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THE SENSIBLE SECONDARY BEAM ELEMENTS FOR THE 200 BEV ACCELERATOR

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1. There are many discourses on quadrupole scaling and beam transport elements for 50 - 150 BeV beams. (See yellow books 1, 2, and 3 and green books 1 and 2). This work adds to that already long list. Although the conclusions reached at the end of this article are not at variance with what is already said, (See M. Longo, UCRL Vol. 2, p. 22) the consequences of particle production at 200 BeV and a small emitance of the external proton beam when carried consistently throughout results in rather sensible quadrupole and bending magnet sizes.

2. To devise a relation between quad aperture length and beam momentum the following relations are used:

$$\vartheta_{\text{Cocconi}} = 0.5/p\,(\text{BeV/c}) \tag{1}$$

$$\text{Quad Gradient: } G = \frac{B_o}{r} = \frac{15\ \text{kG}}{r_{(\text{inches})}} \tag{2}$$

where **r** is quad radius.

Focal length of a doublet:

$$\frac{1}{f_{(\text{inches})}} = \frac{L^3 G^2}{(1312)^2 p^2}\left(\frac{d}{L} + \frac{2}{3}\right) \tag{3}$$

where L is the single quad length; d separation between quads p in BeV/c.

We assume that most beams will have a solid angle

$$\Delta\Omega = \frac{1}{10}\,\vartheta^2_{\text{Cocconi}} \tag{4}$$

(a rather reasonable value at present accelerators viz:

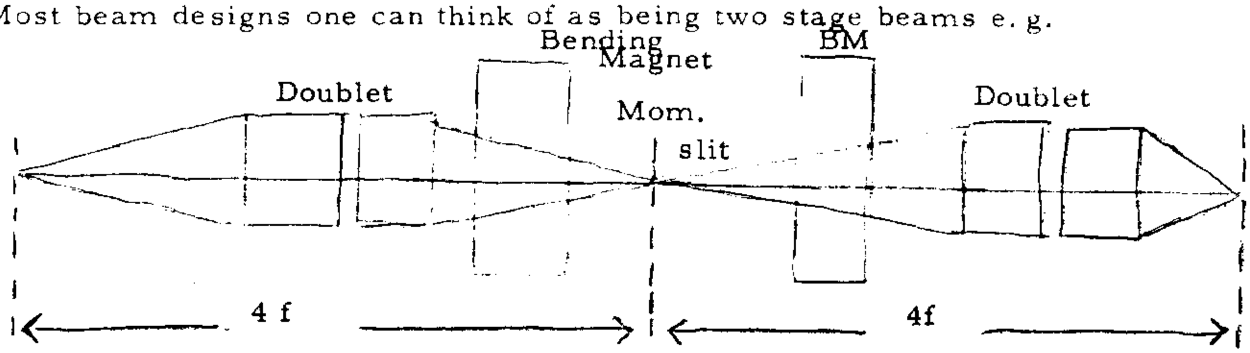
$\Delta\Omega \approx 10^{-3}$ sr at 5 BeV/c)

Using Equations (1), (2), (3), and (4), one obtains the following:

$$p\,(\text{BeV/c}) = \frac{L^3(\text{inches})}{r\ (\text{inches})}\ \frac{1}{1730} \tag{I}$$

Using Equation I, one can plot L (quad length) as a function of p for various r. This is given in Figure 1. One can see that for, say 2 in bore ($r = 1.0$ in.) quadropoles , quad length does not exceed 6 feet (72 inches) for highest practical beam momenta. In other words $L \sim \sqrt[3]{p}$ is slowly varying as a function of p.

3. Most beam designs one can think of as being two stage beams e.g.



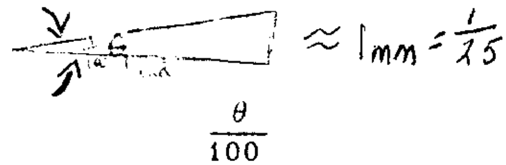
Using same formulas (1-4) one obtains:

$$S\begin{pmatrix}\text{total length}\\ \text{of beam in}\\ \text{inches}\end{pmatrix} = 45\ p\ (\text{BeV/c})\ \ r\ (\text{inches}) \qquad \text{II}$$

Expression II implies that for given sizes of quadrupole elements selected according to I, one varies beam length to match the solid angle. Of course at 100 BeV/c beam one can vary p to ±30% without modifying the beam. For a drastic momentum change, however, one shortens or lengthens the beam accordingly.

4. To arrive at the bending magnet lengths and apertures ~~to~~ the reasoning is complicated by experimental considerations. For what follows we chose to set the minimum momentum bite at ±0.5% $\frac{\Delta p}{p}$ and source size of 1 millimeter diameter. If now at the momentum slit one has 1 : 1 magnification (a condition easily met) the momentum dispersion need not be larger than 100 x source, i.e., 10 cm.

The input relations for these considerations are:



$$\frac{1}{100}\,\theta_{bend} = \frac{1}{25} : \frac{S}{8}$$

$$\theta_{bend} = 0.71\,\frac{1}{p_{BeV/c}\,r_{(in)}} \tag{5}$$

$$\theta_{bend} = \frac{B_o\,\ell}{(B\rho)_p} = \frac{B_o}{1310}\,\frac{\ell}{p} = 1.53 \times 10^{-2}\,\frac{\ell\,(\text{inches})}{p\,(\text{BeV})} \tag{6}$$

Therefore, for 1% $\frac{\Delta p}{p}$ one obtains the following by using (5) and (6):

$$\ell_{\text{inches, bend magnet at 20 kG}} = 46.5\,\frac{1}{r_{in}} \qquad \text{III}$$

Expression III implies bending magnet lengths independent of momentum but rather on quad aperture which in turn dictates the length of lever arms for bending through relation II.

The other way is to think of spot size being a fixed fraction of quadrupole aperature, say 1/2 in. at 10 in. bore quads; i.e., source size: $\frac{1}{10}$ r.

$$\frac{1}{100}\,\theta_{bend} = \frac{r}{10} : \frac{s}{8} \qquad\qquad \theta_{bend} = \frac{80\,r}{s} = \frac{80\,r}{45\,pr} = \frac{1.78}{p}$$

$$\theta_{bend} = 1.53 \times 10^{-2}\,\frac{\ell}{p} = \frac{1.78}{p}$$

$$\ell_{\text{inches, bend magn. length at 20 kG}} = 112 \text{ inches} \qquad \text{IV}$$

Finally, a comparison is made between a well thought of beam such as Longo UCRL-1, p. 137.

M. Longo - beam	This Recipe
p = 150 BeV	p = 150 BeV
4 quads, 2" diameter 64" long + 2 quads 4"	4 quads, 2" diameter, 64" long -------
2 BM 240 in. (15 kG)	2 BM 90" long (20 kG) *
bend at magnets 1.05°	0.52°
image size at 1st focus = 0.17 inches	0.05 inches
$(\Delta p/p)_{min} \approx 0.2\%$	$(\Delta p/p)_{min} = 0.5\%$
length of beam ~ 8000 inches	~ 7000 inches
$\Delta\Omega = 3 \times 10^{-6}$ sr.	$\Delta\Omega \cong 2 \times 10^{-6}$ sr.

*Bending magnets are doubled in length to allow $(\Delta p/p)_{min}$ of 0.5%.