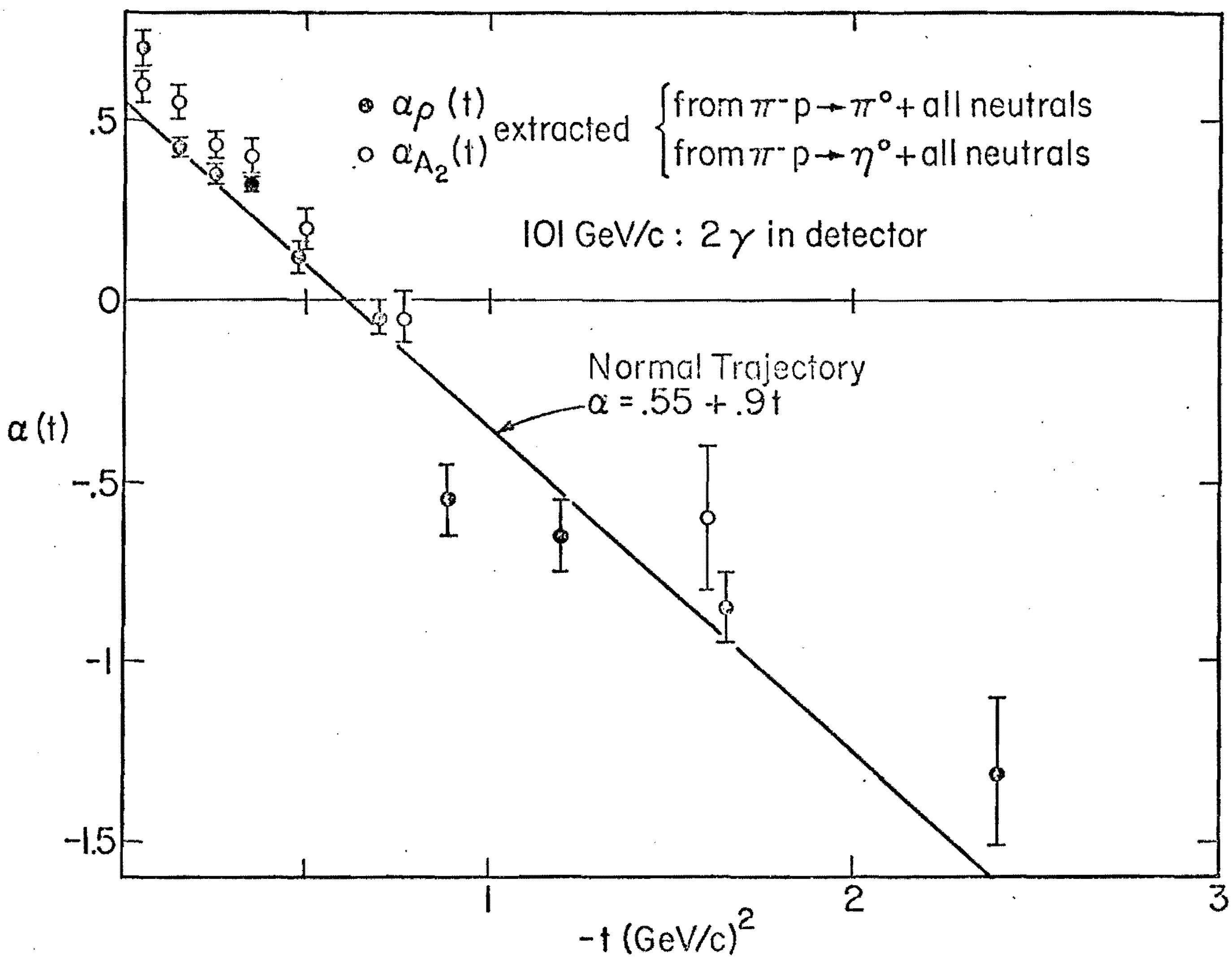


Fig. 6: Values of the trajectory function $\alpha(t)$ determined by fitting the expression

$$\frac{d^2\sigma}{dx\,dt} \propto (1 - x)^{-2\alpha(t)}$$

to the observed energy distributions typified by Figs. 3, 4, and 5 for $\pi^o X^o$ and similar data for ηX^o final states. The data for π^o and η yield the ρ and A_2 trajectories, respectively.

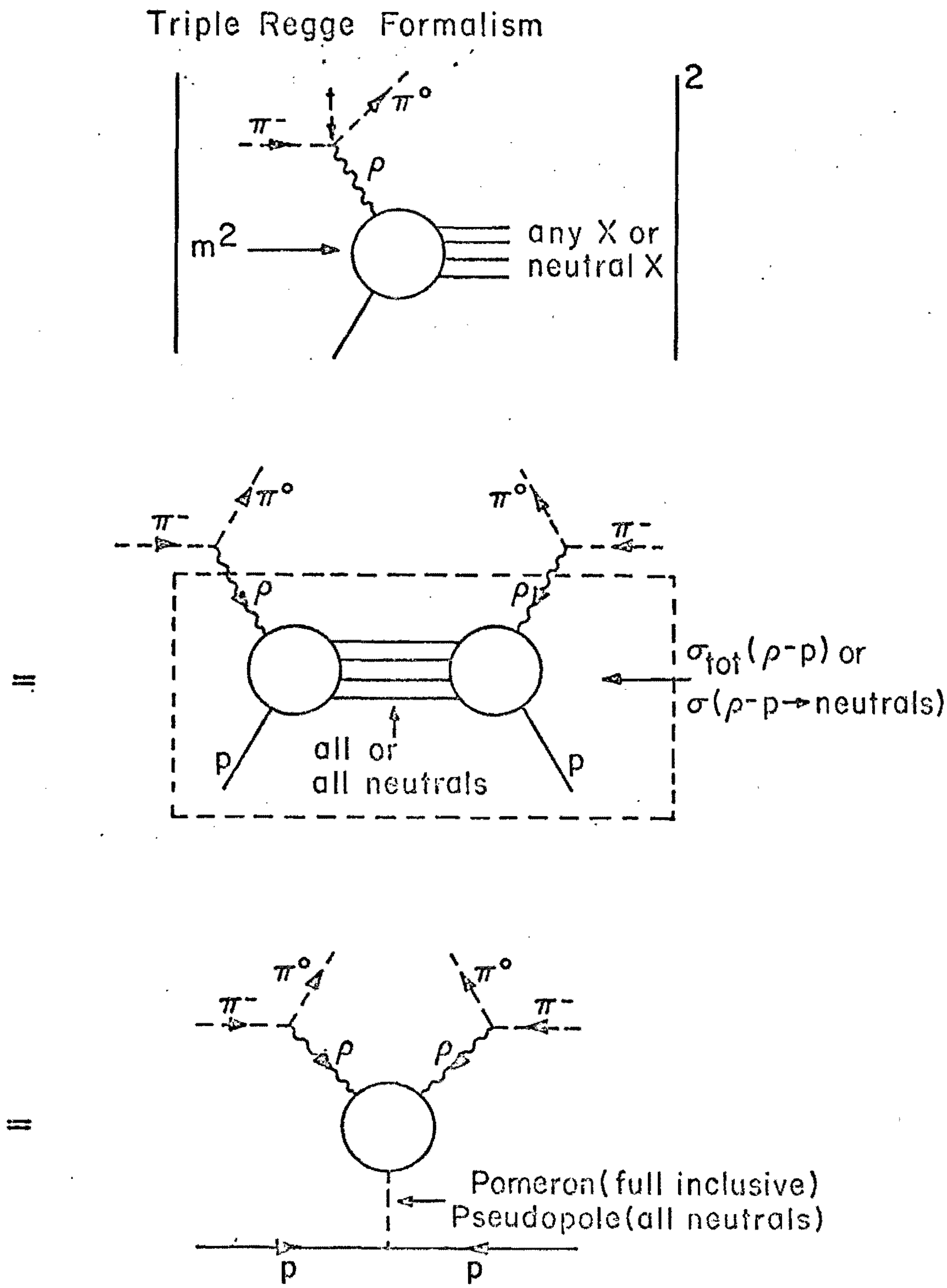


Fig. 7: Diagrams illustrative of triple Regge formalism for the full inclusive reaction or the more restrictive all neutral reactions. The theory is outlined in Sec. II.

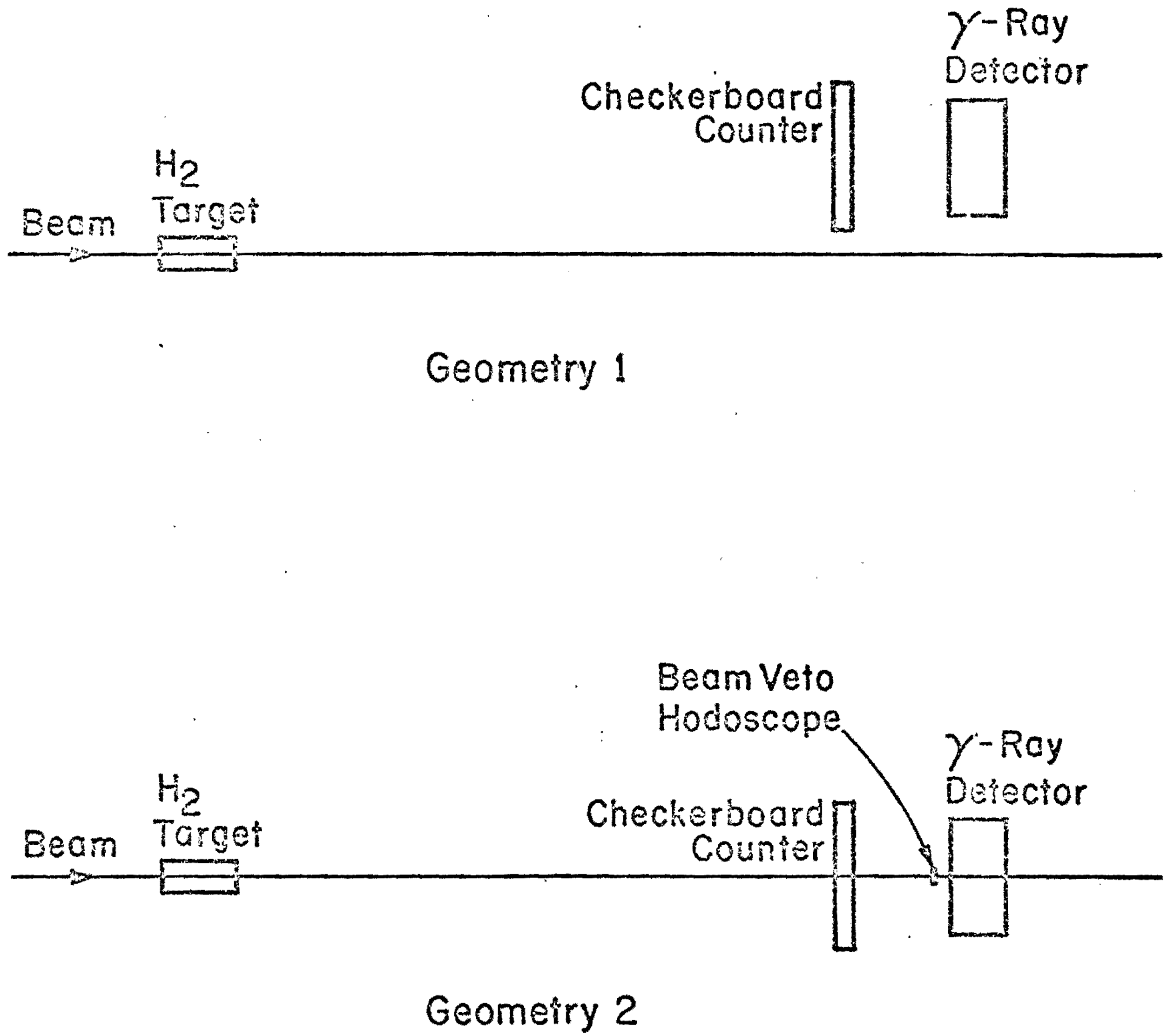


Fig. 8: Schematic of experimental layout for Geometry 1 (detector out of beam) and Geometry 2 (detector in beam), described in Sec. III.

