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A Proposal to Measure

NEUTRAL WEAK CURRENTS IN MUON NUCLEUS INCLUSIVE SCATTERING

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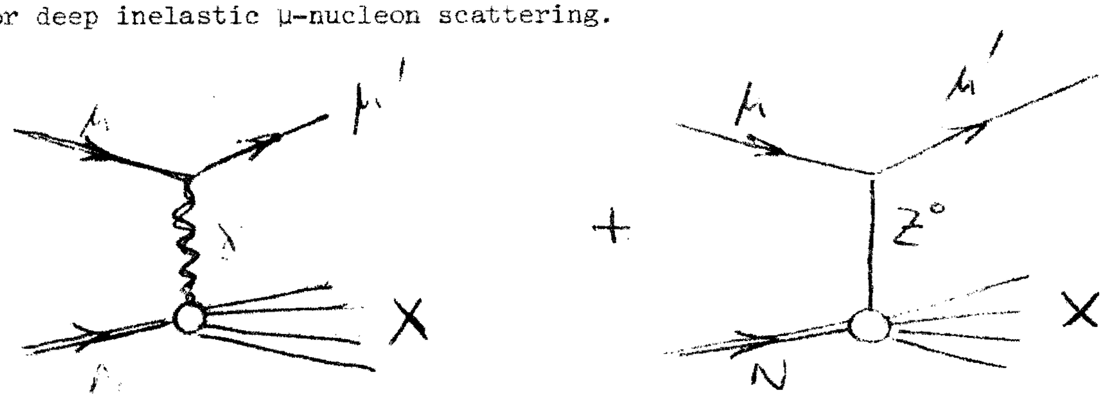
1. Introduction

Recent results of several neutrino experiments provide strong evidence for the existence of a weak neutral current[1]. While the existence of such currents has always been of interest in the phenomenology of weak interactions, recent progress in gauge theories suggests that neutral currents may provide an essential link in the unification of weak and electromagnetic interactions. One is then led to consider the form of the weak neutral current and its relation to the electromagnetic current. Perhaps the most fundamental measure of this relation is the interference between a one photon exchange and a weak neutral current as illustrated in the following Feynman diagrams for deep inelastic μ-nucleon scattering.



The EM coupling dominates because of the large value of α as compared with the Fermi constant G. The kinematics are different in that the EM amplitude has the typical $\frac{1}{Q^2}$ photon propagator dependence while the weak propagator behaves like $\frac{1}{Q^2 + M_z^2}$ which is independent of Q^2 as long as $Q^2 << M_z^2$. Thus at large values of Q^2, the importance of the weak term grows in comparison to the EM term. Indeed, the interference term becomes significant for $Q^2 \simeq 100\ (\mathrm{GeV}/c)^2$, a value obtainable at NAL.

The effect of the weak term has been considered by several authors[2,3,4]. Since the EM term depends on the charge, and the weak term depends on the helicity, the interference is characterized by an asymmetry in the scattering of muons of different sign and/or helicity. The scale for such an asymmetry is set by the ratio of the coupling constants:

$$R = \frac{GQ^2}{2\sqrt{2}\ \pi\alpha} = 0.018\ [\frac{Q^2}{100\ (\text{GeV}/c)^2}]$$

The details involve choices for:

a) form of weak current $j = \bar{\psi}\gamma_\mu(g_V - g_A\gamma_5)\psi$

(namely a choice for g_V, g_A)

b) a model for calculating the nucleon structure functions

(usually a quark model)

Some of the more interesting asymmetries can be listed in a notation where the superscript labels the muon charge and the subscript the helicity.

Unpolarized muons $$\frac{\sigma^- - \sigma^+}{\sigma^- + \sigma^+} = g_A\ \delta R$$

Forward muons $$\frac{\sigma_R^- - \sigma_L^+}{\sigma_R^- + \sigma_L^+} = (g_A - g_V)\delta R$$

Backward muons $$\frac{\sigma_L^- - \sigma_R^+}{\sigma_L^- + \sigma_R^+} = (g_A + g_V)\delta R$$

Helicity flip $$\frac{\sigma_L^- - \sigma_R^-}{\sigma_L^- + \sigma_R^-} = (g_A\delta' + g_V\delta)R = \frac{\sigma_R^+ - \sigma_L^+}{\sigma_R^+ + \sigma_L^+}$$

where δ, δ' depend only on the nucleon structure functions.

Since the NAL muon beams are produced from π-decay, the helicity is a direct function of the ratio E_μ/E_π. Typically, when

$E_\mu/E_\pi \simeq 1$	μ^+_L , μ^-_R unnatural helicity
$E_\mu/E_\pi \simeq 0.8$	unpolarized beam
$E_\mu/E_\pi \simeq 0.6$	μ^+_R , μ^-_L natural helicity

The momentum of the parent pions can be selected by using the triplet train load and the momentum of the muons is selected by the muon transport system. Thus, it is possible to measure all four of the above asymmetries.

We note that the first asymmetry is the average of the second and third and therefore carries no new information in itself. Furthermore, the first three asymmetries include a contribution from two-photon exchange[5]. It is expected that this process will have typically a ℓnQ^2 dependence[5] to be contrasted to the Q^2 dependence of the neutral current so that in principle it can be separated out.

The fourth asymmetry is of great interest because it is purely parity violating and thus contains no contribution from higher order EM effects. Furthermore, it is most sensitive to the detailed form of the neutral current.

We propose to measure the last three asymmetries using muons of energy $E_\mu \sim 170$ GeV, which give adequate rate in the proposed apparatus up to $Q^2 \simeq 100\ (\mathrm{GeV/c})^2$. In all, there are 4 different measurements (μ^+_L , μ^-_R , μ^+_R , μ^-_R) with a requirement of 5×10^{10} muons on target for each measurement.

To estimate the size of these effects we present numerical values for two models of the weak current:

1. Weinberg model (Berman and Primack[3])

$$g_A = -\frac{1}{2\sin^2\theta_w} \quad , \quad g_V = \frac{1 - 2\cos^2\theta_w}{2\sin^2\theta_w}$$

θ_w = Weinberg angle

δ, δ' calculated with 4 quark model

2. V-A model (Derman[2])

$$g_A = g_V = 1$$

δ, δ' calculated with 3 quark (Kuti-Weisskopf[6]) model

TABLE I

Asymmetry		Weinberg	V-A
Unpolarized	$\frac{\sigma^- - \sigma^+}{\sigma^- + \sigma^+}$	- 1.6%	+ 1.1%
Forward	$\frac{\sigma_R^- - \sigma_L^+}{\sigma_R^- + \sigma_L^+}$	- 1.9%	0
Backward	$\frac{\sigma_L^- - \sigma_R^+}{\sigma_L^- + \sigma_R^+}$	- 1.3%	+ 2.2%
Helicity flip	$\frac{\sigma_L^- - \sigma_R^-}{\sigma_L^- + \sigma_R^-}$	see Fig. 1	+ 2.6%

The numerical results given in Table I are calculated for $Q^2 = 100\ (\mathrm{GeV}/c)^2$ and $\nu = \nu_{max}$. (The ν-dependence has been taken into account in the calculation of the rate; section 3.) For the Weinberg model we used the presently accepted value of $\sin^2\theta_w = 0.3$. The helicity flip asymmetry in the Weinberg model is shown in Fig. 1 as a function of $\sin^2\theta_W$. We note that a measurement of the asymmetry to ±0.5% (namely ±1% difference between the respective cross-sections) measures $\sin^2\theta_W$ to ±0.1. The prediction of a V-A model differs then by 5 standard deviations from the Weinberg model for $\sin^2\theta_W = 0.3$.

2. Apparatus and Beam

We propose to use a modification of the apparatus for exp. 98 in the Muon Lab. Our measurements require a heavy target, of approximately 5.0 m of Uranium[7], which will be instrumented so as to provide:

(a) A rough measure of the energy loss of the incident muon.

(b) Minimize multiple scattering in the measurement of the scattered muon angle.

The scattered muon is momentum analyzed in the Chicago Cyclotron magnet for which we assume $p_\perp = 2$ GeV/c. Even at half this value the momentum resolution is such that the overall uncertainty in Q^2 (dominated by the error in the scattering angle) remains less than 10%. We can use the detector of expt. 98 or we can provide our own proportional wire chambers which are presently under construction and will be completed by the end of the year. The layout of the apparatus is shown in Fig. 2.

The target consists of 40 slabs of 5-inch thick uranium 12 × 12 inches square in area. Between each slab a 1-inch scintillator is used to measure pulse height; an X-Y proportional chamber is placed between every fifth slab. Thus, the target serves as a calorimeter and permits a crude energy balance on the sum of the outgoing muon energy and the hadronic shower. This is important to reject contamination of the high Q^2 (and high ν) events from pions which may decay before interacting.

The instrumented target provides us with the position of the interaction vertex and allows the measurement of the scattering angle before it becomes dominated by multiple scattering. We find that $\Delta\theta \simeq 2.4$ miliradians for E' = 50 GeV where the typical scattering angle is of the order of $\theta \simeq 100$ miliradians. Finally, the signal from the target is used as part of the trigger in order to select events with high ν. The trigger is formed by requiring the disappearance of the muon from the beam, the appearance of a scattered muon and the target signal.

The target is followed immediately by an additional 0.5 m of uranium with a beam hole, so as to shield the detection apparatus and the trigger from electrons and hadronic showers. This shield also provides for the absorption of pions from interactions in the downstream end of the target. As usual, scattered muons are required to pass through the 2.5 m iron absorber at the end of the apparatus. Appropriate veto counters are used (as in exp. 98) to protect against beam halo triggers.

The incident beam will be tagged in momentum and space by the use of the existing proportional chambers. Again the momentum resolution is adequate and the chambers can handle the incident beam rate. In Fig. 3 we show νW_2 as obtained in the Brookhaven muon-nucleus experiment using a 0.5 m copper target. The multiple scattering effects were of the same relative magnitude as in our proposed experiment.

3. Sensitivity

The sensitivity of our measurement is limited both from systematic and statistical effects. From the point of view of statistics we note that for a given bin of Q^2 the statistical error on the asymmetry is $\Delta\varepsilon = 1/\sqrt{N_+ + N_-} = 1/\sqrt{N_T}$. Since $N_T \propto 1/Q^4$ and $\varepsilon \propto Q^2$, $\Delta\varepsilon/\varepsilon$ is almost independent of Q^2 and thus measurements at all values of Q^2 have the same statistical weight. We have attempted to reach $\Delta\varepsilon = 0.5\%$ at a $Q^2 \simeq 100$ $(\text{GeV/c})^2$ which implies 40,000 events. Using our target of 10^4 g/cm^2 and a flux of 10^{11} muons, we have calculated the detected events in our spectrometer. Figure 4 shows the statistical error (and the asymmetry) as a function of Q^2 after taking into account the acceptance of the apparatus and the ν-dependence of the effect. We have used 200 GeV incident muons and an asymmetry of 1.9% at $Q^2 = 100$ (2^{nd} entry in Table I). These data would provide a five standard deviation effect as contrasted to zero asymmetry. Namely a sensitivity of the order of 0.5%.

