Chapter XX

Future muon colliders, Higgs and neutrino factories

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Over the past two decades there has been significant progress in developing the concepts and technologies needed to produce, capture, accelerate and collide high intensity beams of muons. At present, a high-luminosity multi-TeV muon collider presents an attractive and very cost-effective option for the next generation lepton-lepton collider, which can allow to fully explore the energy frontier of high energy particle physics in the era following the LHC discoveries. Of growing interest is also a low-energy $\mu^+\mu^-$ collider – Higgs factory - at the center of mass energy $\sqrt{s} = 125$ GeV that will produce copious number of Higgs particles and explore their properties in greater detail. Synergetic to the collider is the neutrino factory concept that is based on racetrack type muon-decay ring production of the neutrinos. Below we briefly review the concepts and feasibility of the muon accelerator facilities for particle physics research, discuss the status of the corresponding accelerator R&D, and outline directions of the future work.

1 Current landscape of accelerator-based particle physics

High energy particle physics aspires to explore a number of critical questions which require next generation colliding beam facilities of two types: a) Higgs Factories with a center of mass energy sufficient for production and precision studies of the Higgs boson and exploration of the Higgs sector in greater detail, including measurements of the Higgs couplings to fermions and vector bosons; self coupling; rare decays; mass and width, etc., and b) colliders for exploration of the energy frontier and potential discoveries through direct searches with the center-of-mass energies significantly beyond that of the LHC to search for new particles/phenomena beyond the Standard Model, reaching mass-scales in the range of tens of TeV and offering a widely extended discovery reach for new gauge bosons $Z'$ and $W'$, colorons, diquark scalars, SUSY, heavy Higgs, test for compositeness of the Standard Model particles, etc [1, 2, 3, 4].

There are four widely-discussed concepts which mostly rely on the currently available technologies of normal-conducting or superconducting RF and/or normal-conducting or superconducting magnets, and generally considered as technically and potentially cost feasible to be constructed over the next several decades: linear $e^+e^-$ colliders, circular $e^+e^-$ colliders, $pp/ep$ colliders and $\mu^+\mu^-$ colliders [5]. They all have limitations in energy, luminosity, efficiency and

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cost. The key requirement for a Higgs factory is high luminosity $O(10^{34-35} \text{ cm}^{-2}\text{s}^{-1})$ and besides a 126 GeV c.m.e. muon Higgs factory, there are four $e^+e^-$ collider proposals which generally satisfy it: 250 GeV c.m.e. International Linear Collider (ILC), 380 GeV Compact Linear Collider (CLIC), Chinese electron positron Collider (CepC) and the Future Circular Collider (FCC-ee). Other important criteria for these proposals include the total facility construction cost, the required AC wall plug power and technical readiness [6].

The most critical requirement for the energy frontier (EF) colliders is the center-of-mass energy reach. Four proposals are under serious consideration: 3 TeV CLIC, energy upgrade of the LHC to 28 TeV (HE-LHC), 6 to 14 TeV Muon Collider and FCC-hh/SppC. Other criteria for the EF machines are (in order): cost, facility's AC wall plug power and the scale duration of the R&D effort needed to bring the concept to the level of construction readiness (e.g., the level of comprehensive TDR, Technical Design Report). Muon collider has a potential of having the lowest construction cost and the lowest AC site power requirement among all. Such prospects justified a very active accelerator R&D on the Muon Collider over the past two decades and call for more investment over the next 15-20 years to prove the feasibility of such facility's construction, performance and cost.

![Figure 1](image_url). Schematics of a 4 TeV Muon Collider on the 6x7 km FNAL site.
2 Muon Colliders

