national accelerator laboratory

FN-226
0402

# MAGNETIC SYSTEM AND ELECTRONIC FEEDBACK SYSTEM TO DAMP TRANSVERSE COHERENT BEAM OSCILLATIONS IN THE NAL MAIN RING

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March 1971

Abstract

A $\nu$-spread of 0.1 in the Main Ring beam is wanted to overcome transverse coherent instabilities expected at full intensity. Computations show the gradients of sextupole or octupole magnets are too large to be realized in practice for this purpose. On the other hand, there is indication that an electronic feedback damper can do the work as only 6 KV driving voltage is required for each centimeter of beam bunch lateral displacement.

# INTRODUCTION

We have seen in a previous paper[1] that the minimum betatron number $\nu$-spread, $\delta$, required to overcome the resistive wall instabilities of a bunched beam circulating in the NAL Main Ring can take a relatively too large value. This depends on the betatron oscillation frequency distribution as one can see in Table I which applies to the most unstable mode, $m = 1092$.[1] In that Table I 2n is the order of the parabola, $\eta$ is a parameter measuring the particle concentration around the center, and $\delta$ the minimum required full spread taken at half of the maximum.[1]

The values shown in Table I correspond to the ideal case of 1113 equally spaced bunches, all with the same geometry and the same number of particles $N = 4.2 \times 10^{10}$. Besides, they apply only in the proximity of the injection where $\gamma$ (the usual relativistic energy factor) is close to 10. On the other hand, it is known[1] that $\delta$ is a linear function of $N/\gamma$.

We only consider the case of a gaussian particle distribution with a required minimum full spread of 0.1. Rough calculations show that the required magnetic field gradient for either octupole or sextupole system to produce this $\nu$-spread in the Main Ring beam is too large to be realized in practice without encountering some difficulties of space and economy.

The second part of the paper, then, is devoted to the theoretical investigation of the feasibility of an electric feedback damper system. The Courant-Sessler equation[2] for the motion of a rigid bunch is again integrated, but this time taking into account the driver electrodes. Calculations for the Main Ring give a driver voltage per unit of bunch lateral displacement reasonably low to encourage undertaking of the construction of this equipment.

## 1. OCTUPOLE AND SEXTUPOLE MAGNETIC DEVICE

### a. Octupole

Let us take $n$ equal octupole magnets distributed around the machine and each of length $\ell$. Let also

$$B = G\, x^3$$

be the field within the magnet gap on the midplane ($x$ can be either the radial or vertical direction).

We can show (see the Appendix) that the $\nu$-spread due to the beam size is

$$\boxed{\delta = \frac{n\ell}{2\pi R}\,\frac{evGa^2}{4m_0\gamma\omega_0^{\,2}\nu_0}} \qquad (1)$$

where

$e$ = particle charge

$v$ = particle longitudinal velocity

$m_0$ = particle rest mass

$\nu_0$ = nominal betatron number

$\omega_0$ = particle angular velocity

R = closed orbit radius

a = beam half-size.

Eq. (1) becomes for the NAL Main Ring

$$G = 25.5 \frac{2\pi R\gamma\delta}{n\ell a^2} \text{ kG/m}^3$$

where a must be given in centimeters. If $\delta$ is matched to the minimum $\nu$-spread for the beam stabilization, $\gamma\delta$ is not dependent on $\gamma$, and in our case is close to 1.

If we take, for example, 6 magnets, each 1 meter long, and we set a = 0.5 cm, then the following gradient is required

$$G = 101{,}600 \text{ kG/m}^3.$$

Obviously this gradient is too large to obtain in practice.

b. Sextupole

It is known that a sextupole field is practically ineffective for pure betatron oscillations, but can give rise to some $\nu$-spread in the beam on both planes for the difference of momentum between particles.

