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MECHANICAL STRESSES IN SUPERCONDUCTING QUADRUPOLES

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Summary

A solution has been found for stresses in a structural composite that models a shell type superconducting quadrupole. The composite consists of three nested hollow cylinders: the innermost cylinder represents the region of the bore tube, the middle cylinder the region of superconductor, and the outermost cylinder the region of the collars. Under zero stress a distribution of current is chosen to give a pure quadrupole field. Subsequent effects of prestress, cool-down and excitation on the state of stress are determined. Each region is characterized by two elastic constants, one thermal constant and one pretension constant. Two different cases are used to produce a pure quadrupole field: (A) two sheet currents nested between the innermost and middle cylinder each with a surface current density varying as cosine two theta; (B) a thick cosine two theta current distribution in the middle region. Numerical results are given for a beam line quadrupole.

Equation for Elastic Displacements

If $\vec{u}$ is the displacement vector then [1]

$$\nabla\times\nabla\times\vec{u}-2\frac{1-\nu}{1-2\nu}\nabla(\nabla\cdot\vec{u}) = 2\frac{1+\nu}{E}\vec{J}\times\vec{B}, \tag{1}$$

where E is Young's modulus, ν is Poisson's ratio, $\vec{J}$ is the current density and $\vec{B}$ is the magnetic induction. If the case of sheet current excitation is used then the RHS of Eq. (1) is zero.

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Generalized Plane Strain

For simplicity consider only the case for which $u_z = \varepsilon_{zz} z$ with ε_{zz} = constant. The remaining components are to be considered functions of (r,θ) only. This is consistent with an excitation in which J_z is the only component of current density. Hence, Eq. (1) becomes

$$\frac{1}{r}\frac{\partial}{\partial\theta}\left[\frac{1}{r}\frac{\partial}{\partial r}(ru_\theta)-\frac{1}{r}\frac{\partial u_r}{\partial\theta}\right]-\beta\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(ru_r)+\frac{1}{r}\frac{\partial u_\theta}{\partial\theta}\right]=-2\frac{1+\nu}{E}J_zB_\theta, \tag{2}$$

$$-\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(ru_\theta)-\frac{1}{r}\frac{\partial u_r}{\partial\theta}\right]-\beta\frac{1}{r}\frac{\partial}{\partial\theta}\left[\frac{1}{r}\frac{\partial}{\partial r}(ru_r)+\frac{1}{r}\frac{\partial u_\theta}{\partial\theta}\right]=2\frac{1+\nu}{E}J_zB_r, \tag{3}$$

where

$$\beta = 2\frac{1-\nu}{1-2\nu} . \tag{4}$$

Magnetic Quantities of Interest

Current Sheets (Model A):

A pure quadrupole may be generated with two sheet currents each varying as cosine two theta. Thus

$$i_z = \begin{Bmatrix} i_b \\ i_c \end{Bmatrix} \cos 2\theta. \quad \begin{matrix} (r=b) \\ (r=c) \end{matrix} \quad (\text{abA/cm}) \tag{5}$$

The vector potential corresponding to this excitation is (emu)

$$A_z = \pi\begin{Bmatrix} \left[(1+b^4r_s^{-4})\frac{i_b}{b}+(1+c^4r_s^{-4})\frac{i_c}{c}\right]r^2 \\ \left[b^4r_s^{-4}\frac{i_b}{b}+(1+c^4r_s^{-4})\frac{i_c}{c}\right]r^2+b^3i_br^{-2} \\ \left[b^4r_s^{-4}\frac{i_b}{b}+c^4r_s^{-4}\frac{i_c}{c}\right]r^2+(b^3i_b+c^3i_c)r^{-2} \end{Bmatrix}\cos 2\theta , \tag{6}$$

where the top entry is for $r<b$, the middle entry for $b<r<c$, and the bottom entry for $c<r<r_s$. See Fig. 1 for geometrical details.

The magnetic induction is given by taking the curl of Eq. (6)

$$B_r = -2\pi \left\{ \begin{array}{l} \left[(1+b^4 r_s^{-4}) \frac{i_b}{b} + (1+c^4 r_s^{-4}) \frac{i_c}{c} \right] r \\ \left[b^4 r_s^{-4} \frac{i_b}{b} + (1+c^4 r_s^{-4}) \frac{i_c}{c} \right] r + b^3 i_b r^{-3} \\ \left[b^4 r_s^{-4} \frac{i_b}{b} + c^4 r_s^{-4} \frac{i_c}{c} \right] r + (b^3 i_b + c^3 i_c) r^{-3} \end{array} \right\} \sin 2\theta \quad (7)$$

$$B_\theta = -2\pi \left\{ \begin{array}{l} \left[(1+b^4 r_s^{-4}) \frac{i_b}{b} + (1+c^4 r_s^{-4}) \frac{i_c}{c} \right] r \\ \left[b^4 r_s^{-4} \frac{i_b}{b} + (1+c^4 r_s^{-4}) \frac{i_c}{c} \right] r - b^3 i_b r^{-3} \\ \left[b^4 r_s^{-4} \frac{i_b}{b} + c^4 r_s^{-4} \frac{i_c}{c} \right] r - (b^3 i_b + c^3 i_c) r^{-3} \end{array} \right\} \cos 2\theta . \quad (8)$$

In order to calculate the forces one needs the average field at the current sheets. Thus

$$B_r = -2\pi \left\{ \begin{array}{ll} (1+b^4 r_s^{-4}) i_b + \frac{b}{c}(1+c^4 r_s^{-4}) i_c & (r=b) \\ (1+c^4 r_s^{-4})(b^3 c^{-3} i_b + i_c) & (r=c) \end{array} \right\} \sin 2\theta \quad (9)$$

$$B_\theta = -2\pi \left\{ \begin{array}{ll} b^4 r_s^{-4} i_b + \frac{b}{c}(1+c^4 r_s^{-4}) i_c & (r=b) \\ -\frac{b^3}{c^3}(1-c^4 r_s^{-4}) i_b + c^4 r_s^{-4} i_c & (r=c) \end{array} \right\} \cos 2\theta . \quad (10)$$

Hence the force per unit area of the current sheet is given by (dynes/ cm^2)

$$f_r = \pi \left\{ \begin{array}{l} b^4 r_s^{-4} i_b^2 + \frac{b}{c}(1+c^4 r_s^{-4}) i_b i_c \\ \frac{-b^3}{c^3}(1-c^4 r_s^{-4}) i_b i_c + c^4 r_s^{-4} i_c^2 \end{array} \right\} (1+\cos 4\theta) , \quad (11)$$

$$f_\theta = -\pi \left\{ \begin{array}{l} (1+b^4 r_s^{-4}) i_b^{\,2} + \frac{b}{c}(1+c^4 r_s^{-4}) i_b i_c \\ (1+c^4 r_s^{-4}) (\frac{b^3}{c^3} i_b i_c + i_c^{\,2}) \end{array} \right\} \sin 4\theta . \qquad (12)$$

The magnetic energy stored in a unit length of the quadrupole is given by

$$W_B = \frac{1}{2}\int A_z i_z r d\theta , \qquad (13)$$

which when contributions from each shell are added gives

$$W_B = \frac{1}{2}\pi^2 \Big\{ b^2 (1+b^4 r_s^{-4}) i_b^{\,2} + 2\frac{b^3}{c}(1+c^4 r_s^{-4}) i_b i_c + c^2 (1+c^4 r_s^{-4}) i_c^{\,2} \Big\} . \quad \text{(ergs/cm)} \qquad (14)$$

If $\overleftrightarrow{\tau}$ designates the Maxwell stress tensor then in utilizing the virial theorem[2] one needs the projection of the outward radial traction on the radius vector. This becomes

$$\vec{r}\cdot\overleftrightarrow{\tau}\cdot\vec{n} = \frac{1}{8\pi}(B_r^{\,2} - B_\theta^{\,2}) r . \qquad (15)$$

Evaluating the integral in the virial theorem gives

$$\int_{r=r_s} \vec{r}\cdot\overleftrightarrow{\tau}\cdot\vec{n} r d\theta = 2\pi^2 (b^3 i_b + c^3 i_c)^2 r_s^{-4} . \qquad (16)$$

Finally the surface current densities are chosen so that both the total current and the radial moment of the current are the same as for the thick cosine two theta quadrupole. Hence

$$2\pi (b i_b + c i_c) = J_o \pi (c^2 - b^2) , \qquad (17)$$

$$2\pi (b^2 i_b + c^2 i_c) = J_o \frac{2}{3}\pi (c^3 - b^3) , \qquad (18)$$

which gives

$$i_b = \frac{1}{6b}(c^2+cb-2b^2)J_o, \quad (19)$$

$$i_c = \frac{1}{6c}(2c^2-cb-b^2)J_o. \quad (20)$$

Equations (8), (19), and (20) may be used to eliminate the current density J_o. Thus

$$B_o' = -\frac{\pi}{3}J_o\left\{\frac{c^2}{b^2}+\frac{c}{b}-\frac{b}{c}-\frac{b^2}{c^2}+[2(c^4-b^4)-bc(c^2-b^2)]r_s^{-4}\right\}. \quad (21)$$

<u>Thick Cosine Two Theta</u> (Model B):

By definition a thick cosine two theta conductor carries an axial current between two radii (b,c) with a current density that varies as

$$J_z = J_o\cos 2\theta. \quad (22)$$

From this it follows that the vector potential[3] corresponding to this excitation is (emu)

$$A_z = \pi J_o\left\{\begin{array}{c}[\ln\frac{c}{b}+\frac{1}{4}(c^4-b^4)r_s^{-4}]r^2 \\ [\frac{1}{4}+\ln\frac{c}{r}+\frac{1}{4}(c^4-b^4)r_s^{-4}]r^2-\frac{1}{4}b^4r^{-2} \\ \frac{1}{4}(c^4-b^4)(r_s^{-4}r^2+r^{-2})\end{array}\right\}\cos 2\theta \quad (23)$$

,

where the three entries are for the three regions explained in Case A.

The magnetic induction becomes

$$B_r = -2\pi J_o \left\{ \begin{array}{c} [\ln\frac{c}{b}+\frac{1}{4}(c^4-b^4)r_s^{-4}]r \\ \\ [\frac{1}{4}+\ln\frac{c}{r}+\frac{1}{4}(c^4-b^4)r_s^{-4}]r-\frac{1}{4}b^4r^{-3} \\ \\ \frac{1}{4}(c^4-b^4)(rr_s^{-4}+r^{-3}) \end{array} \right\} \sin 2\theta \quad , \tag{24}$$

$$B_r = -2\pi J_o \left\{ \begin{array}{c} [\ln\frac{c}{b}+\frac{1}{4}(c^4-b^4)r_s^{-4}]r \\ \\ [-\frac{1}{4}+\ln\frac{c}{r}+\frac{1}{4}(c^4-b^4)r_s^{-4}]r+\frac{1}{4}b^4r^{-3} \\ \\ \frac{1}{4}(c^4-b^4)(rr_s^{-4}-r^{-3}) \end{array} \right\} \cos 2\theta \quad . \tag{25}$$

The Lorentz force in the region of the conductor is (dynes/cm^3)

$$f_r = -J_zB_\theta = \pi J_o{}^2 \left\{ [-\frac{1}{4}+\ln\frac{c}{r}+\frac{1}{4}(c^4-b^4)r_s^{-4}]r+\frac{1}{4}b^4r^{-3} \right\} (1+\cos 4\theta) \quad , \tag{26}$$

$$f_\theta = J_zB_r = -\pi J_o{}^2 \left\{ [\frac{1}{4}+\ln\frac{c}{r}+\frac{1}{4}(c^4-b^4)r_s^{-4}]r-\frac{1}{4}b^4r^{-3} \right\} \sin 4\theta \quad . \tag{27}$$

The magnetic energy stored in a unit length of the quadrupole is given by

$$W_B = \frac{1}{2}\iint A_zJ_z r\,dr\,d\theta \tag{28}$$

Using Eqs. (22-23) one has

$$W_B = \frac{1}{16}\pi^2J_o{}^2 \left\{ -4b^4\ln\frac{c}{b}+(c^4-b^4)[1+\frac{1}{2}(c^4-b^4)r_s^{-4}] \right\} . \quad \text{(ergs/cm)} \tag{29}$$

The integral of interest in the virial theorem is from Eq. (15) and Eqs. (24-25)

$$\int_{r=r_s} \vec{r}\cdot\vec{\vec{\tau}}\cdot\vec{n}\,r\,d\theta = \frac{1}{8}\pi^2J_o{}^2(c^4-b^4)r_s^{-4} \quad . \tag{30}$$

Finally the current density J_o is chosen by relating it to the desired central gradient. From Eq. (25)

$$B_o' = -2\pi J_o[\ln\frac{c}{b}+\frac{1}{4}(c^4-b^4)r_s^{-4}]. \quad (31)$$

Form of Solution

Equations (11-12) and Eqs. (26-27) indicate that a suitable form for the displacement is

$$u_r = P_o(r)+P_4(r)\cos 4\theta \quad u_\theta = Q_4(r)\sin 4\theta. \quad (32)$$

Substituting into Eqs. (2-3) gives

$$-\beta\frac{d}{dr}[\frac{1}{r}\frac{d}{dr}(rP_o)] = -\mu[(1-\lambda)r+4r\ln r-b^4r^{-3}], \quad (33)$$

$$\frac{16}{r^2}P_4-\beta\frac{d}{dr}[\frac{1}{r}\frac{d}{dr}(rP_4)]+\frac{4}{r^2}\frac{d}{dr}(rQ_4)-4\beta\frac{d}{dr}(\frac{Q_4}{r}) =$$

$$-\mu[(1-\lambda)r+4r\ln r-b^4r^{-3}], \quad (34)$$

$$-4\frac{d}{dr}(\frac{P_4}{r})+\frac{4\beta}{r^2}\frac{d}{dr}(rP_4)-\frac{d}{dr}[\frac{1}{r}\frac{d}{dr}(rQ_4)]+\frac{16\beta}{r^2}Q_4 =$$

$$=\mu[-(1+\lambda)r+4r\ln r+b^4r^{-3}], \quad (35)$$

where for convenience

$$\mu = \frac{1}{2}\pi J_o^2\ \frac{1+\nu}{E}, \qquad \lambda = (c^4-b^4)r_s^{-4}+4\ln c. \quad (36)$$

Solutions of the Homogeneous Equations

In general these solutions are of the form

$$P_o = Ar^{-1}+Br \qquad P_4 = Cr^p \qquad Q_4 = Dr^p, \quad (33)$$

where p is found by substituting into Eqs. (34-35) to obtain

$$[16-\beta(p^2-1)]C+4[p+1-\beta(p-1)]D = 0, \quad (38)$$

$$4[-p+1+\beta(p+1)]C-[p^2-1-16\beta]D = 0. \quad (39)$$

The determinant of the coefficients is

$$\Delta(p) = \beta(p^2-9)(p^2-25). \tag{40}$$

Setting this equal to zero gives p = ±3, ±5. Hence there are four solutions which must be added together to give

$$P_4 = -D_1 r^3 + \frac{1-2\beta}{2-\beta} D_2 r^{-3} - \frac{3-2\beta}{2-3\beta} D_3 r^5 + D_4 r^{-5}, \tag{41}$$

$$Q_4 = D_1 r^3 + D_2 r^{-3} + D_3 r^5 + D_4 r^{-5}. \tag{42}$$

In addition to solutions of the form given in Eq. (32) another solution is added of the form

$$u_r = \frac{1}{\beta} G r \ln r \qquad u_\theta = G r \theta, \tag{43}$$

which is utilized in describing pretension.

