national accelerator laboratory

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INFINITE TRANSITION-ENERGY LATTICE USING π-STRAIGHT SECTIONS

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It is generally known that with reverse bending one can push γ_t (transition γ) to ∞ in a lattice (V.V. Vladimirski and E.K. Tarasov, "Theoretical Problems of the Ring Accelerators," Moscow, Academy of Sciences, 1955). We want to show here that the same can be accomplished by judicious insertion of the π-straight sections (A.A. Garren, p. 22, Proceedings of the V International Conference on High Energy Accelerators, Frascati, 1965). This effect was noticed, for example, in the lattice of the 500 MeV booster proposed for the Argonne ZGS (Argonne National Laboratory Accelerator Division Proposal, June 1969).

I. GEOMETRICAL OBSERVATION

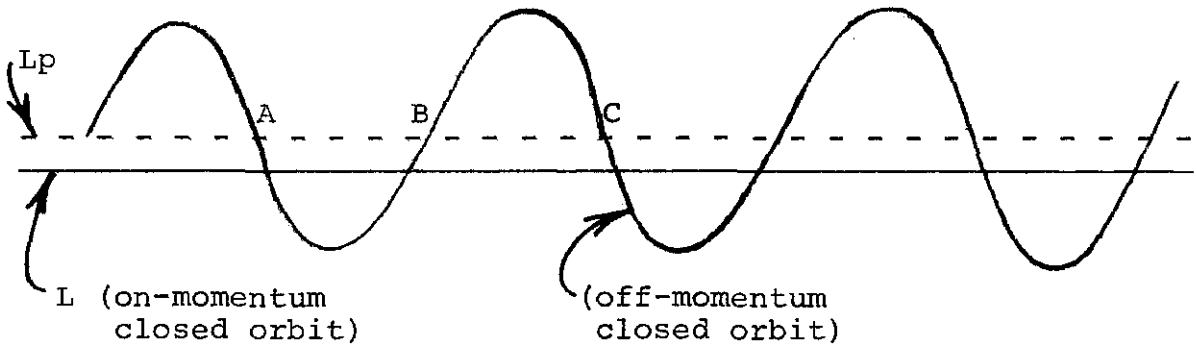

Figure 1

In an AG lattice if the on-momentum closed orbit is shown stretched out as straight line L (Fig. 1), the off-momentum orbit is oscillatory about line L. Because of the curvature of L (say, downward) an inward-swing section such as AB of the off-momentum orbit has a negative first-order length increment and an outward-swing section such as BC has a positive first-order length increment. The net orbit length increment is generally positive but rather small and corresponds roughly to that of the mean orbit shown as dotted straight line Lp. An AG lattice, therefore, generally has a small dispersion orbit-length increment, hence a large momentum compaction.

The concept of reducing (to zero) the dispersion orbit-length increment by introducing reverse bending is as follows: If the curvature of the on-momentum orbit L is reversed over the section BC the first-order orbit-length increment over BC will also reverse sign and become negative. The combined

orbit-length increment over AB and BC will, then, have a large excess negative value. By introducing a few such reverse bends it is possible to totally compensate the small net positive orbit-length increment corresponding to that of Lp and obtain a zero total dispersion orbit-length increment, hence ∞ momentum compaction and infinite γ_t.

An interesting variance of this concept is the following: Instead of a reverse bend over BC one can make the orbit curvature zero, i.e., make section BC into a π-straight section. In this case the first-order orbit-length increment over BC is zero and the combined increment over AB and BC still has a large excess negative value although it is only roughly half that of the case with the reverse bend. Zero total dispersion orbit-length increment and infinite γ_t can similarly be attained. Compared to the case with reverse bending the π-insertion lattice has the following practical advantages:

(a) No reverse bending magnet is required.

(b) Long straight sections in a lattice are needed anyway for injection, extraction, rf cavities, etc.

It is clear from the discussion that any matched long straight section with a large phase advance close to π can be used effectively over section BC. But π-straight section is the simplest and can be analyzed most easily.

II. DISPERSION FORMALISM

We will write the 3 x 3 horizontal optics-dispersion transfer matrix as

$$\begin{pmatrix} a & b & e \\ c & d & f \\ 0 & 0 & 1 \end{pmatrix} \equiv (M|D) \tag{1}$$

where

$$\begin{cases} M = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = 2 \times 2 \text{ optics transfer matrix} \\ D = \begin{pmatrix} e \\ f \end{pmatrix} = \text{dispersion transfer vector.} \end{cases}$$

The following rules of multiplication are obvious:

(a) If a dispersion vector $\begin{pmatrix} \xi \\ \xi' \\ 1 \end{pmatrix}$ is written as

$$\Xi = \begin{pmatrix} \xi \\ \xi' \end{pmatrix}$$

then

$$(M|D)\Xi = M\Xi + D. \tag{2}$$

(b) $$\left(M_2|D_2\right)\left(M_1|D_1\right) = \left(M_2M_1|M_2D_1+D_2\right) \tag{3}$$

(c) $$(M|D)^n = \left(M^n|\left(1-M^n\right)(1-M)^{-1}D\right). \tag{4}$$

(d) If $(M|D)$ is across a periodicity cell and $X = \begin{pmatrix} x \\ x' \end{pmatrix}$ is the periodic dispersion vector (this is conventionally written as $\begin{pmatrix} x_p \\ x'_p \end{pmatrix}$, but since in this paper we will deal

only with the dispersion vector and never with the free oscillation vector we can, without confusion, drop the subscript p), then

$$(M|D)X = MX+D = X$$

and

$$D = (1-M)X. \tag{5}$$

(e) For the π-straight section the optics-dispersion transfer matrix is $(-1|0)$.

