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FERMILAB-FN-615

Beam Loading Due to the Fundamental and Higher Parasitic Modes for a Partially Filled Main Ring

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November 1993

Operated by Universities Research Association Inc. under Contract No. DE-AC02-76CHO3000 with the United States Department of Energy

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BEAM LOADING DUE TO THE FUNDAMENTAL AND HIGHER PARASITIC MODES FOR A PARTIALLY FILLED MAIN RING

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Here, we present a qualitative study of beam loading effects for a partially filled Main Ring, where individual bunches are interacting with their environment via wake fields generated by the fundamental and higher parasitic cavity modes. We employ previously derived[1] simple analytic formula, which describes the beam loading force acting on a given bunch within the train, as a function of the resonant frequency, ω_r, and the quality factor of the coupling impedance, Q, (a single Lorentzian peak). The Main Ring coupling impedance – measured for individual r.f. cavities[2] – is defined as a superposition of many Lorentzian peaks, which makes our analytic formula especially suitable to calculate the net beam loading force experienced by each bunch. The resulting formula reveals resonant frequency regions in the vicinity of the integer multiples of the r.f. frequency, where the beam loading response is equal for all bunches (its absolute value scales as the total number of bunches in the train, M). It also identifies the second set of characteristic resonant frequencies, spaced by the multiples of ω_{rf}/M, at which the beam loading force is not only bunch independent, but it is also suppressed by the factor of Q^{-2}. This formalism gives one an insight into various optimizing schemes; e.g. to modify the detuning of the fundamental mode, or the existing configuration of the parasitic cavity resonances, so that the resulting beam loading force experienced by each bunch is minimized and it is of similar magnitude for all bunches.

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1. INTRODUCTION – MAIN RING IMPEDANCE

A fully populated storage ring is rarely the case for an operational mode of a realistic synchrotron. As a beam is injected and extracted to and from a storage ring there is usually a gap of missing bunches to accommodate injection/extraction devices (even a small gap breaks down the symmetry of a coupled multi-bunch motion). Here, we will discuss an extremely non-symmetric situation, often encountered in high energy colliders, where a relatively short train of bunches is being accelerated in a long storage ring.

We present a quantitative treatment of the beam loading effects in the Main Ring for a typical configuration of M =11 consecutive full buckets in a storage ring of the harmonic number N = 1113. The net beam loading force experienced by each bunch[1] is calculated for a realistic cavity impedance, derived from a measurement[2], which includes the fundamental and higher parasitic cavity modes. The fundamental part of the longitudinal coupling impedance, $Z_{\|}(\omega)$, combines the fundamental modes of 17 r.f. cavities into a single Lorentzian at $\omega_{rf} = N\omega_0$ (f_{rf} = 53 MHz), with the quality factor of Q = 2000 and the net shunt impedance of $R = 4 \times 10^6$ Ohm. Figure 1 summarizes the longitudinal coupling impedance, $Z_{\|}(\omega)$, at higher frequencies. Around the fourth harmonic of the r.f. frequency, $Z_{\|}$, is represented by a cluster of parasitic modes detected in 12 out of 17 r.f. cavities (the fourth harmonic mode was not present for the remaining 5 cavities)[2]. The resonant frequencies were measured for the individual r.f. cavities (they are collected in the table inserted into Figure 1). Each responding cavity contributes a Lorentzian at respective ω_r, with Q = 300 and $R = 3.5 \times 10^4$ Ohm. One can see in Figure 1 two additional clusters of parasitic modes at lower frequencies (around the second harmonic of the r.f. frequency) with the measured signal at least order of magnitude smaller[2] than the fourth harmonic lines. These weak parasitic modes will not be included into our model impedance.

In summary, the Main Ring coupling impedance – measured for individual r.f. cavities[2] – is defined as a superposition of many Lorentzian peaks, which makes our analytic formula especially suitable to calculate the net beam loading force experienced by each bunch. One can use the above formalism to optimize detuning of the fundamental mode and the existing configuration of higher parasitic modes to increase the stability of multi-bunch motion.

2. COUPLED MULTI-BUNCH MOTION – BEAM LOADING

We consider a storage ring of the harmonic number N, which is populated by a train of M consecutive bunches ($M \leq N$). Collective synchrotron motion (the dipole mode) of M bunches coupled via wake fields can be described by the following set of equations of motion[3]

$$\frac{\partial^2}{\partial t^2} y_n(t) + \omega_n^2\, y_n(t) = f_n - A\,\frac{1}{c} \sum_{k=-\infty}^{\infty} \sum_{m=0}^{M-1} W''\Big(-(k + \tfrac{m-n}{N})T_0\Big)\, y_m\Big(t - (k + \tfrac{m-n}{N})T_0\Big),$$

where (2.1)

$$A = \frac{N_0 r_0 \eta \omega_0}{2\pi\gamma}\ .$$

Here W' is the time derivative of the wake function, η is the revolution frequency dispersion function, c is the velocity of light, r_0 is the classical proton radius, ω_0 is the revolution frequency and T_0 is the revolution period.

