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PRINCIPLES AND APPLICATIONS OF MUON COOLING

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Abstract

The basic principles of the application of "ionization cooling" to obtain high phase-space density muon beams are described, and its limitations are outlined. Sample cooling scenarios are presented. Applications of cold muon beams for high energy physics are described. High luminosity $\mu^+\mu^-$ and μ-p colliders at $\lesssim 1$ TeV energy are possible.

I. Introduction

Previous high energy accelerators have used electrons and protons as their basic tools in the pursuit of high collision energies. In extension to higher energies, both have significant liabilities.

Electrons (and positrons) have been useful tools in recent experiments because their "point-like" nature permits their use in studies of simple processes: e^+e^- annihilation to produce new particles ($e^+e^- \to \gamma^* \to$ hadrons, etc.) and the exploration of proton structure in "deep inelastic" ep collisions. Their future use is severely limited by their participation in radiation processes. Synchrotron radiation in storage rings causes electrons to lose an amount of energy, ΔE, per turn of[1]

$$\Delta E(\mathrm{MeV}) = \frac{4\pi e^2}{3R}\left(\frac{E}{mc^2}\right)^4 = \frac{.0885[E(\mathrm{GeV})]^4}{R\ (\mathrm{m})} \qquad (1)$$

where R is the storage ring radius and E the electron energy. Studies based on this equation and the constraint $\frac{\Delta E}{E} \lesssim 1$ indicate that storage rings with E $\gtrsim$ 100 GeV are thoroughly impractical.

Single Pass Linear Accelerators are proposed to circumvent this problem. Radiation processes ("Beamstrahlung" in e^+e^- collisions and Bremstrahlung in ep collisions, as well as "two photon" events obscuring the simpler "one-photon" collisions) still limit their usefulness, and they are quite expensive.

The fourth power of the mass in the radiation formula (1) might suggest that radiation difficulties can be circumvented by use of a heavier probe. Thus protons are used to obtain the highest energy collisions. The liability of protons (and antiprotons) is that they are complex objects with correspondingly complicated interactions which limit their usefulness as probes of short distance structure, both in fixed target and colliding beam ($\bar{p}p$ and pp) experiments.

In this paper, we suggest that muon ($\mu^{\pm}$) beams may be used as primary probes in high energy collisions, since they combine a "point-like" electron-like nature with a large mass which is sufficiently immune to radiation. Muons have, of course, been used in secondary beams as "deep inelastic" probes of hadron structure. Here we note, as Skrinsky[2] and others have noted, that their role can be extended to use in primary beams: $\mu^{\pm}$ storage rings and μ Linacs at high energies.

The principal liabilities of muons are their short lifetimes and the large phase space area of the initial muon beam, which is produced from π-decay. The lifetime τ is given by[3]

$$\tau = 2.197\text{x}10^{-6} \frac{E_\mu}{m_\mu} \text{ seconds.}$$

This is an adequate lifetime for any Linac, and, as we show below, is adequate for some storage ring applications. The short lifetime does preclude long term accumulation of μ's at "low" energies ($\lesssim$ 10 TeV) which will limit μ beam intensity in any scenario.

The large phase space area of a muon beam can be damped using "ionization" cooling[2] to a small value using a muon cooling ring or linac, as will be described below. The damped beam will then be suitable for colliding beam scenarios or acceptance in a high gradient linac or rapid cycling synchrotrons.

Cold μ beams may meet the acceptance requirements of novel high gradient acceleration mechanisms (such as "laser" accelerators)[4] whose basic requirements are a low phase space volume beam. They may be preferred over $e^{\pm}$ or p injection, particularly in scenarios in which the beam passes through a material medium (i.e. "plasma-laser" or "inverse Cerenkov") where e and p usage is limited by electron bremstrahlung and proton-nucleus collisions.

In the following sections, we will describe muon cooling, suggest cooling scenario designs, and describe cooling limitations. In Section VI we describe $\mu^{\pm}$ colliding beam possibilities in various scenarios.

II. Muon Energy Cooling

The basic mechanism of μ cooling is displayed in Figure 1. The muon beam is passed through a material medium in which it loses energy, principally through interactions with atomic electrons. Following this it passes through an accelerating cavity where the average longitudinal energy loss is restored to each μ. Energy cooling occurs following

$$\frac{d(\Delta E_\mu)}{dn} \cong - \frac{\partial \Delta_\mu}{\partial E_\mu} \Delta E_\mu \tag{2}$$

where n is the number of cooling cycles, ΔE_μ is a muon energy deviation from the reference average value, Δ_μ is the energy loss of a muon with energy E_μ in the material absorber, and the derivative is taken at the central energy. Cooling occurs if $\frac{\partial \Delta_\mu}{\partial E_\mu} > 0$. The beam is recirculated through the absorber/accelerator cycle through many turns either by a return path (cooling ring) or repeated structure (linac). Transverse motion is damped as transverse energy is lost in the absorber but not recovered in the accelerator.

The process is basically similar to radiation damping in electron storage rings[1], where energy lost in curved sections of the ring by radiation is recovered in the RF cavities. Radiation damping is limited by quantum fluctuations; analogously muon cooling is limited by statistical fluctuation in muon-atom interactions in the absorber.

The important difference is that muons decay, and therefore cooling must be completed before decay can occur. The muon lifetime is[3]

$$\tau_\mu = 2.197\text{x}10^{-6} \frac{E_\mu}{m_\mu} \text{ seconds}$$

which can be translated to a path length of

$$L = 6.59\text{x}10^{2} \frac{\beta_\mu E_\mu}{m_\mu} \tag{3}$$

where β_μ is the muon speed, which will always be near 1 in this paper. For a storage ring this can be translated to a number of turns of beam storage:

$$N = \frac{L}{2\pi \bar{R}} = \frac{L\bar{B}}{2\pi B\rho} = 297\ \bar{B}(T) \text{ turns} \tag{4}$$

which we note is independent of momentum and only dependent on the ring-averaged bending field.

The energy loss rate for muons passing through a material is, approximately,[3]

$$\frac{dE}{dz} \cong \frac{DZ\rho}{A\beta^2} \left\{ \log\left(\frac{2\, m_e\, \gamma^2 \beta^2 c^2}{I} \right) - \beta^2 \right\} \tag{5}$$

where Z, A are the nuclear charge and atomic number of the absorbing medium, ρ is the density of the material (in gm/cm^3), m_e is the electron mass, γ and β are the muon kinematic factors and D and I are constants with

$$D = .3070\ \text{MeV-cm}^2/\text{gm}$$

$$I \cong 16(Z)^{0.9}\ \text{eV}$$

and c is the velocity of light. The function dE/dz as a function of muon energy for various materials is displayed in Figure 2.

This is a steeply decreasing function of energy for $\gamma \lesssim 3$ but for $\gamma > 5$ it is a slowly increasing function suitable for damping. This weak dependence can be enhanced by placing the absorber in a region where transverse position x depends on energy (a "non-zero dispersion" region) and varying the thickness with position as shown in Figure 3.