2.1 Basic principles of muon colliders

The lifetime of the muon, 2.2 μs in the muon rest frame, is sufficiently long to allow fast acceleration to high energy before the muon decays into an electron, a muon-type neutrino and an electron-type antineutrino ($\mu^- \rightarrow e^- \nu_\mu \overline{\nu}_e$) and then to last for some $300 \times B$ turns in a ring with average field $B$ (Tesla). The muon to electron mass ratio of 207 implies that all synchrotron radiation and beamstrahlung effects are smaller by a factor of about $(m_\mu/m_e)^4 = (207)^4 = 2 \times 10^9$, i.e., essentially negligible, and even a multi-TeV $\mu^+ \mu^-$ collider – see Fig.1 - can be very power efficient because of repetitive acceleration in circular machines (while light electron would powerfully radiate and lose energy) and, therefore, could have quite compact geometry that will fit on existing accelerator sites or tunnels. The center-of-mass energy spread for 3 to 14 TeV $\mu^+ \mu^-$ colliders is very small $dE/E \sim 0.01\%$ - that is an order of magnitude smaller than for an $e^+e^-$ collider of the same energy as shown in Fig.2. The obvious advantage in colliding muons rather than protons is that the muon collider center of mass energy $\sqrt{s}$ is entirely available to produce short-distance reactions rather than being spread among the proton constituents and, e.g., a 14 TeV muon collider with sufficient luminosity might be very effective as a direct exploration machine, with a physics potential similar to that of a 100 TeV proton-proton collider - see Fig.3 from [7].

![Figure 2](image-url). Comparison of the spread of center-of-mass (com) energies for 3 TeV $\mu^+ \mu^-$ collider and 3 TeV $e^+e^-$ collider (CLIC).
In general, the muon colliders are expected to be significantly less expensive than other energy frontier hadron or e+e- machines [8], they need lower AC wall plug power [9, 10] and, due to compact size a smaller number of elements requiring high reliability and individual control for effective operation [11]. In addition, a \( \mu^+ \mu^- \) Higgs factory would have advantages of large Higgs production cross-section via the s-channel production, and of the beam energy of only one half of the e+e- case (i.e., \( 2 \times 63 \text{ GeV} \) for \( \mu^+ \mu^- \rightarrow H_0 \) and, therefore, a small footprint, very small energy spread in non-radiating muon beams \( O(3 \text{ MeV}) \) and low total site power \( \sim 200\text{MW} \) [12, 13]. Finally, a neutrino factory could potentially be realized in the course of construction [14, 15, 10].

Muon colliders were proposed by F.Tikhonin and G.Budker at the end of 1960's [16-18] and conceptually developed later by a number of authors and collaborations (see comprehensive list of references in Refs. [15, 10]). A possible layout of a multi-TeV center-of-mass energy high luminosity \( O(10^{34} \text{ cm}^2\text{s}^{-1}) \) muon collider is shown in Fig.1 and consists of a high power proton driver (SRF 8 GeV 2-4 MW \( H^- \) linac); pre-target accumulation and compressor rings where very high intensity 1-3 ns long proton bunches are formed; a liquid mercury target for converting the proton beam into a tertiary muon beam with energy of about 200 MeV; a multi-stage ionization cooling section that reduces the transverse and longitudinal emittances and creates a low emittance beam; a multistage acceleration (initial and main) system – the latter employing recirculating linear
accelerators (RLA) to accelerate muons in a modest number of turns up to 2 TeV using superconducting RF technology; and, finally, a roughly 2-km diameter collider ring located some 100 meters underground where counter-propagating muon beams are stored and collide over the roughly 1000-2000 turns corresponding to the muon lifetime.

Table 1 presents major parameters of three variants of $\mu^+ \mu^-$ colliders: Higgs factory [19,10], 3 TeV collider [20] and recently proposed 14 TeV c.m.e. collider in the LHC tunnel – see also Fig.4 from [21].

Table 1. The parameters of the low-energy and high-energy Muon Colliders.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Higgs Factory</th>
<th>3 TeV</th>
<th>$\mu^+ \mu^-$ LHC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center-of-mass energy [TeV]</td>
<td>0.126</td>
<td>3</td>
<td>14</td>
</tr>
<tr>
<td>Luminosity [cm$^{-2}$s$^{-1}$]</td>
<td>0.008$\cdot$10$^{34}$</td>
<td>4.4$\cdot$10$^{34}$</td>
<td>33$\cdot$10$^{34}$</td>
</tr>
<tr>
<td>Number of bunches</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Muons/bunch [$10^{12}$]</td>
<td>4</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Circumference [km]</td>
<td>0.3</td>
<td>4.5</td>
<td>26.7</td>
</tr>
<tr>
<td>Focusing at IP $\beta^* / \sigma_z$ [mm]</td>
<td>17/63</td>
<td>5/5</td>
<td>1/1</td>
</tr>
<tr>
<td>Beam energy spread $dp/p$ (rms) [%]</td>
<td>0.004</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Ring depth [m]</td>
<td>~10</td>
<td>~120</td>
<td>~150</td>
</tr>
<tr>
<td>Proton driver pulse rate [Hz]</td>
<td>30</td>
<td>12</td>
<td>5</td>
</tr>
<tr>
<td>Proton driver power [MW]</td>
<td>≈4</td>
<td>≈4</td>
<td>≈1.4</td>
</tr>
<tr>
<td>Transverse emittance $\varepsilon_T$ [$\pi$ $\mu$rad]</td>
<td>300</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>Longitudinal emittance $\varepsilon_L$ [$\pi$ mmrad]</td>
<td>1</td>
<td>70</td>
<td>72</td>
</tr>
</tbody>
</table>