One can identify the first term in the right hand side of Eq.(2.1) with the beam loading force, which drives the n-th bunch. It explicitly depends on the bunch index n and it is given by the following formula[3]

$$f_n = A \sum_{k=-\infty}^{\infty} \sum_{m=0}^{M-1} W'\Big(-(k + \tfrac{m-n}{N})T_0\Big)\ . \qquad (2.2)$$

Furthermore, the perturbed synchrotron frequency, ω_n, in Eq.(2.1) includes the incoherent tune shift correction due to the potential well distortion. The last term in the right hand side of Eq.(2.1) represents pure coherent multi-bunch coupling, which may result in the coupled bunch instability.

The time derivative of the wake function is related to the longitudinal impedance via the inverse Fourier transform as follows[3]

$$W'(t) = \frac{c}{2\pi} \int_{-\infty}^{\infty} d\omega \; e^{i\omega t} \, Z_{\|}(\omega) \quad . \tag{2.3}$$

To express the beam loading force in terms of the coupling impedance one can substitute Eq.(2.3) into Eq.(2.2). Summing over the bunch index, m, yields the following formula[1]

$$f_n = \omega_0 \frac{Ac}{2\pi} \sum_{p=-\infty}^{\infty} e^{2\pi i p \frac{n}{N}} \; Z_{\|}(p\omega_0) \; \frac{1 - e^{-2\pi i p \frac{M}{N}}}{1 - e^{-2\pi i p \frac{1}{N}}} \quad . \tag{2.4}$$

Assuming general form of the longitudinal impedance of a resonant structure, given by the following standard Lorentzian:

$$Z_{\|}(\omega) = \frac{R}{1 + iQ\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right)} \quad , \qquad Q >> 1 \quad , \tag{2.5}$$

where R is the shunt impedance, Q is the quality factor of the resonator and ω_r is its resonant frequency, one can evaluate Eq.(2.4) explicitly via contour integration technique[1]. The resulting general expression is given as follows

$$f_n = \frac{AcR\omega_o}{\pi} \left\{ \left(\frac{\pi\omega_r}{N\omega_o}\right) \delta^2 \; \frac{M\left(\frac{\pi\omega_r}{N\omega_o}\right) - \frac{1}{2} N \sin\left(\frac{2\pi\omega_r}{N\omega_o}\right)}{\sin^2\left(\frac{\pi\omega_r}{N\omega_o}\right) + \left(\frac{\pi\omega_r}{N\omega_o}\right)^2 \delta^2} \right. +$$

$$\left. - \frac{\sin\left(\frac{\pi\omega_r}{N\omega_o}\right) \sin\left(\frac{\pi\omega_r}{N\omega_o} M\right) \cos\left(\frac{\pi\omega_r}{N\omega_o}(2n - M - 1)\right)}{\sin^2\left(\frac{\pi\omega_r}{N\omega_o}\right) + \left(\frac{\pi\omega_r}{N\omega_o}\right)^2 \delta^2} \right\} \quad , \qquad \delta = \frac{1}{2Q} << 1 \quad . \tag{2.6}$$

Denoting the expression in curly bracket by $\tilde{f}_n$, one can introduce a dimensionless beam loading force. Figure 2 illustrates a family of curves for different values of n, calculated according to Eq.(2.6). As discussed in Ref.1, Eq.(2.6) has a simple asymptotics for the resonant frequencies, ω_r, in the vicinity of the integer multiples of the r.f. frequency, $kN\omega_o$, and away from them. These two asymptotic regions are determined by the relative strength of the expressions appearing in the denominator of Eq.(2.6), namely: $\sin^2(\pi x)$ and $(\pi x)^2 \delta^2$. It is convenient to introduce a dimensionless resonant frequency, x, (in units of the r.f. frequency) namely,

$$x = \frac{\omega_r}{N\omega_o} \tag{2.7}$$

Now, 'the immediate vicinity of the integer multiple of the r.f. frequency' is defined by the following inequality

$$\sin^2(\pi x) << (\pi x)^2 \delta^2, \tag{2.8}$$

which can be rewritten into the following simple form

$$\left| x - k \right| << k\delta \ . \tag{2.9}$$

The remainder of the frequency domain, namely resonant frequencies given by

$$\left| x - k \right| >> k\delta \ , \tag{2.10}$$

are considered to be 'away from the multiples of the r.f. frequency' – the inequality given by Eq.(2.8) reversed.

Applying the above asymptotics, Eqs.(2.7)–(2.10), to Eq.(2.6) (neglecting either $\sin^2(\pi x)$ or $(\pi x)^2 \delta^2$, term in the denominator) reduces Eq.(2.6) to the following simple expression

$$
\left(\tilde{f}_n\right)^{asym} = \begin{cases} M - N \dfrac{\sin(2\pi x)}{2\pi x} - 4Q^2 (-1)^{k(M+1)} \dfrac{\sin(\pi x)}{\pi x} \dfrac{\sin(\pi x M)}{\pi x} & \text{for } |x - k| << k\delta \\ -\dfrac{\sin(\pi x M)}{\sin(\pi x)} \cos\left(\pi x(2n - M - 1)\right) & \text{for } |x - k| >> k\delta \end{cases} \tag{2.11}
$$

Figure 3 illustrates a comparison between the exact formula, $\tilde{f}_n(x)$, Eq.(2.6), and its asymptotic version, given by Eq.(2.11). One can notice, that for the resonant frequencies in 'the immediate vicinity of the integer multiple of the r.f. frequency' (the first asymptotic region in Eq.(2.11)) the resulting *beam loading force does not depend on the bunch index*, n, and it is governed by the quality factor, Q. Conversely, for the resonant frequencies 'away from the immediate vicinity of the integer multiple of the r.f. frequency' (the second asymptotic region in Eq.(2.11)) the resulting *beam loading force does not depend on the quality factor*, Q, and it is governed strictly by the bunch index, n. Therefore, for parasitic modes at resonant frequencies 'away from the immediate vicinity of the integer multiple of the r.f. frequency', which is usually the case, the so called 'de-Q-ing' of the modes does not have any effect on the beam loading forces experienced by individual bunches (see Eq.(2.11)).