III. EFFECT OF π-STRAIGHT SECTION INSERTION

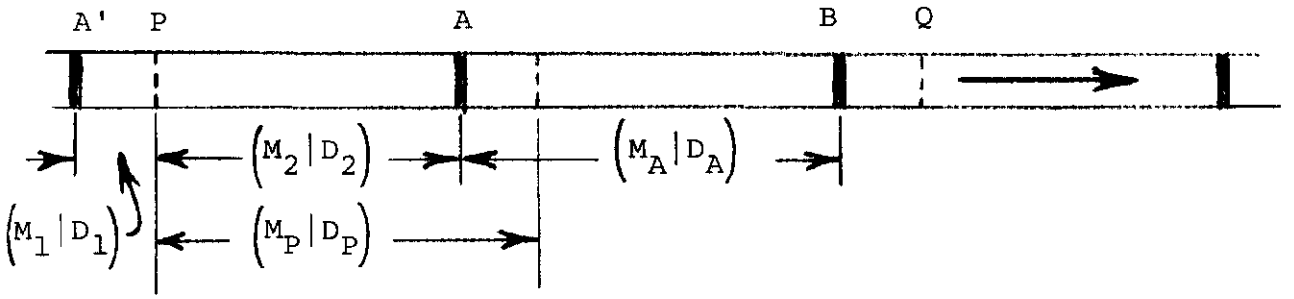

Figure 2

We start with a lattice without π-insertion (Figure 2). The distance between A and B is one repetition length which may contain many cells. We need the following relations between the optics-dispersion transfer matrices defined in Fig. 2. From A to B we have

$$(M_A|D_A) = (M_2|D_2)(M_1|D_1) = (M_2M_1|M_2D_1+D_2)$$

which gives

$$M_2D_1+D_2 = D_A = \left(1-M_A\right) X_A \ . \tag{6}$$

From P to Q we have

$$\left(M_P|D_P\right)^2 = \left(M_1|D_1\right)\left(M_2|D_2\right)\left(M_1|D_1\right)\left(M_2|D_2\right)$$

$$= \left(M_1M_2M_1M_2|M_1M_2M_1D_2+M_1M_2D_1+M_1D_2+D_1\right)$$

which gives

$$M_1M_2M_1D_2+M_1M_2D_1+M_1D_2+D_1 = \left(1-M_P^2\right) X_P \ . \tag{7}$$

Now we insert π-straight sections at A, B, and all other homologous locations. The new transfer matrix from P to Q becomes

$$\left(M_1|D_1\right)\ (-1|0)\ \left(M_2|D_2\right)\left(M_1|D_1\right)\ (-1|0)\ \left(M_2|D_2\right)$$

$$= \left(M_1M_2M_1M_2|M_1M_2M_1D_2-M_1M_2D_1-M_1D_2+D_1\right).$$

Together with Eqs. (6) and (7) this gives for the new periodic dispersion vector at P, $\overline{X}_P = \begin{pmatrix} \overline{x} \\ \overline{x}' \end{pmatrix}_P$,

$$\left(1-M_P^2\right)\overline{X}_P = M_1M_2M_1D_2-M_1M_2D_1-M_1D_2+D_1$$

$$= M_1M_2M_1D_2+M_1M_2D_1+M_1D_2+D_1-2M_1\left(M_2D_1+D_2\right)$$

$$= \left(1-M_P^2\right)X_P-2M_1\left(1-M_A\right)X_A \tag{8}$$

whence, we get

$$\Delta X_P \equiv \overline{X}_P-X_P = -2\left(1-M_P^2\right)^{-1}M_1\left(1-M_A\right)X_A. \tag{9}$$

To simplify the right-hand-side expression we note first

$$\left(1-M_P^2\right)^{-1} = \frac{1-M_P^{-2}}{4\sin^2\mu}$$

where μ is the phase advance across one repetition length (not including the π-insertion). Furthermore,

$$\left(1-M_P^{-2}\right)M_1 = \left(1-M_2^{-1}M_1^{-1}M_2^{-1}M_1^{-1}\right)M_1$$

$$= M_1\left(1-M_1^{-1}M_2^{-1}M_1^{-1}M_2^{-1}\right) = M_1\left(1-M_A^{-2}\right)$$

and

$$\left(1-M_A^{-2}\right)\left(1-M_A\right) = \left(2-M_A-M_A^{-1}\right)\left(1+M_A^{-1}\right)$$

$$= 2(1-\cos\mu)\left(M_A^{1/2}+M_A^{-1/2}\right)M_A^{-1/2}$$

$$= 4\cos\tfrac{\mu}{2}(1-\cos\mu)M_A^{-1/2}$$

where $M_A^{-1/2}$ is the same as M_A except with the phase advance μ replaced by $-\frac{\mu}{2}$. Altogether we have

$$\Delta X_P = -2\left(1-M_P^2\right)^{-1}M_1\left(1-M_A\right)X_A$$

$$= -\frac{1}{2\sin^2\mu}M_1\left(1-M_A^{-2}\right)\left(1-M_A\right)X_A$$

$$= -\frac{1}{\cos\frac{\mu}{2}}M_1M_A^{-1/2}X_A. \qquad (10)$$

M_1 is the matrix from the beginning of the repetition length (point A' in Fig. 1) to P and by definition $M_A^{-1/2}$ reduces the

