national accelerator laboratory

FN-212
0400

DC MAGNETIC MONOPOLE FOCUSING LENSES FOR NEUTRINO BEAM

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June 1, 1970

We assume a point source (target) of mesons (parent particles for neutrinos). Since a magnetic lens is dispersive, it can be designed to give perfect focusing only on one preselected curve in the production angle (θ) versus momentum (p) plane. We take the curve to be θ = Cocconi angle = 0.3/p (p in BeV/c). If we want a parallel beam after the lens the azimuthal field times length ($B\ell$) of the lens is given by

$$B\ell = \left(\frac{100}{3}\,p\right)\theta \text{ kGm} = 10 \text{ kGm}.$$

Note that this is a universal constant valid for all lenses with this design criterion. We are free to choose any radial dependencies for B and ℓ separately; however, for simplicity we shall assume ℓ = 5 m and B = 2 kB. To produce this uniform field we need a volumetric axial current density

$$j = \frac{10}{4\pi}\,\frac{1}{r}\,\frac{d}{dr}\,(rB) = \frac{5000}{\pi r}\ \text{A/cm}^2,$$

where r is in cm. For $r \gtrsim 1$ cm this is a very modest current density in Cu. We should be able to approximate this continuous current distribution by a grid, thereby leaving empty space for the passage of mesons. We consider here only one-dimensional current grids, namely current-sheets. Two natural grid geometries evolve: (1) concentric cylindrical current-sheets (CCC) and (2) radial plane current-sheets (RPC).

For the actual current density in Cu we shall assume $j = 5000\ A/cm^2$. With this current density the power dissipation per unit volume of Cu is (resistivity of Cu = $1.86\times10^{-6}\ \Omega cm$)

$$w = (1.86 \times 10^{-6}) \times (5000)^2\ W/cm^3 = 46.5\ W/cm^3.$$

Without cooling the rate of temperature rise of Cu is (specific heat of Cu = $0.82\ cal/cm^3/°C = 3.43\ J/cm^3/°C$)

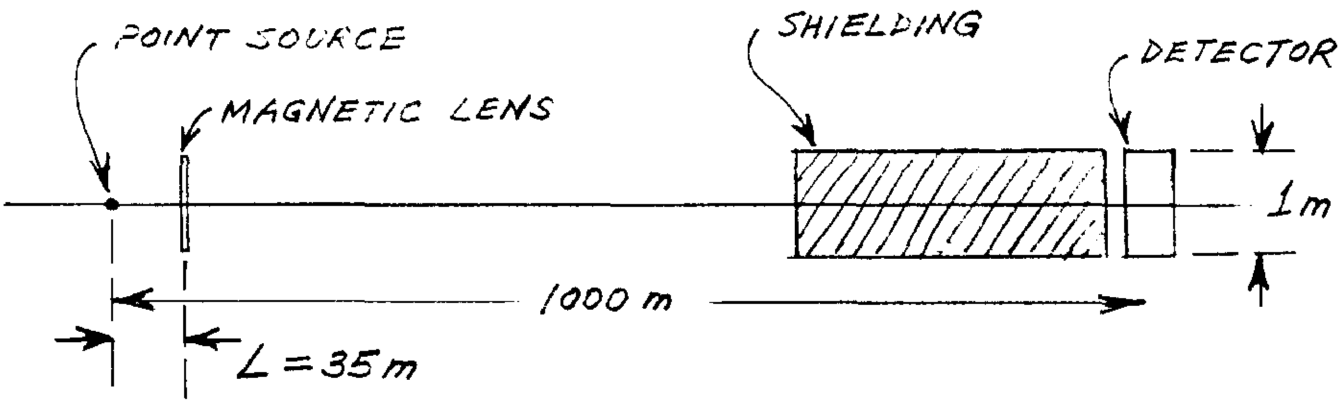
$$\dot{T} = \frac{46.5}{3.43}\ °C/sec = 13.6°C/sec.$$

The radial force per unit volume of Cu conductor toward the axis is

$$f = j \langle B \rangle = 500 \langle B \rangle\ dyne/cm^3,$$

where $\langle B \rangle$ is the average field in gauss on both sides of the current-sheet. For $\langle B \rangle = 2000$ gauss, $f = 10^6\ dyne/cm^3 \simeq 1\ atm/cm$ which is modest.

The geometry of the beam is assumed to be as follows:



The sensitive dimension is the distance from the target (point source of mesons) to the magnetic lens. This distance is assumed to be L = 35 m. For production angle $\theta < 0.5$ m/1000 m = 0.5 mrad, no focusing is needed. Hence, there could be a central hole of radius (0.5 mrad) L = 1.75 cm in the magnetic lens.

