



## MECHANICAL AND THERMAL STRESSES IN DOUBLER DIPOLE MAGNETS

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### Summary

An analytical solution for stresses has been found for a structural composite that models the Doubler dipole. Structural cylinders represent the material inside and outside of the excitation current which is represented by two cosine theta sheet current distributions. A pretensioned structural cylinder surrounds the aforementioned materials. Thermal stresses are represented only in so far as a uniform temperature differing from room temperature alters the stress-strain relation. Temperature gradients are not considered. The mechanical energy stored in the elastic field is calculated. Numerical results are given.

### Thermo Elasticity

The effect of a temperature change in elasticity is obtained by considering the elastic energy density to have the form:<sup>1</sup>

$$W = C_{ij}\epsilon_{ij} + \frac{1}{2}C_{ijkl}\epsilon_{ij}\epsilon_{kl} , \quad (1)$$

where the summation convention for repeated indices is used. The stress tensor is related to the strain tensor using

$$\sigma_{ij} = \frac{\partial W}{\partial \epsilon_{ij}} = C_{ij} + C_{ijkl}\epsilon_{kl} . \quad (2)$$

For homogeneous isotropic materials

$$C_{ij} = -k(3\lambda + 2\mu) \cdot \delta_{ij} \quad (3)$$

and

$$C_{ijkl} = \lambda \delta_{ij} \delta_{kl} + \mu (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}) , \quad (4)$$

where  $k$  is the thermal expansion coefficient integrated from 4.2°K to room temperature,  $\lambda$  and  $\mu$  are the Lamé constants which are related to the more familiar constants  $Y$  (Young's modulus) and  $\nu$  (Poisson's ratio) as follows.

$$\lambda = \frac{\nu}{(1+\nu)(1-2\nu)} Y , \quad \mu = \frac{1}{2(1+\nu)} Y . \quad (5)$$

Under these restrictions Hooke's Law becomes

$$\sigma_{ij} = \frac{\nu}{(1+\nu)(1-2\nu)} Y \delta_{ij} \epsilon_{kk} + \frac{1}{1+\nu} Y \epsilon_{ij} - \frac{kY}{1-2\nu} \delta_{ij} . \quad (6)$$

The condition of equilibrium is then

$$\frac{\partial \sigma_{ij}}{\partial x_i} + f_j = 0 , \quad (7)$$

where  $f_j$  is the body force which in our case will be the Lorentz force  $\vec{J} \times \vec{B}$ . Finally, since some of the boundary conditions relate to the material displacement  $\vec{u}$ , one needs the connection between strain and displacement

$$\epsilon_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) . \quad (8)$$

For the problem at hand the body force is handled by a surface traction and, therefore, the equilibrium condition Eq. (7) may be satisfied identically through the use of the Airy stress function  $\phi$ :

$$\sigma_{ij} = \delta_{ij} \nabla^2 \phi - \frac{\partial^2 \phi}{\partial x_i \partial x_j} . \quad (9)$$

The equation satisfied by the Airy stress function is determined from the equations of compatibility which are imposed on strains since the six strains in Eq. (8) must be interrelated in order to be derivable from three displacements. If Hooke's Law, Eq. (6), is used the equations of compatibility for stresses may be found

$$\nabla^2 \sigma_{ij} + \frac{1}{1+\nu} \frac{\partial^2 \sigma_{kk}}{\partial x_i \partial x_j} = 0 . \quad (10)$$

By introducing the approximation known as generalized plane strain<sup>2</sup> one reduces the problem to manageable proportions. In this approximation one neglects the shear strains connected with the longitudinal or axial direction but permits a free expansion characterized by a uniform strain. For the normal stresses Eq. (4) gives

$$\sigma_{11} = \lambda(\epsilon_{11} + \epsilon_{22} + \epsilon_{33}) + 2\mu\epsilon_{11} - k(3\lambda + 2\mu) \quad (11)$$

$$\sigma_{22} = \lambda(\epsilon_{11} + \epsilon_{22} + \epsilon_{33}) + 2\mu\epsilon_{22} - k(3\lambda + 2\mu) \quad (12)$$

$$\sigma_{33} = \lambda(\epsilon_{11} + \epsilon_{22} + \epsilon_{33}) + 2\mu\epsilon_{33} - k(3\lambda + 2\mu) . \quad (13)$$

Suppose that the index 3 represents the longitudinal direction. Then, from Eqs. (5, 11, 12, 13) one has

$$\sigma_{33} = \nu(\sigma_{11} + \sigma_{22}) + Y\epsilon_{33} - kY. \quad (14)$$

Since  $\epsilon_{33}$  is considered constant in this approximation, Eq. (10) becomes

$$\nabla^2 \sigma_{ij} + \frac{\partial^2}{\partial x_i \partial x_j} (\sigma_{11} + \sigma_{22}) = 0 , \quad (15)$$

or, using Eq. (9) this becomes

$$\delta_{ij} \nabla^4 \phi - \frac{\partial^2}{\partial x_i \partial x_j} \nabla^2 \phi + \frac{\partial^2}{\partial x_i \partial x_j} (2\nabla^2 \phi - \nabla^2 \phi) = 0 . \quad (16)$$

Hence the Airy stress function satisfies the biharmonic equation

$$\nabla^4 \phi = 0. \quad (17)$$

### Virial Theorem

One might expect that the boundary conditions which provide for continuity of displacement and discontinuity of normal and shear stresses according to known surface tractions would be sufficient to specify all the unknowns in solving Eq. (17). However, in this plane strain approximation, information relative to the longitudinal ends is lost and must be supplied by some integral condition. For this problem it is sufficient to invoke the virial theorem which may be found as follows<sup>3</sup>. Let  $\tau_{ij}$  be the Maxwell stress tensor for the magnetic field introduced by the current sheet. The body force  $f_j$  in Eq. (7) is then given by

$$f_j = \frac{\partial \tau_{ij}}{\partial x_i}. \quad (18)$$

To form the virial multiply Eq. (7) by  $x_j$  and utilize Eq. (18). Thus, after integrating over a volume

$$\int x_j \frac{\partial}{\partial x_i} (\sigma_{ij} + \tau_{ij}) d^3x = 0. \quad (19)$$

Integrate by parts using

$$\frac{\partial}{\partial x_i} (x_j \beta_{ij}) = \delta_{ij} \beta_{ij} + x_j \frac{\partial \beta_{ij}}{\partial x_i} \quad (20)$$

to give

$$\int_S x_j (\sigma_{ij} + \tau_{ij}) dS_i - \int \delta_{ij} (\sigma_{ij} + \tau_{ij}) d^3x = 0. \quad (21)$$

Notice that the integrand in the volume integral is just the trace of the tensor. But the trace of the Maxwell stress tensor is the

negative of the energy density<sup>4</sup>. Hence

$$\int \delta_{ij} \tau_{ij} d^3x = -W_B, \quad (22)$$

the magnetic energy. Since, in the problem considered  $\sigma_{ij}$  is zero on the boundary but  $\tau_{ij}$  exists on the iron shield, one has

$$\int \delta_{ij} \sigma_{ij} d^3x = \int_S x_j \tau_{ij} dS_i + W_B. \quad (23)$$

It should be noted that  $\tau_{ij}$  is taken to be zero on the end walls. Hence one visualizes that the magnet structure has terminating ends. Equation (23) indicates that, since the right hand side is positive, a net tensile structure is required to contain magnetic forces in static equilibrium.

#### Pretensioned Band

A method of characterizing a pretensioned member may be found by utilizing the concept of rotational dislocation<sup>5</sup> whereby discontinuity in rotational displacement is permitted. Thus  $u_\theta(2\pi) - u_\theta(0)$  is given a preassigned value. This condition will be used in the outer band rather than the customary continuity of displacement.

#### Magnetic Field

Since a continuously distributed body force as given by the Lorentz force  $\mathbf{J} \times \mathbf{B}$  is more difficult to handle in the equations of elasticity, the region of conduction current in the dipole will be approximated by two current sheets, one at the inner edge of the region and one at the outer edge of the region. Thus the model to be considered consists of two cylindrical current sheets each carrying an axial current density of

$$i = \begin{Bmatrix} i_0 \\ i_1 \end{Bmatrix} \cos\theta \quad \text{at} \quad \begin{Bmatrix} r=b \\ r=c \end{Bmatrix} \quad (24)$$

See Fig. 1 for geometrical details. From the current density as given and an iron shield located at  $r = r_s$ , one finds the following magnetic field components.

$$H_r = -2\pi \begin{Bmatrix} i_0(1+b^2r_s^{-2}) + i_1(1+c^2r_s^{-2}) \\ i_0(b^2r^{-2}+b^2r_s^{-2}) + i_1(1+c^2r_s^{-2}) \\ i_0(b^2r^{-2}+b^2r_s^{-2}) + i_1(c^2r^{-2}+c^2r_s^{-2}) \end{Bmatrix} \sin\theta, \quad (25)$$

$$H_\theta = -2\pi \begin{Bmatrix} i_0(1+b^2r_s^{-2}) + i_1(1+c^2r_s^{-2}) \\ i_0(-b^2r^{-2}+b^2r_s^{-2}) + i_1(1+c^2r_s^{-2}) \\ i_0(-b^2r^{-2}+b^2r_s^{-2}) + i_1(-c^2r^{-2}+c^2r_s^{-2}) \end{Bmatrix} \cos\theta. \quad (26)$$

where the top entry refers to  $0 < r < b$ , the middle entry to  $b < r < c$  and the bottom entry to  $c < r < r_s$ . In order to calculate the forces one needs the average field at the current sheets.

$$\langle H_r \rangle_{Av} = -2\pi \begin{Bmatrix} i_0(1+b^2r_s^{-2}) + i_1(1+c^2r_s^{-2}) \\ i_0(b^2c^{-2}+b^2r_s^{-2}) + i_1(1+c^2r_s^{-2}) \end{Bmatrix} \sin\theta, \quad (27)$$

$$\langle H_\theta \rangle_{Av} = -2\pi \begin{Bmatrix} i_0b^2r_s^{-2} + i_1(1+c^2r_s^{-2}) \\ i_0(-b^2c^{-2}+b^2r_s^{-2}) + i_1c^2r_s^{-2} \end{Bmatrix} \cos\theta. \quad (28)$$

#### Lorentz Force on Current Sheets

The force on a current sheet is given by

$$d\vec{F} = ids \, \vec{k} \times (\vec{i}_r \langle H_r \rangle_{Av} + \vec{i}_\theta \langle H_\theta \rangle_{Av}), \quad (29)$$

where  $i$  is given by Eq. (24) and

$$ds = \left\{ \begin{matrix} b \\ c \end{matrix} \right\} d\theta . \quad (30)$$