phase advance in M_1 by $\frac{\mu}{2}$. Therefore, $M_1 M_A^{-1/2}$ has the form

$$M_1 M_A^{-\frac{1}{2}} = \begin{pmatrix} \sqrt{\frac{\beta_P}{\beta_A}}\left(\cos\phi_P+\alpha_A\sin\phi_P\right) & \sqrt{\beta_A\beta_P}\sin\phi_P \\ -\frac{1}{\sqrt{\beta_A\beta_P}}\left[\left(\alpha_P-\alpha_A\right)\cos\phi_P+\left(1+\alpha_A\alpha_P\right)\sin\phi_P\right] & \sqrt{\frac{\beta_A}{\beta_P}}\left(\cos\phi_P-\alpha_P\sin\phi_P\right) \end{pmatrix} \quad (11)$$

where

$$\phi_P = (\text{phase advance from A' to P}) - \frac{\mu}{2}$$

and where all betatron oscillation functions α_A, β_A, α_P, β_P and the phases are values <u>before</u> the insertion of the π-straight sections. Of course, Eq. (11) can be derived also by straightforward computation. We are only interested in the first row Δx_P of the vector Eq. (10) and since P is the running variable point within the repetition length we can drop the subscript P. This gives

$$\Delta x = \bar{x}-x = -\frac{\sqrt{\beta}}{\cos\frac{\mu}{2}}\left[(\cos\phi+\alpha_A\sin\phi)\frac{x_A}{\sqrt{\beta_A}} + \sin\phi\sqrt{\beta_A}x_A'\right] \quad (12)$$

with

ϕ = (phase measured from beginning of repetition length) $-\frac{\mu}{2}$

= (phase measured from "middle" of repetition length).

Eq. (12) can be derived in different ways, e.g., by matching the solutions of the orbit equations of the orbit equation with inversions at points A, B, etc. to form the closed orbit. The derivation given above seems to be the shortest and most lucid.

The new dispersion orbit-length increment per repetition length with the π-insertions is (The orbit-length increment in

the π-straight section is zero.)

$$\int_{-\mu/2}^{\mu/2} \frac{\bar{x}}{\rho}\,\beta d\phi = \int_{-\mu/2}^{\mu/2} \frac{x}{\rho}\,\beta d\phi + \int_{-\mu/2}^{\mu/2} \frac{\Delta x}{\rho}\,\beta d\phi$$

$$= \int_{-\mu/2}^{\mu/2} \frac{x}{\rho}\,\beta d\phi - \frac{1}{\cos\frac{\mu}{2}} \left[(C+\alpha_A S) \frac{x_A}{\sqrt{\beta_A}} + S\sqrt{\beta_A}\, x_A' \right] \qquad (13)$$

where

$$\begin{cases} C \equiv \int_{-\mu/2}^{\mu/2} \frac{\beta^{3/2}}{\rho} \cos\phi\, d\phi \\ S \equiv \int_{-\mu/2}^{\mu/2} \frac{\beta^{3/2}}{\rho} \sin\phi\, d\phi \end{cases}$$

and $\rho = \rho(\phi)$ is the radius of curvature of the on-momentum orbit. Now we see that because of the denominator $\cos\frac{\mu}{2}$, by judicious choice of μ we can adjust the second term on the right-hand-side of Eq. (13) to cancel the first term to give zero dispersion orbit-length increment or infinite γ_t.

IV. CRUDE ESTIMATE AND EXAMPLES

To give a simple example let us assume that the π-straight sections are inserted at locations where $\alpha_A = 0$, hence $x_A' = 0$. The condition for zero dispersion orbit-length increment Eq. (13) then gives

$$\int_{-\mu/2}^{\mu/2} \frac{x}{\rho}\,\beta d\phi = \frac{C}{\cos\frac{\mu}{2}}\,\frac{x_A}{\sqrt{\beta_A}} \tag{14}$$

Now, for a very crude estimate we have

$$C = \int_{-\mu/2}^{\mu/2} \frac{\beta^{3/2}}{\rho}\cos\Phi\, d\phi = \left\langle \frac{\beta^{3/2}}{\rho} \right\rangle \int_{-\mu/2}^{\mu/2} \cos\phi\, d\phi = 2\left\langle \frac{\beta^{3/2}}{\rho} \right\rangle \sin\frac{\mu}{2} \tag{15}$$

where $\langle\ \rangle$ denotes some kind of average value in the periodicity length from $-\frac{\mu}{2}$ to $\frac{\mu}{2}$. Also

$$\int_{-\mu/2}^{\mu/2} \frac{x}{\rho}\,\beta d\phi = \left\langle \frac{\beta^{3/2}}{\rho} \right\rangle \left\langle \frac{x}{\sqrt{\beta}} \right\rangle \int_{-\mu/2}^{\mu/2} d\phi = \left\langle \frac{\beta^{3/2}}{\rho} \right\rangle \left\langle \frac{x}{\sqrt{\beta}} \right\rangle \mu \tag{16}$$

where $\langle\ \rangle$ again denotes some kind of average. Assuming $\left\langle \frac{\beta^{3/2}}{\rho} \right\rangle$ in Eq. (15) to be roughly equal to that in Eq. (16) and since $\frac{x}{\sqrt{\beta}}$ is more or less constant in a normal lattice, hence $\left\langle \frac{x}{\sqrt{\beta}} \right\rangle \cong \frac{x_A}{\sqrt{\beta_A}}$ we obtain by substituting Eqs. (15) and (16) in Eq. (14)