Let us take again n equal magnets each with length $\ell$, but now the field in the gap is

$$B_y = Gx^2 \qquad\qquad B_x = 2Gxy.$$

We can show (see Appendix) that the $\nu$-spread arising from the beam momentum spread in the radial plane is

$$\boxed{\delta = \frac{n\ell}{2\pi R} \frac{evGR\alpha\Delta p/p}{2m_0\gamma\omega_0^{\;2}\nu_0}} \qquad (2)$$

and the double of that in the vertical plane where

$\alpha$ = momentum compaction factor

$\Delta p/p$ = half momentum spread in percent.

Eq. (2) becomes for the NAL Main Ring

$$G = 1.27 \frac{2\pi\gamma\delta}{n\ell\alpha\Delta p/p} \; G/m^2$$

where $\ell$ is to be expressed in meters. Taking again $\gamma\delta \sim 1$, and $\alpha = 1/400$, $n\ell \sim 6m$, $\Delta p/p = 10^{-3}$ the following gradient is required

$$G = 508 \; kG/m^2.$$

This gradient, although still too large, is considerably less than that with octupole magnets.

In the following we shall consider the feasibility of an electronic feedback damper.

## 2. FEEDBACK DAMPER THEORY

The situation is that shown in Fig. 1. The beam is supposed to rotate clockwise. It crosses two pair of electrodes, the first being a pick-up station (p.u.) and the second a pair of kicker electrodes (k.e.).

At the time a particle bunch crosses the pick-up station, the electrodes provide a voltage difference $V_{p.u.}$, which is linear to the lateral displacement Y of the center of mass of the bunch

$$V_{p.u.} = gY$$

g is the sensitivity of the p.u. electrodes.

$V_{p.u.}$ is then amplified by a factor A (with sign) and sent back (counter clockwise) to the kicker electrodes which are at an angle $\Delta(<\pi)$ apart. The electric field within the electrodes is

$$E = gA\frac{Y}{d}$$

where d is the total aperture of the kicker system. Of course E is zero anywhere around the orbit outside the electrodes which we take to have the total length $\ell$. But supposing that the amplitude of Y is a slow function of the time, i.e., that an appreciable change of this occurs after several beam revolutions, we can take for E an average over one turn, namely

$$E = \eta Y, \qquad \eta = \frac{gA\ell}{2\pi Rd} .$$

We want to investigate whether the systems outlined above can damp the coherent transverse oscillations which could be caused by the resistive vacuum chamber wall of the NAL Main Ring. For this purpose we suppose that the particles have all the same revolution frequency $\omega_0/2\pi$ and the same unperturbed betatron number $\nu_0$.

According to Eq. (3.12) of the Courant and Sessler (CS) paper[2] we can write the following equation to describe the motion of the s-th particle in the n-th bunch

$$m_0\gamma\omega_0^2(\nu_0^2-\nu^2)\xi_s\exp[i(\phi_s+\nu\omega_0 t)]$$

$$= eUP(\theta_s+\omega_0 t,t)$$

$$+ (We^2/2\pi R\omega_0^{1/2})\sum_r\xi_r\exp[i(\phi_r+\nu\omega_0 t)]G(\theta_r-\theta_s,\nu)$$

$$+ e\eta Y_n(t-\tau) \tag{3}$$

where all the symbols have the same meaning as in the CS paper, and $\tau$ is the time that the particle takes to go from the p.u. station to the kicker electrodes assembly.

Let us introduce the particle distribution function in the n-th bunch

$$\psi_n(\theta,\xi,\phi) = N(2\pi R/L)D(\xi,\phi)$$

for $\theta_n-L/4\pi R<\theta<\theta_n+L/4\pi R$, and zero elsewhere ($\theta_n$ is the location of the bunch center, and L is the full length). The function D is normalized to unity and equal for all the bunches.