The net change in a particle energy with respect to the central value in one cycle of passage through an absorber of width $\delta(x)$ plus acceleration is

$$\delta(\Delta E_\mu) \cong -\frac{\partial}{\partial E_\mu}\left[\frac{dE}{dz}\,\delta(x)\right]\Delta E_\mu$$

$$= -\left[\frac{\partial^2 E}{\partial E_\mu \partial_z}\,\delta(0) + \frac{dE}{dz}\,\delta'\,\frac{n}{E_o}\right]\Delta E_\mu = -\frac{\Delta E_\mu}{n_c} \tag{6}$$

where $\delta' = \frac{d\delta}{dx}$, and we have redefined the inverse of the coefficient of Equation (6) as n_c. η is the value of the "Courant-Snyder" dispersion function at the absorber[5]. The damping equation is

$$\frac{d\Delta E_\mu}{dn} = - \left[\frac{\partial^2 E}{\partial E_\mu \, dz} \delta_o + \frac{\eta}{E_o} \frac{dE}{dz} \delta' \right] \Delta E_\mu \tag{7}$$

A limiting requirement of μ cooling is the total acceleration of the central orbit necessary to obtain the desired cooling. The total acceleration to obtain 1/e cooling is

$$E_{cool} = n_c \frac{dE}{dz} \delta(0)$$

which can be compared to E_o, the central energy

$$\frac{E_o}{E_{cool}} = \frac{E_o \dfrac{\partial^2 E}{\partial E_\mu \, \partial z}}{\dfrac{dE}{dz}} + \frac{\eta \dfrac{d\delta}{dx}}{\delta(o)} . \tag{8}$$

The first term has a maximum of ~.2 at E. ≅ 1 GeV with little variation over the broad range .5 GeV < E_o < 10 GeV, which is a reasonable range for μ collection.[6] The second term is constrained by the transverse heating in non-zero dispersion absorption to ≲ 1 with an optimum at ~0.4 with simultaneous transverse damping. These numbers can be compared with the corresponding factor of 2 for electron storage ring radiation damping.

Choosing 0.6 as the sum, we note that $\frac{4.6}{.6} E_o = 7.7\, E_o = 7.7$ GeV of acceleration is necessary to damp 1 GeV muon energy spreads by a factor of 100. Choosing a storage ring system (see equation 4) with $\bar{B} \approx 1$ T and requiring damping within 1 muon lifetime, we find an RF acceleration requirement of ~26 MeV/turn, which is not unreasonably large.

III. Transverse Damping

The mechanism for transverse damping is quite simple. Energy loss is parallel to the particle trajectory but energy gain (in RF) is longitudinal. On each passage through absorber and accelerator

$$x' = \frac{p_x}{p_z} \cong \frac{p_x c}{E_0}$$

is reduced

$$x'_{\text{after}} = \left[1 - \frac{\frac{dE}{dz}\,\delta}{E_0} \right] x'\Big|_{\text{Before}} . \tag{9}$$

Averaging over particle phases and introducing rms emittance:

$$\langle \varepsilon_x \rangle_{rms} = \left\langle \frac{x^2}{\beta} + \beta\, x'^2 \right\rangle \cong 2 \langle \beta\, x'^2 \rangle$$

we obtain

$$\frac{d\ \ \varepsilon_x}{dn} = \frac{-dE}{dz} \cdot \frac{\delta}{E_0} \cdot \varepsilon_x \tag{10}$$

where β here is the Courant-Snyder "betatron" function[5] and the variable "n" indicates the number of passages through an absorber/accelerator cycle. Total acceleration to cool by 1/e is simply E_0 so $E_0/E_{cool} = 1$ for transverse cooling.

The above cooling rate is correct for both x and y transverse cooling where energy loss occurs in zero C-S dispersion regions. However if a "wedge" absorber in a non-zero dispersion region is used to enhance energy cooling then a corresponding degree of transverse heating occurs in that $\eta \neq 0$ dimension, which competes with the damping.

The heating mechanism is shown in Figure 4. In passing through an absorber, particle position (x) does not change (to first order) but energy does. The position is the sum of an energy dependent component ($\eta(\Delta E)$) and a betatron component (x_β) and a decrease in $\eta\ \Delta E$ implies an increase in x_β following:

$$x = \frac{\eta(\Delta E)}{E_o} + x_\beta$$

$$\Delta E \rightarrow \Delta E\Big|_{\text{before}} - \frac{dE}{dz}\,(\delta' x)$$

$$\Delta x_\beta \rightarrow + \frac{\eta}{E_o}\frac{dE}{dz}\,\delta_o + \delta'\left(\frac{\eta \Delta E}{E_o} + x_\beta\right). \tag{11}$$

(We have assumed $d\eta/dz = 0$ at the absorber to simplify discussion.) Assuming energy and betatron amplitudes are uncorrelated and keeping only lowest order terms, we obtain:

$$\Delta\left\langle {x_\beta}^2\right\rangle = + \frac{2\eta}{E_o}\frac{dE}{dz}\,\delta'\left\langle {x_\beta}^2\right\rangle. \tag{12}$$

Note that the first term in Equation (11) has been eliminated since it simply implies a change in the η-function; in a properly matched lattice it does not contribute to $\Delta\left\langle {x_\beta}^2\right\rangle$.

Averaging over betatron phases, we obtain an anti-damping term, which can be added to the previously derived cooling term (Equation 10):

$$\frac{d\left\langle \varepsilon_x\right\rangle_{rms}}{dn} = \left[-\frac{\delta_o}{E_o}\frac{dE}{dz} + \frac{\eta}{E_o}\frac{dE}{dz}\,\delta'\right]\left\langle \varepsilon_x\right\rangle_{rms}. \tag{13}$$

The anti-damping is precisely opposite the corresponding energy damping term, so enhanced energy damping implies decreased betatron damping. μ-cooler design must balance these damping requirements. (The two damping rates are equal at $\eta\Delta' \cong .4\ \delta_0$.)

We note here that the sum of ε_x, ε_y, ΔE damping rates is a constant:

$$\sum_{x,y,\Delta E} \frac{E_0}{E_{cool}} = 2 + \frac{E_0}{\frac{dE}{dz}} \frac{\partial}{\partial E_\mu} \left(\frac{dE}{dz}\right) \cong 2.2 \qquad (14)$$

which parallels a similar invariant for electron radiation damping.[1]

IV. Heating by Statistical Fluctuations

μ cooling is obtained through a finite number of muon-atomic electron collisions in passing through an absorber. Statistical fluctuations in the number of collisions and the energy exchanged in a collision increase the muon energy spread, opposing the damping mechanism.

An estimate of this antidamping mechanism can be obtained by noting that the mean energy exchange is approximately the mean electron ionization energy of the absorber:[7]

$$I \cong 10\ Z_{abs} \quad \text{eV}$$

where I is the mean ionization energy, Z is the nuclear charge of the absorber. The number of collisions is, approximately

$$N_{coll} = \frac{dE}{dz} \frac{\delta_0}{I} \quad \text{per cooling cycle}$$

and the rms energy error is $\sqrt{N_{coll}}\,I$ or

$$\Delta E_{rms} = \sqrt{\frac{dE}{dz}\,\delta_o\, I}\ .$$

Combining cooling with RMS heating we obtain the cooling equation:

$$\frac{d}{dn}\left\langle \Delta E^2 \right\rangle_{rms} = -2\left[\delta_o \frac{\partial}{\partial E_\mu}\left(\frac{dE}{dz}\right) + \frac{\eta}{E_o}\frac{dE}{dz}\,\delta'\right]\left\langle \Delta E^2 \right\rangle_{rms} + I\,\frac{dE}{dz}\,\delta_o . \tag{15}$$

An equilibrium energy spread can be obtained where the above derivative is zero

$$\left\langle \Delta E^2 \right\rangle_{rms} = \frac{I\ E_{cool}}{2}\ . \tag{16}$$

With $E_{cool} \cong 2$ GeV, $I \cong 10$ eV, we obtain $\Delta E_{rms} \cong 10^5$ eV, suggesting an approximate limit for μ cooling. With 1 GeV μ's, $\frac{\Delta E}{E} \sim 10^{-4}\sqrt{Z}$.

