ON CRITICISMS OF EINSTEIN'S EQUIVALENCE PRINCIPLE

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Abstract

Einstein's equivalence principle was initially the equivalence of an accelerated frame and uniform gravity. In spite of being often challenged, Einstein insisted on the fundamental importance of his equivalence principle to general relativity. It is shown that existing criticisms, starting from Synge and Fock, are due to misunderstanding and misconceptions in physics, and/or inconsistent considerations. These include the misinterpretations of Pauli, Bergmann, Tolman, Landau & Lifshitz, Zel'dovich & Novikov, Dirac, Wheeler, Thorne, Hawking, and others. It has been overlooked that Einstein's equivalence principle implies uniqueness of the gauge for a given frame of reference. The recent criticism by Hong has the distinction of starting from his intuitive, though inadequate, observation that "a homogeneous field is characterized by the fact that any part of it is representative of the whole". It is pointed out that his notion of uniform gravity disagrees with experiment on the gravitational redshift. His arguments concerning acceleration also disagree with special relativity, while repeating the same mistake of Landau & Lifshitz. Moreover, it is pointed out that the crucial role of Einstein's equivalence principle in general relativity is firmly established because the Maxwell-Newton Approximation, which is rigorously derived in the theoretical framework of general relativity, is unambiguously supported by experiments. Thus, the Schwarzschild solution is actually invalid in physics.

Key Words: Einstein's equivalence principle, frame of reference, the Euclidean-like structure, physical space-time.

1. Introduction

Since Einstein's three predictions are confirmed by observations [1,2,3,4], many theorists have accepted Einstein's field equation [3-16]. Moreover, it is generally agreed that Einstein's equivalence principle is the theoretical foundation of general relativity. However, Einstein's equivalence principle was also a favorite target for theorists such as Synge [5], Fock [6], Thorne [17], and Ohanian & Ruffini [14] who wanted to demonstrate a difference from Einstein.

Recently, Hong [18] also claimed that the initial form of Einstein's equivalence principle was invalid. In Hong's distinctive approach, he started from his notion of uniform gravity and then showed that uniform gravity is different from an accelerated frame; whereas others showed their failure to obtain a metric for uniform gravity from an accelerated frame. Hong's vague notion of uniform gravity seemed to support the belief that Einstein's equivalence principle was invalid. Unfortunately, this seems to be becoming popular since even Wheeler has been converted to a non-believer of Einstein's principles [14].

In this paper, it will be shown that Hong's claim is also invalid. For convenience of discussion, Einstein's equivalence principle is clarified first. To this end, there are few better ways other than to show what kind of mistakes the previous critics made. The errors can be classified into several types, including misunderstandings, misconceptions, miscalculations, and their combinations. A prevailing but incorrect belief is that Einstein's equivalence principle was non-essential to Einstein's predictions [5,19]. This shows the degree to which Einstein's equivalence principle was misunderstood (see § 5). The other well-known cases will be addressed and analyzed, if this is necessary, to show the errors. It is pointed out also that the Schwarzschild solution is invalid in physics because it is incompatible with Einstein's equivalence principle as well as some observations.

2. The Initial Form of Einstein's Equivalence Principle and Related Misinterpretation.

In 1911, Einstein explained the initial form of his equivalence principle in terms of uniform gravity and acceleration [1]. In 1916, subsequent to his principle of general relativity, Einstein [1] proposed his equivalence principle in the infinitesimal form for the general case of a four dimensional Riemannian physical space-time(1). However, a surprising fact is, as Einstein [19] saw it, that few like Eddington [3] understood Einstein's equivalence principle adequately (2).

Einstein's equivalence principle was challenged by Synge's [5] now popular identification of "true" gravitational fields with metrical curvature. Synge [5] "professed" his misunderstanding of Einstein's equivalence principle as follows:

> "...I have never been able to understand this principle...Does it mean that the effects of a gravitational field are indistinguish-able from the effects of an observer's acceleration? If so, it is false. In Einstein's theory, either there is a gravitational field or there is none, according as the Riemann tensor does or does not vanish. This is an absolute property; it has nothing to do with any observer's world line...The Principle of Equivalence performed the essential office of midwife at the birth of general relativity...I suggest that the midwife be now buried with appropriate honours and the facts of absolute spacetime be faced."