The total dipole moment is

$$Q_n = e\ \exp(i\nu\omega_0 t)\iiint\psi_n(\theta,\xi,\phi)\xi e^{i\phi}d\xi d\phi d\theta$$

$$= NeY_n(t). \tag{4}$$

We multiply both members of Eq. (3) by $e\psi_n$ and perform integration over $\theta$, $\xi$ and $\phi$. Using the same CS's approach we obtain the following linear homogenous system in the $Q_n$'s

$$(NU'-\lambda)Q_n+NW'\sum_m Q_m G_{mm} = 0 \tag{5}$$

$$n = 1,2,\ldots, M,$$

where the symbols have again been defined in the CS paper; but now, taking into account (4),

$$\lambda = m_0\gamma\omega_0^{\ 2}(\nu_0^{\ 2}-\nu^2) - e\eta\ \exp(-i\nu_0\omega_0\tau) \tag{6}$$

$\lambda$ is also known by imposing that the determinant of the system (5) is zero; that gives M eigenvalues, $\lambda_1$, $\lambda_2,\ldots,\lambda_M$, for $\lambda$.

Introducing the complex $\nu$-shift ($\delta\nu = \delta\nu_r + i\delta\nu_i$, $\nu_0+\delta\nu$) and assuming $|\delta| << \nu_0$, we have from Eq. (6),

$$\lambda = \lambda_r + i\lambda_i,$$

$$\delta\nu_r = -\frac{\lambda_r}{2m_0\gamma\omega_0^{\ 2}\nu_0} - \frac{e\eta\ \cos(\nu_0\omega_0\tau)}{2m_0\gamma\omega_0^{\ 2}\nu_0} \tag{7}$$

$$\delta\nu_i = -\frac{\lambda_i}{2m_0\gamma\omega_0^{\ 2}\nu_0} + \frac{e\eta\ \sin(\nu_0\omega_0\tau)}{2m_0\gamma\omega_0^{\ 2}\nu_0} \tag{8}$$

The stability conditions is $\delta\nu_i > 0$, or from (8)

$$\eta\ \sin(\nu_0\omega_0\tau) > (m_0\gamma\omega_0^{\ 2}/e)\ \frac{\lambda_i}{m_0\gamma\omega_0^{\ 2}} \tag{9}$$

and, obviously, $\eta\ \sin(\nu_0\omega_0\tau) > 0$.

The eigenvalues of $\lambda$ have been calculated for the NAL Main Ring in a previous paper.[1] The most unstable mode is m = 1092, for which we have ($\gamma = 10$, $N = 4.2 \times 10^{10}$)

$$\frac{\lambda}{m_0\gamma\omega_0^{\ 2}} = -4.54 + i\ 0.229.$$

For the other modes $\lambda_r$ does not change appreciably, whereas $\lambda_i$ is always less and takes also negative values. Observe that $\lambda_i$ is linear with the number of particles per bunch N and does not depend on $\gamma$. Thus also the condition (9) is linear with N and has no dependence on $\gamma$.

For the NAL Main Ring the condition (9) becomes

$$\boxed{\eta \sin(\nu_0\omega_0\tau) > 0.21 \text{ Volt/cm}^2} \qquad (10)$$

for $N = 4.2 \times 10^{10}$.

To increase the efficiency of the system we want $\sin(\nu_0\omega_0\tau)$ to be very close to $\pm 1$ or

$$\frac{\Delta}{2\pi} = 1 - \frac{1}{\nu_0}\left(K \pm \frac{1}{4}\right), \quad K = \ldots,-2,-1,0,1,2,\ldots \qquad (11)$$

$\Delta/2\pi$ vs. $\nu_0$ has been plotted in Fig. 2 for $20 \le \nu_0 \le 21$ and for several K's.

Let us take

$$|\sin(\nu_0\omega_0\tau)| \sim 1 \qquad (12)$$

$$\ell = 1 \text{ m}$$

$$d = 5 \text{ cm}$$

<u>then we derive from (10) that the kicker electrodes have to be loaded by at least 6.3 kV per each 1 cm of the bunch center of mass lateral displacement</u>.

Also observe that (12) is equivalent to

$$\cos(\nu_0\omega_0\tau) \sim 0$$

and the collective real $\nu$-shift is given only by the first term at the r.h. side of (7).

