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Simulations of the Beam-Beam Interaction with Transient or Stationary Beam Displacement

D. Neuffer, A. Riddiford, A.G. Ruggiero

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Abstract

Results of beam-beam interaction simulations with a constant or transient or periodic displacement of the beam centers are described. No "beam blow-up" or significant emittance increase was observed. We conclude that the beam-beam interaction does not make colliding beams intrinsically unstable in constant or transient displacements at Tevatron parameters[1].

Introduction

During colliding beam operation, significant displacements of colliding beam centers will occur either unintentionally due to collision misalignment errors or intentionally with deliberate beam separation. In particular, it is conceivable to separate the beams in acceleration, and gradually to reduce their separation to centered collisions at full energy. It is important to determine whether beam instability occurs with constant or transient beam displacement.

In previous simulations of beam-beam interactions in the Tevatron[2,3], the centers of the colliding beams were chosen to coincide. Simulations of two types of beam separation are reported here. First, a constant separation of beams was chosen, with separations of 0.1, 0.3, 0.5, 1., and 2. times σ, the rms beam size. The lower values correspond to possible misalignment errors, the larger values to possible deliberate separations at unwanted crossings. Second, a transient separation was chosen, in which initially separated beams are combined over a finite number of turns (1,000 to 50,000) by reducing their separation to zero. This simulates the reduction of separation at full energy described above.

Simulation Procedure

In our simulations we approximate particle circulation around the ring as the product of two transformations: a linear transport around the storage ring followed by a nonlinear beam-beam "kick" at the interaction area.

Transport around the ring can be represented by 2x2 matrices for both transverse (x and y) dimensions:

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{After} = \begin{bmatrix} \cos 2\pi\nu_x & \beta_x \sin 2\pi\nu_x \\ -\dfrac{\sin 2\pi\nu_x}{\beta_x} & \cos 2\pi\nu_x \end{bmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_{Before} \quad (1)$$

In this linear transport x and y motion are decoupled. ν_x, ν_y, β_x, β_y are the usual Courant-Snyder tunes and beta-functions[4]. The beam-beam kick can be represented as

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{After} = \begin{bmatrix} 1 & 0 \\ -\dfrac{4\pi\Delta\nu_x}{\beta_x} F_x(x,y) & 1 \end{bmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_{Before} \quad (2)$$

with a similar expression for y, y'.

The product of these transformations is equivalent to integration of the equation of motion:

$$x'' + K_x(s)\ x = -\frac{4\pi\Delta\nu_x}{\beta_x} F_x(x,y)\ x\ \delta_p(s) \quad (3)$$

s, the distance along the storage ring, is the independent variable, $\delta_p(s)$ is a periodic delta-function, and $K_{x,y}(s)$ is the focusing function.

In the present report we choose parameters which approximate the conditions[1] in the Tevatron: $\beta_x=\beta_y=2$ m and we choose

$$F_x = F_y = \frac{1 - e^{-(x^2+y^2)/2\sigma^2}}{(x^2+y^2)/2\sigma^2} \tag{4}$$

with $\sigma=0.0816$ mm, which is the nonlinear force due to a round, gaussian charge distribution of rms radius σ.

Beam displacement is simulated by replacing x by $x+x_o$ in the beam-beam force, where x_o is the displacement of the beam centers. $F_x=F_y$ is replaced by

$$F = \frac{1 - e^{-((x+x_o)^2+y^2)/2\sigma^2}}{((x+x_o)^2 + y^2)/2\sigma^2} \tag{5}$$

and $x \rightarrow x + x_o$ on the right hand side of equation (3) and $x \rightarrow x + x_o$ in equation (2) so that

$$\begin{pmatrix} x + x_o \\ x' \end{pmatrix}_{After} = \begin{bmatrix} 1 & 0 \\ \frac{4\pi\Delta\nu_x F(x,x_o,y)}{-\beta_x} & 1 \end{bmatrix} \begin{pmatrix} x + x_o \\ x' \end{pmatrix}_{Before}$$

with a similar equation for y, y' except $y_o=0$ is always chosen. x_o is made a function of time to simulate transient displacement.

In the simulations, 100 particle trajectories are calculated with initial positions (x, x', y, y') chosen randomly, within a 4-D gaussian distribution determined by the rms size parameter σ and, as described in reference 1, transported following the above matrices.

