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The masses of the neutrinos

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Abstract

If the cosmological limits on the sum of the neutrino masses are taken seriously we have first measurements of the masses of the neutrinos. Using the Planck experiment's limit $\sum_{i=1}^{3} m_i < 230\ meV$ and some simple assumptions on measurement uncertainties the mass of the heaviest neutrino is $63 \pm 11\ meV$ and the lightest $40 \pm 18\ meV$ for either hierarchy.

A recent seminar at Fermilab reviewed the status of the sum of the masses of neutrinos from various cosmological measurements [1]. If those limits are taken seriously then we already have measurements of the masses of the three neutrinos with some precision. This note reduces the simple algebra of the neutrino masses to graphical form to illustrate this.

The cosmological limits on the sum of neutrino masses is a long and growing list of measurements [2]. I have chosen to use the latest result of the Planck collaboration [3], M = $\sum_{i=1}^{3} m_i < 230\ meV$, to work in units of meV since all the interesting numbers are cleanly represented to appropriate precision as integers, and to use the PDG's averages for the neutrino mass-squared differences:

$$\Delta m_{12}^2 = m_2^2 - m_1^2 = 75.3 \pm 1.8\ meV^2$$
$$\Delta m_{32}^2 = m_3^2 - m_2^2 = 2420 \pm 60\ meV^2\ Normal\ hierarchy$$
$$-\Delta m_{32}^2 = m_2^2 - m_3^2 = 2490 \pm 60\ meV^2\ Inverted\ hierarchy$$
$$M = \sum_{i=1}^{3} m_i = m_1 + m_2 + m_3$$

The dependence of the masses of the three neutrinos as a function of M, the sum of the masses, are shown as the colored curves of figure 1. These curves illustrate the well known results that the minimum mass of the heaviest neutrino is 50 meV for either hierarchy and the minimum sum of the neutrino masses are 59 meV and 98 meV for the Normal and Inverted hierarchies respectively. The bounds on M are shown as dashed lines on figure 1: the lower bound from the non-negativity of the lightest neutrino mass, and the upper bound from the Planck experimental limit.

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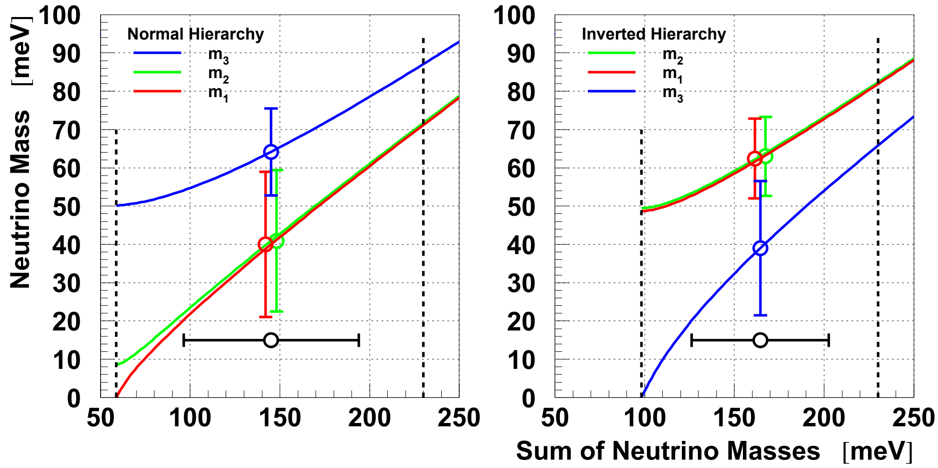

Figure 1: Masses of the neutrinos as a function of the sum of the neutrino masses.

The Planck limit is a $95\%CL$ upper bound; there is a 5% probability that the physical value is higher. Ignoring that 5% the probability distribution for M is taken to be a box with zero probability beyond the limits and a constant between them representing our lack of any prior knowledge of the value of M within the limits. Those who have used wire chambers know by rote that the mean and standard deviation of a box distribution of width w are $< w >= w/2$ and $\sigma = w/\sqrt{12}$ respectively. Applying these to the normal and inverted hierarchy bounds yield measurements of $M = 145 \pm 49\ meV$ and $M = 165 \pm 38\ meV$ respectively. These values are shown as the point with horizontal error bars on figure 1. Propagating those uncertainties to the individual neutrino masses yields the three colored points with error bars in the figure which are tabulated in table 1.

The short summary is that the heaviest neutrino is $63 \pm 11\ meV$ and the lightest $40 \pm 18\ meV$ independent of hierarchy with differences small compared to the uncertainties. The error bars are slightly asymmetric due to the curvature of the mass curves, again with small differences compared to the uncertainties. As figure 1 shows the three neutrino masses measured in this way are completely correlated: if M goes down all three masses m_i go down together.