Taking for the NAL Main Ring

$$\frac{\lambda_r}{m_0\gamma\omega_0^2} = -4.54$$

we obtain

$$\delta\nu_r = 0.11$$

for $N = 4.2 \times 10^{10}$ and $\gamma = 10$.

$\delta\nu_r$ also is linear with N but changes linearly with $\gamma^{-3}$.

3. WHERE TO PLACE THE p.u. AND KICKER ELECTRODES

To increase the efficiency of the damper the pick-up and kicker electrodes have to be placed in sections of the machine exhibiting locally large $\beta$-values.* Indeed there the beam bunch has the maximum of lateral displacement and the p.u. electrodes response is, hence, larger. Besides, to increase the kicking action of the driver electrodes the beam bunch should exhibit the minimum of divergence and this is achieved in a section at low $\gamma$, i.e., again at large $\beta$. That might have the effect of decreasing the factor $\eta$ even for a factor of 2.

Table II is the list of all the drift spaces available in one Main Ring superperiod with a length larger than 35 cm. They are listed starting from the beginning of the Long Straight Section Cell (LC) as they are met moving in the sense of the beam motion (betatron phase advance increasing). NC and MC are the notations, respectively, for the Normal Cell and the Cell with Medium Straight Section. For each drift space the total length, the phase advance $\psi$ and the

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*In this part of the paper $\alpha$, $\beta$ and $\gamma$ are the strong focusing magnetic system functions. Besides we make reference only to the vertical plane.

Twiss parameters $\alpha$ and $\beta$ are given. $\psi$, $\alpha$ and $\beta$ refer to the beginning of the drift space. $\psi$ is normalized in such a way that at the end of a superperiod the phase advance is $\nu_0/6$.

A first selection of the drift elements is made requiring that they are in places with reasonably high $\beta$-values. Taking as criterion $\beta > 80$ m, the selected elements are those marked by an asterisk at the last column of Table II.

The second selection is based upon the request that the pick-up station is a distance apart from the driver station corresponding to the phase advance difference given by Eq. (11)

$$\Delta\psi = \nu_0 \frac{\Delta}{2\pi} = \nu_0 - (K \pm \frac{1}{4}).$$

Let us work with $\nu_0 = 20.25$, thus the minimum of $\Delta\psi$ (apart from the case $\Delta\psi = 0$) is

$$\Delta\psi = 0.50.$$

It results, by inspecting Table II, that a choice can be the following:

1. Place the driver station in the element LC 3. There $\psi$ ranges from 0.1222 up to 0.2378.
2. Place the pick-up station in the element NC 02 2 where $\psi$ ranges only from 0.7099 up to 0.7129.

In this case as the position of the p.u. station is fixed but the driver electrodes can be moved all along the 50 meters of the element LC 3, the damper system can work in the range of $\nu_0$ values

$$20.22 \lesssim \nu_0 \lesssim 20.34.$$

A second choice which leads to a different range for the $\nu_0$ work-point is:

1. Place the p.u. station in the element LC 3.
2. Place the driver station in the element NC 13 2 of the previous superperiod. In this case the range of $\nu_0$ is

$$20.18 \lesssim \nu_0 \lesssim 20.29.$$

Of course, there is also the third choice to have all the damper systems in the same element LC 3 (first the driver and then the p.u. electrodes). In this case $\nu_0$ has the following range

$$20.25 \lesssim \nu_0 \lesssim 20.35.$$

REFERENCES

1. A.G. Ruggiero, "The Transverse Resistive Wall Instabilities in the NAL Main Ring," FN-217 0401 (1970).

2. E.D. Courant and A.M. Sessler, The Rev. of Scient. Instr. 37, No. 11, p. 1579 (1966).

APPENDIX

The equation of motion in the z direction of a particle in the presence of octupole magnets is

$$z'' + \nu_0^2 x + \frac{n\ell}{2\pi R} \frac{evG}{m_0\gamma\omega_0^2} z^3 = 0 \tag{A.1}$$

where the effect of the octupoles has been averaged over one turn. z' is the derivative of z with respect to the angular coordinate $\theta$. z can be either x or y.

