A. PURPOSE

The booster magnet sextupole components were selected to remove the momentum variation of betatron frequencies for small amplitudes and momentum excursion. This note indicates the extent to which this condition is fulfilled using the measured gradients and a momentum range that spans the available aperture.

B. RADIAL MOTION

On the median plane the radial motion is expressed adequately by

\[ \frac{d^2x}{ds^2} = \frac{1}{\rho} (1 + \frac{x}{\rho}) + \frac{e}{p} (1 + \frac{x}{\rho})^2. \]  

Let \( X \) be a periodic solution of Eq. (1) and expand radial motion around this solution,

\[ x = X + u, \]

retaining only terms linear in the betatron amplitude \( u \).

\[ B_y(x) \equiv B_y(X) + uB_y'(X) + \ldots \]  

\[ \frac{1}{\rho} + \frac{e}{p_0} B_y(0) \equiv 0 \]  

\[ \frac{p_0}{p} = \frac{p - \Delta p}{p} = 1 - \frac{\Delta p}{p} \]
Then
\[
\frac{d^2 x}{ds^2} + \frac{d^2 u}{ds^2} = \frac{1}{\rho} + \frac{x}{\rho^2} + \frac{u}{\rho^2} - \frac{1}{\rho}(1-\Delta p)\left(1+\frac{x}{\rho}\right)^2 \left(\frac{B_y'(X)}{B_y(0)} + \frac{B_y''(X)}{B_y(0)}\right).
\]

(6)

But
\[
\frac{d^2 x}{ds^2} = \frac{1}{\rho} + \frac{x}{\rho^2} - \frac{1}{\rho}(1-\Delta p)\left(1+\frac{x}{\rho}\right)^2 \cdot \frac{B_y(X)}{B_y(0)}.
\]

(7)

Let
\[
k_l = \frac{B_y'(0)}{B_y(0)}
\]

(8)
\[
g(X) = \frac{B_y'(X)}{B_y'(0)}
\]

(9)
\[
b(X) = \frac{B_y(X)}{B_y(0)}.
\]

(10)

Then, after subtracting Eq. (7) from Eq. (6) and retaining only terms linear in \( u \), one has
\[
\frac{d^2 u}{ds^2} + \frac{1}{\rho^2}\left[1-\Delta p\right] \left[\left(1+\frac{x}{\rho}\right)^2 \rho k_{1M} g(X) + 2 \left(1+\frac{x}{\rho}\right) b(X)\right] - 1\} u = 0,
\]

(11)

where \( k_l \) has been replaced by \( k_{1M} \) to indicate that the measured gradient is to be used.

For comparison, when \( \Delta p = 0 \), Eq. (11) becomes
\[
\frac{d^2 u}{ds^2} + \frac{1}{\rho^2}(1+\rho k_{1S}) u = 0,
\]

(12)

where \( k_1 \) is now replaced by \( k_{1S} \) to indicate that the betatron frequencies in this case are obtained from the design parameters and were calculated using SYNCH.
Assuming that the motion described by Eq. (11) is not very different from that described by Eq. (12), one has for the change in the radial tune

\[ \Delta \nu_x = \frac{1}{4\pi} \int_c (K_{xM}(s) - K_{xS}(s)) \cdot \beta_x(s) ds, \]  

(13)

where \( K_{xM} \) and \( K_{xS} \) are the coefficients of \( u \) in Eqs. (11) and (12).

C. VERTICAL MOTION

For small betatron amplitudes and median plane symmetry, the vertical motion is described adequately by

\[ \frac{d^2y}{ds^2} = - \frac{e}{\rho} (1 + \frac{2}{\rho} \beta_x) B_x. \]  

(14)

But

\[ B_x(x,y) = B_x(x,0) + y \frac{\partial B_x}{\partial y} x,0 + \ldots \]  

(15)

Using the ampere circuital law and median plane symmetry, one has

\[ B_x(x,y) = y B_y'(x,0). \]  

(16)

Setting \( x = X \) gives

\[ \frac{d^2y}{ds^2} - \frac{1}{\rho} (1 - \frac{\Delta \rho}{\rho}) (1 + \frac{2}{\rho} \beta_x) k_{1M} g(x)y = 0. \]  

(17)

For \( \Delta \rho = 0 \) and \( k_1 \) replaced by \( k_{1S} \), Eq. (17) becomes

\[ \frac{d^2y}{ds^2} - \frac{k_{1S}}{\rho} y = 0. \]  

(18)

Again, assuming that the change from Eq. (18) to Eq. (17)
only causes a small tune shift

\[ \Delta \nu_y = \frac{1}{4\pi} \int_C (K_{YM} - K_{YS}) \beta_y(s) ds, \]  

(19)

where \( K_{YM} \) and \( K_{YS} \) are the coefficients of \( y \) in Eqs. (17) and (18).

