Fermi National Accelerator Laboratory

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Further Parametric Studies of the Accelerator System for Heavy Ion Fusion

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In earlier studies[1,2] we made the seemingly obvious assumption that the heaviest ion with the lowest charge state is most desirable. A closer examination showed that this is not entirely true. Furthermore, little attention was paid to the beam transport lines from the accelerator(s) to the reactor vessel. Simple calculations would reveal that the requirements of the bending dipoles in these transport lines and the final quadrupoles for focusing the beams onto the target are rather unrealistic for the examples[2] given. All these are re-examined in this paper.

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We consider here only the case in which final longitudinal bunching of the beam (in addition to the bunching derived during acceleration) is necessary to obtain the required current. In this case a final bunching ring is needed. In principle the beam could be bunched in the final transport lines on the way to the reactor vessel, but in practice to obtain a significant degree of bunching would require either too long a beam transport or too large a momentum spread. This bunching ring will also serve as a beam distributing ring. The ring can be filled by the beam from a single accelerator. After bunching, the beam segments are extracted from symmetric points around the circumference and transported to strike simultaneously and symmetrically a target located at the center of the ring.

Even if final bunching is not needed, as long as the accelerated beam comes out of a single accelerator from a single spigot (e.g. a linac), the beam must be either split spatially or sectionalized temporally into branches. The branches must then be transported around to strike the target simultaneously from all directions. It is easy to see that the total length and bend angle of all the transport lines would very likely add up to be larger than those of the distributing ring. Hence even in this case a distributing ring may prove to be the most convenient and economical way to distribute and transport the final beams around.

1. Geometry of the bunching/distributing ring

If one is limited by a maximum available bending dipole field intensity the most convenient and economical geometry for the case of n ($\geqslant 2$) beams striking the target is as shown in Fig. 1.

The various geometrical parameters of these figures are

Total bend angle = $(n+1)\pi$

Total arc length = $(n+1)\pi\rho$

Total straight length = $2n\rho \sin\frac{\pi}{n}$

Circumference of ring = $2\pi\rho(1+\frac{n}{\pi} \sin\frac{\pi}{n})$

where ρ = radius of arc.

Here we considered only the planar cases and assumed that it is adequate for the beams to strike the target symmetrically in a plane.

2. Targeting requirement

The targeting requirements are specified by the following 4 parameters:

W = Total energy on target

P = Power on target = Rate of energy deposited on target

E = Specific energy on target = Energy deposited per gram of target

r = Beam spot radius on target

In this paper we will consider two sets of these parameters.

Ion Fusion Power Plant (IFPP)[1]

$$\begin{cases} W = 10 \text{ MJ} \\ P = 600 \text{ TW} \\ E = 30 \text{ MJ/g} \\ r \geq 0.1 \text{ cm} \end{cases}$$

HIDE (Target Coupling Experimental System)

$$\begin{cases} W = 100 \text{ kJ} \\ P = 50 \text{ TW} \\ E = 300 \text{ kJ/g} \\ r \geq 0.1 \text{ cm} \end{cases}$$

3. Space charge effect

In the bunching/distributing ring the tune shift caused by space charge forces is given by[2]

$$\Delta\nu = \frac{(qe)^2}{mc^2} \frac{I}{qec} \frac{R}{\varepsilon_t/\pi} \frac{1}{(\beta\gamma)^2} \tag{1}$$

where

qe = charge of ion $\beta = \frac{1}{c}$ (speed)
m = mass of ion $\gamma = \frac{1}{mc^2}$ (total energy)
I = electric current of beam
ε_t/π = normalized transverse emittance of beam
$R = \frac{1}{2\pi}$ (circumference of ring).

First we transform $\Delta\nu$ into a form which is explicitly independent of the ion specie. We have

$$\frac{I}{qe} = \frac{P/n}{T} = \frac{P/n}{mc^2(\gamma-1)} = \text{particle current} \tag{2}$$

where n = number of beams on target, T = kinetic energy of ion, and

$$R = \frac{pc}{qeB_D} = \frac{mc^2}{qeB_D}\,\beta\gamma = \text{ring radius} \tag{3}$$

where p = momentum of ion, B_D = average bending dipole field in ring. In earlier studies we considered ε_t/π as given by the focusing requirement on target. This led to rather unreasonable demands on the final focusing quadrupoles. It is, therefore, more appropriate to consider ε_t/π as given by realistic final quadrupoles. For this, we have

$$\varepsilon_t/\pi = \beta\gamma r\theta \cong \beta\gamma r\frac{qeB_Q\ell_Q}{pc} = \frac{qeB_Q\ell_Q}{mc^2}\,r \tag{4}$$

where θ = half convergent angle on target, $B_Q\ell_Q$ = pole field times length of final quadrupole. Here we considered only the focusing plane of a single quadrupole. In reality we must use at least a quadrupole doublet for focusing in both planes. This must be kept in mind when one assigns reasonable values for $B_Q \ell_Q$. Substituting Eqs. (2), (3) and (4) in Eq. (1) we get

$$\Delta\nu = \frac{P/n}{B_D (B_Q \ell_Q) c} \frac{1}{\beta\gamma(\gamma-1)} \frac{1}{r} . \tag{5}$$

One set of consistent units for this equation is

P in erg/sec

B in Gauss

all lengths in cm.

The beam spot radius r on target is given by the requisit specific energy deposition E and the range λ of the ions in target through

$$E = \frac{W}{\pi r^2 \lambda} \quad \text{or} \quad \frac{1}{r} = \sqrt{\frac{\pi E}{W}} \sqrt{\lambda} . \tag{6}$$

Implied in this equation is the assumption that whatever the number of beams the total energy W carried by all the beams can be deposited in the target volume $\pi r^2 \lambda$. Substituting Eq. (6) in Eq. (5) we get, finally

$$\Delta\nu = \frac{P\sqrt{\pi E/W}}{B_D (B_Q \ell_Q) c} \frac{F}{n} \qquad F \equiv \frac{\sqrt{\lambda}}{\beta\gamma(\gamma-1)} . \tag{7}$$

In this form the explicit dependence on the charge qe and mass m of the ions vanishes and the dependence on the ion specie is only implicit through the factor

$$F \equiv \frac{\sqrt{\lambda}}{\beta\lambda(\lambda-1)} .$$

This factor is plotted against the normalized kinetic energy $\gamma-1$ in Fig. 2 for Ca^{40}, I^{127} and U^{238} using the range information provided by R. Bangerter.[3] Several interesting features of Eq. (7) are worth mentioning.