Every two thousand turns emittance values ε_x, ε_y, ε_R are calculated using

$$\varepsilon_x = 6 \sqrt{<(x-\bar{x})^2><(x-\bar{x})^2>}$$
$$\varepsilon_y = 6 \sqrt{<(y-\bar{y})^2><(y'-\bar{y}')^2>}$$
$$\varepsilon_R = \sqrt{\varepsilon_x^2 + \varepsilon_y^2}$$

and changes in these values as functions of time are followed.

"Doubling" times for X, Y, and R emittances are obtained from the slopes of the best straight line fits for ε_x, ε_y, and ε_R as functions of time from t=0, using rms emittance values calculated every 2000 turns, as described in reference 2 and 3; a slope b is obtained for each parameter. Also from the statistical fluctuations in emittances an error σ_b is obtained. A slope b is considered significantly different from zero if it is greater than σ_b in magnitude. A "doubling time" T_x is obtained by $T_x=\bar{X}/b_x$ and a "statistically significant doubling time" by $T_{\sigma_x}=\bar{X}/\sigma_{b_x}$.

We present two sets of simulations. In the first set the beams are displaced a constant distance, and six million turns are calculated. In the second set, the transient case, the beams are displaced by 6 σ for 200,000 turns, and the displacement then reduces to zero over 1-50 thousand turns. The centered beams are then tracked for another one million turns.

Results of Constant Displacement Simulations

In tables 1-5, we display the statistical analysis of the 6 million turn simulations with displacements $x_o=.1\ \sigma$, $.3\ \sigma$, $.5\ \sigma$, σ, and $2\ \sigma$. The tunes ν_x, ν_y were chosen at $\nu_x=.3439$, $\nu_y=0.1772$, in a "resonance-free" region and $\Delta\nu=.01$, $\sqrt{3}$ times larger than expected for p-$\bar{p}$ collisions.

As can be seen from the tables, the observed doubling times are greater than the "statistical error times" indicating that beam displacement does not lead to a linear change in ϵ_x, ϵ_y or ϵ_r in time; that is, no beam blow-up occurs.

There is one noticeable effect. With increasing x_o in this set of simulations, the mean value of x-emittance increases slightly while the mean value of y-emittance decreases slightly. The mean value of r-emittance remains unchanged.

This effect is shown in Table 6 and displayed graphically in Figure 1. The effect is established within the first 200,000 turns; we note here that all simulations use the same 100 initial particle positions and therefore start with precisely the same emittances. The net effect is an apparent exchange between x and y emittances to make the beam "rounder" ($\varepsilon_x=\varepsilon_y$) when the collisions become more off-center, at least up to 2 σ. The net effect is up to 2% exchange at $x_o=2\sigma$, of the same magnitude as the x and y emittance difference at the start of the simulation.

Simulations with a Transient Displacement

In Table 7, we display results of the simulations with a transient beam displacement. The results show no significant changes in any of the beam parameters, and show that there is no beam blow-up in the transition from "separated" beams (6σ separation) to colliding beams, independent of the transition rate.

Simulations with a Periodic Displacement

Another possible type of beam displacement is a periodic displacement which could be due to sources such as power supply ripple or synchrotron oscillations with a non-zero dispersion crossing. We have undertaken two simulations to test this possibility. In these, beam displacement varied as

$$x_o(t) = x_{max} \sin \frac{2\pi n(t)}{N}$$

where x_{max} is the maximum oscillation amplitude, n is the turn number and N is the oscillation period. We chose N=1000 to approximate expected periods of synchrotron oscillations and power supply ripple in the Tevatron. We chose two values of x_{max}:

a) x_{max}=0.02 mm which is $\sim\sigma/4$

b) x_{max}=0.1 mm which is $\sim 1.25\sigma$

The results of the simulations are displayed graphically in Figures 2 and 3. Each simulation included a total of 6 million turns, simulations of $\sim$2 Tevatron minutes.

We see no significant changes in x,y, or r emittance, and extrapolate emittance doubling times of >1 day. This demonstrates that periodic crossing modulation with the beam-beam interaction does not lead to large beam blow-up at these parameters.