<u>Displacement</u>

Collecting all the forms together one has after relabeling the constants

$$u_r = \begin{cases} A_1 r^{-1} + B_1 r \\ \quad + [-C_1 r^3 + \frac{1-2\beta_1}{2-\beta_1} D_1 r^{-3} - \frac{3-2\beta_1}{2-3\beta_1} E_1 r^5 + F_1 r^{-5}]\cos 4\theta . \\ A_2 r^{-1} + B_2 r \\ \quad + [-C_2 r^3 + \frac{1-2\beta_2}{2-\beta_2} D_2 r^{-3} - \frac{3-2\beta_2}{2-3\beta_2} E_2 r^5 + F_2 r^{-5}]\cos 4\theta \\ A_3 r^{-1} + \frac{1}{\beta_3} G_3 r \ln r + B_3 r \\ \quad + [-C_3 r^3 + \frac{1-2\beta_3}{2-\beta_3} D_3 r^{-3} - \frac{3-2\beta_3}{2-3\beta_3} E_3 r^5 + F_3 r^{-5}]\cos 4\theta \end{cases} \tag{44}$$

$$u_\theta = \begin{cases} [C_1r^3+D_1r^{-3}+E_1r^5+F_1r^{-5}]\sin 4\theta \\ [C_2r^3+D_2r^{-3}+E_2r^5+F_2r^{-5}]\sin 4\theta \\ G_3r\theta + [C_3r^3+D_3r^{-3}+E_3r^5+F_3r^{-5}]\sin 4\theta \end{cases} . \qquad (45)$$

Strain

Using Eqs. (61-63) from Ref. (2) the strains are given by

$$\varepsilon_{rr} = \begin{cases} -A_1r^{-2} \qquad +B_1 \\ \quad +[-3C_1r^2-3\frac{1-2\beta_1}{2-\beta_1}D_1\,r^{-4}-5\frac{3-2\beta_1}{2-3\beta_1}E_1\,r^4-5F_1r^{-6}]\cos 4\theta \\ -A_2r^{-2} \qquad +B_2 \\ \quad +[-3C_2r^2-3\frac{1-2\beta_2}{2-\beta_2}D_2\,r^{-4}-5\frac{3-2\beta_2}{2-3\beta_2}E_2\,r^4-5F_2r^{-6}]\cos 4\theta \\ -A_3r^{-2}+\frac{1}{\beta_3}G_3(1+\ln r)+B_3 \\ \quad +[-3C_3r^2-3\frac{1-2\beta_3}{2-\beta_3}D_3\,r^{-4}-5\frac{3-2\beta_3}{2-3\beta_3}E_3\,r^4-5F_3r^{-6}]\cos 4\theta \end{cases} , \qquad (46)$$

$$\varepsilon_{\theta\theta} = \begin{cases} A_1r^{-2} \qquad +B_1 \\ \quad +[3C_1r^2+3\frac{3-2\beta_1}{2-\beta_1}D_1\,r^{-4}+5\frac{1-2\beta_1}{2-3\beta_1}E_1\,r^4+5F_1r^{-6}]\cos 4\theta \\ A_2r^{-2} \qquad +B_2 \\ \quad +[3C_2r^2+3\frac{3-2\beta_2}{2-\beta_2}D_2\,r^{-4}+5\frac{1-2\beta_2}{2-3\beta_2}E_2\,r^4+5F_2r^{-6}]\cos 4\theta \\ A_3r^{-2}+G_3(1+\frac{1}{\beta_3}\ln r)+B_3 \\ \quad +[3C_3r^2+3\frac{3-2\beta_3}{2-\beta_3}D_3\,r^{-4}+5\frac{1-2\beta_3}{2-3\beta_3}E_3\,r^4+5F_3r^{-6}]\cos 4\theta \end{cases} , \qquad (47)$$

$$\varepsilon_{r\theta} = \begin{cases} 3C_1r^2-6\frac{1-\beta_1}{2-\beta_1}D_1\,r^{-4}+10\frac{1-\beta_1}{2-3\beta_1}E_1\,r^4-5F_1r^{-6} \\ 3C_2r^2-6\frac{1-\beta_2}{2-\beta_2}D_2r^{-4}+10\frac{1-\beta_2}{2-3\beta_2}E_2\,r^4-5F_2r^{-6} \\ 3C_3r^2-6\frac{1-\beta_3}{2-\beta_3}D_3r^{-4}+10\frac{1-\beta_3}{2-3\beta_3}E_3\,r^4-5F_3r^{-6} \end{cases} \sin 4\theta \; . \qquad (48)$$

Stress

To obtain the stress invert Eqs. (48-50) of Ref. (2). Thus, after using Eq. (4)

$$\sigma_{rr} = \frac{1}{2}\frac{E}{1+\nu}[\beta\varepsilon_{rr}-(2-\beta)\varepsilon_{\theta\theta}-(2-\beta)\varepsilon_{zz}+(4-3\beta)k], \tag{49}$$

$$\sigma_{\theta\theta} = \frac{1}{2}\frac{E}{1+\nu}[-(2-\beta)\varepsilon_{rr}+\beta\varepsilon_{\theta\theta}-(2-\beta)\varepsilon_{zz}+(4-3\beta)k], \tag{50}$$

$$\sigma_{r\theta} = \frac{E}{1+\nu}\varepsilon_{r\theta}, \tag{51}$$

where k is the thermal expansion coefficient integrated from room temperature to say 4.2°k. Using the homogeneous contribution to the strain from Eqs. (46-48) the homogeneous contribution to the stress becomes

$$\frac{1+\nu}{E}\sigma_{rr} = \begin{cases} -A_1r^{-2} -\frac{1}{2}(2-\beta_1)\varepsilon_{zz}+\frac{1}{2}(4-3\beta_1)k_1-(1-\beta_1)B_1 \\ \quad -[3C_1r^2+9\frac{1-\beta_1}{2-\beta_1}D_1 r^{-4}+5\frac{1-\beta_1}{2-3\beta_1}E_1r^4+5F_1r^{-6}]\cos 4\theta \\ -A_2r^{-2} -\frac{1}{2}(2-\beta_2)\varepsilon_{zz}+\frac{1}{2}(4-3\beta_2)k_2-(1-\beta_2)B_2 \\ \quad -[3C_2r^2+9\frac{1-\beta_2}{2-\beta_2}D_2 r^{-4}+5\frac{1-\beta_2}{2-3\beta_2}E_2 r^4+5F_2r^{-6}]\cos 4\theta \\ -A_3r^{-2}-G_3(1-\beta_3)(\frac{1}{2}+\frac{1}{\beta_3}\ln r)-\frac{1}{2}(2-\beta_3)\varepsilon_{zz}+\frac{1}{2}(4-3\beta_3)k_3-(1-\beta_3)B_3 \\ \quad -[3C_3r^2+9\frac{1-\beta_3}{2-\beta_3}D_3 r^{-4}+5\frac{1-\beta_3}{2-3\beta_3}E_3 r^4+5F_3r^{-6}]\cos 4\theta \end{cases}, \tag{52}$$

$$\frac{1+\nu}{E}\sigma_{\theta\theta} = \begin{cases} A_1 r^{-2} -\frac{1}{2}(2-\beta_1)\varepsilon_{zz}+\frac{1}{2}(4-3\beta_1)k_1-(1-\beta_1)B_1 \\ \quad +[3C_1 r^2+3\frac{1-\beta_1}{2-\beta_1}D_1 r^{-4}+15\frac{1-\beta_1}{2-3\beta_1}E_1 r^4+5F_1 r^{-6}]\cos 4\theta \\ A_2 r^{-2} -\frac{1}{2}(2-\beta_2)\varepsilon_{zz}+\frac{1}{2}(4-3\beta_2)k_2-(1-\beta_2)B_2 \\ \quad +[3C_2 r^2+3\frac{1-\beta_2}{2-\beta_2}D_2 r^{-4}+15\frac{1-\beta_2}{2-3\beta_2}E_2 r^4+5F_2 r^{-6}]\cos 4\theta \\ A_3 r^{-2}-G_3\frac{1-\beta_3}{\beta_3}(1+\frac{1}{2}\beta_3+\ln r)-\frac{1}{2}(2-\beta_3)\varepsilon_{zz}+\frac{1}{2}(4-3\beta_3)k_3-(1-\beta_3)B_3 \\ \quad +[3C_3 r^2+3\frac{1-\beta_3}{2-\beta_3}D_3 r^{-4}+15\frac{1-\beta_3}{2-3\beta_3}E_3 r^4+5F_3 r^{-6}]\cos 4\theta, \end{cases} \quad (53)$$

$$\frac{1+\nu}{E}\sigma_{r\theta} = \begin{Bmatrix} 3C_1 r^2-6\frac{1-\beta_1}{2-\beta_1}D_1 r^{-4}+10\frac{1-\beta_1}{2-3\beta_1}E_1 r^4-5F_1 r^{-6} \\ 3C_2 r^2-6\frac{1-\beta_2}{2-\beta_2}D_2 r^{-4}+10\frac{1-\beta_2}{2-3\beta_2}E_2 r^4-5F_2 r^{-6} \\ 3C_3 r^2-6\frac{1-\beta_3}{2-\beta_3}D_3 r^{-4}+10\frac{1-\beta_3}{2-3\beta_3}E_3 r^4-5F_3 r^{-6} \end{Bmatrix} \sin 4\theta . \quad (54)$$

Particular Solution

The particular solution of Eq. (33) may be found by integrating and dropping those terms already included in the homogeneous form. Thus

$$P_o = \frac{\mu}{\beta}[-\frac{1}{8}(2+\lambda)r^3+\frac{1}{2}r^3\ln r+\frac{1}{2}b^4 r^{-1}\ln r] \quad (55)$$

The contribution to the particular solutions (P_4, Q_4) of Eqs. (34-35) arising from a term on the RHS of the form r^{-3} may be found by letting $\begin{pmatrix} P_4 \\ Q_4 \end{pmatrix} = \begin{pmatrix} C \\ D \end{pmatrix} r^{-1}$. Thus

$$16C+8\beta D = \mu b^4 \tag{56}$$

$$8C+16\beta D = \mu b^4 \tag{57}$$

or

$$C = \frac{1}{24}\mu b^4 \qquad D = \frac{1}{24}\frac{\mu}{\beta}b^4. \tag{58}$$

The remaining contribution to the particular solution (P_4, Q_4) arising from the terms on the RHS proportional to r and to rlnr may be found by assuming solutions of the form

$$P_4 = (R+S\ln r+T\ln^2 r)r^3, \tag{59}$$

$$Q_4 = (U+V\ln r+W\ln^2 r)r^3. \tag{60}$$

Substituting into Eq. (34) gives

$$\begin{aligned}&[(16-8\beta)(R+U)-6\beta S+4(1-\beta)V-2\beta T]r\\ &+[(16-8\beta)(S+V)-12\beta T+8(1-\beta)W]r\ln r\\ &+(16-8\beta)(T+W)r\ln^2 r = -\mu[(1-\lambda)r+4r\ln r].\end{aligned} \tag{61}$$

Substituting Eq. (59-60) into Eq. (35) gives

$$\begin{aligned}&[(-8+16\beta)(R+U)-4(1-\beta)S-6V-2W]r\\ &+[(-8+16\beta)(S+V)-8(1-\beta)T-12W]r\ln r\\ &+(-8+16\beta)(T+W)r\ln^2 r = \mu[-(1+\lambda)r+4r\ln r]\end{aligned} \tag{62}$$

In this substitution the term on the RHS of Eqs. (34-35) proportional to r^{-3} has been deleted since this contribution has already been found. Setting

$$T+W = 0 \tag{63}$$

eliminates the term in $r\ln^2 r$ from Eqs. (61-62). Equating the coefficients of rlnr on each side of Eqs. (61-62) gives

$$(16-8\beta)(S+V)-12\beta T+8(1-\beta)W = -4\mu, \tag{64}$$

$$(-8+16\beta)(S+V)-8(1-\beta)T-12W = 4\mu. \tag{65}$$

Solving Eqs. (63-65) simultaneously gives

$$T = \frac{1}{6}(1+\beta)\frac{\mu}{\beta}, \qquad W = -\frac{1}{6}(1+\beta)\frac{\mu}{\beta}, \tag{66}$$

$$S+V = \frac{1}{12}(1-\beta)\frac{\mu}{\beta}. \tag{67}$$

Equating the coefficients of r on each side of Eqs. (61-62) gives

$$[(16-8\beta)(R+U)-6\beta S+4(1-\beta)V-2\beta T] = -\mu(1-\lambda), \tag{68}$$

$$[(-8+16\beta)(R+U)-4(1-\beta)S-6V-2W] = -\mu(1+\lambda). \tag{69}$$

Solving Eqs. (68-69) simultaneously gives

$$S = -\frac{1}{12}\frac{\mu}{\beta}[\frac{1}{6}(19-11\beta)+(1+\beta)\lambda], \; V = \frac{1}{12}\frac{\mu}{\beta}[\frac{1}{6}(25-17\beta)+(1+\beta)\lambda], \tag{70}$$

$$R+U = -\frac{1}{48}\frac{\mu}{\beta}[\frac{1}{6}(25+11\beta)+(1-\beta)\lambda]. \tag{71}$$

Since R and U are coefficients of r^3, a term that is already included in the homogeneous solution, it is possible to choose one relation between R and U arbitrarily. Hence let U = 0. Then

$$R = -\frac{1}{48}\frac{\mu}{\beta}[\frac{1}{6}(25+11\beta)+(1-\beta)\lambda], \qquad U = 0. \tag{72}$$

Collecting all the contributions gives

$$P_4 = \frac{\mu}{\beta}\left\{\frac{1}{24}\beta b^4 r^{-1}-\frac{1}{48}[\frac{1}{6}(25+11\beta)+(1-\beta)\lambda]r^3 -\frac{1}{12}[\frac{1}{6}(19-11\beta)+(1+\beta)\lambda]r^3 \ln r+\frac{1}{6}(1+\beta)r^3\ln^2 r\right\}, \tag{73}$$

$$Q_4 = \frac{\mu}{\beta}\left\{\frac{1}{24}b^4 r^{-1}+\frac{1}{12}[\frac{1}{6}(25-17\beta)+(1+\beta)\lambda]r^3 \ln r -\frac{1}{6}(1+\beta)r^3\ln^2 r\right\}. \tag{74}$$

Displacement

The particular solution exists only in the region of conductor (b<r<c). If this part of the solution is thought of as an additional displacement to be added to the homogeneous forms previously found, then

$$\Delta u_r = P_o + P_4\cos 4\theta \quad \Delta u_\theta = Q_4 \sin 4\theta, \tag{75}$$

where P_o, P_4, Q_4 are given in Eq. (55) and Eqs. (73-74).