a. For the same kinetic energy $T = mc^2(\gamma-1)$ the factor F, hence $\Delta\nu$, is smaller for a lighter ion or, conversely, for the same $\Delta\nu$, hence the same F, the required energy T is lower for a lighter ion. This is because for lighter ions the larger relativistically normalized energy parameters in the denominator override the increase in range in the numerator. Hence as far as $\Delta\nu$ is concerned within the ranges of ion species and energy covered in Fig. 2 lighter ions are preferred.

b. One must also consider the ring aperture, namely the emittance of the beam. To investigate this we get from Eqs. (6) and (7)

$$\frac{1}{r} = \sqrt{\frac{\pi E}{W}}\ \beta\gamma(\gamma-1)F \tag{8}$$

which when substituted in Eq. (4) gives for the un-normalized emittance

$$\frac{\varepsilon_t/\pi}{\beta\gamma} = \frac{qe}{mc^2\beta\gamma}\, B_Q \ell_Q r = \frac{B_Q \ell_Q}{RB_D}\, r$$

$$= \frac{B_Q \ell_Q}{RB_D}\, \frac{1}{F\sqrt{\pi E/W}}\, \frac{1}{\sqrt{(\gamma+1)(\gamma-1)^3}} \tag{9}$$

where we have used Eq. (3). Hence for the same F and the same available field $R\frac{\varepsilon_t/\pi}{\beta\gamma}$ reduces as one goes to a lighter ion. By adjusting the charge qe of the ion we can apply this reduction to either the ring radius or the emittance or both. Therefore, as far as emittance is concerned lighter ions are also preferred.

c. The tune shift $\Delta\nu$ as given by Eq. (7) is independent of the charge qe of the ions. This is because at higher charge the higher space charge forces in the numerator are compensated by the higher magnetic confinement forces in the denominator. The choice of the charge state q of the ion depends, therefore, only on considerations of the ring radius and the beam emittances as given by Eq. (9).

4. Design procedure and considerations

The design considerations and procedure can now be summarized as follows:

a. The desired P and E/W are given by the targeting requirements.

b. In choosing B_D one should keep in mind the ring geometries shown in Fig. 1. This gives

$$B_D = \frac{1}{1+\frac{n}{\pi}\sin\frac{\pi}{n}} \text{ (average field in arc)}$$

$$\cong \frac{1}{1+\frac{n}{\pi}\sin\frac{\pi}{n}} \text{ (~80\% of field in dipole).}$$

Therefore B_D should generally be no greater than half of the field in the dipoles, namely ~10 kG for conventional magnets or ~20-25 kG for superconducting magnets.

c. In choosing $B_Q \ell_Q$ one should keep in mind that we need at least a doublet for focusing in both transverse planes. Hence ℓ_Q should be no larger than 1-2 m. The pole field B_Q could be ~14 kG for conventional magnets or ~40-50 kG for superconducting magnets.

d. If the beam has to be stored in the ring for hundreds or more revolutions the maximum allowable $\Delta\nu$ is $\frac{1}{4}$ in order to avoid major resonances. On the other hand, A. Maschke[4] has shown that if the final bunching is sufficiently fast (tens of revolutions) even integer resonances can be crossed without noticeable deterioration of the transverse emittance. In this case $(\Delta\nu)_{max}$ of several units may even be allowable. The bunching/distributing ring could also be used as a synchrotron for accelerating the ions to the final energy. Throughout acceleration, then, $\Delta\nu$ of the rf-bunched beam must be $<\frac{1}{4}$. Only during the additional fast bunching on the flat-top can this upper limitation of $\frac{1}{4}$ for $\Delta\nu$ be removed.

e. Having chosen B_D, $B_Q \ell_Q$ and $(\Delta\nu)_{max}$; and with the given P and E/W we get from Eq. (7) the maximum allowed value F_{max}/n. In addition, for a given value of F the lower limit (0.1 cm) of r gives an upper limit for γ-1 through Eq. (8). Within these two bounds one should choose the lightest ion from the curves of Fig. 2. The optimum n is that for which these two bounds would permit the use of the lightest ion overall. Of course, the increased cost associated with a larger number of beam transports is also an important consideration. In addition the charge-exchange collision cross-sections for specific ion types should also be considered if the ion lifetime proves to be a concern.

f. The charge number q should be large to reduce the ring radius R. At the same time the beam emittance and hence the ring aperture as given by Eq. (9) must be reasonable. Again, if relevant, the charge-exchange collision cross section should also be considered in choosing the charge state.

This procedure will give a rough cut of the gross parameters for the case in which a final beam bunching/distributing ring is used. The parameters so obtained should be reasonably optimal.

5. Examples

We now demonstrate this design procedure using the two systems with targeting parameters given in section 2 above.

a. IFPP as described in FN-302 (reference 2)

First we check that starting with the assumptions made in FN-302 this procedure will indeed lead to the design developed there. In that paper we ignored the final transport to the reactor vessel, and chose

$$B_D = 65\% \times 50 \text{ kG} \cong 30 \text{ kG} = 3\text{x}10^4 \text{ G}$$

and

$$\varepsilon_t/\pi \cong 4 \text{ cm-mrad.}$$

For r = 0.1 cm and U^{+1}_{238} ions (mc^2 = 222 GeV, q = 1) this emittance gives through Eq. (4)

$$B_Q \ell_Q \cong 300 \text{ kG m} = 3\text{x}10^7 \text{ G cm.}$$

[Even for B_Q = 50 kG this requires ℓ_Q = 6 m. Considering that quadrupole doublets are needed for focusing in both planes this value of $B_Q \ell_Q$ is definitely too large.]