The Cocconi angle for 20 BeV/c particle is 15 mrad which gives a radius of (15 mrad) L = 52.5 cm at the lens. The useful radius of the lens should, therefore, extend to about 50 cm which also agrees with the assumed radius of the detector.

A. Concentric Cylindrical Current-Sheet (CCC)

This is an extension of the geometry of the original van der Meer horn. We shall take the outer radius of the innermost (first) current cylinder to be $r_1 = 2$ cm. To get B = 2 kG outside the cylinder the current i_1 required is

$$i_1 = 10 \frac{r_1 B}{2} = 20 \text{ kA}.$$

With a current density in Cu of 5000 A/cm^2 this current takes 4 cm^2 which gives for the thickness d_1 of the cylinder

$$d_1 = \frac{4\ \text{cm}^2}{2\pi r_1} = 0.32\ \text{cm}.$$

This leaves a central hole of radius 1.68 cm which is approximately equal to the 1.75 cm mentioned earlier.

The field outside the cylinder falls off as 1/r. At $r_2 = 2r_1 = 4$ cm the field falls to 1 kG. We put the second current cylinder there to boost the field back up to 2 kG. The current i_2 required is

$$i_2 = 10\ \frac{r_2 (1\ \text{kG})}{2} = 20\ \text{kA},$$

and the thickness of the second cylinder is

$$d_2 = \frac{4\ \text{cm}^2}{2\pi r_2} = 0.16\ \text{cm}.$$

Scaling in this manner with $r_3 = 8$ cm, $r_4 = 16$ cm, and $r_5 = 32$ cm, we get the parameters given in Table I. The current cylinders and the field shape are shown in Fig. 1. It should be noted that the first cylinder carries twice the current as given by scaling. This is to make up for the absence of current in the central hole. The total current for 5 cylinders is 320 kA. The voltage across each cylinder is $(1.86 \times 10^{-6}\ \Omega\text{cm}) \times (5000\ \text{A/cm}^2) \times (500\ \text{cm}) = 4.65$ V. Electrically the cylinders can be connected either in series or in parallel.

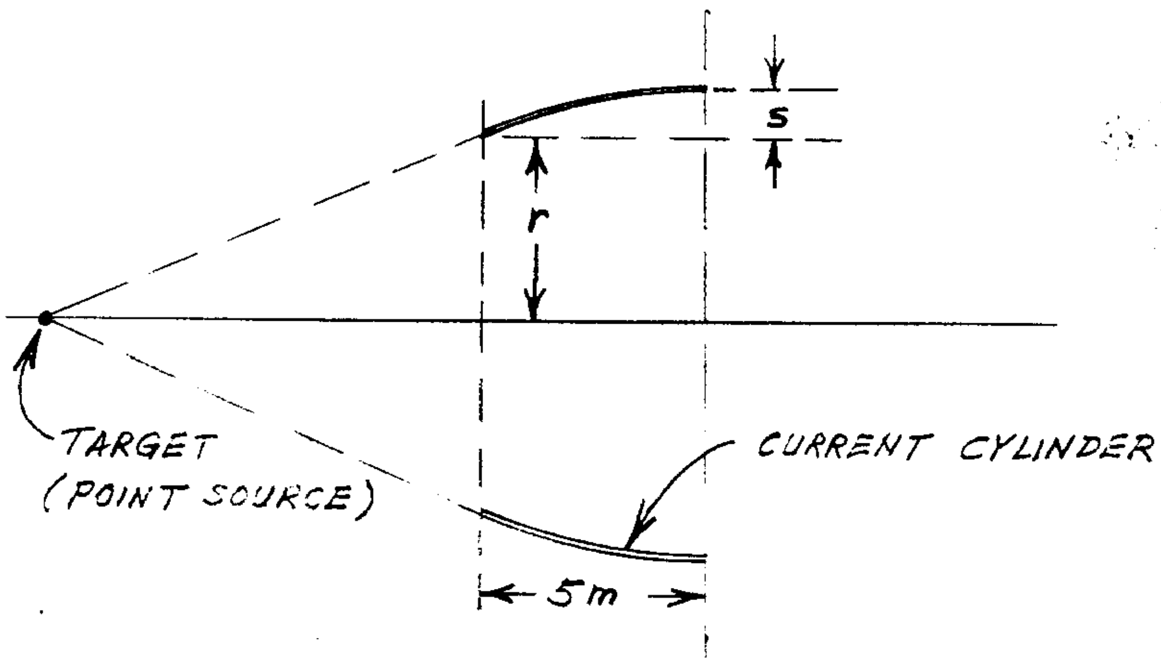
The particle orbit along the CCC has an appreciable sagitta s given by