**D. HARD EDGE APPROXIMATION**

Although Eqs. (13) and (19) are sufficient as expressed, one needs to know the focusing functions \( K_{XM}, K_{XS}, K_{YM}, K_{YS} \) as a function of position. This is most conveniently done by assuming them constant within each magnet and abruptly reduced to zero along some curve that represents the effective termination of the magnet. Justification for this procedure is obtained by appealing to the fact that the reduction in field from full value to zero occurs in a distance that is small compared with a betatron wavelength.

A further simplification is introduced. The curve representing the effective termination of the magnetic field at the entrance to the magnet is a mirror image of the termination curve at the exit end of the magnet. Analytically for one magnet, one has

\[ B_y(x, s) = B_y(x) \cdot \left\{ S_1(s-s_1+f(x)) - S_2(s-s_2-f(x)) \right\}, \]  

(20)

where \( S_1 \) and \( S_2 \) are step functions and

\[ s = s_1 - f(x) \]  

(21)
is the termination curve at the entrance end, and
\[ s = s_2 + f(x) \] (22)
is the termination curve at the exit end. For convenience
\[ s_2 - s_1 = \text{physical magnet length}, \] (23)
and also equals magnet length used in SYNCH. Equation (20) gives also
\[ B_y'(x, s) = B_y'(x) \cdot \left\{ S_1 - S_2 \right\} + B_y(x) f'(x) \left\{ \delta_1 + \delta_2 \right\} \] (24)
where \( \delta_1 \) and \( \delta_2 \) are delta functions corresponding to \( S_1 \) and \( S_2 \).

E. EXPLICIT EXPRESSIONS

In this perturbation calculation the linear orbit functions \( \beta_x(s) \), \( \beta_y(s) \), and \( x_p(s) \) are obtained from the SYNCH program. The periodic solution for the off momentum closed orbit is given then by
\[ X(s) = x_p(s) \cdot \frac{\Delta P}{P} \] (25)
For convenience, let \(^1\)
\[ K_{xM} = \frac{1}{\rho^2} \left\{ \left(1 - \frac{\Delta P}{P}\right) \left[ \left(1 + \frac{X}{\rho}\right)^2 \rho k_{1M} g(X) + 2 \left(1 + \frac{X}{\rho}\right) b(X) \right] - 1 \right\} \] (26)
\[ K_{xS} = \frac{1}{\rho^2} \left(1 + \rho k_{1S}\right) \] (27)
\[ K_{yM} = \frac{1}{\rho} \left(1 - \frac{\Delta P}{P}\right) \left(1 + \frac{X}{\rho}\right)^2 k_{1M} g(X) \] (28)
\[ K_{yS} = \frac{1}{\rho} \left(1 + \rho k_{1S}\right) \] (29)
\[ L_{xM} = \frac{1}{\rho} \left(1 - \frac{\Delta P}{P}\right) \left(1 + \frac{X}{\rho}\right)^2 b(X) \] (30)
\[ L_{xS} = \frac{1}{\rho} \] (31)
\[ L_{yM} = \frac{1}{\rho} \left( 1 - \frac{\Delta D}{\rho} \right) (1 + \frac{X}{\rho}) b(X) \]  
(32)

\[ L_{yS} = \frac{1}{\rho} \]  
(33)

\[ f_{MF} = \Delta S_F + A_F X + B_F X^2 + C_F X^3 + \ldots \]  
(34)

\[ f_{MD} = \Delta S_D + A_D X + B_D X^2 + C_D X^3 + \ldots \]  
(35)

\[ f_{SF} = A_{SF} X \]  
(36)

\[ f_{SD} = A_{SD} X. \]  
(37)

The coefficients \( A_{SF} \) and \( A_{SD} \) are determined such that the effective magnet end shape for the SYNCH run is identical with the physical magnet end. The remaining coefficients in Eqs. (34) and (35) are to be chosen subsequently.