In FN-302 we also assumed that only half of the total beam energy W is deposited in a volume $\pi r^2 \lambda$ of the target. Therefore

$$\frac{E}{W} = \frac{30 \text{ MJ/g}}{5 \text{ MJ}} = 6 \text{ g}^{-1}.$$

Other parameters assumed are

$$P = 600 \text{ TW} = 6\text{x}10^{21} \text{ erg/sec}$$

$$n = 10$$

$$(\Delta\nu)_{max} = \frac{1}{4} \ .$$

Eq. (7) gives

$$F_{max} = \frac{\sqrt{\lambda}}{\beta\gamma(\gamma-1)} = \frac{10\text{x}3\text{x}10^4\text{x}3\text{x}10^7\text{x}3\text{x}10^{10}}{4\text{x}6\text{x}10^{21}\text{x}\sqrt{6\pi}} \text{ g}^{\frac{1}{2}}/\text{cm} = 2.6 \text{ g}^{\frac{1}{2}}/\text{cm.}$$

The U curve of Fig. 2 then gives for the required energy

$$\gamma-1 \cong 0.65 \quad \text{or} \quad T \cong 150 \text{ GeV}$$

agreeing with that given in FN-302.

b. IFPP from this procedure

When the final beam transports to the reactor vessel are taken into account reasonable choices of field intensities are

$$B_D = 20 \text{ kG} = 2\text{x}10^4 \text{ G}$$

$$B_Q \ell_Q = 50 \text{ kG x 1 m} = 5\text{x}10^6 \text{ G cm.}$$

Assuming the final fast bunching on the flat-top to give a factor 4 we can take

$$(\Delta\nu)_{max} = 1.$$

We further assume that the total beam energy of W = 10 MJ is deposited in a target volume of $\pi r^2\lambda$ and obtain

$$\frac{E}{W} = \frac{30 \text{ MJ/g}}{10 \text{ MJ}} = 3 \text{ g}^{-1}.$$

Eq. (7) then gives

$$\frac{F_{max}}{n} = \frac{2\text{x}10^4\text{x}5\text{x}10^6\text{x}3\text{x}10^{10}}{6\text{x}10^{21}\text{x}\sqrt{3\pi}} \text{ g}^{\frac{1}{2}}/\text{cm} = 0.16 \text{ g}^{\frac{1}{2}}/\text{cm}.$$

To get the same $F_{max} = 2.6 \text{ g}^{\frac{1}{2}}/\text{cm}$ we need 16 beams and for $r>0.1$ cm Eq. (8) gives $\gamma-1<0.822$. For this illustrative example we will not bother to optimize n.

It is a little difficult to extrapolate and interpolate the curves in Fig. 2 accurately, but it seems safe to take

$$mc^2 = 175 \text{ GeV} \qquad (\text{Os}^{188}!)$$

and $\gamma - 1 = 0.8$

$T = 140$ GeV $(\beta\gamma = 1.497)$.

Eq. (8) then gives

$$r = 0.105 \text{ cm}$$

and Eq. (9) gives

$$R \frac{\varepsilon_t/\pi}{\beta\gamma} = 26.2 \text{ cm}^2.$$

With a charge number q = 3 we get

$$R = 146 \text{ m} \quad \text{and} \quad \frac{\varepsilon_t/\pi}{\beta\gamma} = 1.80 \text{ cm-mrad}.$$

For this ring we can use 24 m long FODO cells with $\beta_{max} \cong 40$ m. Thus, the beam radius in only ~2.7 cm which is quite modest. On the other hand, if the ring is also used as a synchrotron its aperture must be determined by the larger beam size at injection.

The total particle current is 4286 A or 268 A in each of the 16 beams. With charge number 3 the electric current is 804 A. With

$q = 3$ $\quad \frac{e^2}{mc^2} = 0.823\text{x}10^{-18}$ cm

$I = 804$ A $\quad R = 1.46\text{x}10^4$ cm

$\varepsilon_t/\pi = 2.7\text{x}10^{-3}$ cm $\quad \beta\gamma = 1.497$

Eq. (1) gives indeed $\Delta\nu = 1.0$ as originally assumed.

With an emittance of $\frac{\varepsilon_t/\pi}{\beta\gamma} = 1.80$ cm-mrad and a beam spot radius on target of r = 0.105 cm if the final quadrupole is 10 m away from the target the quadrupole aperture radius r_Q will be

$$r_Q = \frac{10 \text{ m x } 1.80 \text{ cm-mrad}}{0.105 \text{ cm}} = 17 \text{ cm}$$

a quite reasonable value.

c. HIDE

We will try with conventional magnets and take

$$B_D = 10 \text{ kG} = 1\text{x}10^4 \text{ G}$$

$$B_Q \ell_Q = 14 \text{ kG x } 1.4 \text{ m} = 2\text{x}10^6 \text{ G cm.}$$

Again, we assume that the bunching/distributing ring is also used as a synchrotron. Taking the final fast bunching factor to be 6 we can allow the final tune shift to be as large as

$$(\Delta\nu)_{max} = 1.5.$$

Eq. (7) then gives

$$\frac{F_{max}}{n} = \frac{1.5\text{x}10^4\text{x}2\text{x}10^6\text{x}3\text{x}10^{10}}{5\text{x}10^{20}\text{x}\sqrt{3\pi}} \text{ g}^{\frac{1}{2}}/\text{cm} = 0.59 \text{ g}^{\frac{1}{2}}/\text{cm}.$$

Again without bothering to optimize n we shall choose n = 8 which gives $F_{max} = 4.7 \text{ g}^{\frac{1}{2}}/\text{cm}$. Eq. (8) then gives for $r > 0.1$ cm the upper limit $\gamma - 1 < 0.572$.

Extrapolating and interpolating between the curves of Fig. 2 indicate that we can safely choose an ion with

$$mc^2 = 100 \text{ GeV} \qquad (Ag^{107}!)$$

and

$$\gamma - 1 = 0.55$$

$$T = 55 \text{ GeV} \qquad (\beta\gamma = 1.184).$$

Eq. (8) then gives

$$r = 0.107 \text{ cm}$$

and Eq. (9) gives

$$R\frac{\varepsilon_t/\pi}{\beta\gamma} = 21.3 \text{ cm}^2.$$

With a charge number q = 5 we get

$$R = 79 \text{ m} \quad \text{and} \quad \frac{\varepsilon_t/\pi}{\beta\gamma} = 2.7 \text{ cm-mrad.}$$

For such a ring one can use 16 m long FODO cells giving $\beta_{max} \cong 27$ m. Thus the beam radius is only ~2.7 cm which is quite modest. Of course, since the ring is also used as a synchrotron the aperture must be sized to the larger beam radius at injection.

The total particle current is 909 A or 113.6 A in each of 8 beams. With charge number 5 the electric current is 113.6x5 A = 568 A. Substituting

$$q = 5 \qquad \frac{e^2}{mc^2} = 1.44\text{x}10^{-18} \text{ cm}$$

$$I = 568 \text{ A} \qquad R = 7900 \text{ cm}$$

$$\varepsilon_t/\pi = 3.2\text{x}10^{-3} \text{ cm} \qquad \beta\gamma = 1.184$$

in Eq. (1) we get indeed $\Delta\nu = 1.5$ agreeing with the starting design assumption.

