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# COMMENTS ON THE BEHAVIOR OF $\alpha_1$ IN MAIN INJECTOR $\gamma_t$ JUMP SCHEMES

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Tracking studies of transition crossing in the Main Injector have shown that the Johnsen effect is the dominant cause of beam loss and emittance blow up. To suppress this effect one has to have control over  $\alpha_1$  (dispersion of the momentum compaction factor  $\alpha$ ). Various  $\gamma_t$  jump configurations are examined and the resulting changes in  $\alpha_1$  are assessed. These results are further validated by comparison between the simulation and simple analytic  $\alpha_1$ -formulas derived for a model FODO lattice. The lattice assumes full chromaticity compensation and presence of eddy current sextupole component. The last scheme seems to be very promising as one can regard the strength of eddy current sextupole family as an external "knob" to control values of  $\alpha_1$ .

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## INTRODUCTION

Tracking studies of transition crossing in the Main Injector and other Fermilab accelerators, using the code ESME, have shown that the Johnsen effect is the dominant cause of beam loss and emittance blow up [1,2]. This effect is rooted in the variation of  $\gamma_t$  the transition gamma, with  $\delta = \frac{\Delta p}{p_0}$ , the off-momentum parameter. Although transition crossing may be well tuned for the nominal particle, perhaps with an "instantaneous" snap of the radio frequency phase and a  $\gamma_t$  jump scheme, it is not trivial to satisfy the needs of a momentum spectrum of particles. In general, individual off-momentum particles cross transition either too early, or too late. This leads to particle loss and longitudinal emittance growth which is independent of the intensity of the bunch under consideration. A useful parameter characterizing the strength of this effect is the Johnsen time,  $T_J$ , which represents the root mean square spread of the transition crossing time [3-6]. This time is proportional to  $\frac{\sigma_p}{p}$ , the root mean square momentum spread.

The Johnsen time is directly related to the lattice parameter  $\alpha_1$ , which is defined by the equation

$$\frac{\Delta C}{C_0} = \alpha_0 \delta + \alpha_1 \delta^2 + \dots \quad (1)$$

where  $C_0$  is the nominal closed orbit path length, and  $\Delta C$  is the increase in path length for an off momentum particle. Unfortunately, more than one definition of  $\alpha_1$  is common in the literature, as discussed in the Appendix. Let the reader beware! As defined here, the coefficients  $\alpha_0$  and  $\alpha_1$  are geometrical properties of the lattice, given by

$$\alpha_0 = \frac{2\pi}{C_0} \langle \eta_0 \rangle, \quad \alpha_1 = \frac{2\pi}{C_0} \langle \eta_1 \rangle \quad (2)$$

where angle brackets  $\langle \dots \rangle$  denote averaging weighted by bend angle. The quantities being averaged are component dispersions in a momentum expansion of the total dispersion. That is,

$$\eta(s) = \eta_0(s) + \eta_1(s) \delta + \dots \quad (3)$$

explicitly showing the dependence of the dispersion on  $s$ , the accelerator azimuth.

Transition comes for a particle with a macroscopic momentum displacement  $\Delta p$  when a neighboring trajectory, infinitesimally displaced by  $dp$ , has the same revolution frequency. If it is assumed that equation 1 is exact ( $\alpha_2$ , et cetera, are zero), then it can be shown that the exact condition for transition crossing becomes

$$\frac{1}{\gamma_t^2} = \frac{\alpha_0 + 2 \frac{\alpha_1}{\delta}}{1 + \alpha_0 \delta + \alpha_1 \delta^2} (1 + \delta) \quad (4)$$

Keeping only first order terms in  $\delta$ , this equation can be rearranged to give the  $\gamma$  of the off-momentum particle as it passes through transition,

$$\gamma_t = \gamma_{t0} \left[ 1 - \left( \frac{1}{2} + \frac{\alpha_1}{\alpha_0} - \frac{\alpha_0}{2} \right) \delta \right] \quad (5)$$