Figure 4. Schematic layout of a pulsed 14 TeV c.m.e. muon collider in the LHC tunnel.
2.2 Muon cooling and collider design

Since muons decay quickly, large numbers of them must be produced to operate a muon collider at high luminosity. Collection of muons from the decay of pions produced in proton-nucleus interactions results in a large initial 6D phase volume for the muons, which must be reduced (cooled) by a factor of $10^6$ for a practical collider. Without such a cooling, the luminosity reach will not exceed $O(10^{31} \text{ cm}^{-2}\text{s}^{-1})$. The technique of ionization cooling proposed in [22-24] is very fast and uniquely applicable to muons because of their minimal interaction with matter. It involves passing the muon beam through some material absorber in which the particles lose momentum essentially along the direction of motion via ionization energy loss, commonly referred to as ($dE/dx$). Both transverse and longitudinal momentum are reduced via this mechanism, but only the longitudinal momentum is then restored by reacceleration, leaving a net loss of transverse momentum (transverse cooling). The process is repeated many times to achieve a large cooling factor.

![Figure 5](image)

**Figure 5.** Ionization cooling-channel section. 200 MeV muons lose energy in lithium hydride (LiH) absorbers (blue) that is replaced when the muons are reaccelerated in the longitudinal direction in radio frequency (RF) cavities (green). The few-Tesla superconducting (SC) solenoids (red) confine the beam within the channel and radially focus the beam at the absorbers. Some representative component parameters are also shown (from [25]).

The rate of change of the normalised transverse emittance $\varepsilon$ as the beam passes through an absorber is given approximately by
\[
\frac{d\varepsilon}{dz} = -\frac{\varepsilon c^2}{E \nu^2} \left( \frac{dE}{dx} \right) + \frac{\beta c^3 (13.6 \text{MeV}/c)^2}{2 \nu^3 E m x_0}
\]

where \( \nu \) is the muon velocity, \( E \) the energy, \( X_0 \) the radiation length of the absorber and \( \beta \) the transverse betatron function at the absorber. The first term of this equation describes the cooling effect by ionization energy loss and the second describes the heating caused by multiple Coulomb scattering. The cooling effect balances the heating one, leading to small equilibrium emittance. The energy spread acquired in such process due to fluctuation of the ionization losses (Landau straggling) can be reduced by introducing a transverse variation in the absorber density or thickness (e.g., a wedge) at a location where there is dispersion \( D_{x,y} \) (a correlation between transverse position and energy). This method results in a corresponding increase of transverse phase space and represents in an exchange of longitudinal and transverse emittances and allows cooling in all dimensions – so called 6D cooling - due to fast transverse cooling [26]. The cooling effect on the emittance is balanced against stochastic multiple scattering and Landau straggling, leading to an equilibrium emittance.

![Figure 6](image)

**Figure 6.** Simulated 6D cooling path corresponding to one particular candidate MC cooling channel. The first part of the scheme (indicated by “4D Cooling”) is identical to the present baseline Neutrino Factory front-end. Dashed lines indicate approximate luminosity reach of a 3TeV MC. Simulated six-dimensional (6D) cooling path [26] corresponding to one particular candidate muon collider cooling channel.

Theoretical studies [27, 28] and numerical simulations [29] show that, assuming realistic parameters for the cooling hardware, ionization cooling can be expected to reduce the phase space volume occupied by the initial muon beam by a factor of \( 10^{5-6} \). A complete cooling channel would
consist of 20 to 30 cooling stages similar to one depicted in Fig.5, each stage yielding about a factor of 2 in 6D phase space reduction of ~200 MeV muons and at the end providing the transverse and longitudinal emittances suitable for collider application — see Fig.6.