Let us try a solution of (A.1) of the form

$$z = a \sin \psi(\theta) \tag{A.2}$$

where the amplitude <u>a</u> is supposed constant and

$$\psi(\theta) = \int \nu(\theta) d\theta$$

with $\nu(\theta) = \nu_0 + \delta(\theta)$ and $|\delta| << \nu_0$.

Inserting (A.2) in (A.1), neglecting the term in $\psi''$ and $\delta^2$ and replacing $\psi(\theta)$ by $\nu_0\theta$ everywhere gives

$$\delta(\theta,a) = \frac{n\ell}{2\pi R} \frac{evGa^2}{2m_0\gamma\omega_0^2 \nu_0} \sin^2\nu_0\theta \tag{A.3}$$

Average of $\delta(\theta,a)$ over one turn gives Eq. (1) for large value of $\nu_0$.

In the presence of sextupole magnets Eq. (A.1) is replaced by the following

$$z'' + \nu_0^2 z \left(1 + \frac{n\ell}{2\pi R} \frac{\varepsilon evGx}{m_0\gamma\omega_0^2 \nu_0^2}\right) = 0 \tag{A.4}$$

where, again, z can be either x or y, and $\varepsilon = 1$ for z = x and $\varepsilon = -2$ for z = y.

The term in brackets is an odd function of x so that it is fully cancelled after one complete symmetric betatron oscillation. Nevertheless, the particles have different orbits on the radial plane as they have also different energies. Thus we can make use of (A.4) replacing x in the sextupole term by the momentum difference, namely

$$x = R\alpha\delta p/p.$$

That leads easily to Eq. (2) assuming $|\delta| << \nu_0$.

TABLE I

Minimum required full spread, $\delta$, for some distribution functions. $\eta$ measures the relative particle concentration around the distribution center (see ref. 1 for more details).

| Distribution Functions | $\eta$ | $\delta$ |
|---|---|---|
| Lorentz | 0.500 | 0.01 |
| Gauss | 0.76 | 0.10 |
| Parabola, n=5 | 0.792 | 0.12 |
| Parabola, n=4 | 0.802 | 0.13 |
| Parabola, n=3 | 0.812 | 0.14 |
| Squared Cosine | 0.818 | 0.15 |
| Parabola, n=2 | 0.835 | 0.17 |
| Truncated Cosine | 0.866 | 0.22 |
| Parabola, n=1 | 0.886 | 0.26 |
| Rectangular | 1.000 | 7.1 (!) |

TABLE II

LIST OF THE STRAIGHT-SECTION ELEMENTS AVAILABLE IN ONE MAIN RING SUPERPERIOD

| No. | Drift Space | Length (m) | Phase Advance, $\psi$ | $\alpha$ | $\beta$(m) | $\beta$>80m |
|---|---|---|---|---|---|---|
| 1 | LC 1 | 7.30 | 0.0000 | -0.211 | 27.39 | |
| 2 | LC 2 | 1.77 | 0.1141 | -5.265 | 83.56 | * |
| 3 | LC 3 | 50.22 | 0.1222 | 1.310 | 122.37 | * |
| 4 | LC 4 | 1.77 | 0.2518 | -3.342 | 57.61 | |
| 5 | LC 5 | 2.92 | 0.2624 | -0.117 | 82.19 | * |
| 6 | LC 6 | 2.11 | 0.3156 | 1.935 | 96.78 | * |
| 7 | NC 01 1 | 1.80 | 0.4185 | -0.577 | 27.24 | |
| 8 | NC 01 2 | 1.80 | 0.5126 | 1.927 | 96.36 | * |
| 9 | NC 02 1 | 1.80 | 0.6159 | -0.580 | 27.19 | |
| 10 | NC 02 2 | 1.80 | 0.7099 | 1.930 | 96.65 | * |
| 11 | MC 1 | 2.00 | 0.8128 | -0.580 | 27.30 | |
| 12 | MC 2 | 12.86 | 0.8239 | -0.678 | 29.82 | |
| 13 | MC 3 | 2.11 | 0.9065 | 1.934 | 96.66 | * |
| 14 | NC 03 1 | 1.80 | 1.0097 | -0.577 | 27.20 | |
| 15 | NC 03 2 | 1.80 | 1.1038 | 1.926 | 96.39 | * |
| 16 | NC 04 1 | 1.80 | 1.2070 | -0.581 | 27.24 | |
| 17 | NC 04 2 | 1.80 | 1.3008 | 1.933 | 96.73 | * |
| 18 | NC 05 1 | 1.80 | 1.4038 | -0.579 | 27.27 | |
| 19 | NC 05 2 | 1.80 | 1.4977 | 1.930 | 96.46 | * |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | |

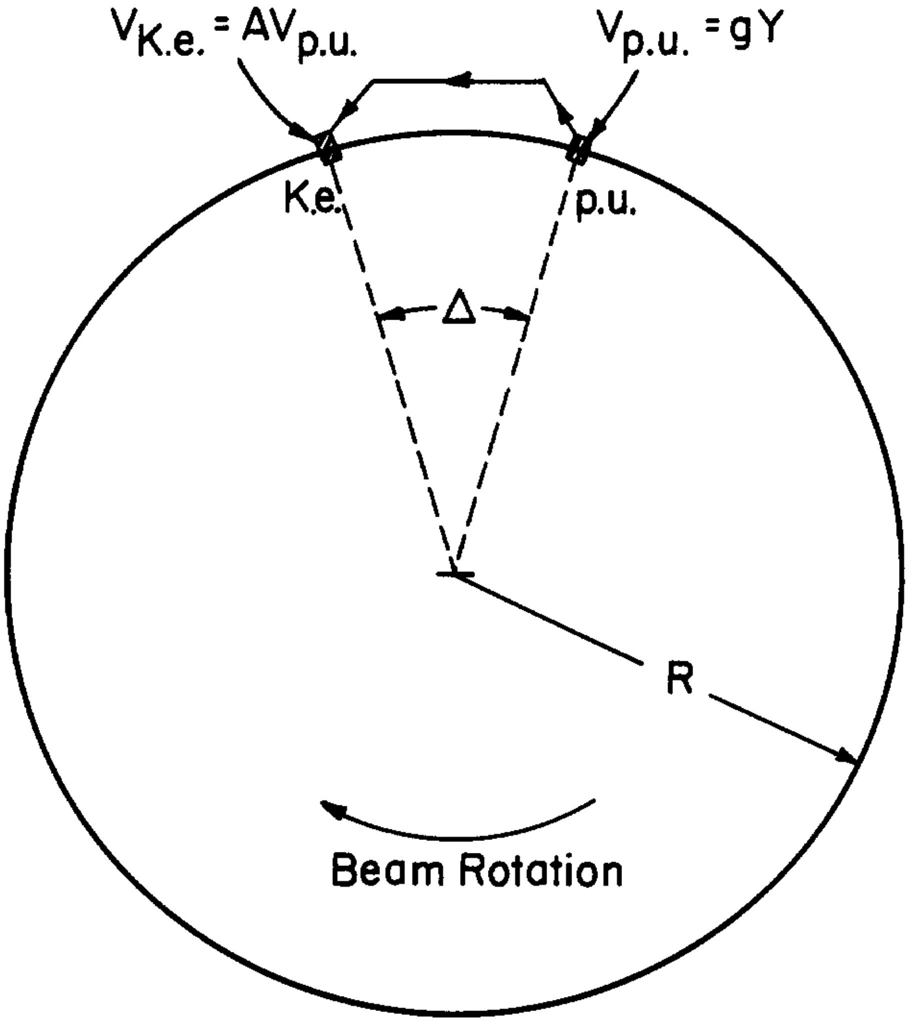


Fig. 1