$$\frac{\mu}{2} \cong \tan\frac{\mu}{2}. \tag{17}$$

Let

$$\mu = (2n-1)\pi - 2\pi\varepsilon \qquad n = \text{integer}.$$

We have

$$\tan\frac{\mu}{2} = \cot\varepsilon\pi$$

and

$$2\pi\nu = N(\mu+\pi) = 2\pi nN - 2\pi\varepsilon N$$

or

$$\nu = nN - \varepsilon N \tag{18}$$

where N is the number of symmetrically inserted π-straight sections and ν is the betatron oscillation wave number including the π-insertions. In the first place ν must have a value (integer) $\pm \frac{1}{4}$. Setting $\varepsilon N = \frac{1}{4}$ we get

$$\varepsilon = \frac{1}{4N} \tag{19}$$

and that the integral part of ν must be a multiple of N. Solving Eq. (18) for n in terms of ν and inserting into μ we get

$$\mu = 2\pi \left(\frac{\nu}{N} - \frac{1}{2}\right). \tag{20}$$

Substituting Eqs. (19) and (20) in Eq. (17) we get

$$\nu \cong N\left(\frac{1}{2} + \frac{1}{\pi} \cot \frac{\pi}{4N}\right). \tag{21}$$

This equation gives an approximate upper limit of ν for a given number N of π-insertions. For low values of N we have

N	ν	N	ν
1	0.82	6	17.5
2	2.54	7	23.3
3	5.06	8	29.9
4	8.40	9	37.2
5	12.5	10	45.4

Suppose we use FODO cells similar to those of the NAL main ring and insert 6 (N = 6) π-straight sections to build up an infinite-γ_t lattice for 400 BeV and $\nu \cong 18 - \frac{1}{4}$ (n = 3). There are 15 normal cells in between two π-insertions. At 400 BeV the orbit parameters obtained from computer runs using SYNCH are

orbit radius = 988.0 m

bending magnet field = 19.225 kG

quadrupole field gradient = ±213.0 kG/m

$\nu_x = 17.772$

$\nu_y = 17.818$

$\gamma_t^2 = \begin{cases} -1811 & \text{mid-D insertion} \\ -4778 & \text{mid-F insertion} \end{cases}$

Other parameters are given in the SYNCH output attached. Instead of γ_t which may be imaginary we print out γ_t^2. The two values of γ_t^2 correspond to the cases in which the π-straight sections are inserted in the middle of either the defocusing or the focusing quadrupoles. The large negative γ_t^2 values (instead of ∞) indicate that we have overshot in reducing the dispersion orbit-length increment and the increment is actually slightly negative in both cases (negative momentum compaction). This is presumably because the ν_x values chosen are closer to 18 than $\frac{1}{4}$.

The β-values are rather large in the π-straight sections which is inherent in the design of the π-straight section. A serious drawback of these lattices is the very large local maximum x (x_p or x_{eq}) which makes very stringent demand on the magnet aperture at these locations. This is, of course, inherent in the method employed to compensate for the dispersion orbit-length increase. These large local maximum x can be reduced by increasing the number of π-insertions (say, from 6 to 9).

The modifications of SYNCH to print out γ_t^2 was made by Dr. J. MacLachlan. His assistance is gratefully acknowledged.

SYNCH RUN MROI MAIN RING WITH NEW L S INSERTION FOR HIGH GAMMA-T MID-D INSERTION

*** GE = // 213.
*** GO = // -213.
*** BRHO = // 13373.8188
*** RO = // 19.225214
*** B MAG // 6.0706 0. BRHO RO
*** QF MAG // 1.0668 GF BRHO 0.
*** QD MAG // 1.0558 GD BRHO 0.
*** O ORF // .3048
*** OO ORF // 2.1082
*** OOO ORF // 4.2672
*** OOOO ORF // 29.183356