**Figure 7.** Candidate scheme for 6D muon cooling (“FOFO snake”) which offers fast reduction of the beam longitudinal and transverse emittances for both signs of muons.

6D ionization cooling channel plays a central role in reaching high luminosity. In one of promising schemes the desired mixing of transverse and longitudinal degrees of freedom is achieved by putting the muons onto a helical trajectory in so called “FOFO-snake” [30] consisting of a series of slightly tilted solenoids as shown in Fig.7. The design simulations of the channels are usually aimed at the attainment of large enough dynamic apertures, taking into account realistic magnetic fields, RF cavities and absorbers, optimization of the B-fields in RF cavities and technological complexity. The design of the final cooling stages – indicated as stage 6 in Fig.6 - is particularly challenging as it requires very high solenoid fields (up to ~50T have been considered [26]). The final MC luminosity is proportional to this field – see Fig.8.

**Figure 8.** Dependence of the ultimate luminosity of high energy muon colliders on the maximum achievable field in short final cooling solenoid magnets.
A Recirculating Linear Accelerator (RLA) with SC RF cavities (e.g. 1.3 GHz ILC like ones) is a very attractive option for acceleration of muons from low energies in cooling sections to the energy of the experiments. It offers small lengths and low wall plug power consumption but requires small beam emittances [31]. Recently, several realistic collider ring beam optics have been designed which boasts a very good dynamic aperture for about \(dP/P\approx\pm 0.5\%\) and small momentum compaction [32, 33, 26].

Under active study are the concepts for the muon collider detectors which must operate in the presence of various backgrounds originating from muon decay [34]. Any straight section within the collider ring produces a beam of muon-decay neutrinos in the direction of the straight section. These neutrinos exit the Earth at some point, perhaps a few tens of kilometers away if the ring is deep. At the exit point, neutrino interactions within the rock create radiation at the surface. Because of the high energy of the muons, the resulting neutrinos are emitted in a narrow plane along the ring orientation \((\theta\sim m_\mu/E_\mu)\) and interact at a rate proportional to \(E_\mu^2\). While the interaction rate is low, it may accumulate into a radiologically significant dose where the beam reaches the earth’s surface. A crude formula for that radiation dose in an idealized ring can be obtained from Refs. [35, 36]:

\[
Dose \approx 0.57 \frac{N_\mu E_\mu^3}{R_x^2} \text{mSv/ year}
\]

where \(N_\mu\) is the number of muons/second (in \(10^{13}/\text{s}\)), \(E_\mu\) is in TeV and \(R_x\) is the distance from the ring to surface exit in km. \(R_x\) is 36 km for a 100m deep ring in an idealized geometry. The dose should not exceed certain level, so that would limit the maximum number of muons circulating in the collider. Besides the straightforward approach of placing the collider ring tunnel at sufficing depth, there are several mitigation ideas how to keep the neutrino radiation below the commonly accepted limit of about 1.5 mSv/yr [37], e.g. the radiation density can be reduced by about an order of magnitude by adding a vertical collider orbit variation of a few mm.