With an emittance of $\frac{\varepsilon_t/\pi}{\beta\gamma} = 2.7$ cm-mrad and a beam spot radius of r = 0.107 cm if the final quadrupole is 10 m away from the target the quadrupole aperture radius r_Q will be

$$r_Q = \frac{10 \text{ m x } 2.7 \text{ cm-mrad}}{0.107 \text{ cm}} = 25 \text{ cm}$$

which is large but not unreasonable.

References

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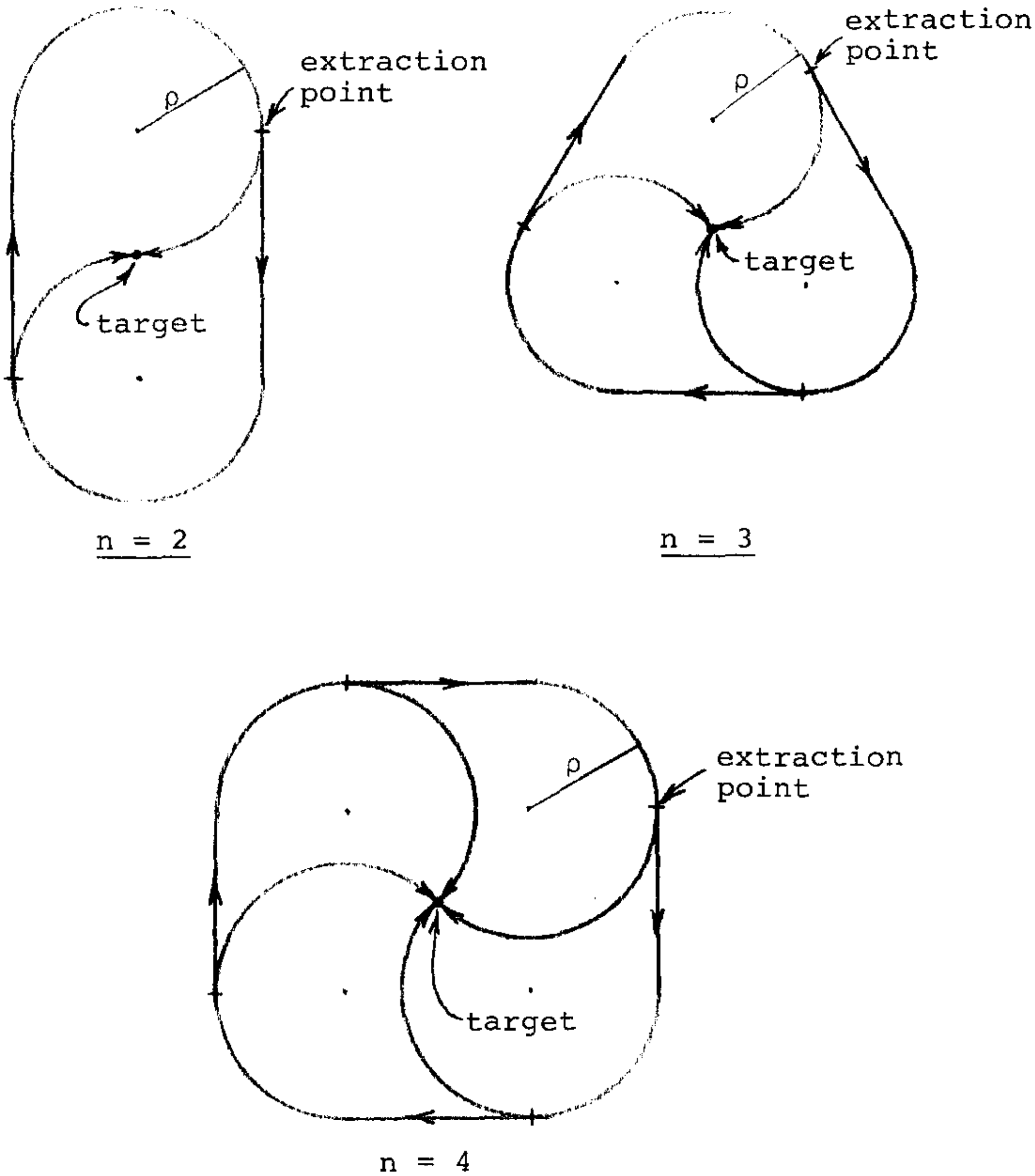
3. R.O. Bangerter, "Target Requirement for Storage Ring Inertial Confinement Fusion", unpublished report distributed at the ERDA Summer Study of Heavy Ions for Inertial Fusion (June 1976) (also contained in reference 1).

4. G. Danby, E. Gill, J. Keane and A.W. Maschke, "Preliminary Results of 200 MeV Bunching Experiments", Brookhaven National Laboratory Report BNL 50643, March 1971



<u>Figure 1</u> Geometry of bunching/distributing ring and beam transport lines to reactor vessel (n = number of beams on target)