If  $\vec{f}$  denotes the force per unit area on the current sheet, then

$$f_r = \pi \left\{ \begin{matrix} i_o^2 b^2 r_s^{-2} + i_o i_l (1 + c^2 r_s^{-2}) \\ i_o i_l (-b^2 c^{-2} + b^2 r_s^{-2}) + i_l^2 c^2 r_s^{-2} \end{matrix} \right\} (1 + \cos 2\theta) , \quad (31)$$

$$f_\theta = -\pi \left\{ \begin{matrix} i_o^2 (1 + b^2 r_s^{-2}) + i_o i_l (1 + c^2 r_s^{-2}) \\ i_o i_l (b^2 c^{-2} + b^2 r_s^{-2}) + i_l^2 (1 + c^2 r_s^{-2}) \end{matrix} \right\} \sin 2\theta \quad (32)$$

### Maxwell Stress Tensor

The Maxwell stress tensor is found by noting that the Lorentz force may be written as the divergence of a tensor. Thus

$$\vec{J} \times \vec{B} = \nabla \cdot \vec{\tau} , \quad (33)$$

where, in cartesian components<sup>4</sup>

$$\vec{\tau} = \frac{1}{4\pi} \begin{pmatrix} B_x^2 - \frac{1}{2}B^2 & B_x B_y & B_x B_z \\ B_x B_y & B_y^2 - \frac{1}{2}B^2 & B_y B_z \\ B_x B_z & B_y B_z & B_z^2 - \frac{1}{2}B^2 \end{pmatrix} . \quad (34)$$

From this it may be seen that the trace of the Maxwell stress tensor is

$$\text{tr } \vec{\tau} = -\frac{1}{8\pi} B^2 , \quad (35)$$

the negative of the energy density.

### Application to Doubler Dipole Magnet

Since generalized plane strain is characterized by

$$\epsilon_{rz} = \epsilon_{\theta z} = 0 , \quad \epsilon_{zz} = \text{constant} , \quad (36)$$

the corresponding stress in Eq. (6) becomes

$$\sigma_{rr} = (\lambda+2\mu)\epsilon_{rr} + \lambda\epsilon_{\theta\theta} + \lambda\epsilon_{zz} - k(3\lambda+2\mu) \quad (37)$$

$$\sigma_{\theta\theta} = \lambda\epsilon_{rr} + (\lambda+2\mu)\epsilon_{\theta\theta} + \lambda\epsilon_{zz} - k(3\lambda+2\mu) \quad (38)$$

$$\sigma_{zz} = \lambda\epsilon_{rr} + \lambda\epsilon_{\theta\theta} + (\lambda+2\mu)\epsilon_{zz} - k(3\lambda+2\mu) . \quad (39)$$

$$\sigma_{r\theta} = 2\mu\epsilon_{r\theta} . \quad (40)$$

Inversion gives

$$\epsilon_{rr} = \frac{1}{2\mu(3\lambda+2\mu)} \cdot [2(\lambda+\mu)\sigma_{rr} - \lambda(\sigma_{\theta\theta} + \sigma_{zz})] + k \quad (41)$$

$$\epsilon_{\theta\theta} = \frac{1}{2\mu(3\lambda+2\mu)} \cdot [2(\lambda+\mu)\sigma_{\theta\theta} - \lambda(\sigma_{rr} + \sigma_{zz})] + k \quad (42)$$

$$\epsilon_{zz} = \frac{1}{2\mu(3\lambda+2\mu)} \cdot [2(\lambda+\mu)\sigma_{zz} - \lambda(\sigma_{rr} + \sigma_{\theta\theta})] + k \quad (43)$$

$$\epsilon_{r\theta} = \frac{1}{2\mu}\sigma_{r\theta} . \quad (44)$$

Utilizing Eq. (5) one has

$$\epsilon_{rr} = \frac{1}{Y}[\sigma_{rr} - \nu\sigma_{\theta\theta} - \nu\sigma_{zz}] + k \quad (45)$$

$$\epsilon_{\theta\theta} = \frac{1}{Y}[-\nu\sigma_{rr} + \sigma_{\theta\theta} - \nu\sigma_{zz}] + k \quad (46)$$

$$\epsilon_{zz} = \frac{1}{Y}[-\nu\sigma_{rr} - \nu\sigma_{\theta\theta} + \sigma_{zz}] + k \quad (47)$$

$$\epsilon_{r\theta} = \frac{1+\nu}{Y}\sigma_{r\theta} . \quad (48)$$

Since  $\epsilon_{zz}$  is taken to be constant in this approximation, Eq. (47)

may be used to eliminate  $\sigma_{zz}$ . Thus

$$\epsilon_{rr} = \frac{1+\nu}{Y} \cdot [(1-\nu)\sigma_{rr} - \nu\sigma_{\theta\theta}] - \nu\epsilon_{zz} + (1+\nu)k \quad (49)$$

$$\epsilon_{\theta\theta} = \frac{1+\nu}{Y}[-\nu\sigma_{rr} + (1-\nu)\sigma_{\theta\theta}] - \nu\epsilon_{zz} + (1+\nu)k . \quad (50)$$



The relation between the stresses and the Airy stress function, Eq. (9), becomes in cylindrical coordinates

$$\sigma_{rr} = \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \theta^2} \quad (51)$$

$$\sigma_{\theta\theta} = \frac{\partial^2 \phi}{\partial r^2} \quad (52)$$

$$\sigma_{r\theta} = - \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial \phi}{\partial \theta} \right) . \quad (53)$$

For the problem under consideration one may take in each annular region functions of the form

$$\phi = A \ln r + G r^2 \ln r + B r^2 + (C r^2 + D r^4 + E r^{-2} + F) \cos 2\theta . \quad (54)$$

The constant  $G$  is related to a multivalued azimuthal displacement and is set equal to zero except in the outer band where it is used to characterize pretension.

#### Stresses, Strains and Displacements

One finds that Eqs. (52-55) gives

$$\sigma_{rr} = \left\{ \begin{array}{l} A_1 r^{-2} \quad + 2B_1 - (2C_1 + 6E_1 r^{-4} + 4F_1 r^{-2}) \cos 2\theta \\ A_2 r^{-2} \quad + 2B_2 - (2C_2 + 6E_2 r^{-4} + 4F_2 r^{-2}) \cos 2\theta \\ A_3 r^{-2} + 2G_3 \ln r + G_3 + 2B_3 - (2C_3 + 6E_3 r^{-4} + 4F_3 r^{-2}) \cos 2\theta \end{array} \right\} \quad (55)$$

$$\sigma_{\theta\theta} = \left\{ \begin{array}{l} -A_1 r^{-2} \quad + 2B_1 + (2C_1 + 12D_1 r^2 + 6E_1 r^{-4}) \cos 2\theta \\ -A_2 r^{-2} \quad + 2B_2 + (2C_2 + 12D_2 r^2 + 6E_2 r^{-4}) \cos 2\theta \\ -A_3 r^{-2} + 2G_3 \ln r + 3G_3 + 2B_3 + (2C_3 + 12D_3 r^2 + 6E_3 r^{-4}) \cos 2\theta \end{array} \right\} \quad (56)$$

$$\sigma_{r\theta} = \left\{ \begin{array}{l} (2C_1 + 6D_1 r^2 - 6E_1 r^{-4} - 2F_1 r^{-2}) \sin 2\theta \\ (2C_2 + 6D_2 r^2 - 6E_2 r^{-4} - 2F_2 r^{-2}) \sin 2\theta \\ (2C_3 + 6D_3 r^2 - 6E_3 r^{-4} - 2F_3 r^{-2}) \sin 2\theta \end{array} \right\} , \quad (57)$$

where the top entry is the bore tube region  $a < r < b$ ; the middle entry is the region of conductors  $b < r < c$ ; and the last entry represents the pretensioned band  $c < r < d$ .

Substituting Eqs. (55-57) into Eqs. (48-50) gives for the strains

$$\frac{\gamma}{1+\nu} \cdot [\epsilon_{rr} + \nu \epsilon_{zz} - (1+\nu)k] =$$

$$\left\{ \begin{array}{l} A_1 r^{-2} + 2(1-2\nu_1)B_1 \\ -[2C_1 + 12\nu_1 D_1 r^2 + 6E_1 r^{-4} + 4(1-\nu_1)F_1 r^{-2}] \cos 2\theta \\ A_2 r^{-2} + 2(1-2\nu_2)B_2 \\ -[2C_2 + 12\nu_2 D_2 r^2 + 6E_2 r^{-4} + 4(1-\nu_2)F_2 r^{-2}] \cos 2\theta \\ A_3 r^{-2} + 2(1-2\nu_3)G_3 \ln r + (1-4\nu_3)G_3 + 2(1-2\nu_3)B_3 \\ -[2C_3 + 12\nu_3 D_3 r^2 + 6E_3 r^{-4} + 4(1-\nu_3)F_3 r^{-2}] \cos 2\theta \end{array} \right\} \quad (58)$$

$$\frac{\gamma}{1+\nu} \cdot [\epsilon_{\theta\theta} + \nu \epsilon_{zz} - (1+\nu)k] =$$

$$\left\{ \begin{array}{l} -A_1 r^{-2} + 2(1-2\nu_1)B_1 \\ + [2C_1 + 12(1-\nu_1)D_1 r^2 + 6E_1 r^{-4} + 4\nu_1 F_1 r^{-2}] \cos 2\theta \\ -A_r r^{-2} + 2(1-2\nu_2)B_2 \\ + [2C_2 + 12(1-\nu_2)D_2 r^2 + 6E_2 r^{-4} + 4\nu_2 F_2 r^{-2}] \cos 2\theta \\ -A_3 r^{-2} + 2(1-2\nu_3)G_3 \ln r + (3-4\nu_3)G_3 + 2(1-2\nu_3)B_3 \\ + [2C_3 + 12(1-\nu_3)D_3 r^2 + 6E_3 r^{-4} + 4\nu_3 F_3 r^{-2}] \cos 2\theta \end{array} \right\} \quad (59)$$

$$\frac{\gamma}{1+\nu} \epsilon_{r\theta} = \left\{ \begin{array}{l} (2C_1 + 6D_1 r^2 - 6E_1 r^{-4} - 2F_1 r^{-2}) \sin 2\theta \\ (2C_2 + 6D_2 r^2 - 6E_2 r^{-4} - 2F_2 r^{-2}) \sin 2\theta \\ (2C_3 + 6D_3 r^2 - 6E_3 r^{-4} - 2F_3 r^{-2}) \sin 2\theta \end{array} \right\} \quad (60)$$