Transverse cooling is severely limited by multiple small angle elastic scattering in the absorber, mostly elastic Coulomb scattering from the nuclei. The scattering angle passing through an absorber can be estimated using the Particle Data Group formula:[3]

$$\theta_{rms} \cong \frac{14\ (\mathrm{MeV})}{E_\mu\ (\mathrm{MeV})}\sqrt{\frac{\delta}{L_R}} \tag{17}$$

where δ is the absorber length, L_R is the PDG radiation length. The change in rms emittance is given by

$$\Delta\,\varepsilon_{rms} \cong \frac{\beta\,\theta^2_{rms}}{2} = \frac{\beta}{2}\left(\frac{14}{E}\right)^2\frac{\delta}{L_R} \tag{18}$$

so the cooling equation for ("zero-dispersion") transverse emittance (unnormalized) is

$$\frac{d\varepsilon}{dn} \cong -\frac{dE}{dx}\frac{\delta}{E_\mu}\,\varepsilon + \frac{\beta}{2}\left(\frac{14}{E}\right)^2\frac{\delta}{L_R}\ . \tag{19}$$

The equilibrium emittance is, therefore,

$$\varepsilon_0 \cong \frac{\beta}{2} \frac{(14)^2}{E_\mu \left(\frac{dE}{dx} L_R\right)} \tag{20}$$

where β is the Courant-Snyder betatron function at the absorber. The product $\left(\frac{dE}{dx} L_R\right)$ depends upon the absorber and is largest for light elements (~100 MeV for Be or C and ~300 MeV for H_2 or D_2 but ~7 MeV for Pb or W).

The appearance of β in Equation (20) demands very strong focusing of the beam to a small radius at the absorber. Suitable optics for this purpose is displayed in Figure 5. Note that β is constrained by the focusing geometry to be $\lesssim$ the length of the absorber, unless the absorber is an active focusing element (Li lens). Shorter absorbers are possible with heavier elements, balancing the constraint of the previous paragraph. In Table 1 we list the critical quantities (I, ρ, $\frac{dE}{dz}$, L_R) influential in choice of an optimum absorber for particular applications. μ cooling design constraints will be further discussed in the next section.

As a relatively modest choice suitable for some cooling goals we choose β = 1. cm, E_μ = 1 GeV and a Beryllium absorber and obtain $\varepsilon_0 \cong 10^{-5}$ meter-radians at 1 GeV, which is matched to typical proton beam emittances.

Somewhat lower emittances are possible, but eventually the conflict between focusing and absorber requirements will limit the cooling.

V. μ Cooler Design Outlines

The cooling principles can be used to develop muon cooling scenarios and in this section we outline some possibilities.

We first consider a μ storage ring scenario. The major requirements are shown in Figure 5: a high intensity multi-GeV primary (proton) beam, a target for π/μ-pair production, a π-decay channel (or "stochastic injection")[6] and a high acceptance μ storage ring. The general properties of muon collection have been discussed previously for a different application.[6] We only quote the approximate result that $\gtrsim 10^{-2}$-10^{-3} 1 GeV μ per primary proton can be stored within an acceptance of ±5% momentum, ~100 π mm-mr emittance. (Stronger focusing at the target and in the decay channel than that of reference 6 is desired to maintain low μ phase space.) Our design goal is a reduction in $\frac{\Delta p}{p}$ to $\lesssim \pm .2\%$ and $\varepsilon_\perp \lesssim 10\,\pi$. (This reduced momentum spread can be rotated into a reduced muon bunch length by a "compressor arc" similar to that used for compressing radiation cooled $e^\pm$ beams for Linac Colliders.[8])

Table 2 and Figure 6 show parameters of a system which meets these requirements: a 30 meter storage ring requiring ~15 MeV/turn RF with three "low-beta" insertions for Be absorbers. These insertions can have non-zero C-S dispersion for enhanced $\frac{\Delta p}{p}$ cooling.

The storage ring design is most severely by the focusing requirements of the "low-beta" insertions. "Low-beta" at the absorber implies "large-beta" at the focusing magnets and therefore large apertures which must be large enough to accommodate the hot injected beam:

$$a > \sqrt{\beta_{max}\,\varepsilon_{initial}}\ . \tag{21}$$

This constrains the focusing strength of the lens. The lens gradient G is limited by

$$G \lesssim \frac{B_{max}}{a_{max}}\ . \tag{22}$$

We require a magnet doublet of length $\gtrsim 1.5$ m to obtain $\beta_{min} \approx 1$ cm with conventional 2 T magnets.

Stronger focusing (lower β or shorter lenses) can be obtained with complex lenses and/or higher fields ($\gtrsim$4 T superconducting magnets). However, the transverse cooling remains substantially constrained by this aperture requirement.

This constraint can be circumvented by use of a Linear muon cooler. A sample design is outlined in Figure (7) with parameters displayed in Table 3. This design consists of alternating "low-beta" insertions with accelerator cavity sections. The insertion length is kept constant while the lens aperture decreases as the beam shrinks (from emittance cooling). This provides stronger focusing, decreasing β_{min}, while the absorber length, $L_{abs} = 2\ \beta_{min}$ also decreases. The focusing parameter k_F with

$$k_F = \frac{B_o\ L_{ins}}{B_p \sqrt{\varepsilon/\beta_{min}}} \tag{23}$$

kept constant at $k_F \cong 8$. The mean acceleration per structure period decreases from 10 MeV to 0.7 MeV following L_{abs}. The cooling rate decreases correspondingly. In fact cooling in this design follows the equation

$$\frac{d\varepsilon}{ds} \cong -A\ \varepsilon^2 \tag{24}$$

where $A\ \varepsilon \cong \frac{dE_\mu}{ds}/E_\mu$ is the cooling rate.

The limitation $L_{abs} \lesssim 2\ \beta_{min}$ may be circumvented in some parameter regions by use of an actively focusing absorber such as a Lithium lens. Lithium lenses can obtain high fields[9] (equivalent to >15 T quads) focusing in both x and y directions, which can maintain the beam at low beta provided

$$\frac{B_{eq}}{B_p\ a} \gtrsim \frac{1}{\beta_{min}^2} \tag{25}$$

where a is the lens radius, B_{eq} is the equivalent focusing field and B_p

is the magnetic rigidity, 3.3 E_μ(GeV) T-m. We have not included Li lens absorbers in our sample designs, but we expect optimum designs to incorporate them in some portion of the structure.

The major advantages of the Linear Cooler are:

1. Transverse cooling is not limited by the large aperture required to accommodate injected uncooled beam.

2. Losses through decay are less because of the shorter particle path length.

The disadvantages are:

1. The linac does not naturally contain non-zero C-S dispersion for enhanced momentum cooling; however dispersion can be introduced with bending magnets.

2. The single pass linac structure is longer and therefore more expensive.

We note that a linac structure is more naturally suited for transverse cooling, while a storage ring more naturally obtains enhanced $\frac{\Delta p}{p}$ cooling. An optimum cooling scheme may combine these with a linac "precooler" followed by a storage ring. Multiple storage rings or more complicated structures similar to "race-track microtrons" may also be possible. Optimum systems can reach $\varepsilon_\perp \lesssim 2\pi$ mm-mr at 1 GeV and $\frac{\Delta p}{p} \lesssim 0.1\%$, or perhaps substantially less.