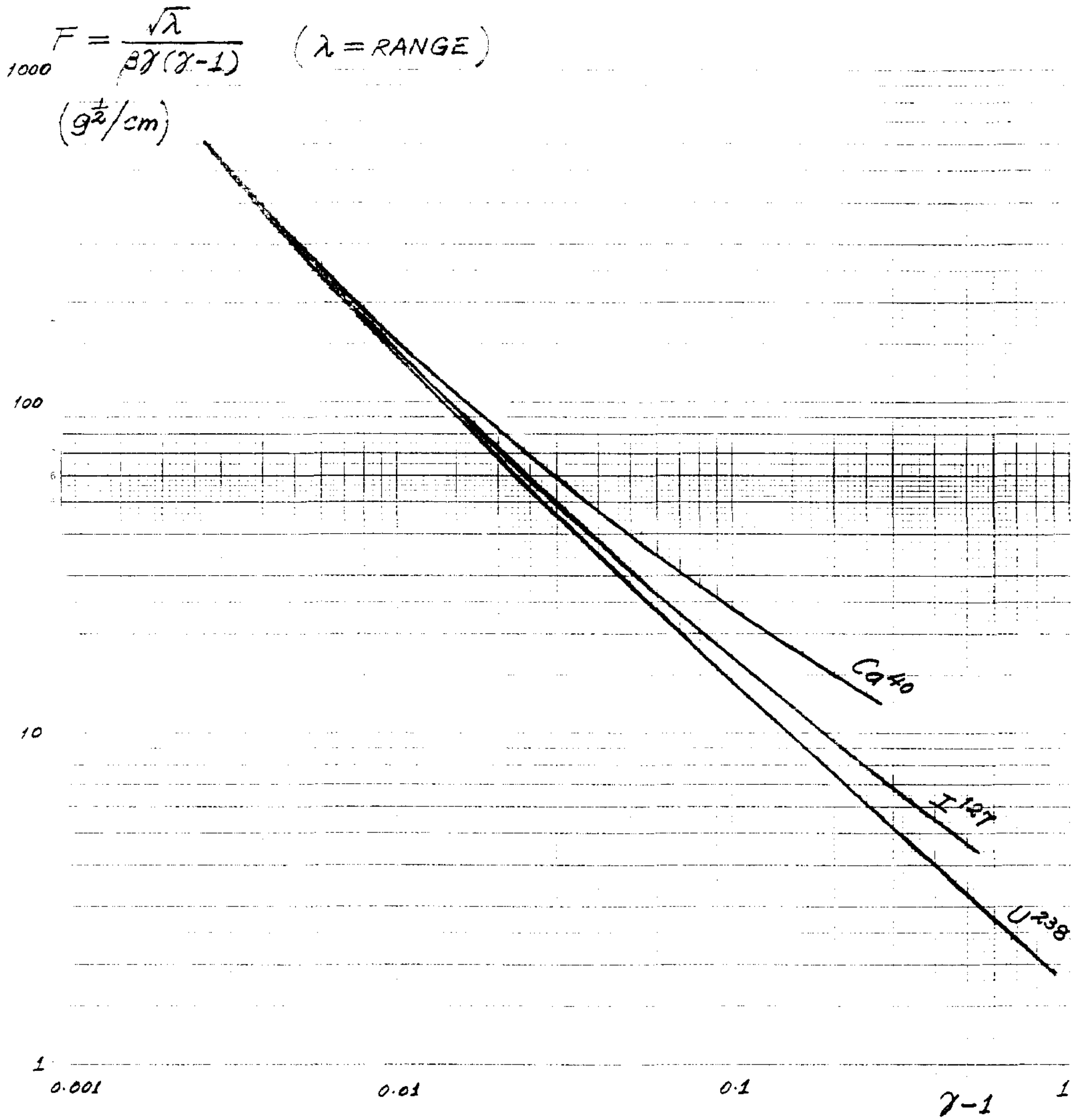


Figure 2 Range-energy relations for U^{238}, I^{127} and Ca^{40} ions in gold - F versus γ-1.

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Further Parametric Studies of the

Accelerator System for Heavy Ion Fusion - Addendum

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We consider the case of filling N rings from one injector at kinetic energy T_i. For the time being we shall assume the injector to be an rf linac. The possibility of using other types of accelerator as injector will be discussed later. The beam in each ring is divided into k/N segments, compressed by a bunching factor b, and extracted at kinetic energy T_f to form k/N beams. Thus, the total number of beams striking the target simultaneously is k. If the injection energy T_i equals the final energy T_f the rings are d.c. and serve only as accumulator/compressor rings. If $T_i<T_f$, the rings are synchrotrons and serve as accumulator/accelerator/compressor rings.

The targeting requirements are given by

P = total power on target

W = total energy on target

E = specific energy deposition in target

r = target radius

The principal design considerations are the following.

A. First, we have to deliver the required specific energy deposition. This gives a condition on the range λ of the ions in the target material

$$\lambda = \frac{1}{\pi r^2}\frac{W}{E} . \qquad \text{(I)}$$

The kinetic energy T_f for a given ion type is, then, given by the range/energy curve.[1] The heaviest ion gives the highest T_f, hence the least demand on the current.

B. Then, we have the two conditions for delivering the required total energy and power. In terms of the number of ions n in the ring, the tune shift $\Delta\nu$ at injection is given by

$$\Delta\nu = q^2 \frac{r_o}{2\pi} \frac{n}{\varepsilon/\pi} \frac{1}{\eta_i\gamma_i} \qquad (0)$$

where

$$\begin{cases} r_o = \dfrac{e^2}{mc^2} \ (= 1.44\mathrm{x}10^{-18}\,\mathrm{m} \ \text{for } mc^2 = 1\ \mathrm{GeV}) \\ q = \text{charge number of ion} \\ \varepsilon = \text{normalized emittance} \\ \gamma,\ \beta,\ \eta \equiv \beta\gamma = \dfrac{p}{mc} \ \text{are conventional relativistic parameters} \\ \text{subscript i denotes injection value} \end{cases}$$

But the required number of ions per ring is

$$n = \frac{1}{N}\frac{W}{T_f} \qquad (N = \text{number of rings}).$$

Hence

$$\frac{N}{q^2} = \frac{r_o}{2\pi} \frac{W/T_f}{(\varepsilon/\pi)\Delta\nu} \frac{1}{\eta_i\gamma_i} . \qquad (II)$$

In terms of the final d.c. particle current i_f we have

$$n = \frac{2\pi R}{c\beta_f} i_f$$

where the ring radius R is given by

$$R = \frac{mc^2}{qe}\frac{\eta_f}{B_f} = \frac{1}{qr_o}\frac{e}{B_f}\eta_f \qquad (1)$$

$(\frac{mc^2}{e} = \frac{100}{3}$ kGm for $mc^2 = 1$ GeV) with B_f = average final bending field in ring. Thus

$$\Delta\nu = q \frac{e\gamma_f}{cB_f} \frac{i_f}{\varepsilon/\pi} \frac{1}{\eta_i\gamma_i}.$$

But the required final particle current per beam before bunching is

$$i_f = \frac{1}{kb} \frac{P}{T_f}$$

where

$$\begin{cases} k = \text{total number of beams on target} \\ b = \text{bunching factor} = \dfrac{\text{peak current of bunched beam}}{\text{current of d.c. beam}}. \end{cases}$$

Hence

$$\frac{kb}{q} = \frac{e\gamma_f}{cB_f} \frac{P/T_f}{(\varepsilon/\pi)\Delta\nu} \frac{1}{\eta_i\gamma_i} . \qquad \text{(III)}$$

We have assumed here that the tune-shift limitation needs not be applied during the fast final bunching as was shown to be true by the BNL experiment.[2]

We might add here the electric current of the d.c. beam in the ring at injection

$$I_i = qei_f \frac{\beta_i}{\beta_f} = \frac{qe}{kb} \frac{P}{T_f} \frac{\beta_i}{\beta_f} \qquad (2)$$

and the voltage of the injector linac

$$V_{linac} = \frac{T_i}{qe} \qquad (3)$$

Clearly we would like the left-hand-side quantities in conditions (II) and (III) to be small. This implies that we should have