In terms of these symbols, the tune shifts for the booster lattice whose period is \( N \) becomes

\[
\Delta v_x = \frac{2N}{4\pi} \left\{ \int_{s_{1F}}^{s_{2F}} (K_{xM} - K_{xS}) \beta_x ds + \int_{s_{1D}}^{s_{2D}} (K_{xM} - K_{xS}) \beta_x ds \right. \\
+ \left[ (K_{xM} \beta_x)_{\text{entr.}(F)} + (K_{xM} \beta_x)_{\text{exit}(F)} \right] \Delta S_F \\
+ \left[ (K_{xM} \beta_x)_{\text{entr.}(D)} + (K_{xM} \beta_x)_{\text{exit}(D)} \right] \Delta S_D \\
+ \left( L_{xM} \beta_x f'_M \right)_{\text{entr.}(F)} + \left( L_{xM} \beta_x f'_M \right)_{\text{exit}(F)} \\
+ \left( L_{xM} \beta_x f'_M \right)_{\text{entr.}(D)} + \left( L_{xM} \beta_x f'_M \right)_{\text{exit}(D)} \\
- \left( L_{xS} \beta_x f'_S \right)_{\text{entr.}(F)} - \left( L_{xS} \beta_x f'_S \right)_{\text{exit}(F)} \\
- \left( L_{xS} \beta_x f'_S \right)_{\text{entr.}(D)} - \left( L_{xS} \beta_x f'_S \right)_{\text{exit}(D)} \right\} \]  
(38)

and
\[
\Delta \nu_y = \frac{2N}{4\pi} \left\{ \int_{s_{1F}}^{s_{2F}} (K_{yM} - K_{yS}) \beta_y ds + \int_{s_{1D}}^{s_{2D}} (K_{yM} - K_{yS}) \beta_y ds \\
+ \left[ (K_{yM} \beta_y)' \text{entr.}(F) + (K_{yM} \beta_y)' \text{exit}(F) \right] S_{F} \\
+ \left[ (K_{yM} \beta_y)' \text{entr.}(D) + (K_{yM} \beta_y)' \text{exit}(D) \right] S_{D} \\
+ (L_{yM} \beta_y f'M)' \text{entr.}(F) + (L_{yM} \beta_y f'M)' \text{exit}(F) \\
+ (L_{yM} \beta_y f'M)' \text{entr.}(D) + (L_{yM} \beta_y f'M)' \text{exit}(D) \\
- (L_{yS} \beta_y f'S)' \text{entr.}(F) - (L_{yS} \beta_y f'S)' \text{exit}(F) \\
- (L_{yS} \beta_y f'S)' \text{entr.}(D) - (L_{yS} \beta_y f'S)' \text{exit}(D) \right\}
\] 

Note that \( s_{1F}, s_{2F}, s_{1D}, s_{2D} \) are the effective magnet ends on the central orbit as used in SYNCH. Thus, positive \( S_{F} \) and \( S_{D} \) indicate that the effective magnet ends at the central orbit increase the magnet length in the perturbed case.

F. OPTIMUM END SHAPES

Equations (38) and (39) show that the tune shifts \( \Delta \nu_x \) and \( \Delta \nu_y \) away from the corresponding SYNCH tunes are linearly related to the coefficients in the power series expansion of the effective end shapes as given in Eqs. (34) and (35). One might consider, therefore, that an optimum end shape could be obtained by adjusting \( S_{F}' A_F, B_F, C_F, \) etc., and \( S_{D}' A_D, B_D, C_D, \) etc., such that the quantity
\[ \int \left( W_x \frac{\Delta p}{p} \cdot (\Delta v_x) \right)^2 + W_y \frac{\Delta p}{p} \cdot (\Delta v_y) \right)^2 \, d\left( \frac{\Delta p}{p} \right) = \text{minimum}. \quad (40) \]

In this way any design or fabrication difficulties in the body of the magnet could be rectified by finding suitable end shaping that yields the desired effective end shape. Notice, however, that, although this procedure will work, one should be careful about interpreting the obtained \( \Delta S'_F, \Delta S'_D, A'_F, A'_D \) so obtained. The difficulty stems from the fact that, in the linear theory, additional focusing may be obtained either by increasing the magnet length or by changing the edge angle. The functions of momentum that multiply \( \Delta S_F \) or \( \Delta S_D \) are only negligibly different from those that multiply \( A_F \) and \( A_D \). If they had been identical, the least squares procedure would have failed. To obtain realistic results, it is preferable to remove this difficulty by choosing fixed values for \( \Delta S_F \) and \( \Delta S_D \) and minimize Eq. (40) with respect to the remaining coefficients.