What is equally important is the elimination of systematic biases. These can arise mainly from

(a) Different characteristics of the incident beams

(b) Asymmetries in the detection apparatus

(c) Drifts of the detection efficiency with time.

As far as point (a), the direct sampling of the beam provides an exact knowledge of the beam configurations. Since all measurements will be made at the same momentum, there is no reason why the beams should not be identical. Furthermore, we plan to trigger (on a prescaled basis) on $\nu \simeq 0$ events, i.e., straight through tracks, to monitor the beam performance.

As far as (b), we propose to reverse the polarity of the spectrometer magnet periodically. We also note that the long target allows us to populate the same region of the detector with low Q^2 and high Q^2 events. One of the intrinsic advantages of this procedure lies in the fact that at low Q^2 no asymmetry should be present and this can be used to calibrate the instrumental effects.

Finally, as far as (c) is concerned, it is customary to make frequent interchanges in the measurement of the two configurations for which the asymmetry is sought. We do not feel that this is essential in the present case, since low Q^2 events can be used to calibrate the apparatus efficiency between two measurements.

Our proposed sequence of measurements is shown in Table II. Given the reversal of the spectrometer magnet, the μ^+ and μ^- events sample the same parts of the apparatus with equal weight and instrumental biases are eliminated. In the notation of Table II

Forward asymmetry
$$2a_F = \frac{\sigma_R^-(+) - \sigma_L^+(-)}{\sigma_R^-(+) + \sigma_L^+(-)} + \frac{\sigma_R^-(-) - \sigma_L^+(+)}{\sigma_R^+(-) + \sigma_L^-(+)}$$

Forward bias
$$2b_F = \frac{\sigma_R^-(+) - \sigma_L^+(-)}{\sigma_R^-(+) + \sigma_L^+(-)} - \frac{\sigma_R^-(-) - \sigma_L^+(+)}{\sigma_R^-(-) + \sigma_L^+(+)}$$

In a similar manner we define the backward muon asymmetry a_B and the backward muon bias b_B. Finally, the helicity flip asymmetry is obtained from the appropriate combination of the above measurements

TABLE II

A. Forward muons

Accelerator			300 GeV
Pions (triplet load)			170 GeV
Muons			170 GeV
1.	μ_L^+	spectrometer	+ } 5×10^{10}
2.	μ_L^+		− }
3.	μ_R^-		+ } 5×10^{10}
4.	μ_R^-		− }

B. Backward muons

Accelerator			400 GeV
Pions (triplet load)			280 GeV
Muons			170 GeV
5.	μ_R^+	spectrometer	+ } 5×10^{10}
6.	μ_R^+		− }
7.	μ_L^-		+ } 5×10^{10}
8.	μ_L^-		− }

$$2a_H = \frac{\sigma_L^-(+) - \sigma_R^-(+)}{\sigma_L^-(+) + \sigma_R^-(+)} + \frac{\sigma_L^-(-) - \sigma_R^-(-)}{\sigma_L^-(-) + \sigma_L^-(-)} =$$

$$= \frac{\sigma_R^+(+) - \sigma_L^+(+)}{\sigma_R^+(+) + \sigma_R^+(+)} + \frac{\sigma_R^+(-) - \sigma_L^+(-)}{\sigma_R^+(-) + \sigma_L^+(-)}$$

and again the bias of the apparatus is determined by subtracting the measurements with opposite spectrometer polarity.

4. Beam and Running Time Requirements

As indicated in Table II, the beam requirements are 10^{11} muons with the accelerator at 300 GeV and 10^{11} muons with the accelerator at 400 GeV. Presently the muon flux with the doublet train load is of the order of 10^6/pulse. We estimate that the selection of a pion momentum band reduces the rate by a factor of 2 to 2.5. For the backward muons, a loss factor of ~1.6 is introduced from the increased decay length of the pions; when the muon capture efficiency is taken into account, we estimate an overall loss of 2. On the other hand, at 400 GeV we expect that the primary beam delivered on the neutrino target will be twice that of the 300 GeV operation. Therefore, a beam of 4×10^5 muons/pulse seems reasonable for both configurations. On that basis the experiment can be completed in 1000 hours of beam time.

5. Conclusions

The proposed measurement of the interference of the E.M. and weak neutral current in muon-nucleus inclusive scattering is feasible with the present NAL muon beam. Of particular interest is a measurement of the difference in the cross-section for the same charge but opposite helicity of the incident muons. This asymmetry can be measured at the level of ±0.5%.

In order to reach high Q^2, advantage is taken of a massive target and for backward muons it is necessary to use 400 GeV incident protons. Since the asymmetry is a function of Q^2, it is possible to normalize internally the data obtained in the two configurations of the leptons. Additional internal checks of the consistency of the data are provided for. By triggering on the energy loss in the target it is possible to select high ν (and thus also high Q^2) events.

An improvement in beam intensity will benefit the experiment but this is not essential at the present time. The detection of the scattered muon can be performed with the existing apparatus of experiment 98, and the instrumented target can be available in approximately six months. We feel that the proposed measurement can clarify the ideas of a unified weak and EM interaction and should not be delayed unduly.

References

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   F. Hasert et al., Phys. Letters 46B, 138 (1973).
2. E. Derman, Phys. Rev. D7, 2755 (1973).
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   (The published paper includes certain corrections from the original version, SLAC-PUB-1360.)
4. M. Suzuki, L.B.L. preprint, Oct. 16 (1973) to be published in Nuclear Physics; also D. Dicus, private communication.
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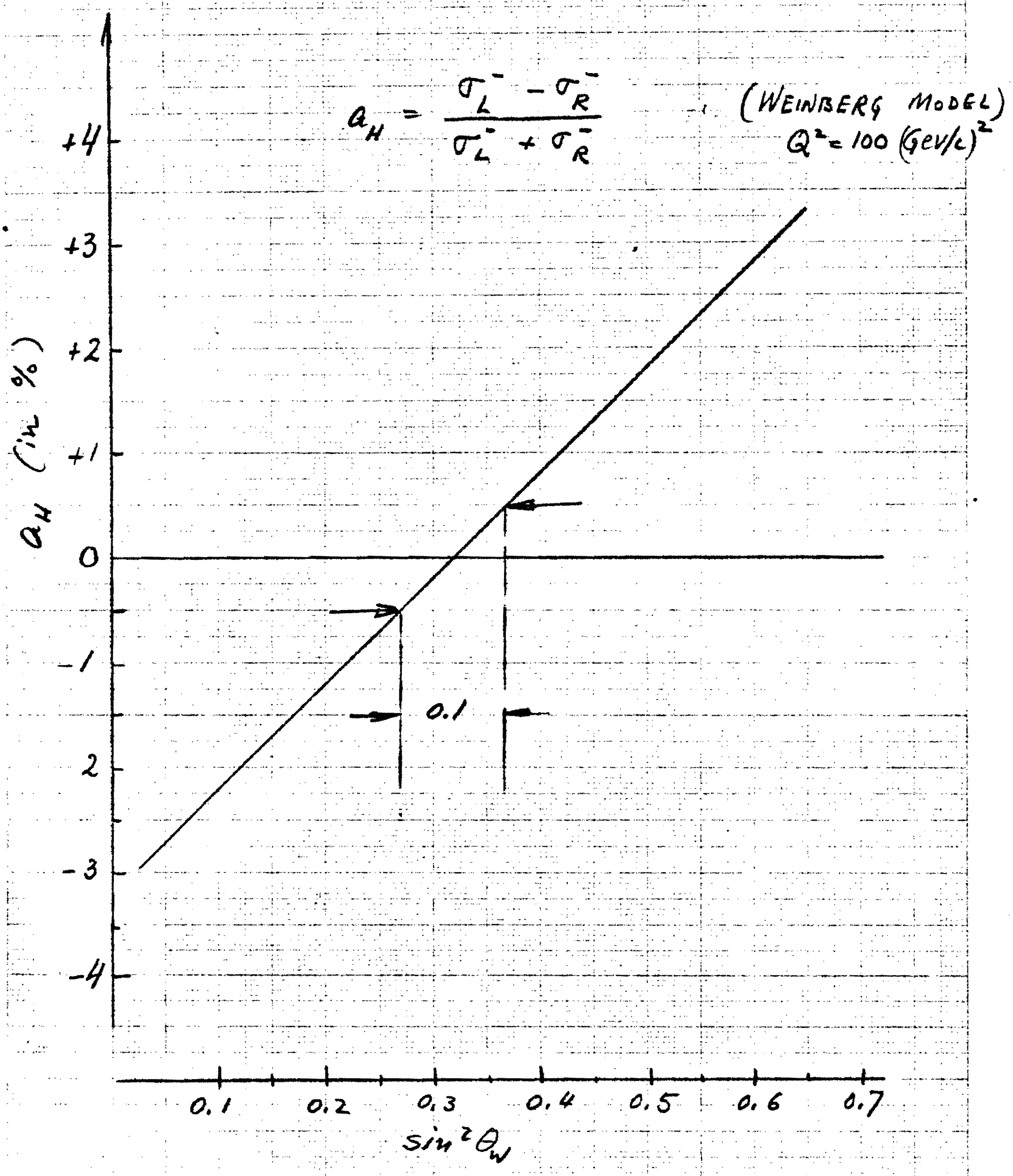
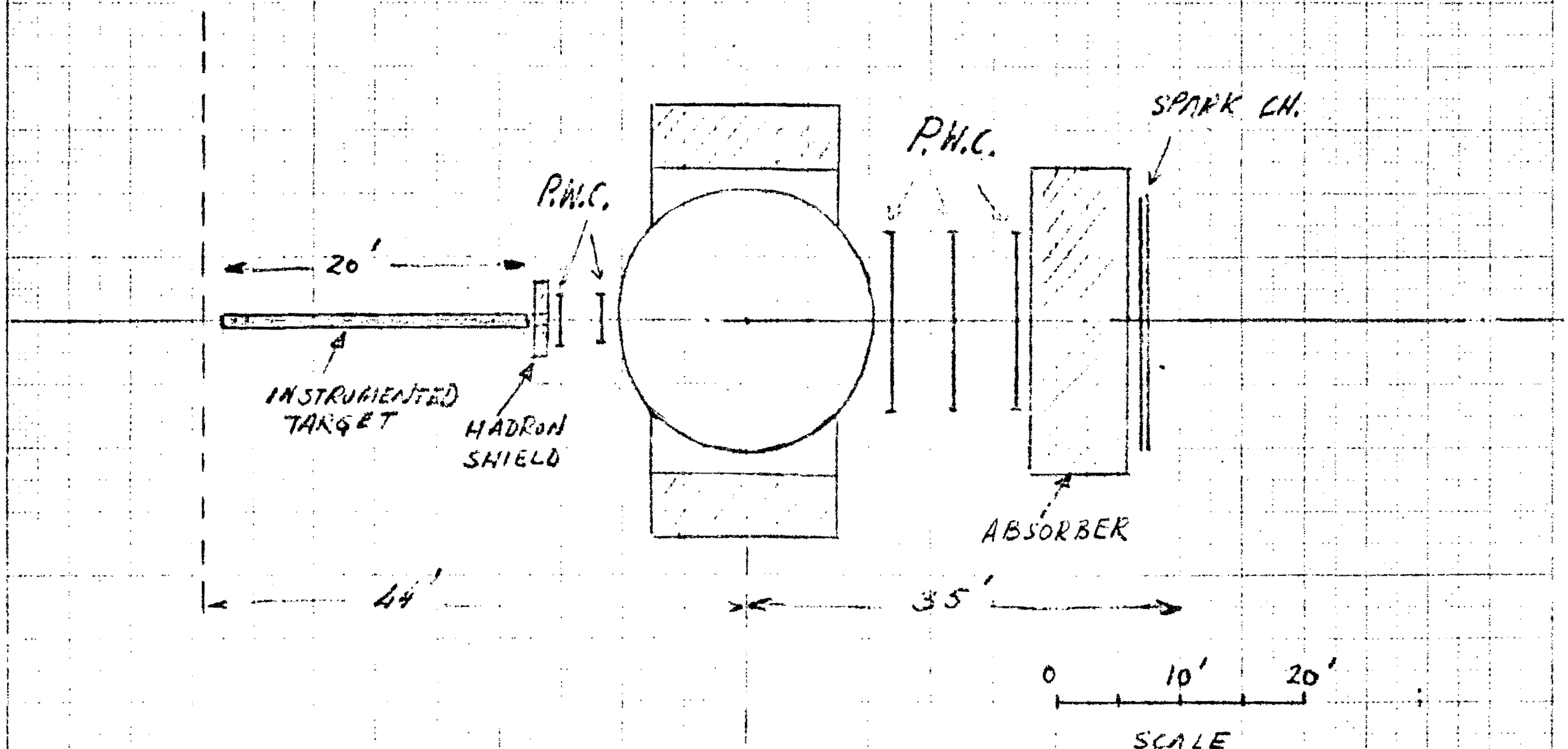
6. J. Kuti and V. F. Weisskopf, Phys. Rev. D4, 3418 (1971).
7. The advantage of uranium over steel, apart from its higher density, is that it is non-magnetic.