Here  $\gamma_{t0}$  is the  $\gamma$  of the nominal particle as it passes through its transition. However, a more useful quantity is the  $\gamma$  of the nominal particle at the time that the off momentum particle passes through transition. This is given by

$$\gamma_0(\delta) = \gamma_{t0} [ 1 - \delta ] \quad (6)$$

so that the Johnsen time becomes

$$T_J = \frac{\gamma_0(\frac{\sigma_p}{p})}{\dot{\gamma}} = \frac{\gamma_{t0}}{\dot{\gamma}} \left[ \frac{3}{2} + \frac{\alpha_1}{\alpha_0} - \frac{\alpha_0}{2} \right] \frac{\sigma_p}{p} \quad (7)$$

where  $\dot{\gamma}$  is the ramp rate through transition. The Main Injector, for example, has (approximately)  $\gamma_{t0} = 20$ ,  $\dot{\gamma} = 240 \text{ sec}^{-1}$ ,  $\alpha_1 = 0$ , and  $\frac{\sigma_p}{p} = 5.10^{-3}$ , so that  $T_J$  is about 0.6 milliseconds, or about 60 accelerator turns.

Clearly, if it is not possible to control the Johnsen time, it is futile to arrange for a  $\gamma_t$  jump on a time scale much faster than this. Also, analysis of how  $\alpha_1$  changes should be included as an additional topic in the evaluation of  $\gamma_t$  jump schemes, to ensure that  $T_J$  does not rise significantly. On the positive side, if it is possible to measure and control  $\alpha_1$ , then it should be possible to make  $T_J = 0$ , and ameliorate the damage done by the Johnsen effect, by setting

$$\alpha_1 = -\frac{3}{2}\alpha_0 + \frac{1}{2}\alpha_0^2 \quad (8)$$

The second term on the right is essentially negligible. The improvement in transition performance might then be dramatic enough that a  $\gamma_t$  jump would no longer be necessary. This may be true especially if  $\alpha_1$  control is combined with rf gymnastic tricks, such as the use of a synchronous phase of  $90^\circ$  and a second harmonic cavity, as now being discussed elsewhere [7].

# THE DIFFERENTIAL EQUATION FOR THE SUM DISPERSION, $\eta_+$

The horizontal closed orbit  $h(s)$  is found by solving the differential equation

$$h'' + \frac{K(s)}{1 + \delta} h + \frac{S(s)}{1 + \delta} h^2 = G(1 - \frac{1}{1 + \delta}) \quad (9)$$

with periodic boundary conditions. A prime indicates differentiation with respect to  $s$ ,  $K$  is the quadrupole strength,  $S$  is the sextupole strength, and  $G$  is the dipole bending strength. If  $h$  is expanded in a dispersion function series

$$h = x_{CO} + \eta_0 + \eta_1 \delta + \dots \quad (10)$$

which is substituted into 9, three differential equations are obtained by grouping terms according to their order in  $\delta$ , up to second order. The solution of the lowest order equation is trivial when there are no closed orbit perturbations,  $x_{CO} = 0$ , so that the remaining two equations become

$$\eta_0'' + K \eta_0 = G \quad (11a)$$

$$\eta_1'' + K \eta_1 = -G + K \eta_0 - S \eta_0^2 \quad (11b)$$

The differential equation for the convenient "sum dispersion", defined by

$$\eta_+ = \eta_0 + \eta_1 \quad (12)$$

is obtained by adding equations 11a and 11b, to give

$$\eta_+'' + K \eta_+ = K \eta_0 - S \eta_0^2 \quad (13)$$

The source terms on the right hand side of equation 13 depend only on the solution of 11a for the "normal" dispersion. Solving this differential equation is slightly easier than solving 11b for  $\eta_1$ , and a subsequent calculation of  $\langle \eta_+ \rangle$  allows for a direct knowledge of  $\alpha_1$ , since

$$\alpha_1 = \alpha_+ - \alpha_0 = \frac{2\pi}{C_0} \langle \eta_+ \rangle - \alpha_0 \quad (14)$$

In order to set the Johnsen time to zero, therefore, it is necessary to set

$$\alpha_+ = -\frac{1}{2} \alpha_0 \quad (15)$$

The next section solves equation 13 in a naive FODO representation of the Main Injector, and quantitatively identifies the major factors which affect  $\eta_+$  and hence  $\alpha_+$  and  $\alpha_1$ .