As a closing comment in this section we note that it may be desirable to modify existing ~1 GeV storage rings with low beta insertions and increased RF to obtain measurable μ cooling and explore the practicality of this technique.

VI. Possible Uses for Cooled Muons

In this section we outline some suggested possible uses for suitably cooled muons, which may be developed in greater detail in future notes. The reader is invited to develop these possibilities as well as to suggest his own ideas.

A. The "Proton Klystron": Skrinsky[2] suggested the acceleration of cold muons (as well as π, e, etc.) in a high gradient linac structure which is loaded by injection of a high-intensity, high-energy proton beam ($>10^{11}$ p, 400-1000 GeV) A muon bunch following in the wake of the protons can be accelerated to high energy ($\gtrsim$500 GeV, say) and then used in fixed target (μ-p) or colliding with other linac (μ^+, μ^-, etc.) bunches traveling in the opposite direction.

We have shown that muon bunches with $\gtrsim 10^9 \mu$, $\varepsilon \lesssim 10^{-4}\pi$ (normalized) m-R can be produced in a μ cooling ring using some proton bunches from the primary proton ring, and a compressor arc can rotate the beam to a length ($\lesssim$ a few mm) suitable for the klystron linac.

B Other Linac Structures

Cold muon bunches can be injected into a linac structure for acceleration to suitably high energies, provided, of course, the linac structure is constructed. Acceleration in a SLAC-type linac is possible with 1 GeV cold muons. Other innovative structures (laser accelerators, ...) may have substantially lower phase space requirements which may or may not be achievable with muon cooling. It is left as an exercise for the reader to invent a practical innovative linac (with $\gtrsim$1 GeV/m gradient), determine its phase space requirements, and design a μ cooler matched to those requirements. (Homework is due Monday!) The muons could be used in external beam or collider modes.

We merely list some possible advantages of muons (over $e^{\pm}$) in such structures.

1. Their relative immunity to bremstrahlung and synchrotron radiation may make them more suitable in schemes with a material medium (plasma wave accelerator) or beam bending fields ("inverse free electron laser", "two wave device")[4] or beam focusing fields (almost any scheme).

2. Muons are relatively immune to "bremstrahlung" in μ-X collisions.

3. The higher mass of muons may make matching of beam and accelerator phase velocities easier than with electrons.

C. Linac-Storage Ring Systems

A real advantage of $\mu^{\mp}$ beams from a Linac is that the high energy beams can be circulated in a storage ring for hundreds of turns to magnify luminosity over a single pass mode. In Figure 8 we display a sample system of this type which uses an existing Linac and mimics the SLC design.[8] In the system SLAC $e^{\pm}$ bunches at ~50 GeV are used to produce muons which are collected and cooled. Separate μ^+, μ^- bunches are reinjected into the Linac, accelerated to 50 GeV and injected into a superconducting 50 GeV storage ring with $\bar{B} = 4\,T$ (R = 40 m).

The luminosity L can be obtained from the formula:

$$L = \frac{f\, N^+ N^-}{4\pi\, \varepsilon_{rms}\, \beta^*} \tag{26}$$

where f is the collision repetition rate, N^+, N^- are the number of muons, ε_{rms} is the emittance and β^* is the collision C-S betatron function ($\beta_x^* \cong \beta_y^*$). With parameters achievable in the above SLC-like scenario ($f = 2\times10^5$, $N^+ = N^- \cong 3\times10^8$ from ~10^{11} e^-, $\varepsilon = 2\times10^{-8}$ m-R at 50 GeV, $\beta^* = .1$ cm) we obtain $L = 10^{28}/\mathrm{cm}^2$-sec which is sufficient to observe the Z_o, but not quite competitive with SLC. A higher intensity muon source and a higher brightness final muon beam can provide $\mu^{\pm}$ collider parameters competitive with single pass $e^{\pm}$ colliders, particularly at higher energies.

We note here that the short lifetime of muons is in some ways an advantage, in that the small collision tune shifts and large aperture (20 σ) needed for e^+e^- storage ring colliders can be avoided.

D. Rapid Cycling Muon Accelerator - Storage Rings

The short lifetimes of muons would seem to preclude conventional synchrotron acceleration of muons. However, at high energies ($\gtrsim$100 GeV) the requirements of a muon accelerator do not seem unreasonable, particularly when compared with the requirements of e^+e^- storage rings at $E \gtrsim 50$ GeV.

The basic requirements are a storage ring which can cycle from low to full energy within ~100 turns with an RF system which can accelerate at $\gtrsim$.01 of full energy per turn. The RF requirement is met by only ~10 GeV/turn of acceleration for a 1 TeV $\mu^+ \mu^-$ collider (100 GeV $e^\pm$ storage rings need ~10 GeV of RF) and the rapid cycling requirement (for 1 TeV) is not as great as that in rapid cycling proton synchrotrons (60 Hz, 16 GeV K-factories). Conventional rather than superconducting magnets are preferred for rapid cycling.

The components of a 1 TeV $\mu^+ \mu^-$ collider are outlined in Figure 9. They are: a rapid-cycling proton synchrotron (100 Hz) to produce high intensity μ beams, a μ cooler at ~1 GeV, a ~20 GeV Linac for μ injection and an ~3 km-radius "rapid-cycling" muon storage ring. Luminosities of $\gtrsim 10^{32}$ cm^{-2} sec^{-1} seem practical with little extrapolation from readily achievable parameters. With $f \cong 100$ Hz x 300 turns storage, $n_B N^+ N^- \cong 10^{22}$ (100 bunches of $3\text{x}10^9$ μ^+, μ^-), $\beta^* = 0.1$ cm, $\varepsilon_{rms} = 10^{-9}\pi$ m-R at 1 TeV, we obtain $L \cong 3\text{x}10^{33}$ cm^{-2} sec^{-1}. Some of these parameters may be mildly optimistic, but the $\mu^+ \mu^-$ accelerator seems reasonably attractive.

E. μ-p Collider

A significant advantage of muons (over electrons) is that they can be stored in the same storage ring as protons for lepton-proton collisions. In Figure 8 the storage ring could contain protons before μ injection at full energy or in Figure 9 protons could be injected with μ^- for acceleration. Luminosities can be higher than in $\mu^+ \mu^-$ scenarios because of the larger number of protons, and as discussed above it is easy to match μ and p emittances.

The revolution frequency is naturally mismatched because of the different velocities at equal energies. However they could be rematched by displacing the two beams in energy under the condition

$$\frac{\Delta p}{p}\left[\frac{1}{\gamma_T^{\,2}} - \frac{1}{\gamma^2}\right] \cong \frac{1}{2\,\gamma_{proton}^{\,2}} - \frac{1}{2\,\gamma_\mu^{\,2}} \tag{27}$$

where γ_T is the transition energy, γ_μ, γ_p are the muon and proton kinetic factors, $\frac{\Delta p}{p}$ is the momentum offset. This works in a highly relativistic system with $\gamma_p >> \gamma_T >> 1$.