Strain

Using Eqs. (61-63) from Ref. (2) the incremental strains are given by

$$\Delta\varepsilon_{rr} = \frac{\mu}{\beta}\left\{ -\frac{1}{8}(2+3\lambda)r^2+\frac{3}{2}r^2\ln r+\frac{1}{2}b^4r^{-2}-\frac{1}{2}b^4r^{-2}\ln r \right.$$
$$+\left[-\frac{1}{24}\beta b^4r^{-2}-\frac{1}{48}[\frac{1}{6}(151-11\beta)+(7+\beta)\lambda]r^2\right.$$
$$\left.\left. -\frac{1}{24}[11-19\beta+6(1+\beta)\lambda]r^2\ln r+\frac{1}{2}(1+\beta)r^2\ln^2 r\right]\cos 4\theta\right\}, \tag{76}$$

$$\Delta\varepsilon_{\theta\theta} = \frac{\mu}{\beta}\left\{ -\frac{1}{8}(2+\lambda)r^2+\frac{1}{2}r^2\ln r+\frac{1}{2}b^4r^{-2}\ln r \right.$$
$$+\left[\frac{1}{24}(4+\beta)b^4r^{-2}-\frac{1}{48}[\frac{1}{6}(25+11\beta)+(1-\beta)\lambda]r^2\right.$$
$$\left.\left. +\frac{1}{24}[27-19\beta+6(1+\beta)\lambda]r^2\ln r-\frac{1}{2}(1+\beta)r^2\ln^2 r\right]\cos 4\theta\right\}, \tag{77}$$

$$\Delta\varepsilon_{r\theta} = \frac{\mu}{\beta}\left\{ -\frac{1}{24}(1+2\beta)b^4r^{-2}+\frac{1}{12}[\frac{1}{6}(25-3\beta)+\lambda]r^2 \right.$$
$$\left. +\frac{1}{12}[\frac{51}{6}(1-\beta)+3(1+\beta)\lambda]r^2\ln r-\frac{1}{2}(1+\beta)r^2\ln^2 r\right\}\sin 4\theta \quad . \tag{78}$$

Stress

The incremental stress is related to the incremental strain using Eqs. (49-51) after dropping the terms in ε_{zz} and k. Thus

$$\Delta\sigma_{rr} = \frac{E}{1+\nu}[\tfrac{1}{2}\beta\Delta\varepsilon_{rr}-\tfrac{1}{2}(2-\beta)\Delta\varepsilon_{\theta\theta}] \tag{79}$$

$$\Delta\sigma_{\theta\theta} = \frac{E}{1+\nu}[-\tfrac{1}{2}(2-\beta)\Delta\varepsilon_{rr}+\tfrac{1}{2}\beta\Delta\varepsilon_{\theta\theta}], \tag{80}$$

$$\Delta\sigma_{r\theta} = \frac{E}{1+\nu}\Delta\varepsilon_{r\theta}. \tag{81}$$

Applying these relations to Eqs. (76-78)gives

$$\begin{aligned}\Delta\sigma_{rr} = \frac{E}{1+\nu}\cdot\frac{\mu}{\beta}\Big\{&\tfrac{1}{8}[2(1-\beta)+(1-2\beta)\lambda]r^2-\tfrac{1}{2}(1-2\beta)r^2\ln r\\&+\tfrac{1}{4}\beta b^4r^{-2}-\tfrac{1}{2}b^4r^{-2}\ln r\\&+\Big[-\tfrac{1}{24}(4-\beta)b^4r^{-2}+\tfrac{1}{96}[\tfrac{1}{3}(25-77\beta)+2(1-5\beta)\lambda]r^2\\&-\tfrac{1}{8}[9(1-\beta)+2(1+\beta)\lambda]r^2\ln r+\tfrac{1}{2}(1+\beta)r^2\ln^2 r\Big]\cos 4\theta\Big\}\end{aligned}, \tag{82}$$

$$\begin{aligned}\Delta\sigma_{\theta\theta} = \frac{E}{1+\nu}\cdot\frac{\mu}{\beta}\Big\{&\tfrac{1}{8}[2(1-\beta)+(3-2\beta)\lambda]r^2-\tfrac{1}{2}(3-2\beta)r^2\ln r\\&-\tfrac{1}{4}(2-\beta)b^4r^{-2}+\tfrac{1}{2}b^4r^{-2}\ln r\\&+\Big[\tfrac{1}{8}\beta b^4r^{-2}+\tfrac{1}{96}[\tfrac{1}{3}(151-99\beta)+2(7-3\beta)\lambda]r^2\\&+\tfrac{1}{48}[22(1-\beta)+12(1+\beta)\lambda]r^2\ln r-\tfrac{1}{2}(1+\beta)r^2\ln^2 r\Big]\cos 4\theta\Big\}.\end{aligned} \tag{83}$$

$$\begin{aligned}\Delta\sigma_{r\theta} = \frac{E}{1+\nu}\cdot\frac{\mu}{\beta}\Big\{&-\tfrac{1}{24}(1+2\beta)b^4r^{-2}+\tfrac{1}{12}[\tfrac{1}{6}(25-3\beta)+\lambda]r^2\\&+\tfrac{1}{12}[\tfrac{51}{6}(1-\beta)+3(1+\beta)\lambda]r^2\ln r-\tfrac{1}{2}(1+\beta)r^2\ln^2 r\Big\}\sin 4\theta.\end{aligned} \tag{84}$$

Boundary Conditions

Within the innermost cylinder (a<r<b) and within the outermost cylinder (c<r<d) Eqs. (44-45) and Eqs. (52-54) are complete expressions for the displacement and stress. Inside the middle cylinder and for Model B Eqs. (44-45) must be supplemented by Eqs. (55, 73-75) to give the general expression for displacment. In addition, Eqs. (52-54) must be augmented by Eqs. (82-84) to give the complete form for stress. Using the complete forms the boundary conditions are as follows.

At r=a, the innermost radius

$$\sigma_{rr}^{(+)} = \sigma_{r\theta}^{(+)} = 0, \tag{85}$$

At r=b,

$$\sigma_{rr}^{(+)} - \sigma_{rr}^{(-)} = -f_r(b), \tag{86}$$

$$\sigma_{r\theta}^{(+)} - \sigma_{r\theta}^{(-)} = -f_\theta(b), \tag{87}$$

$$u_r^{(+)} - u_r^{(-)} = 0, \tag{88}$$

$$u_\theta^{(+)} - u_\theta^{(-)} = 0, \tag{89}$$

At r=c

$$\sigma_{rr}^{(+)} - \sigma_{rr}^{(-)} = -f_r(c), \tag{90}$$

$$\sigma_{r\theta}^{(+)} - \sigma_{r\theta}^{(-)} = -f_\theta(c), \tag{91}$$

$$u_r^{(+)} - u_r^{(-)} = 0, \tag{92}$$

$$u_\theta^{(+)}(2\pi) - u_\theta^{(+)}(0) = c\alpha, \tag{93}$$

and, after removing term proportional to θ

$$u_\theta^{(+)} - u_\theta^{(-)} = 0, \tag{94}$$

At r=d, the outermost radius

$$\sigma_{rr}^{(-)} = \sigma_{r\theta}^{(-)} = 0. \tag{95}$$

Note that f_r and f_θ are given by Eqs. (11-12) and may be written as

$$f_r = f_{ro} + f_{r4}\cos 4\theta, \tag{96}$$

$$f_\theta = f_{\theta 4}\sin 4\theta. \tag{97}$$

Since the normal stress σ_{rr} and the radial displacement u_r involve both isotropic terms and terms proportional to $\cos 4\theta$ there are 19 relations to be obtained from Eqs. (85-95). An examination of Eq. (52) indicates that there are twenty unknowns ($A_1 B_1 C_1 D_1 E_1 F_1 A_2 B_2 C_2 D_2 E_2 F_2$ $A_3 G_3 B_3 C_3 D_3 E_3 F_3 \varepsilon_{zz}$). The viral theroem will be used to supply the last relation.

In detail Eq. (85) gives

$$\frac{E_1}{1+\nu_1}[-A_1 a^{-2} - (1-\beta_1)B_1 - \tfrac{1}{2}(2-\beta_1)\varepsilon_{zz}]$$

$$= -\frac{E_1}{1+\nu_1}\,\frac{1}{2}(4-3\beta_1)k_1, \tag{98}$$

$$\frac{E_1}{1+\nu_1}[-3C_1 a^2 - 9\frac{1-\beta_1}{2-\beta_1}D_1 a^{-4} - 5\frac{1-\beta_1}{2-3\beta_1}E_1 a^4 - 5F_1 a^{-6}] = 0, \tag{99}$$

$$\frac{E_1}{1+\nu_1}\,[3C_1 a^2 - 6\frac{1-\beta_1}{2-\beta_1}D_1 a^{-4} + 10\frac{1-\beta_1}{2-3\beta_1}E_1 a^4 - 5F_1 a^{-6}] = 0. \tag{100}$$

At r=b, Eqs. (86-89) give

$$\frac{E_1}{1+\nu_1}[A_1 b^{-2} + (1-\beta_1)B_1 + \tfrac{1}{2}(2-\beta_1)\varepsilon_{zz}]$$

$$+\frac{E_2}{1+\nu_2}[-A_2 b^{-2} - (1-\beta_2)B_2 - \tfrac{1}{2}(2-\beta_2)\varepsilon_{zz}]$$

$$= -\frac{E_2}{1+\nu_2}\,\frac{1}{2}(4-3\beta_2)k_2 + \frac{E_1}{1+\nu_1}\cdot\frac{1}{2}(4-3\beta_1)k_1 - \Delta\sigma_{rro}(b) - f_{ro}(b), \tag{101}$$

$$\frac{E_1}{1+\nu_1}[3C_1b^2+9\frac{1-\beta_1}{2-\beta_1}D_1b^{-4}+5\frac{1-\beta_1}{2-3\beta_1}E_1b^4+5F_1b^{-6}]$$

$$+\frac{E_2}{1+\nu_2}[-3C_2b^2-9\frac{1-\beta_2}{2-\beta_2}D_2b^{-4}-5\frac{1-\beta_2}{2-3\beta_2}E_2b^4-5F_2b^{-6}]$$

$$= -\Delta\sigma_{rr4}(b)-f_{r4}(b), \tag{102}$$

$$\frac{E_1}{1+\nu_1}[-3C_1b^2+6\frac{1-\beta_1}{2-\beta_1}D_1b^{-4}-10\frac{1-\beta_1}{2-3\beta_1}E_1b^4+5F_1b^{-6}]$$

$$+\frac{E_2}{1+\nu_2}[3C_2b^2-6\frac{1-\beta_2}{2-\beta_2}D_2b^{-4}+10\frac{1-\beta_2}{2-3\beta_2}E_2b^4-5F_2b^{-6}]$$

$$= -\Delta\sigma_{r\theta 4}(b)-f_{\theta 4}(b), \tag{103}$$

$$-A_1b^{-1}-B_1b+A_2b^{-1}+B_2b = -\Delta u_{ro}(b), \tag{104}$$

$$C_1b^3-\frac{1-2\beta_1}{2-\beta_1}D_1b^{-3}+\frac{3-2\beta_1}{2-3\beta_1}E_1b^5-F_1b^{-5}$$

$$-C_2b^3+\frac{1-2\beta_2}{2-\beta_2}D_2b^{-3}-\frac{3-2\beta_2}{2-3\beta_2}E_2b^5+F_2b^{-5} = -\Delta u_{r4}(b), \tag{105}$$

$$-C_1b^3-D_1b^{-3}-E_1b^5-F_1b^{-5}$$

$$+C_2b^3+D_2b^{-3}+E_2b^5+F_2b^{-5} = -\Delta u_{\theta 4}(b). \tag{106}$$

At r=c Eqs. (90-93) give

$$\frac{E_2}{1+\nu_2}[A_2c^{-2}+(1-\beta_2)B_2+\frac{1}{2}(2-\beta_2)\varepsilon_{zz}]$$

$$+\frac{E_3}{1+\nu_3}[-A_3c^{-2}-G_3(1-\beta_3)(\frac{1}{2}+\frac{1}{\beta_3}\ln c)-(1-\beta_3)B_3-\frac{1}{2}(2-\beta_3)\varepsilon_{zz}]$$

$$= -\frac{E_3}{1+\nu_3}\cdot\frac{1}{2}(4-3\beta_3)k_3+\frac{E_2}{1+\nu_2}\cdot\frac{1}{2}(4-3\beta_2)k_2+\Delta\sigma_{rro}(c)-f_{ro}(c), \tag{107}$$

$$\frac{E_2}{1+\nu_2}[3C_2c^2+9\frac{1-\beta_2}{2-\beta_2}D_2c^{-4}+5\frac{1-\beta_2}{2-3\beta_2}E_2c^4+5F_2c^{-6}]$$

$$+\frac{E_3}{1+\nu_3}[-3C_3c^2-9\frac{1-\beta_3}{2-\beta_3}D_3c^{-4}-5\frac{1-\beta_3}{2-3\beta_3}E_3c^4-5F_3c^{-6}] = \Delta\sigma_{rr4}(c)-f_{r4}(c). \quad (108)$$

$$\frac{E_2}{1+\nu_2}[-3C_2c^2+6\frac{1-\beta_2}{2-\beta_2}D_2c^{-4}-10\frac{1-\beta_2}{2-3\beta_2}E_2c^4+5F_2c^{-6}]$$

$$+\frac{E_3}{1+\nu_3}[3C_3c^2-6\frac{1-\beta_3}{2-\beta_3}D_3c^{-4}+10\frac{1-\beta_3}{2-3\beta_3}E_3c^4-5F_3c^{-6}] = \Delta\sigma_{r\theta 4}(c)-f_{\theta 4}(c), \quad (109)$$

$$-A_2c^{-1} \qquad -B_2c$$

$$+A_3c^{-1}+\frac{1}{\beta_3}cG_3\ln c+B_3c = \Delta u_{ro}(c). \quad (110)$$

$$C_2c^3-\frac{1-2\beta_2}{2-\beta_2}D_2c^{-3}+\frac{3-2\beta_2}{2-3\beta_2}E_2c^5-F_2c^{-5}$$

$$-C_3c^3+\frac{1-2\beta_3}{2-\beta_3}D_3c^{-3}-\frac{3-2\beta_3}{2-3\beta_3}E_3c^5+F_3c^{-5} = \Delta u_{r4}(c), \quad (111)$$

$$2\pi G_3c = c\alpha, \quad (112)$$

$$-C_2c^3-D_2c^{-3}-E_2c^5-F_2c^{-5}$$

$$+C_3c^3+D_3c^{-3}+E_3c^5+F_3c^{-5} = \Delta u_{\theta 4}(c). \quad (113)$$

At r=d Eq. (95) gives

$$\frac{E_3}{1+\nu_3}[-A_3d^{-2}-G_3(1-\beta_3)(\frac{1}{2}+\frac{1}{\beta_3}\ln d)-(1-\beta_3)B_3-\frac{1}{2}(2-\beta_3)\epsilon_{zz}]$$

$$= -\frac{E_3}{1+\nu_3}\,\frac{1}{2}(4-3\beta_3)k_3\,, \quad (114)$$

$$\frac{E_3}{1+\nu_3}[-3C_3c^2-9\frac{1-\beta_3}{2-\beta_3}D_3c^{-4}-5\frac{1-\beta_3}{2-3\beta_3}E_3c^4-5F_3c^{-6}] = 0, \tag{115}$$

$$\frac{E_3}{1+\nu_3}[3C_3c^2-6\frac{1-\beta_3}{2-\beta_3}D_3c^{-4}+10\frac{1-\beta_3}{2-3\beta_3}E_3c^4-5F_3c^{-6}] = 0. \tag{116}$$