Furthermore, the structure of Eq.(2.11) (zeros of $\sin(\pi x M)$) reveals another finer level of symmetry governed by the fractional, $\frac{\ell}{M}$, multiples of $N\omega_o$. Indeed, as seen in Figure 4, the beam loading force vanishes up to terms of $O(\delta^2)$, for a discrete set of resonant frequencies defined by

$$
\omega_r = \left(k + \frac{\ell}{M}\right) N\omega_o , \quad \ell = 1, 2, \ldots, M - 1 . \tag{2.12}
$$

These resonant frequencies are clearly marked in Figure 4 (arrows). Similarly, one can find frequency regions where bunch-to-bunch variation of the beam loading force is the strongest – they are defined by the extremes of $\sin(\pi x M)$, which is also illustrated in Figure 4.

3. BEAM LOADING FORCE – NUMERICAL EXAMPLE

The Main Ring coupling impedance is represented as a superposition of many Lorentzian peaks (one fundamental mode plus 12 higher parasitic modes described in Figure 1),which makes our asymptotic formula, Eq.(2.11), especially suitable to calculate the net beam loading force experienced by each bunch. The fundamental mode, by definition, is right at the first harmonic, or perhaps slightly detuned to stabilize Robinson instability. Therefore, the first asymptotic region in Eq.(2.11) will apply. One can use the above formalism to optimize detuning of the fundamental mode and the existing configuration of higher parasitic modes to increase the stability of multi-bunch motion. Assuming the same R/Q ratio for the fundamental and all higher parasitic modes[2], one can express the beam loading force due to the fundamental mode (bunch index independent) by the following simple formula

$$\left(\tilde{f}\right)^{\mathrm{fund}} = \frac{17}{12} M \; \frac{Q_{\mathrm{fund}}}{Q_{\mathrm{para}}} = 52, \qquad (3.1)$$

providing that the fundamental frequency detuning, $\Delta\omega_{rf}$, is very small

$$\Delta\omega_{rf} << \frac{\omega_{rf}}{2Q} \; . \qquad (3.2)$$

Figure 5 shows a family of beam loading curves for individual bunches in the resonant frequency region, which contains all 12 parasitic cavity modes. Their resonant frequencies are marked by arrows. One can see, that the resonant frequencies of all 12 parasitic modes are 'away from the immediate vicinity of the integer multiple of the r.f. frequency', therefore the second asymptotic region in Eq.(2.11) will apply. The Q-independent beam loading force is given by the following formula

$$\left(\tilde{f}_n\right)^{\mathrm{para}} = \sum_{\ell=1}^{12} \tilde{f}_n(x_\ell) \; . \qquad (3.3)$$

Figure 6 summarizes values of $\left(\tilde{f}_n\right)^{para}$ for all 11 bunches. One can see from Figure 6, that the total bunch-to-bunch spread of the beam loading force is equal to about 14 of our dimensionless units, which is quite a substantial effect (about 27% of the beam loading force due to the fundamental mode).

4. SUMMARY

The starting simple asymptotic formula, which describes the beam loading force experienced by a given bunch, as a function of the resonant frequency, ω_r, and the quality factor, Q, of a simple Lorentzian impedance is employed to study beam loading effects for a realistic (measured) configuration of cavity resonance. As was demonstrated in this paper, one can get immediately a simple quantitative answer in terms of the beam loading experienced by each bunch along the train. Superimposing many parasitic cavity modes one can use the above formulas to choose appropriate tuning of existing configuration of parasitic modes to minimize the beam loading formalism reveals resonant frequency regions in the vicinity of the integer multiples of the r.f. frequency, $N\omega_o$, where the beam loading response is equal for all bunches (its absolute value scales as M). The complimentary asymptotic region, 'away from the immediate vicinity of the integer multiple of the r.f. frequency' is especially relevant for studying beam loading effects due to higher order parasitic cavity modes. The resulting *beam loading force does not depend on the quality factor*, Q, and it is governed strictly by the bunch index, n. The formula also identifies the second set of characteristic resonant frequencies, spaced by the multiples of $N\omega_o/M$, at which the beam loading force is not only bunch independent, but also considerably smaller (suppressed by the factor of Q^{-2}). Similarly, our analytic formula identifies frequency regions, where bunch-to-bunch variation of the beam loading force is the strongest (ω_r at odd multiples of $N\omega_o/2M$). Presented numerical example gives one an insight into various optimizing schemes; e.g. to modify the existing configuration of parasitic cavity resonances, or to change number of bunches in the train, so that the resulting bunch-to-bunch spread of the beam loading force is minimized, which could be instrumental in stabilizing multi-bunch motion.