$$s = \rho \frac{\theta_c^2}{2} = \frac{500\ \text{cm}}{2}\ \theta_c,$$

since $\rho\theta_c = 5$ m and where θ_c = Cocconi angle. But the radius of the cylinder is given by $r = L\theta_c$; therefore

$$s = \frac{500\ \text{cm}}{2}\ \frac{r}{L} = 0.075\ r.$$

The cylinders should be tapered by s as shown exaggerated below.



Values of s for each cylinder are given in Table I.

The force per unit area F on the cylinder is $F = fd = 500 \langle B \rangle d$ dyne/cm^2. The values of F for each cylinder are also given in Table I, where the unit 10^6 dyne/cm^2 is approximately 1 atm. If the cylinder is perfectly symmetrical, the force is balanced and tends only to buckle the cylinder. On the other hand, if the cylinder is out of round or off axis the unbalanced force will be rather alarming.

Neglecting the power dissipation in the current return path the total power required for 5 cylinders is 1.49 MW. This power could be supplied by a homopolar generator. The cylinders should be cooled by water sprays which is turned off during the, say, 1-sec beam-spill time. Since the temperature rise without cooling is only 13.6°C/sec this interruption is quite acceptable.

For this geometry since there is quite a lot of material (Cu) through which the beam has to penetrate the overall efficiency of the CCC could only be studied using the computer. The temperature rise is given is rather modest. We can perhaps increase the current density and reduce the amount of Cu in the cylinders. But the mechanical force on the cylinders should be carefully investigated especially since the suspensions of the cylinders at the ends have to be heavy enough to carry the return current, rigid enough to provide the necessary alignment accuracy, but flexible enough to take up the thermal expansion.

B. Radial Plane Current-Sheets (RPC)

This geometry is reminiscent of that discussed by R. B. Palmer (National Accelerator Laboratory Summer Study Report, Vol. I, p. 383, 1969) and by D. H. Frisch and Y. W. Kang (Ibid., p. 389). We assume 20 radial plane current-sheets evenly spaced around the 2π azimuth and running the length of 5 m as shown in cross section in Fig. 2. Each conductor sheet has a length of 5 m, a radial width of 48.5 cm (extending from r = 1.5 cm to r = 50 cm) and a uniform thickness of 0.1 cm except for a double thickness (0.2 cm) strip of 1.5 cm radial width (extending from r = 1.5 cm to r = 3.0 cm) along the edge closest to the axis. As before, the double thickness edge strip will compensate for the absence of current in the 1.5 cm radius central hole.

With a current density of 5000 A/cm^2 the current enclosed within a radius r (for r > 3.0 cm) is $(0.1\ r) \times 20 \times 5000$ A $= 10^4$ rA giving a uniform field of $B = 2 \times 10^4\ r/10\ r = 2$ kG. Clearly, the field is zero for r < 1.5 cm and rises linearly to 2 kG at r = 3.0 cm.

The current per sheet is $0.1 \times 50 \times 5000$ A = 25 kA and the total current for 20 sheets is 500 kA. The voltage drop across the length of each sheet is, again, 4.65 V. The total power is 2.33 MW which can be supplied by a homopolar generator. The 20 sheets can again be connected either in series or in parallel. The parallel connection leads to a simpler mechanical structure.

With the continuous radial current distribution replaced by currents concentrated in the 20 radial planes the field lines are distorted from circles to 20 sectored rosettes with cusps at the current planes. At the curent sheet the radial field component B_r is given by

$$(2\ \text{cm})\ B_r = \frac{4\pi}{10}\ (0.1\ \text{cm} \times 1\ \text{cm} \times 5000\ \text{A/cm}^2)$$

or

$$B_r = \frac{1}{2}\ \frac{4\pi}{10} \times 500\ \text{G} = 314\ \text{G}.$$

The sense of this field component is to deflect the particles into the current plane on both sides. The magnitude is such as to give an azimuthal sagitta in the 5 m length of the RPC of

$$\frac{500\text{ cm}}{2} \; \frac{0.314 \times 5}{\frac{100}{3}\,p} = \frac{11.8}{p}\text{ cm},$$

(where p in in BeV/c) which is only 0.59 cm for the lowest momentum of p = 20 BeV/c. Anyways, the particles deflected toward the current-sheet will very likely go through the 0.1 cm thickness of the Cu conductor.