The Planck collaboration has published the profile likelihood distribution for their previous limit of $M < 260\ meV$ [4]. That result looks quite

	Normal	Inverted
m_1	$40^{+18}_{-20}\ meV$	$62^{+11}_{-10}\ meV$
m_2	$41^{+18}_{-19}\ meV$	$63^{+11}_{-10}\ meV$
m_3	$64^{+13}_{-10}\ meV$	$39^{+16}_{-19}\ meV$
$M = \sum_{i=1}^{3} m_i$	$145 \pm 49\ meV$	$164 \pm 38\ meV$

Table 1: Computed mass values and uncertainties assuming the sum of masses shown

Gaussian ($\Delta\chi^2$ is parabolic) however $M \sim -50 \pm 155\ meV$. They apply the Feldman-Cousins prescription to evaluate their limit based on the constraint that $M > 0$. This limit is physically too low, given what we know about neutrino oscillations, and the resulting PDF after the Feldman-Cousins prescription is not available.

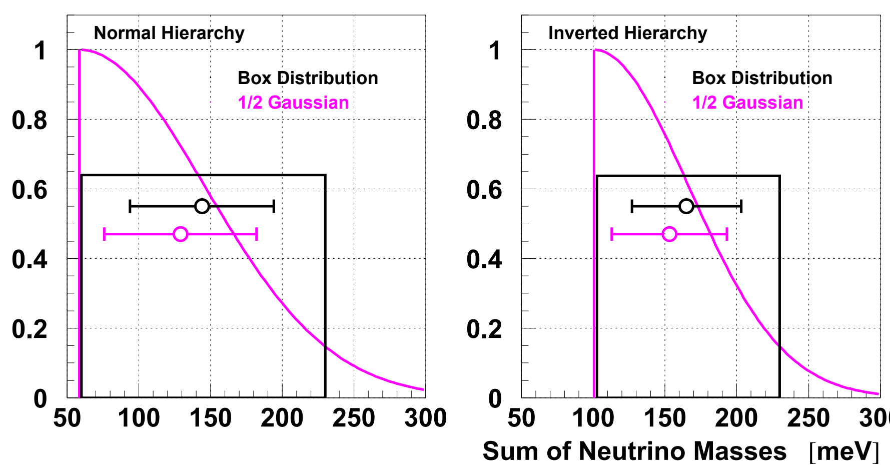

Figure 2: PDFs for the sum of the neutrino masses.

In order to assess the effect of the 5% probability that $M > 230\ meV$ a toy probability distribution, based on a Gaussian distribution function, is constructed, following what Planck has reported. A normalized Gaussian has two parameters: one must be fixed with the other determined by requiring a 5% tail above 230 meV. Setting the mean of the Gaussian to the lower bound this is called a "1/2 Gaussian" distribution. A more positive mean leads to what these distribution functions will begin to look like when Planck is able to make measurements rather that just set limits. These distribution

	limits	Normal	limits	Inverted
Box	$59 - 230$	$149 \pm 49\ meV$	$98 - 230$	$164 \pm 38\ meV$
1/2 Gaussian	$59 - 230$	$122 \pm 43\ meV$	$98 - 230$	$148 \pm 33\ meV$
1/2 Gaussian	$59 - \infty$	$129 \pm 52\ meV$	$98 - \infty$	$153 \pm 40\ meV$

Table 2: sum of masses mean and standard deviation for different distribution choices.

functions are shown in figure 2 with their means and standard deviations tabulated in table 2. The standard deviations for the 1/2 Gaussians increase $\sim 20\%$ when evaluated out to $+\infty$ relative to just the bounded region. However, those larger values are well within 10% of the standard deviation of the box distributions. For a first measurement of the heaviest and lightest neutrino masses, with 20% and 50% precision respectively, uncertainties that are known to 10% of themselves will be taken as good enough.

A most important consideration is the subjunctive with which this paper began: *If the cosmological limits on the sum of the neutrino masses are taken seriously.* If these limits get down to $< 100\ meV$, and are correct, then the Inverted hierarchy is ruled out and the neutrino masses would be measured with lower values and precisions of a few percent for the heaviest state. These would be very important results whose reliability can, and will, be vigorously questioned. The connection between the observables in experiments like Planck and the sum of the neutrino masses includes Physics modeling which is subject to systematic uncertainties. Critical evaluation of those systematics are serious questions which must be left to, and defended by, the experts in those models and experiments. Improvements to the $< 100\ meV$ level in direct measurements of a neutrino mass; the endpoint in Tritium beta decay, for example, will require advancements in experimental techniques which are, thus far, un-achieved. People are trying [5].

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References

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[5] N. S. Oblath, *Project 8: Using Radio-Frequency Techniques to Measure Neutrino Mass*, arXiv:1310.0397v1.