This shows that Synge does not understand the crucial role of applying Einstein's equivalence principle. In spite of this, Synge's view on Einstein's equivalence principle is currently dominant (see Sections 5 & 6), and misunderstandings persist. For instance, Thorne [17] criticized Einstein's principle as follows:

> "In deducing his principle of equivalence, Einstein ignored tidal gravitation forces; he pretended they do not exist. Einstein justified ignoring tidal forces by imagining that you (and your reference frame) are very small."

However, Einstein has already explained these problems. For instance, the problem of tidal forces had been answered in Einstein's July 12, 1953 letter to A. Rehtz [20] as follows:

> "The equivalence principle does not assert that every gravitational field (e.g., the one associated with the Earth) can be produced by acceleration of the coordinate system. It only asserts that the qualities of physical space, as they present themselves from an accelerated coordinate system, represent a special case of the gravitational field."

Here, Einstein has made clear that this principle is proposed for a physical space, where all physical requirements are sufficiently satisfied. As Einstein [19] explained to Laue, "What characterizes the existence of a gravitational field, from the empirical standpoint, is the non-vanishing of the Γ^l_{ik} (field strength), not the non-vanishing of the R_{iklm}." and no gravity is a special case of gravity. This view is crucial in general relativity because it allows Einstein to conclude that the geodesic equation is also the equation of motion of a massive particle under gravity, and this made it possible to conceive a field equation for the metric. Einstein insisted, throughout his life, on the fundamental importance of the principle to his general theory of relativity [19].

Einstein [1] derived the gravitational red shift from the initial form of his principle: the equivalence of a uniformly accelerated frame and uniform gravity. This is independent of the need for a Riemannian space with a Lorentz signature. A known deficiency of his results was his initial, incorrect formula for light deflection under gravity. Einstein [21] corrected this formula in 1915. The cause of this deficiency has been identified as account not being taken of the effects of curved space.

Nevertheless, an unverified belief advocated by Fock [6] and others [14,22] is that Einstein's equivalence principle could be intrinsically incompatible with the notion of a curved space. Such a belief must be very absurd to Einstein since his argument for a Riemannian space is based on the application of his equivalence principle to the uniformly rotating disk [1,2]. Recently, such a belief has been proven to be fundamentally incorrect since the Maxwell-Newton Approximation, a linear field equation for weak gravity, that produces light bending, has been derived [22] from Einstein's equivalence principle together with the notion of a Riemannian space assuming Newtonian theory is taken as a form of first order approximation[3).

On the other hand, uniform acceleration cannot exist forever; otherwise the resulting speed would exceed the velocity of light. Thus, uniform acceleration must be started at some time, and then decreased some time afterward. Moreover, uniform gravity must be confined in a finite region since it is equivalent to an accelerated frame; otherwise the light speed as the maximum velocity would be violated. Like an electromagnetic plane wave, uniform gravity also does not really exist in nature. Thus, the equivalence of acceleration and uniform gravity is best described, as Einstein did, in terms of a uniformly accelerated chest.[4)

Einstein assumed that the mechanical equivalence of an inertial system K under a uniform gravitational field, which generates a gravitational acceleration γ (but, system K is free from acceleration), and a system K' accelerated by γ in the opposite direction, can be extended to other physical processes. In Einstein's derivation of gravitational red shift [1], the equivalence principle is used only locally. This was somewhat unclear since the obvious, though crucial, step of replacing the infinitesimal energy γh by $\Delta\Phi$ (that is the difference of gravitational potential Φ for an infinitesimal distance h) is omitted. In practice, uniform gravity is essentially a local idealization of a non-uniform gravity, and a large-scale region of uniform gravity does not really exist. Thus, it is unrealistic to consider observing the bending of light in a field of uniform gravity at different locations as identical. Unfortunately, this is exactly the starting point of Hong's thesis [18].