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Table 1

Properties of Some Absorber Materials

Material	Density $\rho\ \frac{gm}{cm^3}$	Mean Ionization Energy I(eV)	$\frac{dE}{dz}$ Minimum Energy Loss (MeV/cm)	L_{Rad} Radiation Length (cm)	$\frac{dE}{dz} \cdot L_{Rad}$ (MeV)
H_2	.071	18	0.29	890	260
Li	0.53	40	0.84	155	130
Be	1.85	64	2.97	35.3	105
C	2.27	80	4.03	18.8	76
Cu	9.0	315	12.9	1.43	18.5
W	19.3	750	22.4	0.35	7.8
Pb	11.4	790	12.8	0.56	7.2

Table 2

Muon Cooling
Storage Ring Parameters

μ Storage Energy	1 GeV
Circumference	30 m
β_{min} (minimum β value)	~1 cm
Absorber Sections	3x4m (FDOODF)
RF Requirements	18 MV/turn
Transverse Cooling	$100\pi \rightarrow 10\pi$
Longitudinal Cooling	$\pm 5\% \rightarrow \sim \pm .2\%$
Cooling Time	$\lessapprox$ 200 turns
Absorber	3x2 cm Be
Decay Channel Length	60 m

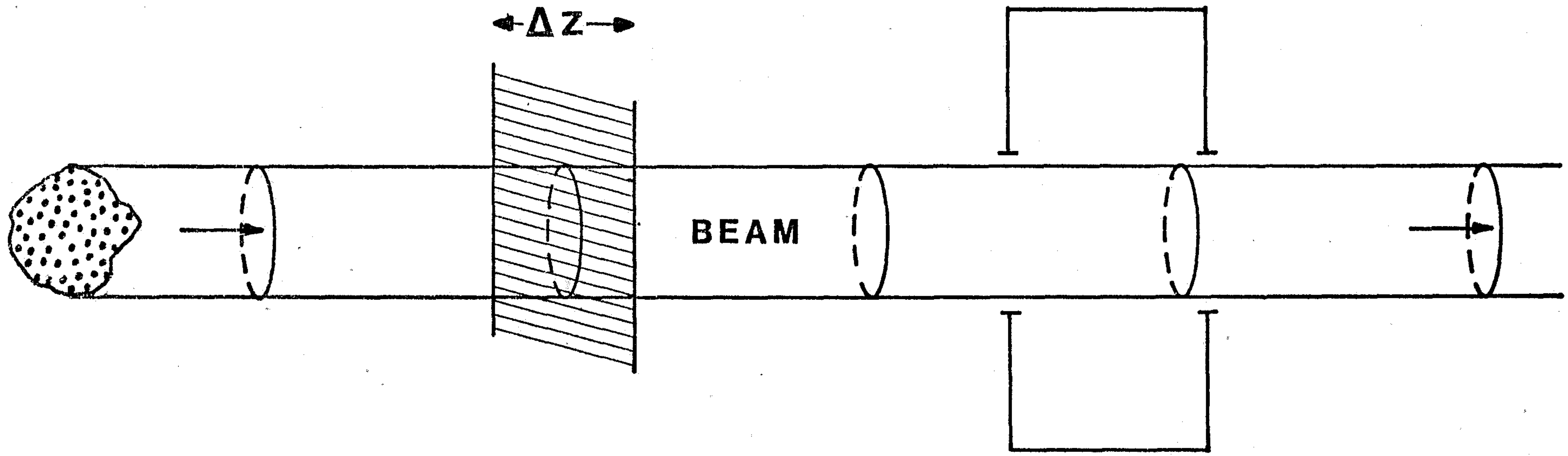
Table 3

Muon Cooler Linac Parameters

Muon Energy	500 MeV
Total Linac Acceleration	1.5 GeV
Total Length	1 km
Transverse Cooling	$100\,\pi \rightarrow 7\,\pi$ mm-mr
Momentum Cooling	$\pm 6\% \rightarrow \pm .5\%$
Focusing Period	2 m
Magnet Aperture	$.2\,\text{m} \rightarrow .05\,\text{m}$
β_{min} (Absorber Length = 2β/Focusing Period)	$.01 \rightarrow .0007$
Absorber	Be

FIGURE 1

SKETCH OF "IONIZATION COOLING" PRINCIPLE



ABSORBER

ENERGY LOSS IS

$\frac{dE}{dz}\Delta z$

AND DEPENDS ON
MUON ENERGY

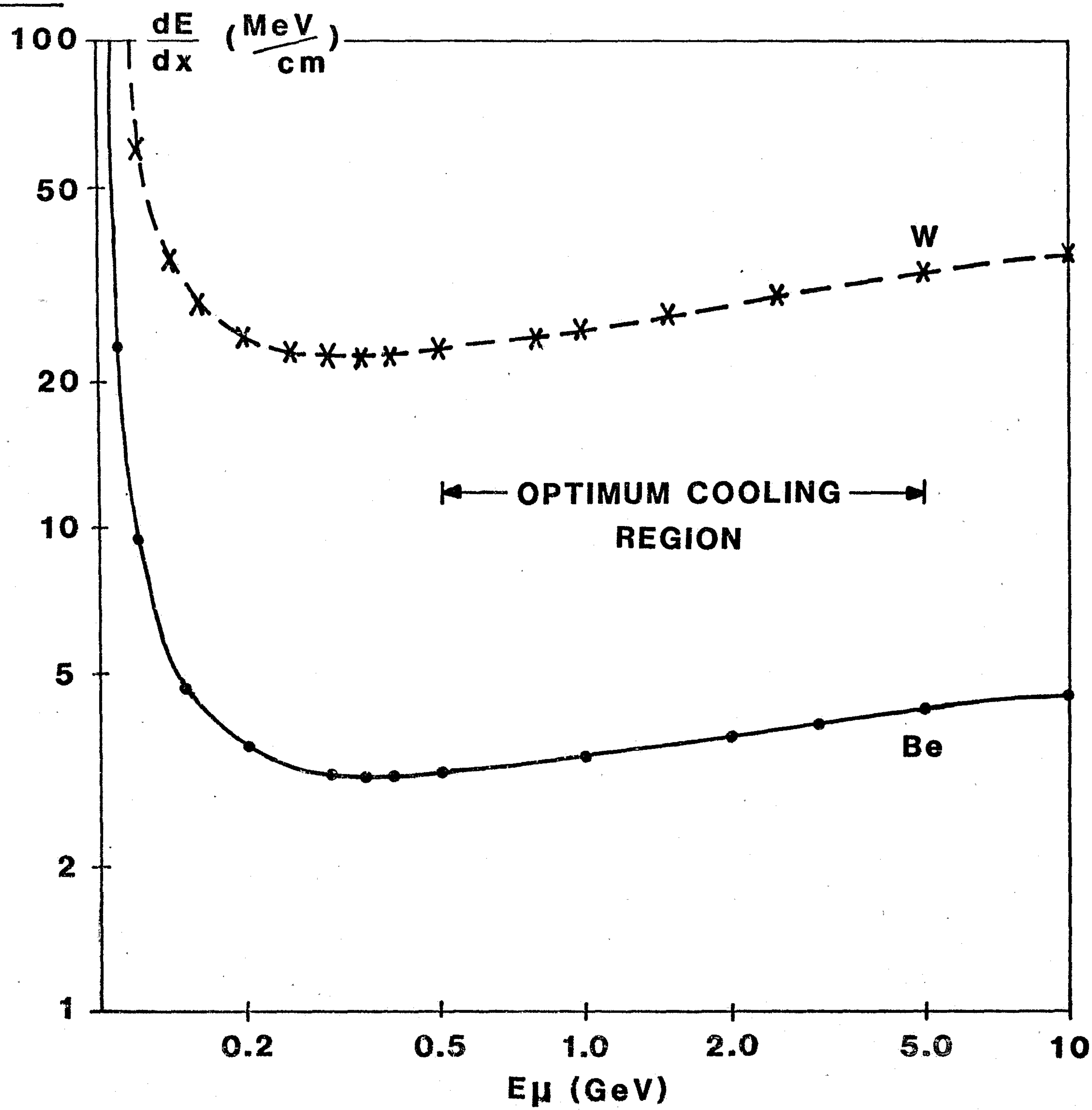
ACCELERATOR

ENERGY GAIN IS

$\overline{\Delta E}$

INDEPENDENT
OF MUON ENERGY

FIGURE 2



dE/dx (MeV/cm)
100
50
20
10
5
2
1
W
OPTIMUM COOLING REGION
Be
0.2
0.5
1.0
2.0
5.0
10
Eμ (GeV)