The relation between strain and displacement given in Eq. (8) expressed in cylindrical coordinates becomes

$$\epsilon_{rr} = \frac{\partial u_r}{\partial r} \quad (61)$$

$$\epsilon_{\theta\theta} = \frac{u_r}{r} + \frac{1}{r} \frac{\partial u_\theta}{\partial \theta} \quad (62)$$

$$\epsilon_{r\theta} = \frac{1}{2} \left( \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r} + \frac{1}{r} \frac{\partial u_r}{\partial \theta} \right) \quad (63)$$

Utilizing Eqs. (58-60) one finds by partial integration

$$\frac{Y}{1+\nu} [u_r + \nu \epsilon_{zz} r - (1+\nu) k r] = \left\{ \begin{array}{l} -A_1 r^{-1} \quad +2(1-2\nu_1) B_1 r \\ -[2C_1 r + 4\nu_1 D_1 r^3 - 2E_1 r^{-3} - 4(1-\nu_1) F_1 r^{-1}] \cos 2\theta \\ -A_2 r^{-1} \quad +2(1-2\nu_2) B_2 r \\ -[2C_2 r + 4\nu_2 D_2 r^3 - 2E_2 r^{-3} - 4(1-\nu_2) F_2 r^{-1}] \cos 2\theta \\ -A_3 r^{-1} + 2(1-2\nu_3) G_3 r (\ln r - 1) + (1-4\nu_3) G_3 r + 2(1-2\nu_3) B_3 r \\ -[2C_3 r + 4\nu_3 D_3 r^3 - 2E_3 r^{-3} - 4(1-\nu_3) F_3 r^{-1}] \cos 2\theta \end{array} \right\} \quad (64)$$

$$\frac{Y}{1+\nu} \cdot u_\theta = \left\{ \begin{array}{l} 2[C_1 r + (3-2\nu_1) D_1 r^3 + E_1 r^{-3} - (1-2\nu_1) F_1 r^{-1}] \sin 2\theta \\ 2[C_2 r + (3-2\nu_1) D_2 r^3 + E_2 r^{-3} - (1-2\nu_2) F_2 r^{-1}] \sin 2\theta \\ 4(1-\nu_3) G_3 r \theta + 2[C_3 r + (3-2\nu_3) D_3 r^3 + E_3 r^{-3} - (1-2\nu_3) F_3 r^{-1}] \sin 2\theta \end{array} \right\} \quad (65)$$

$$u_z = \epsilon_{zz} z \quad (66)$$

The unknown functions in the partial integration are set to zero in order not to introduce rigid body rotations.

Boundary Conditions

At  $r=a$  no traction is transmitted. Hence

$$\sigma_{rr} = \sigma_{r\theta} = 0 , \quad (67)$$

or

$$A_1 a^{-2} + 2B_1 = 0 \quad (68)$$

$$-2C_1 - 6E_1 a^{-4} - 4F_1 a^{-2} = 0 \quad (69)$$

$$2C_1 + 6D_1 a^2 - 6E_1 a^{-4} - 2F_1 a^{-2} = 0 . \quad (70)$$

At  $r=b$  the equilibrium condition is

$$\sigma_{rr}^{(+)} - \sigma_{rr}^{(-)} + f_r = 0 , \quad (71)$$

and

$$\sigma_{r\theta}^{(+)} - \sigma_{r\theta}^{(-)} + f_\theta = 0 , \quad (72)$$

where  $f_r$  and  $f_\theta$  are given by Eqs. (31-32). Thus

$$(A_2 - A_1)b^{-2} + 2(B_2 - B_1) = -\pi i_0 [i_0 b^2 r_s^{-2} + i_1 (1 + c^2 r_s^{-2})] \quad (73)$$

$$-2(C_2 - C_1) - 6(E_2 - E_1)b^{-4} - 4(F_2 - F_1)b^{-2} = -\pi i_0 [i_0 b^2 r_s^{-2} + i_1 (1 + c^2 r_s^{-2})] \quad (74)$$

$$2(C_2 - C_1) + 6(D_2 - D_1)b^2 - 6(E_2 - E_1)b^{-4} - 2(F_2 - F_1)b^{-2} = \pi i_0 [i_0 (1 + b^2 r_s^{-2}) + i_1 (1 + c^2 r_s^{-2})] \quad (75)$$

Also at  $r=b$  the displacements are continuous.

$$u_r^{(+)} - u_r^{(-)} = u_\theta^{(+)} - u_\theta^{(-)} = 0 . \quad (76)$$

From Eqs. (64-65) one obtains

$$\frac{1+\nu_2}{Y_2} [-A_2 b^{-1} + 2(1-2\nu_2)B_2 b] - \frac{1+\nu_1}{Y_1} [-A_1 b^{-1} + 2(1-2\nu_1)B_1 b]$$

$$-(v_2 - v_1)\epsilon_{zz}b = -(1+v_2)k_2b + (1+v_1)k_1b \quad (77)$$

$$\begin{aligned} & \frac{1+v_2}{Y_2} [-2C_2b - 4v_2D_2b^3 + 2E_2b^{-3} + 4(1-v_2)F_2b^{-1}] \\ & - \frac{1+v_1}{Y_1} [-2C_1b - 4v_1D_1b^3 + 2E_1b^{-3} + 4(1-v_1)F_1b^{-1}] = 0 \end{aligned} \quad (78)$$

$$\begin{aligned} & \frac{1+v_2}{Y_2} [2C_2b + 2(3-2v_2)D_2b^3 + 2E_2b^{-3} - 2(1-2v_2)F_2b^{-1}] \\ & - \frac{1+v_1}{Y_1} [2C_1b + 2(3-2v_1)D_1b^3 + 2E_1b^{-3} - 2(1-2v_1)F_1b^{-1}] = 0 . \end{aligned} \quad (79)$$

At  $r=c$  the equilibrium condition is

$$\sigma_{rr}^{(+)} - \sigma_{rr}^{(-)} + f_r = 0, \quad \sigma_{r\theta}^{(+)} - \sigma_{r\theta}^{(-)} + f_\theta = 0 . \quad (80)$$

Hence, using Eqs. (55) and (57)

$$(A_3 - A_2)c^{-2} + (2\ln c + 1)G_3 + 2(B_3 - B_2) = -\pi i_1 [i_o(-b^2c^{-2} + b^2r_s^{-2}) + i_1c^2r_s^{-2}] \quad (81)$$

$$\begin{aligned} & -2(C_3 - C_2) - 6(E_3 - E_2)c^{-4} - 4(F_3 - F_2)c^{-2} = \\ & -\pi i_1 [i_o(-b^2c^{-2} + b^2r_s^{-2}) + i_1c^2r_s^{-2}] \end{aligned} \quad (82)$$

$$\begin{aligned} & 2(C_3 - C_2) + 6(D_3 - D_2)c^2 - 6(E_3 - E_2)c^{-4} - 2(F_3 - F_2)c^{-2} \\ & = \pi i_1 [i_o(b^2c^{-2} + b^2r_s^{-2}) + i_1(1 + c^2r_s^{-2})] . \end{aligned} \quad (83)$$

At  $r=c$   $u_r$  is continuous or

$$u_r^{(+)} - u_r^{(-)} = 0 . \quad (84)$$

The pretension condition in the band is formulated by utilizing

the notion of rotational dislocation<sup>5</sup> whereby a small angle  $\alpha$  is removed from the band. Subsequently this angle is closed up and held by welding, slippage between the band and region 2 being permitted. Thus, for region 3 and  $r=c$

$$u_{\theta}(2\pi) - u_{\theta}(0) = c\alpha . \quad (85)$$

After removal of the term responsible for pretensioning, subsequent slippage is not allowed and then for  $r=c$

$$u_{\theta}^{(+)} \text{ minus term prop. to } \theta = u_{\theta}^{(-)} . \quad (86)$$

The conditions of Eqs. (84-86) yield

$$\begin{aligned} & \frac{1+\nu_3}{Y_3} [-A_3 c^{-1} + 2(1-2\nu_3)G_3 c(\ln c - 1) + (1-4\nu_3)G_3 c + 2(1-2\nu_3)B_3 c] \\ & - \frac{1+\nu_2}{Y_2} [-A_2 c^{-1} + 2(1-2\nu_2)B_2 c] - (\nu_3 - \nu_2)\epsilon_{zz} c = - (1+\nu_3)k_3 c + (1+\nu_2)k_2 c \end{aligned} \quad (87)$$

$$\begin{aligned} & \frac{1+\nu_3}{Y_3} [-2C_3 c - 4\nu_3 D_3 c^3 + 2E_3 c^{-3} + 4(1-\nu_3)F_3 c^{-1}] \\ & - \frac{1+\nu_2}{Y_2} [-2C_2 c - 4\nu_2 D_2 c^3 + 2E_2 c^{-3} + 4(1-\nu_2)F_2 c^{-1}] = 0 \end{aligned} \quad (88)$$

$$4 \frac{(1-\nu_3^2)}{Y_3} G_3 c 2\pi = c\alpha \quad (89)$$

$$\begin{aligned} & 2 \frac{1+\nu_3}{Y_3} [C_3 c + (3-2\nu_3)D_3 c^3 + E_3 c^{-3} - (1-2\nu_3)F_3 c^{-1}] \\ & - 2 \frac{1+\nu_2}{Y_2} [C_2 c + (3-2\nu_2)D_2 c^3 + E_2 c^{-3} - (1-2\nu_2)F_2 c^{-1}] = 0 . \end{aligned} \quad (90)$$

At  $r=d$  no traction is transmitted. Hence

$$\sigma_{rr} = \sigma_{r\theta} = 0 \quad (91)$$

or

$$A_3 d^{-2} + (2\ell n d + 1)G_3 + 2B_3 = 0 \quad (92)$$

$$-2C_3 - 6E_3 d^{-4} - 4F_3 d^{-2} = 0 \quad (93)$$

$$2C_3 + 6D_3 d^2 - 6E_3 d^{-4} - 2F_3 d^{-2} = 0 . \quad (94)$$

Note that Eqs. (68-70, 73-75, 77-79, 81-83, 87-90, 92-94) provide 19 conditions among the 20 variables  $A_1 \ B_1 \ C_1 \ D_1 \ E_1 \ F_1 \ A_2 \ B_2 \ C_2 \ D_2 \ E_2 \ F_2 \ A_3 \ G_3 \ B_3 \ C_3 \ D_3 \ E_3 \ F_3 \ \epsilon_{zz}$ .