NORMAL CELL
*** C CYC 22 // QD QO B O B O B O B O QF QF
B // QO B O B O B C B O QD

BETATRON FUNCTIONS THROUGH C

POS	S	PSIX	BETAX	ALPHAX	XEQ	DXEQ	BETAX	PSIY	BETAY	ALPHAY	YEQ	DYEQ	BET
0	0.00000	0.00000	35.3505	-.000	3.23260	.00105	5.94564	0.00000	102.6260	.000	0.00000	0.00000	10.1
1	1.06680 QD	.00477	36.0276	-.638	3.26306	.05615	6.00230	.00166	100.7878	1.713	0.00000	0.00000	10.0
2	3.17500 QO	.01374	38.8932	-.721	3.38143	.05615	6.23644	.00512	93.7934	1.630	0.00000	0.00000	9.6
3	9.24560 B	.03593	49.0846	-.958	3.74877	.06487	7.00604	.01662	75.3782	1.394	0.00000	0.00000	8.6
4	9.55040 O	.03691	49.6722	-.970	3.76854	.06497	7.04785	.01727	74.5319	1.382	0.00000	0.00000	8.6
5	15.62100 B	.05423	62.8877	-1.207	4.18885	.07360	7.93018	.03184	59.1817	1.146	0.00000	0.00000	7.6
6	15.92580 O	.05500	63.6272	-1.219	4.21128	.07360	7.97667	.03266	58.4867	1.134	0.00000	0.00000	7.6
7	21.99640 B	.06856	78.8659	-1.456	4.68457	.08233	8.93683	.05130	46.1534	.897	0.00000	0.00000	5.7
8	22.30120 O	.06917	80.7582	-1.468	4.70966	.08233	8.98656	.05236	45.6100	.885	0.00000	0.00000	5.7
9	28.37180 B	.07993	100.0221	-1.705	5.23593	.09105	10.00111	.07619	36.2975	.648	0.00000	0.00000	4.9
10	28.67660 O	.08041	101.0853	-1.717	5.26368	-.09105	10.05312	.07753	35.9058	.637	0.00000	0.00000	4.9
11	29.74340 QF	.08207	102.9083	-.000	5.31289	.00107	10.14437	.08232	35.2307	.000	0.00000	0.00000	4.9
12	30.81020 QF	.08373	101.0853	1.717	5.26595	-.08894	10.05313	.08711	35.9056	-.636	0.00000	0.00000	4.9
13	32.91840 QO	.08717	93.9986	1.635	5.07845	-.08894	9.69529	.09611	38.7630	-.719	0.00000	0.00000	4.2
14	38.98300 B	.09864	75.5902	1.398	4.56504	-.08021	8.69426	.11837	48.9306	-.956	0.00000	0.00000	4.9
15	39.29380 O	.09929	74.7418	1.386	4.54059	-.08021	8.64534	.11935	49.5169	-.968	0.00000	0.00000	7.0
16	45.36480 B	.11391	59.3576	1.149	4.08015	-.07148	7.70439	.13673	62.7044	-1.204	0.00000	0.00000	7.9
17	45.66920 O	.11454	58.6611	1.137	4.05836	-.07148	7.65305	.13750	63.4423	-1.216	0.00000	0.00000	7.9
18	51.73980 B	.13122	46.3013	.899	3.65090	-.06276	6.80448	.15113	79.6455	-1.453	0.00000	0.00000	8.9
19	52.04460 O	.13187	45.7563	.888	3.63177	-.06276	6.76434	.15171	80.5346	-1.465	0.00000	0.00000	8.9
20	58.11520 B	.15003	36.4204	.650	3.27729	-.05403	6.03493	.16250	99.7483	-1.700	0.00000	0.00000	9.9
21	58.42000 O	.15937	36.0276	.638	3.26082	-.05403	6.00230	.16298	100.7885	-1.712	0.00000	0.00000	10.0
22	59.48680 QO	.16414	35.3506	.000	3.23260	.00105	5.94564	.16465	102.6260	.000	0.00000	0.00000	10.1

THETA = .0598[illegible]17 QX = .16413848 QY = .16464660
GAMMA-T SQUARE = 20.4271[illegible]3E+01

MAXIMA BETAX(11)= 102.90832138 XEQ(11)= 5.31289263 BETAY(1)= 102.62603720

-13-

FN-232
0402

SUPERPERIOD CONSISTS OF CSTD *15C

*** S CYC 37 6 // 0000 0F 0F 0F 0F 0F 000 0D 0D 0D 0D 0000 0000
• // 0F 0F 0F 0F 000 0D 0D 0D 0000 C C
• // C C C C C C C C C C C
• // C