G. LEAST SQUARES ANALYSIS

In Eq. (38) let the quantities not subject to adjustment be designated by \( \text{DNUX}(J) \) where uniformly incremented values of \( \Delta p/p \) are represented by the index \( J \). Thus
By utilizing Eqs. (34) and (35) and defining the array

\[ \{D(K)\} = \{ A_F, A_D, B_F, B_D, C_F, C_D, \ldots \} \]  

(42)

and the array \( T(J,K) \) to be

\[
T(J,1) = (L_x M_x \beta_x)_{\text{entr.}(F)} + (L_x M_x \beta_x)_{\text{exit}(F)}
\]

\[
T(J,2) = (L_x M_x \beta_x)_{\text{entr.}(D)} + (L_x M_x \beta_x)_{\text{exit}(D)}
\]

\[
T(J,3) = 2(L_x M_x \beta_x \beta_x)_{\text{entr.}(F)} + 2(L_x M_x \beta_x \beta_x)_{\text{exit}(F)}
\]

\[
T(J,4) = 2(L_x M_x \beta_x \beta_x)_{\text{entr.}(D)} + 2(L_x M_x \beta_x \beta_x)_{\text{exit}(D)}
\]

\[
T(J,5) = 3(L_x M_x \beta_x \beta_x^2)_{\text{entr.}(F)} + 3(L_x M_x \beta_x \beta_x^2)_{\text{exit}(F)}
\]

\[
T(J,6) = 3(L_x M_x \beta_x \beta_x^2)_{\text{entr.}(D)} + 3(L_x M_x \beta_x \beta_x^2)_{\text{exit}(D)}
\]

\[
T(J,7) = \text{etc.},
\]

(43)
Equation (38) becomes
\[
\Delta v_x(J) = DNX(J) + \sum_K T(J,K)D(K). \tag{44}
\]

In an analogous manner let the fixed quantities in Eq. (39) be designated by DNYU(J). Then
\[
DNYU(J) = \frac{2N}{4n} \left\{ \int_{s_1F}^{s_2F} (K_{YM} - K_{YS}) \beta_y ds + \int_{s_1D}^{s_2D} (K_{YM} - K_{YS}) \beta_y ds \right. \\
+ \left[ (K_{YM} \beta_y)^{entr.}(F) + (K_{YM} \beta_y)^{exit}(F) \right] \Delta S_F \\
+ \left[ (K_{YM} \beta_y)^{entr.}(D) + (K_{YM} \beta_y)^{exit}(D) \right] \Delta S_D \\
- (L_{YS} \beta_y f')^{entr.}(F) - (L_{YS} \beta_y f')^{exit}(F) \\
- (L_{YS} \beta_y f')^{entr.}(D) - (L_{YS} \beta_y f')^{exit}(D) \right\}. \tag{45}
\]

Further, let the array \( S(J,K) \) be
\[
S(J,1) = (L_{YM} \beta_y)^{entr.}(F) + (L_{YM} \beta_y)^{exit}(F) \\
S(J,2) = (L_{YM} \beta_y)^{entr.}(D) + (L_{YM} \beta_y)^{exit}(D) \\
S(J,3) = 2(L_{YM} \beta_y X)^{entr.}(F) + 2(L_{YM} \beta_y X)^{exit}(F) \\
S(J,4) = 2(L_{YM} \beta_y X)^{entr.}(D) + 2(L_{YM} \beta_y X)^{exit}(D) \\
S(J,5) = 3(L_{YM} \beta_y X^2)^{entr.}(F) + 3(L_{YM} \beta_y X^2)^{exit}(F) \\
S(J,6) = 3(L_{YM} \beta_y X^2)^{entr.}(D) + 3(L_{YM} \beta_y X^2)^{exit}(D) \\
S(J,7) = etc. \tag{47}
\]
Equation (39) then becomes
\[ \Delta v_y(J) = DNUY(J) + \sum_K S(J,K)D(K) \]  
(48)

If Eqs. (44) and (48) are substituted into Eq. (40) and the minimization carried out one finds for \( D(K) \)
\[ D(K) = -\sum_L C^{-1}(K,L)A(L) \]  
(49)

where
\[ A(L) = \sum_J (\bar{W}^x(J)DNUX(J)T(J,L) + \bar{W}^y(J)DNUY(J)S(J,L)) \]  
(50)
and
\[ C(K,L) = \sum_J (\bar{W}^x(J)T(J,K)T(J,L) + \bar{W}^y(J)S(J,K)S(J,L)) \]  
(51)

Equation (40) at the minimum becomes
\[ \text{SUM} = \sum_J (\bar{W}^x(J)DNUX^2(J) + \bar{W}^y(J)DNUY^2(J)) + \sum_K A(K)D(K) \]  
(52)

The operations indicated in Eqs. (40) to (52) have been coded in the program TUNA. In order to obtain the inverse matrix, MATINV by Garbow has been included as a subroutine.