#### Use of Virial Theorem

The virial theorem in Eq. (23) may be expressed as

$$\int_V \text{tr} \vec{\tau} dV = \int_S \vec{r} \cdot \vec{\tau} \cdot \vec{n} dS + W_B . \quad (95)$$

But the traction on the surface is<sup>6</sup>

$$\vec{\tau} \cdot \vec{n} = \frac{1}{4\pi} [\vec{H}(\vec{H} \cdot \vec{n}) - \frac{1}{2} H^2 \vec{n}] . \quad (96)$$

It is assumed that the magnet is of finite length and that the end surfaces used to specify S are sufficiently far removed so that no fields are present. On the cylindrical iron surface  $r=r_s$

$$\vec{r} \cdot \vec{\tau} \cdot \vec{n} = \frac{1}{8\pi} (H_r^2 - H_\theta^2) r_s . \quad (97)$$

Using Eq. (25-26) for the fields

$$\int_S \vec{r} \cdot \vec{\tau} \cdot \vec{n} ds = \frac{2\ell\pi^2}{r_s^2} (i_o b^2 + i_l c^2)^2 . \quad (98)$$

The magnetic energy is given by Eqs. (25-26) and (35)

$$W_B = \ell\pi^2 [i_o^2 b^2 + 2i_o i_l b^2 + i_l^2 c^2 + (i_o b^2 + i_l c^2)^2 r_s^{-2}] , \quad (99)$$

which, together with Eq. (98), gives

$$\int_V \text{tr} \vec{\sigma} dV = \ell \pi^2 [i_o^2 b^2 + 2i_o i_1 b^2 + i_1^2 c^2 + 3(i_o b^2 + i_1 c^2)^2 r_s^{-2}] . \quad (100)$$

Then, using Eq. (14) to eliminate  $\sigma_{zz}$  one has after cancelling the effective length  $\ell$

$$\begin{aligned} \iint [(1+v)(\sigma_{rr} + \sigma_{\theta\theta}) + Y(\epsilon_{zz} - k)] r dr d\theta = \\ \pi^2 [i_o^2 b^2 + 2i_o i_1 b^2 + i_1^2 c^2 + 3(i_o b^2 + i_1 c^2)^2 r_s^{-2}] . \end{aligned} \quad (101)$$

Using Eqs. (55-56) this gives the condition

$$\begin{aligned} 4\pi(1+v_1)(b^2 - a^2)B_1 + 4\pi(1+v_2)(c^2 - b^2)B_2 \\ + 2\pi(1+v_3)[d^2(2\ell nd + 1) - c^2(2\ell nc + 1)]G_3 + 4\pi(1+v_3)(d^2 - c^2)B_3 \\ + \pi[Y_1(b^2 - a^2) + Y_2(c^2 - b^2) + Y_3(d^2 - c^2)]\epsilon_{zz} \\ = \pi[k_1 Y_1(b^2 - a^2) + k_2 Y_2(c^2 - b^2) + k_3 Y_3(d^2 - c^2)] \\ + \pi^2 [i_o^2 b^2 + 2i_o i_1 b^2 + i_1^2 c^2 + 3(i_o b^2 + i_1 c^2)^2 r_s^{-2}] , \end{aligned} \quad (102)$$

which provides the last condition necessary for determining all of the unknowns. The current densities  $i_o$  and  $i_1$  can be chosen in many ways. The following choice comes from equating respectively the current and radial moment of the current in the two sheet dipoles to the same quantities in the thick cosine theta dipole and expressing the result in terms of the central magnetic field  $H_o$ . Thus Eq. (24) becomes

$$i = \left\{ \begin{matrix} cb^{-1+2} \\ bc^{-1+2} \end{matrix} \right\} \cdot \frac{H_o \cos \theta}{12\pi [1 + \frac{1}{3}(b^2 + bc + c^2)r_s^{-2}]} . \quad (103)$$



### Internal Energy

The expression for the internal energy in Eq. (1) may be obtained using the strains in cylindrical coordinates by partially integrating Eq. (2) using Eqs. (37 - 40). Thus

$$W = \frac{1}{2}(\lambda+2\mu)(\epsilon_{rr}^2 + \epsilon_{\theta\theta}^2 + \epsilon_{zz}^2) + \lambda(\epsilon_{rr}\epsilon_{\theta\theta} + \epsilon_{\theta\theta}\epsilon_{zz} + \epsilon_{zz}\epsilon_{rr}) + \mu\epsilon_{r\theta}^2 - k(3\lambda+2\mu)(\epsilon_{rr} + \epsilon_{\theta\theta} + \epsilon_{zz}) . \quad (104)$$

After rearrangement and use of Eqs. (37-40), the strain energy may be written in terms of stresses and strains:

$$W = \frac{1}{2} \left\{ [\sigma_{rr} - k(3\lambda+2\mu)]\epsilon_{rr} + [\sigma_{\theta\theta} - k(3\lambda+2\mu)]\epsilon_{\theta\theta} + [\sigma_{zz} - k(3\lambda+2\mu)]\epsilon_{zz} + \sigma_{r\theta} \epsilon_{r\theta} \right\} . \quad (105)$$

### Stress Distribution in Iron Shield

In the region of the iron shield between  $r=r_s$  and  $r=R_s$  the Airy stress function may be taken as

$$\phi = A \ln r + Br^2 + (Cr^2 + Dr^4 + Er^{-2} + F) \cos 2\theta . \quad (106)$$

Equations (51-54) then give

$$\sigma_{rr} = Ar^{-2} + 2B - (2C + 6Er^{-4} + 4Fr^{-2}) \cos 2\theta \quad (107)$$

$$\sigma_{\theta\theta} = -Ar^{-2} + 2B + (2C + 12Dr^2 + 6Er^{-4}) \cos 2\theta \quad (108)$$

$$\sigma_{r\theta} = (2C + 6Dr^2 - 6Er^{-4} - 2Fr^{-2}) \sin 2\theta \quad (109)$$

The boundary conditions at  $r=r_s$  are  $(\sigma_{rr}^{(-)} = \tau_{rr}^{(+)} = 0)$

$$\sigma_{rr}^{(+)} - \tau_{rr}^{(-)} = 0 \quad (110)$$

$$\sigma_{r\theta}^{(+)} = 0 , \quad (111)$$

where, from Eqs. (96) and (25)

$$\tau_{rr}^{(-)} = \frac{1}{8\pi} H_r^2 = \pi(i_0 b^2 + i_1 c^2)^2 r_s^{-4} (1 - \cos 2\theta) . \quad (112)$$

Hence

$$A r_s^{-2} + 2B = - \pi(i_0 b^2 + i_1 c^2)^2 r_s^{-4} \quad (113)$$

$$-2C - 6E r_s^{-4} - 4F r_s^{-2} = \pi(i_0 b^2 + i_1 c^2)^2 r_s^{-4} \quad (114)$$

$$2C + 6D r_s^2 - 6E r_s^{-4} - 2F r_s^{-2} = 0 . \quad (115)$$

At  $r=R_s$  the boundary conditions are

$$\sigma_{rr} = \sigma_{r\theta} = 0 . \quad (116)$$

Hence

$$A R_s^{-2} + 2B = 0 \quad (117)$$

$$2C + 6E R_s^{-4} + 4F R_s^{-2} = 0 \quad (118)$$

$$2C + 6D R_s^2 - 6E R_s^{-4} - 2F R_s^{-2} = 0 . \quad (119)$$

Note that Eqs. (113-115, 117-119) provide six equations for determining the six unknowns (A-F).

As in the previous problem the generalized plane strain approximation will be used. Since this introduces one more unknown, the longitudinal strain, the virial theorem will be used to provide the last condition. Thus using Eq. (95) and (97) with an inwardly directed normal gives

$$\iint (\sigma_{rr} + \sigma_{\theta\theta} + \sigma_{zz}) r dr d\theta = - 2\pi^2 (i_0 b^2 + i_1 c^2)^2 r_s^{-2} . \quad (120)$$

Equation (47) may be used to eliminate  $\sigma_{zz}$ . In this case since the iron shield remains at room temperature  $k=0$ . Thus

$$\iint [(1+\nu)(\sigma_{rr} + \sigma_{\theta\theta}) + Y\epsilon_{zz}] r dr d\theta = - 2\pi^2 (i_o b^2 + i_l c^2)^2 r_s^{-2} . \quad (121)$$

Integrating after using Eqs. (107-108) for the stresses gives

$$\pi(R_s^2 - r_s^2)[4(1+\nu)B + Y\epsilon_{zz}] = - 2\pi^2 (i_o b^2 + i_l c^2)^2 r_s^{-2} . \quad (122)$$

Thus the longitudinal strain is determined.

### Numerical Calculations

The stresses and strains that exist in the three nested hollow cylinders have been calculated as a function of the central magnetic field. Twenty algebraic relations in Eqs. (68, 69, 70, 73, 74, 75, 77, 78, 79, 81, 82, 83, 87, 88, 89, 90, 92, 93, 94, 102) among the nineteen coefficients in the Airy stress functions  $(A_1, B_1, C_1, D_1, E_1, F_1, A_2, B_2, C_2, D_2, E_2, F_2, A_3, G_3, B_3, C_3, D_3, E_3, F_3)$  and the longitudinal strain  $\epsilon_{zz}$  have been solved. Thus the stress and strain of any point in the dipole model structure may be found. For simplicity in the presentation of numerical results, however, only the values on the median plane are given. It is usually clear whether a quantity is stress or strain. Otherwise, R is radial, T is theta or azimuthal, Z is axial or longitudinal. With regard to position A, B, C, D are the points on the median plane at the cylindrical boundaries between the various media. To indicate the side of the point, P is used for positive and M for negative. Thus, for example, RTBP indicates the  $(r, \theta)$  component at the positive side of point B.

The boundary between elastic and plastic isotropic media is a function of the invariants of the tensor representing the deviation of stress from the mean stress. A generally accepted simplification

of this condition regards the onset of plastic flow as being determined only by the second invariant of this tensor<sup>6</sup>

$$J_2 = \frac{1}{6}[(\sigma_{rr}-\sigma_{\theta\theta})^2 + (\sigma_{\theta\theta}-\sigma_{zz})^2 + (\sigma_{zz}-\sigma_{rr})^2] + \sigma_{r\theta}^2 . \quad (123)$$

Since the condition may be stated as

$$3J_2 = Y_t^2 , \quad (124)$$

where  $Y_t$  is the yield stress in tension, the  $\sqrt{3J_2}$  has been tabulated for ready comparison of the state of stress with the yield point. Note that for 45 kG the band stress slightly exceeds the elastic limit.

A comment relative to the appearance of negative elastic energies is in order. Equation (1) is actually an expression for the density of free energy ( $u-T\eta$ ) where  $u$  is the internal energy and  $\eta$  is the entropy density<sup>7</sup>. However, the term in  $T\eta$  that depends only on the temperature has been dropped since it does not affect the state of stress. Hence, negative values of the free energy are caused by positive values of the entropy density.