### 2.3 Progress toward a muon collider

The ionization cooling method though relatively straightforward in principle, has some practical implementation challenges such as RF breakdown suppression and attainment of high accelerating gradients in relatively low frequency normal-conducting RF cavities immersed in strong magnetic fields. The International Muon Ionization Cooling Experiment (MICE) [38, 39] at RAL (UK) – see Fig.9 - has recently demonstrated effective \(O(10\%)\) reduction of transverse emittance of initially dispersed 140 MeV/c muons passing through an ionization cooling channel cell consisting of a sequence of LiH or liquid Hydrogen absorbers within a lattice of up to 3.5 T solenoids that provide the required particle focusing [40, 41].
One of the key challenges for muon collider facilities is the need for high power multi-MW proton targets to generate intense muon beams. Proper target would consist of a free-liquid mercury jet flowing at 20m/s within the confines of a 15-20T solenoid field. This causes pions, produced by the impact of a proton beam, to be confined within the strong solenoid field so that they can be conducted into a decay channel where the muon decay products are collected. The jet is impacted by a multi-megawatt proton beam with a beam spot size of only a few mm. The resulting energy deposition within the target is equivalent to depositing the energy from hundreds of 1KW heaters in a cylinder 1cm diameter and 30cm long. This target concept has been validated by the MERIT (MERcury Intense Target) experiment at CERN [42]. The goal of this experiment was to demonstrate that the proposed target system could, in principle, sustain an intense primary proton beam of up to 4MW power. The experiment consisted of a free-flowing, 1cm diameter mercury jet with velocities up to 20m/s. A pulsed solenoid, capable of sustaining a 15T peak field for 1 second, was developed and used in the experiment. In order to fully contain the mercury jet, the magnet was constructed to provide a generous bore with a 15cm diameter and a length of 1m. Proton beam pulses of 14 and 24 GeV from the CERN PS with intensities up to $30 \times 10^{12}$ protons per pulse impacted this liquid jet. At 24 GeV, this maximal beam intensity corresponds to an energy of 115kJ in a single pulse. The mercury jet is destroyed by such intense beam pulses but only after the beam is gone. A key result of the MERIT experiment – see Fig.10 - showed that the disruption of the mercury from even the most intense proton pulses did not extend beyond 30cm, hence this portion of the dispersed mercury jet could be replaced in 15 thousandths of a second when the jet is flowing at 20m/s. Thus this result points to the possible eventual operation of the system at a repetition rate of more than 60 times a second.
Other notable accomplishments of the US Muon Accelerator Program program [19, 20] include attainment of accelerating gradients of 50 MV/m in vacuum and pressurized gas-filled NC RF cavities immersed in 3 T magnetic field at Fermilab [43, 44] – see Fig.11; also at Fermilab - rapid cycling HTS magnets achieved record field ramping rate of 12 T/s [45]; some 16-20 T small bore HTS solenoids were built at BNL - an important step toward the 30-40 T magnets needed for the final muon cooling stage [46]; the collaboration and its international partners have successfully carried out complete 6D muon ionization cooling and overall collider start-to-end simulations as well overall facility feasibility studies, shown that muon colliders can be built with the present day SC magnet and RF technologies and developed initial designs for 1.5 TeV, 3 TeV, 6 TeV and 14 TeV colliders (see Table 1 and series of articles in JINST Muon Accelerators Special Issue [48]).
The highest energy circular muon collider proposal [21] deals with pulsed 14 TeV c.m.e. 

relatively low facility construction cost is one of the main attractions of the 14 TeV μ+μ− collider – see Fig. 12. The total project cost (TPC, α β γ) of a 14 TeV μ+μ− collider is expected to be feasible because of the re-use of existing tunnels and the CERN injection complex, as well as the use of cost-efficient magnets and a very limited use of expensive SRF accelerators. It will also offer an outstanding energy efficiency (luminosity per MW of wall-plug electric power) [6].
where $\alpha, \beta, \gamma$ are technology dependent constants, fitted from previous projects (correspondingly, $\alpha \approx 2B$ for civil construction, $\beta \approx 1, 2$ or $10$ B$\$ for technical components like NC and SC magnets and SRF, respectively, and $\gamma = 2B$ for the total required site “wall-plug” power $P$), $L$ is the total length of the accelerator tunnels, $E_{cm}$ is the center of mass energy. In the $\mu+\mu$- LHC proposal the civil construction costs can be reduced by reusing the existing 27 km LHC tunnel, the 7 km SPS tunnel and the accompanying CERN infrastructure. The incremental cost to build the proposed collider in its least expensive proton source configuration with existing CERN proton complex (providing some 160 kW of beam power on muon production target) and with combined system of SC and pulsed magnets to get to $7+7 = 14$ TeV using up to 20 GeV of the SRF acceleration would be about $8.9 \pm 3$ B$. The luminosity of such collider will be $O(10^{33} \text{ cm}^{-2}\text{s}^{-1})$. A high power proton driver (2 MW 8GeV beam, some 20MW of site power) is needed in the high-luminosity collider configuration and that will cost an extra $1.8 \pm 0.6$ B$. It is interesting to note that the main cost driver of this collider – as well as many other facilities – is for the systems needed to reach the maximum energy (magnets, tunnels, RF, etc), while the cost of the subsystems which define the performance (luminosity) of the collider – such as power supplies, injectors, cooling systems, etc – is relatively small, as illustrated in Fig.13.

**Figure 13.** Illustration of the dependence of the muon collider total construction cost on the center of mass energy and luminosity.
3 Neutrino Factory and νSTORM

Any intense muon source by definition will be an intense source of neutrino’s as the muon decays into an electron, a muon-type neutrino and an electron-type antineutrino (\(\mu^- \rightarrow e^- \nu_\mu \bar{\nu}_e\)).