Einstein assumed that the mechanical equivalence of an inertial system K under a uniform gravitational field, which generates a gravitational acceleration γ (but, system K is free from acceleration), and a system K' accelerated by γ in the opposite direction, can be extended to other physical processes. This initial form was further elaborated to an infinitesimal form for a curved space due additionally to the principle of general relativity [1,2]. Currently, the derivation of the Maxwell-Newton Approximation, which shows that the gravitational red shifts are directly related to g_{tt}, the time-time component of the space-time metric [1], should have removed any remaining doubt on the validity of Einstein's equivalence principle [22,23].

There are three important points related to Einstein's equivalence principle: 1) The equivalence principle is proposed for a physical space (-time), in which all physical requirements are sufficiently satisfied. 2) The equivalence principle is defined in terms of acceleration with respect to a frame of reference of Einstein's physical space [1,2]. 3) In a free fall, the resulting local space must be Minkowski. These points were clear in the initial form, but are often neglected by others in the infinitesimal form, and thus, Einstein's equivalence principle has been mistaken to be the same as Pauli's version [4].

Moreover, Pauli's version is still popular in spite of Einstein objected as a misinterpretation [19]. This is due to that the physical meaning of space-time coordinates is ambiguous in Einstein's theory [1,2,6,24]. Some theorists even regarded Einstein's equivalence principles as merely a heuristic argument [25] because they believed that the space-time coordinates were arbitrary. This is discussed in a paper that shows the uniqueness of the gauge for a given frame of reference [26].

3. Misunderstandings and Misrepresentations of Tolman and Fock.

Einstein concluded that physical reality involves curved space-time. However, the connection between an accelerated frame and a space-time metric has not been established as Einstein envisioned. Here, it will be shown that the main reason is conceptual errors. Theorists have incorrectly assumed that an accelerated frame must be related to a Euclidean subspace.

To apply Einstein's equivalence principle, it is crucial that the space-time under consideration must be a physical space. Theorists, both for and against general relativity, have made mistakes by ignoring this. For example, Logunov and Mestvirshvili [27] showed that inconsistent results would be obtained through a coordinate transformation. On the other hand, Tolman [12] also ignored this problem in his illustration of Einstein's equivalence principle. Thus, instead of the validity of Einstein's theory, Tolman seemed to show the opposite, i.e., arbitrariness and invalidity just as Logunov et al. claimed.

Tolman claimed that his treatment [12] is based on the relation of the principle of equivalence to the fundamental idea of the relativity of all kinds of motion. Tolman is a good example of those theorists who apply Einstein's equivalence principle without adequate understanding. To illustrate the equivalence principle, Tolman started with system K_0 with the flat metric,

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2, \tag{1}$$

for the first observer. Consider a second observer in a system K', which can be taken as moving relative to the first with acceleration a in the x-direction, and use coordinates x', y', z', and t' as given by

$$x' = x - \frac{1}{2}at^2 \qquad y' = y \qquad z' = z \qquad t' = t \tag{2}$$

according to the usual transformation to accelerated axes, which Tolman regards as a reasonable change at least at low velocities. Substituting from (2) into (1), Tolman thought that he obtained the formula for the interval for the second observer as

$$ds^2 = (c^2 - a^2t'^2)\, dt'^2 - 2at'dx'dt' - dx'^2 - dy'^2 - dz'^2. \tag{3}$$

Then, according to the geodesic equation, from metric (3) Tolman obtained

$$\frac{d^2x'}{ds^2} = \frac{-a}{c^2 - a^2t'^2}, \tag{4a}$$

and

$$\frac{d^2y'}{ds^2} = \frac{d^2z'}{ds^2} = \frac{d^2t'}{ds^2} = 0 \tag{4b}$$

as approximately the equations of motion for the case of particles having negligible velocity. Thus, Tolman claimed that the equivalence principle was illustrated by (4).