1. r_o small, hence heavy ions.
2. T_f large, hence heavy ions.
3. γ_f small. But within the range of interest $\gamma_f \cong 1$ anyway.
4. B_f large. Conventional magnets give $B_f \cong 10$ kG. Superconducting magnets give $B_f \cong 20$ kG and consume no power.
5. $\eta_i\gamma_i$ large. This is largest when injecting at the final energy ($T_i = T_f$), but must be compromised with the cost of the injector.
6. $\Delta\nu$ large. If the rings are used as synchrotrons ($T_i<T_f$) rf acceleration requires some bunching, hence $\Delta\nu$ must be $<\frac{1}{4}$. We shall take $\Delta\nu \cong \frac{1}{8}$. If the rings are used only as accumulator/compressors we can take $\Delta\nu \cong \frac{1}{4}$. (It may be possible to waive the tune-shift limitation also during the rather fast process of multiturn injection. If, further, space charge neutralization is applied, we can ignore the space charge effect altogether. But these possibilities have yet to be demonstrated.)
7. ε/π large. This is limited either by the reactor vessel and beam port dimensions or by the strength of the final focusing quadrupoles

$$B_Q \ell_Q \cong \frac{mc^2}{qe} \frac{\varepsilon/\pi}{r} \tag{4}$$

where B_Q and ℓ_Q are respectively the pole-tip field and the length of the quadrupole.

C. In addition to the conditions (II) and (III) the requirements on the bunching and accelerating rf systems will help in making a choice of the parameters q, N, k, b and T_i. The peak momentum spread (in mc units) $\Delta\eta_i$ of the beam from the injector linac is given by[3]

$$\frac{\Delta\eta_i}{\eta_i} \cong 2\text{x}10^{-4} \frac{\sqrt{q}}{\left[T_i\,(\text{GeV})\right]^{5/8}} \; . \qquad (5)$$

This momentum spread of the injected beam in the ring (considered as a d.c. beam) is conserved through acceleration. That of the final bunched beam is, then, $\Delta\eta_b = b\Delta\eta_f = b\Delta\eta_i$. First, we must check that $\Delta\eta_b$ is allowed by chromatic aberration in the final transport elements. The condition is

$$\frac{\Delta\eta_b}{\eta_f} = \frac{b\Delta\eta_i}{\eta_f} < \frac{r}{a} \qquad (6)$$

where a is the radius of the beam port. Second, to bunch the beam on the ring flattop the momentum width of the stationary bucket of the bunching rf must at least be equal to $\Delta\eta_b$. This gives

$$\Delta\eta_b = \left(\frac{2}{\pi h} \frac{qeV_b}{mc^2} \frac{\gamma_f}{\frac{1}{\gamma_f^{\,2}} - \frac{1}{\gamma_t^{\,2}}}\right)^{1/2}$$

where

$$\begin{cases} h = \frac{k}{N} = \text{harmonic number} \\ V_b = \text{peak rf voltage per turn} \\ \gamma_t = \text{transition } \gamma \text{ (generally >>1).} \end{cases}$$

Neglecting $\frac{1}{\gamma_t^{\,2}}$ compared to $\frac{1}{\gamma_f^{\,2}}$ we get

$$V_b \cong \frac{\pi h}{2} \frac{mc^2}{qe} \frac{(\Delta\eta_b)^2}{\gamma_f^3} \, . \tag{7}$$

We note that since $\Delta\eta_b \propto \sqrt{q}$, V_b is independent of q. The bunching rf runs at the fixed final frequency

$$F_f = h\frac{c\beta_f}{2\pi R} \, . \tag{8}$$

The accelerating rf is given by the desired energy gain per turn. Assuming a linear ramp we have

$$V_a \sin\phi_s = \frac{2\pi R}{c} \frac{\eta_f - \eta_i}{\Delta t} \frac{mc^2}{qe} \tag{9}$$

where

V_a = peak rf voltage per turn

ϕ_s = synchronous phase (determined by the necessary bucket area)

Δt = acceleration time (generally must be <0.1 sec to avoid excessive beam loss due to charge exchange interactions)

The accelerating rf must be frequency modulated between the injection frequency

$$F_i = h\frac{c\beta_i}{2\pi R} \tag{10}$$

and the final frequency F_f given by Eq. (8). Therefore it is much more difficult to produce the accelerating rf voltage V_a than the fixed-frequency bunching rf voltage V_b. But if V_b is not much larger than V_a it may be economically advantageous to combine the two rf systems.

There are, in addition, a large number of other accelerator parameters whose choice depends on compromises between performance, reliability, economy etc. based on the experience of the designers. We shall not enter into these discussions here.

We now apply the procedure outlined above to a few interesting examples.

D. Example 1 - Prototype power plant

$$\begin{cases} P = 100 \text{ TW} \\ W = 1 \text{ MJ} \\ E = 40 \text{ MJ/g} \\ r = 1 \text{ mm} \end{cases}$$

Condition (I) gives

$$\lambda = 0.796 \text{ g/cm}^2$$

and the range/energy curve gives for U_{238} (mc^2 = 222 GeV, $r_o = 6.5\text{x}10^{-21}$ m, $\frac{mc^2}{e}$ = 7400 kGm)

$$T_f = 26.2 \text{ GeV} = 4.2\text{x}10^{-9}\text{J} \qquad \begin{cases} \gamma_f = 1.118 \\ \eta_f = 0.500 \\ \beta_f = 0.447 \end{cases}$$

<u>Case A</u> $T_i=T_f$. Taking

$$\begin{cases} \Delta\nu = \frac{1}{4} \\ B_f = 20 \text{ kG (superconducting magnets)} \\ \varepsilon/\pi = 0.5\text{x}3\text{x}10^{-5} \text{ m} = 1.5\text{x}10^{-5} \text{ m (beam port radius} \\ \quad a = 0.3 \text{ m, 10 m away from target)} \end{cases}$$

we get for conditions (II) and (III)

$$\frac{N}{q^2} = 0.118, \qquad \frac{kb}{q} = 102.$$

A reasonable choice is

$$\boxed{q = 4,\ N = 2,\ k = 16,\ b = 25.5}$$

(The fact that $N/q^2 = 0.125$ is slightly larger than 0.118 simply means that we will be able to deliver slightly more energy than W = 1 MJ.) Eqs. (1) through (8) give

$$
\begin{aligned}
R &= 46.25\ \text{m} \\
I_i &= 37.3\ \text{A (37 turns @ 1 A/turn)} \\
V_{linac} &= 6.55\ \text{GV} \\
B_Q \ell_Q &= 27.75\ \text{kGm} \qquad \text{(quite o.k.)} \\
\Delta\eta_i/\eta_i &= 0.52\text{x}10^{-4} \\
\Delta\eta_b/\eta_f &= 1.3\text{x}10^{-3} \qquad (<\tfrac{r}{a} = 3.3\text{x}10^{-3}\ \text{o.k.}) \\
V_b &= 219\ \text{kV} \qquad \text{(quite o.k.)} \\
F_f &= 3.69\ \text{MHz}
\end{aligned}
$$

Two remarks should be made.