FIGURE 3

USE OF VARYING THICKNESS ABSORBER TO ENHANCE ENERGY DEPENDENCE OF ENERGY LOSS

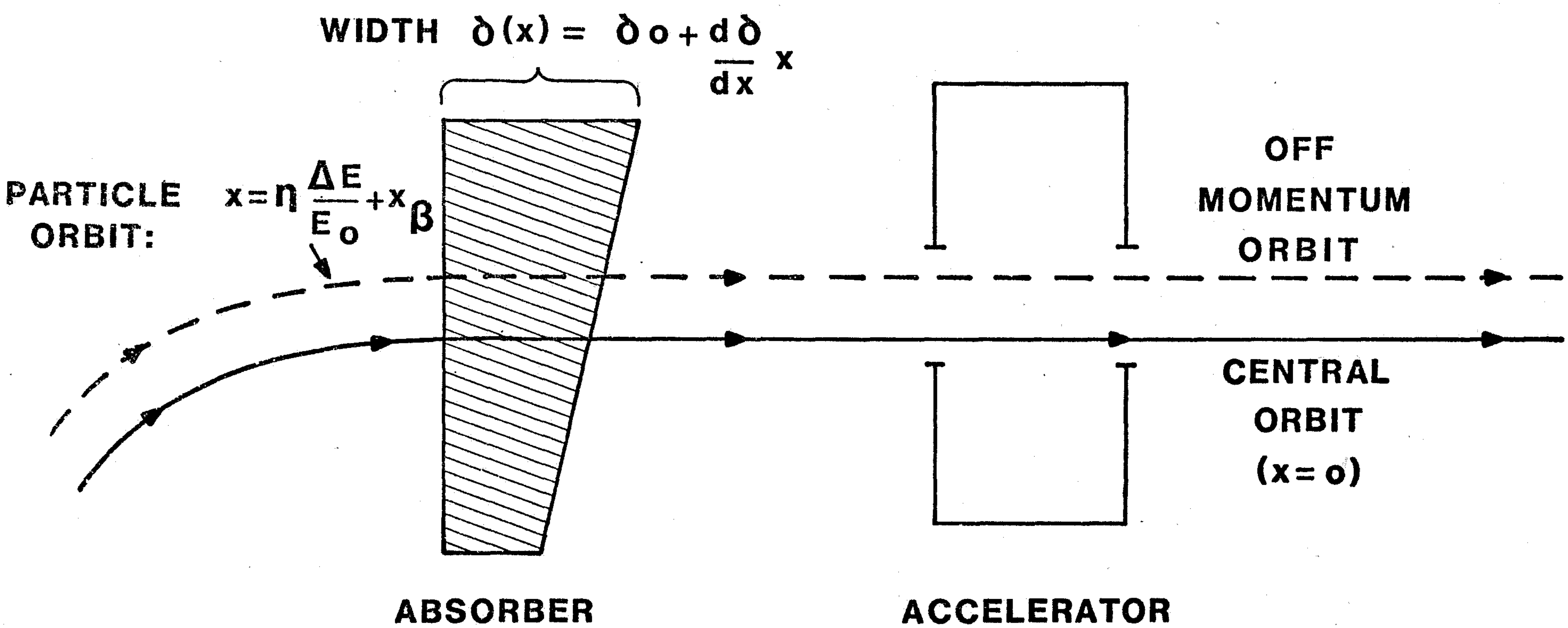
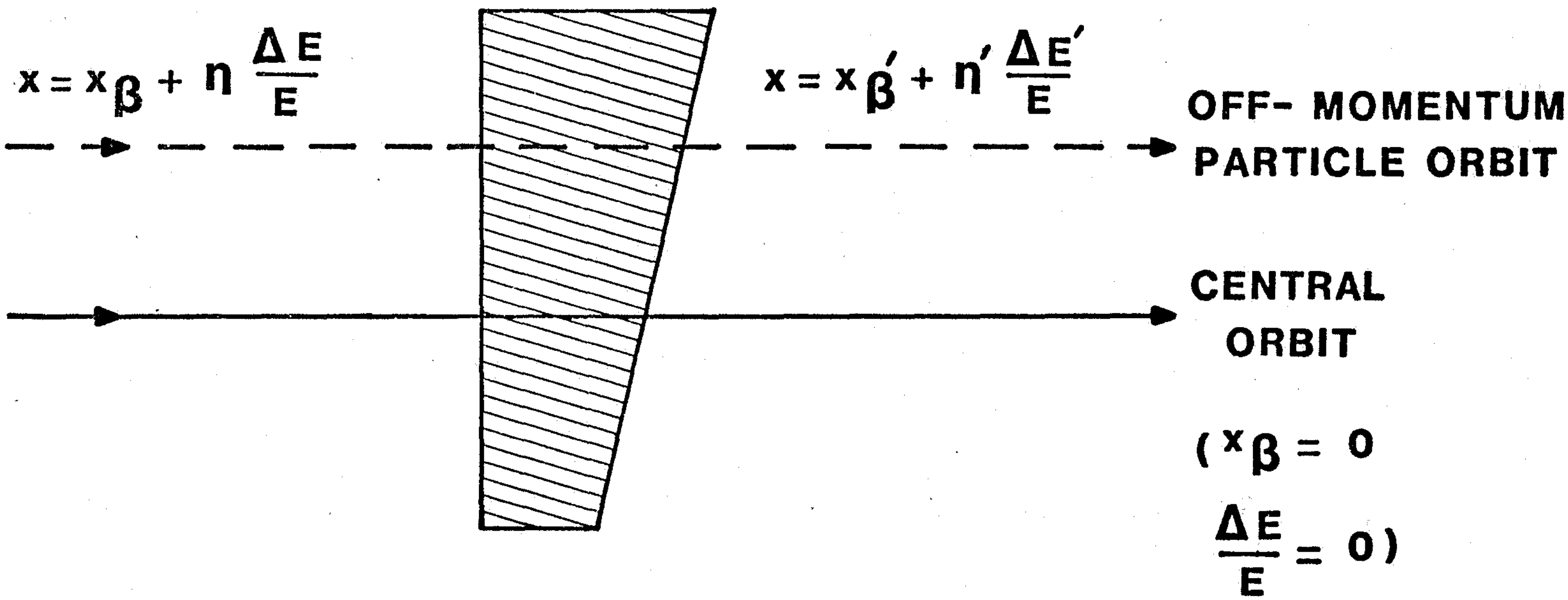


FIGURE 4

CHANGE IN COURANT- SNYDER BETATRON AMPLITUDE IN PASSING THROUGH A "WEDGE" ABSORBER AT $\eta \neq 0$



$$x_\beta' = x_\beta + \eta \frac{\Delta E}{E_0} - \frac{\eta' \Delta E'}{E_0}$$

FIGURE 5 SKETCH OF LOW-BETA ABSORBER SECTION (FDOODF DESIGN)