REFERENCES

1. S.A. Bogacz, Beam Loading and Potential Well Distortion for a Partially Filled Ring, FN-611, (1993).

2. X. Lu, (private communication).

3. A.W. Chao, Physics of Collective Beam Instabilities in High Energy Accelerators, (John Wiley & Sons, Inc. 1993) Ch. IV,

ACKNOWLEDGMENT

The author wishes to thank Francois Ostiguy, Bill Ng and Pat Colestock for stimulating discussions and comments.

FIGURE CAPTIONS

Figure 1. Measured longitudinal coupling impedance, $Z_{\parallel}(\omega)$, at higher frequencies. A cluster of 12 parasitic modes around the fourth harmonic of the r.f. frequency (detected in 12 out of 17 r.f. cavities). Each cavity contributes a Lorentzian at respective ω_r, with Q = 300 and R = 3.5×10^4 Ohm. Two additional clusters of parasitic modes at lower frequencies with the measured signal at least order of magnitude smaller[2] than the fourth harmonic lines.

Figure 2. Dimensionless beam loading force, $\tilde{f}_n$, acting on the n-th bunch as a function of the resonant frequency, ω_r, of the coupling impedance.

Figure 3. Asymptotics of the dimensionless beam loading force, $\tilde{f}_n$, acting on the 0-th bunch for resonant frequencies, ω_r , at the *'immediate vicinity'* and *'away '* from the multiples of the r.f. frequency.

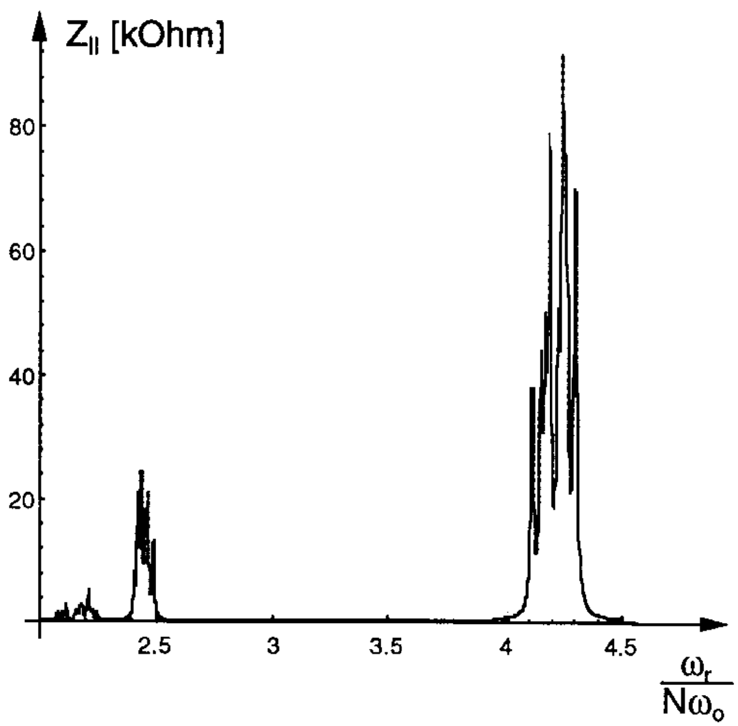
Figure 4. Dimensionless beam loading force, $\tilde{f}_n$, acting on the n-th bunch for resonant frequencies, ω_r , *'away from the immediate vicinity'* of the multiple of the r.f. frequency. The beam loading force vanishes for a discrete set of resonant frequencies defined by the fractional, $\frac{\ell}{M}$, multiples of $N\omega_o$.

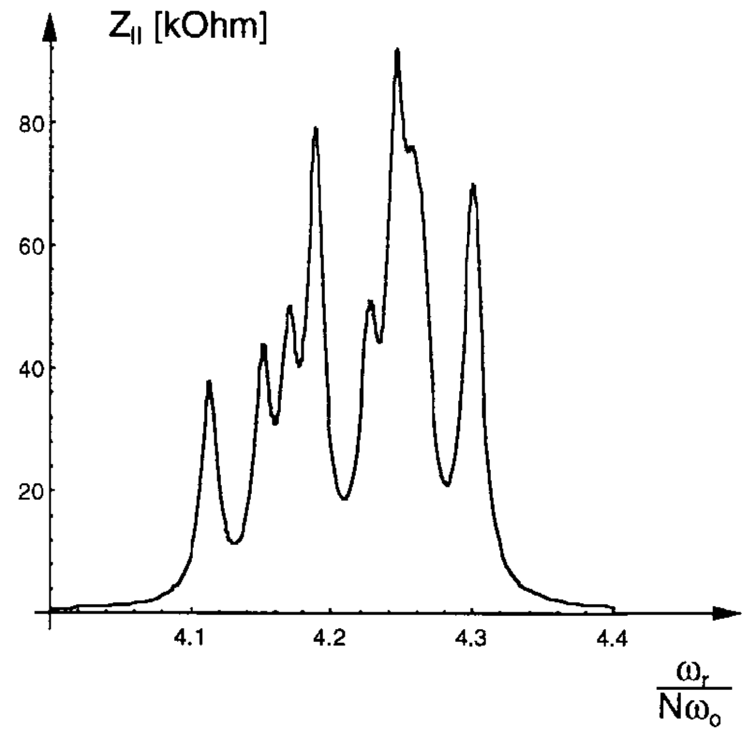
Figure 5. A family of beam loading curves for individual bunches in the resonant frequency region, containing all 12 parasitic cavity modes (marked by arrows). The resonant frequencies of all parasitic modes are *'away from the immediate vicinity of the integer multiple of the r.f. frequency'* – the Q-independent asymptotic for the beam loading force will apply.