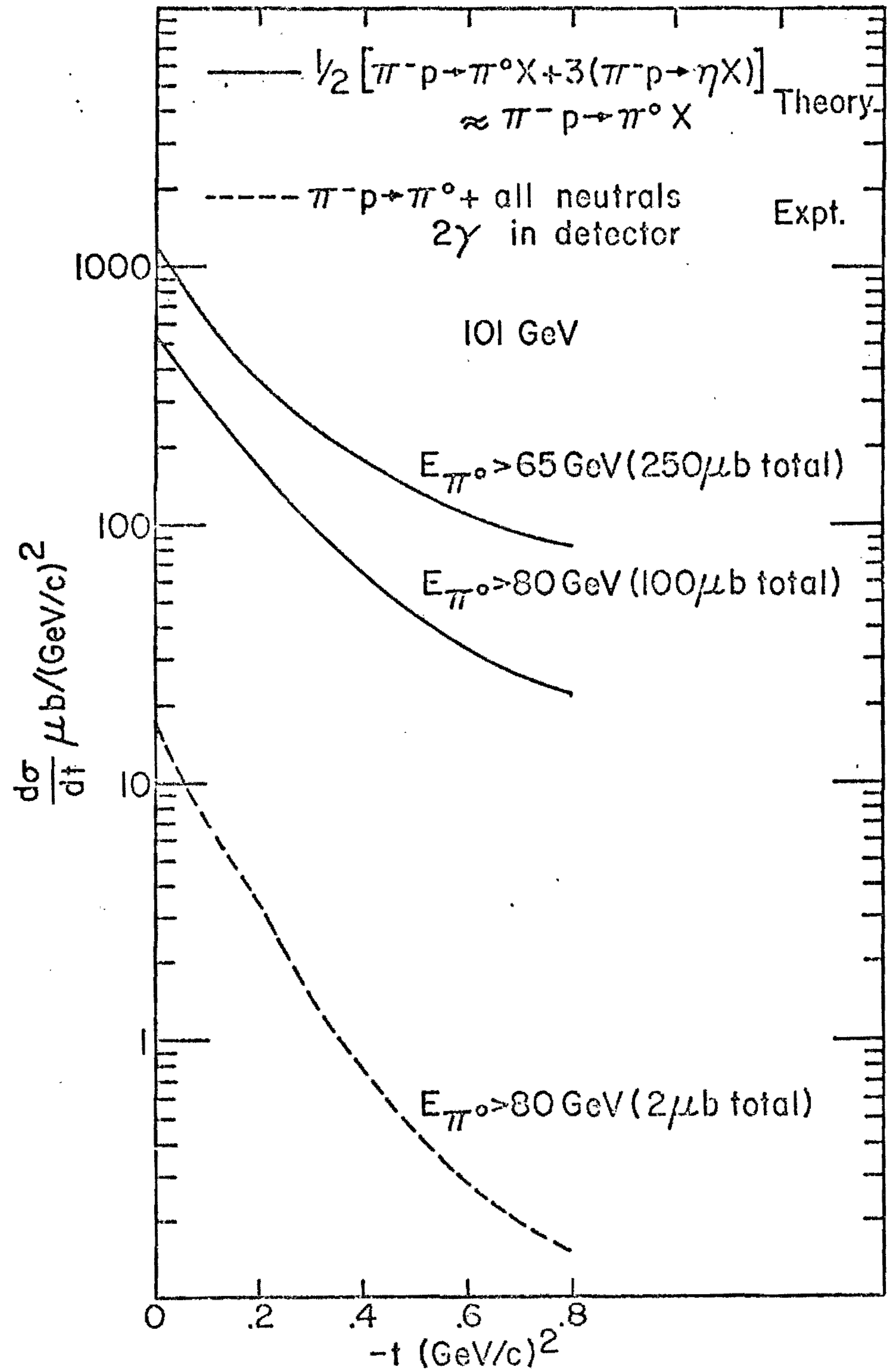


Fig. 9: Theoretical estimate of $\pi^- p \to \pi^0 X$ at 101 GeV/c for $E_{\pi^0} \geq 80$ GeV ("signal") and $E_{\pi^0} \geq 65$ GeV ("trigger'), using factorization and $pp \to pX$ data, as described in Sec. III of the Feasibility Study submitted as a supplement to this proposal. This is compared with the triple Regge contribution to $\pi^- p \to \pi^0 X^0$ at 101 GeV/c for $E_{\pi^0} \geq 80$ GeV.

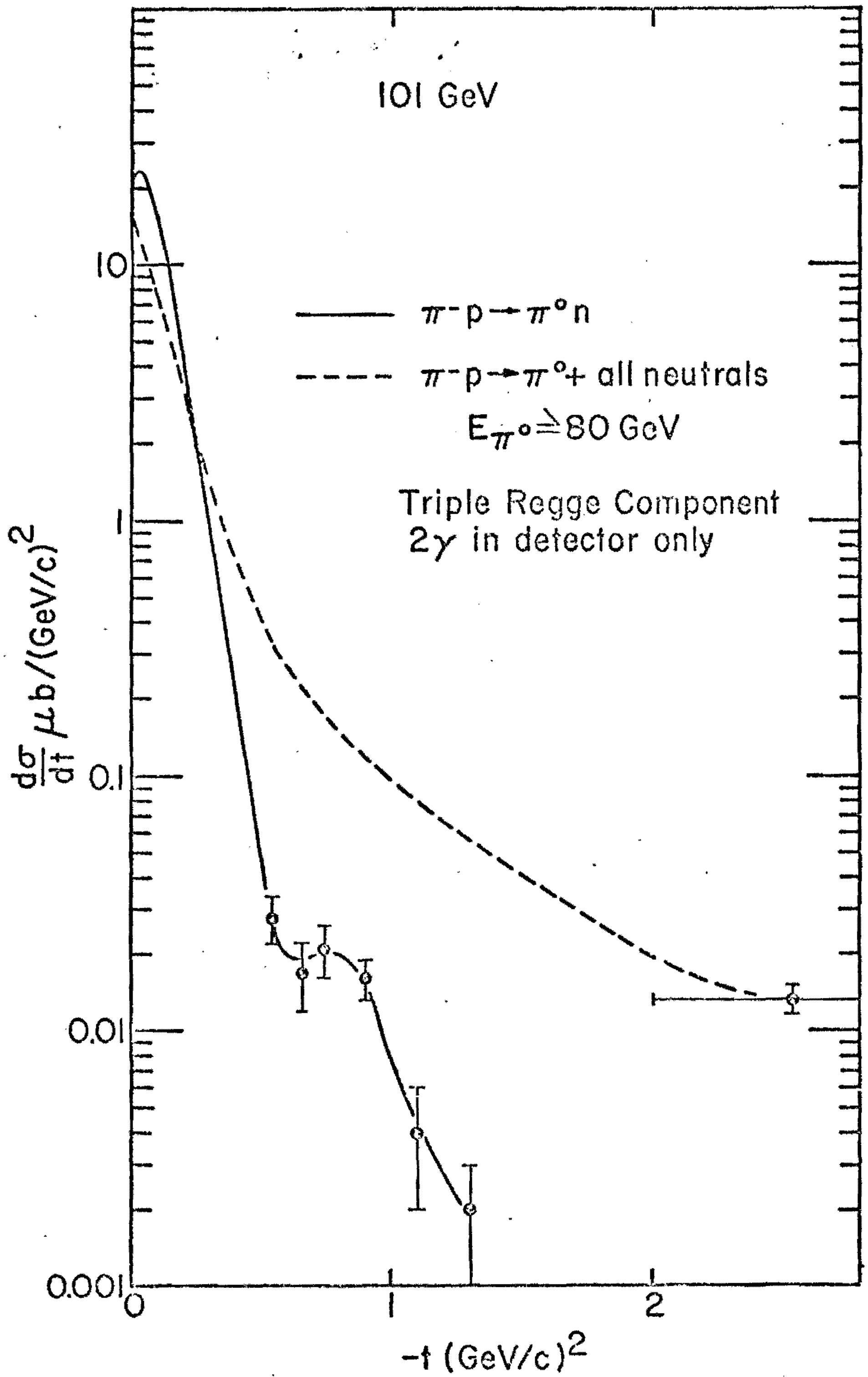


Fig. 10: Comparison of $\pi^- p \to \pi^o X^o$ for $E_{\pi^o} > 80$ GeV and $\pi^- p \to \pi^o n$ as a function of t at beam momentum of 101 GeV/c. The curve for $\pi^- p \to \pi^o X^o$ represents only the inferred triple Regge contribution and not the low mass resonance term $\pi^- p \to \pi^o$"Δ^o". The latter contribution would be important only at small -t.

Addendum to Proposal 350

PHYSICS AND FEASIBILITY STUDY FOR ANALYSIS OF INCLUSIVE π^0 AND η PRODUCTION IN π^-p AND pp INTERACTIONS.*

G. C. Fox and D. J. Mellema

California Institute of Technology
Pasadena, California

*Work supported in part by the U.S. Atomic Energy Commission under Contract No. AT(11-1)-68.

Addendum 350

PHYSICS AND FEASIBILITY STUDY FOR ANALYSIS OF INCLUSIVE π^0 AND η PRODUCTION IN π^-p AND pp INTERACTIONS.*

G. C. Fox and D. J. Mellema

California Institute of Technology
Pasadena, California

*Work supported in part by the U.S. Atomic Energy Commission under Contract No. AT(11-1)-68.

Abstract

This study represents the background theoretical and analysis details of a proposed Fermilab experiment to measure inclusive π^0 and η production in $\pi^- p$ and (later) pp collisions.

We summarize the triple Regge formalism for both the full inclusive and the rather amazing extension to $\pi^- p \to \pi^0$ + all neutrals reaction. We describe the proposed experiment in sufficient detail to motivate the theory and discuss detection efficiencies and energy resolution. Finally we estimate event rates for many of the reactions of interest.

I: Introduction

In this note, we present background material for the accompanying experiment proposal to measure inclusive meson production. This concentrates on inclusive π^0 and η reactions but will also obtain more limited information on ω, η' and $\pi^0\pi^0$ pairs. The mesons are detected through their neutral decay in the apparatus used in the $\pi^- p \to (\pi^0, \eta) n$ experiment E111. We are interested in the relatively unexplored region where the meson (say π^0 to be specific) carries off most of the energy of the beam. Precisely we study region $x \gtrsim 0.8$, where the Feynman x-value is essentially the laboratory energy ratio E_{π^0}/E_{beam}, and relatively small momentum transfer t, $0 \lesssim -t \lesssim 3\,(\mathrm{GeV}/c)^2$ (GeV/c)^2, between beam and π^0. Theoretically, as detailed in Section II and figure 1, this region is described by the triple Regge formalism[1] which predicts the cross-section

$$d^2\sigma/dt\,dx = g_1\,(1-x)^{1-2\alpha(t)} + g_2/\sqrt{s}\,(1-x)^{1/2-2\alpha(t)} \qquad (1)$$

Here α is trajectory function of exchanged Regge pole (or cut) and the first term dominates at Fermilab energies. One determines α as a function of t by fitting x dependence of data to the above form. The accompanying experiment has been optimized for the reations

$$\pi^- p \to \pi^0 + \text{anything} \qquad (2a)$$

$$\pi^- p \to \eta + \text{anything} \qquad (2b)$$

which select pure ρ or A_2 exchange respectively. These provide the cleanest tests of triple Regge theory because only a single Regge pole is thought to be exchanged. Thus one can trivially determine its trajectory from (1), and so the test for the presence of cuts. One can also check the prediction of the seemingly successful interchange model[2] of high p_T processes that $\alpha_\rho(t)$ deviates from a linear form at large -t and asymptotes to -1. Independent of such specific theories, we feel that because of their simplicity the inclusive processes (2) are of basic importance to theory. Here the situation is quite analogous to exclusive processes

$$\pi^- p \rightarrow (\pi^0, \eta) n \qquad (3)$$

which first provided solid evidence for relevance of Regge theory and still form basis of most phenomenological work on strong interaction dynamics. As emphasized in Section II, although the triple Regge formalism has been successfully compared with available experiments, good data is at present confined to diffractive processes. Unfortunately these allow such a host of exchanged trajectories, including the rather obsure Pomeron, that the neat tests possible in our reactions (2) are not allowed. A similar situation occurs in exclusive processes where diffractive pp elastic scattering has never provided good tests of Regge theory whereas the clean reations (3) have been vitally important.

An important advantage of the inclusive reaction (2) over exclusive (3) is that it has a much larger cross-section. For instance in 400 hours, we estimate that one can collect upwards of 4,000 events with $x \geq 0.8$ and $2 < -t < 3$ $(\text{GeV/c})^2$ for $\pi^- p \rightarrow \pi^0$ inclusive. Actually this number is very uncertain. Another crude estimate gave an order of magnitude more events. In any case the proposed experiment can reasonably hope to determine $\alpha_\rho(t)$ out to $t \gtrsim -3$ and provide more sensitive tests of theory than is possible in exclusive processes which run out of cross-section for $t \gtrsim -1.5$ $(\text{GeV/c})^2$.

This rate estimate is described in Section III as is that for: $\pi^- p \rightarrow \eta'$ + anything which is A_2 exchange and measured η'/η ratio allows neat SU(3) symmetry tests.[3]

Also discussed there are the baryon exchange reations

$$pp \rightarrow (\pi^o, \eta, \eta') + \text{anything},$$

which are smaller in size and so more difficult to study than their $\pi^- p$ analogues. However even the limited information it is possible to gather in a modest inclusive experiment, should be able to push study of baryon exchange out to larger $-u$ than has so far been achieved. For instance the whole question of shrinkage (i.e. a linear or flat baryon pole/cut trajectory) is still open but testable in an experiment of 100 hours out to $u \approx -1.5\ (\text{GeV/c})^2$.

In Section II, we also emphasize that the reaction

$$\pi^- p \rightarrow \pi^o + \text{all neutrals}$$

provides new tests of both the triple Regge formalism and some neat physics suggested by the multiperipheral model. In fact we have already performed a preliminary analysis of a (biased) sample of all neutral data at 100 GeV/c. Theory suggests the approximate dependence,

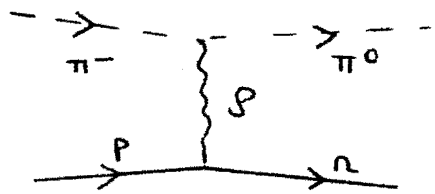
$$d^2\sigma/dt\,dx \propto (1-x)^{-2\alpha_\rho(t)} \qquad (4)$$

and the appendix and figures 2 to 6 record the results of our analysis. As predicted by (4) we find for $t=0$ an x distribution peaked at $x=1$ (fig. 2, (4) becomes $1/(1-x)$ for $t=0$, $\alpha_\rho(0)=.5$), for $t=-.5$ a roughly flat shape in x (fig. 3, $\alpha_\rho(.6)\simeq 0$, (4) becomes constant in x) and for largest t bin (fig. 4, $2 \le -t \le 3\ (\text{Gev/c})^2$) a distribution peaked at $x = 0{\cdot}8$. This change in x shape with t directly reflects non zero slope of ρ trajectory and corresponds to well known shrinkage in $\pi^- p \to \pi^0 n$. Fig. 5 summarizes the ρ and A_2 trajectories determined in this way: this is really just meant to indicate our ability to perform our new proposed experiments. The analysis suffers from both haste in data analysis (which will be improved quite soon) and inevitable biases in the data inherent from the CEX setup. (See appendix). Fig. 6 compares the π^0+ all neutral cross-section (for $x \geq 0{\cdot}8$) with $\pi^- p \to \pi^0 n$ at the same energy. The larger inclusive cross-section at high t — and hence better α determination here — is obvious. We estimate that the full inclusive π^0 cross-section is yet some 10 to 50 times bigger again than its all neutral analogue. At small -t, the theoretical estimate (which should be good to a factor of 2) is shown in fig. 7 and compared with our measured (biased) all neutral cross-section. Within its accuracy, this prediction agrees with crude measurements we made during the running of E111 to estimate trigger rate for this proposal. All the rate estimates for $\pi^- p$ and pp inclusive are described in Section III.