On the other hand, consider a particle P in K' at the beginning of a free fall. Since the velocity of K' relative to K_0 is v = at, for the local Minkowski space (X, Y, Z, T) of P, we have $dx = \gamma[dX + v\, dT]$, $cdt = \gamma[cdT + (v/c)dX]$, where $\gamma = [1 - (v/c)^2]^{-1/2}$. It thus follows from (3) that there is no time dilation although $\gamma dx' = dX$. Thus, Einstein's equivalence principle is not satisfied and K' is not a physical space. In addition, if metric (3) were valid in physics, $ds^2 = 0$ would imply the light speed to be at ± c in the x'-direction, and thus metric (3) violates coordinate relativistic causality. These demonstrate that Tolman does not understand the need of a local time in Einstein's theory [1,2].

Moreover, if Tolman's calculation were valid, he actually showed that Einstein's equivalence principle was invalid. In Einstein's [1] analysis, the effects of an accelerated frame can be related to a gravitational potential Φ, which is a function of spatial variables in Newtonian theory. But, all the metric elements of (3) are functions of time t'. Although $\Gamma^{x'}{}_{t't'} \neq 0$, the non-zero term in (4a) comes from $g_{t'x'}$ but not from $g_{t't'}$ (since $\partial_\mu g_{t't'} = 0$ for $\mu \neq t'$).

Tolman simply ignored that Einstein's later paper [1,2] confirms his 1911 analysis, and one has the relations,

$$g_{t't'} \approx 1 + 2\Phi/c^2, \qquad \text{and} \qquad a_i \approx -\partial\Phi/\partial x^i \tag{5}$$

where Φ is the negative gravitational potential and a function of x'. Obviously, (5) is not consistent with equation (4). Thus, if Einstein's equivalence principle is valid, metric (3) cannot be a physical space. Since Tolman's calculation is valid in Pauli's version [4] (see also Section 5), this illustrates that Pauli's version is actually incompatible with Einstein's theory.

In an attempt to overcome the deficiency of metric (3), in 1958 Fock [6] modified transformation (2) with

$$x = x' - \frac{1}{2} a t'^2, \qquad t = t' - at'x'/c^2. \tag{6}$$

Then, he obtained

$$ds^2 = (c^2 - 2ax' - a^2t'^2)\, dt'^2 - dx'^2 - dy'^2 - dz'^2 + a^2 (t'dx' + x'dt')^2/c^2. \tag{7}$$

The term 2ax' seems to serve the purpose, and metric (7) would be superficially compatible with relation (5).

However, the equation of motion even for dx'/ds = 0 is very complicated as follows:

$$\frac{d^2x'}{ds^2} = a\, c^{-2}\, (1 - a^2t'^2/c^2)^{-1}\, [(1 - ax'/c^2)^2 - a^2t'^2/c^2]^{-1}. \tag{8}$$

It is clear that (8) is not a uniform acceleration for a static particle since the right hand side depends on both space and time. Nevertheless, Fock [6] believed that the problem of time-dependence could be resolved within the speculated metric,

$$ds^2 = (c + ax'/c)^2\, dt'^2 - dx'^2 - dy'^2 - dz'^2, \tag{9}$$

Fock [11] proposed the following mathematically ingenious transformation,

$$x = x' \cosh (at'/c) + (c/a)[\cosh (at'/c) - 1] \tag{10a}$$

$$y = y'; \qquad z = z' \tag{10b}$$

$$t = (c/a) \sinh (at'/c) + (x'/c) \sinh (at'/c), \tag{10c}$$

although its physics is not clear. Under the condition at'/c << 1, the above equation can be written approximately as

$$x = x' + at'^2/2; \qquad y = y'; \qquad z = z'; \; t = t' \tag{11}$$

Substituting (10) into the flat metric, one obtains the metric (9) exactly. Finally, Fock has obtained a metric whose time dilation seems to be compatible with Einstein's paper of 1911. An important difference is that Einstein's is based on physical considerations, whereas Fock gave only a purely mathematical manipulation.