$a_H = \frac{\sigma_L^- - \sigma_R^-}{\sigma_L^- + \sigma_R^-}$
(WEINBERG MODEL)
$Q^2 = 100\ (GeV/c)^2$
a_H (in %)
+4
+3
+2
+1
0
-1
2
-3
-4
0.1
0.1
0.2
0.3
0.4
0.5
0.6
0.7
$\sin^2\theta_W$

Fig. 1



SPARK CH.
P.W.C.
P.W.C.
20'
INSTRUMENTED TARGET
HADRON SHIELD
ABSORBER
44'
35'
0
10'
20'
SCALE

FIG. 2

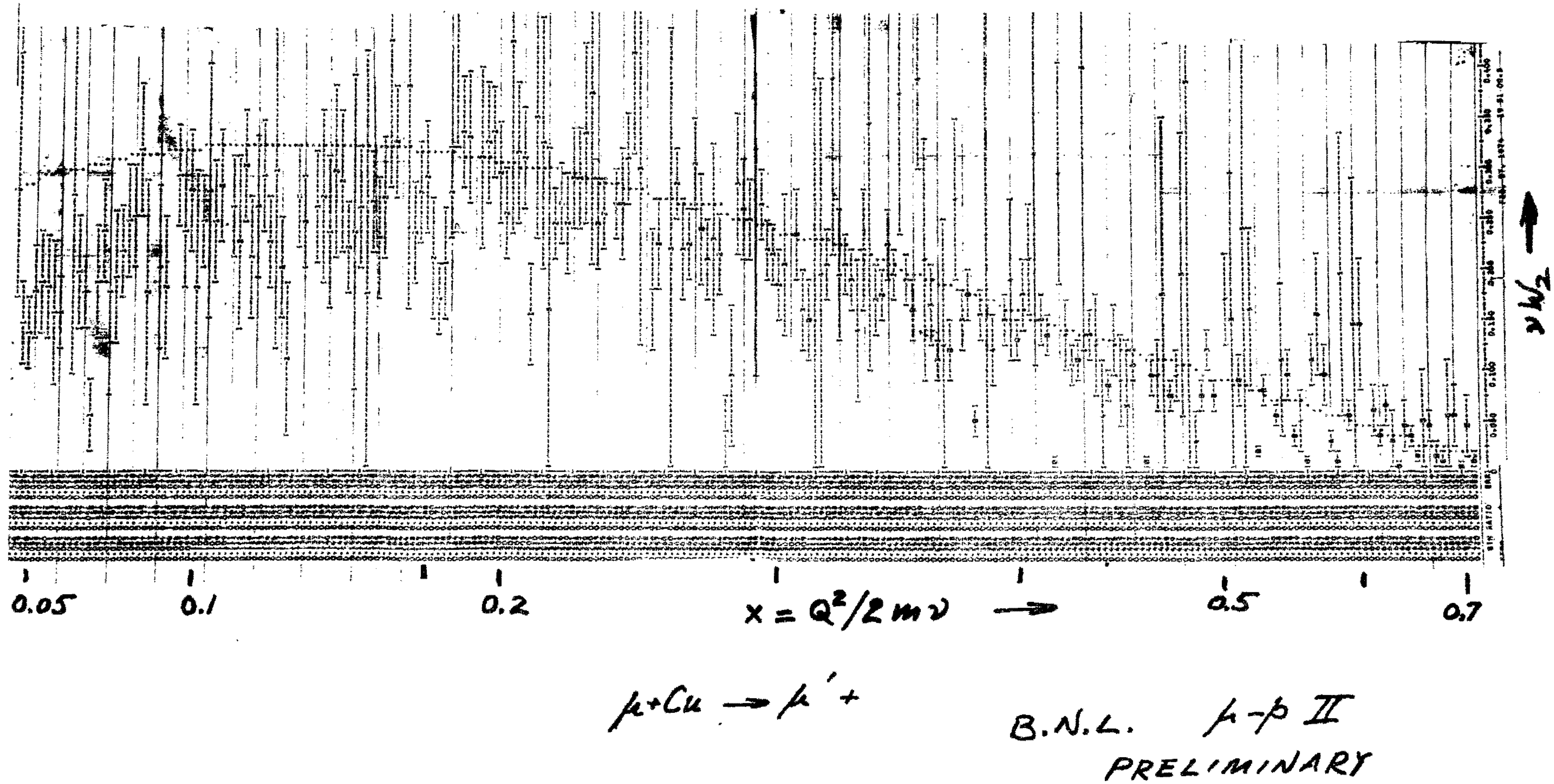


Figure 3

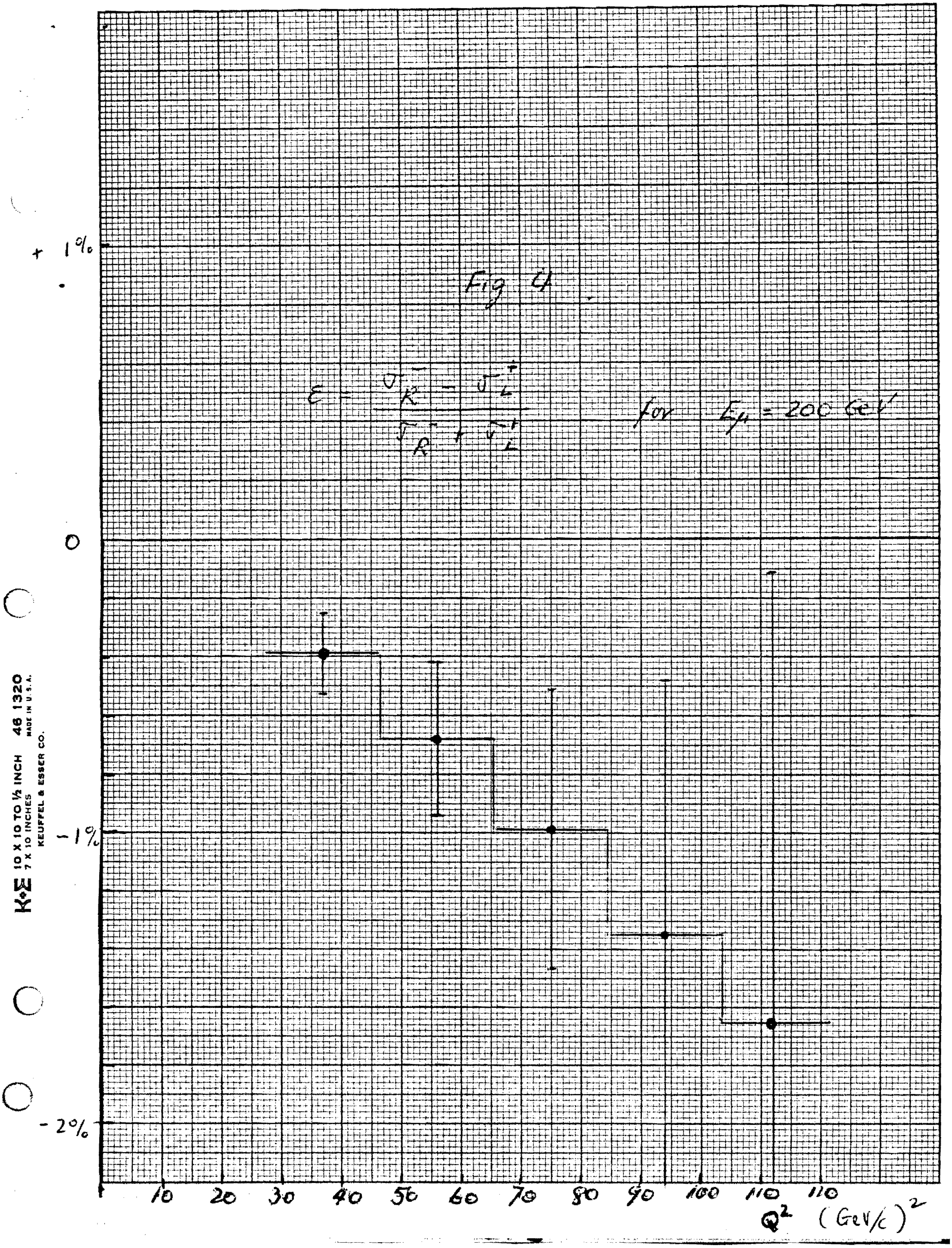


Fig 4
ε = (σ_R^- − σ_L^+) / (σ_R^- + σ_L^+)
for E_μ = 200 GeV
+1%
0
-1%
-2%
10 20 30 40 50 60 70 80 90 100 110 110
Q² (GeV/c)²
K+E 10 X 10 TO 1/2 INCH 46 1320
7 X 10 INCHES MADE IN U.S.A.
KEUFFEL & ESSER CO.