References

1. The Fermilab Antiproton Source Design Report, February 1982.

2. D. Neuffer, A. Riddiford, A. Ruggiero, FN-333, Fermilab, April 1981.

3. D. Neuffer, A. Riddiford, A. Ruggiero, FN-343, Fermilab, July 1981.

4. E.D. Courant and H.S. Snyder, Annals of Physics, 3 , 1 (1958).

TABLE 1 Emittance data for Case C/X0 = 0.1 σ, σ = 0.08165 mm, ν_x = 0.3439, ν_y = 0.1772, $\Delta\nu$ = 0.01

Million Turns	X Average (mm-mrad)	T_X Doubling Time-X (days)	Y Average (mm-mrad)	T_Y Doubling Time-Y (days)	R Average (mm-mrad)	T_R Doubling Time-R (days)	Correlation Coefficient Bar Average	Correlation Coefficient Cumulative
0.2		(±0.0111)		(±0.00933)		(±0.0137)		
0.2	0.0175712	-0.0518	0.0187775	-0.0150	0.0257176	-0.0224	0.0640	0.0640
1.0	0.0175833	0.182	0.0187683	-0.0794	0.0257191	-0.242	0.0233	-0.0124
2.0	0.0175908	0.378	0.0187605	-0.233	0.0257185	-0.969	-0.1118	0.0082
3.0	0.0175859	-1.27	0.0187604	-1.17	0.0257151	-1.22	0.0013	0.0148
4.0	0.0175825	-0.802	0.0187601	-4.05	0.0257126	-1.40	0.0116	0.0041
5.0	0.0175844	-9.66	0.0187612	6.84	0.0257147	34.8	0.1497	0.0275
5.2	0.0175851	9.17	0.0187601	-6.10	0.0257144	-26.7	0.0466	0.0277
5.4	0.0175852	9.17	0.0187604	-13.1	0.0257146	87.8	-0.0768	0.0233
5.6	0.0175861	3.19	0.0187609	18.2	0.0257156	5.66	-0.1233	0.0196
5.8	0.0175872	1.78	0.0187614	5.92	0.0257168	2.84	0.1570	0.0248
6.0	0.0175868	2.42	0.0187615	5.71	0.0257166	3.49	-0.1904	0.0185
6.0		(±1.79)		(±1.69)		(±2.43)		

TABLE 2 Emittance data for Case C/X0 = 0.3σ, σ = 0.08165 mm, ν_x = 0.3439, ν_y = 0.1772, $\Delta\nu$ = 0.01

Million Turns	X Average (mm-mrad)	T_X Doubling Time-X (days)	Y Average (mm-mrad)	T_Y Doubling Time-Y (days)	R Average (mm-mrad)	T_R Doubling Time-R (days)	Correlation Coefficient Bar Average	Correlation Coefficient Cumulative
0.2		(±0.00928)		(±0.00978)		(±0.0135)		
0.2	0.0176536	-0.00811	0.0186662	-0.0113	0.0256932	-0.00962	0.0128	0.0128
1.0	0.0176617	0.250	0.0186773	-0.199	0.0257069	-1.40	-0.0925	-0.0626
2.0	0.0176735	0.278	0.0186859	0.648	0.0257211	0.400	0.0057	-0.0525
3.0	0.0176714	18.2	0.0186925	0.401	0.0257245	0.750	-0.0433	-0.0618
4.0	0.0176707	-17.3	0.0186914	1.32	0.0257233	2.70	-0.0302	-0.0635
5.0	0.0176648	-0.671	0.0186881	-2.66	0.0257168	-1.11	0.0342	-0.0346
5.2	0.0176630	-0.547	0.0186882	-3.31	0.0257156	-0.977	0.0344	-0.0320
5.4	0.0176636	-0.686	0.0186871	-1.85	0.0257152	-1.03	0.0860	-0.0281
5.6	0.0176634	-0.735	0.0186865	-1.58	0.0257147	-1.02	-0.1940	-0.0347
5.8	0.0176622	-0.658	0.0186853	-1.20	0.0257130	-0.864	0.0316	-0.0313
6.0	0.0176626	-0.782	0.0186839	-0.922	0.0257122	-0.852	-0.1087	-0.0342
6.0		(±1.70)		(±1.56)		(±2.34)		

TABLE 3   Emittance data for Case $C/X_o = 0.5\,\sigma$, $\sigma = 0.08165$ mm, $\nu_x = 0.3439$, $\nu_y = 0.1772$, $\Delta\nu = 0.01$