To determine the validity of a manifold as a physical space, the physics must be considered. Apparently, the mathematical requirement, $at'/c \ll 1$, instead of just $at'/c < 1$, is to make (11) approximately valid, but it does not seem to have a physical basis. Moreover, metric (9), in addition to being incompatible with the observed light bending, does not produce a uniform acceleration as claimed. The equation of motion for $dx'/ds = 0$, though better than (8), is not a uniform gravity as follows:

$$\frac{d^2x'}{ds^2} = -a\,c^{-2}[1 + ax'/c^2]^{-1}. \qquad (12)$$

As expected, Fock cannot find a valid interpretation for (12). Nevertheless, Fock believed that this is due to an intrinsic deficiency of Einstein's equivalence principle. This is a good example of a failure due to misconception being blamed on Einstein's equivalence principle. Unfortunately, Hong made the same kind of error [18].

The crucial conceptual error of Tolman and Fock is that they seem to believe that a frame of reference corresponds to a Euclidean subspace. This is not true since the metric of Einstein's rotating disk [1,2] with an angular velocity Ω is

$$ds^2 = (c^2 - \Omega^2 r'^2)\, dt'^2 - dr'^2 - (1 - \Omega^2 r'^2/c^2)^{-1} r'^2\, d\phi'^2 - dz'^2 \qquad (13a)$$

where

$$x' = r' \cos\phi', \quad y' = r' \sin\phi'. \qquad (13b)$$

Although metric (13a) does not have a Euclidean subspace, (13b) is a Euclidean-like structure, which is necessary for the angle ϕ' to be well defined. Similarly, a Euclidean-like structure is also included in other solutions such as the Schwarzschild solution,

$$ds^2 = (1 - 2MG/\rho)\, c^2dt^2 - (1 - 2MG/\rho)^{-1}d\rho^2 - \rho^2 d\theta^2 - \rho^2 \sin^2\theta d\varphi^2, \text{ for } \rho > 2MG \qquad (14a)$$

where

$$x' = \rho \sin\theta \cos\varphi, \qquad y' = \rho \sin\theta \sin\varphi, \qquad \text{and} \qquad z' = \rho \cos\theta. \qquad (14b)$$

Metric (14a) is a function of ρ ($= [x'^2 + y'^2 + z'^2]^{1/2}$). The radical ρ is related to Euclidean coordinates (x', y', z'), and thus the metric is defined in terms characteristic of a Euclidean-like structure, which corresponds to the metric when $M = 0$. The physical reason for the Euclidean-like structure is that a measuring rod, if attached to a frame, would also be under the influence of gravity [26]. Thus, the space-time coordinates are actually restricted (see also Appendix).

When a frame K', which is linearly accelerated in the x direction, has a relative velocity v towards the inertial coordinate system K, which was initially at rest relative to K', according to special relativity the metric for K' has the form,

$$ds^2 = (c^2 - v^2)\, dt'^2 - dr'^2 - (1 - v^2/c^2)^{-1}dx^2 - dy'^2 - dz'^2 \qquad (15)$$

by similar arguments used for the uniformly rotating disk. Thus, it is incorrect to assume that the frame of reference of K' corresponds to a Euclidean subspace. The metric for accelerated frames will be presented in a separate paper [28].

4. The Problems in Hong's Approach.

First, Hong [18] assumes unrealistically the existence of an extensive field of uniform gravity, whereas Einstein realistically considered an accelerated frame [1,2]. Then, Hong considered a light ray emitted horizontally in a field of vertical uniform gravity (see figure 1.). Assuming this light ray travels from O to A and that OB is the horizontal component and BA is the vertical component, then the time at O is t_0 and the light arrives at A at time t_A. At time t_0 the velocity of a free falling cabin has a velocity v_0 that is increased to v_A at time t_A. During the time interval $\Delta t = t_A - t_0$, a cabin travels from O' to A' whereas the cabin would have to travel to B' if its velocity remained unchanged at v_0. Then Hong claimed, "it is clear that the validity of this assertion requires the equality of B'A' and BA and the constancy of B'A'." (In general relativity, BA actually depends on the location.)

Figure 1. The path of a horizontally emitted ray of light and that of a free-falling cabin in vertically directed homogeneous gravitational field.