In Section IV, we collect together some notes on proposed experimental set-up (schematized in Fig. 8) and discuss difficulties in analysis. It will require a lot more attention to detail to change the preliminary analysis, reported in Figs. 2 to 6, into solid results. However we believe that we can quite straightforwardly consolidate these beautiful results and extend them to the full inclusive reaction.

II: Physics of the Triple Regge Region

An important part of the experimental high energy physics effort in strong interactions is devoted to the study of dynamics of simple exclusive processes such as $\pi^- p \to \pi^0 n$ or $\pi^+ p \to \rho^0 \Delta^{++}$. The Regge exchange picture has proved to be a surprisingly successful theory for such reactions. For instance completed experiment E111 indicates[5] that the single ρ exchange diagram,



describes $\pi^- p \to \pi^0 n$ from 5 to 200 GeV with amazing accuracy. Further the ρ trajectory obtained from this analysis is linear

$$\alpha_\rho(t) \approx 0.5 + t \tag{5}$$

out to the largest $t (\simeq -1.5\ (\mathrm{GeV}/c)^2)$ studied. Again this trajectory extrapolates nicely through the ρ and g poles. Theoretically this success is rather surprising for general results indicate presence of cuts in the j plane which lie higher than the pole (5) and should dominate at high energy. Further there are problems in other reactions with simple pole models which are popularly explained by absorption effects[6]. These give rise to the cuts alluded to before and so spoil the simple expectation (5). The success then of the pole model in $\pi^- p \to \pi^0 n$ is reduced to an accident (e.g. many believe that cuts are small in spin flip amplitudes which dominate $\pi^- p \to (\pi^0, \eta) n$.) Thus it is important to investigate other reactions to see if simple Regge theory still works. Questions one must ask not only concern energy dependence $s^{\alpha(t)}$ for amplitudes but also t dependence: for instance is the dip at $t \simeq -.6\ (\mathrm{GeV}/c)^2$ present in all ρ exchange processes? Again the recent interchange model[2] for large p_T predicts that $\alpha_\rho(t)$ will break away from its simple form (5) past $t \simeq -1{\cdot}5$ $(\mathrm{GeV}/c)^2$ and in fact asymptote to -1 as $t \to -\infty$. Both this and cut modifications of ρ amplitudes are best examined at large $-t$ where cross sections are small and so very hard to measure in exclusive processes at Fermilab energies.

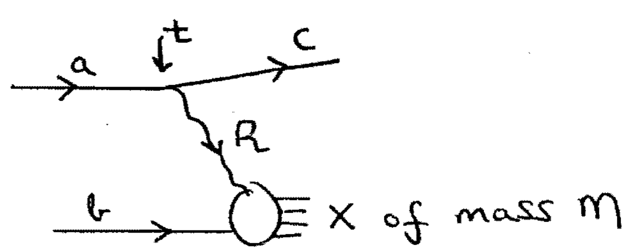
However it has recently been demonstrated that the same Regge phenomena may be studied in inclusive processes where the cross-sections are larger and so more amenable to study at large $-t$. The reactions studied so far are essentially[7] the diffractive processes, mainly[8]

$$pp \to p + \text{anything}$$

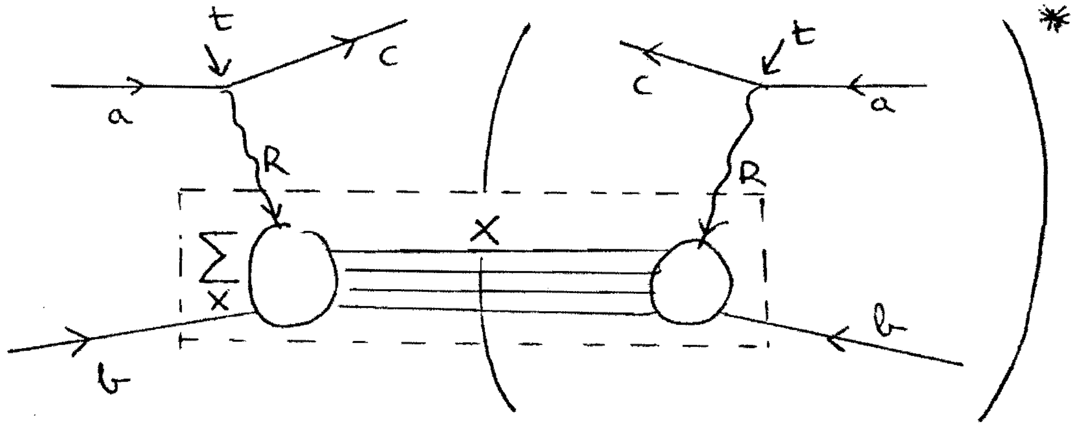
at both Fermilab[9] and ISR energies[10]. The general inclusive process $a + b \to c + \text{anything}$ can be described in terms of the two variables - the normal Feynman x (or fraction of maximum energy $\sqrt{s}/2$ carried by c in ab c.m.s) and the momentum transfer t. In the region of interest - high s, x near 1, and small t- two other useful variables - transverse momentum p_T and missing mass M - are given by

$$t \simeq -p_T^2$$
$$1 - x \approx M^2/s.$$

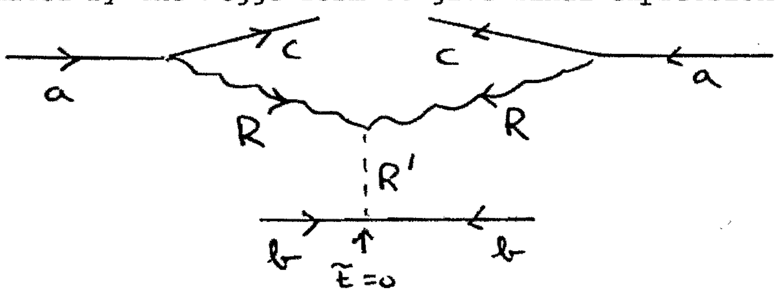
In this region, the amplitude for our inclusive process can be described by the diagram



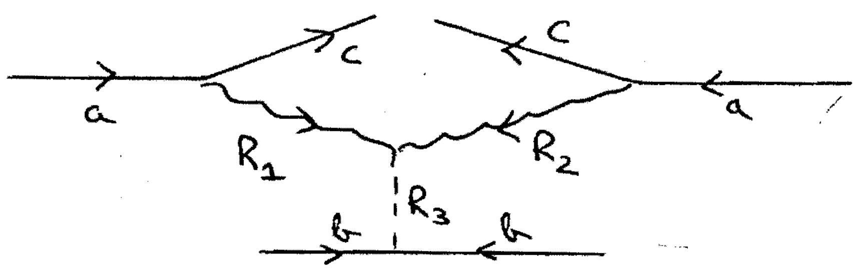
with some Regge pole R with trajectory $\alpha(t)$ exchanged. Squaring and summing this amplitude leads to the cross-section form:



where dotted box is formally proportional to the Reggeon particle total cross-section and so can be equated by the generalized optical theorem to imaginary part of forward elastic Reggeon-particle amplitude. The latter is evaluated at invariant energy M and so at large M can itself be approximated by the Regge form to give final expression.



which is summarized in figure 1 for the special case of $\pi^- p \to \pi^0$ + anything. Now the general case is exceedingly complicated because one must sum over all diagrams



allowed by conservation laws. Unfortunately the current data - which as we said - was predominately diffractive reaction $pp \to p + X$, allows a multitude of Regge exchanges. (Namely R_1, R_2 can independently be Pomeron, ρ, ω, A_2, f, π, B ... exchange). One can still do very interesting analyses of the data, for each term has distinctive behavior

$$d^2\sigma/dt\,dx \propto (1-x)^{1-\alpha_1(t)-\alpha_2(t)}$$

for R_3 = Pomeron,

$$\text{or}\quad d^2\sigma/dt\,dx \propto \frac{1}{\sqrt{s}}(1-x)^{1/2-\alpha_1(t)-\alpha_2(t)}$$

for $R_3 = f, \rho$ etc,

and as the data covers a range of x, s and t, detailed fits can uncover the relative contributions of each term without unreasonable assumptions. The fits[8] do show neat agreement between theory and experiment but the large number of pole terms - and the necessity to neglect some of them - (namely so-called interference terms when $R_1 \neq R_2$) - makes the fits both untransparent and unconstrained. In particular it is impossible to ask about niceties of the theory and test for violations, so that there is no clear test for cut contributions. The tremendous advantage of the reactions

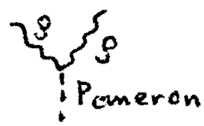
$$\pi^- p \to \pi^0 + \text{anything}$$

$$\pi^- p \to \eta + \text{anything}$$

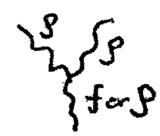
lies in fact that both only allow a single trajectory to be exchanged - ρ for π^0 and A_2 exchange for η production. Going back to exclusive processes, there is still no convincing Regge analysis of pp elastic scattering (simply because of the large number of spin states and exchanges just as in inclusive case) whereas $\pi^- p \to (\pi^0, \eta) + n$ have formed backbone of the theoretical analyses of exchange processes because of their extreme simplicity. The inclusive π^0 and η production processes, discussed in this note, represent by far the cleanest test of triple Regge formalism and best hope to search for cuts and other interesting effects in strong interaction dynamics. Note that if cut contributions to the inclusive process do turn out experimentally to be small, then it is not possible to use the standard explanation of a dominant spin flip amplitude as in $\pi^- p \to \pi^0 \eta$. In fact a radical re-thinking of current absorption and cut theories will be necessary. The measured cross-section is given by

$$d\sigma/dt\,dx = g_1 (1-x)^{1-2\alpha(t)} + \frac{g_2}{\sqrt{s}} (1-x)^{1/2-2\alpha(t)} \qquad (1)$$

where g_1, g_2 represent

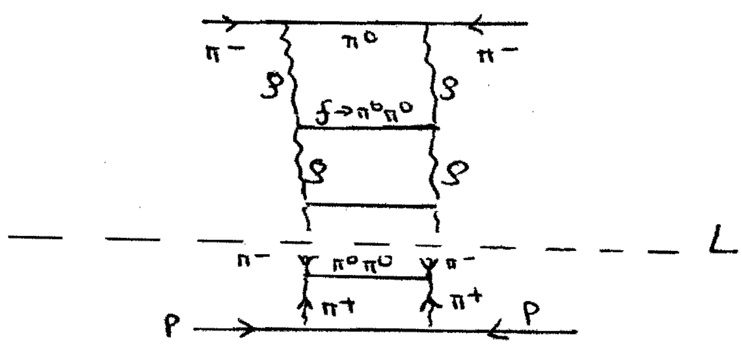


and



couplings. We can distinguish terms by comparing x dependence at two or more energies. Actually we expect the g_1 term to dominate at 100 GeV/c and above. This result based on factorization and $pp \to pX$ analysis is described in Section III but it is not surprising for s ($\approx 2m\,p_{lab}$) is huge and the $1/\sqrt{s}$ coefficient suppresses the g_2 term. Thus we will have no problem in identifying $\alpha(t)$ - and so testing for cuts, linear trajectories etc. - from our measured x dependence at fixed s. We can also extract t dependence of coupling constants g_1 for ρ and A_2 exchange. This will enable us to test for the dip at $t \approx -.6$ $(GeV/c)^2$ (familiar from $\pi^- p \to \pi^0 n$) expected for ρ exchange in naive Regge models. More generally we can examine the relation between ρ and A_2 coupling constants - in exclusive channels this is given by exchange degeneracy, but this formalism has as yet not been extended to inclusive reactions.

We now pass to an interesting extension of the theory. This extension was originally born of necessity as we had only a (biased) sample of all neutral data taken in conjunction with E111 and not full inclusive data for our feasability studies. However the all neutral inclusive reaction has since turned out to be so intriguing that it would be nice to a proper job of it. First we extract from the slightly doubtful if popular multiperipheral model the result[11] that the s-dependence of reactions like $\pi^- p \to$ all neutrals can be described by poles in j-plane and hence have a power law behavior. Maybe the model is wrong, but its prediction both agrees with experiment and can be expressed in such general terms that it might also be true in more realistic theories. The multiperipheral model result follows at once from fact that ladder diagrams like



split into two separate ladders having same form when cut by any line L. This implies we can sum them by a t-channel integral equation and obtain j-plane poles. The same technique will work for any cross-section of form:
ab → any particle not satisfying some given property P.
In our case, the property P is being charged, while in another example $\bar{p}p$ annihilation, P is property of being a baryon or antibaryon. Examination of experimental data, confirms an approximate power law behavior for $\pi^- p \to$ all neutrals: in fact

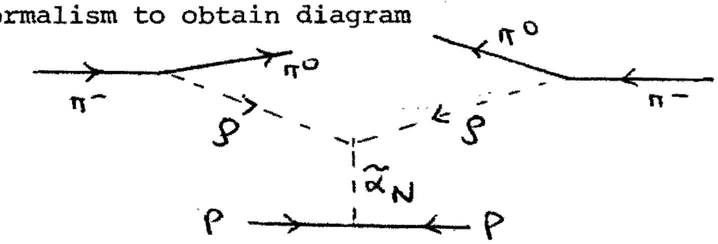
$$\sigma(\pi^- p \to \text{all neutrals}) \sim s^{\tilde{\alpha}_N - 1}$$

where (pseudo) trajectory $\tilde{\alpha}_N \gtrsim 0$. This contrasts with full total cross-section which is roughly constant with trajectory $\alpha = 1$ (the Pomeron).

Now consider the process:

$$\pi^- p \rightarrow \pi^0 + \text{all neutrals}$$

where the π^0 is emitted in the triple Regge region, x near 1 and small t. As already illustrated in figure 1, one can clearly analyse this in the triple Regge formalism to obtain diagram



where in bottom leg one has not the conventional Pomeron, f, ρ trajectories but rather our new all neutral pole with intercept $\tilde{\alpha}_N$. The corresponding cross-section is given by $d^2\sigma/dt\,dx \sim (1/M^2)^{1-\tilde{\alpha}_N} \cdot (1-x)^{1-2\alpha_\rho(t)}$

$$\text{or} \quad d^2\sigma/dt\,dx \sim 1/s \; (1-x)^{-2\alpha_\rho(t)} \qquad (6)$$

in approximation $\tilde{\alpha}_N = 0$. As described in appendix , we have used this formalism to analyse our present all neutral data and find the beautiful results displayed in figures 2 to 6 . The formalism cantrivially be extended to the general reaction $\pi^- p \rightarrow (\eta, \omega^0$ or indeed any $M^{*0}) +$ all neutrals.