The mechanical force toward the axis on each sheet is $25\text{ kA}/10 \times 2\text{ kG} = 5 \times 10^6$ dyne/cm length $\approx$ 5 kg/cm length or about 2500 kg/sheet. Since the tensile strength of Cu is about 4500 kg/cm^2 this force is not at all excessive. Compared to the CCC where the force is perpendicular to the current sheet the RPC geometry giving a force in the plane of the current sheet is much more desirable. Of course, asymmetries in the positioning of the conductor sheets will produce a small force perpendicular to the conductor plane. But, this force is entirely negligible compared, for instance, to even the weight of the conductor sheet.

A possible structural arrangement is shown in Figs. 2 and 3. For the particular end connections to the Cu sheet shown, the current in the sheet within about 0.5 m of each end will not be uniform or axial in direction. The effect due to this error should be negligible. However, if desired, the ends of the sheet can certainly be shaped to either eliminate or at least greatly reduce the improper end current distributions.

Also indicated in Fig. 2 are possible locations of water nozzles for spray-cooling the Cu sheets. Here, again, the slow temperature rise of 13.6°C/sec without cooling permits interrupting the cooling water spray during the beam spill time at a duty factor of, say, 25%.

In conclusion, the RPC geometry is more desirable and gives a simple, reliable, and economical design. The overall efficiency of the lens should, of course, be studied in detail using a computer. It is possible to use a geometry with a two-dimensional current grid, namely 5 m long current-carrying wires strung between two end plates and properly distributed over the cylindrical volume of the lens to give a uniform azimuthal field of 2 kG. But this geometry offers no advantage over the RPC geometry.

It is a pleasure to acknowledge several helpful discussions with D. Edwards.

Table I.

Current Cylinder	r(cm)	i(kA)	d(cm)	s(cm)	F(10^6 dyne/cm^2)	W(kW)
1st	2	20	0.32	0.15	0.16	93
2nd	4	20	0.16	0.30	0.12	93
3rd	8	40	0.16	0.60	0.12	186
4th	16	80	0.16	1.20	0.12	372
5th	32	160	0.16	2.40	0.12	744