Note that $\Delta\sigma_{rro}$ and $\Delta\sigma_{rr4}$ are the isotropic and cos4θ terms of Eq. (82). Similarly $\Delta\sigma_{r\theta 4}$ is the sin4θ term of Eq. (84). Likewise $\Delta u_{ro} = P_o$, $\Delta u_{r4} = P_4$ and $\Delta u_{\theta 4} = Q_4$ of Eq. (75). Note also that the double current sheet excitation (Model A) is obtained by setting $\Delta\sigma_{rr} = \Delta\sigma_{\theta\theta} = \Delta\sigma_{r\theta} = \Delta u_r = \Delta u_\theta = 0$ while the thick cosine two theta excitation (Model B) is obtained by retaining the additional stresses and displacements but setting $f_{ro} = f_{r4} = f_{\theta 4} = 0$.

Use of the Virial Theorem

The virial theorem is used to obtain the final relation among the unknowns. It states that

$$\iint(\sigma_{rr}+\sigma_{\theta\theta}+\sigma_{zz})rdrd\theta = \int_{r=rs}\vec{r}\cdot\vec{\vec{\tau}}\cdot\vec{n}rd\theta+W_B, \tag{117}$$

where the double integral is throughout the cross section of the material under stress. The single integral is over the cylinder at $r=r_s$ (iron shield) and W_B is the magnetic energy per unit length contained within the region bounded by $r=r_s$. The RHS of Eq. (117) has been evaluated in Eqs. (14) and (16) for Model A and in Eqs. (29-30) for Model B. From Eq. (47) in Ref. (2) one remembers that

$$\sigma_{zz} = \nu(\sigma_{rr}+\sigma_{\theta\theta})+E(\varepsilon_{zz}-k). \tag{118}$$

The LHS of Eq. (117) becomes

$$LHS(117) = \iint[1+\nu)(\sigma_{rr}+\sigma_{\theta\theta})+E(\varepsilon_{zz}-k)]rdrd\theta, \tag{119}$$

or using Eqs. (52-53)

$$\mathrm{LHS}(117) = 2\pi\int E(1-\beta)[-G(1+\tfrac{1}{\beta}+\tfrac{2}{\beta}\ln r)-\varepsilon_{zz}+3k-2B]r\,dr$$

$$+2\pi\int(1+\nu)(\Delta\sigma_{rr}+\Delta\sigma_{\theta\theta})r\,dr, \qquad (120)$$

where the subscripts have been dropped since it is to be summed over each structural region. The indefinite integral is

Indefinite LHS(117)

$$= \pi E(1-\beta)[-G(1+\tfrac{2}{\beta}\ln r)-\varepsilon_{zz}+3k-2B]r^2$$

$$+\pi E\tfrac{\mu}{\beta}(1-\beta)[\tfrac{1}{4}(2+\lambda)r^4-r^4\ln r-b^4\ln r], \qquad (121)$$

where Eqs. (82-83) were utilized to obtain the second term. Placing this second term on the RHS and summing over all regions gives the final relation.

$$\pi E_1(1-\beta_1)[-\varepsilon_{zz}+3k_1-2B_1](b^2-a^2)$$

$$+\pi E_2(1-\beta_2)[-\varepsilon_{zz}+3k_2-2B_2](c^2-b^2)$$

$$+\pi E_3(1-\beta_3)\Big\{[-G_3-\varepsilon_{zz}+3k_3-2B_3](d^2-c^2)$$

$$-2\frac{G_3}{\beta_3}[d^2\ln d-c^2\ln c]\Big\} = \mathrm{RHS}(122), \qquad (122)$$

where for Model A (double sheet cosine two theta)

$$\mathrm{RHS}(122) = \tfrac{1}{2}\pi^2\Big\{b^2(1+5b^4r_s^{-4})i_b^2$$

$$+2\frac{b^3}{c}(1+5c^4r_s^{-4})i_b i_c$$

$$+c^2(1+5c^4r_s^{-4})i_c^2\Big\}. \qquad (123)$$

In Eq. (123) i_b and i_c are given by Eqs. (19-21).

For Model B (thick cosine two theta)

$$RHS(122) = \frac{1}{16}\pi^2 J_o^{\,2}\left\{-4b^4 \ln\frac{c}{b} + (c^4-b^4)\left[1+\frac{5}{2}(c^4-b^4)r_s^{-4}\right]\right\}$$

$$+\frac{1}{8}\pi^2 J_o^{\,2}\frac{1+\nu_2}{1-\nu_2}\left\{-4b^4 \ln\frac{c}{b} + (c^4-b^4)\left[1+\frac{1}{2}(c^4-b^4)r_s^{-4}\right]\right\}, \qquad (124)$$

where J_o is given in Eq. (31).

Numerical Calculations

The condition of stress, strain and displacement that exists in three nested hollow cylinders as a result of thermal cooldown, pre-tension in the outer cylinder, and two different cosine two theta axial current distributions in the middle cylinder has been calculated as a function of the central quadrupole gradient. Twenty algebraic relations in Eqs.(98-116) and Eq. (122) among the twenty unknown coefficients ($A_1 B_1 C_1 D_1 E_1 F_1 A_2 B_2 C_2 D_2 E_2 F_2 A_3 G_3 B_3 C_3 D_3 E_3 F_3 \varepsilon_{zz}$) have been solved. Thus, for example, the state of stress at any point in the quadrupole structure may be found. It is usually clear whether a quantity is stress or strain. Otherwise, R is radial, T is theta or aximuthal, Z is axial or longitudinal. With regard to position A,B,C,D are the radii bounding the various media. To indicate which side of a boundary radius, P is used for positive and M for negative. Thus, for example, RTBP indicates the (r,θ) component at radius B but in the material between B and C. As explained previously[2] $\sqrt{3J_2}$ is a stress which is to be compared with the yield stress in tension.

Numerical results are given relative to a beam line quadrupole. The following cases were calculated

Case	Cool Down	Pretension	Excitation
1	No	No	No
2	No	No	Yes
3	No	Yes	No
4	No	Yes	Yes
5	Yes	No	No
6	Yes	No	Yes
7	Yes	Yes	No
8	Yes	Yes	Yes

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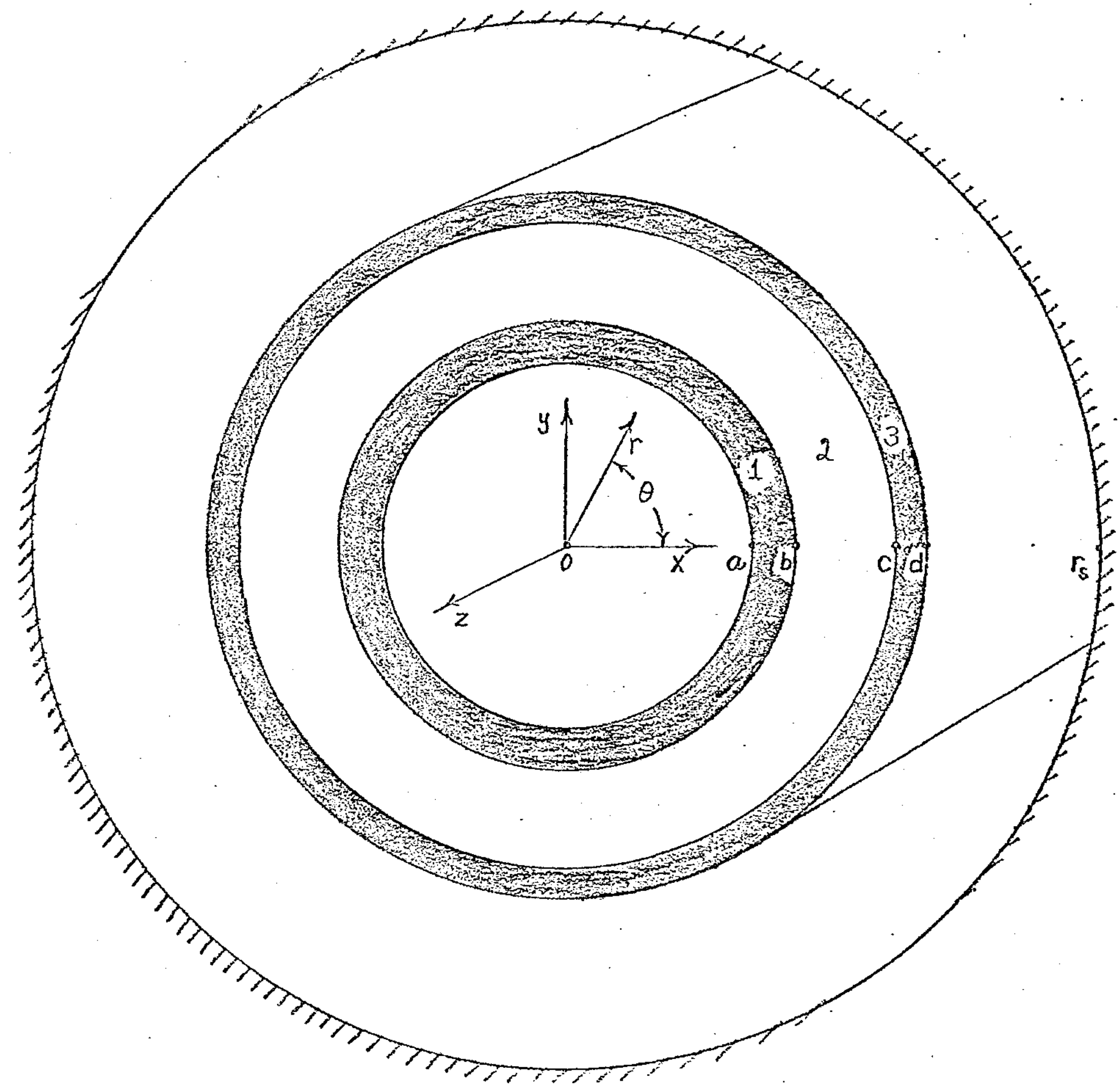

Fig. 1. Geometric Details of Doubler Dipole Model

ELASTIC STRESS AND STRAIN IN BEAM LINE QUADRUPOLE ELQUAD

CENTRAL GRADIENT(KG/IN)	= 0.0000	INNER BORE TUBE RADIUS(IN)	= 3.0000	INNER CONDUCTOR RADIUS(IN))	= 3.5000
OUTER CONDUCTOR RADIUS(IN))	= 5.8130	OUTER RADIUS OF BANDING(IN)	= 6.5000	RADIUS OF IRON SHIELD(IN)	= 9.0000
BORE YOUNGS MODULUS(LBS/IN/IN)	= 29000000.	COND. YOUNGS MODULUS(LBS/IN/IN)	= 1000000.	BAND YOUNGS MODULUS(LBS/IN/IN)	= 11000000.
BORE POISSONS RATIO	= .3330	COND. POISSONS RATIO	= .1000	BAND POISSONS RATIO	= .3330
BORE THERMAL STRAIN(IN/IN)	= 0.0000	COND. THERMAL STRAIN(IN/IN)	= 0.0000	BAND THERMAL STRAIN(IN/IN)	= 0.0000
BAND DISLOCATION(MILLIRAD)	= 0.000	MAGNETIC ENERGY((IN-LBS)/IN)	= 0.00	MAGNETIC ENERGY(J/M)	= 0.00
TYPE OF COMPUTATION	= ELQUAS				

TRANSVERSE STRESS(LB/IN/IN)

ANGLE (DEG)	RRAP	TTAP	RTAP	RRBM	TTBM	RTBM	RRBP	TTBP	RTBP	RRCM	TTCM	RTCM	RRCP	TTCP	RTCP	RRDM	TTDM	RTDM
45.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
40.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
35.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
30.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
25.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
20.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
15.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
10.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
5.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
-.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

TRANSVERSE STRAIN (MUIN/IN)

ANGLE (DEG)	RRAP	TTAP	RTAP	RRBM	TTBM	RTBM	RRBP	TTBP	RTBP	RRCM	TTCM	RTCM	RRCP	TTCP	RTCP	RRDM	TTDM	RTDM
45.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
40.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
35.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
30.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
25.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
20.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
15.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
10.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
5.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
-.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

ANGLE	LONG. STRESS(LB/IN/IN) AND STRAIN(MUIN/IN)							SQRT(3*J2) IN (KLB/IN/IN)						TRANSVERSE DISPLACEMENT(IN)							
(DEG)	ZZAP	ZZBM	ZZBP	ZZCM	ZZCP	ZZDM	EZZ	YAP	YBM	YBP	YCM	YCP	YDM	URA	UTA	URB	UTB	URC	UTC	URD	UTD
45.0	0.	0.	0.	0.	0.	0.	0.	0.0	0.0	0.0	0.0	0.0	0.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
40.0	0.	0.	0.	0.	0.	0.	0.	0.0	0.0	0.0	0.0	0.0	0.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
35.0	0.	0.	0.	0.	0.	0.	0.	0.0	0.0	0.0	0.0	0.0	0.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
30.0	0.	0.	0.	0.	0.	0.	0.	0.0	0.0	0.0	0.0	0.0	0.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
25.0	0.	0.	0.	0.	0.	0.	0.	0.0	0.0	0.0	0.0	0.0	0.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
20.0	0.	0.	0.	0.	0.	0.	0.	0.0	0.0	0.0	0.0	0.0	0.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
15.0	0.	0.	0.	0.	0.	0.	0.	0.0	0.0	0.0	0.0	0.0	0.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
10.0	0.	0.	0.	0.	0.	0.	0.	0.0	0.0	0.0	0.0	0.0	0.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5.0	0.	0.	0.	0.	0.	0.	0.	0.0	0.0	0.0	0.0	0.0	0.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-.0	0.	0.	0.	0.	0.	0.	0.	0.0	0.0	0.0	0.0	0.0	0.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Case 1. No Cool Down, No Pre Load, No Excitation