On a final theoretical note, we remark that the triple Regge formalism is valid in region of small t, and large M^2 and S/M^2. The latter condition implies x near 1 (as $S/M^2 = 1/(1-x)$) and experience with $pp \rightarrow pX$ has identified the region $x > \cdot 8$ (or $s/M^2 > 5$). It is important to investigate whether the triple-Regge behavior occurs over a larger or small x-region in reactions (2). Since only a single trajectory is exchanged in these reactions, the range of validity of the triple - Regge formalism can be studied with greater precision.

III. Expected Cross-Sections

Define triple Regge cross-section

$$\frac{d\sigma_{TR}}{dt}(x_L, t) = \int_{x_L}^{1} dx \, d^2\sigma/dt\,dx \tag{7}$$

from which it is easy to calculate the expected number of events in a given experiment by multiplying by t - bin, geometric efficiency (cf. Fig. 9) and sensitivity in evs/μb.

$A: \pi^- p \to (\pi^0, \eta, \eta') + \text{anything}$

For small t, we can estimate this quite reliably using factorization and SU(3) to relate g_1 and g_2 in (1) to the analogous terms extracted from fits to pp → pX data[8]. We first dispose of the g_2 term which is only 15% of g_1 (integrated in x from ·8 to 1) at 100 GeV/c and decreasing with energy in accordance with $1/\sqrt{s}$ in (1). It turns out that the ratio r,

$$r = \frac{g_1(A_2 A_2 - \text{Pomeron})}{g_1(\rho\rho - \text{Pomeron})} \tag{8}$$

is not known as exchange degeneracy (EXD) is not believed to be valid for such couplings. Thus one cannot distinguish ρ and A_2 using pp → pX data and one can only predict their sum which is

$$\sigma^{\pi^0 + 3\eta} = \sigma(\pi^- p \to \pi^0 n) + 3\,\sigma(\pi^- p \to \eta n) \tag{9}$$

and the individual reactions are given in terms of (8) and (9) by

$$\sigma^{\pi^0} = \frac{\sin^2(\pi\alpha_\rho(t)/2)\;\sigma^{\pi^0+3\eta}}{\sin^2(\pi\alpha_\rho(t)/2) + r\cos^2(\pi\alpha_\rho(t)/2)} \tag{10}$$

$$\sigma^{\eta} = \frac{r\cos^2(\pi\alpha_\rho(t)/2)\ \sigma^{\pi^0+3\eta}}{3\{\sin^2(\pi\alpha_\rho(t)/2) + r\cos^2(\pi\alpha_\rho(t)/2)\}} \tag{11}$$

One of the aims of the small t data analysis (where $\alpha_{\rho,A_2}(t)$ is not so interesting) will be to find r (and so examine for unexpected EXD) and search for zero at $\alpha_\rho = 0$ from (10). In any case, figure 7 shows

$\frac{1}{2}\ d\sigma_{TR}^{\pi^0+3\eta}/dt\ (x_L = \cdot 65 \text{ and } \cdot 8, t)$

extracted from our pp → pX fits[8]. If r = 1, this is a good estimate of σ^{π^0} at t = o: whatever r, the rates are so large that we will have plenty of events.

For r ≈ 1, we find that

$$\int_0^\infty d(-t)\, d\sigma_{TR}/dt\,(\cdot 8, t) \approx \left\{\begin{array}{ll} 100\ \mu b & \pi^- p \to \pi^0, \\ 10\ \mu b & \pi^- p \to \eta, \\ 0{\cdot}25\ \mu b & \pi^- p \to \eta'. \end{array}\right\} \text{inclusive}$$

At large −t (≈2.5 (GeV/c)^2), we don't really know what to do and have tried two methods. First we use factorization and pp → pX data[10] at $x = \cdot 8$. This gives estimate

$$\int_2^3 d(-t)\ d\sigma_{TR}^{\pi^0}/dt\ (\cdot 8, t) \approx 1/5\ \mu b \tag{12}$$

Another method is to use our $\pi^- p \to \pi^0$ + all neutral data and assume

$$\frac{\sigma(\rho^- p \to \text{all neutrals})}{\sigma(\rho^- p \to \text{all})} = \frac{\sigma(\pi^- p \to \text{all neutrals})}{\sigma(\pi^- p \to \text{all})} \tag{13}$$

where ρ is our exchanged Regge pole in figure 1. At 100 GeV/c and x = ·8, this gives

$$\frac{d\sigma/dt\,dx\,(\pi^- p \to \pi^0 + \text{all neutrals})}{d\sigma/dt\,dx\,(\pi^- p \to \pi^0 + \text{all})} \approx 1/250 \qquad (14)$$

independent of t. Using the appropriate x dependence ((1) or (4)), this agrees rather well with ratio of curves in fig. 7. Applying it at larger t gives

$$\int_2^3 d(-t)\,\frac{d\sigma^{\pi^0}_{TR}}{dt}\,(.8,t) \approx 3\ \text{pb}. \qquad (15)$$

(15) is of course much bigger than (12) and the difference reflects our theoretical incompetence at large -t. Note that in the interchange model[2], (12) would be an underestimate as $pp \to pX$ decreases with t faster than $\pi^- p \to \pi^0 X$ (former is $1/t^6$, latter $1/t^4$).

B: $pp \to (\pi^0, \eta, \eta') +$ anything

The proton induced π^0 production is easier to estimate. Thus we assume $pp \to \pi^0$ is a half $pp \to \pi^+$ which is a slight underestimate based on neglect of I = 3/2 exchange. Using a conventional linear nuclear trajectory and the ISR measurements* of $pp \to \pi^+$ at $x = .8$, we find

$$d\sigma_{TR}/du\,(.8, u=-1) = 0.5\ \mu b/(GeV/c)^2$$
$$d\sigma_{TR}/du\,(.8, u=-1.5) = 0.01\ \mu b/(GeV/c)^2 \qquad (16)$$

and $$\int_{0.4}^{\infty} d(-u)\, d\sigma_{TR}/du\,(.8, u) = 1\ \mu b$$

The 26,000 ev/μb study possible in a 100 hours of running should provide a useful first study of baryon exchange for $.4 < |u| \lesssim 1.5$.

We expect very few η events because of the small $p\bar{p} \to \eta$ coupling. Any η' events would be very exciting because of the neat SU (3) phenomenology of $\frac{1}{2}^+ \overline{\frac{1}{2}^+} \to 0^-$ couplings that it allows.

*The rapid decrease with u in (16) corresponds to rather normal exp $(-6\, p_T)$ behavior in $x = .8$ data.

C $\pi^- p \to (\pi^0\pi^0)$ + anything

This reaction for low mass $\pi^0\pi^0$ pairs is particularly interesting as it is pure π exchange. Shrinkage or not of the π Regge trajectory is still a debated point. If we make stringent cut of $270 \leq M_{\pi^0\pi^0} \leq 570$ MeV, we find:

$$\frac{d\sigma_{TR}}{dt}(.8, t=-.1) \approx 90\ \mu b/(GeV/c)^2$$

$$\frac{d\sigma_{TR}}{dt}(.8, t=-.4) \approx 2.5\ \mu b/(GeV/c)^2$$

which estimates come from formula

$$d\sigma/dt\,dx^{(\pi^- p \to \pi^0\pi^0 n)} \approx \frac{0.003}{(t-m_\pi^2)^2} \cdot (1-x)^{1-2\alpha_\pi(t)} \cdot \sigma_{tot}^{\pi^- p}(m^2)$$

derived using π exchange inclusive formalism of ref. 8, and unpublished data from Sonderegger on $\pi^- p \to (\pi^0\pi^0)n$ at 5 GeV/c. These cross-sections look large enough for one to use this inclusive reaction to determine π trajectory in interesting $-t \geq .4\ (GeV/c)^2$ region. Note that 570 MeV upper mass-cut was solely determined by availability of Sonderegger's data for quick calculation. In our experiment, mass limit will be determined by necessity to both avoid A_2 exchange (which is allowed for higher mass, non S wave, $\pi^0\pi^0$ pairs) and to get reasonable acceptance in apparatus. 570 MeV seems quite conservative.

IV. Summary of Experimental Setup and Analysis

In figure 8 we sketch the layout of the accompanying proposed experiment. It is similar to E 111 except the shower vetoes have been removed. This allows clean studies of the all neutral final state. See appendix for discussion of back scattering difficulties in all neutral final state for E 111 setup which later was of course optimized for CEX reactions. The apparatus will be arranged in two configurations:

Geometry 1 - The detector is taken out of the beam to emphasize the large -t region and avoid saturation of data collection by the multitudinous small t triggers. In this setting the geometrical acceptance for π^0's ranges from 15% to 35% for $1 < -t < 3$ $(GeV/c)^2$. (Figure 9).

Geometry 2 - The detector is left in the beam with ~100% geometrical acceptance for $-t < 2$ $(GeV/c)^2$, but the data rates at 100 and 150 GeV/c exceed collection capacity by a factor of about six.

The proposed trigger will require $\geq$ 65% of the beam energy in the detector ($x \geq 0.65$), to ensure that sample is entirely free of trigger bias for $x \gtrsim 0.8$. In Geometry 1, one may be able to lower the trigger threshold and investigate a wider range of x without exceeding our data collecting capability. As discussed in Section II, it would be important to try to understand region of validity of triple Regge formalism.

Geometry 1 should provide a cleaner sample of events than Geometry 2 in that there should be fewer unwanted particles in the detector (i.e., particles other than the γ's from the neutral mesons of interest). Actually, such backgrounds are not a serious effect in either geometry - for instance in our data for $\pi^- p \to \pi^0$ + all neutral, using Geometry 2, only 25% of the events with x>0.8 have additional photons besides those from the desired π^0. In ordinary multiparticle events, geometry 1 has one-third as many particles in the detector as geometry 2. We expect at least this amount of improvement for inclusive data with unwanted particles decreasing with t as p_T conservation tends to force other particles to the other side of the beam line.

Let us now discuss analysis of the data. First we must dispose of charged particles which are a new feature not present in our much studied all neutral test data. As has already been discovered in analyzing data from E268, these are not a serious problem because they typically give puny pulse heights in the detector. Moreover, their position is tagged using a charged particle hodoscope placed in front of the γ-ray detector.

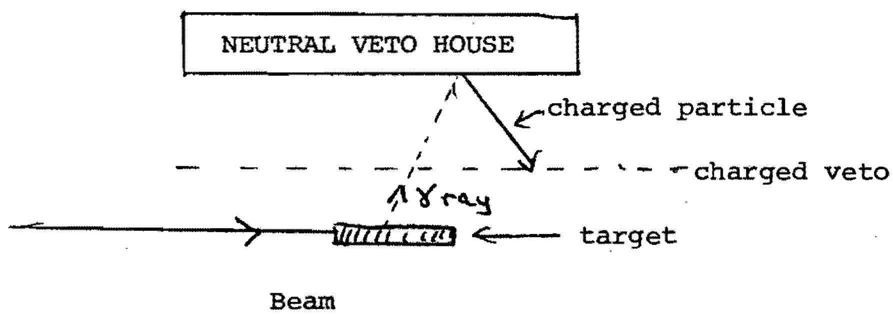
However additional neutral particles are more tiresome. Thus no longer can we use the total energy in detector as a measure of π^0 energy: rather we must fit the measured pulse heights in 70 x and y counters to our observed photon shower shape. This, which is roughly independent of photon energy is shown in fig. 10 - note our photons of interest are very distinctive. Thus π^0 has $\geq$80% of beam energy (as we study $x \gtrsim .8$) and there is only 20% left to divide among all the other particles. Further the central three fingers hold 80% of energy of a given photon. Thereby if we make a decay angle cut ($\cos\theta < .5$), to ensure both photons from π^0 decay have substantial energies, they will stand out if not like a rose in the desert, at least like a sore thumb, which is sufficient for our humble computer. So an essential feature of photon detector is that it will have comparable energy resolution whether or not additional photons enter detector. In the current E111 set up, we have found (see appendix) a 2% energy resolution for 100 GeV π^0's and even better for η's. That this is sufficient for the accompanying experiment is best illustrated by figures 2 to 4 but to further convince you, we show in figure 11 the predicted shapes (1) for various $\alpha_\rho(t)$ smeared by our π^0 energy resolution. Clearly we can easily distinguish say a linear trajectory ($\alpha_\rho(-3) = -2.2$) from a flat $\alpha_\rho(-3) = -1$. Thus figures 2 to 5 illustrate that resolution only gives systematic errors of $\approx \pm .1$ in α which are much smaller than effects being studied.

Appendix: Analysis of $\pi^- p \rightarrow (\pi^0, \eta)$ + all neutrals from E111

To aid the study of background in the charge exchange reactions (CEX)

$$\pi^- p \rightarrow (\pi^0, \eta..) + n \qquad \text{(A1)}$$

which were measured in experiment 111, data on the all neutral final states were recorded simultaneously. Whereas the CEX reactions were characterized by no signal in either charged or neutral vetoes, the all neutral final state reaction was signified by no signal in just the charged vetoes. Unfortunately the sample has some bias from the backscattering of γ showers from the (neutral) veto house into charged veto counters thereby falsely vetoing an all neutral reaction i.e.



This difficulty is not present in accompanying proposal, as neutral vetoes are removed. We took the sample of 101 GeV/c all neutral data and searched for the triple Regge reaction

$$\pi^- p \rightarrow (\pi^0, \eta) + \text{all neutrals} \qquad \text{(A2)}$$

characterized by a π^0 or η with energy greater than 80 GeV/c in the π^0 detector. We have undertaken a provisional study of backscattering bias using pulse height information in charged and neutral vetoes. The effect is definitely present but it does not look as if it will affect our analysis too much as the size is not strongly correlated with π^0 or η energy in range needed. However for this and other reasons that will soon appear, the analysis is preliminary.