CONSIDERATIONS ON THE FERMILAB'S MUON BEAM

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10/21/74

ABSTRACT: We propose that magnetized (but not saturated) iron lenses can be used in the Fermilab's muon beam, increasing its momentum acceptance tenfold so that fluxes of the order 5×10^{-6} muons/proton can be achieved. Furthermore, polarized beams with 80% polarization can be achieved at flux levels of 2×10^{-7} muons/proton. The proposed scheme, if successful, will use substantially less power than presently, and involves no new elements except for the iron lenses. These considerations are equally applicable to the muon beam designs proposed for the CERN II accelerator.

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I. INTRODUCTION

As part of our proposal[1] #314 and of our interest in muon physics[2] we have examined possibilities for increasing the flux of the existing Fermilab muon beam. The beam is shown in Fig. 1 where we label the five legs of the beam by A–E and the enclosures by N100–N104. One concludes the following:

(a) The present momentum bite is very small $\Delta p/p \simeq \pm 1\%$, being limited primarily by the acceptance of the dipoles at the bend points. This achromatism can be overcome by focusing (through one whole betatron wavelength) from N100 to N101 and from N102 to N104. Momentum bites of $\Delta p/p = \pm 10\%$ can then be achieved.

(b) All of the muons produced in the decay pipe (leg A) can be focused onto the aperture of the first bend (N100). To see this note that since the c.m. momentum of the muon in π-decay is $\bar{p} = 29.7$ MeV the maximum decay angle for a 150 GeV muon is

$$\theta_{max} = \frac{29.7 \times 10^{-3}}{150} = 0.2 \text{ mrad}$$

Thus, if the parent pions are focused onto N100, the maximum transverse displacement of the decay muons at the focal point is

$$X_{max} = Z\theta_{max} \simeq 400 \times (2 \times 10^{-4}) = 0.08 \text{ m}$$

This matches the horizontal acceptance at N100 and misses slightly in the vertical plane.

Furthermore, if the pion focusing elements are close the the proton target all muons, irrespective of their momentum, will obey the above quasi-focusing condition. One concludes that instead of a "muon-channel" which is useful for low energy muon beams, one needs a "horn" type device, albeit of a D.C. variety.*

(c) In the absence of the D.C. "horn" we explore the possibilities of the existing equipment. The drawback here is the achromatism which results from the large magnification (of the order of 20) of the source as seen at N100. For instance, for 150 GeV central momentum and if a symmetric triplet is located at 20 m from the target and since N100 is at 400 m from the target, a ray produced at 2.5 mr reaches N100 with the following transverse displacements

p = 135	x = -7 cm
p = 150	x = 0
p = 165	x = 9 cm

Thus a $\Delta p/p = \pm 10\%$ in parent pions can be achieved. This is of the desired order for a polarized muon beam. For a maximum intensity unpolarized beam $\Delta p/p \simeq \pm 25\%$ would be preferable, and thus the horn is advantageous.

We also note that in the existing "doublet train-load" the angular acceptance in the horizontal plane is limited to < 1 mr so that a wider momentum band can be accepted.

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* For instance a superconducting horn may be the answer.

(d) All the phase space of muons reaching N100 must be transported to the experiment with a $\Delta p/p \approx \pm 10\%$ as mentioned in (a). This can be easily achieved by the iron lens transport system.

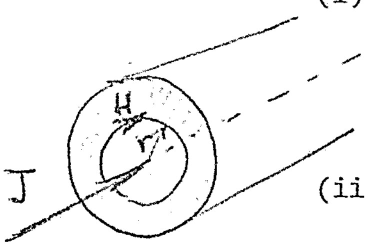
(e) For a polarized muon beam, pions must be eliminated from the beam immediately after N100.

In the following section we discuss the axial current iron lens, and review the properties of the FOFO channel that results. Based on these assumptions we make estimates of the muon flux and outline our plans for the implementation of this program.

II. THE AXIAL CURRENT IRON LENS

A. It is well known that an axially symmetric current of uniform density J results in focusing of charged particles moving in the direction of the current.

(i) The azimouthal magnetic field is a function of radius



$$B(r) = \mu_o \frac{J}{2} r \qquad (2.1)$$

(ii) The betatron wavelength is given by

$$\lambda = 2\pi\sqrt{\frac{pc}{0.3\ B_o}\ \frac{1}{\beta}} \qquad (2.2)$$

with λ(cm), pc (MeV), B_o (KG/cm)

(iii) Typically, for $\lambda = 628$ m and pc = 150 GeV

$B_o = 5$ gauss/cm

therefore $J = 0.08$ Amps/mm^2

and for a 15 cm radius lens, I = 5600 Amps

(iv) The maximum angular acceptance of the lens for a ray starting on axis with slope α_o' is

$$\alpha_o' = \left.\frac{dx}{dz}\right|_{z=0} = R\,\frac{2\pi}{\lambda} \qquad (2.3)$$

where R is the radius. For instance for $\lambda = 628$m and R = 15 cm we obtain $\alpha_o' =$ 1.5 mr.

B. For the present purposes we require $\lambda \sim 150$ m which implies an increase in total current of the order of 16 or close to 100 KAmps. This is too high; however the magnetic field can be reinforced using iron. Consider an iron cylinder of radius R which has the axial current I flowing through it. The homogeneous current density $J = I/(\pi R^2)$ produces an azimouthal field

$$B(r) = \mu\mu_o \frac{J}{2} r \qquad (2.1')$$

with $\mu \simeq 3000$.

(i) Now the current densities are very low and the limit is saturation of the field at the outer radius. If we assume the saturation value B(R) = 20 KGauss, we obtain for a 15 cm radius lens

$B(r) = B_o r$ with $B_o = 1.33$ kG/cm

which is typical of air quadrupoles.

(ii) The corresponding current through the cylinder is

I = 500 A

at practically zero voltage ($R \sim 1.4 \times 10^{-5}$ Ω/m).

(iii) Note that this is an "infinite pole" magnet since it focuses in all planes through its axis exactly as an optical lens.

(iv) The focal length is given by

$$\boxed{f = \frac{pc}{3B_o\,\Lambda} \quad (m)} \qquad (2.2')$$

with pc (in GeV), B_o in (KGauss/cm) and f, Λ the length of the lens in meters. Thus for pc = 150 GeV and Λ = 1 m we obtain f = 37.5 m.

(v) Such a lens is useful only for high energy muons since hadrons are absorbed while low energy muons suffer excessive energy loss. For high energy muons use

$dE/dx \simeq 1$ GeV/m in steel

(vi) Multiple scattering is a problem but the arrangement in a continuous channel obviates this to a large extent. Typically, for a lens of length $\Lambda = 0.50$ m, and 150 GeV muons

$$\theta_{r.m.s.} \simeq 0.75 \text{ mrad} \tag{2.4}$$

This is well within the angular acceptance of the proposed system.

(vii) The lenses serve also as a filter for the hadrons in the beam.

III. THE FOFO CHANNEL

A. We consider a lattice of "iron lenses" of equal strength spaced at a distance ℓ from one another. In the thin lens approximation the transfer matrix for one element of the lattice (i.e. FO), is

$$\begin{pmatrix} 1-\ell/f & \ell \\ -1/f & 1 \end{pmatrix} \tag{3.1}$$

where f is the focal length. The lattice is stable if

$$0 < \alpha = \ell/f < 4$$

with the acceptance being reasonable only for $\alpha < 2$.

If we consider an arbitrary ray which enters the first element with

$$Z_o = \begin{bmatrix} a_o \\ a'_o \end{bmatrix}$$

its displacement at the exit of the m^{th} element of the lattice is given by

$$a_m = a_o \cos m\phi + \frac{(2a'_o f - a_o)}{\sqrt{(4/\alpha)-1}} \sin m\phi \tag{3.2}$$

where $\cos\phi = 1 - (\alpha/2)$

B. We are more interested in the case where we approximate a continuous distribution. Then $\ell \to o$, $f \to \infty$ as ℓf = constant.

In this limit $\alpha \to o$ (the system remains stable) eq. 3.2 reads

$$x(m) = x_o \cos m\phi + (x_o' \sqrt{\ell f} - \frac{x_o\sqrt{\alpha}}{2}) \sin m \phi \qquad (3.2')$$

Further, since $m = z/\ell$ and $\phi \to \sqrt{\alpha}$ one obtains

$$m\,\phi = z/\sqrt{\ell f}$$

A continuous distribution has the oscillatory solution

$$x(z) = x_o \cos(2\pi z/\lambda) + (2\pi x_o'/\lambda) \sin(2\pi z/\lambda) \qquad (3.2'')$$

Comparing the two equations 3.2' and 3.2'' we see that in the limit $\alpha \to o$ the F0F0 channel has a betatron wavelength given by

$$\boxed{\lambda/2\pi = \sqrt{\ell f}} \qquad (3.3)$$

C. We now estimate the amount of steel required to perform one complete betatron oscillation in a beam of length L. Let the number of lenses be N. Then

$$\ell = L/N \quad \text{with} \quad \lambda = L$$

From eq. 2.2 $f = \frac{pc}{3B_o\Lambda}$ and using $B_o = 1.33$ KG/cm $f = \frac{pc}{4\Lambda}$ with Λ the length of each lens

Then

$$\Lambda N = \frac{(2\,\pi)^2}{L} \cdot \frac{pc}{4} \qquad (3.4)$$

with Λ, L in meters, and pc in GeV.

(i) For instance, four our previous example of L = 628m and pc = 150 GeV, the total length of steel is

$$\Lambda N = 2.4 \text{ (m)}$$

(ii) For the more realistic case of L = 150 m as required between N101 and N102 we obtain for pc = 150 GeV

$$\Lambda N = 9.2 \text{ (m)}$$

which is still acceptable.

(iii) Increasing the steel also increases the acceptance of the system. For $\lambda = L = 150$ m eq. 2.3 yields an angular acceptance of

$$\alpha_o' = 0.15 \frac{2\,\pi}{150} = 6.28 \text{ mr} \qquad (3.5)$$

This is adequate to compensate for multiple scattering and can transport a substantial phase space, and include the dispersion at a bend point.