Figure 6. The net beam loading force, $\left(\tilde{f}_n\right)^{para}$, for all 11 bunches. The total bunch-to-bunch spread of the beam loading force is equal to about 14 dimensionless units.

Main Ring Cavities (17) - Parasitic Modes

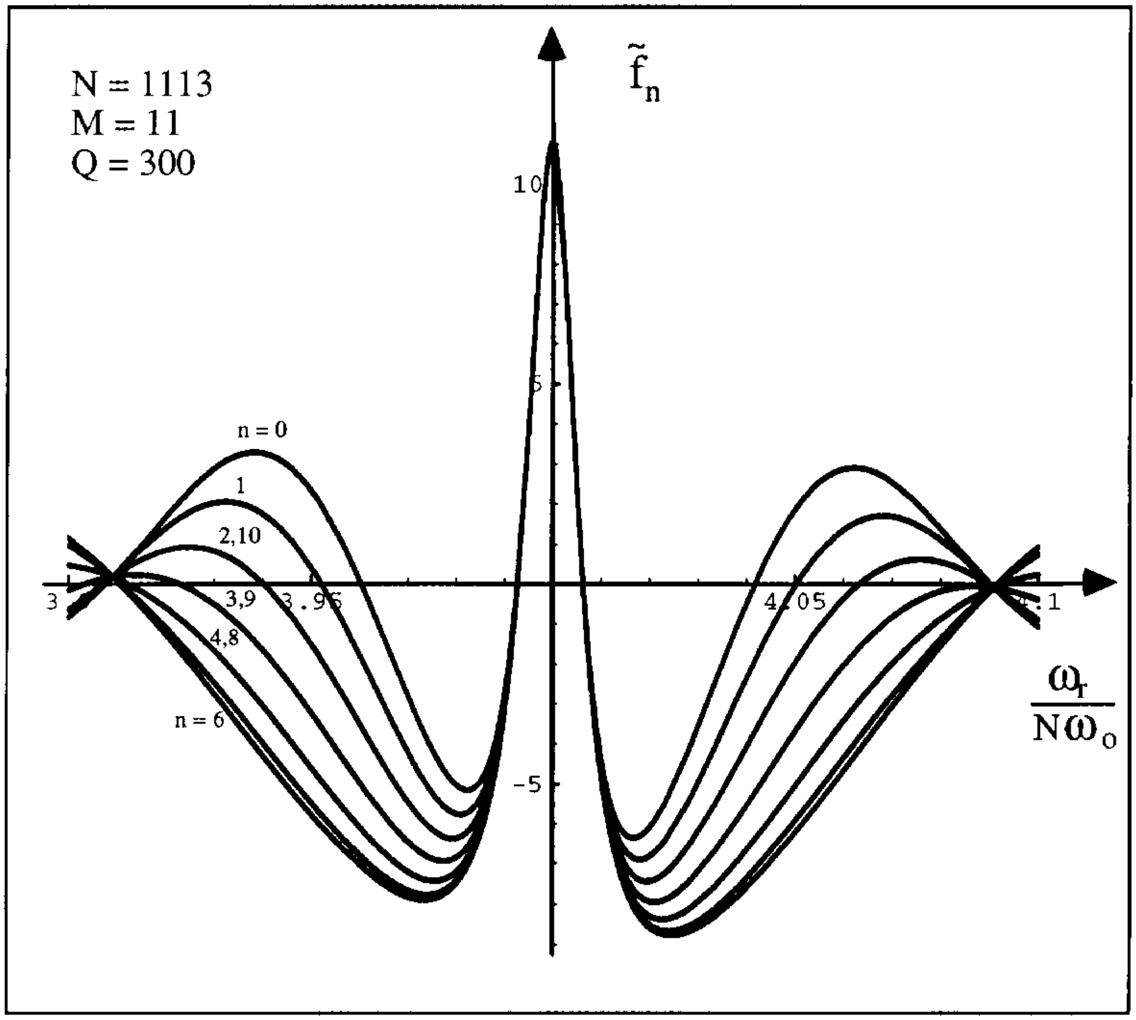
Q = 300