Million Turns	X Average (mm-mrad)	T_X Doubling Time-X (days)	Y Average (mm-mrad)	T_Y Doubling Time-Y (days)	R Average (mm-mrad)	T_R Doubling Time-R (days)	Correlation Coefficient Bar Average	Correlation Coefficient Cumulative
0.2		(±0.00983)		(±0.00904)		(±0.0135)		
0.2	0.0177637	-0.00734	0.0186398	-0.123	0.0257499	-0.00931	-0.0141	-0.0141
1.0	0.0177518	-0.0809	0.0186176	-0.508	0.0257259	-0.145	-0.0833	-0.0577
2.0	0.0177595	0.737	0.0186249	1.13	0.0257365	0.891	-0.1648	-0.1014
3.0	0.0177587	-2.50	0.0186291	0.535	0.0257389	1.26	-0.1101	-0.1080
4.0	0.0177583	-4.86	0.0186260	-125	0.0257364	-9.88	-0.0664	-0.0894
5.0	0.0177581	-57.5	0.0186241	-1.89	0.0257349	-3.50	-0.1738	-0.0921
5.2	0.0177591	4.33	0.0186235	-1.59	0.0257351	-4.54	-0.0688	-0.0914
5.4	0.0177600	2.28	0.0186232	-1.56	0.0257355	-7.81	-0.1069	-0.0920
5.6	0.0177602	2.22	0.0186239	-2.52	0.0257362	269.	-0.1637	-0.0942
5.8	0.0177595	4.75	0.0186256	39.1	0.0257369	9.04	-0.0631	-0.0936
6.0	0.0177595	4.78	0.0186260	8.15	0.0257373	6.22	-0.2348	-0.0976
6.0		(±1.56)		(±1.50)		(±2.27)		

TABLE 4 Emittance data for Case $C/X_o = \sigma$, $\sigma = 0.08165$ mm, $\nu_x = 0.3439$, $\nu_y = 0.1772$, $\Delta\nu = 0.01$

Million Turns	X Average (mm-mrad)	T_X Doubling Time-X (days)	Y Average (mm-mrad)	T_Y Doubling Time-Y (days)	R Average (mm-mrad)	T_R Doubling Time-R (days)	Correlation Coefficient Bar Average	Correlation Coefficient Cumulative
0.2		(±0.00781)		(±0.00865)		(±0.0123)		
0.2	0.0179439	-0.0139	0.0184924	-0.0109	0.0257692	-0.0123	-0.106	-0.106
1.0	0.0179583	0.0858	0.0185040	-0.582	0.0257878	0.205	-0.250	-0.201
2.0	0.0179522	-0.874	0.0185084	31.9	0.0257867	-1.89	-0.342	-0.200
3.0	0.0179499	-0.744	0.0185096	2.11	0.0257859	-2.34	-0.186	-0.193
4.0	0.0179490	-1.12	0.0185070	-1.42	0.0257834	-1.28	-0.246	-0.201
5.0	0.0179474	-1.36	0.0185097	3.88	0.0257844	-4.76	-0.118	-0.201
5.2	0.0179473	-1.45	0.0185084	-9.04	0.0257834	-2.66	-0.276	-0.204
5.4	0.0179492	-9.31	0.0185091	20.8	0.0257851	-52.9	-0.069	-0.200
5.6	0.0179494	-14.4	0.0185101	3.28	0.0257860	7.65	-0.419	-0.206
5.8	0.0179495	-63.7	0.0185081	-5.12	0.0257846	-9.84	-0.182	-0.206
6.0	0.0179489	-5.97	0.0185023	-7.39	0.0257843	-6.95	-0.180	-0.205
6.0		(±1.30)		(±1.32)		(±2.09)		

TABLE 5 Emittance data for Case $C/X_o = 2\sigma$, σ = 0.08165 mm, ν_x = 0.3439, ν_y = 0.1772, $\Delta\nu$ = 0.01