ELASTIC STRESS AND STRAIN IN BEAM LINE QUADRUPOLE ELQUAD

CENTRAL GRADIENT(KG/IN)	= 14.9000	INNER BORE TUBE RADIUS(IN)	= 3.0000	INNER CONDUCTOR RADIUS(IN))	= 3.5000
OUTER CONDUCTOR RADIUS(IN))	= 5.8130	OUTER RADIUS OF BANDING(IN)	= 6.5000	RADIUS OF IRON SHIELD(IN)	= 9.0000
BORE YOUNGS MODULUS(LBS/IN/IN)	= 29000000.	COND. YOUNGS MODULUS(LBS/IN/IN)	= 1000000.	BAND YOUNGS MODULUS(LBS/IN/IN)	= 11000000.
BORE POISSONS RATIO	= .3330	COND. POISSONS RATIO	= .1000	BAND POISSONS RATIO	= .3330
BORE THERMAL STRAIN(IN/IN)	= 0.0000	COND. THERMAL STRAIN(IN/IN)	= 0.0000	BAND THERMAL STRAIN(IN/IN)	= 0.0000
BAND DISLOCATION(MILLIRAD)	= 0.000	MAGNETIC ENERGY((IN-LBS)/IN)	= 127140.73	MAGNETIC ENERGY(J/M)	= 566354.17
TYPE OF COMPUTATION	= ELQUA5				

TRANSVERSE STRESS(LB/IN/IN)

ANGLE (DEG)	RRAP	TTAP	RTAP	RRBM	TTBM	RTBM	RRBP	TTBP	RTBP	RRCM	TTCM	RTCM	RRCP	TTCP	RTCP	RRDM	TTDM	RTDM
45.0	-0.	-11804.	-0.	243.	11110.	0.	243.	43.	0.	-193.	275.	-0.	-193.	4012.	-0.	-0.	-999.	0.
40.0	-0.	-10776.	0.	271.	10714.	-656.	271.	31.	-656.	-184.	258.	179.	-184.	3783.	179.	-0.	-912.	0.
35.0	-0.	-7916.	0.	349.	9574.	-1233.	349.	73.	-1233.	-156.	207.	336.	-156.	3146.	336.	-0.	-692.	0.
30.0	-0.	-3281.	0.	469.	7323.	-1661.	469.	106.	-1661.	-114.	130.	453.	-114.	2161.	453.	-0.	-357.	0.
25.0	-0.	2281.	0.	617.	5685.	-1889.	617.	147.	-1889.	-62.	35.	515.	-62.	952.	515.	-0.	55.	0.
20.0	0.	8201.	0.	774.	3406.	-1889.	774.	190.	-1889.	-7.	-66.	515.	-7.	-334.	515.	-0.	494.	0.
15.0	0.	13763.	0.	921.	1264.	-1661.	921.	231.	-1661.	45.	-162.	453.	45.	-1543.	453.	-0.	905.	0.
10.0	0.	18298.	0.	1041.	-483.	-1233.	1041.	264.	-1233.	87.	-239.	336.	87.	-2528.	336.	-0.	1242.	0.
5.0	0.	21258.	0.	1120.	-1623.	-656.	1120.	285.	-656.	115.	-290.	179.	115.	-3171.	179.	-0.	1461.	0.
-.0	0.	22286.	-0.	1147.	-2019.	0.	1147.	294.	0.	125.	-307.	-0.	125.	-3394.	-0.	-0.	1537.	0.

TRANSVERSE STRAIN (MUIN/IN)

ANGLE (DEG)	RRAP	TTAP	RTAP	RRBM	TTBM	RTBM	RRBP	TTBP	RTBP	RRCM	TTCM	RTCM	RRCP	TTCP	RTCP	RRDM	TTDM	RTDM
45.0	126.	-417.	-0.	-217.	292.	0.	220.	-0.	0.	-238.	277.	-0.	-232.	277.	-0.	-15.	-135.	0.
40.0	110.	-385.	0.	-211.	270.	-30.	245.	4.	-722.	-227.	259.	197.	-223.	259.	22.	-18.	-128.	0.
35.0	65.	-294.	0.	-191.	233.	-57.	321.	17.	-1356.	-194.	206.	370.	-194.	206.	41.	-27.	-111.	0.
30.0	-5.	-155.	0.	-160.	178.	-76.	437.	37.	-1827.	-143.	124.	498.	-151.	124.	55.	-40.	-94.	0.
25.0	-90.	15.	0.	-123.	110.	-87.	573.	61.	-2078.	-82.	25.	566.	-98.	25.	62.	-57.	-53.	0.
20.0	-180.	197.	0.	-83.	38.	-87.	729.	87.	-2078.	-15.	-92.	566.	-42.	-82.	62.	-75.	-15.	0.
15.0	-265.	367.	0.	-46.	-30.	-76.	870.	111.	-1827.	46.	-181.	498.	11.	-181.	55.	-91.	18.	0.
10.0	-335.	506.	0.	-15.	-36.	-57.	935.	131.	-1356.	96.	-263.	370.	54.	-263.	41.	-105.	46.	0.
5.0	-380.	597.	0.	4.	-122.	-30.	1061.	144.	-722.	129.	-316.	197.	82.	-316.	22.	-114.	63.	0.
-.0	-396.	628.	-0.	11.	-134.	0.	1087.	149.	0.	141.	-334.	-0.	92.	-334.	-0.	-117.	69.	0.

LONG. STRESS(LB/IN/IN) AND STRAIN(MLIN/IN); SQRT(3*J2) IN (KLB/IN/IN); TRANSVERSE DISPLACEMENT(IN)

ANGLE (DEG)	ZZAP	ZZBM	ZZBP	ZZCM	ZZCP	ZZDM	EZZ	YAP	YBM	YBP	YCM	YCP	YDM	URA	UTA	UPB	UTB	URC	UTC	URD	UTD
45.0	842.	8553.	193.	173.	3082.	1431.	165.	12.2	9.8	.2	.4	3.8	2.2	-.0004	.0000	-.0005	.0000	-.0016	.0000	-.0017	.0000
40.0	1184.	8430.	197.	172.	3011.	1507.	165.	11.4	9.6	1.2	.5	3.7	2.1	-.0003	-.0001	-.0004	-.0000	-.0015	-.0003	-.0016	-.0001
35.0	2170.	8077.	207.	170.	2806.	1580.	165.	9.1	8.8	2.1	.7	3.2	2.0	-.0002	-.0003	-.0003	-.0001	-.0013	-.0005	-.0013	-.0001
30.0	3580.	7535.	222.	166.	2492.	1692.	165.	6.0	7.8	2.9	.8	2.6	1.9	-.0000	-.0004	-.0001	-.0001	-.0009	-.0007	-.0009	-.0002
25.0	5532.	6871.	241.	162.	2107.	1829.	165.	4.8	6.6	3.3	.9	2.1	1.8	.0002	-.0004	.0001	-.0001	-.0004	-.0008	-.0005	-.0002
20.0	7503.	6164.	261.	157.	1597.	1975.	165.	7.9	5.7	3.3	.9	2.1	1.8	.0004	-.0004	.0004	-.0001	.0001	-.0008	.0000	-.0002
15.0	9356.	5500.	280.	153.	1312.	2112.	165.	12.2	5.3	3.0	.8	2.6	1.8	.0007	-.0004	.0006	-.0001	.0005	-.0007	.0005	-.0002
10.0	10865.	4958.	295.	149.	938.	2224.	165.	15.9	5.3	2.3	.7	3.2	1.9	.0008	-.0003	.0008	-.0001	.0009	-.0005	.0009	-.0001
5.0	11851.	4605.	305.	147.	793.	2297.	165.	18.5	5.5	1.4	.5	3.7	2.0	.0010	-.0001	.0009	-.0000	.0012	-.0003	.0012	-.0001
-.0	12194.	4482.	309.	146.	721.	2322.	165.	19.3	5.6	.8	.4	3.9	2.0	.0010	.0000	.0010	.0000	.0013	.0000	.0012	.0000

Case 2. No Cool Down, No Pre Load, With Excitation

ELASTIC STRESS AND STRAIN IN BEAM LINE QUADRUPOLE ELQUAD

CENTRAL GRADIENT(KG/IN)	= 0.0000	INNER BORE TUBE RADIUS(IN)	= 3.0000	INNER CONDUCTOR RADIUS(IN))	= 3.5000
OUTER CONDUCTOR RADIUS(IN))	= 5.8130	OUTER RADIUS OF BANDING(IN)	= 6.5000	RADIUS OF IRON SHIELD(IN)	= 9.0000
BORE YOUNGS MODULUS(LBS/IN/IN)	= 29000000.	COND. YOUNGS MODULUS(LBS/IN/IN)	= 1000000.	BAND YOUNGS MODULUS(LBS/IN/IN)	= 11000000.
BORE POISSONS RATIO	= .3330	COND. POISSONS RATIO	= .1000	BAND POISSONS RATIO	= .3330
BORE THERMAL STRAIN(IN/IN)	= 0.0000	COND. THERMAL STRAIN(IN/IN)	= 0.0000	BAND THERMAL STRAIN(IN/IN)	= 0.0000
BAND DISLOCATION(MILLIRAD)	= 2.500	MAGNETIC ENERGY((IN-LBS)/IN)	= 0.00	MAGNETIC ENERGY(J/M)	= 0.00
TYPE OF COMPUTATION	= ELQUA5				

TRANSVERSE STRESS(LB/IN/IN)

ANGLE (DEG)	RRAP	TTAP	RTAP	RRBM	TTBM	RTBM	RRBP	TTBP	RTBP	RRCM	TTCM	RTCM	RRCP	TTCP	RTCP	RRDM	TTDM	RTDM
45.0	0.	-3089.	0.	-410.	-2679.	0.	-410.	-119.	0.	-317.	-212.	0.	-317.	2565.	0.	-0.	2798.	0.
40.0	0.	-3089.	0.	-410.	-2679.	0.	-410.	-119.	0.	-317.	-212.	0.	-317.	2565.	0.	-0.	2798.	0.
35.0	0.	-3089.	0.	-410.	-2679.	0.	-410.	-119.	0.	-317.	-212.	0.	-317.	2565.	0.	-0.	2798.	0.
30.0	0.	-3089.	0.	-410.	-2679.	0.	-410.	-119.	0.	-317.	-212.	0.	-317.	2565.	0.	-0.	2798.	0.
25.0	0.	-3089.	0.	-410.	-2679.	0.	-410.	-119.	0.	-317.	-212.	0.	-317.	2565.	0.	-0.	2798.	0.
20.0	0.	-3089.	0.	-410.	-2679.	0.	-410.	-119.	0.	-317.	-212.	0.	-317.	2565.	0.	-0.	2798.	0.
15.0	0.	-3089.	0.	-410.	-2679.	0.	-410.	-119.	0.	-317.	-212.	0.	-317.	2565.	0.	-0.	2798.	0.
10.0	0.	-3089.	0.	-410.	-2679.	0.	-410.	-119.	0.	-317.	-212.	0.	-317.	2565.	0.	-0.	2798.	0.
5.0	0.	-3089.	0.	-410.	-2679.	0.	-410.	-119.	0.	-317.	-212.	0.	-317.	2565.	0.	-0.	2798.	0.
-.0	0.	-3089.	0.	-410.	-2679.	0.	-410.	-119.	0.	-317.	-212.	0.	-317.	2565.	0.	-0.	2798.	0.

TRANSVERSE STRAIN (MUIN/IN)

ANGLE (DEG)	RRAP	TTAP	RTAP	RRBM	TTBM	RTBM	RRBP	TTBP	RTBP	RRCM	TTCM	RTCM	RRCP	TTCP	RTCP	RRDM	TTDM	RTDM
45.0	52.	-90.	0.	33.	-72.	0.	-391.	-72.	0.	-289.	-174.	0.	-125.	224.	0.	-109.	230.	0.
40.0	52.	-90.	0.	33.	-72.	0.	-391.	-72.	0.	-289.	-174.	0.	-125.	224.	0.	-109.	230.	0.
35.0	52.	-90.	0.	33.	-72.	0.	-391.	-72.	0.	-289.	-174.	0.	-125.	224.	0.	-109.	230.	0.
30.0	52.	-90.	0.	33.	-72.	0.	-391.	-72.	0.	-289.	-174.	0.	-125.	224.	0.	-109.	230.	0.
25.0	52.	-90.	0.	33.	-72.	0.	-391.	-72.	0.	-289.	-174.	0.	-125.	224.	0.	-109.	230.	0.
20.0	52.	-90.	0.	33.	-72.	0.	-391.	-72.	0.	-289.	-174.	0.	-125.	224.	0.	-109.	230.	0.
15.0	52.	-90.	0.	33.	-72.	0.	-391.	-72.	0.	-289.	-174.	0.	-125.	224.	0.	-109.	230.	0.
10.0	52.	-90.	0.	33.	-72.	0.	-391.	-72.	0.	-289.	-174.	0.	-125.	224.	0.	-109.	230.	0.
5.0	52.	-90.	0.	33.	-72.	0.	-391.	-72.	0.	-289.	-174.	0.	-125.	224.	0.	-109.	230.	0.
-.0	52.	-90.	0.	33.	-72.	0.	-391.	-72.	0.	-289.	-174.	0.	-125.	224.	0.	-109.	230.	0.