(iv) The momentum acceptance is good. After one betatron oscillation the transverse displacement of the aperture limited ray (i.e. the one that reaches maximum excursion) is

$$x = \pi R \frac{dp}{p}$$

IV. THE PROPOSED BEAM

A. Maximum intensity

The general ideas have been discussed in section 1 and are shown in Fig. 1. To summarize we consider an explicit case of 300 GeV incident protons and using the triplet load to focus pions of momentum

$$p = 150 \pm 15 \text{ GeV/c}$$

onto the N100 bend. We assume an acceptance in both horizontal and vertical planes of

$$\Theta_{max} = 2.5 \text{ m rad}$$

and assume the mean production angle to be at $\Theta = 0$. The integrated pion yield is of the order of

$$2 \times 10^{-3} \ \pi^{+}/\text{proton}$$

using the standard 30 cm long Aℓ target.

The decay probability in leg A (400 m) is

decay → 4.8%

The bend in N100 is set for 150 GeV and is followed in leg B by an iron lens lattice which produces one betatron oscillation between N100 and N101. In our present design we are using 11 elements approximately 80 cm long each. This results in a momentum acceptance of the order of ± 15% about the central momentum.

The bend at N101 removes the dispersion in angle produced by the first bend and the beam drifts onto N102. Between N102 and N104 a similar lattice is introduced and the beam into the muon lab is again dispersion free. The quadrupoles in enclosures N100, N101 and N103 can be removed to save power. On the other hand, they could be maintained as part of the lattice to reduce multiple scattering and energy loss. The filter in N102 will be removed.

Since the muon spectrum extends only to 0.43 E_π the beam transports on the average 1/3 of all muons reaching N100. Thus the flux is

$$(2 \times 10^{-3}) \times (4.8 \times 10^{-2}) \times (1/3) = 3 \times 10^{-5} \mu/\text{proton}$$

This does not account for losses due to multiple scattering which however should not exceed a factor of 3. If in addition we include a factor of 2 for losses due to the 4-inch vertical aperature presently available, we can estimate

$$5 \times 10^{-6} \ \mu/\text{proton}$$

In any event, it is reasonable that fluxes of the order of 10^{-5} μ^{+}/proton can be achieved, at a muon momentum 130 ± 20 GeV, without major modifications of the beam. Such fluxes (i.e. ~ 10^{8}/pulse) are probably above the rate-handling capabilities of the existing detectors.

B. Polarized beam

The forward polarization presents no real problem. We propose to keep the muon transport system tuned to the same momentum, which is also important for reducing biases in the detection system. Instead, the incident energy as well as the pion focus are changed. While this is not necessary, if the extraction had a front porch one could measure simultaneously both polarizations, by simply ramping the pion focusing train load. The proton, pion and muon momenta for the two configurations are indicated below.

	Forward	Backward
E_p	300	400
p_π	130-160	210-260
p_μ	130-150	130-150
<P>	70%	80%
μ^+/p	1.2×10^{-6}	1.4×10^{-7}

The existing doublet train load is not optimum for momentum selection because it requires a limitation on the production angle.(3) For a momentum half width at half max. of $\pm$ 10% the production angle in the vertical plane is reduced to $\pm$ 0.5 m rad with a loss of flux by a factor of 4 from our previous estimate. The momentum band of the pions is unaffected and this is also true for the momentum band of the muons. Thus we expected $\mu/p \simeq 1.2 \times 10^{-6}$.

For the backward decays we have the following additional losses.

a. Production spectrum $(\langle x\rangle = 0.48) \rightarrow (\langle x\rangle = 0.58)$ estimate 1/3

b. Fraction of accepted muons 3/5

c. Decay probability 0.58

Namely $\mu(\text{backward})/\mu(\text{forward}) \simeq 0.12$ resulting in a ratio $\mu^+/p \simeq 1.4 \times 10^{-7}$.

Since we propose to run with approximately 10^6 muons/pulse the forward muon polarization can be further sharpened at the expense of flux. In any case the fluxes proposed here will reduce the running time of the experiment to 250 hours.

V. IMPLEMENTATION

Clearly, the preceding analysis depends on whether or not an"iron lens" can be successfully constructed in practice. We propose that the current into and out of the iron cylinder be fed through a tapered copper plug which will be connected to the iron with an indium or similar seal. All the lenses in a lattice will be connected in series, the voltage drop appearing mainly across the cable.

The lenses can be equipped with rollers so that they can be pushed into the muon beam pipe. This will also permit their removal when it is desired to use a hadron beam. There may be some difficulty if the pipe is out of round but this is not insurmountable.

As far as alignment of the lenses is concerned, errors in angular placement are not very serious since they are of higher order. On the other hand if the axis of the lens is displaced from the beam axis, by a distance d, first order effects appear. Again, these are not serious as long as $\ell/f \ll R/d$, since in this case the beam follows the lattice axis. Namely if the lenses are properly centered in the beam pipe the beam will follow the contour of the pipe.

We are now performing detailed Monte Carlo calculations to determine the exact effect of multiple scattering and of the existing apertures of the beam. However, if the iron lenses perform as predicted, our previous conclusions are of the correct order of magnitude and must be true.

To check the performance of the iron lenses we propose that with the assistance of the neutrino lab's staff we construct one such element. One can envisage measuring the magnetic field inside the lens by making small radial slots perpendicular to the field lines and inserting a hall probe. This procedure, (even though widely used in textbooks) is probably of marginal value. We feel that the best way to test the performance of the lens is to use muons and measure the focal properties of the lens.

We can perform such a test during the next running periof of the muons beam in January. We would install the lens behind the apparatus of E-98 in the muon lab and use proportional chambers to measure the direction of the muons before and after the lens. This should definitively settle whether the proposed scheme works or not.

VI. ACKNOWLEDGMENTS

It is a pleasure to acknowledge discussions and information on the muon beam, given to us cheerfully by T. Yamanouchi, P. Limon, R. Stefanski and A. Skuza. We also thank D. McCall, W. Metcalf and I-J. Kim for assistance with Monte Carlo calculations. Finally, the encouragement of J. Sanford is much appreciated.

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   F. Lobkowicz, D. Green and S. Serio "Design Study for a muon and antiproton facility at SLAC" University of Rochester Report UR-875-161, (1966) unpublished.
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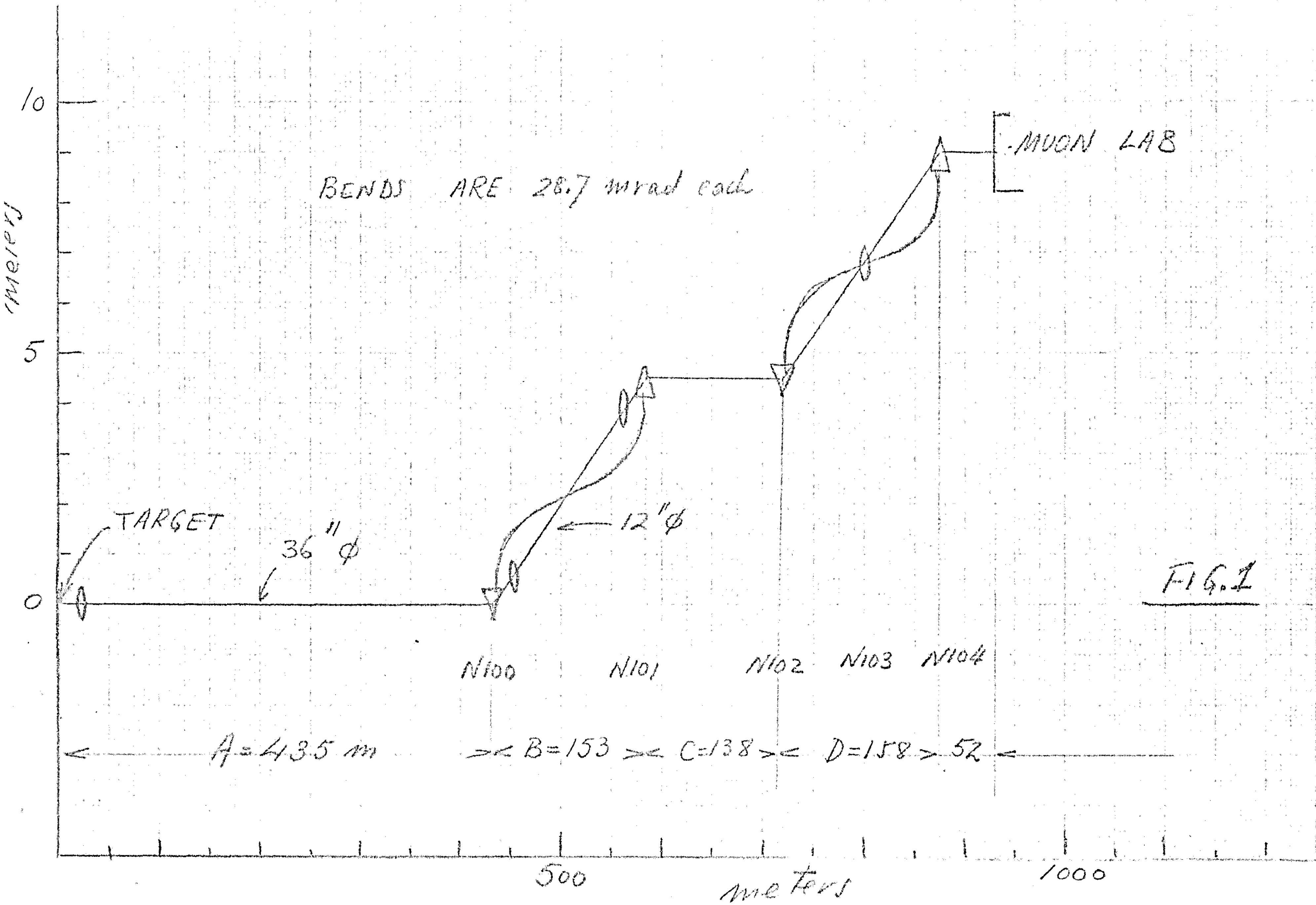

10
5
0
meters
BENDS ARE 28.7 mrad each
MUON LAB
TARGET
36" ϕ
12" ϕ
N100
N101
N102
N103
N104
A=435 m
B=153
C=138
D=158
52
500
1000
meters

FIG. 1

CONSIDERATIONS ON THE FERMILAB'S MUON BEAM

F. Lobkowicz and A. C. Melissinos
University of Rochester

(Updated 12/11/74)

ABSTRACT:

We propose that magnetized (but not saturated) iron lenses can be used in the Fermilab's muon beam, increasing its momentum acceptance tenfold so that fluxes of the order 5×10^{-6} muons/proton can be achieved. Furthermore, polarized beams with 80% polarization can be achieved at flux levels of 2×10^{-7} muons/proton. We propose that a 1-ft. diameter pipe containing the iron lenses be laid between enclosures N100 and N104. The new beam will be able to transport 250 GeV muons and use substantially less power than the existing beam.

1. Introduction

As part of our proposal[1] #314 and of our interest in muon physics[2] we have examined possibilities for increasing the flux of the Fermilab's muon beam.