Million Turns	X Average (mm-mrad)	T_X Doubling Time-X (days)	Y Average (mm-mrad)	T_Y Doubling Time-Y (days)	R Average (mm-mrad)	T_R Doubling Time-R (days)	Correlation Coefficient Bar Average	Correlation Coefficient Cumulative
0.2		(±0.00744)		(±0.00744)		(±0.0125)		
0.2	0.0179738	-0.0104	0.0183784	-0.0215	0.0257091	-0.0141	-0.286	-0.286
1.0	0.0180010	0.138	0.0183733	3.56	0.0257247	0.274	-0.194	-0.253
2.0	0.0179969	-1.17	0.0183802	0.802	0.0257263	4.45	-0.252	-0.250
3.0	0.0179993	1.79	0.0183821	1.59	0.0257293	1.70	-0.336	-0.272
4.0	0.0179949	-0.996	0.0183874	0.722	0.0257301	4.66	-0.283	-0.282
5.0	0.0179911	-0.777	0.0183903	0.719	0.0257294	14.9	-0.216	-0.279
5.2	0.0179913	-0.921	0.0183901	0.843	0.0257294	15.0	-0.203	-0.275
5.4	0.0179909	-0.944	0.0183918	0.668	0.0257304	4.22	-0.215	-0.273
5.6	0.0179909	-1.05	0.0183907	0.922	0.0257296	12.3	-0.475	-0.280
5.8	0.0179920	-1.67	0.0183913	0.856	0.0257309	3.35	-0.376	-0.283
6.0	0.0179917	-1.65	0.0182918	0.892	0.0257309	3.68	-0.280	-0.282
6.0		(±1.36)		(±1.25)		(±2.17)		

Table 6

Six Million Turn Mean Emittances

Displacement	$\bar{\varepsilon}_x$	$\bar{\varepsilon}_y$	$\bar{\varepsilon}_R$
0.1 σ	.017587	.018762	.025717
0.3 σ	.017663	.018684	.025712
0.5 σ	.017760	.018626	.025737
1.0 σ	.017949	.018508	.025784
2.0 σ	.017992	.018292	.025731

TABLE 7 Emittance data for transient XO. For 200,000 turns XO = 6σ; σ = 0.08165 mm. Then XO was brought linearly to zero in 1000; 10,000; 50,000 turns. The emittances were followed for another million turns. The transient numbers below refer to the transient turns only. The sums were begun again at the end of the transient, and are then cumulative.

Thousand Turns	X Average (mm-mrad)	Doubling Time-X (days)	Significant Time-X (days)	Y Average (mm-mrad)	Doubling Time-Y (days)	Significant Time-Y (days)	R Average (mm-mrad)	Doubling Time-R (days)	Significant Time-R (days)
1T	0.0178043	0.000174	0.000111	0.0184525	-0.000068	0.00048	0.0256426	-0.000214	0.000117
201	0.0175502	0.0120	0.107	0.0187033	0.0076	0.123	0.0256494	0.0093	0.176
401	0.0175461	-0.235	0.108	0.0186673	-0.0174	0.113	0.0256202	-0.0304	0.155
601	0.0175560	0.0949	0.114	0.0186683	-0.0589	0.113	0.0256276	-0.235	0.156
801	0.0175603	0.0806	0.112	0.0186791	0.139	0.118	0.0256383	0.105	0.159
1001	0.0175545	-6.09	0.111	0.0186771	3.08	0.117	0.0256330	16.5	0.159
10T	0.0176651	-0.000527	0.000633	0.0185461	0.000081	0.000466	0.0256142	0.000180	0.000823
210	0.0175219	0.00678	0.100	0.0186992	0.00559	0.110	0.0256269	0.00614	0.146
410	0.0175556	0.0145	0.114	0.0186787	0.411	0.110	0.0256349	0.0301	0.159
610	0.0175620	0.0305	0.115	0.0186801	0.343	0.111	0.0256403	0.0599	0.159
810	0.0175659	0.0506	0.119	0.0186880	0.0791	0.111	0.0256487	0.0631	0.160
1010	0.0175650	0.101	0.121	0.0186863	0.160	0.107	0.0256469	0.126	0.158
50T	0.0176049	0.00158	0.00242	0.0186692	0.000713	0.00223	0.0256624	0.000959	0.00385
250	0.0176341	-0.0163	0.141	0.0185647	0.00530	0.0959	0.0256061	0.0144	0.165
450	0.0176041	-0.0158	0.131	0.0185888	0.0101	0.107	0.0256103	0.0461	0.173
650	0.0176091	-0.0800	0.138	0.0186201	0.0168	0.106	0.0256291	0.0394	0.174
850	0.0176034	-0.0738	0.125	0.0186254	0.0352	0.105	0.0256291	0.116	0.169
1050	0.0176046	-0.205	0.127	0.0186330	0.0436	0.109	0.0256353	0.102	0.174

TABLE 7

Figure 1

Six million turn mean emittances as a function of beam displacement.