MID-D INSERTION

BETATRON FUNCTIONS THROUGH S

POS	S	PSIX	BETAX	ALPHAX	XEQ	DXEQ	BETAX	PSIY	BETAY	ALPHAY	YEQ	DYEQ	PET
0	0.00000	0.00000	35.3506	.000	-.30970	.76391	5.94564	0.00000	102.6260	.000	0.00000	0.00000	10.1
1	29.18336 0000	.10994	59.4426	-.826	21.98382	.76391	7.70991	.04913	110.9155	-.284	0.00000	0.00000	10.5
2	30.25016 0F	.11267	60.1439	.172	22.59737	.38461	7.75525	.04561	113.5630	-2.212	0.00000	0.00000	10.6
3	31.31696 0F	.11552	58.7167	1.158	22.80195	-.00165	7.66268	.04707	120.4710	-4.302	0.00000	0.00000	10.9
4	32.38376 0F	.11849	55.2640	2.059	22.59185	-.38789	7.43397	.04842	132.1435	-6.706	0.00000	0.00000	11.4
5	33.45056 0F	.12171	50.0345	2.813	21.97685	-.76710	7.07351	.04963	149.4313	-9.598	0.00000	0.00000	12.2
6	37.71776 000	.13949	29.2714	2.053	18.70348	-.76710	5.41031	.05320	242.5932	-12.257	0.00000	0.00000	15.5
7	38.78456 0D	.14572	25.5761	1.432	18.05243	-.45532	5.05729	.05387	264.8631	-8.399	0.00000	0.00000	15.2
8	39.85136 0D	.15273	23.0868	.916	17.72908	-.15180	4.80487	.05449	278.1003	-3.935	0.00000	0.00000	16.6
9	40.91816 0D	.16035	21.6217	.466	17.72756	.14896	4.64991	.05509	281.4509	.813	0.00000	0.00000	16.7
10	41.98496 0D	.16833	21.0741	.050	18.04785	.45242	4.59065	.05570	274.6736	5.502	0.00000	0.00000	16.5
11	71.16831 0000	.32419	58.6482	-1.338	31.25098	.45242	7.65821	.09555	50.5097	2.179	0.00000	0.00000	7.1
12	100.35167 0000	.37039	137.2536	-2.726	44.45410	.45242	13.31366	.41268	20.2563	-1.143	0.00000	0.00000	4.5
13	101.41847 0F	.37134	179.8606	.297	44.53302	-.30469	13.41121	.42053	23.2239	-1.656	0.00000	0.00000	4.8
14	102.48527 0F	.37229	176.0005	3.299	43.80597	-1.05629	13.26652	.42729	27.4074	-2.289	0.00000	0.00000	5.2
15	103.55207 0F	.37328	165.9514	5.064	42.28611	-1.78878	12.88221	.43294	33.1119	-3.090	0.00000	0.00000	5.7
16	104.61887 0F	.37436	150.4376	6.391	40.00096	-2.48888	12.26530	.43758	40.7536	-4.116	0.00000	0.00000	6.3
17	105.68607 000	.37928	87.4695	6.365	29.38039	-2.48888	9.35251	.44920	83.9007	-5.995	0.00000	0.00000	9.1
18	109.95287 0D	.38237	75.8621	4.581	26.98389	-2.01078	8.70988	.45109	95.5243	-4.835	0.00000	0.00000	9.7
19	111.01967 0D	.38474	67.6849	3.130	25.07723	-1.56917	8.22709	.45279	104.2832	-3.326	0.00000	0.00000	10.2
20	112.08647 0D	.38737	62.3417	1.908	23.62579	-1.15605	7.89569	.45437	109.5462	-1.578	0.00000	0.00000	10.4
21	113.15327 0D	.39016	59.4426	.826	22.60323	-.76391	7.70391	.45591	110.9341	.285	0.00000	0.00000	10.5
22	142.33662 0000	.50000	35.3506	.000	.30970	-.76391	5.94564	.50000	102.6261	.000	0.00000	0.00000	10.1
23	201.82342 C	.66414	35.3506	-.000	-21.47005	-.32097	5.94564	.66465	102.6260	.000	0.00000	0.00000	10.1
24	261.31022 C	.82828	35.3506	-.000	-19.22367	.43518	5.94564	.82329	102.6260	.000	0.00000	0.00000	10.1
25	320.79702 C	.99242	35.3506	.000	4.86398	.76908	5.94564	.99394	102.6261	.000	0.00000	0.00000	10.1
26	380.28382 C	1.15655	35.3506	-.000	27.36492	.35599	5.94564	1.15859	102.6260	.000	0.00000	0.00000	10.1
27	439.77062 C	1.32069	35.3506	-.000	26.39444	-.40232	5.94564	1.32323	102.6260	.000	0.00000	0.00000	10.1
28	499.25742 C	1.48483	35.3506	.000	2.89643	-.76831	5.94564	1.48788	102.6260	.000	0.00000	0.00000	10.1
29	558.74422 C	1.64897	35.3506	-.000	-20.27461	-.38601	5.94564	1.65253	102.6260	.000	0.00000	0.00000	10.1
30	618.23102 C	1.81311	35.3506	-.000	-20.58223	.37275	5.94564	1.81717	102.6260	.000	0.00000	0.00000	10.1
31	677.71782 C	1.97725	35.3506	.000	2.27278	.76939	5.94564	1.98182	102.6260	.000	0.00000	0.00000	10.1
32	737.20462 C	2.14138	35.3506	-.000	26.06132	.41934	5.94564	2.14647	102.6260	.000	0.00000	0.00000	10.1
33	796.69142 C	2.30552	35.3506	-.000	27.64633	-.33814	5.94564	2.31111	102.6260	.000	0.00000	0.00000	10.1
34	856.17822 C	2.46966	35.3506	.000	5.48622	-.76572	5.94564	2.47576	102.6260	.000	0.00000	0.00000	10.1
35	915.66502 C	2.63380	35.3506	-.000	-18.86580	-.44753	5.94564	2.64041	102.6260	.000	0.00000	0.00000	10.1
36	975.15182 C	2.79794	35.3506	-.000	-21.72462	.30695	5.94564	2.80505	102.6260	.000	0.00000	0.00000	10.1
37	1034.63862 C	2.96208	35.3506	.000	-.30970	.76391	5.94564	2.96970	102.6260	.000	0.00000	0.00000	10.1

THETA = 6.28318522 QX = 17.77246329 QY = 17.81819445

GAMMA-T SQUARE = -18.110907E+02

MAXIMA BETAX(131= 179.86062717 XEQ(131= 44.53301928 BETAY(91= 281.45094544

Insertion

```
*** QF      =          //  213.
*** QD      =          //  -213.
*** BRHO    =          //  17777.9188
*** BQ      =          //  19.225214
*** B       MAG        //  6.0706     0.        BRHO      BQ
*** QF      MAG        //  1.0568     QF        BRHO      0.
*** QD      MAG        //  1.0568     QD        BRHO      0.
*** 0       DRF        //  .7948
*** 00      DRF        //  2.1082
*** 000     DRF        //  4.2672
*** 0000    DRF        //  29.183356

        NORMAL CELL
*** C       CYC   22   //  QF   00   B   0   B   0   B   0   B   0   QD   QD
  *                    //  00   B   0   B   0   B   0   B   0   QF
```