## SOLUTION IN A FODO LATTICE WITH EDDY CURRENT SEXTUPOLES

Suppose that an accelerator like the Main Injector is represented as made up purely of FODO cells, as illustrated in Figure 1. The quadrupoles are thin, and there is no drift space. All of the half cell length  $L$  is filled with two identical dipoles of bend radius  $R$ , which are separated by a thin sextupole representing the field due to vacuum chamber eddy currents, induced during the ramp. There are also two thin chromatic correction sextupoles per half cell, immediately adjacent to the focussing and defocusing quadrupoles. The strength of the (half) quadrupoles is  $\pm q$ , and of the sextupoles is  $g_F$ ,  $g_D$ , and  $g_E$ , where

$$q = \frac{s}{L} = \frac{\sin(\phi_{1/2})}{L} \quad (16a)$$

$$g_F = \frac{(S_F \Delta l) \eta_{0F}}{q}, \quad g_E = \frac{(S_E \Delta l) \eta_{0E}}{q}, \quad g_D = \frac{(S_D \Delta l) \eta_{0D}}{q} \quad (16b)$$

In these expressions  $\phi_{1/2}$  is the half cell phase advance (about 44 degrees in the Main Injector), while  $S_F$  and  $\eta_{0F}$  (for example) are the sextupole gradient and the lowest order dispersion, at the F chromatic sextupole of thin length  $\Delta l$ . From here on  $s$ , as defined in equation 16a, is the sine of the half cell phase advance, and not the azimuthal coordinate.



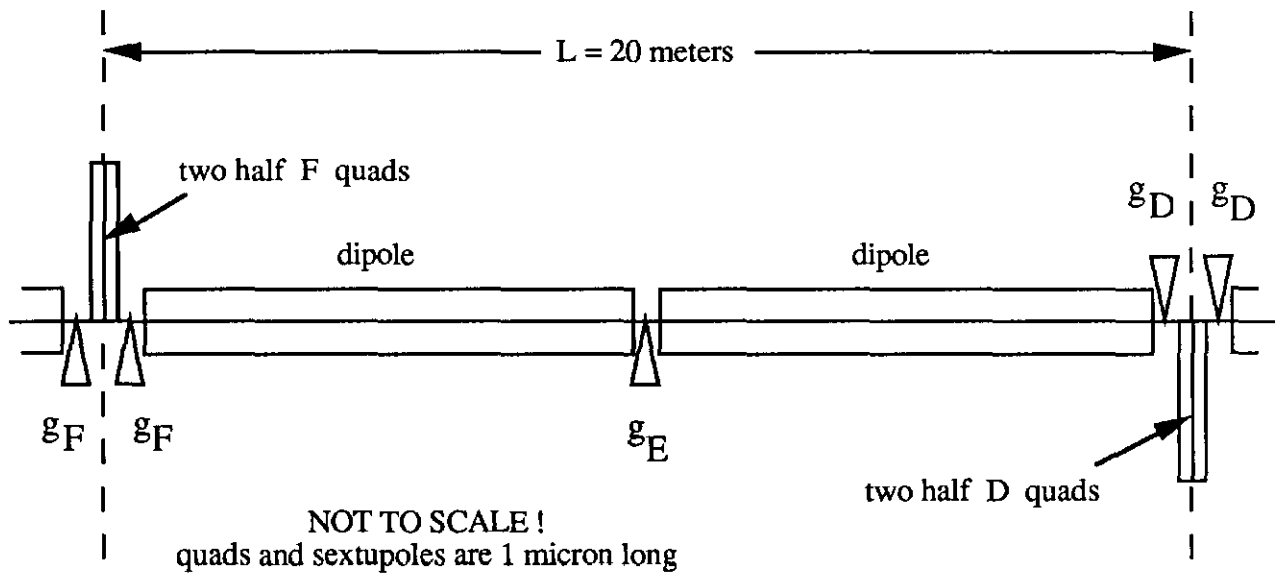


Figure 1 Half of a FODO cell, in a model representing the Main Injector with three sextupole families.

Using these convenient definitions, it can be shown by solving equation 13 that the matched values of the sum dispersion at F, E, and D are

$$\eta_{+F} = \frac{1}{s} [ \eta_{0F} (1 + s)(1 - g_F) - \eta_{0D}(1 + g_D) - \frac{\eta_{0E}}{2} (2 + s)g_E ] \quad (17a)$$

$$\eta_{+E} = \frac{1}{2s} [ \eta_{0F} (2 + s)(1 - g_F) - \eta_{0D}(2 - s)(1 + g_D) - \frac{\eta_{0E}}{2} (2 + s)(2 - s)g_E ] \quad (17b)$$

$$\eta_{+D} = \frac{1}{s} [ \eta_{0F} (1 - g_F) - \eta_{0D}(1 - s)(1 + g_D) - \frac{\eta_{0E}}{2} (2 - s)g_E ] \quad (17c)$$