In principle the analysis is simple: one need only collect together all data on (2) in a fixed t range and fit the energy dependence of the final π^0 or η to the approximate theoretical form

$$d\sigma/dt\,dx \approx \text{Const}.\ (1-x)^{-2\alpha(t)} \qquad (6)$$

to find the desired $\rho(\pi^0)$ or A_2 (η reaction) trajectory. There are of course technical difficulties. In the actual data processing we must find a reliable method of determining the π^0,η energy and its error (the determination of t presents no problems since the experimental t resolution is much better than needed). We have solved this problem for the special case when only two photons hit the detector (≈75% of the events). In this circumstance, there are no other particles in the detector besides the π^0 (or η) of interest, and a simple moment technique developed by Walker for E111 is applicable.

Define the first three spatial moments of energy measured in the x or y counters

$$E_0^{x,y} = \sum_i E_i^{x,y} \qquad (A.3)$$

: $E_i^{x,y}$ is the energy measured by ith counter at position x_i, y_i.

$$\bar{z} = 1/E_0^z \sum_i z_i E_i^z \qquad (A.4)$$

$$\sigma_z^2 = 1/E_0^z \sum_i (z_i - \bar{z})^2 E_i^z \qquad (A.5)$$

} $z = x$ or y

Then one can easily show that a particle of mass m and energy E decaying into photons, satisfies

$$(m/E)^2 = 1/L^2 \left\{ \sigma_x^2 + \sigma_y^2 - 2\,\delta^2 \right\} \qquad (A6)$$

where detector is distance L from the target and the $\delta^2 \approx 0.65\ \text{cm}^2$ term comes from the intrinsic spatial spread of a single photon shower. In the moment method, we combine the values of E determined by (A.6) (where δ_x, σ_y are measured and m fixed at its conventional value* m_π or m_η) and the simple zero moment

$$E_0 = \tfrac{1}{2}\left(E_0^x + E_0^y\right) \qquad \text{(A.7)}$$

The energy resolution from (A.6), (A.7) or their combination was determined by analysing π^0's and η's of known energy from $\pi^- p \to (\pi^0, \eta)n$ at both 50 and 100 GeV/c taken in the same geometry. We find (standard deviation) resolutions

$$\delta E_0 \sim 2.8\sqrt{E/100} \quad \text{for } \pi^0 \text{ or } \eta \qquad \text{(A.8)}$$

$$\begin{aligned} \delta E_2 &\sim 3.2\,(E/100) \quad \text{for } \pi^0 \\ \delta E_2 &\sim 1.8\,(E/100) \quad \text{for } \eta \end{aligned} \qquad \text{(A.9)}$$

where E_2 represents energy determined by second moment (A.6) and η has a better E_2 determination due to the greater photon separation in detector. Combining (A.8) and (A.9) we get typical combined errors** of

$$\begin{aligned} \delta E_{tot} &\sim 2\ \text{GeV} \quad \text{for } \pi^0 \\ \delta E_{tot} &\sim 1.4\ \text{GeV} \quad \text{for } \eta \end{aligned} \qquad \text{(A.10)}$$

This is more than adequate as is shown in figures 2 to 4 which plots the energy distribution from three t-bins along with the result of a fit to

---

*Identification as π^0 or η determined by rough m value found from (A.6) using $E = E_0$.

**It should be pointed out that this resolution was achieved on the data sets on which the program was tuned. In the hasty analysis of the entire data sample shown in Figs. 2-4, the resolution was broadened by many factors which will eventually disappear on a careful treatment.

formula (6) smeared with an appropriate resolution function. Actually we must not only include a term (6) which describes high $M^2 \gtrsim 5$ (GeV/c)^2 region but also a low mass contribution

$$\pi^- p \rightarrow (\pi^0, \eta) + (N^{*0} \rightarrow \text{neutrals} - n\pi^0 \text{ etc.})$$

where the normal nucleon resonances (1234, 1520, 1688...) are excited. In the lowest t-bin (fig. 2) this is sufficiently important to render extraction of triple Regge term very hard*. However this difficulty disappears rapidly with t and the fits shown in figures estimate

N^{*0} / Triple Regge = 1.4, .24, $\lesssim$.02 for $-t$ = .05, .5 and 2.5 respectively.

Figure 5 summarizes the ρ and A_2 trajectory up to $t \simeq -2.5$ (GeV/c)^2 determined in this way. The analysis is so crude that this should be regarded as a feasability study and not a determination of trajectories on which we can lavish theoretical interpretation.

Further progress demands the analysis of the multiphoton ($N_\gamma > 2$) events in the detector. These contain events from reaction (A.2) analysed above where a fast π^0, η ($E_{\pi^0,\eta} \gtrsim 80$ GeV) and a soft photon ($E_\gamma < 20$ GeV) enter detector. Several other interesting reactions also contribute to the multiphoton event sample - an obvious example is the π exchange process

$$\pi\text{-}p \rightarrow (\pi^0 \pi^0) + \text{all neutrals} \qquad \text{(A.11)}$$

where a π^0 pair with total energy >80 GeV hits detector.

*This difficulty will <u>not</u> be present at small $-t$ in full inclusive reaction $\pi^- p \rightarrow (\pi^0 ...)$ + anything.

The cross-sections in table 1 give one a good feeling as to magnitude of various components of neutral final state cross-sections. We see from this table that we have analysed 5.5 μb of general triple Regge reaction

$$\pi^- p \rightarrow (X, E_X \gtrsim 75 \text{ GeV}) + \text{all neutrals} \qquad (A.12)$$

The remaining 4 μb is divided between reactions like (A.11) and fast π^0, η's accompanied by a soft photon - the latter being estimated as around 1 μb.

Analysis of these events is now in progress using explicit fits of the 140 detector pulse heights to the electromagnetic shower shapes. Rather preliminary fits give energy resolutions very similar to (A.10). Note that the actual identification of photons of interest is very easy because if we select $|\cos\theta_{\pi^0,\eta\ \text{decay}}| < .5$, the energies of photons from the π^0,ηdecay must be larger than any possible background photons i.e. for the extreme case $E_{\pi^0} = 80$, $\cos\theta = .5$, we get photon decay energies of 60 and 20 and there is at most 20 GeV of energy available for background particles. All other parameters are even more favourable.

Table 1: Makeup of 101 GeV/c Observed All Neutral Cross-Section

(The all neutral cross-section is not corrected for biases e.g. back scattering. Further the simple CEX reactions $\pi^- p \to (\pi^0, \eta ..) n$ are conventionally excluded from quoted all neutral cross-sections.)

Reaction	σ(μb)	
$\pi^- p \to \pi^0 n$	3.3	
$\pi^- p \to (\eta \to 2\gamma)\ n$	0.3	5.6 μb total CEX
$\pi^- p \to (X \neq \text{above})\ n$	2	
$\pi^- p \to$ all neutrals (E>75 GeV in detector)	9.5	total all neutral cross-section 17.5 μb
$\pi^- p \to$ all neutrals (E<75)	8	
$\pi^- p \to \pi^0$ + all neutrals (2γ and E>75 in detector)	5	this is essentially all 2 γ cross-section
$\pi^- p \to \eta$ + all neutrals (2γ + E>75)	0.5	
$\pi^- p \to$ all neutrals (>2γ and E>75 in detector	4	
$\pi^- p \to \pi^0$ + all neutrals (Triple Regge term, 2γ in detector)	2	

References

1. For reviews of triple Regge formalism see G. C. Fox, invited talk at "5th international conference on high energy collisions", Stony Brook (1973); D.W.G.S. Leith, lecture notes at 1974 SLAC Summer School.

2. R.Blankenbecler and S.J. Brodsky, "Unified description of inclusive and exclusive reactions at all momentum transfers", SLAC - PUB - 1430 preprint (1974).

3. A.D. Martin and C. Michael, Physics Letters 37B, 513 (1971).

4. E.L. Berger and G.C. Fox, Nucl. Phys. B26, 1(1971). J.K. Storrow and G.A. Winbow, "Analysis of high energy backward πN Scattering, II", Daresbury preprint DL/P 205 (1974).

5. Preliminary E111 results were described in preprints submitted to London conference.

6. G.C. Fox and C. Quigg, Ann. Rev. Nucl. Sci. 23, 219 (73) and hosts of references both therein and thereout.

7. Nondiffractive data in triple Regge region is available for $pp \to nX$ from ISR and Fermilab (E188A). However this both allows several exchanges (π, B; ρ, A_2..) and there is no data in the interesting large $-t$ region. The ISR data quotes large $-t$ points but they are irrelevant as x is small.

8. An analysis of all diffractive triple Regge data (with exception of recent results from ref. 9 (d)) may be found in: R.D. Field and G.C. Fox, "Triple Regge and Finite Mass Sum rule analysis of the inclusive reaction $pp \to pX$", Nuclear physics, to be published.

9. Fermilab results on diffractive Scattering in triple Regge region come from

(a) NAL πp and pp 30" bubble chamber data.

(b) Experiment 14A (Columbia, Stony Brook).

(c) Experiment 188A (Imperial College, Rutgers, Upsala).

(d) Experiment 186 (Dubna, Fermilab, Rockefeller, Rochester).

Further diffractive triple Regge data can be expected in

(e) Experiment 221 (Columbia, Stony Brook).

(f) Experiment 51A (Northeastern)

(g) Experiment 96 (Focussing Spectrometer Facility)

(h) Experiment 86A (Washington, Orsay)

10. ISR results on pp → p + anything in triple Regge region come from CHLM collaboration, Nucl. Phys B73, 40 (1974) and earlier work quoted there.

11. P.W. Coulter and D.R. Snider, Phys. Rev. D8, 4055 (1973) is a typical reference.

Figure Captions

1) Illustration of triple Regge formalism for either $\pi^- p \to \pi^0$ + anything (the full inclusive reaction) or $\pi^- p \to \pi^0$ + all neutrals. The theory is detailed in Section II.

2) Energy distribution of π^0 from very preliminary analysis of $\pi^- p \to \pi^0$ plus all neutrals at 101 GeV/c described in appendix. This figure corresponds to momentum transfer cut $0 \leq -t \leq .1$ (GeV/c)^2.

3) As figure 2, but for $.4 \leq -t \leq .6$ (GeV/c)^2.

4) As figure 2, but for $2 \leq -t \leq 3$ (GeV/c)^2.

5) The values of trajectory function $\alpha(t)$ determined by fitting (4) to energy distributions (typified in figures 2 to 4) of π^0, η in $\pi^- p \to (\pi^0, \eta)$ + all neutrals. The π^0 data gives ρ and η the A_2 trajectory.

6) Comparison of $\pi^- p \to \pi^0$ + all neutral cross-section for $E_{\pi^0} \geq 80$ GeV and $\pi^- p \to \pi^0 n$ as a function of t at 101 GeV/c. The all neutral data only has triple Regge contribution (4) and not the low mass resonance term $\pi^- p \to \pi^0$ "Δ^0". As is evident from figures 2 to 4 which mark ratios of these terms, the resonance contribution is only important at small -t.

7) Theoretical estimate of $\pi^- p \to \pi^0$ + anything at 101 GeV for $E_{\pi^0} \geq 80$ GeV ("Signal") and $E_{\pi^0} \geq 65$ GeV (trigger) using factorization and pp $\to$ p + anything data described in section III. This is compared with same $\pi^- p \to \pi^0$ + all neutral curve shown in fig. 6.

8) Schematic of experimental layout for geometry 1 (detector out of beam) and geometry 2 (detector in beam) described in section IV.

9) Geometric efficiency for detection of π^0's and η's in geometries 1 and 2. These values must be multiplied by a half, as in text, for cut $|\cos\theta| < \cdot 5$ in analysis.

10) Fraction of Energy in photon shower plotted against distance of detector finger from shower centre.

11) Theoretical x dependence smeared with current π^0 energy resolution. Plotted is $\pi^- p \rightarrow \pi^0$ + all neutral expectation with various values of $\alpha_\rho(t)$.