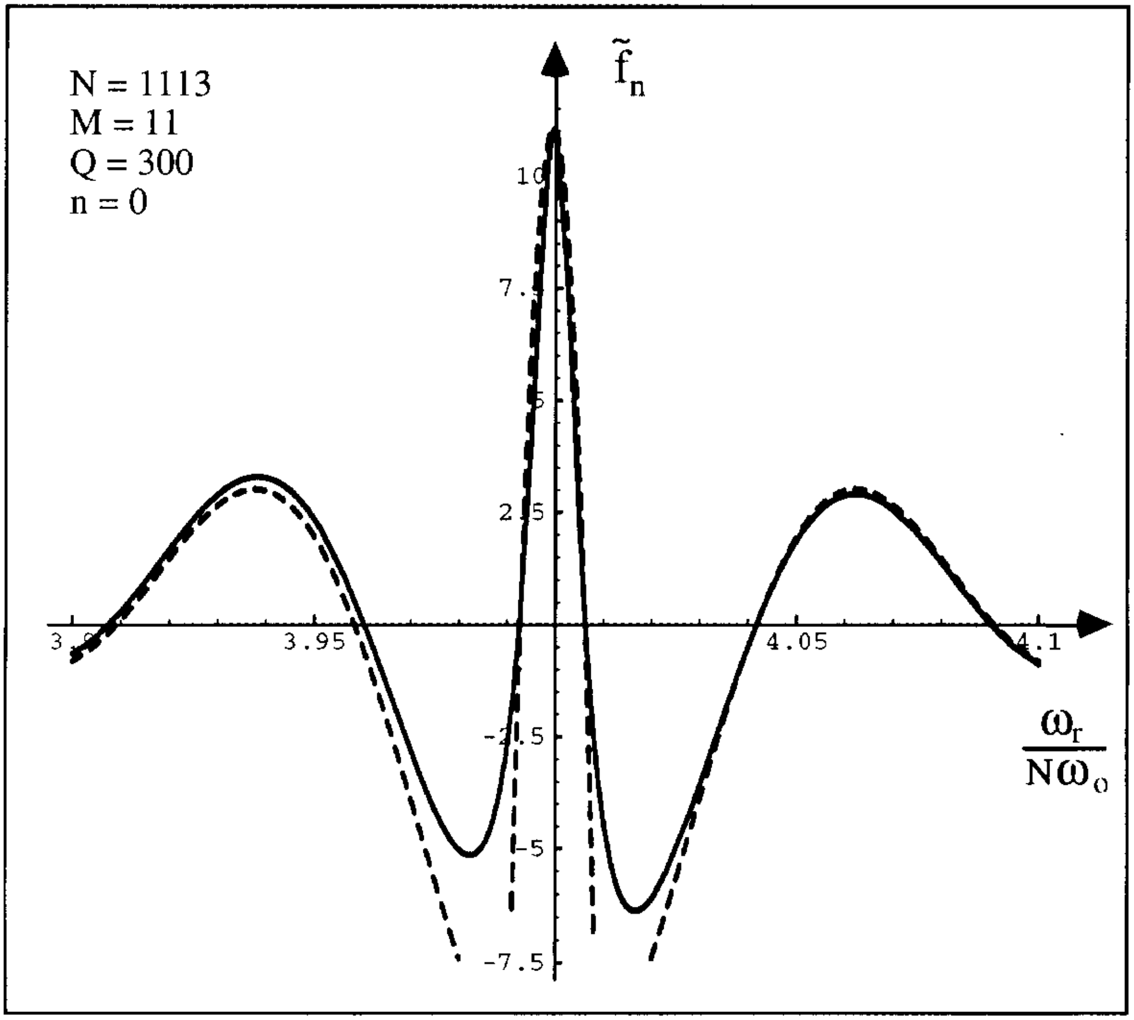
cavity number	1	2	3	4	5	6	7	8	9	10	11	12
frequency [MHz]	226	222	220	228	224	227.8	225	222	218	221	225.6	225

Figure 1



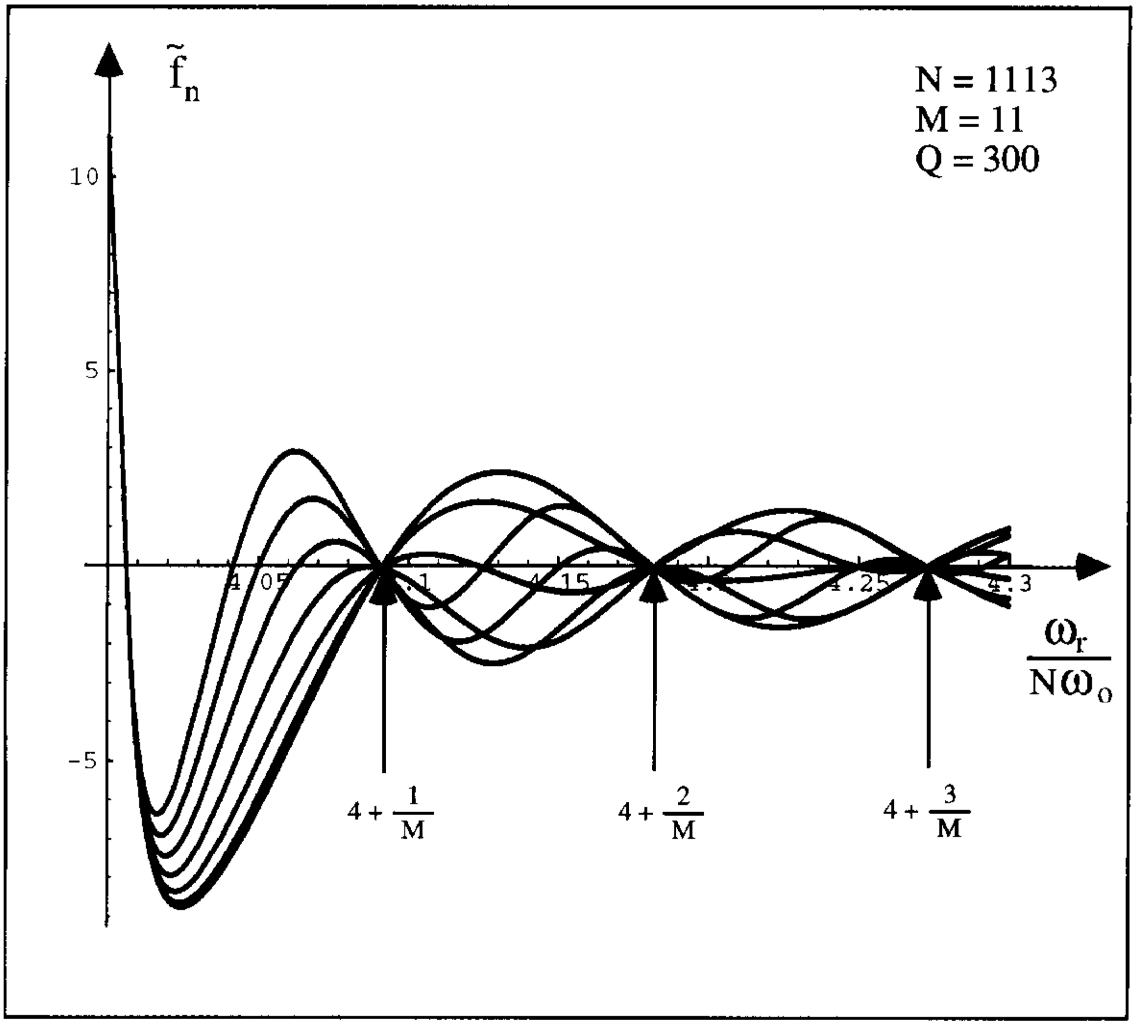
N = 1113
M = 11
Q = 300
$\tilde{f}_n$
10
5
-5
n = 0
1
2,10
3,9
4,8
n = 6
3
3.95
4.05
4.1
$\frac{\omega_r}{N\omega_o}$