Note that  $\eta_{+E} = \frac{1}{2}(\eta_{+F} + \eta_{+D})$  when  $g_E = 0$ , as required by equation 13, which says that the sum dispersion propagates linearly in a quadrupole and sextupole free region. These solutions can be combined to evaluate  $\alpha_+$ , since

$$\alpha_+ = \frac{\langle \eta_+ \rangle}{R} = \frac{1}{4R} (\eta_{+F} + 2\eta_{+E} + \eta_{+D}) \quad (18)$$

$$= \frac{1}{2sR} [ \eta_{0F} (2 + s)(1 - g_F) - \eta_{0D}(2 - s)(1 + g_D) - \frac{\eta_{0E}}{4} (8 - s^2)g_E ]$$

This expression becomes more direct when the  $\eta_0$  terms are replaced by their explicit matched solutions

$$\eta_{0F} = \frac{L^2}{R} \frac{2 + s}{2 s^2} \quad (19a)$$

$$\eta_{0E} = \frac{L^2}{R} \frac{8 - s^2}{8 s^2} \quad (19b)$$

$$\eta_{0D} = \frac{L^2}{R} \frac{2 - s}{2 s^2} \quad (19c)$$

Thus, an alternative form for equation 18, parameterized primarily by the sextupoles strengths, is

$$\alpha_+ = \frac{L^2}{R^2} \frac{1}{4s^3} [ 8s - (2 + s)^2 g_F - (2 - s)^2 g_D - \frac{(8 - s^2)^2}{16} g_E ] \quad (20)$$

In order to take stock of the meaning of equation 20, consider the case when only the F and D sextupoles are turned on, at a strength to correct for  $f$  times the natural chromaticity.

Independent of the strength of any sextupoles, it can easily be shown that

$$\alpha_0 = \frac{L^2}{R^2} \frac{1}{s^2} [ 1 - \frac{s^2}{12} ] \quad (21)$$

It can also easily be shown that, in order to correct  $f$  times the natural FODO chromaticity,  $g_F = \frac{f}{2}$  and  $g_D = -\frac{f}{2}$ , demonstrating the utility of the natural scaling that was introduced, apparently arbitrarily, in equation 16b. Substituting these values (and  $g_E = 0$ ) into equation 20 gives

$$\alpha_+ = \frac{L^2}{R^2} \frac{1}{s^2} [ 2 - f ] \quad (22)$$

which immediately leads to

$$\alpha_1 = \frac{L^2}{R^2} \frac{1}{s^2} [ 1 - f + \frac{s^2}{12} ] \quad (23)$$

showing that  $\alpha_1 \ll \alpha_0$  in a simple FODO lattice with the net chromaticity set to zero ( $f = 1$ ).

## CONTROLLING $\alpha_1$ WITH SEXTUPOLES

The middle sextupole, of strength  $g_E$ , can be thought of in (at least) two ways. In the first point of view, it represents the sextupole field caused by eddy currents induced in the vacuum chamber of the dipoles. (Note in passing that this representation is not perfect, since the changes in horizontal and vertical chromaticities are modeled as equal and opposite, whereas in reality they are unequal.) In the second point of view,  $g_E$  represents a free knob with which  $\eta_+$ , and hence also  $\alpha_+$  and  $\alpha_1$ , can be controlled. The reader may choose either perspective in what follows - or a combination, in which  $g_E$  represents the net strength after an independent correction sextupole is powered to over or under compensate the local eddy current sextupole fields. From any perspective, the task of setting the net chromaticities to their desired values is left to the F and D sextupoles. For the sake of a semi-quantitative interpretation, suppose that the F and D sextupoles have their strengths set to compensate for the sum of the chromaticity induced by an eddy current sextupole of strength  $g_E$ , plus  $f$  times the natural chromaticity.