H. NUMERICAL RESULTS FOR BOOSTER

All numerical results are expressed in the coordinate system \((x,y,s)\) where \(s\) is distance measured along the equilibrium orbit for \(p = p_o\), \(x\) is measured in the direction radially normal to the equilibrium orbit, and \(y\) is the vertical direction. The fractional momentum change \(\Delta p/p\) is converted to equilibrium orbit position at the entrance to the
F-magnet.

Ideal design gradients as a function of $x$ were chosen previously and are characterized by a selection of sextupole moments, $k_2(P) = 0.6079 m^{-2}$, $k_2(D) = -1.256 m^{-2}$. Figure 1 indicates the variation of the idealized gradients with $x$. Figure 2 shows that both the radial and vertical tune variation with momentum has indeed been reduced to zero. In addition, the increment in the gradient length at each end of the F and the D magnets is shown. This was obtained by fixing $\Delta S_F = \Delta S_D = 0$ and using the least squares adjustment to find the coefficients $A_F, A_D, B_F, B_D$, etc. The incremental gradient lengths so obtained correspond to an effective termination of the magnetic fields characterized principally by $A_F = -0.0354, A_D = -0.0301$, the conditions that make the end faces parallel.

The normalized gradients at 8 GeV excitation measured by R. E. Peters are shown in Figure 3. These gradients together with the effective termination used in the magnet design, namely, that the effective entrance and exit planes of the magnet are parallel and coincide with the end laminations, yield tune variations as shown in Figure 4. Clearly the effects of the finite pole width cause fluctuations in the gradient that are reflected in the tune variations.

In order to realize the design effective endings for the magnets, end packs were machined by numerically controlled contour milling. The surfaces chosen were derived basically
from two-dimensional reasoning, the variation with the radial dimension being introduced in such a manner as to permit the surface to match the design body contours smoothly. The effective termination of the magnetic fields after installation of these end packs was measured by Peters. Figure 5 shows the variation of the tunes with momentum for this case in which measured gradients and measured field terminations are employed. It is clear that the end packs have over compensated for the difficulty shown in Figure 4.

Figure 6 shows the result of asking the question, "What is the best shape for the effective field termination?" The least squares minimization mode of TUNA was activated using the measured values $\Delta S_F = 0.007689 \, \text{m}$ and $\Delta S_D = 0.01162 \, \text{m}$ for each of several polynomial degrees from 4 through 9. Only the results for an eighth degree fit are shown since all lower degrees gave tune variations outside of the band ±0.1. Also shown are the incremental gradient lengths that are derived from the eighth order effective termination shapes.

Table 1 presents the power series coefficients that express the shape of the effective magnetic field terminations according to Eqs. (34) and (35). Three cases are shown for each magnet: (1) measured gradients-design terminations, (2) measured gradients-measured terminations, (3) measured gradients-adjusted terminations. Table 2 presents the numerical calculation of the radial tune variation with momentum for each of the cases just mentioned and in addition the test case
in which the idealized gradient was used together with the design termination. Table 3 gives the same information as Table 2 except that it relates to the vertical tune.

In summary, then, the measurements of curves along which the interior fields of the magnets effectively terminate show that the simplified method of deriving an end pack shape needs modification. To date, a method of using the difference between the measured terminating curve and the desired terminating curve to generate a new iron shape has been devised. Its basic limitation stems from the fact that the pole width is larger than the aperture width and, hence, it is impossible to determine completely the pole shape. This problem is being considered further.
Ideal Normalized Gradients

\[ k_1(F) = 2.2147 \text{ m}^{-1} \]
\[ k_1(D) = -2.7719 \text{ m}^{-1} \]
BETATRON FREQUENCY SHIFT VS. MOMENTUM SHIFT

IDEAL GRADIENTS

DESIGN ENDS
MEASURED NORMALIZED GRADIENTS (PETERS)

\[ k_1(F) = 2.1006 \text{ m}^{-1} \]
\[ k_1(D) = -2.7627 \text{ m}^{-1} \]
\[ B_0(F) = 725.7 \text{ G} \]
\[ B_0(D) = 6152 \text{ G} \]