For completeness the effect of the distortions caused by banding, cooldown, and magnetic excitation are indicated by their multipole contribution<sup>8</sup> to an otherwise pure dipole field. At the reference radius let

$$\Delta B = \Delta B_1 + \Delta B_3 \quad (125)$$

where  $\Delta B_1$  is the change in the dipole component and  $\Delta B_3$  is the change in the sextupole component of the resulting field. Further,

let

$$\Delta B_1 = \Delta B_{1c} + \Delta B_{1s} \quad \Delta B_3 = \Delta B_{3c} + \Delta B_{3s} , \quad (126)$$

where the subscript c refers to the contribution due to the conductor alone and the subscript s refers to the contribution from the shield. Also let

$$R_1 = \frac{\Delta B_1}{B} \quad R_3 = \frac{\Delta B_3}{B} , \quad (127)$$

where B is the original magnetic field for zero mechanical displacement field. Thus, the output lists  $\Delta B_{1c}$ ,  $\Delta B_{1s}$ ,  $R_1$  and  $\Delta B_{3c}$ ,  $\Delta B_{3s}$ ,  $R_3$  for the displacement field that results from each state of strain.

No calculations have been made for the stresses in the iron since the inner iron surface field is modest.

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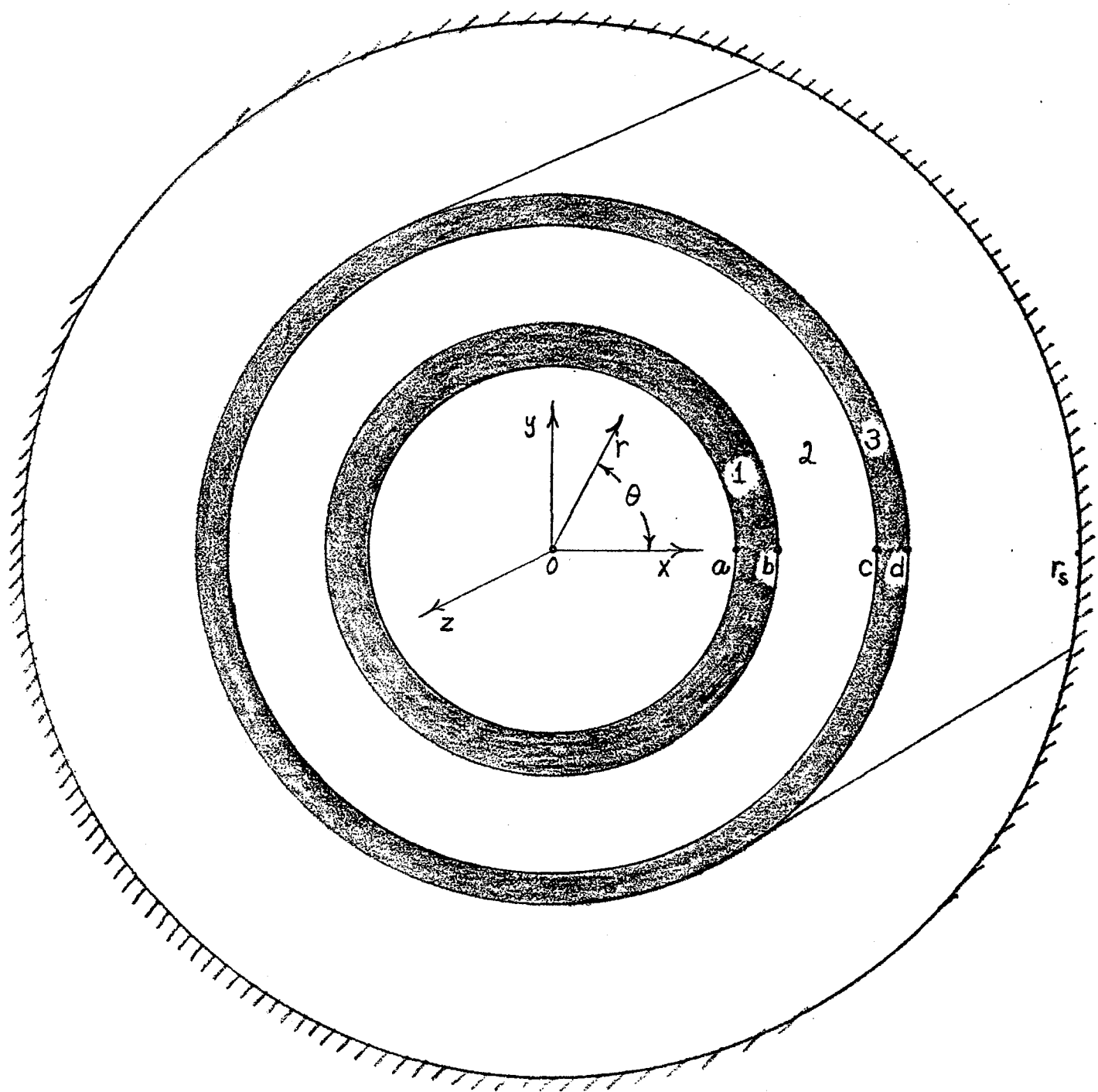


Fig. 1. Geometric Details of Doubler Dipole Model

ELASTIC STRESS AND STRAIN IN DOUBLER MODEL---ELAST3

CENTRAL MAGNETIC FIELD(KG)	=	.0000	INNER BORE TUBE RADIUS(IN)	=	1.2375	INNER CONDUCTOR RADIUS(IN)	=	1.5000
OUTER CONDUCTOR RADIUS(IN)	=	2.1490	OUTER RADIUS OF BANDING(IN)	=	2.5240	RADIUS OF IRON SHIELD(IN)	=	3.7500
BORE YOUNGS MODULUS(LBS/IN/IN)	=	11330000.	COND. YOUNGS MODULUS(LBS/IN/IN)	=	1000000.	BAND YOUNGS MODULUS(LBS/IN/IN)	=	18670000.
BORE POISSONS RATIO	=	.3330	COND. POISSONS RATIO	=	.1000	BAND POISSONS RATIO	=	.3330
BORE THERMAL STRAIN(IN/IN)	=	0.0000	COND. THERMAL STRAIN(IN/IN)	=	0.0000	BAND THERMAL STRAIN(IN/IN)	=	0.0000
REFERENCE RADIUS(IN)	=	1.0000	MAGNETIC ENERGY((IN-LB)/IN)	=	.00	MAGNETIC ENERGY(J/M)	=	.00
BORE YIELD STRESS(LB/IN/IN)	=	120000.	COND. YIELD STRESS(LB/IN/IN)	=	35000.	BAND YIELD STRESS(LB/IN/IN)	=	20000.
BAND DISLOCATION(MILLI RAD)	=	5.250	INT.(SPR+STT+SZZ)DA((IN-LB)/IN)	=	0.	INTEGRAL OF SZZ*DA((IN-LB)/IN)	=	0.
BORE ELAST. ENERGY((IN-LB)/IN)	=	3.41	COND. ELAST. ENERGY((IN-LB)/IN)	=	4.98	BAND ELAST. ENERGY((IN-LB)/IN)	=	4.67
DISPL. DIPOL WITH NO SHIELD(KG)	=	-.000303	SHLD CONTR. TO DISPL. DIPOL(KG)	=	-.000000	DIPOLE DELTAB/B AT REF. RADIUS	=	-.000303
DISPL. SEXT. WITH NO SHIELD(KG)	=	-.000000	SHLD CONTR. TO DISPL. SEXT.(KG)	=	.000000	SEXTU. DELTAB/B AT REF. RADIUS	=	-.000000

ANGLE (DEG)	RRAP	TTAP	RTAP	RRRM	TTBM	RTBM	RRBP	TRANSVERSE STRESS(LB/IN/IN)				RTCH	RRCP	TTCP	RTCP	RRDM	TTDM	RTDM
								TTBP	RTBP	RRCM	TTCM							
90.0	0.	-7104.	-0.	-1134.	-5969.	-0.	-1134.	-549.	-0.	-984.	-699.	-0.	-984.	4686.	-0.	0.	6524.	-0.
80.0	0.	-7104.	-0.	-1134.	-5969.	-0.	-1134.	-549.	-0.	-984.	-699.	-0.	-984.	4686.	-0.	0.	6524.	-0.
70.0	0.	-7104.	-0.	-1134.	-5969.	-0.	-1134.	-549.	-0.	-984.	-699.	-0.	-984.	4686.	-0.	0.	6524.	-0.
60.0	0.	-7104.	-0.	-1134.	-5969.	-0.	-1134.	-549.	-0.	-984.	-699.	-0.	-984.	4686.	-0.	0.	6524.	-0.
50.0	0.	-7104.	-0.	-1134.	-5969.	-0.	-1134.	-549.	-0.	-984.	-699.	-0.	-984.	4686.	-0.	0.	6524.	-0.
40.0	0.	-7104.	-0.	-1134.	-5969.	-0.	-1134.	-549.	-0.	-984.	-699.	-0.	-984.	4686.	-0.	0.	6524.	-0.
30.0	0.	-7104.	-0.	-1134.	-5969.	-0.	-1134.	-549.	-0.	-984.	-699.	-0.	-984.	4686.	-0.	0.	6524.	-0.
20.0	0.	-7104.	-0.	-1134.	-5969.	-0.	-1134.	-549.	-0.	-984.	-699.	-0.	-984.	4686.	-0.	0.	6524.	-0.
10.0	0.	-7104.	-0.	-1134.	-5969.	-0.	-1134.	-549.	-0.	-984.	-699.	-0.	-984.	4686.	-0.	0.	6524.	-0.
0.	0.	-7104.	-0.	-1134.	-5969.	-0.	-1134.	-549.	-0.	-984.	-699.	-0.	-984.	4686.	-0.	0.	6524.	-0.

ANGLE (DEG)	RRAP	TTAP	RTAP	RRRM	TTBM	RTBM	RRBP	TRANSVERSE STRAIN (MUIN/IN)				RTCH	RRCP	TTCP	RTCP	RRDM	TTDM	RTDM
								TTBP	RTBP	RRCM	TTCM							
90.0	285.	-550.	-0.	152.	-417.	-0.	-1060.	-417.	-0.	-895.	-582.	-0.	-151.	254.	-0.	-148.	318.	-0.
80.0	285.	-550.	-0.	152.	-417.	-0.	-1060.	-417.	-0.	-895.	-582.	-0.	-151.	254.	-0.	-148.	318.	-0.
70.0	285.	-550.	-0.	152.	-417.	-0.	-1060.	-417.	-0.	-895.	-582.	-0.	-151.	254.	-0.	-148.	318.	-0.
60.0	285.	-550.	-0.	152.	-417.	-0.	-1060.	-417.	-0.	-895.	-582.	-0.	-151.	254.	-0.	-148.	318.	-0.
50.0	285.	-550.	-0.	152.	-417.	-0.	-1060.	-417.	-0.	-895.	-582.	-0.	-151.	254.	-0.	-148.	318.	-0.
40.0	285.	-550.	-0.	152.	-417.	-0.	-1060.	-417.	-0.	-895.	-582.	-0.	-151.	254.	-0.	-148.	318.	-0.
30.0	285.	-550.	-0.	152.	-417.	-0.	-1060.	-417.	-0.	-895.	-582.	-0.	-151.	254.	-0.	-148.	318.	-0.
20.0	285.	-550.	-0.	152.	-417.	-0.	-1060.	-417.	-0.	-895.	-582.	-0.	-151.	254.	-0.	-148.	318.	-0.
10.0	285.	-550.	-0.	152.	-417.	-0.	-1060.	-417.	-0.	-895.	-582.	-0.	-151.	254.	-0.	-148.	318.	-0.
0.	285.	-550.	-0.	152.	-417.	-0.	-1060.	-417.	-0.	-895.	-582.	-0.	-151.	254.	-0.	-148.	318.	-0.