Triple Regge Formalism

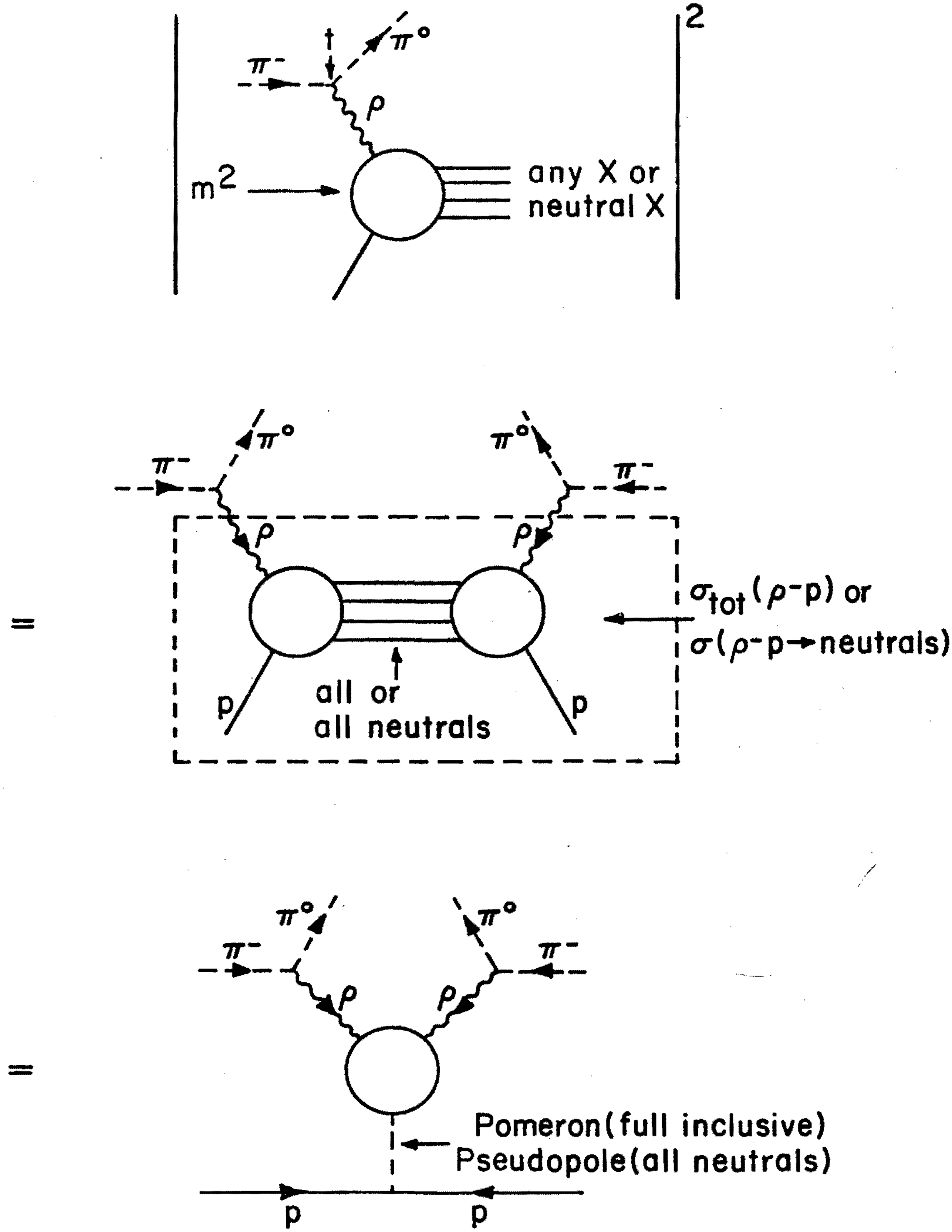


Fig. 1

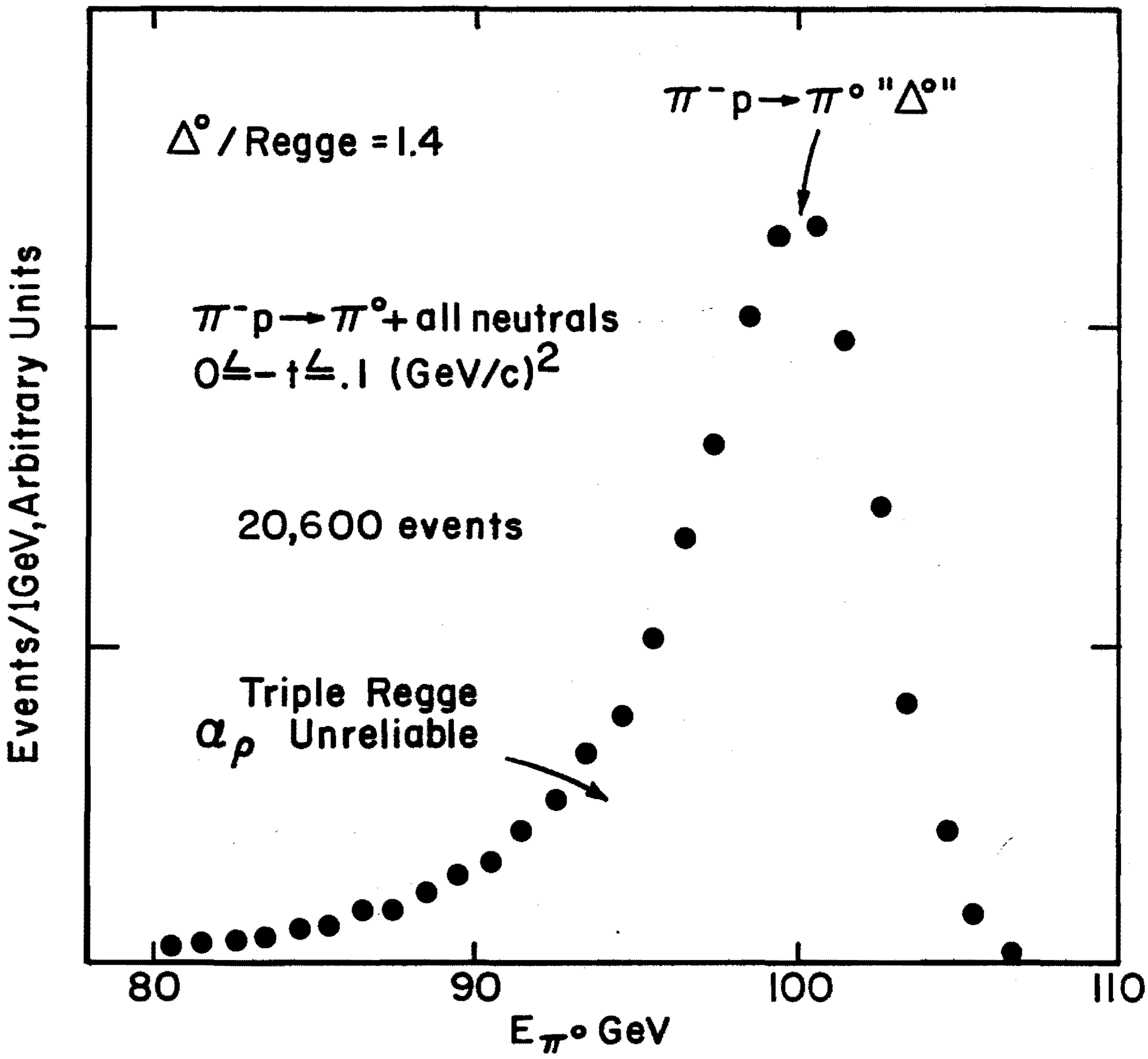
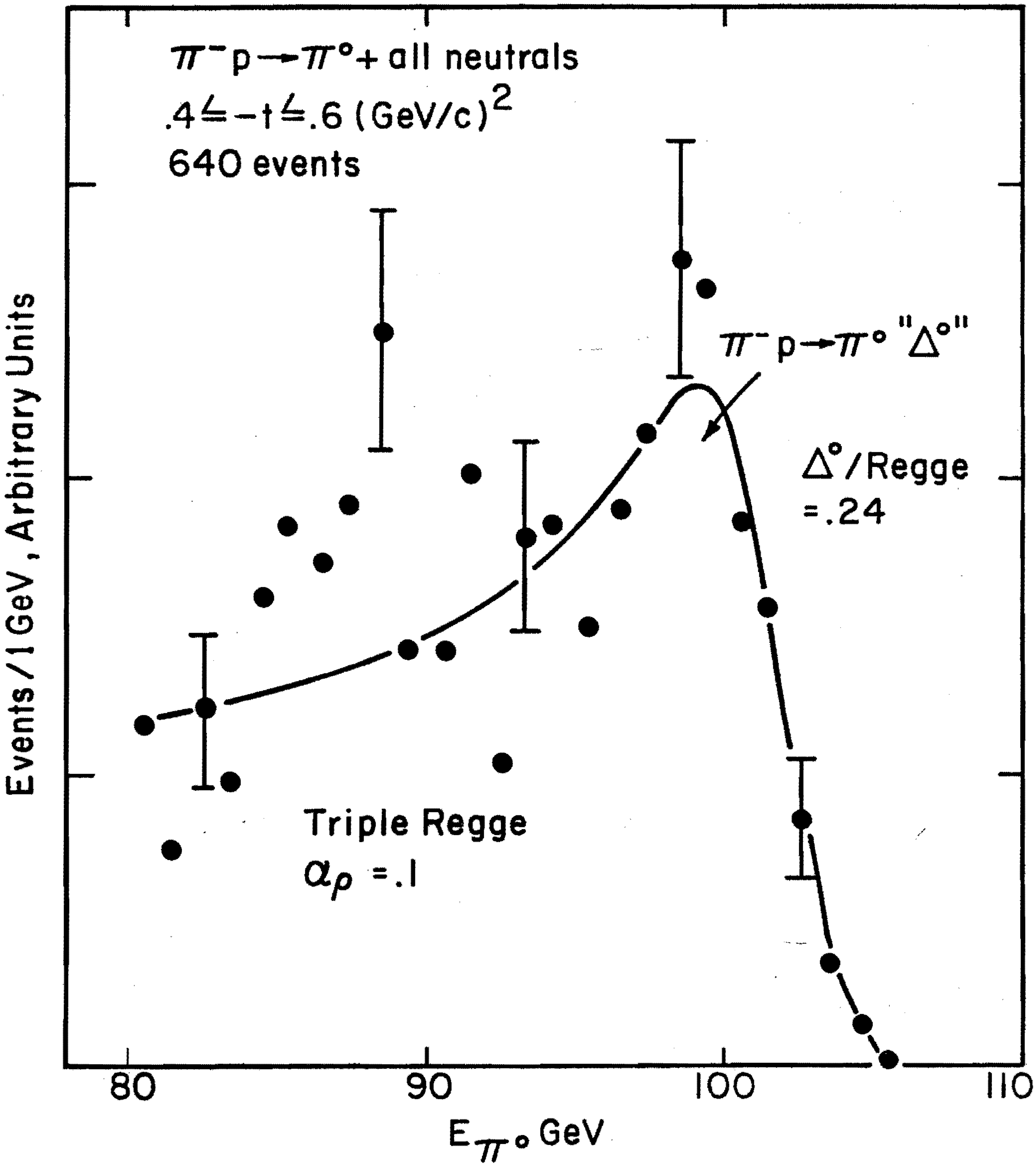

$\pi^- p \rightarrow \pi^\circ$ "Δ°"
Δ° / Regge = 1.4
$\pi^- p \rightarrow \pi^\circ$ + all neutrals
$0 \leq -t \leq .1$ (GeV/c)^2
20,600 events
Triple Regge
α_ρ Unreliable
Events/1GeV, Arbitrary Units
80
90
100
110
E_{π° GeV

Fig. 2



$\pi^- p \to \pi^\circ$ + all neutrals
$.4 \leq -t \leq .6\ (GeV/c)^2$
640 events
$\pi^- p \to \pi^\circ$ "Δ°"
Δ°/Regge
=.24
Triple Regge
α_ρ = .1
Events / 1 GeV, Arbitrary Units
80
90
100
110
E_{π° GeV