Figure 2



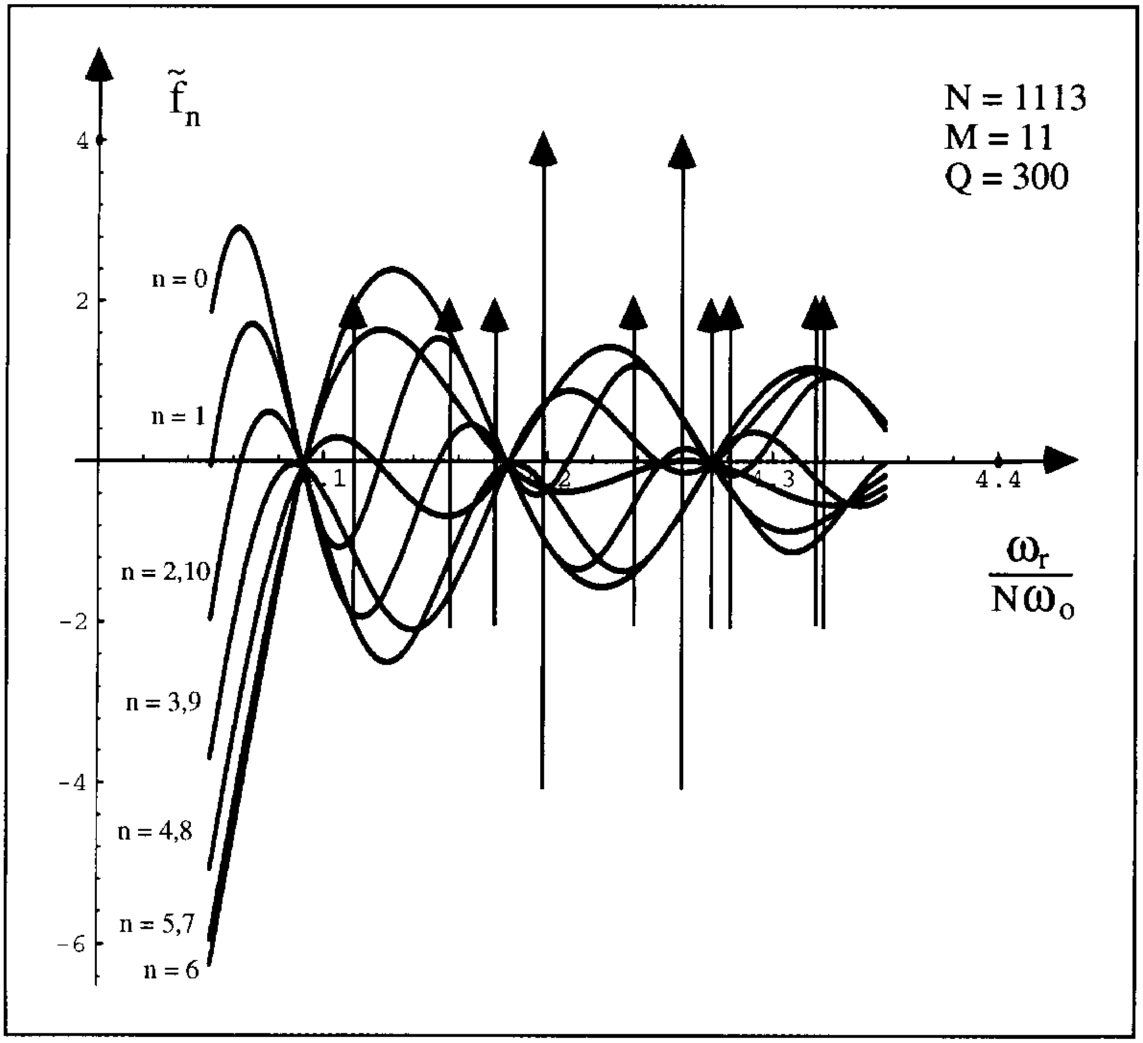
N = 1113
M = 11
Q = 300
n = 0
$\tilde{f}_n$
10
7.5
5
2.5
-2.5
-5
-7.5
3.9
3.95
4.05
4.1
$\frac{\omega_r}{N\omega_o}$