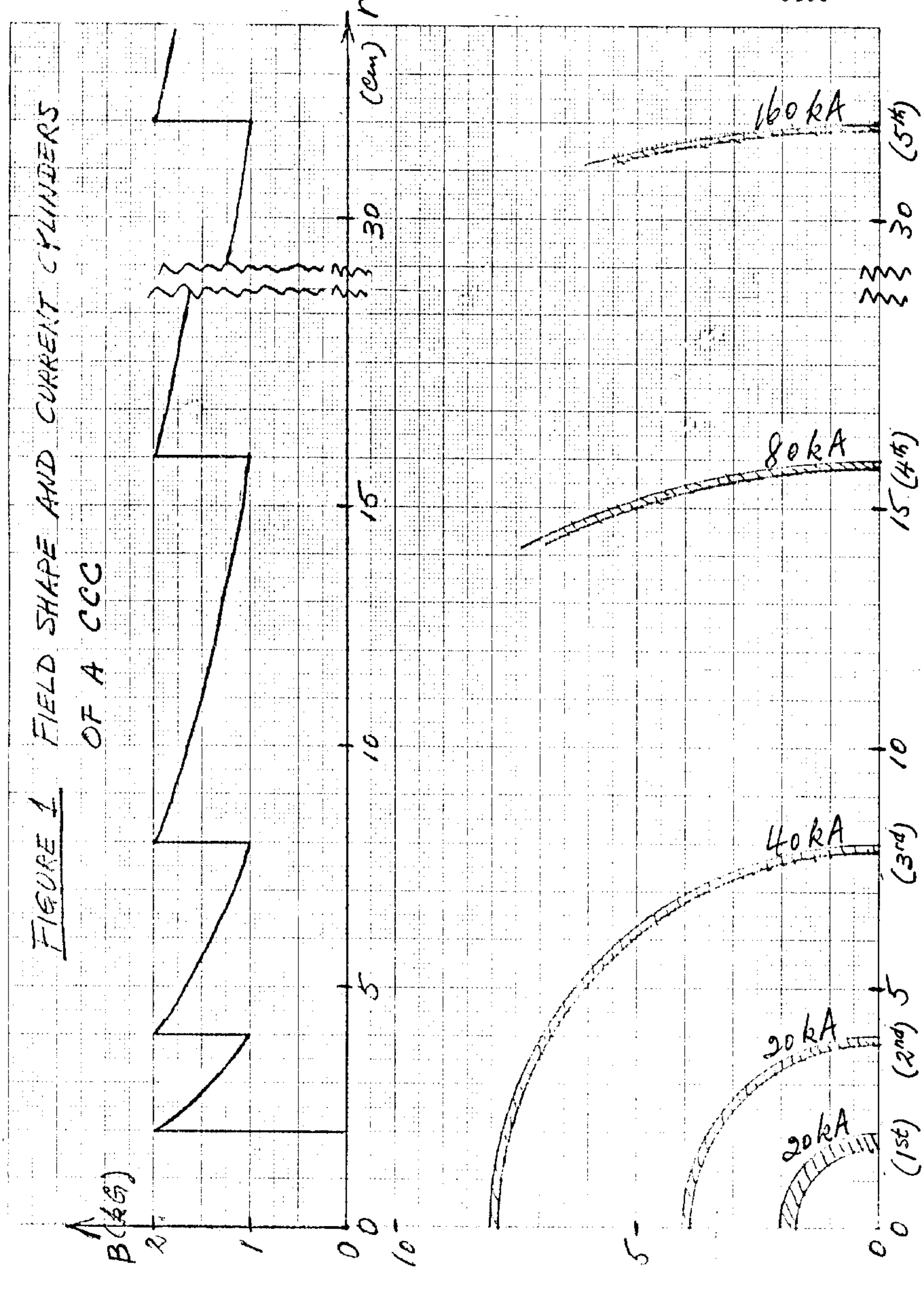


FIGURE 1 FIELD SHAPE AND CURRENT CYLINDERS OF A CCC