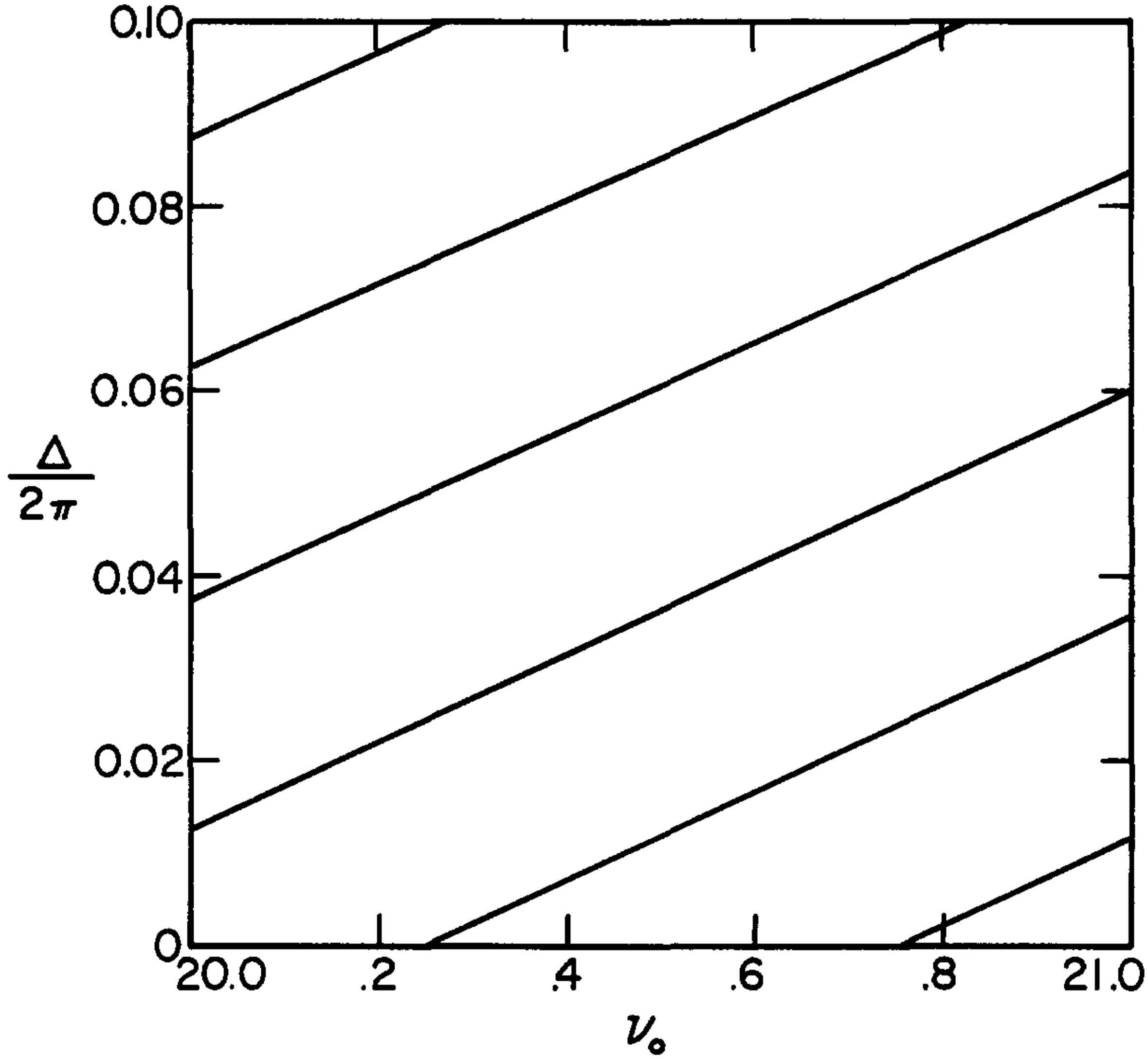


Fig. 2