In this case it is readily shown that the F and D sextupole strengths are given by

$$g_F = \frac{f}{2} - \frac{2 - s^2}{4} g_E \quad (24a)$$

$$g_D = -\frac{f}{2} - \frac{2 - s^2}{4} g_E \quad (24b)$$

Substituting these expressions into equation 20 gives

$$\alpha_+ = \frac{L^2}{R^2} \frac{1}{s^2} \left[ (2 - f) - \left(\frac{3}{8}\right)^2 s^3 g_E \right] \quad (25)$$

or, equivalently,

$$\alpha_1 = \frac{L^2}{R^2} \frac{1}{s^2} \left[ \left( 1 - f + \frac{s^2}{12} \right) - \left( \frac{3}{8} \right)^2 s^3 g_E \right] \quad (26)$$

These equations reduce to equations 22 and 23 when  $g_E = 0$ , as they should. If it is also assumed that the phase advance per cell is (approximately)  $90^\circ$ , then  $s = 1/\sqrt{2}$ , and the three compaction factors become simply

$$\alpha_0 = \frac{L^2}{R^2} \times 1.917 \quad (27a)$$

$$\alpha_+ = \frac{L^2}{R^2} \left[ 4 - 2f - 0.0994 \times g_E \right] \quad (27b)$$

$$\alpha_1 = \frac{L^2}{R^2} \left[ 2.083 - 2f - 0.0994 \times g_E \right] \quad (27c)$$

Typical values for  $g_E$  due to eddy current sextupoles in the Main Injector are 2 or 3, showing through equation 24 that they more or less dominate the strength of the F and D families [8].

To test the results of equation 27, and to gain some insight into the prospect of controlling  $\alpha_1$  in the Main Injector, consider a lattice made up of 80 simple FODO cells. In the parameterization introduced above the half cell has a length  $L = 20$  meters, and is filled with dipoles of bending radius  $R = 3200/2\pi = 509.30$  meters. Figure 2 summarizes results of simulations of such a lattice, using the program MAD (version 8.17)[9] to study the variation in closed orbit path-length as a function of  $\Delta p/p$ , over a range from  $-0.003$  to  $+0.003$ . The momentum compaction factor  $\alpha_p$  defined by equation A-3 in the Appendix is plotted. Equations A-3 and A-6 are then used to yield values of  $\alpha_0$  and  $\alpha_1$ , ready for comparison with the analytic predictions of equations 27a and 27c. Eddy current multipoles higher than sextupole are neglected.

Three cases are considered; no eddy currents with and without complete compensation of the natural chromaticities,  $(f, g_E) = (0,0)$  and  $(1,0)$ , and then complete chromaticity compensation with large but realistic eddy current sextupole strengths,  $(f, g_E) = (1,5)$ . Each of the plots is linear to a very good approximation, showing that  $\alpha_0$  and  $\alpha_1$  are the dominant coefficients in the  $\alpha p$  expansion. Table 1 shows excellent agreement between the simulated and the predicted values of  $\alpha_0$  and  $\alpha_1$ , except for what appears to be a systematic error in  $\alpha_1$  of  $0.120 \pm 0.005 \times 10^{-3}$ . The source of this small difference is not known, but is not considered to be important. Further comparisons using other design programs are anticipated, to see if the error persists.

If the sextupole family strength parameter  $g_E$  is regarded as an external "knob" to control values of  $\alpha_1$ , the inevitable conclusion is that the sensitivity to the family is too weak to reduce the Johnsen time to zero, short of using a very large strength or relinquishing control of the net chromaticities. Recall from equation 8 and from Table 1 that the desired value of  $\alpha_1$  is approximately  $-4.5$ , but notice that the  $\alpha_1$  sensitivity coefficient in equation 27c is only 0.0994, disappointingly small. This is reflected in the minor changes of  $\alpha_1$  between the second and third rows of Table 1.