(a) The present limitation in muon flux is associated with the very small momentum acceptance, $\Delta p/p = \pm 1\%$, of the transport system from the end of the decay region to the muon lab. In our proposed scheme the momentum acceptance is increased tenfold to $\Delta p/p = \pm 10\%$.

(b) We note that all of the muons produced in the decay pipe can be focused onto the aperture of the first bend at N100. Since the c.m. momentum of the muon in π-decay is $\bar{p} = 29.7$ MeV the maximum decay angle for a 150 GeV muon is

$$\theta_{max} = \frac{29.7 \times 10^{-3}}{150} = 0.2 \text{ mrad}$$

Thus, if the parent pions are focused onto N100, the maximum transverse displacement of the decay muons at the focal point is

$$x_{max} = z\theta_{max} \simeq 400 \times (2 \times 10^{-4}) = \pm 0.08 \text{ m}$$

This matches the horizontal acceptance at N100 and misses slightly in the vertical plane.

Furthermore, if the pion focusing elements are close to the proton target all muons, irrespective of their momentum, will obey the above quasi-focusing condition. One concludes that instead of a "muon-channel" which is useful for low energy muon beams, one needs a "horn" type device, albeit of a D.C. variety.*

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*For instance a superconducting horn may be the answer.

(c) In the absence of the D.C. "horn" we explore the possibilities of the existing equipment. The drawback here is the achromatism which results from the large magnification (of the order of 20) of the source as seen at N100. For instance, for 150 GeV central momentum and if a symmetric triplet is located at 20 m from the target and since N100 is at 400 m from the target, a ray produced at 2.5 mr reaches N100 with the following transverse displacements

p = 135	x = -7 cm
p = 150	x = 0
p = 165	x = 9 cm

Thus a $\Delta p/p = \pm 10\%$ in parent pions can be achieved. This is of the desired order for a polarized muon beam. For a maximum intensity unpolarized beam $\Delta p/p \simeq \pm 25\%$ would be preferable, and thus the horn is advantageous.

We also note that in the existing "doublet train-load" the angular acceptance in the horizontal plane is limited to <1 mr so that a wider momentum band can be accepted.

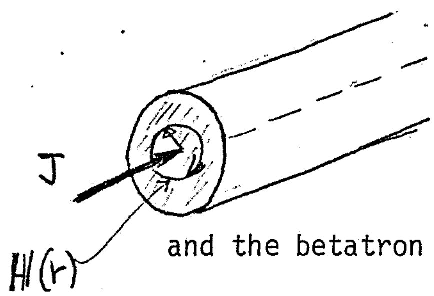
(d) All the phase space of muons reaching N100 must be transported to the experiment with a $\Delta p/p \simeq \pm 10\%$ as mentioned in (a) above. This can be easily achieved by the iron lens transport system.

(e) For a polarized muon beam, pions must be eliminated from the beam <u>immediately</u> after N100.

In the following section we discuss the axial current iron lens, and review the properties of the FOFO channel that results. Based on these assumptions we make estimates of the muon flux.

2. The Axial Current Lens

(a) It is well known that an axially symmetric current of uniform density J results in focusing of charged particles moving in the direction of the current. The azimuthal magnetic field is a function of radius



$$B(r) = \mu_0 \frac{J}{2} r = B_0 r \tag{2.1}$$

and the betatron wavelength is given by

$$\lambda = 2\pi \sqrt{\frac{pc}{0.3\ B_0}} \tag{2.2}$$

with λ(cm), pc (MeV), B_0 (kG/cm). The angular acceptance of the system is given by $\pm x_0'$ where

$$x_0' = (dx/dz)_{z=0} = R\,\frac{2\pi}{\lambda} = R\sqrt{(0.3\ B_0)/pc} \tag{2.3}$$

with R the radius in cm. Typically for $\lambda \simeq 450$ m, and R = 15 cm, the required current is I = 10 kA, and $x_0' = 2$ mrad.

(b) For practical purposes and for the special case of transporting muons one can consider reinforcing the magnetic field by using iron. Consider then an iron cylinder or radius R with the axial current I flowing through it. If we assume for the moment that B is strictly proportional to H, then $B = B_0 r$ as in (a) above. Namely, we obtain a lens which focuses simultaneously in all planes. We call this device a "pinch" magnet, and its focal length is

$$f = \frac{pc}{3B_0\Lambda} \tag{2.4}$$

with f in meters, pc in GeV, B_0 in kG/cm and Λ the length of the lens in meters.

To achieve the linearity of B with H various approaches can be used. The simplest is to feed the current close to the periphery of the lens and consider the integral of $\int Bdz$ along the length of the lens, as a function of the radial position. We have modeled this current feed for a lens made of cold rolled steel for the magnetization curve shown in Fig. 1. For R = 15 cm and Λ = 15 cm we obtain for the average field $\langle B \rangle = (\int Bdz)/\Lambda$ the curves shown in Fig. 2. Namely for a current I = 5000 Amps gradients of the order of B_0 = 1.1 kG/cm can be achieved. We note that the power requirements are small since the current flows at practically zero voltage.

The use of iron restricts the use of this type of lens to muons. It introduces energy loss, which for 150 GeV muons in steel we take as 24 MeV/cm. It also introduces multiple scattering which must be compensated for. However, the lenses serve also as a filter for the hadrons in the muon beam.

3. The FOFO Channel

(a) We consider a lattice of "iron lenses" of equal strength spaced at a distance ℓ from one another. In the thin lens approximation the transfer matrix for one element of the lattice (i.e. FO), is

$$\begin{pmatrix} 1-\ell/f & \ell \\ -1/f & 1 \end{pmatrix} \tag{3.1}$$

where f is the focal length. The lattice is stable if

$$0 < \alpha = \ell/f < 4$$

with the acceptance being reasonable only for $\alpha < 2$.

If we consider an arbitrary ray which enters the first element with

$$Z_o = \begin{bmatrix} a_o \\ a_o' \end{bmatrix}$$

its displacement at the exit of the m^{th} element of the lattice is given by

$$a_m = a_o \cos m\phi + \frac{(2a_o' f - a_o)}{\sqrt{(4/\alpha) - 1}} \sin m\phi \qquad (3.2)$$

where $\cos\phi = 1 - (\alpha/2)$.

(b) We are more interested in the case where we approximate a continuous distribution. Then $\ell \to o$, $f \to \infty$ as ℓf = constant. In this limit $\alpha \to o$ (the system remains stable) Eq. 3.2 reads

$$x(m) = x_o \cos m\phi + (x_o' \sqrt{\ell f} - \frac{x_o\sqrt{\alpha}}{2}) \sin m\phi \qquad (3.2')$$

Further, since $m = z/\ell$ and $\phi \to \sqrt{\alpha}$ one obtains

$$m\phi = z/\sqrt{\ell f}$$

A continuous distribution has the oscillatory solution

$$x(z) = x_o \cos(2\pi z/\lambda) + x_o'(\lambda/2\pi)\sin(2\pi z/\lambda) \qquad (3.2'')$$

Comparing the two equations 3.2' and 3.2" we see that in the limit $\alpha \to o$ the FOFO channel has a betatron wavelength given by

$$\lambda = 2\pi\sqrt{\ell f} \qquad (3.3)$$

(c) Consider then the requirements for performing one complete betatron oscillation in a beam of length L. Let the number of lenses be N so that

$$\ell = L/N \qquad \text{and } \lambda = L$$

The total length of steel required is

$$\Delta N = \frac{pc}{3B_o f} N = \frac{pc\ L}{3B_o \ell f} = \frac{pc}{3B_o} \frac{L}{(L/2\pi)^2} = \frac{4\pi^2}{3B_o} \frac{pc}{L} \tag{3.4}$$

where we used Eqs. (2.4), (3.3) and the condition $\lambda = L$. Typically for $L = 450$ m, $B_o = 1.1$ kG/cm and pc = 150 GeV, Eq. (3.4) gives $\Delta N = 4$ m. Namely an energy loss of 10 GeV and an r.m.s. multiple scattering angle of $\theta_{rms} \doteq 1.5$ mrad (assuming all the steel was lumped in one section). This is within the acceptance of the system and comparable to the angular dispersion of the original beam.

4. Proposed Beam Design

(a) Maximum intensity: Let us consider the case of 300 GeV incident protons and use the triplet to focus pions of momentum

$$p = 150 \pm 15 \text{ GeV/c}$$

onto the N100 bend. Assume acceptance in both horizontal and vertical planes of

$$\theta_{max} = \pm 2.5 \text{ mrad}$$

at a mean production angle $\theta = 0$. Using the 30 cm long Al target we estimate 2×10^{-3} π^+/proton and the probability for decay before reaching N100 is 4.8%.

If the muon transport accepts $\Delta p = \pm 10\%$ and since the muon spectrum extends only to $0.43\ E_\pi$, approximately 1/4 of all the muons reaching N100 will be captured. Namely

$$\mu^+/\text{proton} \simeq 2 \times 10^{-5}$$

We must now consider the transport of the muons to the experimental area. For this we propose that a standard 1 ft. diameter pipe be laid from N100 to N104, as shown in Fig. 3. This will permit the use of N100 and N104 for the bending magnets which now provide a reduced bend of 20 mrad. Thus, muons up to 225 GeV/c can be transported without modification of the present system.

In this pipe we propose to install 22 pinch magnets 18 cm long each, which provides for one complete betatron oscillation. We have investigated the effect of multiple scattering and estimate the efficiency of the transport to be 61% into the 4 × 4 aperture of the existing benders at N104. In Fig. 4 we show the beam profile as calculated for a similar system which has been focused through one betatron oscillation but with L = 150 m and ΛN = 8.8 m. The angular dispersion at the entrance of the system was taken as ±1 mrad and the spot size ±0.5-inches. The first bend was 30 mrad (instead of the proposed 20 mr). In Fig. 5 we show the resulting momentum spectrum under the above conditions.

In our previous estimate of the muon flux we did not account for the 4 × 4 aperture of the benders at N100 and we include for this a factor of 2 in addition to the transport efficiency. Namely one obtains at the target

$$\mu^+/\text{proton} = 5 \times 10^{-6}$$

for muon momenta of 140 ± 15 GeV/c.

(b) Polarized beam: The forward polarization presents no real problem. We propose to keep the muon transport system tuned to the same momentum, which is also important for reducing biases in the detection system. Instead, the incident energy as well as the pion focus are changed. While this is not necessary, if the extraction had a front porch one could measure simultaneously both polarizations, by simply ramping the pion focusing train load. The proton, pion and muon momenta for the two configurations are indicated below.

	Forward	Backward
E_p	300	400
p_π	130-160	210-260
p_μ	130-150	130-150
$\langle p \rangle$	70%	80%
μ^+/p	1.2×10^{-6}	1.4×10^{-7}

The existing doublet train load is not optimum for momentum selection because it requires a limitation on the production angle.(3) For a momentum half width at half max. of ±10% the production angle in the vertical plane is reduced to ±0.5 mrad with a loss of flux by a factor of 4 from our previous estimate. The momentum band of the pions is unaffected and this is also true for the momentum band of the muons. Thus we expect $\mu/p \simeq 1.2 \times 10^{-6}$.