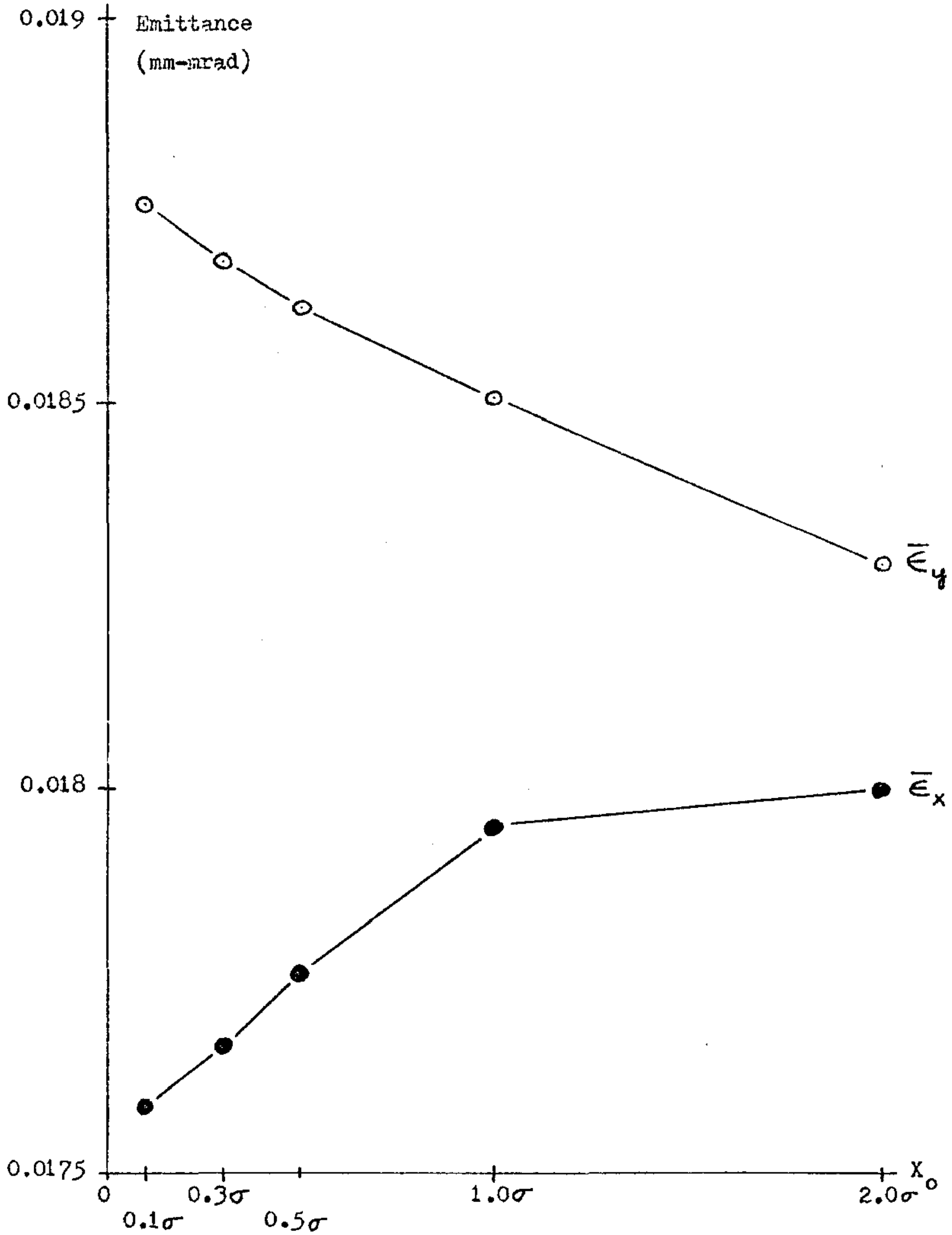
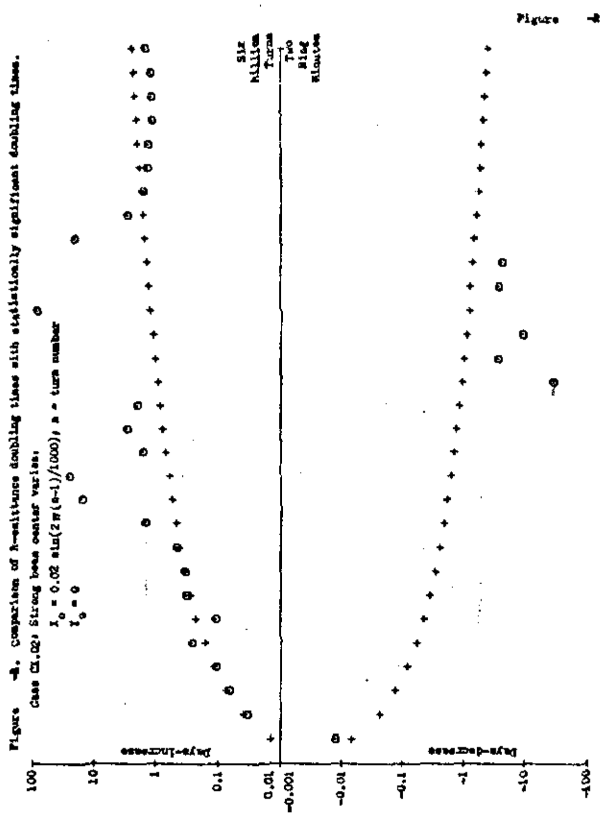
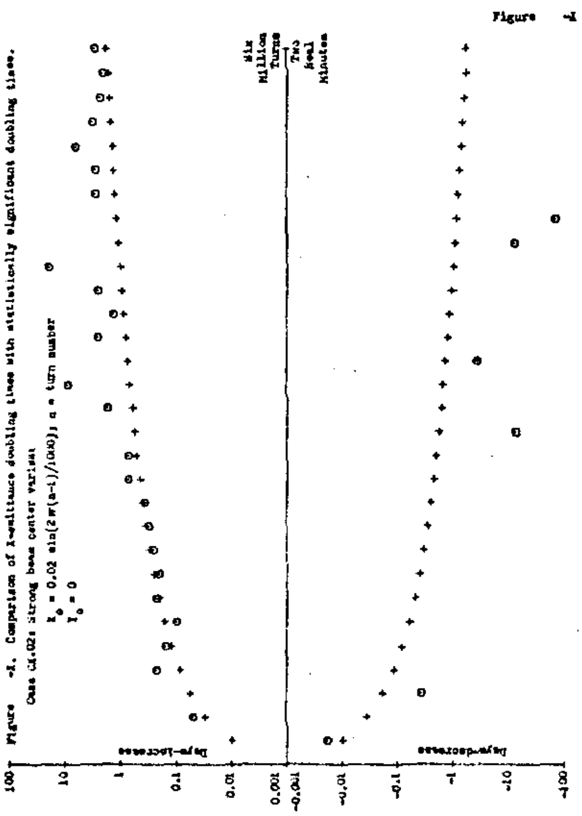