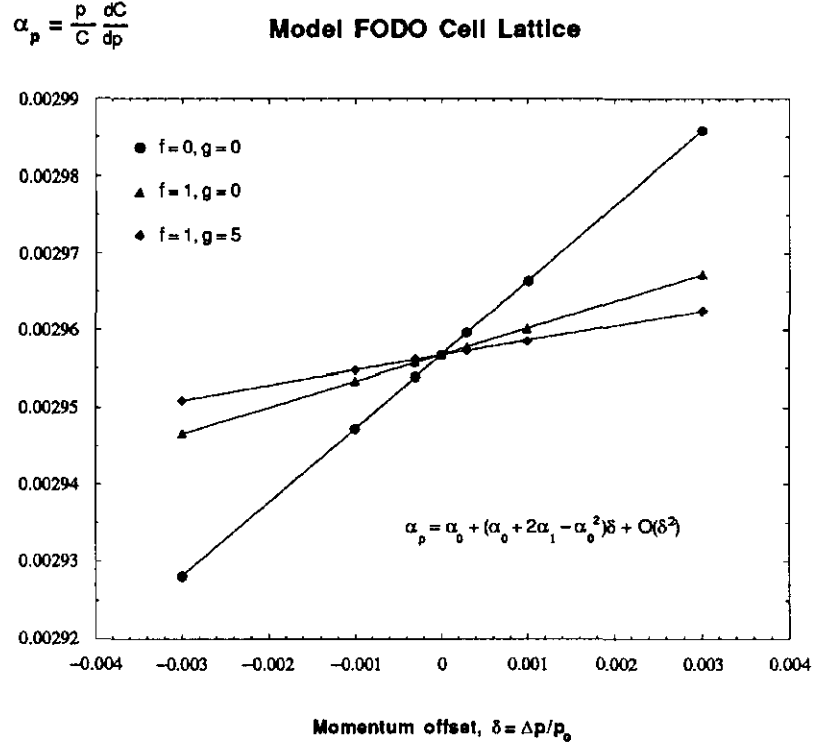


Figure 2 Numerical simulation of momentum compaction factor variation with  $\delta$  for three configurations of eddy current and chromaticity compensation sextupole strengths.

$(f, g_E)$	$\alpha_0$		$\alpha_1$	
	predicted	simulated	predicted	simulated
	$(\times 10^{-3})$	$(\times 10^{-3})$	$(\times 10^{-3})$	$(\times 10^{-3})$
(0,0)	2.956	2.956	3.213	3.332
(1,0)	2.956	2.956	.129	.244
(1,5)	2.956	2.956	-.638	-.512

Table 1 Comparison of predicted and simulated  $\alpha_0$  and  $\alpha_1$  values.

Investigations are currently under way to find an optical configuration that will significantly increase the orthogonality of the three families beyond the unfortunate results of the FODO lattice. An apparently promising candidate involves the introduction of a dispersion wave in the arcs of the Main Injector, around transition time. This begins to resemble an unmatched  $\gamma_t$  jump scheme - except that the lattice perturbation can be introduced slowly, and that the needed size of the dispersion wave is expected to be relatively modest.



## BEHAVIOR OF MATCHED AND UNMATCHED $\gamma_t$ JUMP SCHEMES

The satisfactory agreement between MAD817 simulations and analytic predictions reported in the previous section, for the simple case of a FODO lattice, encourages the use of the program to study the behavior of momentum compaction factors for more realistic Main Injector lattices, where analytic results are no longer tractable. Here we consider two families of Main Injector lattices representing matched and unmatched  $\gamma_t$  jump schemes. These schemes are described in detail elsewhere [10]. It is important to check that the resulting change of  $\alpha_1$  does not greatly affect the Johnsen time  $T_J$ , extending the variation of transition crossing time for different parts of a bunch.

The simulation places one thin eddy current sextupoles of strength  $g_E$  at the middle of each dipole, with a multipole strength of  $b_2 = 0.561 \text{ m}^{-2}$ . Two families of chromatic sextupoles are used to compensate for both natural and eddy current chromaticities. It is assumed, for the sake of definiteness, that the F and D sextupole strengths are not changed while jumping through transition. Figure 3a summarizes the behavior of the matched scheme with bipolar ( $\Delta\gamma_t = \pm 0.65$ ) and unipolar ( $\Delta\gamma_t = -1.3$ ) excitations. Figure 3b examines an unmatched unipolar excitation - a bipolar jump is not possible in this scheme. The linear character of  $\alpha p(\delta)$  in the realistic range  $\delta = -0.01$  to  $+0.01$  is apparent in all cases. Note that the vertical scales are significantly different in the two figures - compare the same  $\Delta\gamma_t = 0$  case shown in both figures.

Table 2 summarizes the simulation results. Since the arcs in a matched scheme are essentially just a sequence of FODO cells slightly retuned by a quadrupole perturbation, it is expected to produce qualitatively the same results as the FODO lattice in the previous section. Indeed, the values of  $\alpha_1$  recorded in Table 2 are an order of magnitude smaller than  $\alpha_0$ , and do not pose any danger to the Johnsen time. By contrast, the unmatched scheme produces large value of  $\alpha_1 \approx 1.68 \alpha_0$ , which, according to equation 8, more than doubles the Johnsen time. Also included in Table 2 are the uncorrected chromaticities,  $\xi_H$  and  $\xi_V$ , with the F and D sextupoles turned off, but with the eddy current sextupoles turned on.