For the backward decays we have the following additional losses.

a. Production spectrum ($\langle x \rangle = 0.48$) → ($\langle x \rangle = 0.58$)
   estimate 1/3

b. Fraction of accepted muons 3/5

c. Decay probability 0.58

Namely $\mu(\text{backward})/\mu(\text{forward}) \simeq 0.12$ resulting in a ratio $\mu^+/p \simeq 1.4 \times 10^{-7}$.

Since we propose to run with approximately 10^6 muons/pulse the forward muon polarization can be further sharpened at the expense of flux. In any case the fluxes proposed here will reduce the running time of the experiment to 250 hours.

5. Present Status

We are continuing model calculations of the proposed muon beam, and in particular on the path of the muons lost from the beam, the so called "halo".

We intend to construct a prototype of a "pinch" magnet and check its performance by using either the BNL or Fermilab muon beam. This seems to be the best way for probing the magnetic field inside the lens. Various approaches to the current feed will also be investigated.

Physically the lenses can be permanently installed in the new beam pipe. Exact alignment is not serious since the angular placement produces only higher order effects. Displacements of the lens axis are of first order but they are compensated by the adiabatic transport properties of the FOFO channel.

Finally, we mention that the placement of the new pipe will make it possible to bring both muon and hadron beams to the experimental area without any changes in apparatus.

Acknowledgements

It is a pleasure to acknowledge discussions and information on the muon beam, given to us cheerfully by T. Yamanouchi, P. Limon, R. Stefanski and A. Skuza. We also thank D. McCall, W. Metcalf and I-J. Kim for assistance with Monte Carlo calculations, and W. Carithers for critical discussions. The encouragement of J. Sanford is much appreciated.

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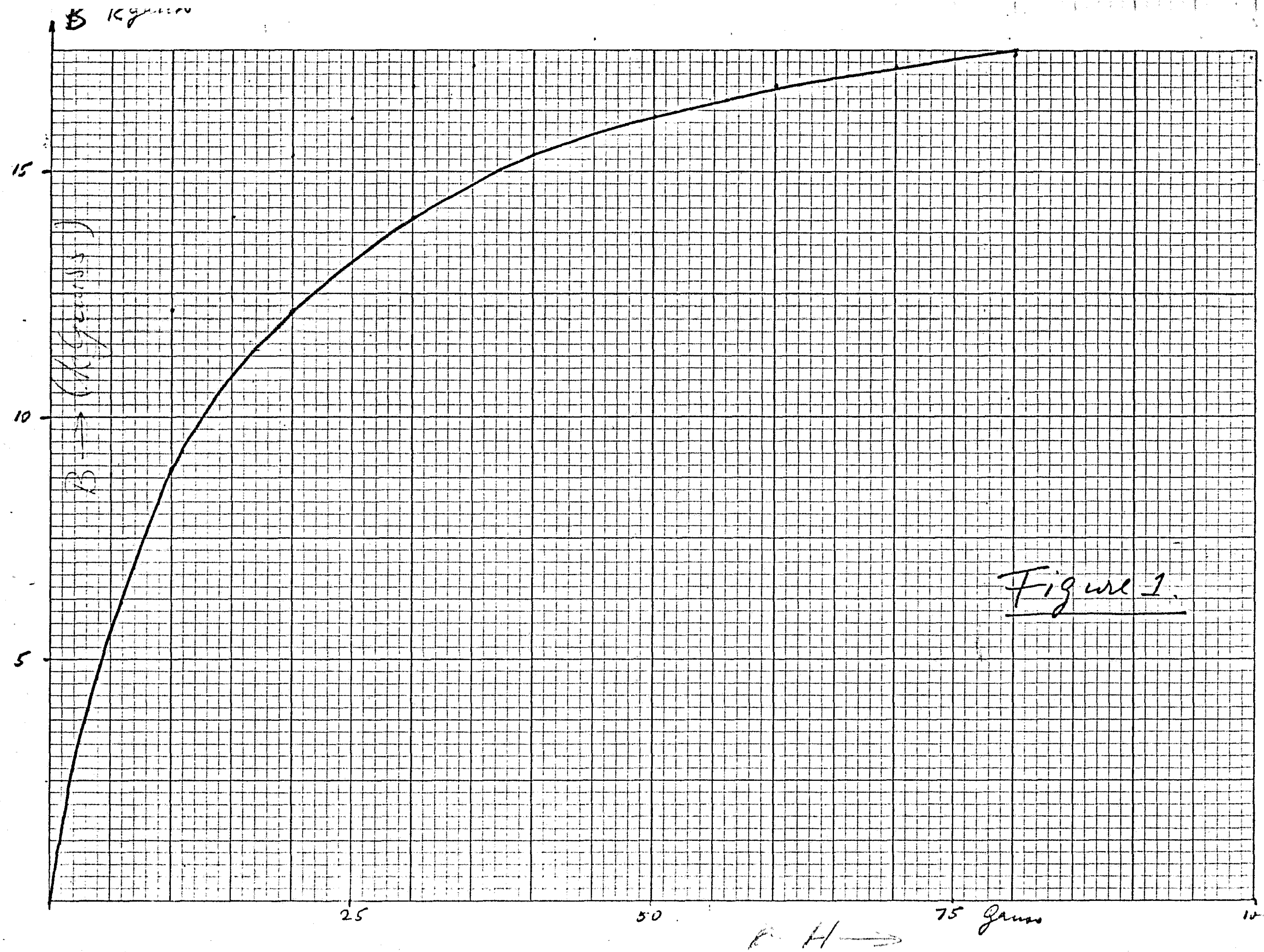
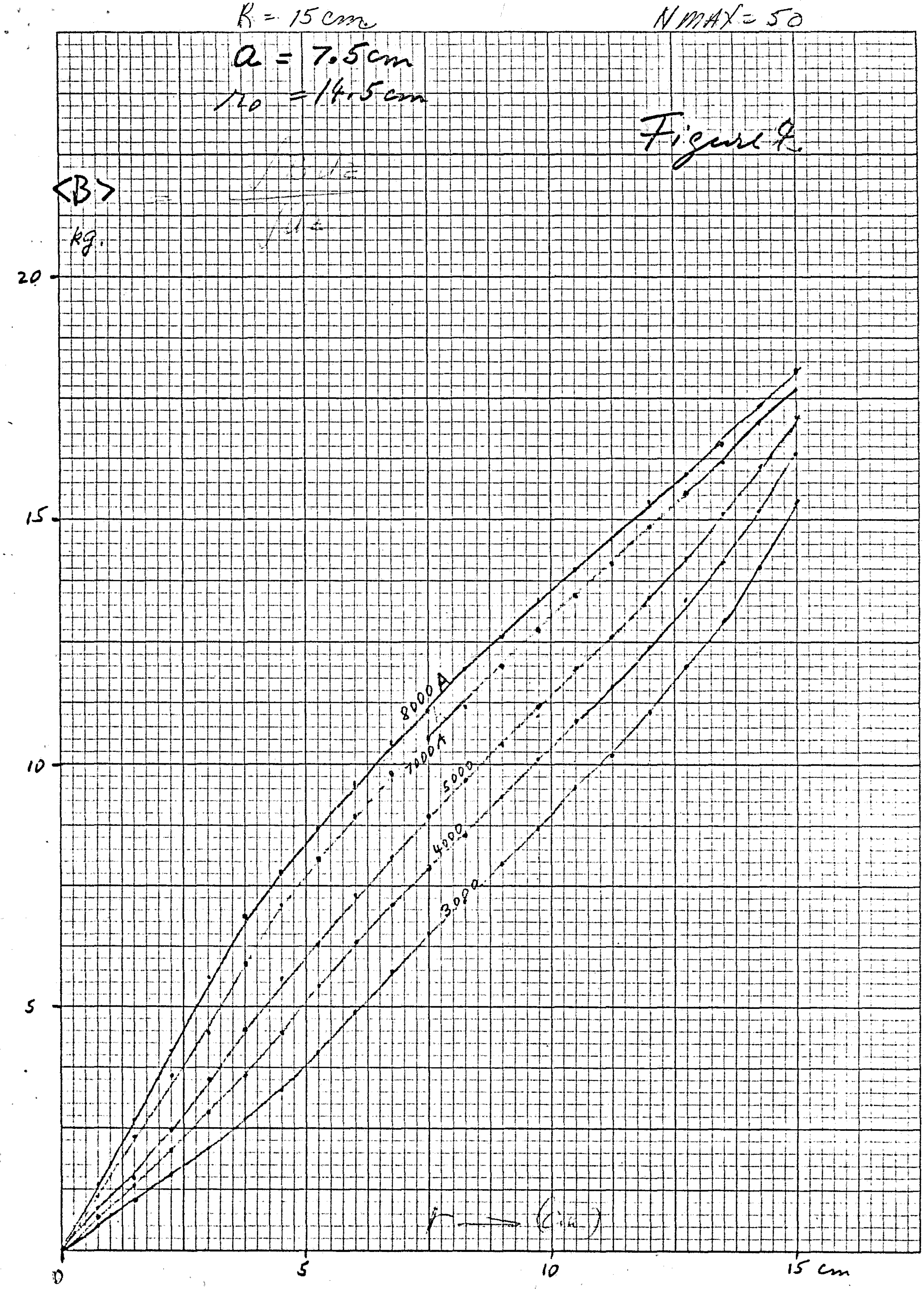


Figure 1.