Fig. 3

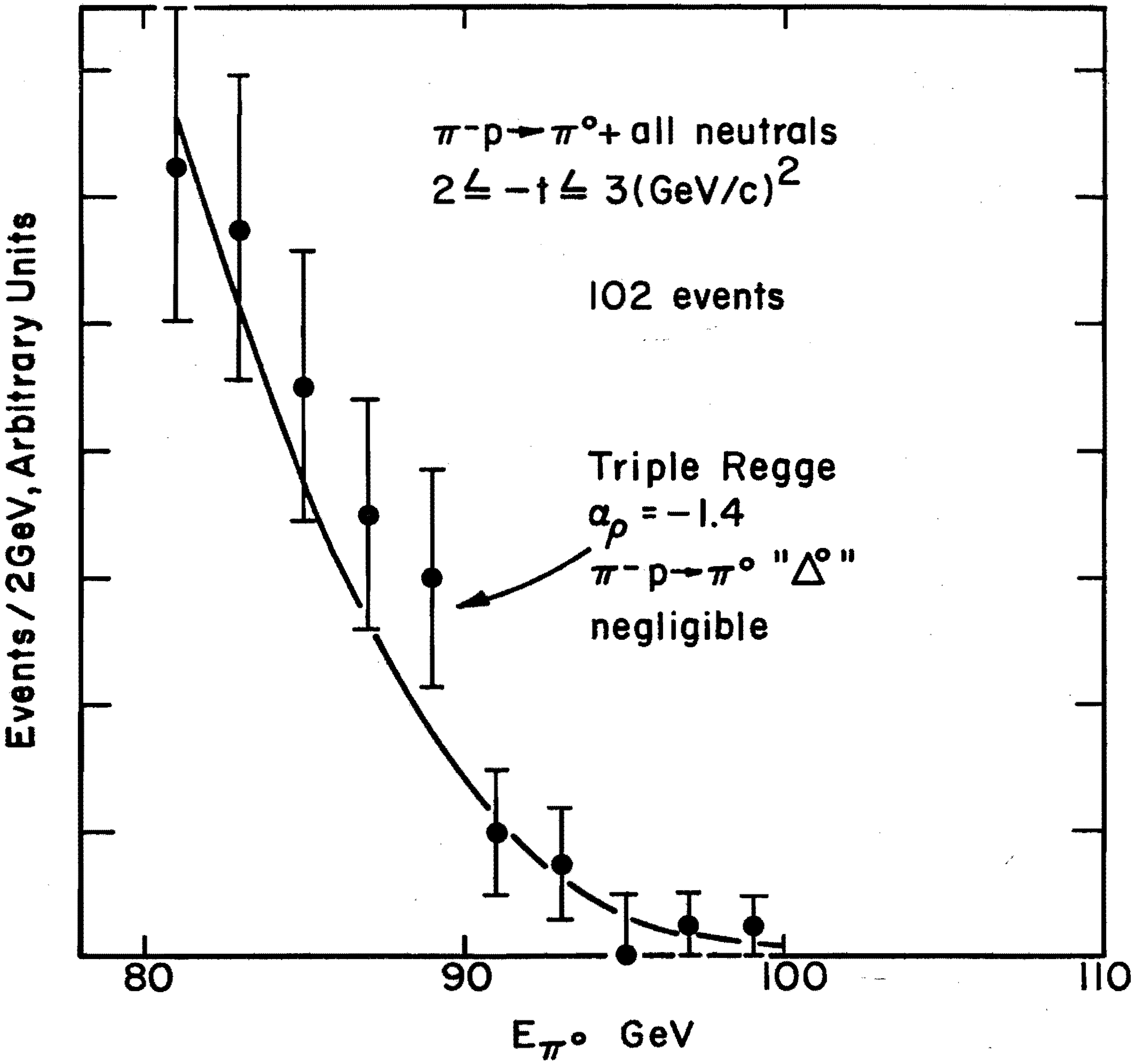
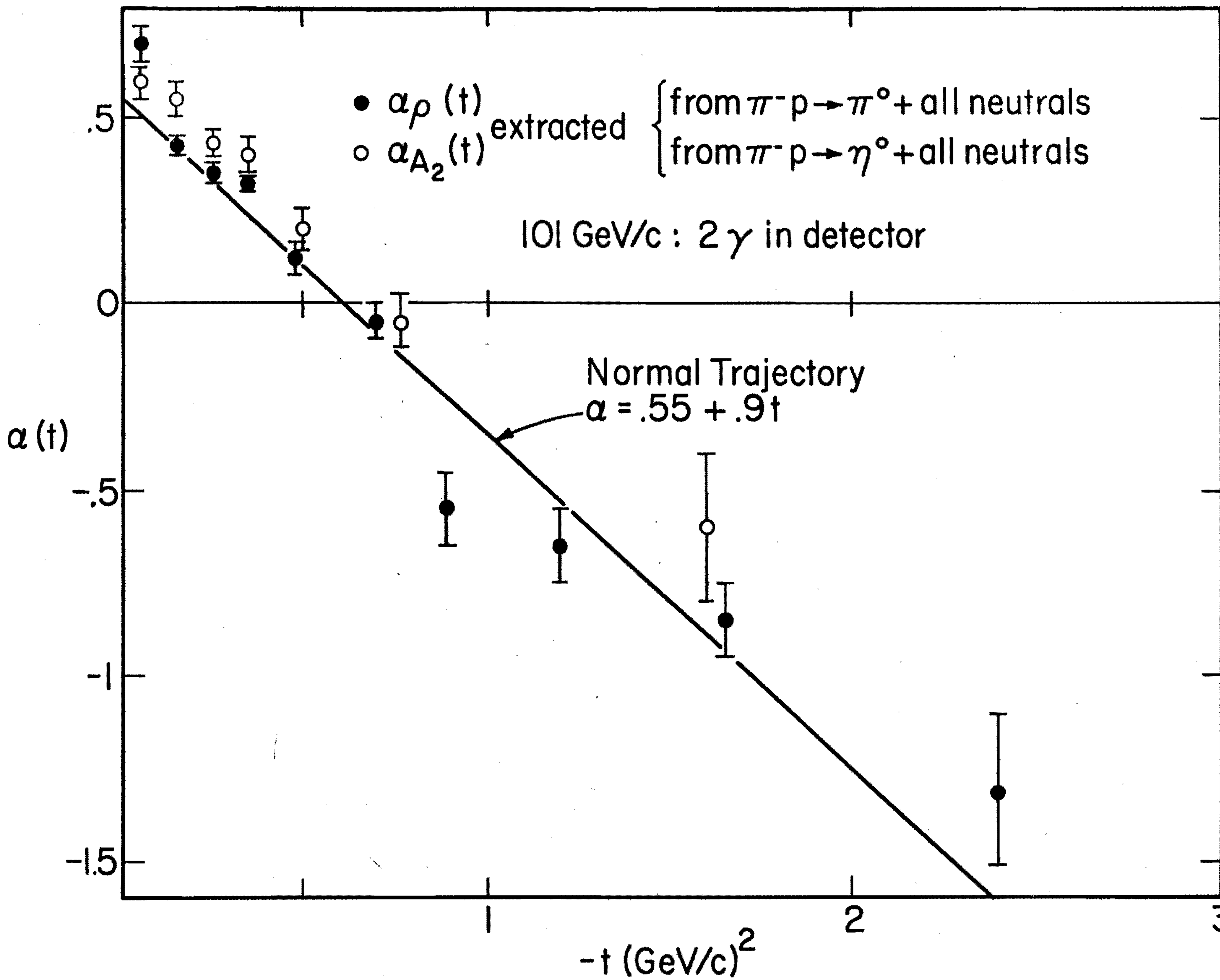
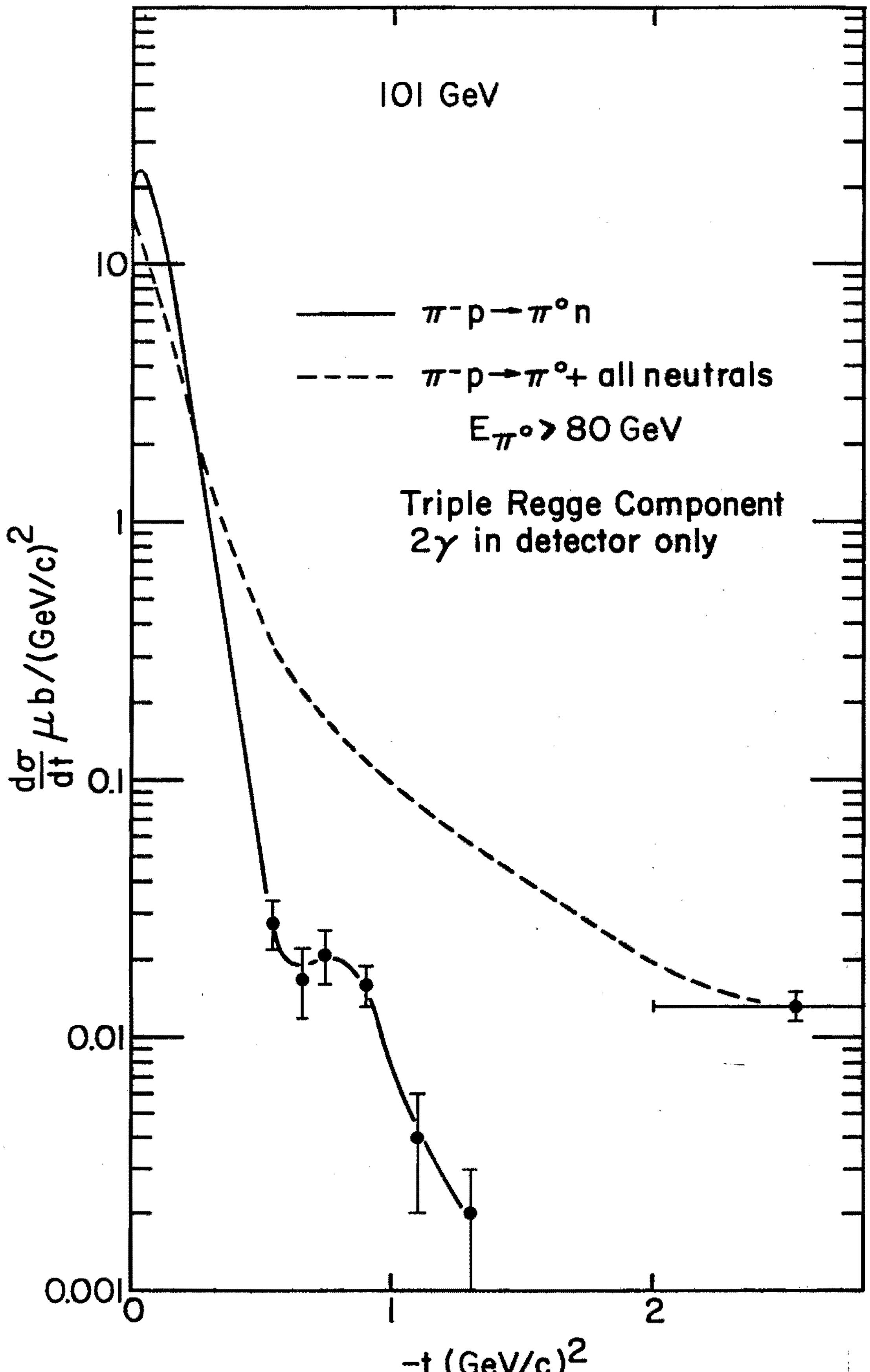

π⁻p → π°+ all neutrals
2 ≤ −t ≤ 3(GeV/c)²
102 events
Triple Regge
α_ρ = −1.4
π⁻p → π° "Δ°"
negligible
80
90
100
110
E_π° GeV
Events / 2GeV, Arbitrary Units

Fig. 4



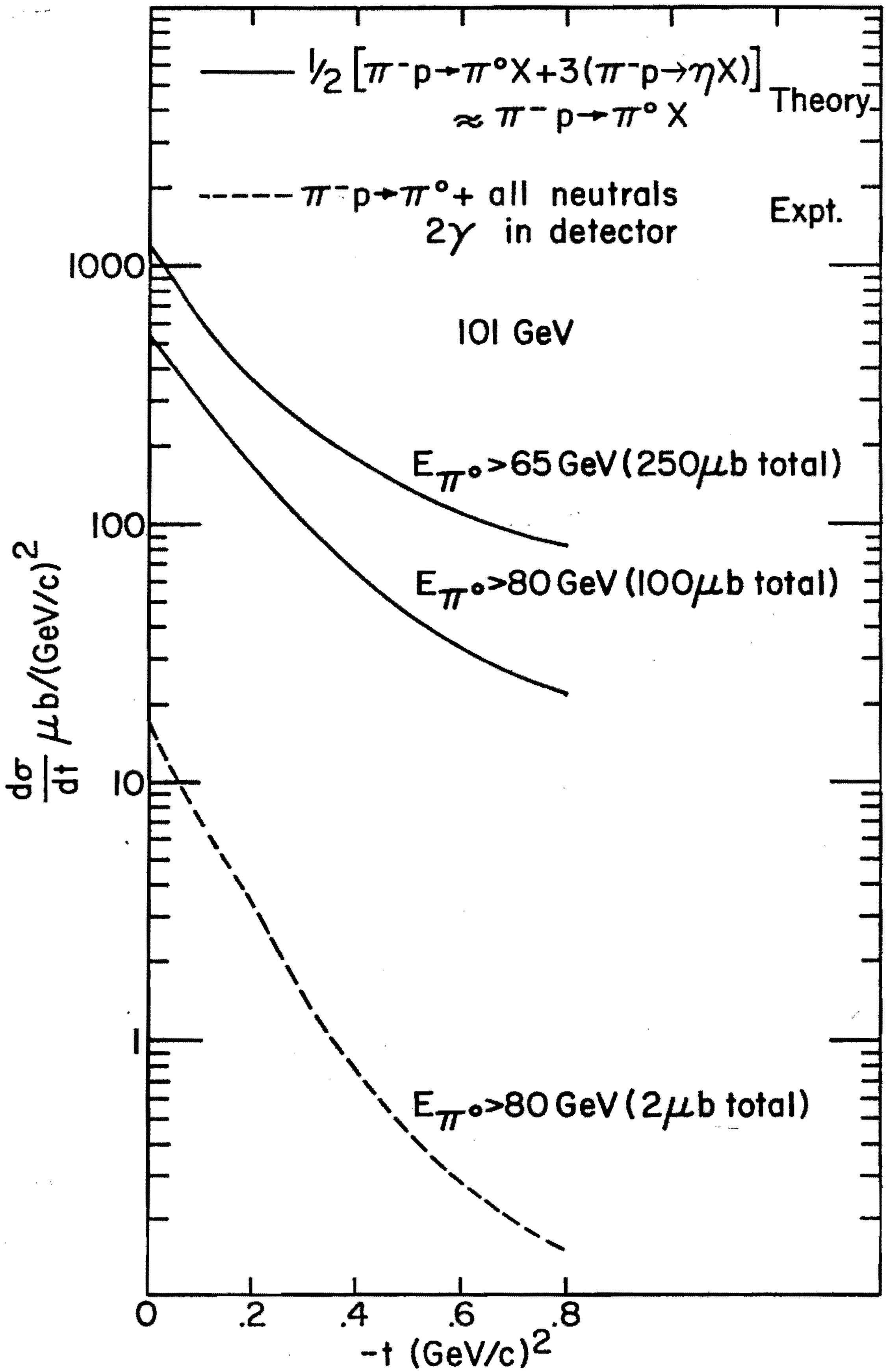
● α_ρ (t) extracted from $\pi^- p \to \pi^\circ$ + all neutrals
○ α_{A_2} (t) extracted from $\pi^- p \to \eta^\circ$ + all neutrals
101 GeV/c : 2 γ in detector
Normal Trajectory
$\alpha = .55 + .9t$
α (t)
.5
0
-.5
-1
-1.5
1
2
3
$-t$ (GeV/c)^2

Fig. 5



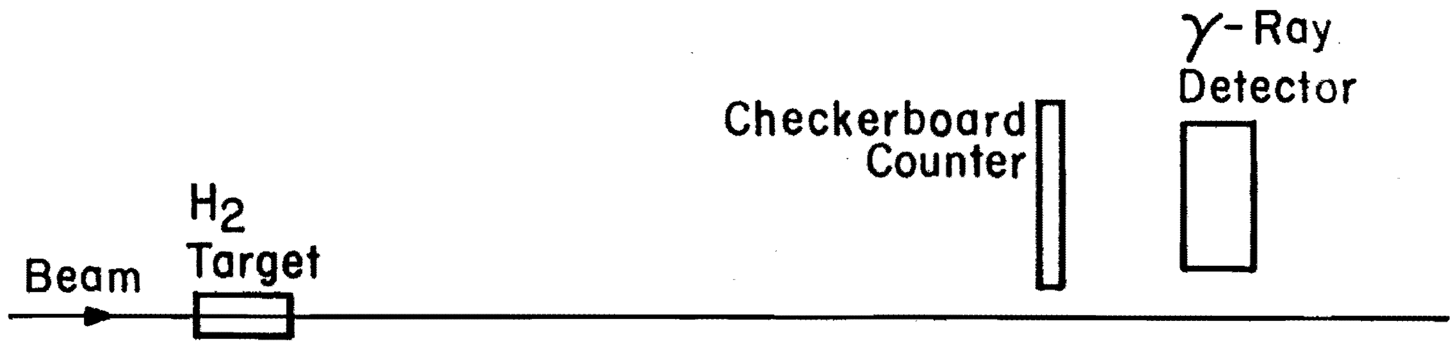
101 GeV
$\pi^- p \rightarrow \pi^\circ n$
$\pi^- p \rightarrow \pi^\circ$+ all neutrals
$E_{\pi^\circ} > 80$ GeV
Triple Regge Component
2γ in detector only
$\frac{d\sigma}{dt}$ $\mu b/(GeV/c)^2$
10
1
0.1
0.01
0.001
0
1
2
$-t\ (GeV/c)^2$