Figure 3



$\tilde{f}_n$
N = 1113
M = 11
Q = 300
10
5
-5
4.05
4.15
4.25
4.3
$\frac{\omega_r}{N\omega_o}$
$4+\frac{1}{M}$
$4+\frac{2}{M}$
$4+\frac{3}{M}$

Figure 4



$\tilde{f}_n$
N = 1113
M = 11
Q = 300
4
2
-2
-4
-6
n = 0
n = 1
n = 2,10
n = 3,9
n = 4,8
n = 5,7
n = 6
1
2
3
4.4
$\frac{\omega_r}{N\omega_o}$

Figure 5

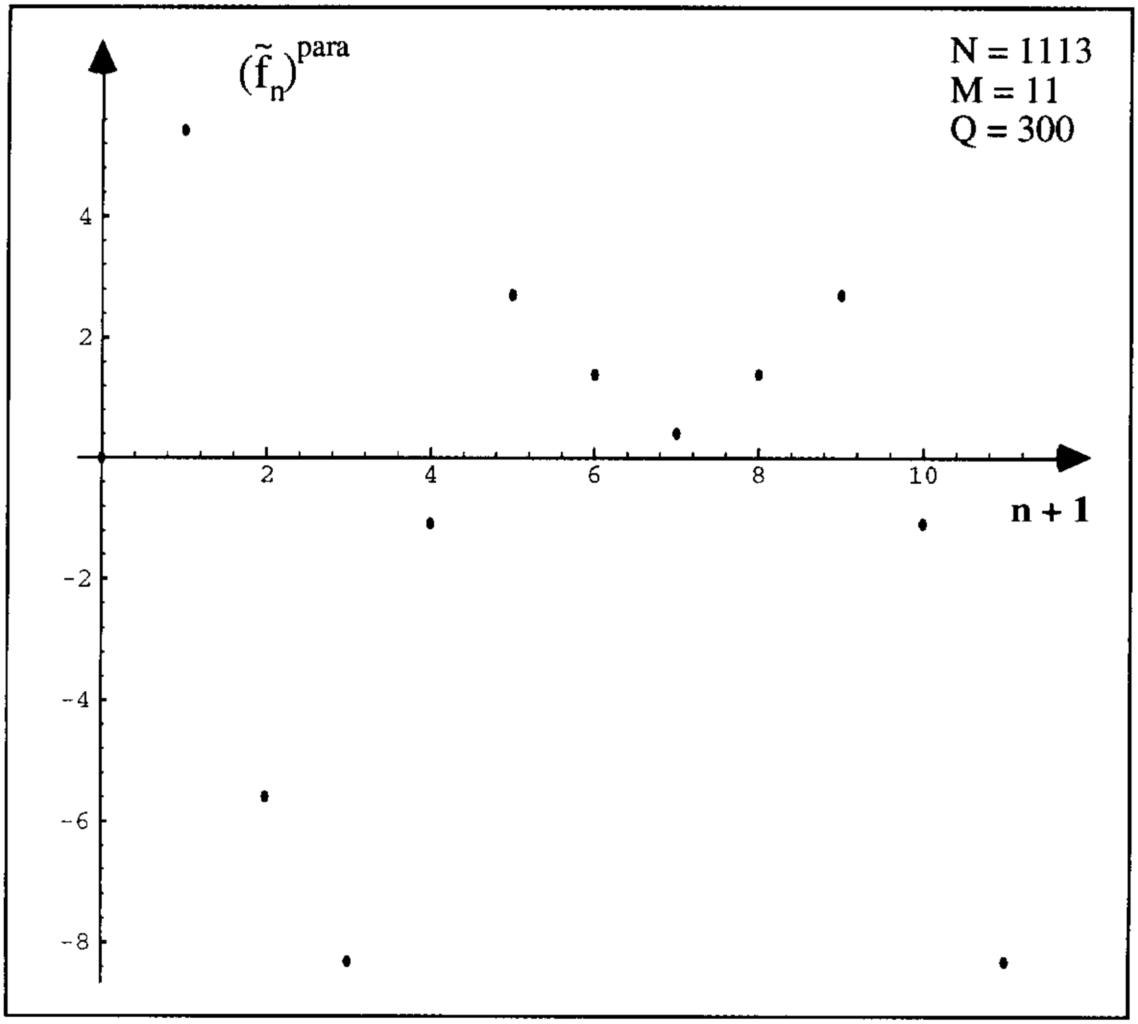


Figure 6