ANGLE (DEG)	LONG.	STRESS(LB/IN/IN) AND STRAIN(MUIN/IN)						SQRT(3*J2) IN (KLB/IN/IN)						TRANSVERSE DISPLACEMENT(IN)					
	ZZAP	ZZBP	ZZCP	ZZDM	ZZEP	ZZFM	EZZ	YAP	YBP	YCP	YDM	YEP	YFM	UPA	UTA	URB	UTB	URC	UTC
90.0	-2509.	-2609.	-190.	-190.	831.	1771.	-21.	6.2	4.3	.8	.7	5.0	5.8	-.0007	-.0000	-.0006	-.0000	-.0013	-.0000
80.0	-2509.	-2609.	-190.	-190.	831.	1771.	-21.	6.2	4.3	.8	.7	5.0	5.8	-.0007	-.0000	-.0006	-.0000	-.0013	-.0000
70.0	-2509.	-2609.	-190.	-190.	831.	1771.	-21.	6.2	4.3	.8	.7	5.0	5.8	-.0007	-.0000	-.0006	-.0000	-.0013	-.0000
60.0	-2509.	-2609.	-190.	-190.	831.	1771.	-21.	6.2	4.3	.8	.7	5.0	5.8	-.0007	-.0000	-.0006	-.0000	-.0013	-.0000
50.0	-2509.	-2609.	-190.	-190.	831.	1771.	-21.	6.2	4.3	.8	.7	5.0	5.8	-.0007	-.0000	-.0006	-.0000	-.0013	-.0000
40.0	-2509.	-2609.	-190.	-190.	831.	1771.	-21.	6.2	4.3	.8	.7	5.0	5.8	-.0007	-.0000	-.0006	-.0000	-.0013	-.0000
30.0	-2509.	-2609.	-190.	-190.	831.	1771.	-21.	6.2	4.3	.8	.7	5.0	5.8	-.0007	-.0000	-.0006	-.0000	-.0013	-.0000
20.0	-2509.	-2609.	-190.	-190.	831.	1771.	-21.	6.2	4.3	.8	.7	5.0	5.8	-.0007	-.0000	-.0006	-.0000	-.0013	-.0000
10.0	-2509.	-2609.	-190.	-190.	831.	1771.	-21.	6.2	4.3	.8	.7	5.0	5.8	-.0007	-.0000	-.0006	-.0000	-.0013	-.0000
0.	-2509.	-2609.	-190.	-190.	831.	1771.	-21.	6.2	4.3	.8	.7	5.0	5.8	-.0007	-.0000	-.0006	-.0000	-.0013	-.0000

Table 1. Pre-Stress, No Field



ELASTIC STRESS AND STRAIN IN DOUBLER MODEL---ELAST3

CENTRAL MAGNETIC FIELD(KG)	=	45.0000	INNER ROPE TUBE RADIUS(IN)	=	1.2375	INNER CONDUCTOR RADIUS(IN)	=	1.5000
OUTER CONDUCTOR RADIUS(IN)	=	2.1490	OUTER RADIUS OF BANDING(IN)	=	2.5240	RADIUS OF IRON SHIELD(IN)	=	3.7500
ROPE YOUNGS MODULUS(LBS/IN/IN)	=	11330000.	COND. YOUNGS MODULUS(LBS/IN/IN)	=	1000000.	BAND YOUNGS MODULUS(LBS/IN/IN)	=	18670000.
ROPE POISSONS RATIO	=	.3333	COND. POISSONS RATIO	=	.1030	BAND POISSONS RATIO	=	.3330
ROPE THERMAL STRAIN(IN/IN)	=	0.0000	COND. THERMAL STRAIN(IN/IN)	=	0.0000	BAND THERMAL STRAIN(IN/IN)	=	0.0000
REFERENCE RADIUS(IN)	=	1.0000	MAGNETIC ENERGY((IN-LBS)/IN)	=	17344.27	MAGNETIC ENERGY(J/M)	=	77260.85
ROPE YIELD STRESS(LB/IN/IN)	=	120000.	COND. YIELD STRESS(LB/IN/IN)	=	35000.	BAND YIELD STRESS(LB/IN/IN)	=	20000.
BAND DISLOCATION(MILLIRAD)	=	5.250	INT. (SRR+STT+SZZ)DA((IN-LB)/IN)	=	25.28	INTEGRAL OF SZZ*DA((IN-LB)/IN)	=	17344.
ROPE ELAST. ENERGY((IN-LB)/IN)	=	6.48	COND. ELAST. ENERGY((IN-LB)/IN)	=	11.08	BAND ELAST. ENERGY((IN-LB)/IN)	=	7.74
DISPL. DIPOL WITH NO SHIELD(KG)	=	-.001577	SHLD CONTR. TO DISPL. DIPOL(KG)	=	.008538	DIPOLE DELTAB/B AT REF. RADIUS	=	-.000155
DISPL. SEXT. WITH NO SHIELD(KG)	=	-.020088	SHLD CONTR. TO DISPL. SEXT.(KG)	=	.000665	SEXTU. DELTAB/B AT REF. RADIUS	=	-.000432

ANGLE (DEG)	RRAP	TTAP	RTAP	RRBM	TTBM	RTBM	RRBP	TTBP	PTBP	RRCH	TTCH	RTCH	RRCP	TTCP	RTCP	RRDM	TTDM	RTDM
90.0	-0.	13981.	-0.	77.	-3150.	0.	77.	-264.	0.	-812.	-358.	0.	-812.	11995.	0.	0.	20.	-0.
80.0	-0.	12845.	0.	26.	-3206.	-963.	-10.	-276.	-584.	-832.	-378.	-341.	-830.	11604.	-115.	0.	454.	0.
70.0	-0.	9573.	0.	-122.	-3359.	-1809.	-263.	-311.	-1104.	-889.	-438.	-641.	-883.	10477.	-216.	0.	1703.	0.
60.0	-0.	4559.	0.	-350.	-3618.	-2438.	-549.	-365.	-1488.	-977.	-529.	-863.	-964.	8751.	-291.	0.	3617.	0.
50.0	-0.	-1590.	0.	-628.	-3924.	-2772.	-1123.	-431.	-1692.	-1085.	-640.	-982.	-1063.	6634.	-330.	0.	5965.	0.
40.0	0.	-8135.	0.	-925.	-4249.	-2772.	-1629.	-501.	-1692.	-1200.	-759.	-982.	-1168.	4381.	-330.	0.	8464.	0.
30.0	0.	-14285.	0.	-1204.	-4555.	-2438.	-2102.	-567.	-1488.	-1308.	-870.	-863.	-1267.	2264.	-291.	0.	10811.	0.
20.0	0.	-19299.	0.	-1431.	-4804.	-1809.	-2489.	-621.	-1105.	-1395.	-961.	-641.	-1348.	538.	-216.	0.	12725.	0.
10.0	0.	-22571.	0.	-1579.	-4967.	-963.	-2741.	-656.	-584.	-1453.	-1020.	-341.	-1400.	-588.	-115.	0.	13975.	0.
0.0	0.	-23708.	0.	-1631.	-5023.	-0.	-2829.	-668.	-0.	-1473.	-1041.	-0.	-1419.	-980.	-0.	0.	14409.	0.
ANGLE (DEG)	RRAP	TTAP	RTAP	RRBM	TTBM	RTBM	RRBP	TTBP	PTBP	RRCH	TTCH	RTCH	RRCP	TTCP	RTCP	RRDM	TTDM	RTDM
90.0	-576.	1069.	-0.	101.	-279.	0.	97.	-279.	0.	-773.	-273.	0.	-352.	562.	0.	-29.	-27.	-0.
80.0	-532.	980.	0.	99.	-281.	-113.	12.	-281.	-646.	-791.	-292.	-375.	-344.	544.	-8.	-39.	-7.	0.
70.0	-403.	723.	0.	94.	-283.	-213.	12.	-288.	-1215.	-841.	-344.	-705.	-320.	492.	-15.	-69.	53.	0.
60.0	-207.	329.	0.	86.	-299.	-287.	-611.	-299.	-1637.	-913.	-424.	-950.	-282.	411.	-21.	-114.	144.	0.
50.0	34.	-153.	0.	76.	-312.	-325.	-1073.	-312.	-1861.	-1012.	-523.	-1080.	-237.	313.	-24.	-170.	256.	0.
40.0	290.	-667.	0.	65.	-326.	-326.	-1565.	-326.	-1851.	-1113.	-628.	-1090.	-188.	208.	-24.	-230.	375.	0.
30.0	531.	-1149.	0.	56.	-339.	-287.	-2027.	-339.	-1637.	-1207.	-726.	-950.	-143.	110.	-21.	-285.	486.	0.
20.0	729.	-1543.	0.	48.	-349.	-213.	-2404.	-349.	-1215.	-1284.	-806.	-705.	-105.	29.	-15.	-331.	578.	0.
10.0	856.	-1800.	0.	42.	-356.	-113.	-2650.	-356.	-646.	-1335.	-859.	-375.	-81.	-23.	-8.	-361.	637.	0.
0.0	900.	-1889.	0.	40.	-359.	-0.	-2735.	-359.	-0.	-1352.	-877.	-0.	-73.	-41.	-0.	-371.	658.	0.