R = 15 cm
NMAX = 50
a = 7.5 cm
r_o = 14.5 cm
Figure 9.
<B>
kg.
20
15
10
5
8000 A
7000 A
5000
4000
3000
0
5
10
15 cm
r ⟶ (cm)

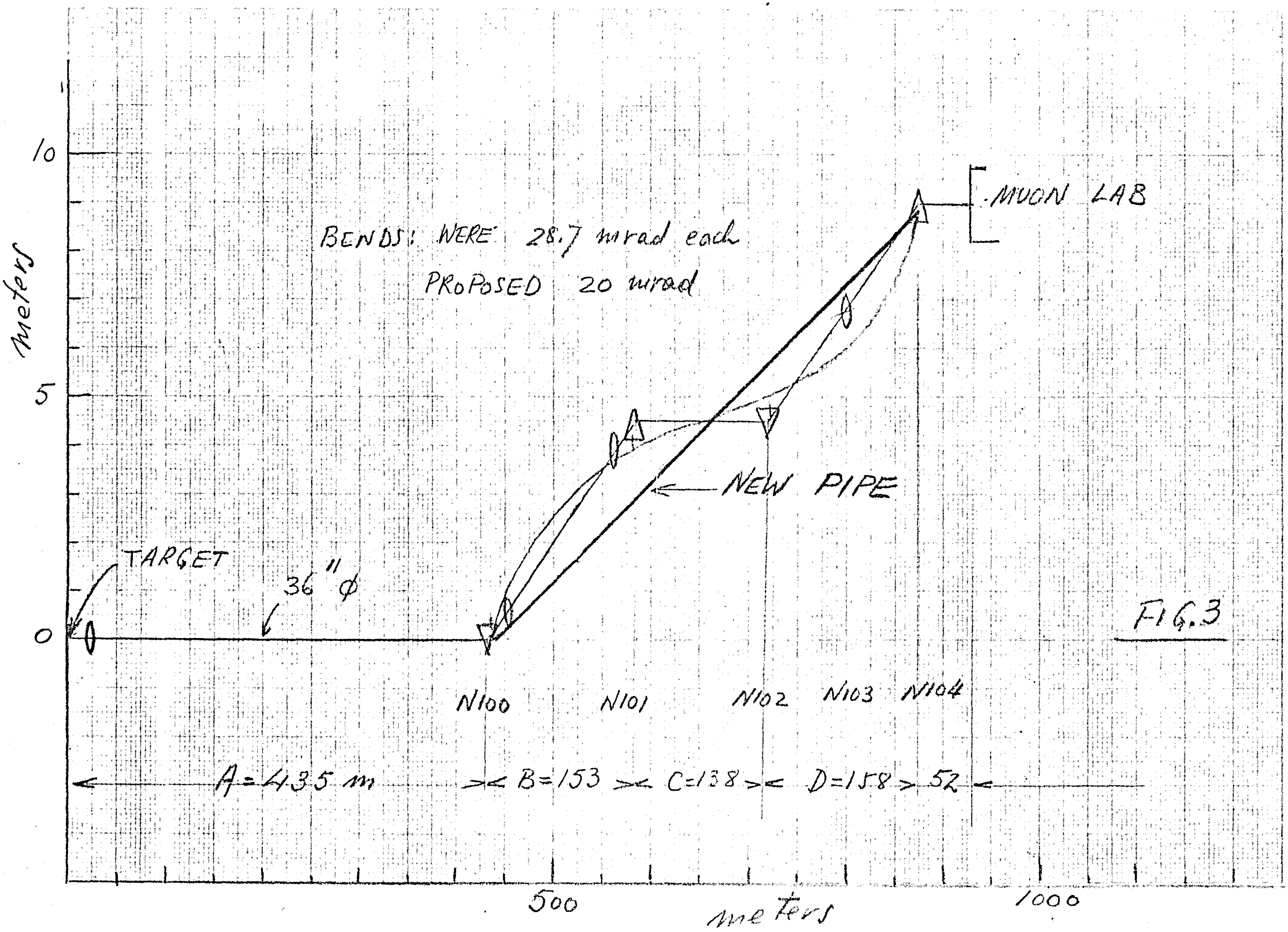

BENDS: WERE 28.7 mrad each
PROPOSED 20 mrad
MUON LAB
NEW PIPE
TARGET
36" φ
10
5
0
meters
N100
N101
N102
N103
N104
A=435 m
B=153
C=138
D=158
52
500
1000
meters

FIG. 3

FIG. 4

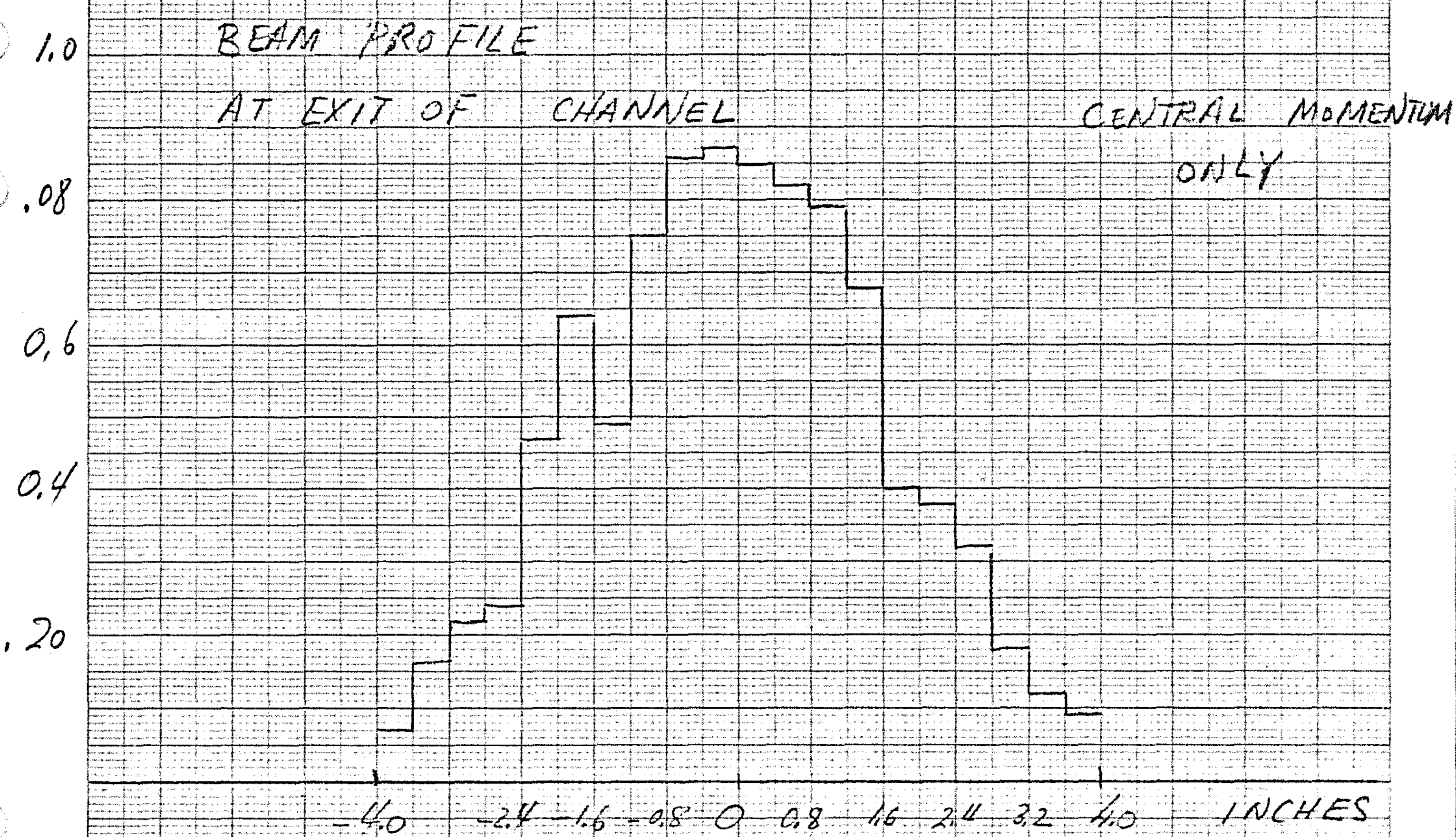
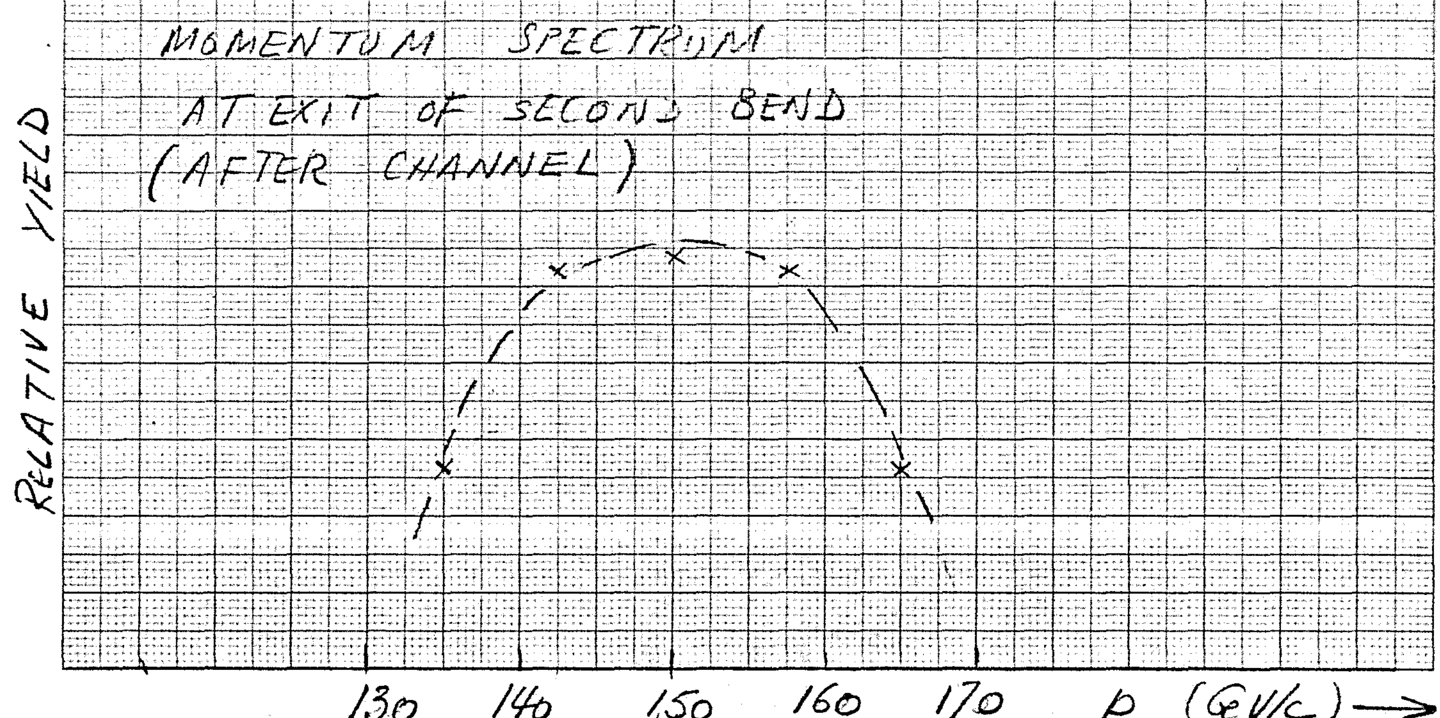

BEAM PROFILE
AT EXIT OF CHANNEL
CENTRAL MOMENTUM ONLY
1.0
.08
0.6
0.4
.20
-4.0
-2.4
-1.6
-0.8
0
0.8
1.6
2.4
3.2
4.0
INCHES

FIG 5


MOMENTUM SPECTRUM
AT EXIT OF SECOND BEND
(AFTER CHANNEL)
RELATIVE YIELD
130
140
150
160
170
p (GeV/c)