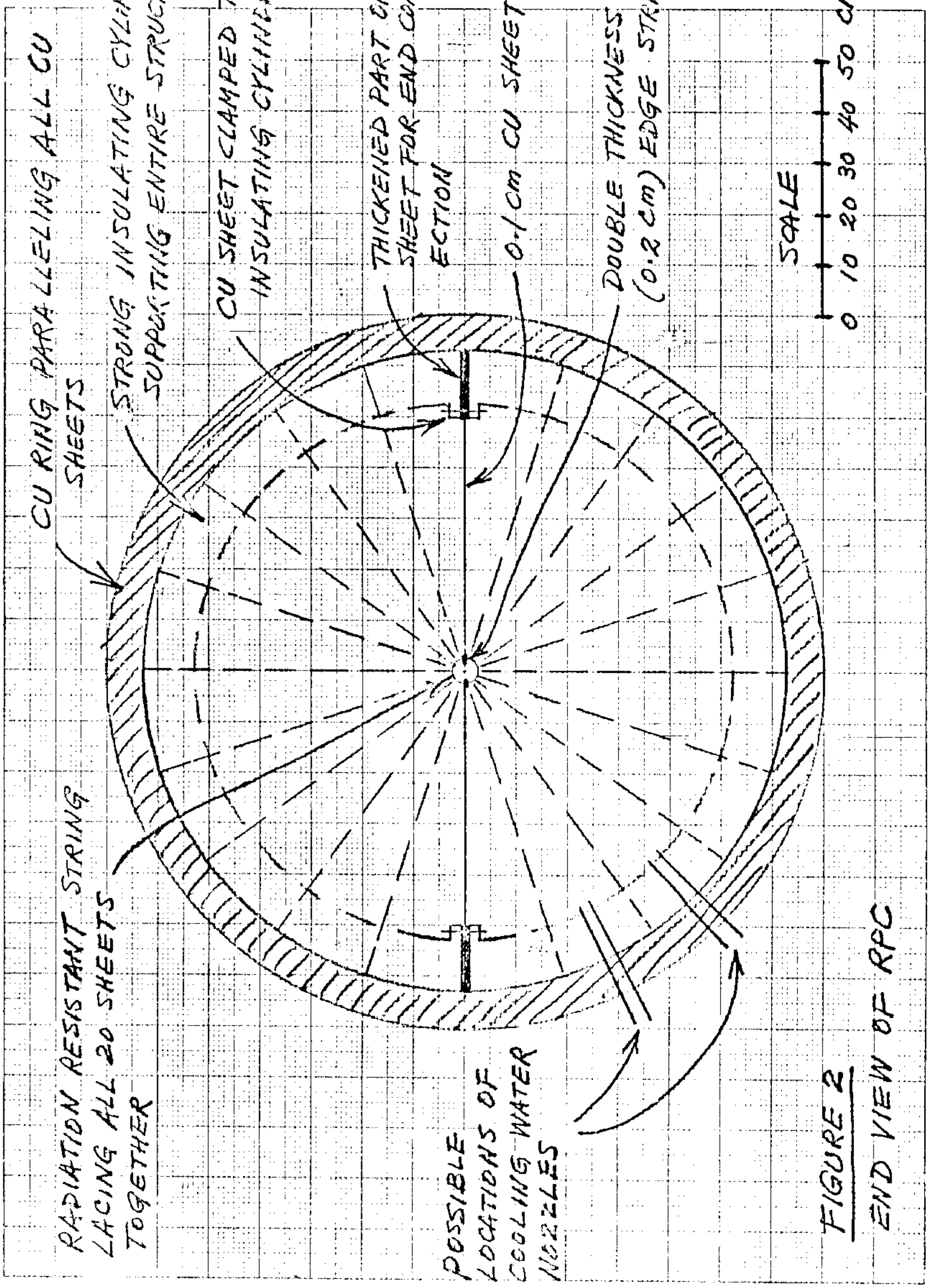
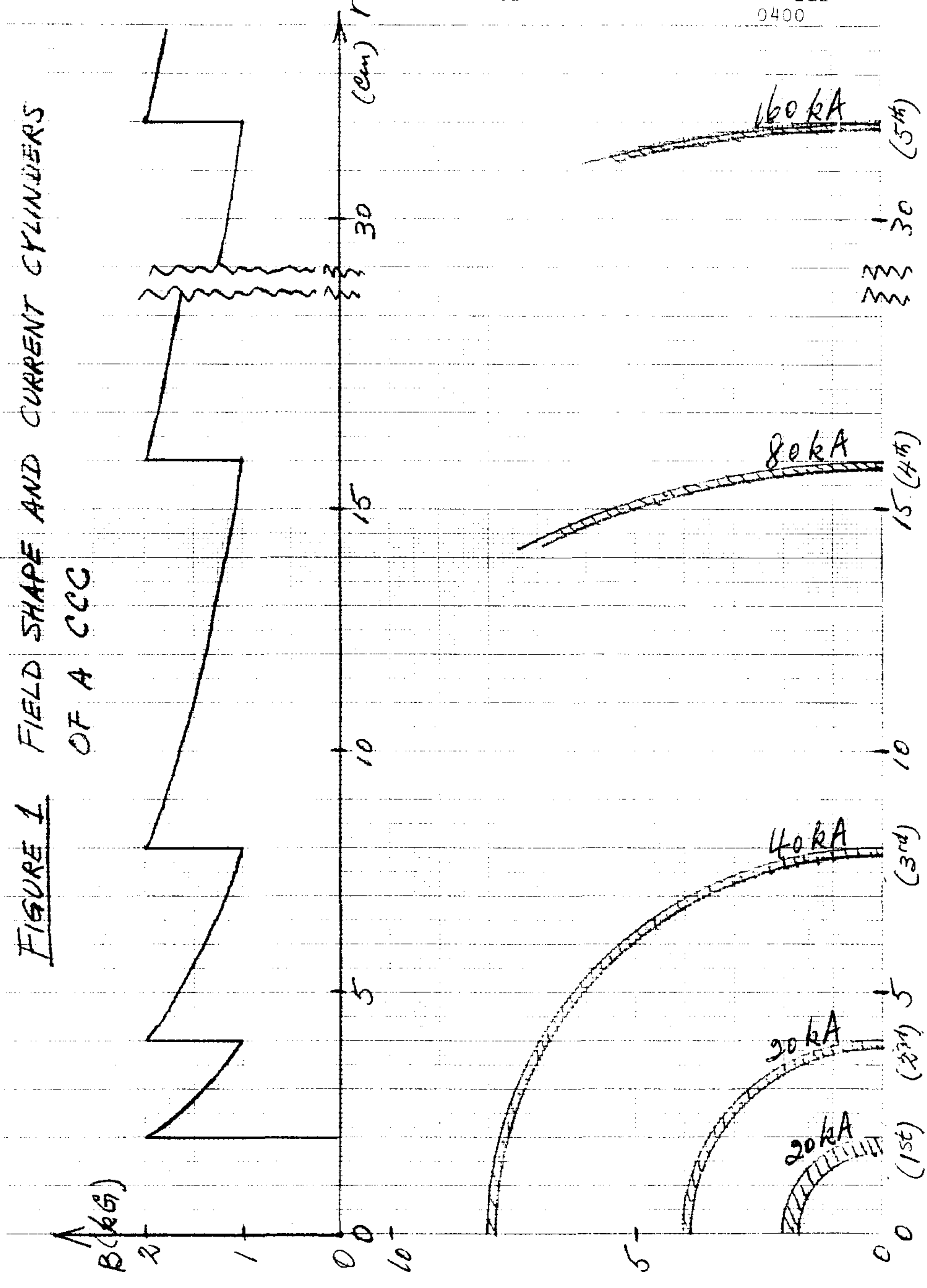