A crucial point is, however, that the rules governing the motions of the light and the cabin under uniform gravity have not yet been specified [18]. On the other hand, the above claim based on his intuition, requires a proof. This claim seems obvious only for the case $v_0 = 0$. Moreover, the time interval Δt may not be constant for traveling the same horizontal distance since, in general relativity [1,2], light speed depends on the metric, although a gravitational force depends on the differentials of the metric. Thus, in a homogeneous gravitational field, only for some aspects, any part of it is representative of the whole. Moreover, since a homogenous field must have a beginning end and a finishing end, any part of it does not really represent the whole.

Hong argued that B'A' cannot be a constant since Δv must decrease in the same time interval. An implicit assumption is that the region of uniform gravity is infinite, but this is not the reality. Uniform gravity exists only as a local idealization in nature. If uniform gravity is no longer valid beyond a region as reality suggests, then there is no reason for Δv to decrease.

Hong's other major argument is based on his belief that the accelerations of the cabin, which is an inertial system, and a particle with a velocity relative to the cabin, must be the same [29]. However, the constant acceleration of the cabin requires only a particle at rest in the cabin to have the same acceleration. Under the same force, both special and general relativity imply that particles with different velocity have different acceleration. (Hong seems also to be a victim of blind faith 5) to Landau & Lifshitz [29].). Thus, Hong's belief is a consequence of not accounting for the effects of special relativity. On the other hand, it is based on special relativity that Hong claimed his objection to a uniform acceleration of the frame.

In conclusion, since none of the arguments of Hong is valid, his objection to Einstein's equivalence principle is baseless.

5. Einstein's Equivalence Principle and Invalidity of the Schwarzschild Solution

From current textbooks, there are numerous versions of Einstein's equivalence principle [5-16]. However, they are essentially Pauli's version [4] in various forms. Pauli's [4, p.145] version of the equivalence principle is as follows:

> "For every infinitely small world region (i.e. a world region which is so small that the space- and time-variation of gravity can be neglected in it) there always exists a coordinate system K_0 (X_1, X_2, X_3, X_4) in which gravitation has no influence either in the motion of particles or any physical process."

Apparently, Pauli overlooked or disagreed with Einstein's [1, p.144] remark, "For it is clear that, e.g., the gravitational field generated by a material point in its environment certainly cannot be 'transformed away' by any choice of the system of coordinates, i.e. it cannot be transformed to the case of constant $g_{\mu\nu}$." The initial form of Einstein's principle stands against Pauli's version since the equivalence of uniform gravity and acceleration clearly requires considering the frame of reference. Also, there are unphysical solutions that have the proper signature [30], although they violate the principle of causality 6) [31].

Note that Galileo established the equivalence of inertial mass and gravitational mass. And mathematical theorems [5] established the existence of local Minkowski space. Thus, Einstein's contribution is only that in a free fall, the local space *must* be Minkowski. However, his contribution is an important one because it makes special relativity as a simple special case. Consequently, general relativity can be continuously and smoothly developed from special relativity. Thus, the Michelson-Morley experiment [1] should be considered as the first experimental support of Einstein's equivalence principle.

Moreover, mathematical covariance 7), which also does not require any physical meaning for coordinates, had to be used to replace the principle of general relativity [1,2]. It should be noted that unrestricted covariance is also incompatible with Einstein's equivalence principle. For example, Einstein [2] remarked, "As in the special theory of relativity, we have to discriminate between time-like and space-like line elements in the four-dimensional continuum; owing to the change of sign introduced, time-like line elements have a real, space-like line elements an imaginary ds. The time-like ds can be measured directly by a suitably chosen clock." On the other hand, Hawking [32] declared, "In relativity, there is no real distinction between space and time coordinates, just as there is no real difference between any two space coordinates." Nevertheless, Hawking [32] also inconsistently wrote, "an arrow of time, something that distinguished the past from the future, giving a direction of time". Thus, there is a distinction between a time coordinate and a space coordinate. A basic problem neglected in unrestricted covariance is that a tensor equation has no physical meaning unless a physically valid space-time coordinate system is used.