ANGLE (DEG)	LONG. STRESS(LB/IN/IN) AND STRAIN(MUIN/IN) ZZAP	ZZBM	ZZBP	ZZCM	ZZCP	ZZDM	EZZ	SQRT(3*J2) IN (KLP/IN/IN) YAP	YBM	YBP	YCM	YCP	YDM	TRANSVERSE DISPLACEMENT(IN) URA	UTA	URB	UTB	URC	UTC	URD	UTD
45.0	-1397.	-1397.	-66.	-66.	609.	792.	-13.	2.7	2.0	.3	.2	2.5	2.5	-.00030	.0000	-.00030	.0000	-.00100	.0000	-.0011	.0023
40.0	-1397.	-1397.	-66.	-66.	609.	792.	-13.	2.7	2.0	.3	.2	2.5	2.5	-.00030	.0000	-.00030	.0000	-.00100	.0000	-.0011	.0013
35.0	-1397.	-1397.	-66.	-66.	609.	792.	-13.	2.7	2.0	.3	.2	2.5	2.5	-.00030	.0000	-.00030	.0000	-.00100	.0000	-.0011	.0016
30.0	-1397.	-1397.	-66.	-66.	609.	792.	-13.	2.7	2.0	.3	.2	2.5	2.5	-.00030	.0000	-.00030	.0000	-.00100	.0000	-.0011	.0014
25.0	-1397.	-1397.	-66.	-66.	609.	792.	-13.	2.7	2.0	.3	.2	2.5	2.5	-.00030	.0000	-.00030	.0000	-.00100	.0000	-.0011	.0011
20.0	-1397.	-1397.	-66.	-66.	609.	792.	-13.	2.7	2.0	.3	.2	2.5	2.5	-.00030	.0000	-.00030	.0000	-.00100	.0000	-.0011	.0009
15.0	-1397.	-1397.	-66.	-66.	609.	792.	-13.	2.7	2.0	.3	.2	2.5	2.5	-.00030	.0000	-.00030	.0000	-.00100	.0000	-.0011	.0007
10.0	-1397.	-1397.	-66.	-66.	609.	792.	-13.	2.7	2.0	.3	.2	2.5	2.5	-.00030	.0000	-.00030	.0000	-.00100	.0000	-.0011	.0005
5.0	-1397.	-1397.	-66.	-66.	609.	792.	-13.	2.7	2.0	.3	.2	2.5	2.5	-.00030	.0000	-.00030	.0000	-.00100	.0000	-.0011	.0002
-.0	-1397.	-1397.	-66.	-66.	609.	792.	-13.	2.7	2.0	.3	.2	2.5	2.5	-.00030	.0000	-.00030	.0000	-.00100	.0000	-.0011	-.0000

Case 3. No Cool Down, With Pre Load, No Excitation

ELASTIC STRESS AND STRAIN IN BEAM LINE QUADRUPOLE   ELQUAD

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CENTRAL GRADIENT(KG/IN)           =    14.8000 INNER BORE TUBE RADIUS(IN)         =    3.0000 INNER CONDUCTOR RADIUS(IN))        =    3.5000
OUTER CONDUCTOR RADIUS(IN))       =     5.8130 OUTER RADIUS OF BANDING(IN)        =    6.5000 RADIUS OF IRON SHIELD(IN)          =    9.0000
BORE YOUNGS MODULUS(LBS/IN/IN)    = 29000000. COND. YOUNGS MODULUS(LBS/IN/IN)=   1000000. BAND YOUNGS MODULUS(LBS/IN/IN) = 11000000.
BORE POISSONS RATIO               =      .3330 COND. POISSONS RATIO               =     .1000 BAND POISSONS RATIO                =     .3330
BORE THERMAL STRAIN(IN/IN)        =     0.0000 COND. THERMAL STRAIN(IN/IN)        =    0.0000 BAND THERMAL STRAIN(IN/IN)         =    0.0000
BAND DISLOCATION(MILLIRAD)        =      2.500 MAGNETIC ENERGY((IN-LBS)/IN)       = 127140.73 MAGNETIC ENERGY(J/M)               = 566354.17
TYPE OF COMPUTATION               =  ELQUAS
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TRANSVERSE STRESS(LB/IN/IN)

ANGLE (DEG)	RRAP	TTAP	RTAP	RRBM	TTBM	RTBM	RRBP	TTBP	RTBP	RRCM	TTCM	RTCM	RRCP	TTCP	RTCP	RRDM	TTDM	RTDM
45.0	-0.	-14892.	-0.	-166.	6431.	0.	-166.	-76.	0.	-510.	63.	-0.	-510.	6578.	-0.	-0.	1810.	0.
40.0	-0.	-13864.	0.	-139.	8035.	-656.	-139.	-68.	-656.	-501.	46.	179.	-501.	6354.	179.	-0.	1837.	0.
35.0	-0.	-10905.	0.	-61.	6895.	-1233.	-61.	-46.	-1233.	-473.	-5.	336.	-473.	5711.	336.	-0.	2106.	0.
30.0	-0.	-6370.	0.	60.	5149.	-1661.	60.	-13.	-1651.	-431.	-82.	453.	-431.	4725.	453.	-0.	2442.	0.
25.0	-0.	-807.	0.	207.	3007.	-1889.	207.	28.	-1889.	-379.	-177.	515.	-379.	3517.	515.	-0.	2854.	0.
20.0	0.	5112.	0.	364.	727.	-1889.	364.	71.	-1889.	-324.	-278.	515.	-324.	2231.	515.	-0.	3292.	0.
15.0	0.	10675.	0.	511.	-1415.	-1651.	511.	112.	-1651.	-272.	-373.	453.	-272.	1023.	453.	-0.	3704.	0.
10.0	0.	15209.	0.	632.	-3162.	-1233.	632.	145.	-1233.	-230.	-451.	336.	-230.	37.	335.	-0.	4040.	0.
5.0	0.	18169.	0.	710.	-4302.	-656.	710.	167.	-656.	-202.	-501.	179.	-202.	-606.	179.	-0.	4259.	0.
-.0	0.	19197.	-0.	737.	-4698.	0.	737.	175.	0.	-193.	-519.	-0.	-193.	-829.	-0.	-0.	4335.	0.

TRANSVERSE STRAIN (MUIN/IN)

ANGLE (DEG)	RRAP	TTAP	RTAP	RRBM	TTBM	RTBM	RRBP	TTBP	RTBP	RRCM	TTCM	RTCM	RRCP	TTCP	RTCP	RRDM	TTDM	RTDM
45.0	177.	-597.	-0.	-195.	210.	0.	-171.	-72.	0.	-528.	104.	-0.	-357.	502.	-0.	-124.	95.	0.
40.0	162.	-476.	0.	-178.	198.	-30.	-145.	-67.	-722.	-516.	85.	197.	-347.	483.	22.	-127.	102.	0.
35.0	116.	-385.	0.	-158.	162.	-57.	-70.	-55.	-1356.	-483.	32.	370.	-319.	430.	41.	-136.	120.	0.
30.0	47.	-246.	0.	-128.	105.	-75.	45.	-35.	-1827.	-433.	-49.	498.	-276.	349.	55.	-149.	147.	0.
25.0	-38.	-75.	0.	-90.	38.	-87.	187.	-11.	-2079.	-371.	-149.	566.	-223.	249.	62.	-166.	180.	0.
20.0	-129.	106.	0.	-51.	-34.	-87.	337.	15.	-2078.	-305.	-255.	566.	-167.	143.	62.	-183.	215.	0.
15.0	-214.	277.	0.	-13.	-102.	-76.	479.	39.	-1827.	-243.	-355.	498.	-114.	43.	55.	-200.	249.	0.
10.0	-283.	416.	0.	17.	-157.	-57.	594.	59.	-1356.	-193.	-436.	370.	-71.	-38.	41.	-214.	275.	0.
5.0	-329.	506.	0.	37.	-193.	-30.	669.	72.	-722.	-160.	-489.	197.	-42.	-91.	22.	-222.	294.	0.
-.0	-344.	536.	-0.	44.	-206.	0.	696.	76.	0.	-149.	-508.	-0.	-33.	-110.	-0.	-226.	300.	0.

LONG. STRESS(LB/IN/IN) AND STRAIN(MUIN/IN) — SQRT(3*J2) IN (KLB/IN/IN) — TRANSVERSE DISPLACEMENT(IN)

ANGLE (DEG)	ZZAP	ZZBM	ZZBP	ZZCM	ZZCP	ZZDM	EZZ	YAP	YBM	YBP	YCM	YCP	YDM	URA	UTA	URB	UTB	URC	UTC	URD	UTD
45.0	-555.	7156.	128.	107.	3691.	2273.	152.	14.6	8.0	.3	.6	6.2	2.1	-.0006	.0000	-.0007	.0000	-.0026	.0000	-.0028	.0020
40.0	-213.	7033.	131.	106.	3520.	2299.	152.	13.8	7.8	1.2	.7	6.0	2.1	-.0006	-.0001	-.0007	-.0000	-.0025	-.0003	-.0027	.0017
35.0	773.	6680.	141.	104.	3415.	2372.	152.	11.3	7.2	2.1	.8	5.4	2.3	-.0005	-.0003	-.0005	-.0001	-.0023	-.0005	-.0024	.0015
30.0	2283.	6138.	156.	101.	3101.	2483.	152.	7.8	6.3	2.9	.9	4.6	2.5	-.0003	-.0004	-.0004	-.0001	-.0019	-.0007	-.0020	.0012
25.0	4135.	5474.	175.	96.	2716.	2621.	152.	4.6	5.6	3.3	1.0	3.7	2.7	-.0001	-.0004	-.0001	-.0001	-.0014	-.0003	-.0016	.0009
20.0	6106.	4767.	195.	92.	2306.	2767.	152.	5.7	5.4	3.3	1.0	2.7	3.1	.0002	-.0004	.0001	-.0001	-.0009	-.0008	-.0010	.0007
15.0	7958.	4103.	214.	87.	1920.	2904.	152.	9.6	5.6	2.9	.9	2.1	3.4	.0004	-.0004	.0004	-.0001	-.0005	-.0007	-.0006	.0005
10.0	9469.	3561.	230.	84.	1606.	3015.	152.	13.3	6.2	2.2	.7	1.6	3.6	.0006	-.0003	.0005	-.0001	-.0001	-.0005	-.0002	.0003
5.0	10454.	3208.	240.	82.	1401.	3089.	152.	15.3	5.7	1.2	.6	1.9	3.8	.0007	-.0001	.0007	-.0000	.0002	-.0003	.0001	.0002
-.0	10796.	3085.	243.	81.	1330.	3114.	152.	16.7	6.9	.5	.5	1.9	3.9	.0007	.0000	.0007	.0000	.0002	.0000	.0002	-.0000

Case 4. No Cool Down, With Pre Load, With Excitation

ELASTIC STPESS AND STRAIN IN BEAM LINE QUADRUPOLE ELQUAD

CENTRAL GRADIENT(KG/IN)	= 0.0000	INNER BORE TUBE RADIUS(IN)	= 3.0000	INNER CONDUCTOR RADIUS(IN))	= 3.5000
OUTER CONDUCTOR RADIUS(IN))	= 5.8130	OUTER RADIUS OF BANDING(IN)	= 6.5000	RADIUS OF IRON SHIELD(IN)	= 9.0000
BORE YOUNGS MODULUS(LBS/IN/IN)	= 29000000.	COND. YOUNGS MODULUS(LBS/IN/IN)	= 1000000.	BAND YOUNGS MODULUS(LBS/IN/IN)	= 11000000.
BORE POISSONS RATIO	= .3330	COND. POISSONS RATIO	= .1000	BAND POISSONS RATIO	= .3330
BORE THERMAL STRAIN(IN/IN)	= -.0030	COND. THERMAL STRAIN(IN/IN)	= -.0033	BAND THERMAL STRAIN(IN/IN)	= -.0042
BAND DISLOCATION(MILLIRAD)	= 0.000	MAGNETIC ENERGY((IN-LBS)/IN)	= 0.00	MAGNETIC ENERGY(J/M)	= 0.00
TYPE OF COMPUTATION	= ELQUA5				

TRANSVERSE STRESS(LB/IN/IN)

ANGLE (DEG)	RRAP	TTAP	RTAP	RRBM	TTBM	RTBM	RRBP	TTBP	RTBP	RRCM	TTCM	RTCM	RRCP	TTCP	RTCP	RRDM	TTDM	RTDM
45.0	0.	-10935.	0.	-1451.	-9484.	0.	-1451.	-20.	0.	-994.	-476.	0.	-994.	8940.	0.	0.	7945.	0.
40.0	0.	-10935.	0.	-1451.	-9484.	0.	-1451.	-20.	0.	-994.	-476.	0.	-994.	8940.	0.	0.	7945.	0.
35.0	0.	-10935.	0.	-1451.	-9484.	0.	-1451.	-20.	0.	-994.	-476.	0.	-994.	8940.	0.	0.	7945.	0.
30.0	0.	-10935.	0.	-1451.	-9484.	0.	-1451.	-20.	0.	-994.	-476.	0.	-994.	8940.	0.	0.	7945.	0.
25.0	0.	-10935.	0.	-1451.	-9484.	0.	-1451.	-20.	0.	-994.	-476.	0.	-994.	8940.	0.	0.	7945.	0.
20.0	0.	-10935.	0.	-1451.	-9484.	0.	-1451.	-20.	0.	-994.	-476.	0.	-994.	8940.	0.	0.	7945.	0.
15.0	0.	-10935.	0.	-1451.	-9484.	0.	-1451.	-20.	0.	-994.	-476.	0.	-994.	8940.	0.	0.	7945.	0.
10.0	0.	-10935.	0.	-1451.	-9484.	0.	-1451.	-20.	0.	-994.	-476.	0.	-994.	8940.	0.	0.	7945.	0.
5.0	0.	-10935.	0.	-1451.	-9484.	0.	-1451.	-20.	0.	-994.	-476.	0.	-994.	8940.	0.	0.	7945.	0.
-.0	0.	-10935.	0.	-1451.	-9484.	0.	-1451.	-20.	0.	-994.	-476.	0.	-994.	8940.	0.	0.	7945.	0.

TRANSVERSE STRAIN (MUIN/IN)

ANGLE (DEG)	RRAP	TTAP	RTAP	RRBM	TTBM	RTBM	RRBP	TTBP	RTBP	RRCM	TTCM	RTCM	RRCP	TTCP	RTCP	RRDM	TTDM	RTDM
45.0	-2642.	-3144.	0.	-2708.	-3078.	0.	-4652.	-3078.	0.	-4150.	-3579.	0.	-4783.	-3579.	0.	-4663.	-3700.	0.
40.0	-2642.	-3144.	0.	-2708.	-3078.	0.	-4652.	-3078.	0.	-4150.	-3579.	0.	-4783.	-3579.	0.	-4663.	-3700.	0.
35.0	-2642.	-3144.	0.	-2708.	-3078.	0.	-4652.	-3078.	0.	-4150.	-3579.	0.	-4783.	-3579.	0.	-4663.	-3700.	0.
30.0	-2642.	-3144.	0.	-2708.	-3078.	0.	-4652.	-3078.	0.	-4150.	-3579.	0.	-4783.	-3579.	0.	-4663.	-3700.	0.
25.0	-2642.	-3144.	0.	-2708.	-3078.	0.	-4652.	-3078.	0.	-4150.	-3579.	0.	-4783.	-3579.	0.	-4663.	-3700.	0.
20.0	-2642.	-3144.	0.	-2708.	-3078.	0.	-4652.	-3078.	0.	-4150.	-3579.	0.	-4783.	-3579.	0.	-4663.	-3700.	0.
15.0	-2642.	-3144.	0.	-2708.	-3078.	0.	-4652.	-3078.	0.	-4150.	-3579.	0.	-4783.	-3579.	0.	-4663.	-3700.	0.
10.0	-2642.	-3144.	0.	-2708.	-3078.	0.	-4652.	-3078.	0.	-4150.	-3579.	0.	-4783.	-3579.	0.	-4663.	-3700.	0.
5.0	-2642.	-3144.	0.	-2708.	-3078.	0.	-4652.	-3078.	0.	-4150.	-3579.	0.	-4783.	-3579.	0.	-4663.	-3700.	0.
-.0	-2642.	-3144.	0.	-2708.	-3078.	0.	-4652.	-3078.	0.	-4150.	-3579.	0.	-4783.	-3579.	0.	-4663.	-3700.	0.