FIGURE 2

END VIEW OF RPC



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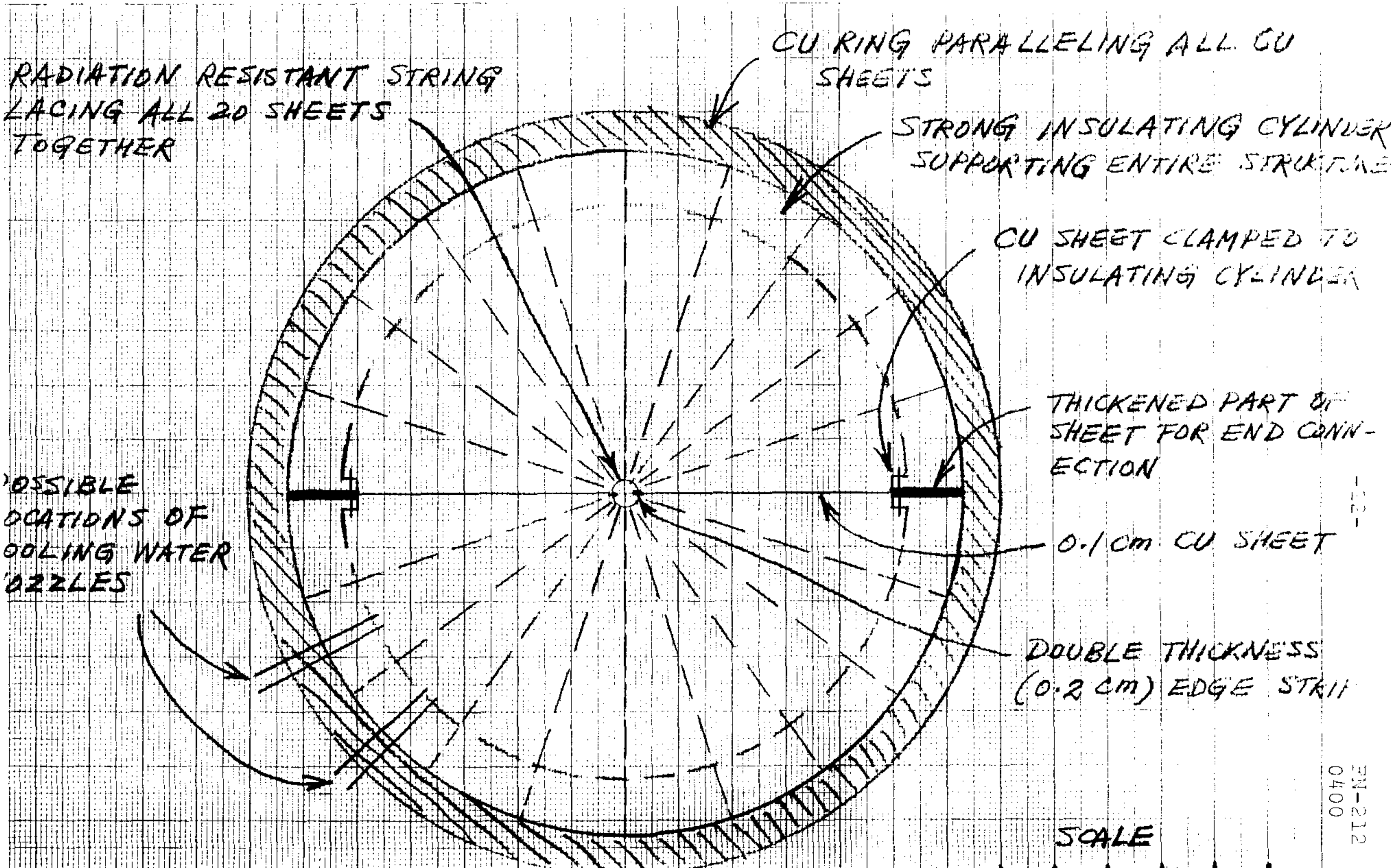