The Neutrino Factory (NF) concept [14] assumes that the neutrinos can be focused and pointed out in one or several directions toward distant neutrino detectors – see Fig. 14. The NF is attractive since it provides very high intensity neutrino and antineutrino beams which are exact CP conjugates - a total of \(O(10^{21})\) muon decays per year. The flavor content and energy spectrum as well as the total flux can be determined to better than 1%, which, combined with the great flexibility in neutrino energy, makes NF the ideal source for precision neutrino physics. Moreover, the beam contains equal numbers of muon and electron flavors and therefore, it is possible to directly measure the relevant cross sections, including nuclear effects, in the near detector. As a result it is widely recognized that the Neutrino Factory is the only concept that will allow an accuracy in the determination of leptonic mixing parameters that can compete with that in the quark sector [48].

![Figure 14. Schematic diagram of the Neutrino Factory accelerator facility [49]. The design is a development of a 25 GeV NF described in [48].](image-url)
The functional elements of a Neutrino Factory, illustrated schematically in Figure 15, are very much similar to those in the front end of a muon collider [25, 49], and include: i) a proton source producing a high-power (1-4 MW) multi-GeV bunched proton beam; ii) a pion production target that operates within a high-field solenoid. The solenoid confines the pions radially, guiding them into a decay channel; iii) a solenoid decay channel; iv) a system of RF cavities that captures the muons longitudinally into a bunch train, and then applies a time-dependent acceleration that increases the energy of the slower (low-energy) bunches and decreases the energy of the faster (high-energy) bunches; v) a cooling channel that uses ionization cooling to reduce the transverse phase space occupied by the beam, so that it fits within the acceptance of the first acceleration stage; vi) an acceleration scheme that accelerates the muons to the final energy of 5 – 20 GeV; vii) 5-20 GeV “racetrack” storage ring with long straight sections. Performance of the NF depends on the power of the proton driver and the degree or absence of the muon cooling.

Figure 15. (a) Neutrino Factory and (b) Muon Collider schematics.

Speaking of intense high energy proton beams - they are widely used to uncover the elusive properties of neutrinos and observe rare processes that probe physics beyond the Standard Model. The neutrino beams from high-energy proton accelerators, derived from the decays of charged \( \pi \) and \( K \)-mesons, which in turn are created from proton beams striking thick nuclear targets, have been instrumental discovery tools in particle physics. Currently, the most powerful accelerators for the neutrino research are under operation at Fermilab (Batavia, IL, USA) and at J-
PARC (Tokai, Japan) with proton beam energies and average power levels of 120 GeV and 0.75 MW and 30 GeV and 0.5 MW, correspondingly [50].

The Neutrinos from Stored Muons (νSTORM) facility might serve as the first step towards the Neutrino Factory concept [51]. Its option proposed for CERN offers a definitive neutrino-nucleus scattering programme using beams of electron and muon antineutrinos from the decay of muons confined within a storage ring [52]. Protons from the CERN's SPS are sent onto a target and converted into muons. The $\mu^+/\mu^-$-beams with a central momentum of between 1 GeV/c and 6 GeV/c and a momentum spread of 16% are injected and stored in a racetrack-shaped ring, oriented toward neutrino near-detector some 50 m away and a far-detector 2 km away - see Fig.16. At νSTORM, the flavour composition of the beam and the neutrino-energy spectrum are both precisely known and the storage-ring instrumentation will allow the neutrino flux to be determined to a precision of 1% or better.

![Diagram of the νSTORM accelerators](image)

**Figure 16.** Scheme of the νSTORM accelerators: proton beam from the CERN SPS (or other high power proton accelerator) hits the target, the resulting muons are collected and injected into a 585 m racetrack storage ring where they circulate for hundred turns decaying into the very well directed neutrino beam.

The facility does not call for the record power out of the SPS - it requires only 156kW of 100 GeV protons ($4 \times 10^{13}$ protons per pulse in two 10 μs fast extractions with $T_{\text{cycle}}=3.6$ s. The major challenges of the proposal are the necessity to have a large diameter (0.5 m) magnets to accept most of the secondary muons and a sophisticated focusing lattice which should assure survival of about 60% of muons after 100 turns with the 10%rms beam momentum spread. Cost estimate for such a facility if built at CERN is 160 MCHF.