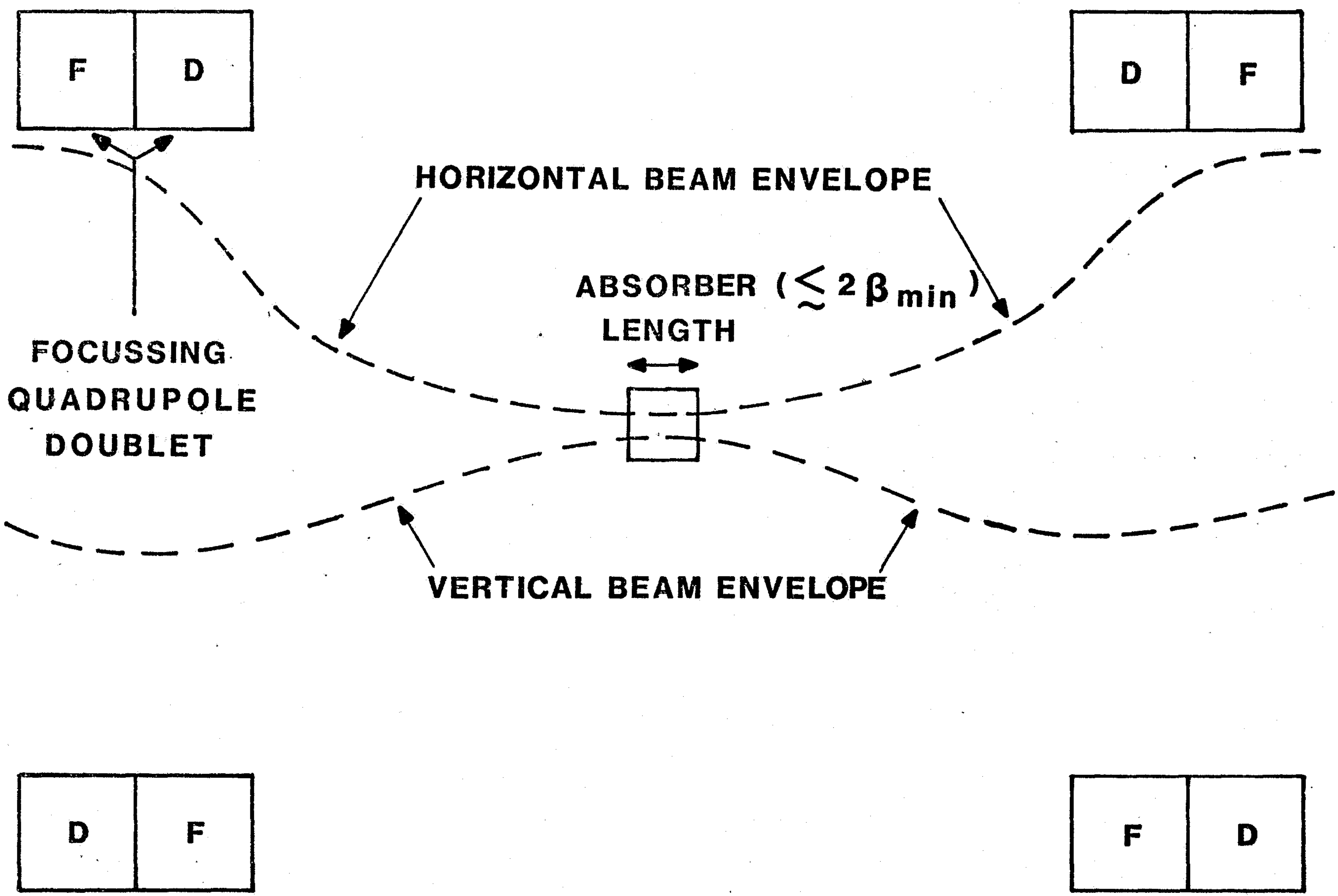


FIGURE 6 MUON COOLING RING

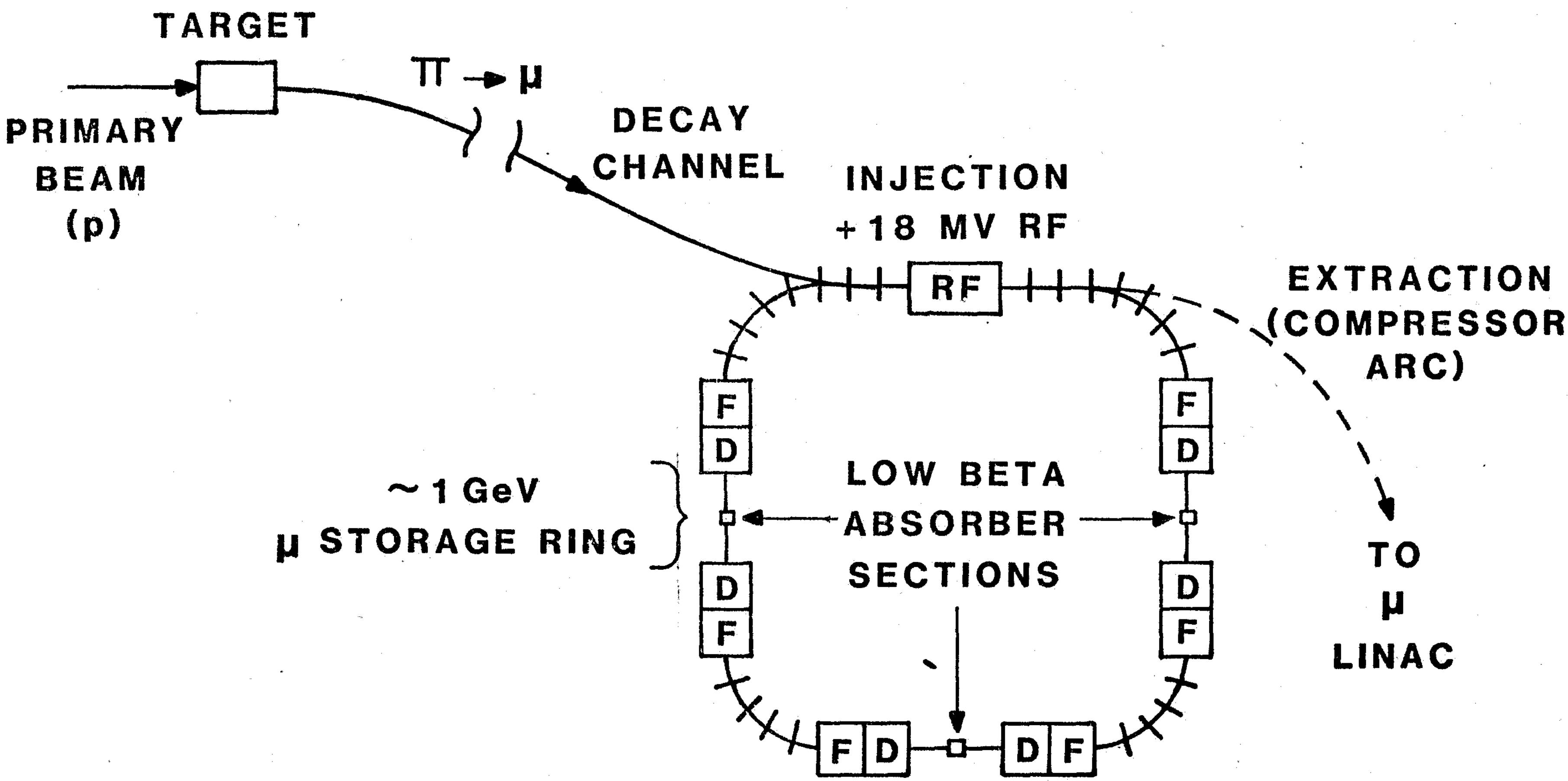


FIGURE 7

SKETCH OF MUON COOLING LINAC