LONG. STRESS(LB/IN/IN) AND STRAIN(MUIN/IN) — SQRT(3*J2) IN (KLB/IN/IN) — TRANSVERSE DISPLACEMENT(IN)

ANGLE (DEG)	ZZAP	ZZBM	ZZBP	ZZCM	ZZCP	ZZDM	EZZ	YAP	YBM	YBP	YCM	YCP	YDM	URA	UTA	URB	UTB	URC	UTC	URD	UTD
45.0	-20273.	-20273.	-471.	-471.	8987.	8987.	-3574.	17.6	16.4	1.3	.5	10.0	8.5	-.00940	.0000	-.01080	.0000	-.02080	.0000	-.0240	-.0000
40.0	-20273.	-20273.	-471.	-471.	8987.	8987.	-3574.	17.6	16.4	1.3	.5	10.0	8.5	-.00940	.0000	-.01080	.0000	-.02080	.0000	-.0240	-.0000
35.0	-20273.	-20273.	-471.	-471.	8987.	8987.	-3574.	17.6	16.4	1.3	.5	10.0	8.5	-.00940	.0000	-.01080	.0000	-.02080	.0000	-.0240	-.0000
30.0	-20273.	-20273.	-471.	-471.	8987.	8987.	-3574.	17.6	16.4	1.3	.5	10.0	8.5	-.00940	.0000	-.01080	.0000	-.02080	.0000	-.0240	-.0000
25.0	-20273.	-20273.	-471.	-471.	8987.	8987.	-3574.	17.6	16.4	1.3	.5	10.0	8.5	-.00940	.0000	-.01080	.0000	-.02080	.0000	-.0240	-.0000
20.0	-20273.	-20273.	-471.	-471.	8987.	8987.	-3574.	17.6	16.4	1.3	.5	10.0	8.5	-.00940	.0000	-.01080	.0000	-.02080	.0000	-.0240	-.0000
15.0	-20273.	-20273.	-471.	-471.	8987.	8987.	-3574.	17.6	16.4	1.3	.5	10.0	8.5	-.00940	.0000	-.01080	.0000	-.02080	.0000	-.0240	-.0000
10.0	-20273.	-20273.	-471.	-471.	8987.	8987.	-3574.	17.6	16.4	1.3	.5	10.0	8.5	-.00940	.0000	-.01080	.0000	-.02080	.0000	-.0240	-.0000
5.0	-20273.	-20273.	-471.	-471.	8987.	8987.	-3574.	17.6	16.4	1.3	.5	10.0	8.5	-.00940	.0000	-.01080	.0000	-.02080	.0000	-.0240	-.0000
-.0	-20273.	-20273.	-471.	-471.	8987.	8987.	-3574.	17.6	16.4	1.3	.5	10.0	8.5	-.00940	.0000	-.01080	.0000	-.02080	.0000	-.0240	.0000

Case 5. Cool Down, No Pre Load, No Excitation

ELASTIC STRESS AND STRAIN IN BEAM LINE QUADRUPOLE ELQUAD

CENTRAL GRADIENT(KG/IN)	= 14.8000	INNER BORE TUBE RADIUS(IN)	= 3.0000	INNER CONDUCTOR RADIUS(IN))	= 3.5000
OUTER CONDUCTOR RADIUS(IN))	= 5.8130	OUTER RADIUS OF BANDING(IN)	= 6.5000	RADIUS OF IRON SHIELD(IN)	= 0.0000
BORE YOUNGS MODULUS(LBS/IN/IN)	= 29000000.	COND. YOUNGS MODULUS(LBS/IN/IN)	= 1000000.	BAND YOUNGS MODULUS(LBS/IN/IN)	= 11000000.
BORE POISSONS RATIO	= .3330	COND. POISSONS RATIO	= .1000	BAND POISSONS RATIO	= .3330
BORE THERMAL STRAIN(IN/IN)	= -.0030	COND. THERMAL STRAIN(IN/IN)	= -.0033	BAND THERMAL STRAIN(IN/IN)	= -.0042
BAND DISLOCATION(MILLIRAD)	= 0.000	MAGNETIC ENERGY((IN-LBS)/IN)	= 127140.73	MAGNETIC ENERGY(J/M)	= 566354.17
TYPE OF COMPUTATION	= ELQUA5				

TRANSVERSE STRESS(LB/IN/IN)

ANGLE (DEG)	RRAP	TTAP	RTAP	RRBM	TTBM	RTBM	RRBP	TTBP	RTBP	RRCM	TTCM	RTCM	RRCP	TTCP	RTCP	RRDM	TTDM	RTDM
45.0	0.	-22739.	-0.	-1207.	1626.	0.	-1207.	24.	0.	-1183.	-201.	-0.	-1188.	12952.	-0.	0.	6957.	0.
40.0	0.	-21711.	0.	-1180.	1230.	-656.	-1180.	31.	-656.	-1178.	-218.	179.	-1175.	12729.	179.	0.	7034.	0.
35.0	0.	-15751.	0.	-1101.	90.	-1233.	-1101.	53.	-1233.	-1151.	-269.	336.	-1151.	12085.	336.	0.	7253.	0.
30.0	0.	-14216.	0.	-981.	-1656.	-1661.	-981.	86.	-1651.	-1108.	-346.	453.	-1108.	11160.	453.	0.	7589.	0.
25.0	0.	-8654.	0.	-834.	-3799.	-1889.	-834.	127.	-1889.	-1056.	-441.	515.	-1056.	9892.	515.	0.	8101.	0.
20.0	0.	-2734.	0.	-677.	-6078.	-1889.	-677.	171.	-1889.	-1001.	-542.	515.	-1001.	8605.	515.	0.	8439.	0.
15.0	0.	2829.	0.	-529.	-8221.	-1661.	-529.	211.	-1651.	-949.	-637.	453.	-949.	7397.	453.	0.	6851.	0.
10.0	0.	7363.	0.	-409.	-9967.	-1233.	-409.	245.	-1233.	-907.	-715.	336.	-907.	6412.	336.	0.	9157.	0.
5.0	0.	10323.	0.	-331.	-11107.	-656.	-331.	266.	-656.	-879.	-765.	179.	-879.	5769.	179.	0.	9406.	0.
-.0	0.	11351.	-0.	-304.	-11503.	0.	-304.	274.	0.	-670.	-783.	-0.	-870.	5545.	-0.	0.	9482.	0.

TRANSVERSE STRAIN (MUIN/IN)

ANGLE (DEG)	RRAP	TTAP	RTAP	RRBM	TTBM	RTBM	RRBP	TTBP	RTBP	RRCM	TTCM	RTCM	RRCP	TTCP	RTCP	RRDM	TTDM	RTDM
45.0	-2516.	-3561.	-0.	-2926.	-2795.	0.	-4432.	-3078.	0.	-4388.	-3302.	-0.	-5015.	-3302.	-0.	-4678.	-3834.	0.
40.0	-2532.	-3529.	0.	-2919.	-2808.	-30.	-4406.	-3073.	-722.	-4377.	-3320.	197.	-5006.	-3320.	22.	-4681.	-3828.	0.
35.0	-2577.	-3439.	0.	-2899.	-2844.	-57.	-4330.	-3061.	-1356.	-4344.	-3373.	370.	-4977.	-3373.	41.	-4699.	-3811.	0.
30.0	-2646.	-3300.	0.	-2869.	-2900.	-76.	-4215.	-3041.	-1827.	-4293.	-3455.	498.	-4934.	-3455.	55.	-4703.	-3783.	0.
25.0	-2731.	-3129.	0.	-2831.	-2968.	-87.	-4074.	-3017.	-2078.	-4231.	-3555.	566.	-4881.	-3555.	62.	-4720.	-3750.	0.
20.0	-2822.	-2948.	0.	-2792.	-3040.	-87.	-3923.	-2991.	-2078.	-4165.	-3661.	566.	-4825.	-3661.	62.	-4737.	-3715.	0.
15.0	-2907.	-2777.	0.	-2754.	-3108.	-76.	-3761.	-2967.	-1827.	-4104.	-3761.	498.	-4772.	-3761.	55.	-4754.	-3661.	0.
10.0	-2977.	-2636.	0.	-2724.	-3163.	-57.	-3666.	-2947.	-1356.	-4053.	-3842.	370.	-4729.	-3842.	41.	-4767.	-3654.	0.
5.0	-3022.	-2547.	0.	-2704.	-3199.	-30.	-3591.	-2934.	-722.	-4021.	-3895.	197.	-4701.	-3895.	22.	-4776.	-3636.	0.
-.0	-3038.	-2516.	-0.	-2697.	-3212.	0.	-3565.	-2929.	0.	-4009.	-3913.	-0.	-4691.	-3913.	-0.	-4779.	-3630.	0.

ANGLE (DEG)	LONG. STRESS(LB/IN/IN) AND STRAIN(MUIN/IN) ZZAP	ZZBM	ZZBP	ZZCM	ZZCP	ZZDM	EZZ	SQRT(3*J2) IN (KLB/IN/IN) YAP	YBM	YBP	YCM	YCP	YDM	TRANSVERSE DISPLACEMENT(IN) URA	UTA	URB	UTB	URC	UTC	URD	UTD
45.0	-19431.	-11720.	-277.	-298.	12069.	10468.	-3409.	21.3	12.2	1.1	.9	13.7	9.2	-.0098	.0000	-.0112	.0000	-.0224	.0000	-.0257	.0000
40.0	-19089.	-11843.	-274.	-299.	11999.	10494.	-3409.	20.5	12.1	1.6	1.0	13.6	9.3	-.0098	-.0001	-.0112	-.0000	-.0223	-.0003	-.0256	-.0001
35.0	-18103.	-12196.	-264.	-301.	11793.	10567.	-3409.	18.4	11.9	2.4	1.0	13.1	9.4	-.0096	-.0003	-.0111	-.0001	-.0221	-.0005	-.0254	-.0001
30.0	-16593.	-12738.	-248.	-304.	11479.	10679.	-3409.	15.5	11.8	3.0	1.1	12.4	9.5	-.0095	-.0004	-.0109	-.0001	-.0217	-.0007	-.0250	-.0002
25.0	-14741.	-13402.	-230.	-309.	11094.	10816.	-3409.	12.8	11.8	3.4	1.1	11.6	9.7	-.0092	-.0004	-.0106	-.0001	-.0212	-.0008	-.0245	-.0002
20.0	-12770.	-14109.	-210.	-313.	10684.	10962.	-3409.	11.6	12.2	3.4	1.1	10.8	9.9	-.0090	-.0004	-.0104	-.0001	-.0207	-.0008	-.0240	-.0002
15.0	-10917.	-14773.	-191.	-318.	10299.	11099.	-3409.	12.6	12.7	2.9	1.0	10.1	10.2	-.0088	-.0004	-.0102	-.0001	-.0203	-.0007	-.0235	-.0002
10.0	-9407.	-15315.	-175.	-321.	9985.	11211.	-3409.	14.6	13.3	2.2	.8	9.6	10.3	-.0086	-.0003	-.0100	-.0001	-.0199	-.0005	-.0231	-.0001
5.0	-8422.	-15668.	-165.	-323.	9780.	11284.	-3409.	16.3	13.7	1.3	.6	9.3	10.5	-.0085	-.0001	-.0098	-.0000	-.0196	-.0003	-.0229	-.0001
-.0	-8079.	-15791.	-162.	-324.	9709.	11309.	-3409.	16.9	13.9	.5	.5	9.2	10.5	-.0084	.0000	-.0098	.0000	-.0195	.0000	-.0228	.0000

Case 6. Cool Down, No Pre Load, No Excitation

ELASTIC STRESS AND STRAIN IN BEAM LINE QUADRUPOLE   ELQUAD

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CENTRAL GRADIENT(KG/IN)            =      0.0000 INNER BORE TUBE RADIUS(IN)        =      3.0000 INNER CONDUCTOR RADIUS(IN))        =      3.5000
OUTER CONDUCTOR RADIUS(IN))        =      5.8130 OUTER RADIUS OF BANDING(IN)       =      6.5000 RADIUS OF IRON SHIELD(IN)          =      9.0000
BORE YOUNGS MODULUS(LBS/IN/IN)     =   29000000. COND. YOUNGS MODULUS(LBS/IN/IN)=   1000000. BAND YOUNGS MODULUS(LBS/IN/IN)     =  11000000.
BORE POISSONS RATIO                =       .3330 COND. POISSONS RATIO              =      .1000 BAND POISSONS RATIO                =      .3330
BORE THERMAL STRAIN(IN/IN)         =     -.0030 COND. THERMAL STRAIN(IN/IN)       =     -.0033 BAND THERMAL STRAIN(IN/IN)         =     -.0042
BAND DISLOCATION(MILLIRAD)         =       2.500 MAGNETIC ENERGY((IN-LBS)/IN)      =       0.00 MAGNETIC ENERGY(J/M)               =       0.00
TYPE OF COMPUTATION                =  ELQUAS
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TRANSVERSE STRESS(LB/IN/IN)

ANGLE (DEG)	RRAP	TTAP	RTAP	RRBM	TTBM	RTBM	RRBP	TTBP	RTBP	RRCM	TTCM	RTCM	RRCP	TTCP	RTCP	RRDM	TTDM	RTDM
45.0	0.	-14023.	0.	-1860.	-12163.	0.	-1860.	-139.	0.	-1312.	-688.	0.	-1312.	11505.	0.	0.	10743.	0.
40.0	0.	-14023.	0.	-1860.	-12163.	0.	-1860.	-139.	0.	-1312.	-688.	0.	-1312.	11505.	0.	0.	10743.	0.
35.0	0.	-14023.	0.	-1860.	-12163.	0.	-1860.	-139.	0.	-1312.	-688.	0.	-1312.	11505.	0.	0.	10743.	0.
30.0	0.	-14023.	0.	-1860.	-12163.	0.	-1860.	-139.	0.	-1312.	-688.	0.	-1312.	11505.	0.	0.	10743.	0.
25.0	0.	-14023.	0.	-1860.	-12163.	0.	-1860.	-139.	0.	-1312.	-688.	0.	-1312.	11505.	0.	0.	10743.	0.
20.0	0.	-14023.	0.	-1860.	-12163.	0.	-1860.	-139.	0.	-1312.	-688.	0.	-1312.	11505.	0.	0.	10743.	0.
15.0	0.	-14023.	0.	-1860.	-12163.	0.	-1860.	-139.	0.	-1312.	-688.	0.	-1312.	11505.	0.	0.	10743.	0.
10.0	0.	-14023.	0.	-1860.	-12163.	0.	-1860.	-139.	0.	-1312.	-688.	0.	-1312.	11505.	0.	0.	10743.	0.
5.0	0.	-14023.	0.	-1860.	-12163.	0.	-1860.	-139.	0.	-1312.	-688.	0.	-1312.	11505.	0.	0.	10743.	0.
-.0	0.	-14023.	0.	-1860.	-12163.	0.	-1860.	-139.	0.	-1312.	-688.	0.	-1312.	11505.	0.	0.	10743.	0.