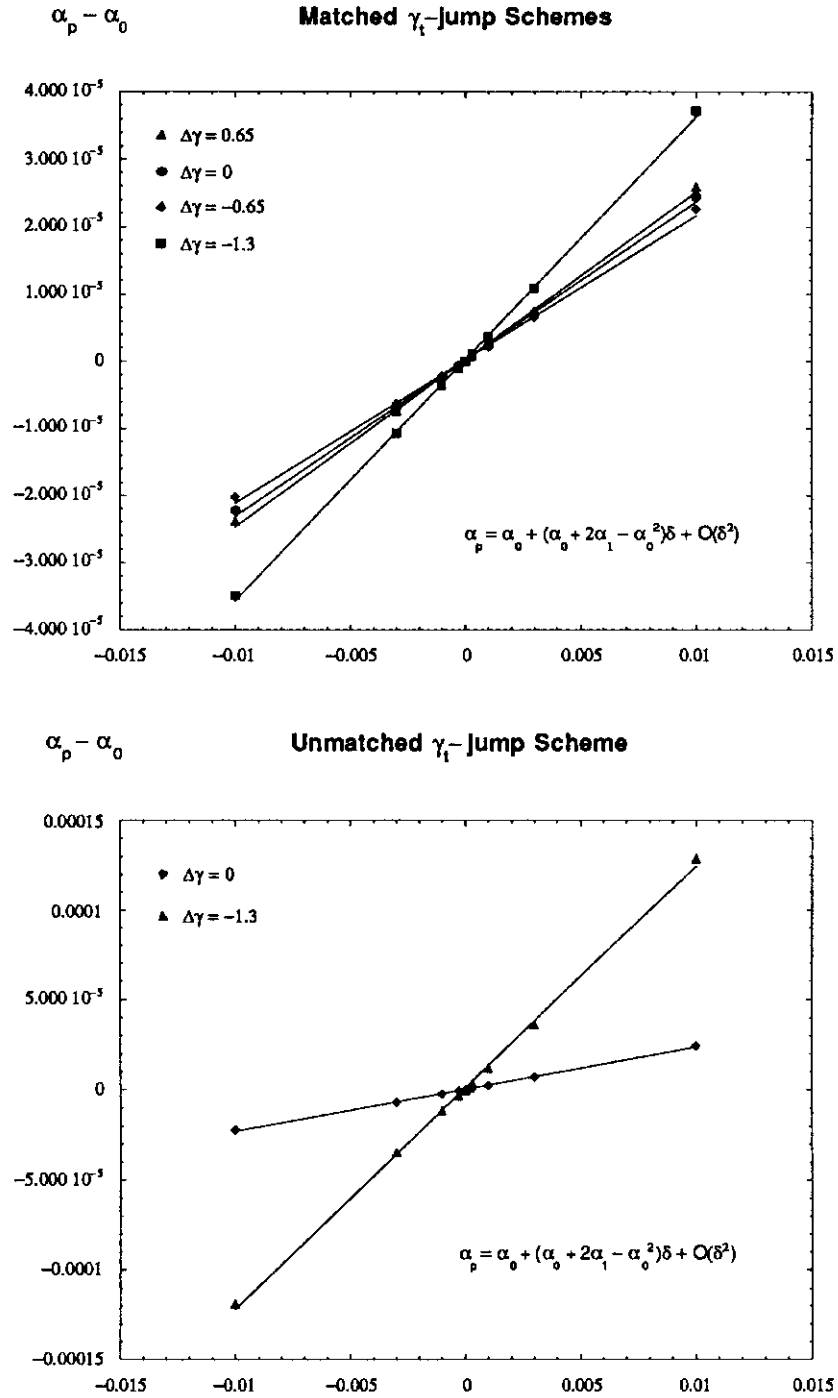


Figure 3 Numerical simulation of momentum compaction factor variation versus  $\delta$  carried out for various  $\gamma_t$ -jump configurations. The simulation includes eddy current sextupoles and full chromaticity compensation.

Scheme	$\Delta\gamma_t$	$\alpha_0$ ( $\times 10^{-3}$ )	$\alpha_1$ ( $\times 10^{-3}$ )	$\xi_H$ uncorrected	$\xi_V$
Matched	.65	2.23	.14	-10.6	-46.4
	.0	2.37	-.01	-7.6	-47.3
	-.65	2.53	-.19	-6.4	-48.3
	-1.3	2.70	-.45	-8.5	-49.7
Unmatched	.0	2.37	-.01	-7.6	-47.3
	-1.3	2.70	-4.82	-3.9	-50.1

Table 2 The behavior of momentum compaction coefficients, and of the uncorrected chromaticities, in matched and unmatched Main Injector transition jump schemes.