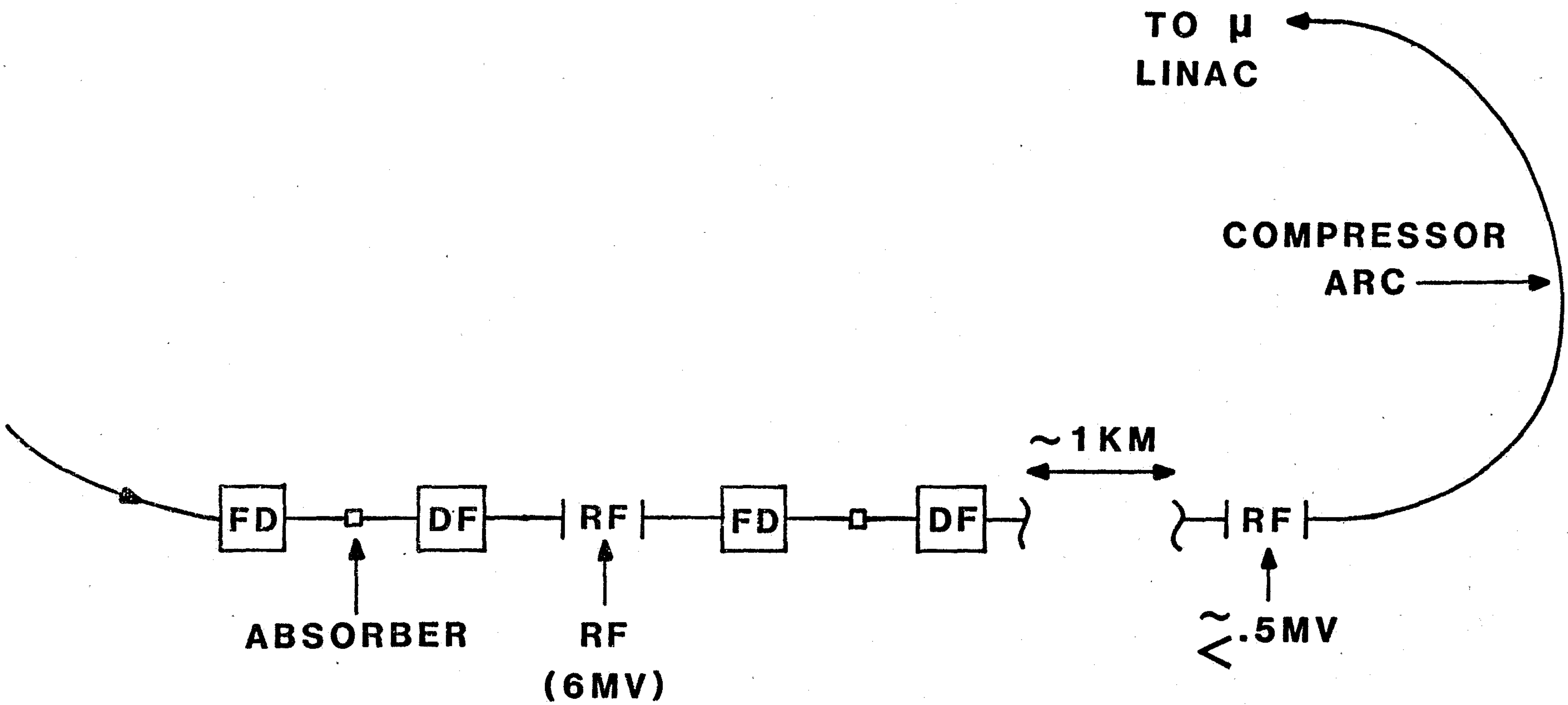


FIGURE 8

A SAMPLE LINAC-STORAGE RING SYSTEM

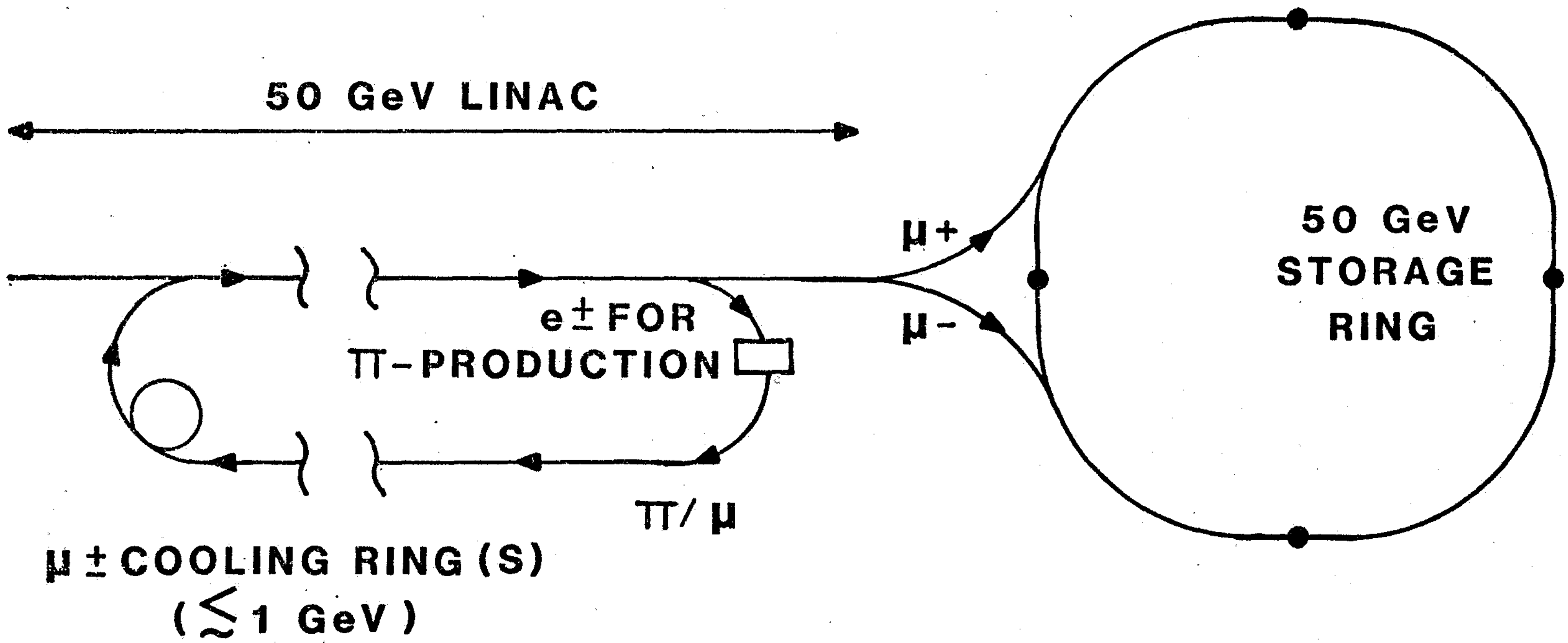


FIGURE 9 1 TEV μ RAPID CYCLING SYNCHROTRON