(i) One can reduce k to 8 (thereby saving 8 final transport lines) and increase b to 51, but then $\Delta\eta_b/\eta_f$ will be $2.6\text{x}10^{-3}$ which is too close to the limit of $3.3\text{x}10^{-3}$.

(ii) One can reduce V_{linac}, say, by a factor 2 by increasing q to 8, N to 8, and k to 32. But the increased cost of 6 rings and 16 beam transport lines may offset the reduced cost of the injector linac.

<u>Case B</u> $T_i < T_f$. With $\Delta\nu = \frac{1}{8}$, $B_f = 20$ kG, and $\varepsilon/\pi = 1.5\text{x}10^{-5}$ m conditions (II) and (III) give

$$\frac{N}{q^2} = 0.235\ \frac{\eta_f \gamma_f}{\eta_i \gamma_i}, \qquad \frac{kb}{q} = 204\ \frac{\eta_f \gamma_f}{\eta_i \gamma_i}$$

In this case, although it is possible to operate several synchrotrons in a synchronized fashion, there is a definite preference for N = 1. A reasonable choice is

$$q = 1,\ N = 1,\ k = 32,\ b = 27.1$$

$$\frac{\eta_i \gamma_i}{\eta_f \gamma_f} = 0.235$$

The injection energy is given by $\eta_i \gamma_i = 0.235\ \eta_f \gamma_f = 0.1315$ to be

$$T_i = 1.88 \text{ GeV} \qquad \begin{cases} \gamma_i = 1.00847 \\ \eta_i = 0.130 \\ \beta_i = 0.129 \end{cases}$$

Eqs. (1) through (10) give

R = 185 m

I_i = 1.27 A (10 turns @ 127 mA/turn)

V_{linac} = 1.88 GV

$B_Q \ell_Q$ = 111 kGm (just o.k. with superconducting quadrupoles)

$\Delta\eta_i/\eta_i = 1.35 \times 10^{-4}$

$\Delta\eta_b/\eta_f = 0.95 \times 10^{-3}$ (o.k.)

V_b = 1.81 MV (o.k.)

F_f = 3.69 MHz, F_i = 1.07 MHz

$V_a \sin\phi_s$ = 3.18 MV (for Δt = 0.1 sec) (too high)

<u>Case C</u> $T_i < T_f$. To reduce V_a we can either increase the acceleration time Δt or give up on N = 1 and choose

$$q = 2,\ N = 2,\ k = 32,\ b = 27.1$$

$$\frac{\eta_i \gamma_i}{\eta_f \gamma_f} = 0.471$$

The injection energy is

$$T_i = 7.10 \text{ GeV} \qquad \begin{cases} \gamma_i = 1.0320 \\ \eta_i = 0.255 \\ \beta_i = 0.247 \end{cases}$$

Eqs. (1) through (10) give

$R = 92.5$ m

$I_i = 4.86$ A (20 turns @ 243 mA/turn)

$V_{linac} = 3.55$ GV

$B_Q \ell_Q = 55$ kGm (o.k.)

$\Delta\eta_i/\eta_i = 0.83\text{x}10^{-4}$

$\Delta\eta_b/\eta_f = 1.15\text{x}10^{-3}$ (o.k.)

$V_b = 658$ kV (o.k.)

$F_f = 3.69$ MHz, $F_i = 2.11$ MHz

$V_a \sin\phi_s = 527$ kV (for $\Delta t = 0.1$ sec) (o.k.)

Compared to Case B these parameters are easier to achieve, except now we need a higher voltage injector linac and two synchrotrons.

E. Example 2 - Heavy ion demonstration experiment (HIDE)

$$\begin{cases} P = 5 \text{ TW} \\ W = 100 \text{ kJ} \\ E = 20 \text{ MJ/g} \\ r = 1 \text{ mm} \end{cases}$$

Condition (I) gives

$$\lambda = 0.159 \text{ g/cm}^2$$

and the range/energy curve gives for U_{238}

$$T_f = 7.4 \text{ GeV} \qquad \left\{ \begin{array}{l} \gamma_f = 1.0333 \\ \eta_f = 0.260 \\ \beta_f = 0.252 \end{array} \right.$$
$$= 1.18\text{x}10^{-9}\text{J}$$

Case A $T_i = T_f$. Taking

$$\left\{ \begin{array}{l} \Delta\nu = \frac{1}{4} \\ B_f = 10 \text{ kG (conventional magnets)} \\ \varepsilon/\pi = 0.260\text{x}5\text{x}10^{-5} \text{ m} = 1.3\text{x}10^{-5} \text{ m (beam port} \\ \qquad \text{radius a = 0.25 m, 5 m away from target)} \end{array} \right.$$

we get for conditions (II) and (III)

$$\frac{N}{q^2} = 0.100, \qquad \frac{kb}{q} = 79.9 \ .$$

A reasonable choice is

q = 3, N = 1, k = 12, b = 20

Eqs. (1) through (8) give

$R = 64.2$ m

$I_i = 8.46$ A (20 turns @ 423 mA/turn)

$V_{linac} = 2.47$ GV

$B_Q \ell_Q = 32.1$ kGm (just o.k. for conventional quadrupoles)

$\Delta\eta_i/\eta_i = 1.0\text{x}10^{-4}$

$\Delta\eta_b/\eta_f = 2.0\text{x}10^{-3}$ ($< \frac{r}{a} = 4\text{x}10^{-3}$ o.k.)

$V_b = 337$ kV (o.k.)