## CONCLUSIONS

General analytical expressions are reported above for the variation of the dispersion function to first order in the off-momentum parameter  $\delta$ , and for the variation of the closed orbit circumference to second order. This makes it possible to evaluate how the critical Johnsen time (for example) depends on effects like eddy current sextupoles in the Main Injector dipoles, or on transition jump configurations. In the simple but relevant case of a FODO lattice representation of the Main Injector, analytic results are in good quantitative agreement with a lattice design code.

If there is no  $\gamma_t$  jump in the Main Injector, the Johnsen time is typically expected to be about 0.6 milliseconds, or about 60 machine turns. There is no benefit from turning on a  $\gamma_t$  jump much faster than this. Examination of nominal Main Injector transition jump schemes reveals that a matched scheme produces little change in the Johnsen time, but that  $T_J$  is more than doubled in the unmatched scheme.

A third family of sextupoles might be used to deliberately and practically control the Johnsen time, without modifying the nominal transition momentum. (Two other sextupole families are used to achieve the desired net chromaticities.) Such control, especially when used in conjunction with rf gymnastics, may make transition crossing so innocuous that it becomes unnecessary to include a transition jump in Main Injector designs.

However, the Johnsen time in a FODO lattice is quite insensitive to a third family of sextupoles located at the middle of the half cells. The good news is, then, that eddy current sextupoles are not expected to significantly affect transition crossing performance. The bad news is that using mid-cell sextupoles to control the Johnsen time is not practical. Continuing investigations suggest that a modest dispersion wave significantly improves the orthogonality of three families of sextupoles.

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## APPENDIX – RELATIONS BETWEEN $\alpha_1$ AND OTHER PARAMETERS

For reference purposes, this appendix describes how the three definitions of  $\alpha_1$  and the one definition of  $\alpha_p$ , all of which are common in the literature, are related. It has already been released as Fermilab Main Injector note MI-0038, by MacLachlan, Ng, and Peggs, with only minor differences.

The "circumference" definition of  $\alpha_1$ , as defined in equation 1 and used throughout above, comes from expanding the difference in the circumference of the closed orbit,  $\Delta C = C - C_0$ , as a polynomial in the off momentum parameter,  $\delta = (p - p_0) / p_0$ ,

$$\frac{\Delta C}{C_0} = \alpha_0 \delta + \alpha_{C1} \delta^2 + \dots \quad (\text{A-1})$$

The definition introduced by Johnsen is very similar

$$\frac{\Delta C}{C_0} = \alpha_0 \delta + \alpha_0 \alpha_{J1} \delta^2 + \dots \quad (\text{A-2})$$

The variation of the transition energy  $\gamma_t$  with  $\delta$  is directly described by  $\alpha_p$ , defined through

$$\alpha_p(\delta) = \frac{1}{\gamma_t^2(\delta)} = \frac{p}{C} \frac{dC}{d\delta} = \alpha_0 + \alpha_{E1} \delta + \dots \quad (\text{A-3})$$

This introduces the ESME definition of  $\alpha_1$ .

Note that equation A-3 represents the local derivative at some momentum  $p$ . WARNING - when used with a constant momentum offset, some lattice design codes return  $\alpha_p(\delta)$ , but some return

$$\frac{p_0}{C_0} \frac{dC}{d\delta} = \alpha_0 + \alpha_{C1} \delta + \dots \quad (\text{A-4})$$

Comparing equations A-1 and A-2 gives

$$\alpha_{F1} = \frac{\alpha_{C1}}{\alpha_0} \quad (A-5)$$

Performing the differentiation in equation A-3 and expanding gives

$$\alpha_{E1} = \alpha_0 + 2 \alpha_{C1} - \alpha_0^2 \quad (A-6)$$

Although this paper has conveniently taken equation 1 to define  $\alpha_1 = \alpha_{C1}$ , the selection is (somewhat) arbitrary. The reader is NOT implored to adopt one or another of the definitions introduced here, but rather is asked to be careful to specify which definition he or she is using. He or she IS implored not to invent any more definitions for  $\alpha_1$ .