FIG. 3
BETATRON FREQUENCY SHIFT VS. MOMENTUM SHIFT
MEASURED GRADIENTS (PETERS)
DESIGN ENDS

FIG. 4
BETATRON FREQUENCY SHIFT VS. MOMENTUM SHIFT

MEASURED GRADIENTS (PETERS)
MEASURED ENDS (PETERS)

$\Delta \gamma_x$

$\Delta \gamma_y$

$\Delta L_e (in)$

$\chi_F (inches)$

(F)

(D)

 FIG. 6
BETATRON FREQUENCY SHIFT VS. MOMENTUM SHIFT
MEASURED GRADIENTS (PETERS)
ADJUSTED ENDS

\[ \Delta \beta_x, \Delta \beta_y \]

\[ X_F \text{(Inches)} \]

\[ \Delta L_e \text{(in)} \]

\[ \text{(D)} \]

\[ \text{(F)} \]

\[ \text{(inches)} \]

FIG. 6
Table 1. Coefficients for Effective Field Termination

\[ s_2 - s_1 = 2.8896m \]

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<th>F-Magnet</th>
<th>D-Magnet</th>
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*R. E. Peters*
Table 2. Radial Betatron Frequency Variation with Momentum

SYNCH tune = 6.700

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* R. E. Peters
Table 3. Vertical Betatron Frequency Variation with Momentum

SYNCH tune = 6.800

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*Note: R. E. Peters
REFERENCES

1. Note that near the magnet ends one does not know a priori whether to use the curvilinear coordinate system or a rectangular coordinate system until the actual location of the effective termination of the magnetic field has been found. Assuming that the terminating surface is about 1 centimeter from the last lamination, an estimate of the uncertainty introduced into the tune calculations in using Eqs. (26) and (28) instead of their cartesian equivalents is approximately 0.001. This effect is therefore neglected.


Purpose

Although the results of a previous note with the same title as above (FN-192) are correct when the magnet lengths are chosen to be identical with the design or comparison case (SYNCH), a small correction must be made for the reshaping of the closed orbits, if the magnet lengths are changed. This correction is included below.

Closed Orbit Correction

If $L_s$ designates the magnet length used in the design (SYNCH) of the booster lattice, then for increments $\Delta S$ to this length at each end of the magnet the net bending relation becomes

$$2NB_yF(0)(L_s + 2\Delta S_F) + 2NB_yD(0)(L_s + 2\Delta S_D) = 2\pi<B_B>,$$  \hspace{1cm} (1)

where $N$ is the sector number. From the turns ratio and the gap ratio of $F$ to $D$ magnets, one has

$$\frac{B_{yF}(0)}{B_{yD}(0)} = \frac{N_F}{N_D} \cdot \frac{G_D}{G_F} = \frac{48}{56} \times \frac{2.25}{1.64} = 1.175958. \hspace{1cm} (2)$$

The radii of curvature in the $F$ and $D$ magnets are
\[ \rho_F = \frac{\langle B_R \rangle}{B'_{YF}(0)} ; \quad \rho_D = \frac{\langle B_R \rangle}{B'_{YD}(0)} . \]  

Thus, for booster parameters, since \( L_{SF} = L_{SD} = L_s \)

\[ \rho_F = 18.77192 \left( 1 + \frac{2\Delta S}{L} + 1.175958 \left( 1 + \frac{2\Delta S}{L} \right) \right) , \]  

and

\[ \rho_D = 22.07492 \left( 1 + \frac{2\Delta S}{L} + 1.175958 \left( 1 + \frac{2\Delta S}{L} \right) \right) , \]

where the radii of curvature are measured in meters.

Since \( \Delta S_F \) and \( \Delta S_D \) were not considered adjustable in the least squares fitting procedure of FN-192, the formulation there is correct except that the radii of curvature associated with all measured quantities (M), for example \( k_{xM} \), should be changed to those given by Eqs. (4-5). All radii of curvature associated with the design or comparison quantities such as \( k_{xS} \) remain unchanged. These modifications have been incorporated into the TUNA code.

**Results**

Two changes are occasioned by the above corrections

1. Figure 5 (FN-192) -- \( \Delta v_x \) and \( \Delta v_y \) are both lowered at all points by approximately 0.05.

2. Table 1 (FN-192) -- The adjusted coefficients representing the best end shape become
3. Further corrections having no effect on results

   a. Equation (29) in FN-192 should have a capital S subscript.

   b. Equation (31) in FN-192 should read

\[ L_{xs} = \frac{1}{\rho} \]