$F_f = 2.25$ MHz

Case B $T_i < T_f$. With $\Delta\nu = \frac{1}{8}$, B_f = 10 kG, and $\varepsilon/\pi = 1.3 \times 10^{-5}$ m conditions (II) and (III) give

$$\frac{N}{q^2} = 0.200 \frac{\eta_f \gamma_f}{\eta_i \gamma_i}, \qquad \frac{kb}{q} = 160 \frac{\eta_f \gamma_f}{\eta_i \gamma_i} .$$

A reasonable choice is

$$\boxed{\begin{array}{c} q = 1,\ N = 1,\ k = 32,\ b = 25 \\ \dfrac{\eta_i \gamma_i}{\eta_f \gamma_f} = 0.200 \end{array}}$$

The injection energy is given by $\eta_i \gamma_i = 0.2\ \eta_f \gamma_f = 0.0538$ to be

$$T_i = 320 \text{ MeV} \qquad \begin{cases} \gamma_i = 1.00144 \\ \eta_i = 0.0537 \\ \beta_i = 0.0536 \end{cases}$$

Eqs. (1) through (10) give

R = 192.6 m

I_i = 0.180 A (10 turns @ 18 mA/turn)

V_{linac} = 320 MV

$B_Q \ell_Q$ = 96.2 kGm (o.k. only with superconducting quadrupoles)

$\Delta\eta_i/\eta_i = 4.08 \times 10^{-4}$

$\Delta\eta_b/\eta_f = 2.10 \times 10^{-3}$ ($< 4 \times 10^{-3}$ o.k.)

V_b = 3.03 MV (rather high)

F_f = 2.00 MHz, F_i = 0.426 MHz

$V_a \sin\phi_s$ = 1.85 MV (for Δt = 0.1 sec)(rather high)

Case C $T_i < T_f$. To reduce V_a and V_b we can give up on N = 1 and choose

$q = 2,\ N = 2,\ k = 32,\ b = 25$
$\frac{\eta_i \gamma_i}{\eta_f \gamma_f} = 0.400$

The injection energy is

$$T_i = 1.267 \text{ GeV} \qquad \begin{cases} \gamma_i = 1.00571 \\ \eta_i = 0.107 \\ \beta_i = 0.106 \end{cases}$$

Eqs. (1) through (10) give

$R = 96.3$ m

$I_i = 0.713$ A (20 turns @ 36 mA/turns)

$V_{linac} = 633$ MV

$B_Q \ell_Q = 48.1$ kG m (o.k. with superconducting quadrupoles)

$\Delta\eta_i/\eta_i = 2.44\text{x}10^{-4}$

$\Delta\eta_b/\eta_f = 2.50\text{x}10^{-3}$ ($<4\text{x}10^{-3}$ just o.k.)

$V_b = 1.077$ MV (o.k.)

$F_f = 2.00$ MHz, $F_i = 0.844$ MHz

$V_a \sin\phi_s = 343$ kV (for $\Delta t = 0.1$ sec) (o.k.)

These parameters are easier to obtain than those of Case B but now we need a higher voltage linac and two synchrotrons.

For an rf linac the normalized emittance of the beam is approximately[3] $\varepsilon/\pi \cong 10^{-6}$ m. With multiturn injection, stacking in both horizontal and vertical planes the number of injected turns assumed in both Examples above can easily be accommodated within the allowed emittances.

F. We now discuss briefly the possibilities of using other types of accelerator as injectors.

Synchrotron

In Cases B and C discussed above we already have, in essence, the system consisting of a d.c. accumulator/compressor (A/C) ring combined with a synchrotron injector. The only additional feature introduced by separating the A/C ring and the synchrotron is the possibility of filling the A/C ring to its tune-shift limit with several pulses from the synchrotron injector. From Eq. (0) we see that for the same emittance ε/π, the tune-shift limited number of ions in the A/C ring is $2\frac{\eta_f\gamma_f}{\eta_i\gamma_i}$ times that in the synchrotron where the factor 2 represents the ratio between the allowed $\Delta\nu$ values of $\frac{1}{4}$ and $\frac{1}{8}$ for the A/C ring and the synchrotron respectively. For ease of discussion we shall concentrate on Example 2 and take $\frac{\eta_f\gamma_f}{\eta_i\gamma_i} = 2.5$ as in Case C. (This corresponds to an injection energy into the synchrotron of $T_i = 1.267$ GeV. With $q = 3$, that for the optimal A/C ring of Case A, the synchrotron injector-linac will need a voltage of $V_{linac} = 422$ MV.)

With identical R and ε/π, to inject 5 pulses from the synchrotron to fill the same phase-space volume in the A/C ring we have to resort to a non-Liouvillean injection process such as the charge-exchange injection. Such an injection process does not seem to be available for U ions (indeed, perhaps unavailable for anything other than H^- and HI^+).

With more conventional phase-space conserving injection processes we must reduce the phase space volume occupied by the beam in the synchrotron by a factor 5. We can reduce the longitudinal emittance by reducing the circumference of the synchrotron by a factor 5. The 5 pulses can then be injected end-to-end to fill the circumference of the A/C ring. But this requires B_f = 50 kG in the synchrotron which is, again, not available. Indeed, if this field were available it would have been used for the A/C ring.

Finally, we can reduce the transverse emittance ε/π in the synchrotron by a factor 5 (in both planes). The number of ions per pulse is, then, reduced also by a factor 5. But, in principle, we can now inject 5 (horizontal) x 5 (vertical) = 25 pulses into the A/C ring and effectively increase the number of injected ions 5-fold. However, since coherence in transverse oscillation is invariably lost between pulses, contrary to multi-turn injection a practical scheme for multipulse injection without significant dilution of the phase-space density does not exist. In any case, this process will require the synchrotron to have a repetition rate of 250 Hz (25 pulses in 0.1 sec) which is also impractical.

We conclude, therefore, that separating the A/C ring and the synchrotron is not likely to lead to any practical advantage.

For N>1, a simple minded advantage of the separation is the possibility of using the same synchrotron to fill N A/C rings in succession. The repetition rate of the synchrotron must be increased N-fold. On the last pulse the synchrotron can also serve as one of the N A/C rings. Whether the cost savings in making N-1 rings d.c. are off-set by the additional costs incurred in raising the repetition rate of the synchrotron and in the beam transfer lines (at T_f) to the N-1 A/C rings can only be answered by a detailed cost analysis.

Induction Linac

The need of A/C rings implies that the injector can deliver only low current beams. On the other hand, the pulse length is sufficiently long (tens of μsec) to supply the requisit total number of ions. Multiturn injection and longitudinal compression in the A/C ring, then, converts the beam into the desire short (~10 nsec) and high-current (10^2 to 10^3 A) pulses. An induction linac is expected to be capable of delivering just these short and high-current beam pulses. Therefore, with an induction linac we do not anticipate the need for A/C rings at all.

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