## 4 Advanced Muon Production and Acceleration Schemes

### 4.1 Low emittance muon production scheme

Recently, a scenario that uses resonant production of $\mu^+\mu^-$ pairs at threshold from $e^+e^-$ collisions has been developed [53 - 55]. This has the advantage of producing $\mu^+\mu^-$ with very small emittances; it has the disadvantage of low production rate. The primary engine for such low
emittance source (LEMMA) is a ~45 GeV positron storage ring. Bunches of positrons collide with an electron target (within a material slab) producing $\mu^+$ and $\mu^-$ at threshold, each with ~22 GeV/c momentum. A small transverse-momentum at threshold production and small spot creates $\mu$ beams with small emittance. With a slab target of 0.3mm Berillium, $\sim 10^{-7}$ muon pairs per $e^+$ bunch pass are obtained. A 6.3km circumference ring with 100 bunches of $3 \times 10^{11}$ $e^+$ feeds 63m circumference 22 GeV $\mu$ rings, which accumulate muons for ~2500 turns, obtaining bunches of $\sim 4.5 \times 10^{7}$ muons at ~2200Hz ($10^{11} \mu$/s). The $e^+$ storage ring in this scenario is quite challenging. At $3 \times 10^{13}$ $e^+$ stored, it produces ~140MW of synchrotron radiation. An $e^+$ lifetime of ~250 turns implies a beam source of 40 MW of 45 GeV $e^+$ is required and $\sim 5 \times 10^{15}$ $e^+/s$, much larger than that readily available from modern day positron sources. Beam dynamics simulation [56] showed lifetimes of only ~40 turns in a model lattice; this would then require $\sim 3 \times 10^{16}$ $e^+/s$ (250 MW).

The scenario accumulates muons at 22 GeV, and therefore has a natural cycle time of ~0.45 ms (~2.2kHz). Initial analysis of such a scheme for a 14 TeV cme muon collider has shown its limited luminosity reach and significant cost (indeed, even if the SPS tunnel is reused, a new 45 GeV positron ring with 120 MW of SR power requiring 1 GV of SRF with some 250MW of total wall plug power will be expensive) [21]. Of course, the advantage of the LEMMA concept is low muon intensity and, therefore, no neutrino radiation concerns.

4.2 Linear Crystal Muon Collider

Electromagnetic wakefield waves in an ionized plasma media, excited by short relativistic bunches of charged particles or by short high power laser pulses, have been of great interest due to the promise to offer extremely high acceleration gradients of $G$ (max. gradient) = $m_e c \omega_p/e \approx 96 \times n_0^{1/2}$ [V/m], where $\omega_p = (4\pi n_p e^2/m_e)^{1/2}$ is the electron plasma frequency and $n_p$ is the ambient plasma density of [cm$^3$], $m_e$ and $e$ are the electron mass and charge, respectively, and $c$ is the speed of light in vacuum [57]. A practically obtainable plasma density ($n_p$) in ionized gas is about $\sim 10^{18}$ cm$^3$ and plasma wakefield accelerating gradients up to $\sim 100$ GV/m are demonstrated, see references in [6]. The ultimate density of charge carriers (conduction electrons) is in solids $n_0 = \sim 10^{20} - 10^{23}$ cm$^3$ and it is significantly higher than in the gaseous plasma, and correspondingly the wakefield strength of conduction electrons in solids, if excited, can possibly reach 10 TV/m.

Wakefield acceleration of muons (instead of electrons or hadrons) channeling between the planes in crystals [58, 59] or inside nanostructures like carbon nanotubes (CNT) [60] allows to envision a compact 1 PeV linear crystal muon collider [61]. The choice of muons is beneficial because of small scattering on the solid media electrons, absence of the beamstrahlung effects at the IP and continuous focusing while channeling in crystals, i.e., the acceleration to the final energy can be done in a single stage. Muon decay becomes practically irrelevant in such very fast acceleration gradients as the muon lifetime quickly grows with energy as $2.2 \mu s \times \gamma$. Initial luminosity analysis of such machine assumes small number of muons per bunch $\sim 10^3$, small number of bunches $\sim 100$, high repetition rate $\sim$1 MHz and ultimately small sizes and overlap of the colliding beams $\sim 1$ Angstrom. Excitation of the plasma wakefields in crystals or/and
nanostructures can be possible by either short sub-\(\mu\)m high density bunches of charged particles or X-ray laser pulses [62]; by heavy high-Z ions or by pre-modulated or self-modulated very high current bunches [63]. The concept of acceleration in the crystal or CNT plasma needs proof-of-principle demonstration, extensive theoretical analysis, modeling and simulations [64].