BETATRON FUNCTIONS THROUGH C

POS	S	PSIX	BETAX	ALPHAX	XEQ	DXEQ	BETAX	PSIY	BETAY	ALPHAY	YEQ	DYEQ	BET
0	0.00000	0.00000	102.9083	-.000	5.31289	.00107	10.14437	0.00000	35.2307	.000	0.00000	0.00000	5.9
1	1.05680 QF	.00166	101.0653	1.717	5.26595	-.08894	10.05312	.00479	35.9056	-.636	0.00000	0.00000	5.9
2	3.17500 QD	.00510	93.9586	1.635	5.07845	-.08894	9.69529	.01379	38.7630	-.719	0.00000	0.00000	6.2
3	9.24560 B	.01657	75.5902	1.398	4.56504	-.08021	8.69426	.03604	48.9306	-.956	0.00000	0.00000	6.9
4	9.55240 0	.01722	74.7418	1.386	4.54059	-.08021	8.64534	.03703	49.5169	-.968	0.00000	0.00000	7.0
5	15.67100 B	.03175	59.3576	1.149	4.08015	-.07148	7.70439	.05440	62.7044	-1.204	0.00000	0.00000	7.9
6	15.92587 0	.03257	58.6611	1.137	4.05836	-.07148	7.65905	.05517	63.4423	-1.216	0.00000	0.00000	7.9
7	21.93640 B	.05115	46.3010	.899	3.65090	-.06276	6.80448	.06878	79.6455	-1.453	0.00000	0.00000	8.9
8	22.30120 0	.05220	45.7563	.888	3.63177	-.06276	6.76434	.06939	80.5346	-1.465	0.00000	0.00000	8.9
9	28.37180 B	.07596	36.8204	.650	3.27729	-.05403	6.03493	.08018	99.7483	-1.700	0.00000	0.00000	9.9
10	28.67660 0	.07730	36.0276	.638	3.26082	-.05403	6.00230	.08066	100.7885	-1.712	0.00000	0.00000	10.0
11	29.74340 QD	.08207	35.3506	.000	3.23260	.00105	5.94564	.08232	102.6260	.000	0.00000	0.00000	10.1
12	30.81020 QD	.08684	36.0276	-.638	3.26306	.05615	6.00230	.08399	100.7878	1.713	0.00000	0.00000	10.0
13	32.91840 00	.09591	38.8932	-.721	3.38143	.05615	6.23644	.08744	93.7404	1.630	0.00000	0.00000	9.6
14	38.88900 B	.11799	49.0946	-.958	3.74877	.06487	7.00604	.09894	75.3782	1.394	0.00000	0.00000	8.6
15	39.29380 0	.11998	49.6722	-.970	3.76854	.06487	7.04785	.09959	74.5319	1.382	0.00000	0.00000	8.6
16	45.36440 B	.13530	62.8877	-1.207	4.18885	.07360	7.93018	.11416	59.1817	1.146	0.00000	0.00000	7.6
17	45.66920 0	.13706	63.6272	-1.219	4.21128	.07360	7.97667	.11498	58.4867	1.134	0.00000	0.00000	7.6
18	51.77980 B	.15063	79.8669	-1.456	4.68457	.08233	8.93683	.13362	46.1534	.897	0.00000	0.00000	6.7
19	52.04460 0	.15124	80.7582	-1.468	4.70966	.08233	8.99656	.13468	45.6100	.885	0.00000	0.00000	6.7
20	58.11520 B	.16209	100.0221	-1.705	5.23593	.09105	10.00111	.15851	36.2975	.648	0.00000	0.00000	6.0
21	58.42000 0	.16248	101.0653	-1.717	5.26368	.09105	10.05312	.15386	35.9058	.637	0.00000	0.00000	5.9
22	59.48680 QF	.16414	102.9083	.000	5.31289	-.00107	10.14437	.16465	35.2307	.000	0.00000	0.00000	5.9

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          THETA =     .06981717  QX =        .16413848  QY =        .16464660
GAMMA-T SQUARE =  20.427163E+01
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MAXIMA   BETAX( 11)= 102.908332138  XEQ( 11)=   5.31289263  BETAY( 11)= 102.626037271

        STRAIGHT CELL
*** CSTR    CYC   22   //  0000 QF   QF   QF   QF   000  00   00   00   00   0000 0000
  *                    //  QF   QF   QF   QF   000  00   00   00   00   0000
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SUPERPERIOD CONSISTS OF CSTR +15C