ANGLE (DEG)	LONG. STRESS(LB/IN/IN)	STRAIN(MUIN/IN)	STRESS(LB/IN/IN)	STRAIN(MUIN/IN)	SQRT(3*J2) IN (KLB/IN/IN)	YAP	YBM	YBP	YCH	YCP	YOM	UFA	UTA	URB	UTR	URC	UTC	URD	UTD
90.0	5623.	-56.	67.	-32.	5318.	1601.	85.	12.2	3.2	.3	.7	11.1	1.6	-.0041	.0000	-.0042	.0000	-.0045	.0033
80.0	5245.	-92.	57.	-36.	5181.	1745.	85.	11.2	3.6	1.1	.9	10.8	1.6	-.0039	.0000	-.0040	.0000	-.0043	.0025
70.0	4155.	-149.	23.	-47.	4789.	2161.	85.	8.3	4.5	1.9	1.3	9.8	1.6	-.0033	.0018	-.0033	.0012	-.0037	.0018
60.0	2436.	-325.	16.	-55.	4187.	2793.	85.	4.0	5.5	3.2	1.6	8.4	1.6	-.0023	.0024	-.0023	.0016	-.0029	.0012
50.0	438.	-545.	-170.	-81.	3449.	3590.	85.	1.8	5.8	3.1	1.9	6.0	1.6	-.0011	.0027	-.0011	.0019	-.0019	.0007
40.0	-1742.	-735.	-123.	-110.	2664.	4412.	85.	7.4	5.9	3.2	1.9	5.0	1.6	-.0001	.0027	-.0002	.0019	-.0007	.0007
30.0	-3790.	-955.	-182.	-132.	1926.	5194.	85.	12.8	5.5	3.1	1.8	3.4	1.6	-.0013	.0024	-.0014	.0016	-.0004	.0017
20.0	-5459.	-1109.	-226.	-150.	1324.	5931.	85.	17.2	4.7	2.8	1.6	1.1	1.0	-.0023	.0018	-.0024	.0012	-.0012	.0012
10.0	-6549.	-1214.	-254.	-162.	932.	6247.	85.	20.1	4.0	2.5	1.4	0.8	1.1	-.0029	.0009	-.0030	.0006	-.0018	.0007
0.0	-6927.	-1245.	-264.	-166.	795.	6392.	85.	21.1	4.6	2.4	1.3	0.8	1.2	-.0031	.0000	-.0032	.0000	-.0020	.0000

Table 2. Pre-Stress, 45 kG Field

ELASTIC STRESS AND STRAIN IN DOUBLER MODEL---ELAST3

CENTRAL MAGNETIC FIELD(KG)	=	.9000	INNER ROPE TUBE RADIUS(IN)	=	1.2375	INNER CONDUCTOR RADIUS(IN)	=	1.5000
OUTER CONDUCTOR RADIUS(IN)	=	2.1490	OUTER RADIUS OF BANDING(IN)	=	2.5240	RADIUS OF IRON SHIELD(IN)	=	3.7500
BOFF YOUNGS MODULUS(LBS/IN/IN)	=	11330000.	COND. YOUNGS MODULUS(LBS/IN/IN)	=	1000000.	BAND YOUNGS MODULUS(LBS/IN/IN)	=	18670000.
BOFF POISSONS RATIO	=	.3330	COND. POISSONS RATIO	=	.1000	BAND POISSONS RATIO	=	.3330
BOFF THERMAL STRAIN(IN/IN)	=	-.0015	COND. THERMAL STRAIN(IN/IN)	=	-.0030	BAND THERMAL STRAIN(IN/IN)	=	-.0030
REFERENCE RADIUS(IN)	=	1.0000	MAGNETIC ENERGY((IN-LBS)/IN)	=	.00	MAGNETIC ENERGY(J/M)	=	.00
BOFF YIELD STRESS(LB/IN/IN)	=	120000.	COND. YIELD STRESS(LB/IN/IN)	=	35000.	BAND YIELD STRESS(LB/IN/IN)	=	20000.
BAND DISLOCATION(MILLIRAD)	=	5.250	INT. (SRR+STT+SZZ)DA((IN-LB)/IN)	=	0.	INTEGRAL OF SZZ*DA((IN-LB)/IN)	=	0.
BOFF ELAST. ENERGY((IN-LB)/TN)	=	-450.57	COND. ELAST. ENERGY((IN-LB)/IN)	=	-225.73	BAND ELAST. ENERGY((IN-LB)/IN)	=	-8279.05
DISP. DIPOL WITH NO SHIELD(KG)	=	.000000	SHLD CONTR. TO DISPL. DIPOL(KG)	=	-.000000	DIPOLE DELTA8/B AT REF. RADIUS	=	.001673
DISPL. SEXT. WITH NO SHIELD(KG)	=	-.000000	SHLD CONTR. TO DISPL. SEXT.(KG)	=	.000000	SEXTU. DELTA8/B AT REF. RADIUS	=	-.000000

ANGLE (DEG)	RRAP	TTAP	RTAP	RPRM	TTBM	RTBM	RRBP	TTBP	RTBP	RRCH	TTCH	RTCH	RRCP	TTCP	RTCP	RRDM	TTDM	RTDM
90.0	-0.	-19732.	-0.	-3151.	-16581.	-0.	-3151.	422.	-0.	-2235.	-494.	-0.	-2235.	12528.	-0.	0.	13115.	-0.
80.0	-0.	-19732.	-0.	-3151.	-16581.	-0.	-3151.	422.	-0.	-2235.	-494.	-0.	-2235.	12528.	-0.	0.	13115.	-0.
70.0	-0.	-19732.	-0.	-3151.	-16581.	-0.	-3151.	422.	-0.	-2235.	-494.	-0.	-2235.	12528.	-0.	0.	13115.	-0.
60.0	-0.	-19732.	-0.	-3151.	-16581.	-0.	-3151.	422.	-0.	-2235.	-494.	-0.	-2235.	12528.	-0.	0.	13115.	-0.
50.0	-0.	-19732.	-0.	-3151.	-16581.	-0.	-3151.	422.	-0.	-2235.	-494.	-0.	-2235.	12528.	-0.	0.	13115.	-0.
40.0	-0.	-19732.	-0.	-3151.	-16581.	-0.	-3151.	422.	-0.	-2235.	-494.	-0.	-2235.	12528.	-0.	0.	13115.	-0.
30.0	-0.	-19732.	-0.	-3151.	-16581.	-0.	-3151.	422.	-0.	-2235.	-494.	-0.	-2235.	12528.	-0.	0.	13115.	-0.
20.0	-0.	-19732.	-0.	-3151.	-16581.	-0.	-3151.	422.	-0.	-2235.	-494.	-0.	-2235.	12528.	-0.	0.	13115.	-0.
10.0	-0.	-19732.	-0.	-3151.	-16581.	-0.	-3151.	422.	-0.	-2235.	-494.	-0.	-2235.	12528.	-0.	0.	13115.	-0.
0.	-0.	-19732.	-0.	-3151.	-16581.	-0.	-3151.	422.	-0.	-2235.	-494.	-0.	-2235.	12528.	-0.	0.	13115.	-0.

ANGLE (DEG)	RRAP	TTAP	RTAP	RPRM	TTBM	RTBM	RRBP	TTBP	RTBP	RRCH	TTCH	RTCH	RRCP	TTCP	RTCP	RRDM	TTDM	RTDM
90.0	-310.	-2631.	-0.	-681.	-2261.	-0.	-6191.	-2261.	-0.	-5183.	-3268.	-0.	-3487.	-2433.	-0.	-3394.	-2458.	-0.
80.0	-310.	-2631.	-0.	-681.	-2261.	-0.	-6191.	-2261.	-0.	-5183.	-3268.	-0.	-3487.	-2433.	-0.	-3394.	-2458.	-0.
70.0	-310.	-2631.	-0.	-681.	-2261.	-0.	-6191.	-2261.	-0.	-5183.	-3268.	-0.	-3487.	-2433.	-0.	-3394.	-2458.	-0.
60.0	-310.	-2631.	-0.	-681.	-2261.	-0.	-6191.	-2261.	-0.	-5183.	-3268.	-0.	-3487.	-2433.	-0.	-3394.	-2458.	-0.
50.0	-310.	-2631.	-0.	-681.	-2261.	-0.	-6191.	-2261.	-0.	-5183.	-3268.	-0.	-3487.	-2433.	-0.	-3394.	-2458.	-0.
40.0	-310.	-2631.	-0.	-681.	-2261.	-0.	-6191.	-2261.	-0.	-5183.	-3268.	-0.	-3487.	-2433.	-0.	-3394.	-2458.	-0.
30.0	-310.	-2631.	-0.	-681.	-2261.	-0.	-6191.	-2261.	-0.	-5183.	-3268.	-0.	-3487.	-2433.	-0.	-3394.	-2458.	-0.
20.0	-310.	-2631.	-0.	-681.	-2261.	-0.	-6191.	-2261.	-0.	-5183.	-3268.	-0.	-3487.	-2433.	-0.	-3394.	-2458.	-0.
10.0	-310.	-2631.	-0.	-681.	-2261.	-0.	-6191.	-2261.	-0.	-5183.	-3268.	-0.	-3487.	-2433.	-0.	-3394.	-2458.	-0.
0.	-310.	-2631.	-0.	-681.	-2261.	-0.	-6191.	-2261.	-0.	-5183.	-3268.	-0.	-3487.	-2433.	-0.	-3394.	-2458.	-0.

ANGLE (DEG)	LONG.	STRESS (LB/IN/IN) AND STRAIN (MUIN/IN)						SQRT(3*J2) IN (KLB/IN/IN)						TRANSVERSE DISPLACEMENT (IN)							
	ZZAP	ZZBP	ZZCP	ZZCM	ZZCP	ZZDM	EZZ	YAP	YBM	YBP	YCM	YCP	YDM	URA	UTA	UPB	UTB	UPC	UTC	URD	UTD
90.0	-20759.	-20759.	-25.	-25.	8052.	8992.	-2752.	20.3	15.9	3.4	2.0	13.1	11.6	-.0033	.0000	-.0034	.0000	-.0070	.0000	-.0083	.0033
80.0	-20759.	-20759.	-25.	-25.	8052.	8992.	-2752.	20.3	15.9	3.4	2.0	13.1	11.6	-.0033	.0000	-.0034	.0000	-.0070	.0000	-.0083	.0029
70.0	-20759.	-20759.	-25.	-25.	8052.	8992.	-2752.	20.3	15.9	3.4	2.0	13.1	11.6	-.0033	.0000	-.0034	.0000	-.0070	.0000	-.0083	.0026
60.0	-20759.	-20759.	-25.	-25.	8052.	8992.	-2752.	20.3	15.9	3.4	2.0	13.1	11.6	-.0033	.0000	-.0034	.0000	-.0070	.0000	-.0083	.0023
50.0	-20759.	-20759.	-25.	-25.	8052.	8992.	-2752.	20.3	15.9	3.4	2.0	13.1	11.6	-.0033	.0000	-.0034	.0000	-.0070	.0000	-.0083	.0020
40.0	-20759.	-20759.	-25.	-25.	8052.	8992.	-2752.	20.3	15.9	3.4	2.0	13.1	11.6	-.0033	.0000	-.0034	.0000	-.0070	.0000	-.0083	.0017
30.0	-20759.	-20759.	-25.	-25.	8052.	8992.	-2752.	20.3	15.9	3.4	2.0	13.1	11.6	-.0033	.0000	-.0034	.0000	-.0070	.0000	-.0083	.0014
20.0	-20759.	-20759.	-25.	-25.	8052.	8992.	-2752.	20.3	15.9	3.4	2.0	13.1	11.6	-.0033	.0000	-.0034	.0000	-.0070	.0000	-.0083	.0011
10.0	-20759.	-20759.	-25.	-25.	8052.	8992.	-2752.	20.3	15.9	3.4	2.0	13.1	11.6	-.0033	.0000	-.0034	.0000	-.0070	.0000	-.0083	.0008
0.0	-20759.	-20759.	-25.	-25.	8052.	8992.	-2752.	20.3	15.9	3.4	2.0	13.1	11.6	-.0033	.0000	-.0034	.0000	-.0070	.0000	-.0083	.0004