Fig. 6

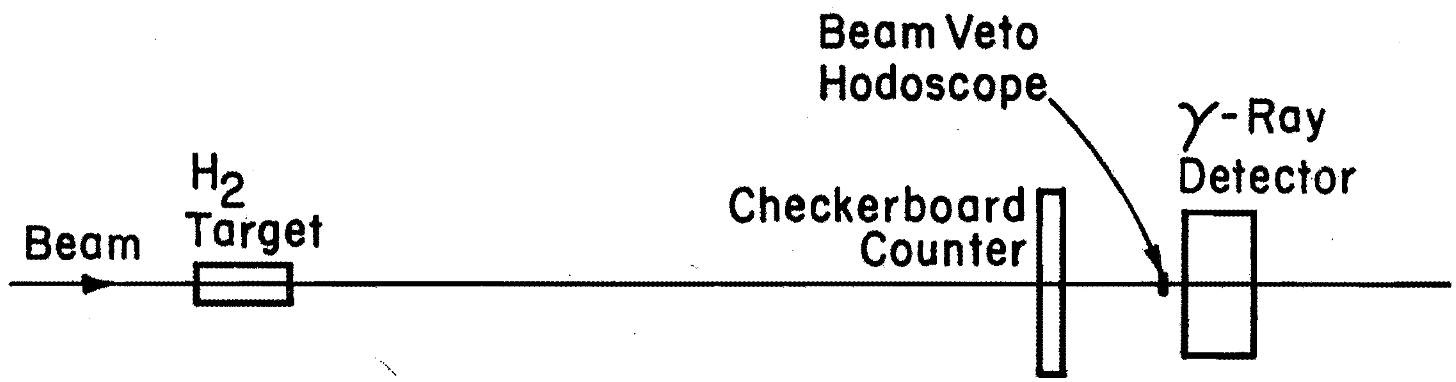


1/2 [π⁻p → π°X + 3(π⁻p → ηX)]
≈ π⁻ p → π° X
Theory
π⁻p → π° + all neutrals
2γ in detector
Expt.
1000
101 GeV
Eπ° >65 GeV (250μb total)
100
Eπ° >80 GeV (100μb total)
10
1
Eπ° >80 GeV (2μb total)
dσ/dt μb/(GeV/c)²
0
.2
.4
.6
.8
−t (GeV/c)²

Fig. 7

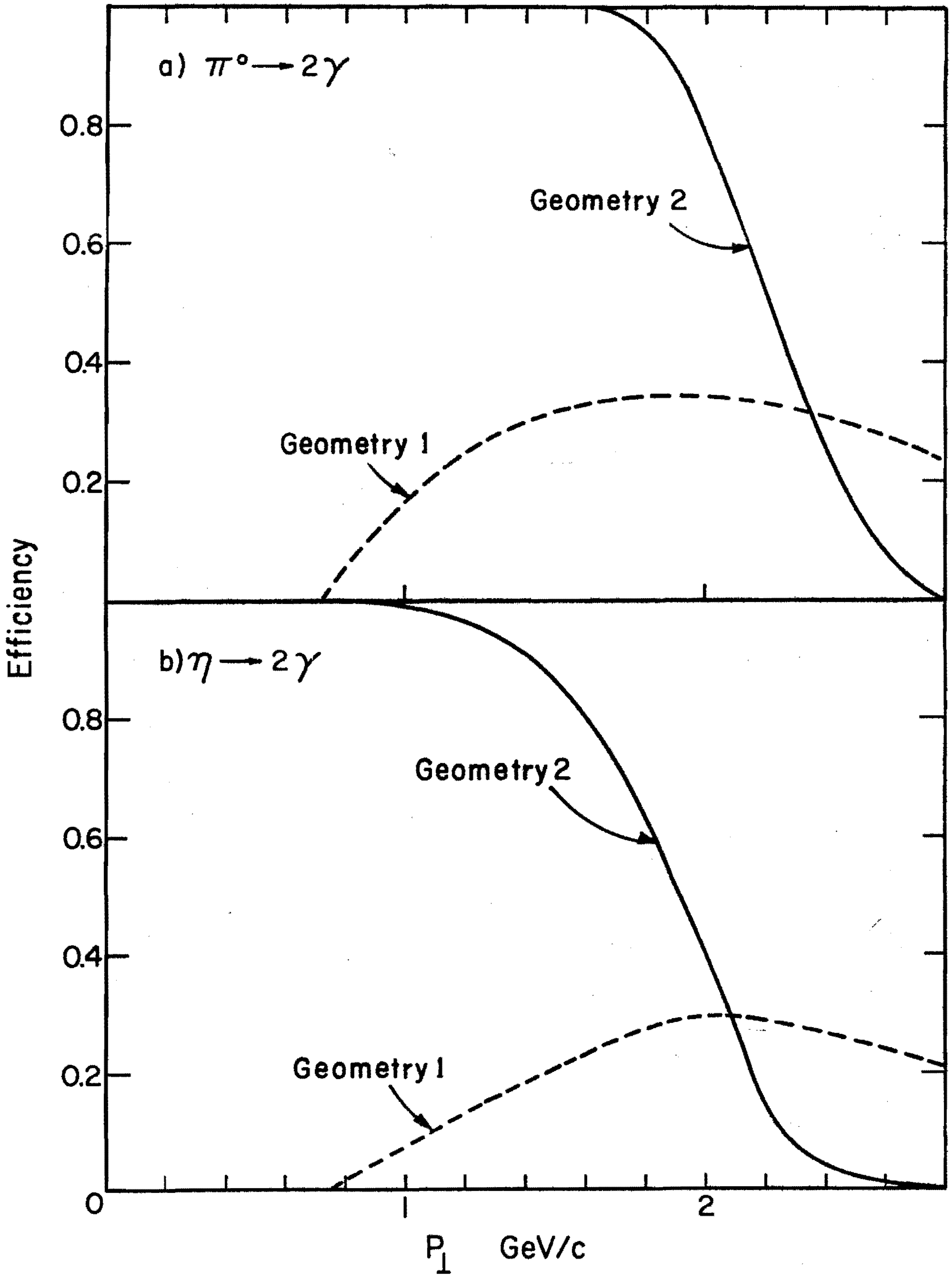


Geometry 1



Geometry 2

Fig. 8



a) $\pi^\circ \rightarrow 2\gamma$
Geometry 2
Geometry 1
b) $\eta \rightarrow 2\gamma$
Geometry 2
Geometry 1
Efficiency
0.8
0.6
0.4
0.2
0
1
2
$P_\perp$ GeV/c

Fig. 9

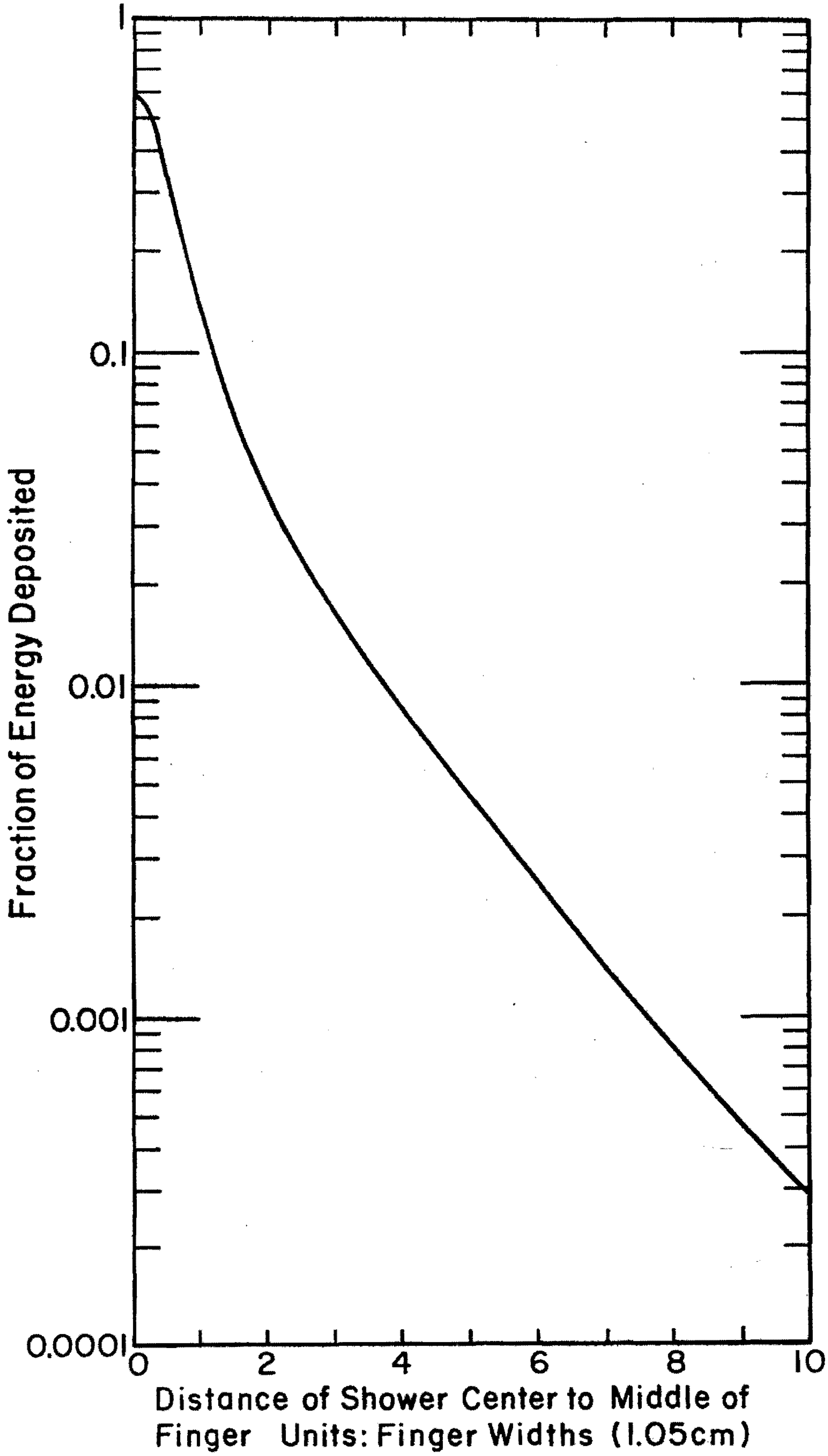
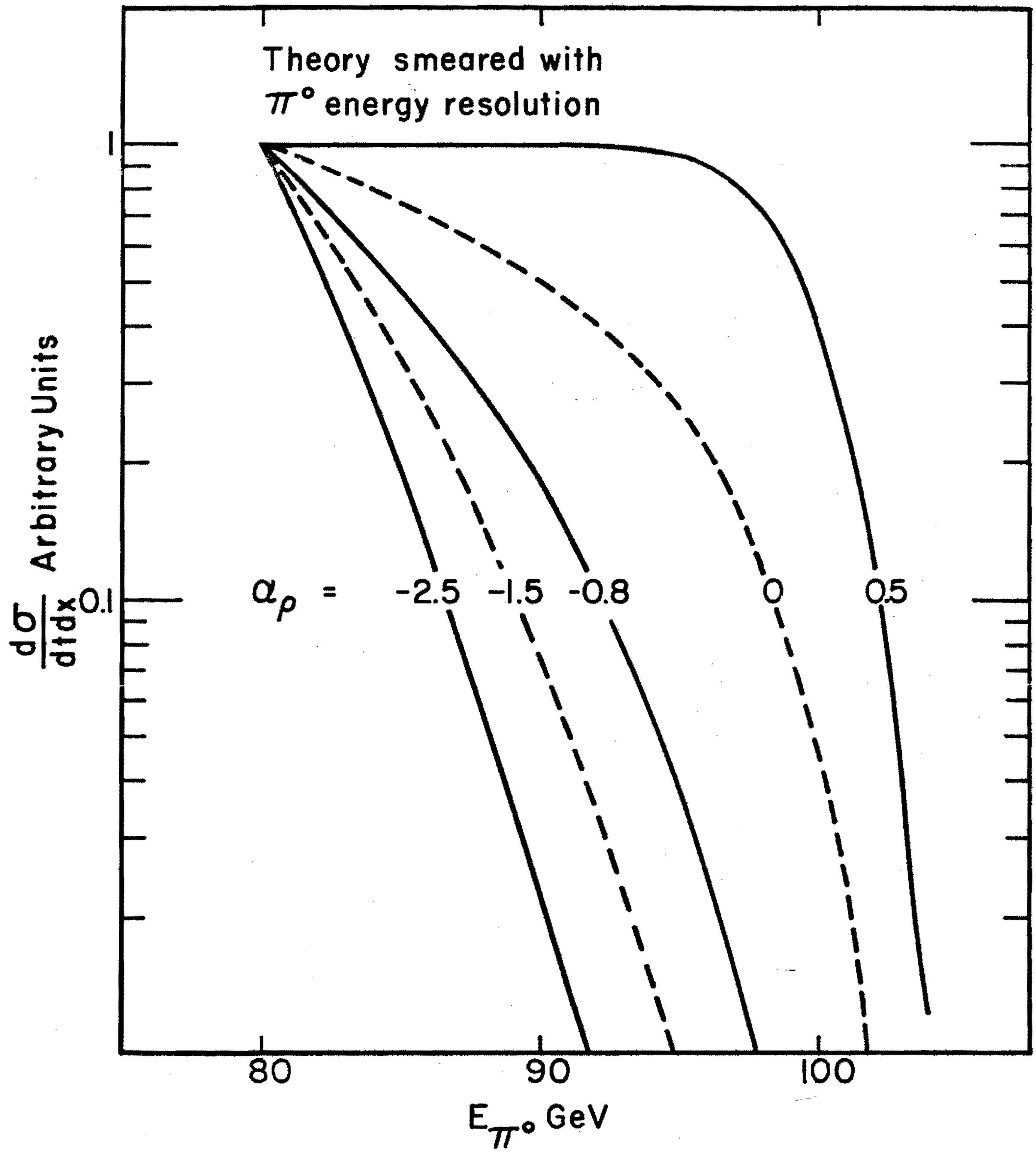

1
0.1
0.01
0.001
0.0001
Fraction of Energy Deposited
0
2
4
6
8
10
Distance of Shower Center to Middle of
Finger Units: Finger Widths (1.05cm)

Fig. 10



Theory smeared with
π° energy resolution
1
0.1
$\frac{d\sigma}{dt\,dx}$ Arbitrary Units
α_ρ = -2.5 -1.5 -0.8 0 0.5
80
90
100
E_{π° GeV

Fig. 11