![Figure 17. Concept of a linear X-ray crystal muon collider.](image)

5 Conclusions

Muons are unique particles which offer unmatched promise for the future directions of particle physics. On one hand muons are point-like leptons and their collisions at ultrahigh energies will outperform those of hadrons due to negligible spread of the center-of-mass energies of the constituents. That gives an enormous energy advantage to muon colliders and, e.g., a 14 TeV cme muon collider will have the energy reach equivalent to that of a 100 TeV proton-proton collider. The 14 TeV muon collider can be of the size of the LHC or less while the 100 TeV \(pp\) collider needs a 100 km or longer tunnel. Correspondingly, the cost of the energy frontier muon collider is expected to be 2.5-3 times lower than for the hadron collider. The cost estimates for the latter one are widely discussed now and the publicly available cost estimates raise serious doubts in its feasibility. Another alternative is to collide electrons and positrons. The problem with them is that light leptons radiate in EM fields and the only possible way to very high energies is linear acceleration (i.e. linear colliders). Disadvantage of linear colliders is that the beams are spent after the collisions (instead of being re-used in the circular colliders) and in order to achieve high luminosity one need high beam powers, or, equivalently, high AC wall-plug power. As the result, the energy efficiency of linear colliders – e.g., in the units of (integrated luminosity/MW of the accelerator facility wall-plug power) - is very low compared to that of circular ones. Also, the cost of accelerating elements such as SRF or NC RF cavities, is significantly higher than that of bending and focusing magnets and efficient re-use of them (sequential acceleration over many turns) is critical, especially when very high multi-TeV beam energies are needed. Of course, being 207
times heavier than electrons, muons do lose energy being bent by magnets and they do not suffer from the detrimental beamstrahlung effects due radiation in the process of collision itself.

All in all, muons look like the only particles of choice for future high energy physics – they are heavy leptons, they can be accelerated to previously unmatched energies at still reasonable cost, e.g, that of the LHC. Such an advantage is probably impossible not to use, despite many serious, but lower-level, performance related concerns. The latter are many – muons are unstable particles, they need to be created, collected, cooled and accelerated to the energy of the experiments – and all that quickly, several ms for the highest energy colliders. Then they decay while circulating in the collider ring for ~1000 turns and essentially deposit the energy of decay products in the nearby environment (magnets, vacuum chambers, tunnels) and in the remote areas due to neutrino radiation. All these issues are well understood and a number of feasible solutions offered over the past two decades of active exploration of the muon collider concept. Moreover, many solutions have been already demonstrated, with the most sought one being the demonstration of the ionization cooling of muons at RAL (MICE experiment).

In order to proceed to practical realization of the muon colliders, these issues can be addressed in a staged approach, in which each step naturally offers particle physics breakthroughs. The stages may include low energy single beam νSTORM facility, then a Neutrino Factory, then a 126 GeV cme μ+ μ− Higgs factory followed by multi-TeV (say, 14 TeV) energy frontier muon collider. In far future, one can also envision a PeV scale linear muon collider based on acceleration in crystals or nanostructures (CNTs).

6 Further Reading

The above overview only scratches a surface of the concepts and issues regarding muon accelerators for high energy particle physics research. For those interested in more comprehensive knowledge of the history, methods, technologies, applications, successes and future potential of the muon accelerators many excellent reviews are available, such as Refs. [12, 14, 15, 19, 26, 47, 48, 61, 65].

Acknowledgements

I would like to thank my many collaborators whom I worked with on and discussed the variety of issues related to muon accelerators and colliders, including the members of the NFMCC (Neutrino Factory and Muon Collider Collaboration), the IDS-NF (International Design Study – Neutrino Factory), Fermilab’s MCTF (Muon Collider Task Force) and the European/CERN MCWG (Muon Collider Working Group). Fermi National Accelerator Laboratory which is operated by Fermi Research Alliance, LLC under Contract No. DE-AC02-07CH11359 with the United States Department of Energy.
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