Table 3. Pre-Stress, Cool Down, No Field

# ELASTIC STRESS AND STRAIN IN DOUBLER MODEL---ELAST3

CENTRAL MAGNETIC FIELD(KG)	=	45.0000	INNER BOPE TUBE RADIUS(IN)	=	1.2375	INNER CONDUCTOR RADIUS(IN)	=	1.5000
OUTER CONDUCTOR RADIUS(IN)	=	2.1490	OUTER RADIUS OF BANDING(IN)	=	2.5240	RADIUS OF IPON SHIELD(IN)	=	3.7500
BOPE YOUNGS MODULUS(LBS/IN/IN)	=	11330000.	COND. YOUNGS MODULUS(LBS/IN/IN)	=	1000000.	BAND YOUNGS MODULUS(LBS/IN/IN)	=	18670000.
BOPE POISSONS RATIO	=	.3333	COND. POISSONS RATIO	=	.1030	BAND POISSONS RATIO	=	.3330
BOPE THERMAL STRAIN(IN/IN)	=	-1.0115	COND. THERMAL STRAIN(IN/IN)	=	-1.0030	BAND THERMAL STRAIN(IN/IN)	=	-1.0030
REFERENCE RADIUS(IN)	=	1.0000	MAGNETIC ENERGY((IN-LB)/IN)	=	17344.27	MAGNETIC ENERGY(J/M)	=	77260.85
BOPE YIELD STRESS(LB/IN/IN)	=	120000.	COND. YIELD STRESS(LB/IN/IN)	=	35000.	BAND YIELD STRESS(LB/IN/IN)	=	20000.
BAND DISLOCATION(MILLIRAD)	=	5.251	INT. (SRR+STT+SZZ)DA((IN-LB)/IN)	=	25028.	INTEGRAL OF SZZ*DA((IN-LB)/IN)	=	17344.
BOPE ELAST. ENERGY((IN-LB)/IN)	=	-458.05	COND. ELAST. ENERGY((IN-LB)/IN)	=	-216.73	BAND ELAST. ENERGY((IN-LB)/IN)	=	-8268.53
DISPL. DIPOL WITH NO SHIELD(KG)	=	-.040597	SHLD CONTR. TO DISPL. DIPOL(KG)	=	-.011997	DIPOL DELTAB/B AT REF. RADIUS	=	-.001524
DISPL. SEXT. WITH NO SHIELD(KG)	=	-.020083	SHLD CONTR. TO DISPL. SEXT.(KG)	=	.000665	SEXTU. DELTAB/B AT REF. RADIUS	=	-.000432

ANGLE (DFG)	RRAP	TTAP	RTAP	RRBM	TTBM	RTBM	RRBP	TRANSVERSE STRESS (LB/IN/IN)				RTCH	RRCP	TTCP	RTCP	RRDM	TTDM	RTDM			
								TTBP	RTBP	RRCM	TTCH										
90.0	-0.	1353.	-0.	-1939.	-13761.	0.	-1939.	707.	0.	-2063.	-153.	0.	-2063.	19837.	0.	0.	6612.	-0.			
80.0	-0.	217.	0.	-1991.	-13818.	-963.	-2027.	695.	-588.	-2082.	-173.	-341.	-2081.	19446.	-115.	0.	7045.	0.			
70.0	-0.	-3855.	0.	-2139.	-13980.	-1909.	-2279.	659.	-1104.	-2140.	-233.	-641.	-2134.	18319.	-216.	0.	8295.	0.			
60.0	-0.	-8069.	0.	-2366.	-14230.	-2438.	-2666.	606.	-1488.	-2228.	-324.	-863.	-2214.	16593.	-291.	0.	10209.	0.			
50.0	-0.	-14218.	0.	-2645.	-14535.	-2772.	-3140.	540.	-1692.	-2336.	-435.	-982.	-2313.	14476.	-330.	0.	12556.	0.			
40.0	0.	-27763.	0.	-2941.	-14861.	-2772.	-3644.	470.	-1692.	-2450.	-554.	-982.	-2419.	12223.	-330.	0.	15055.	0.			
30.0	0.	-26913.	0.	-3220.	-15167.	-2438.	-4119.	404.	-1488.	-2558.	-665.	-863.	-2518.	10106.	-291.	0.	17403.	0.			
20.0	0.	-31927.	0.	-3447.	-15416.	-1909.	-4505.	350.	-1105.	-2646.	-756.	-641.	-2598.	8380.	-216.	0.	19317.	0.			
10.0	0.	-35199.	0.	-3596.	-15578.	-963.	-4757.	315.	-588.	-2703.	-815.	-341.	-2651.	7254.	-115.	0.	20566.	0.			
0	0.	-36335.	0.	-3647.	-15635.	0.	-4845.	303.	0.	-2723.	-836.	0.	-2669.	6862.	0.	0.	21000.	0.			
ANGLE (DFG)	RRAP	TTAP	RTAP	RRBM	TTBM	RTBM	RRBP	TRANSVERSE STRAIN (MUIN/IN)				RTCH	RRCP	TTCP	RTCP	RRDM	TTDM	RTDM			
								TTBP	RTBP	RRCM	TTCH										
90.0	-1172.	-1012.	-0.	-732.	-2122.	0.	-5033.	-2122.	0.	-5061.	-2960.	0.	-3688.	-2124.	0.	-3275.	-283.	-0.			
80.0	-1127.	-1102.	0.	-733.	-2125.	-113.	-5118.	-2125.	-646.	-5078.	-2978.	-375.	-3679.	-2143.	-8.	-3286.	-2783.	0.			
70.0	-999.	-1358.	0.	-739.	-2132.	-213.	-5364.	-2132.	-1215.	-5128.	-3030.	-705.	-3655.	-2195.	-15.	-3315.	-2723.	0.			
60.0	-802.	-1752.	0.	-747.	-2143.	-287.	-5741.	-2143.	-1637.	-5205.	-3111.	-950.	-3618.	-2275.	-21.	-3361.	-2632.	0.			
50.0	-552.	-2234.	0.	-757.	-2156.	-326.	-6203.	-2156.	-1861.	-5300.	-3209.	-1080.	-3572.	-2374.	-24.	-3417.	-2520.	0.			
40.0	-305.	-2748.	0.	-767.	-2170.	-326.	-6695.	-2170.	-1861.	-5480.	-3314.	-1080.	-3524.	-2478.	-24.	-3476.	-2401.	0.			
30.0	-54.	-3231.	0.	-777.	-2183.	-287.	-7157.	-2183.	-1637.	-5495.	-3412.	-950.	-3478.	-2577.	-21.	-3532.	-2289.	0.			
20.0	132.	-3624.	0.	-785.	-2193.	-213.	-7534.	-2193.	-1215.	-5572.	-3493.	-705.	-3441.	-2657.	-15.	-3577.	-2198.	0.			
10.0	250.	-3881.	0.	-790.	-2200.	-113.	-7789.	-2200.	-646.	-5622.	-3545.	-375.	-3417.	-2710.	-8.	-3607.	-2139.	0.			
0	305.	-3970.	0.	-792.	-2203.	0.	-7865.	-2203.	0.	-5640.	-3563.	0.	-3408.	-2728.	0.	-3617.	-2118.	0.			
ANGLE (DFG)	LONG.	STRESS (LB/IN/IN) AND STRAIN (MUIN/IN)						SQRT(3*J2) IN (KL9/IN/IN)						TRANSVERSE DISPLACEMENT (IN)							
	ZZAP	ZZBP	ZZCP	ZZDM	ZZEP	ZZFM	ZZG	YAP	YBP	YCP	YDM	YEP	YFM	URA	UTA	URB	UTB	URC	UTC	URD	UTD
90.0	-12527.	-18206.	231.	133.	12538.	8821.	-2645.	13.3	14.6	2.4	2.1	19.3	7.9	-.0067	.0000	-.0070	.0000	-.0102	.0000	-.0115	.0033
80.0	-12306.	-18242.	221.	129.	12432.	8965.	-2645.	13.0	14.6	2.7	2.2	19.3	8.2	-.0065	.0009	-.0067	.0006	-.0101	.0007	-.0113	.0025
70.0	-13995.	-18346.	193.	117.	12009.	9381.	-2645.	12.7	14.9	3.3	2.4	18.1	8.9	-.0059	.0018	-.0061	.0012	-.0095	.0012	-.0108	.0018
60.0	-15566.	-18504.	149.	99.	11408.	10019.	-2645.	13.6	15.1	4.0	2.6	16.8	10.1	-.0049	.0024	-.0051	.0016	-.0086	.0017	-.0099	.0012
50.0	-17713.	-18699.	95.	77.	10570.	10801.	-2645.	16.2	15.2	4.4	3.5	15.3	11.8	-.0037	.0027	-.0039	.0019	-.0076	.0019	-.0088	.0007
40.0	-19832.	-18900.	37.	54.	9884.	11633.	-2645.	20.3	15.2	4.9	3.8	13.6	13.7	-.0025	.0027	-.0026	.0019	-.0064	.0019	-.0077	.0003
30.0	-21947.	-19101.	-17.	32.	9146.	12414.	-2645.	24.8	14.9	5.0	3.8	12.2	15.5	-.0013	.0024	-.0014	.0016	-.0054	.0017	-.0067	.0001
20.0	-23510.	-19259.	-61.	14.	8545.	13052.	-2645.	28.7	14.6	5.0	2.6	11.1	17.1	-.0003	.0018	-.0004	.0012	-.0045	.0012	-.0058	.0000
10.0	-24599.	-19363.	-90.	3.	8152.	13468.	-2645.	31.3	14.4	5.0	2.5	10.4	18.1	-.0003	.0009	-.0003	.0006	-.0040	.0007	-.0053	.0000
0	-25078.	-19396.	-100.	-1.	8016.	13612.	-2645.	32.2	14.2	5.0	2.4	10.2	18.5	-.0006	.0000	-.0005	.0000	-.0038	.0000	-.0051	.0000

FN-22  
1620

Table 4. Pre-Stress, Cool Down, 45 kG Field