*** S CYC 37 6 // 0000 OF OF OF OF 000 OD OD OD OD 0000 0b00 C C

MID-F INSERTION

RETATRON FUNCTIONS THROUGH S

POS	S		PSIX	BETAX	ALPHAX	XFQ	DXEQ	BETAX	PSIY	BETAY	ALPHAY	YEQ	DYEQ	BET
0	0.00000		0.00000	102.9083	.000	-.91806	.43129	10.14437	0.00000	35.2307	.000	0.00000	0.00000	5.9
1	29.14336	0000	.04394	111.1843	-.284	11.66841	.43129	10.54440	.11011	59.4017	-.828	0.00000	0.00000	7.7
2	30.25016	OF	.04551	109.7900	1.583	12.02153	.22973	10.47807	.11293	62.3761	-1.911	0.00000	0.00000	7.8
3	31.31636	OF	.04709	104.5118	3.335	12.15709	.02402	10.22310	.11553	67.6537	-3.132	0.00000	0.00000	8.2
4	32.39376	OF	.04878	95.7303	4.847	12.17262	-.18213	9.78418	.11791	75.8347	-4.583	0.00000	0.00000	8.7
5	33.45055	OF	.05067	84.0781	6.010	11.75966	-.38498	9.16941	.12000	87.4457	-6.367	0.00000	0.00000	9.3
6	37.71776	000	.06227	40.8284	4.126	10.12686	-.38498	6.39971	.12592	150.4347	-8.334	0.00000	0.00000	12.2
7	38.78456	OD	.06590	33.1686	3.098	9.80683	-.21590	5.75922	.12699	165.9558	-8.067	0.00000	0.00000	12.8
8	39.85136	OD	.07255	27.4500	2.295	9.66482	-.05073	5.23927	.12798	176.0126	-3.303	0.00000	0.00000	13.2
9	40.91816	OD	.07929	27.2554	1.660	9.69826	.11352	4.82238	.12893	179.8803	-.301	0.00000	0.00000	13.4
10	81.83995	OD	.08713	20.2789	1.147	9.30775	.27982	4.50321	.12988	177.2801	2.723	0.00000	0.00000	13.3
11	71.15831	0000	.09465	50.5675	-2.184	18.07394	.27982	7.11109	.17603	58.7648	1.338	0.00000	0.00000	7.6
12	100.35167	0000	.44444	275.2799	-5.516	26.24012	.27982	16.59156	.33144	21.1123	-.048	0.00000	0.00000	4.5
13	101.41847	OF	.44505	282.3757	-.816	26.30029	-.16720	16.79511	.33941	21.6544	-.464	0.00000	0.00000	4.6
14	102.48527	OF	.44565	278.7211	3.942	25.88447	-.61119	16.69494	.34703	23.1148	-.914	0.00000	0.00000	4.8
15	103.55207	OF	.44627	265.9577	8.416	25.00319	-1.04412	16.29287	.35403	25.5999	-1.430	0.00000	0.00000	5.0
16	104.61887	OF	.44694	243.2413	12.283	23.66345	-1.45816	15.59619	.36025	29.2911	-2.051	0.00000	0.00000	5.4
17	108.86607	000	.45058	149.7796	9.619	17.44118	-1.45816	12.23945	.37803	50.0304	-2.809	0.00000	0.00000	7.0
18	109.95287	OD	.45171	132.4539	6.720	16.03922	-1.17416	11.50886	.38125	55.2519	-2.056	0.00000	0.00000	7.4
19	111.01967	OD	.45305	120.7569	4.311	14.92841	-.81148	10.98894	.38422	58.6964	-1.154	0.00000	0.00000	7.6
20	112.08647	OD	.45451	113.8353	2.216	14.08960	-.66535	10.66936	.38707	60.1158	-.169	0.00000	0.00000	7.7
21	113.15327	OD	.45602	111.1843	.284	13.50454	-.43129	10.54440	.38990	59.4078	.828	0.00000	0.00000	7.7
22	142.37662	0000	.50000	102.9083	.000	.91806	-.43129	10.14437	.50000	35.2307	.000	0.00000	0.00000	5.9
23	201.82342	C	.66414	102.9083	.000	-35.11868	-.18439	10.14437	.66465	35.2307	.000	0.00000	0.00000	5.9
24	261.31022	C	.82928	102.9083	-.000	-31.83113	.24289	10.14437	.82929	35.2307	.000	0.00000	0.00000	5.9
25	320.79702	C	.99242	102.9083	.000	7.58321	.43497	10.14437	.99394	35.2307	.000	0.00000	0.00000	5.9
26	380.29382	C	1.15855	102.9083	.000	44.78940	.20503	10.14437	1.15859	35.2307	.000	0.00000	0.00000	5.9
27	439.77062	C	1.32069	102.9083	-.000	43.60221	-.22328	10.14437	1.32323	35.2307	.000	0.00000	0.00000	5.9
28	499.25742	C	1.48483	102.9083	.000	5.17225	-.43339	10.14437	1.48788	35.2307	.000	0.00000	0.00000	5.9
29	558.74422	C	1.64897	102.9083	.000	-33.11891	-.22094	10.14437	1.65253	35.2307	.000	0.00000	0.00000	5.9
30	618.23102	C	1.81311	102.9083	-.000	-34.03078	.20744	10.14437	1.81717	35.2307	.000	0.00000	0.00000	5.9
31	677.71782	C	1.97725	102.9083	.000	3.32354	.43510	10.14437	1.98182	35.2307	.000	0.00000	0.00000	5.9
32	737.20462	C	2.14139	102.9083	.000	42.61274	.24061	10.14437	2.14647	35.2307	.000	0.00000	0.00000	5.9
33	796.69142	C	2.30552	102.9083	-.000	45.62360	-.18686	10.14437	2.31111	35.2307	.000	0.00000	0.00000	5.9
34	856.17822	C	2.46966	102.9083	.000	9.42772	-.43155	10.14437	2.47576	35.2307	.000	0.00000	0.00000	5.9
35	915.66502	C	2.63380	102.9083	.000	-30.77030	-.25547	10.14437	2.64041	35.2307	.000	0.00000	0.00000	5.9
36	975.15182	C	2.79794	102.9083	-.000	-35.87331	.17012	10.14437	2.80505	35.2307	.000	0.00000	0.00000	5.9
37	1034.63862	C	2.96208	102.9083	.000	-.91806	.43129	10.14437	2.96970	35.2307	.000	0.00000	0.00000	5.9

THETA = 6.29318522 GX = 17.77246330 GY = 17.61819444

GAMMA-T SQUARE = -47.781238E+02

MAXIMA BETAX(13)= 282.07573930 XEQ(33)= 45.62359546 BETAY(91)= 179.88027938