Figure 2





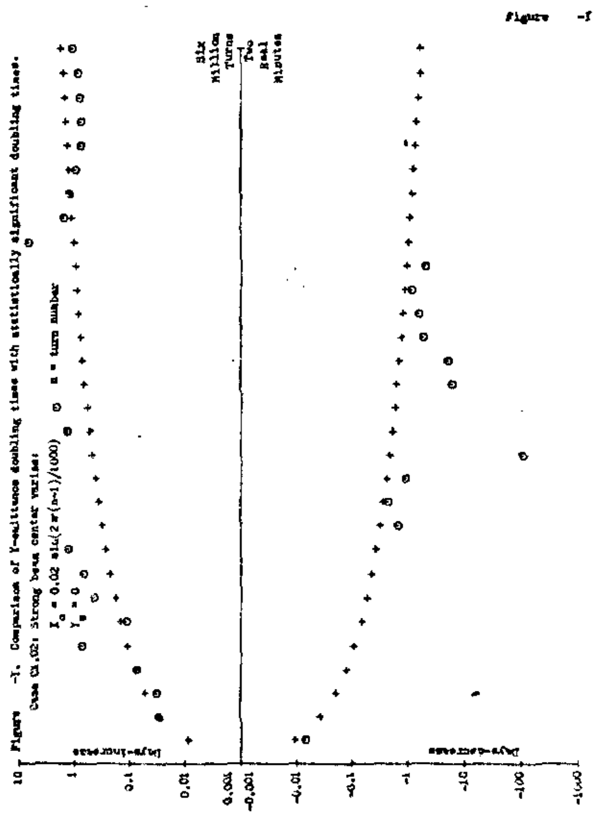


Figure 3