TRANSVERSE STRAIN (MUIN/IN)

ANGLE (DEG)	RRAP	TTAP	RTAP	RRBM	TTBM	RTBM	RRBP	TTBP	RTBP	RRCM	TTCM	RTCM	RRCP	TTCP	RTCP	RRDM	TTDM	RTDM
45.0	-2590.	-3235.	0.	-2676.	-3149.	0.	-5043.	-3149.	0.	-4439.	-3753.	0.	-4908.	-3355.	0.	-4771.	-3469.	0.
40.0	-2590.	-3235.	0.	-2676.	-3149.	0.	-5043.	-3149.	0.	-4439.	-3753.	0.	-4908.	-3355.	0.	-4771.	-3469.	0.
35.0	-2590.	-3235.	0.	-2676.	-3149.	0.	-5043.	-3149.	0.	-4439.	-3753.	0.	-4908.	-3355.	0.	-4771.	-3469.	0.
30.0	-2590.	-3235.	0.	-2676.	-3149.	0.	-5043.	-3149.	0.	-4439.	-3753.	0.	-4908.	-3355.	0.	-4771.	-3469.	0.
25.0	-2590.	-3235.	0.	-2676.	-3149.	0.	-5043.	-3149.	0.	-4439.	-3753.	0.	-4908.	-3355.	0.	-4771.	-3469.	0.
20.0	-2590.	-3235.	0.	-2676.	-3149.	0.	-5043.	-3149.	0.	-4439.	-3753.	0.	-4908.	-3355.	0.	-4771.	-3469.	0.
15.0	-2590.	-3235.	0.	-2676.	-3149.	0.	-5043.	-3149.	0.	-4439.	-3753.	0.	-4908.	-3355.	0.	-4771.	-3469.	0.
10.0	-2590.	-3235.	0.	-2676.	-3149.	0.	-5043.	-3149.	0.	-4439.	-3753.	0.	-4908.	-3355.	0.	-4771.	-3469.	0.
5.0	-2590.	-3235.	0.	-2676.	-3149.	0.	-5043.	-3149.	0.	-4439.	-3753.	0.	-4908.	-3355.	0.	-4771.	-3469.	0.
-.0	-2590.	-3235.	0.	-2676.	-3149.	0.	-5043.	-3149.	0.	-4439.	-3753.	0.	-4908.	-3355.	0.	-4771.	-3469.	0.

LONG. STRESS(LB/IN/IN) AND STRAIN(MUIN/IN) — SQRT(3*J2) IN (KLB/IN/IN) — TRANSVERSE DISPLACEMENT(IN)

ANGLE (DEG)	ZZAP	ZZBM	ZZBP	ZZCM	ZZCP	ZZDM	EZZ	YAP	YBM	YBP	YCM	YCP	YDM	URA	UTA	URB	UTB	URC	UTC	URD	UTD
45.0	-21670.	-21670.	-536.	-536.	9596.	9779.	-3586.	19.0	17.2	1.6	.7	12.0	10.3	-.00970	.0000	-.01100	.0000	-.02180	.0000	-.0251	.0020
40.0	-21670.	-21670.	-536.	-536.	9596.	9779.	-3586.	19.0	17.2	1.6	.7	12.0	10.3	-.00970	.0000	-.01100	.0000	-.02180	.0000	-.0251	.0018
35.0	-21670.	-21670.	-536.	-536.	9596.	9779.	-3586.	19.0	17.2	1.6	.7	12.0	10.3	-.00970	.0000	-.01100	.0000	-.02180	.0000	-.0251	.0016
30.0	-21670.	-21670.	-536.	-536.	9596.	9779.	-3586.	19.0	17.2	1.6	.7	12.0	10.3	-.00970	.0000	-.01100	.0000	-.02180	.0000	-.0251	.0014
25.0	-21670.	-21670.	-536.	-536.	9596.	9779.	-3586.	19.0	17.2	1.6	.7	12.0	10.3	-.00970	.0000	-.01100	.0000	-.02180	.0000	-.0251	.0011
20.0	-21670.	-21670.	-536.	-536.	9596.	9779.	-3586.	19.0	17.2	1.6	.7	12.0	10.3	-.00970	.0000	-.01100	.0000	-.02180	.0000	-.0251	.0009
15.0	-21670.	-21670.	-536.	-536.	9596.	9779.	-3586.	19.0	17.2	1.6	.7	12.0	10.3	-.00970	.0000	-.01100	.0000	-.02180	.0000	-.0251	.0007
10.0	-21670.	-21670.	-536.	-536.	9596.	9779.	-3586.	19.0	17.2	1.6	.7	12.0	10.3	-.00970	.0000	-.01100	.0000	-.02180	.0000	-.0251	.0005
5.0	-21670.	-21670.	-536.	-536.	9596.	9779.	-3586.	19.0	17.2	1.6	.7	12.0	10.3	-.00970	.0000	-.01100	.0000	-.02180	.0000	-.0251	.0002
-.0	-21670.	-21670.	-536.	-536.	9596.	9779.	-3586.	19.0	17.2	1.6	.7	12.0	10.3	-.00970	.0000	-.01100	.0000	-.02180	.0000	-.0251	-.0000

Case 7. Cool Down, With Pre Load, No Excitation

ELASTIC STRESS AND STRAIN IN BEAM LINE QUADRUPOLE ELOQUAD

CENTRAL GRADIENT(KG/IN)	= 14.8000	INNER BORE TUBE RADIUS(IN)	= 3.0000	INNER CONDUCTOR RADIUS(IN))	= 3.5000
OUTER CONDUCTOR RADIUS(IN))	= 5.8130	OUTER RADIUS OF BANDING(IN)	= 6.5000	RADIUS OF IRON SHIELD(IN)	= 9.0000
BORE YOUNGS MODULUS(LBS/IN/IN)	= 29000000.	COND. YOUNGS MODULUS(LBS/IN/IN)	= 1000000.	BAND YOUNGS MODULUS(LBS/IN/IN)	= 11000000.
BORE POISSONS RATIO	= .3330	COND. POISSONS RATIO	= .1000	BAND POISSONS RATIO	= .3330
BORE THERMAL STRAIN(IN/IN)	= -.0030	COND. THERMAL STRAIN(IN/IN)	= -.0033	BAND THERMAL STRAIN(IN/IN)	= -.0042
BAND DISLOCATION(MILLIRAD)	= 2.500	MAGNETIC ENERGY((IN-LBS)/IN)	= 127140.73	MAGNETIC ENERGY(J/M)	= 566354.17
TYPE OF COMPUTATION	= ELOQUAD				

TRANSVERSE STRESS(LB/IN/IN)

ANGLE (DEG)	RRAP	TTAP	RTAP	RRBM	TTBM	RTBM	RRBP	TTBP	RTBP	RRCM	TTCM	RTCM	RRCP	TTCP	RTCP	RRDM	TTDM	RTDM
45.0	0.	-25827.	-0.	-1617.	-1053.	0.	-1617.	-95.	0.	-1505.	-412.	-0.	-1505.	15517.	-0.	0.	9756.	0.
40.0	0.	-24799.	0.	-1590.	-1449.	-656.	-1590.	-88.	-656.	-1495.	-430.	179.	-1495.	15294.	179.	0.	9832.	0.
35.0	0.	-21839.	0.	-1511.	-2599.	-1233.	-1511.	-66.	-1233.	-1468.	-430.	336.	-1468.	14651.	336.	0.	10051.	0.
30.0	0.	-17305.	0.	-1391.	-4335.	-1661.	-1391.	-33.	-1661.	-1425.	-558.	453.	-1425.	13666.	453.	0.	10387.	0.
25.0	0.	-11742.	0.	-1243.	-6478.	-1889.	-1243.	8.	-1889.	-1374.	-653.	515.	-1374.	12457.	515.	0.	10799.	0.
20.0	0.	-5823.	0.	-1087.	-8757.	-1889.	-1087.	51.	-1889.	-1318.	-754.	515.	-1318.	11171.	515.	0.	11237.	0.
15.0	0.	-260.	0.	-939.	-10900.	-1661.	-939.	92.	-1661.	-1265.	-849.	453.	-1265.	9962.	453.	0.	11649.	0.
10.0	0.	4275.	0.	-819.	-12646.	-1233.	-819.	125.	-1233.	-1224.	-927.	336.	-1224.	8977.	336.	0.	11985.	0.
5.0	0.	7234.	0.	-740.	-13786.	-656.	-740.	147.	-656.	-1197.	-977.	179.	-1197.	8334.	179.	0.	12204.	0.
-.0	0.	8262.	-0.	-713.	-14182.	0.	-713.	155.	0.	-1187.	-995.	-0.	-1187.	8111.	-0.	0.	12280.	0.

TRANSVERSE STRAIN (MUIN/IN)

ANGLE (DEG)	RRAP	TTAP	RTAP	RRBM	TTBM	RTBM	RRBP	TTBP	RTBP	RRCM	TTCM	RTCM	RRCP	TTCP	RTCP	RRDM	TTDM	RTDM
45.0	-2464.	-3651.	-0.	-2893.	-2867.	0.	-4823.	-3149.	0.	-4677.	-3475.	-0.	-5140.	-3078.	-0.	-4786.	-3604.	0.
40.0	-2480.	-3620.	0.	-2886.	-2880.	-30.	-4797.	-3145.	-722.	-4666.	-3494.	197.	-5131.	-3096.	22.	-4789.	-3598.	0.
35.0	-2525.	-3529.	0.	-2866.	-2916.	-57.	-4722.	-3132.	-1356.	-4633.	-3547.	370.	-5102.	-3149.	41.	-4798.	-3580.	0.
30.0	-2595.	-3390.	0.	-2836.	-2971.	-76.	-4606.	-3112.	-1827.	-4583.	-3628.	498.	-5059.	-3230.	55.	-4812.	-3553.	0.
25.0	-2680.	-3220.	0.	-2799.	-3039.	-87.	-4465.	-3088.	-2078.	-4521.	-3728.	566.	-5006.	-3330.	62.	-4828.	-3520.	0.
20.0	-2770.	-3038.	0.	-2759.	-3111.	-87.	-4314.	-3062.	-2078.	-4455.	-3834.	566.	-4950.	-3436.	62.	-4846.	-3484.	0.
15.0	-2856.	-2858.	0.	-2722.	-3179.	-76.	-4173.	-3038.	-1827.	-4393.	-3934.	498.	-4897.	-3536.	55.	-4863.	-3451.	0.
10.0	-2925.	-2729.	0.	-2691.	-3235.	-57.	-4057.	-3018.	-1356.	-4343.	-4015.	370.	-4854.	-3618.	41.	-4876.	-3424.	0.
5.0	-2970.	-2638.	0.	-2671.	-3271.	-30.	-3982.	-3006.	-722.	-4310.	-4069.	197.	-4826.	-3671.	22.	-4885.	-3406.	0.
-.0	-2986.	-2606.	-0.	-2664.	-3283.	0.	-3956.	-3001.	0.	-4299.	-4087.	-0.	-4816.	-3689.	-0.	-4888.	-3400.	0.

LONG. STRESS(LB/IN/IN) AND STRAIN(MUIN/IN) · SQRT(3*J2) IN (KLB/IN/IN) · TRANSVERSE DISPLACEMENT(IN)

ANGLE (DEG)	ZZAP	ZZBM	ZZBP	ZZCM	ZZCP	ZZDM	EZZ	YAP	YBM	YBP	YCM	YCP	YDM	URA	UTA	UPB	UTB	UPC	UTC	URD	UTD
45.0	-20828.	-13117.	-343.	-363.	12678.	11260.	-3422.	23.7	11.8	1.4	1.1	15.8	10.6	-.0101	.0000	-.0115	.0000	-.0234	.0000	-.0268	.0020
40.0	-20486.	-13240.	-339.	-364.	12607.	11286.	-3422.	22.9	11.8	1.8	1.1	15.5	10.6	-.0100	-.0001	-.0114	-.0000	-.0233	-.0003	-.0267	.0017
35.0	-19500.	-13593.	-329.	-366.	12402.	11359.	-3422.	20.8	11.8	2.5	1.2	15.1	10.8	-.0099	-.0003	-.0113	-.0001	-.0231	-.0005	-.0265	.0015
30.0	-17990.	-14135.	-314.	-370.	12038.	11471.	-3422.	17.7	11.9	3.1	1.3	14.4	11.0	-.0097	-.0004	-.0111	-.0001	-.0227	-.0007	-.0261	.0012
25.0	-16138.	-14799.	-295.	-374.	11703.	11608.	-3422.	14.5	12.3	3.5	1.3	13.5	11.2	-.0095	-.0004	-.0109	-.0001	-.0222	-.0008	-.0256	.0009
20.0	-14167.	-15506.	-275.	-379.	11293.	11754.	-3422.	12.3	12.9	3.4	1.2	12.6	11.5	-.0093	-.0004	-.0106	-.0001	-.0217	-.0008	-.0251	.0007
15.0	-12314.	-16170.	-256.	-383.	10908.	11891.	-3422.	12.2	13.7	3.0	1.1	11.8	11.9	-.0090	-.0004	-.0104	-.0001	-.0213	-.0007	-.0246	.0005
10.0	-10804.	-16712.	-241.	-387.	10534.	12033.	-3422.	13.5	14.5	2.3	.9	11.1	12.0	-.0089	-.0003	-.0102	-.0001	-.0209	-.0005	-.0242	.0003
5.0	-9819.	-17065.	-231.	-389.	10399.	12076.	-3422.	14.8	15.0	1.4	.8	10.7	12.1	-.0087	-.0001	-.0101	-.0000	-.0206	-.0003	-.0240	.0002
-.0	-9477.	-17188.	-227.	-390.	10317.	12101.	-3422.	15.4	15.2	.8	.7	10.6	12.2	-.0087	.0000	-.0100	.0000	-.0206	.0000	-.0239	-.0000

Case 8. Cool Down, With Pre Load, With Excitation