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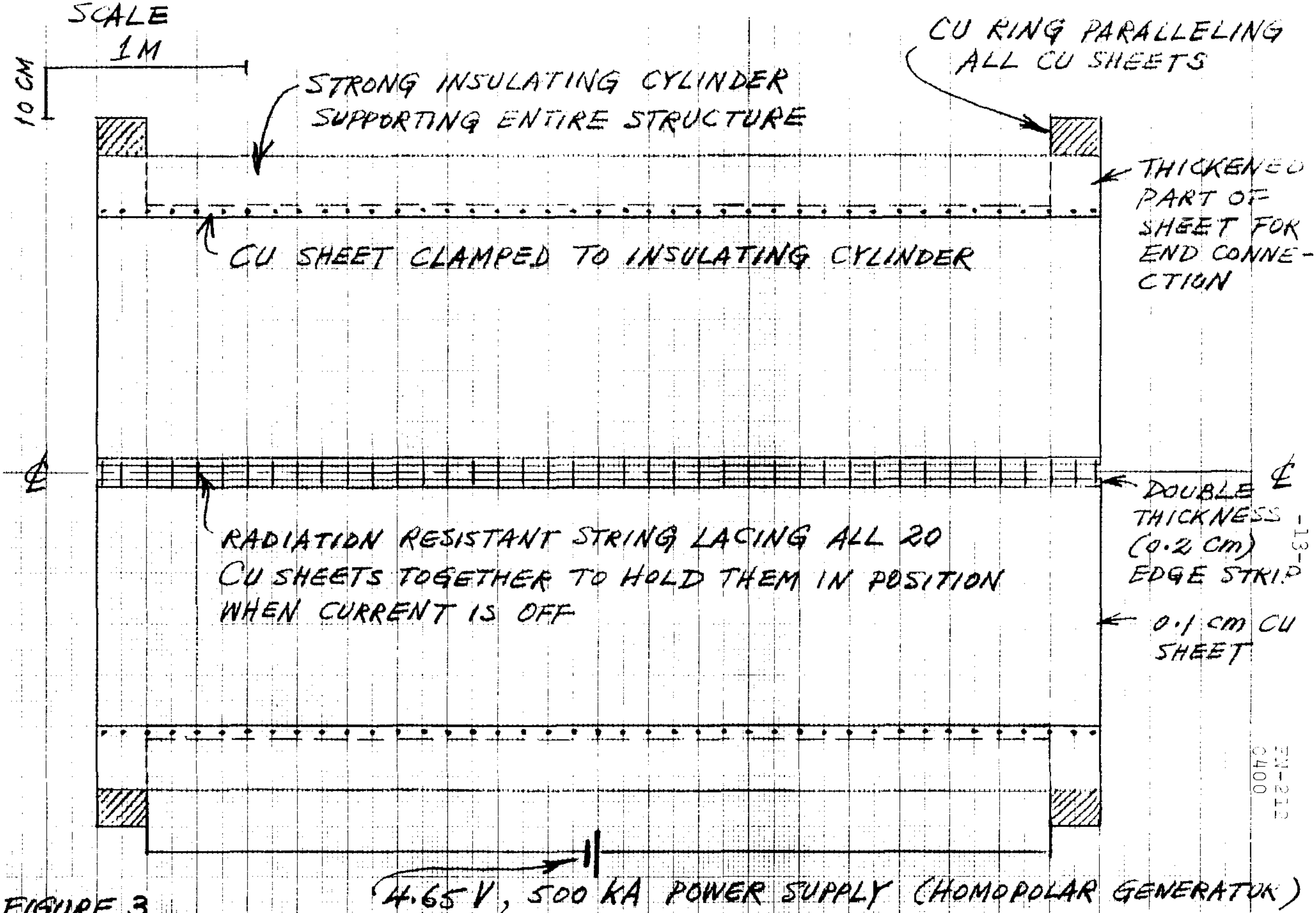

FIGURE 3
SIDE VIEW OF RPC

Addendum to FN-212

Parameters for a Small RPC Lens

Distance from target to detector	900 m
Radius of detector	1 m
Length of RPC	10 m
Distance from target to leading edge of RPC	10 m
Azimuthal magnetic field in RPC	1 kG
Number of radial current sheets	20
Radius of central hole	1 cm
Maximum radius of RPC (for max. production angle of 15 mrad corresponding to 20 BeV/c)	25 cm
Thickness of Cu sheet	1 mm
Current density in Cu	2500 A/cm^2
Current per sheet	6250 A
Voltage per sheet	4.65 V
Total voltage with 20 current sheets in series	93 V
Total power	581 kW