However, the erroneous viewpoints of Synge [5] and Fock [6], in particular their view that Einstein's equivalence principle is non-essential, have misled many. Theorists such as Friedman [31,33] advocated Pauli's version and theorists such as E. J. Weinberg [26,34] believed that the difference between these two versions is only philosophical and not physical (i.e., meaningless or at least unimportant). Moreover, the importance of Einstein's equivalence principle was somewhat unclear since its use in Einstein's predictions can be presented indirectly [8-15], and thus needs some clarification.

For instance, the gravitational red shift was incorrectly regarded as derivable with a coordinate-free method [10,11] since a valid space-time coordinate system, which satisfies Einstein's equivalence principle, must be assumed [26]. The deflection of a light ray can be derived without referring to local light speeds, and the connection between perihelion and Einstein's equivalence principle is neither direct nor obvious. Moreover, different gauges would give the same first order result for the integrated effect of light bending [2]. However, a coordinate system with certain physical meaning is needed because a perihelion and a deflection angle are defined in terms of the Euclidean-like structure.

Note that if accuracy to the second order is considered, the calculated results are no longer gauge invariant. Consider the Schwarzschild solution, the isotropic solution, and the harmonic solution respectively as follows:

$$ds^2 = (1 - 2M\kappa/r)\, c^2dt^2 - (1 - 2M\kappa/r)^{-1}dr^2 - r^2d\theta^2 - r^2 \sin^2\theta d\varphi^2, \tag{16a}$$

$$ds^2 = [(1 - M\kappa/2r)^2/(1 + M\kappa/2r)^2]\, c^2dt^2 - (1 + M\kappa/2r)^4\, (dr^2 + r^2d\theta^2 + r^2 \sin^2\theta d\varphi^2) \tag{16b}$$

$$ds^2 = [(1 - M\kappa/r)/(1 + M\kappa/r)]c^2dt^2 - [(1 + M\kappa/r)/(1 - M\kappa/r)]dr^2 - (1 + M\kappa/r)^2\rho^2(d\theta^2 + \sin^2\theta\, d\varphi^2) \tag{16c}$$

where

$$x = r \sin\theta \cos\varphi,\ y = r \sin\theta \sin\varphi,\ z = r \cos\theta, \text{ and } r = [x'^2 + y'^2 + z'^2]^{1/2}, \tag{16d}$$

$\kappa = G/c^2$ ($G = 6.67 \times 10^{-8}$ erg cm/gm^2), and M is the total mass of a spherical mass distribution with the center at the origin of the frame of reference. Since these metrics have different time-time components, they give different gravitational red shifts.

All three predictions of Einstein are expressed in terms of the Euclidean-like structure (16d) [26] (i.e. the usual measurement [1]), whose physical existence depends on a satisfaction of Einstein's equivalence principle. Thus, Einstein's predictions are inextricably related to Einstein's equivalence principle, but not Pauli's version.

The crucial role of Einstein's equivalence principle in general relativity becomes even more obvious after the physical meaning of space-time coordinates are clarified [26]. Einstein's equivalence principle implies that the realistic gauge is unique for a given frame of reference. Thus, Einstein's equivalence principle leads to the need to find out experimentally the gauge of the gravitational field of the earth [26]. This also means that validity of diffeomorphic solutions must be justified individually.

In view of this, the Schwarzschild solution is in a very unfavorable position because there is no specific experimental support other than the four standard tests. Although gravity can be theoretically anisotropic, for the sourceless cases, different situations give different space contractions. On the other hand, the first order of the isotropic and the harmonic solutions is,

$$ds^2 = (1 - 2M\kappa/r)\, c^2dt^2 - (1 + 2M\kappa/r)\, (dx^2 + dy^2 + dz^2), \tag{17}$$

which is compatible with Einstein's equivalence principle (i.e., eq. (106) in [2]). Moreover, metric (17) can be derived with the Maxwell-Newton Approximation, which is independent of Einstein equation and is supported by observations [8,23].

The Maxwell-Newton Approximation with a massive energy-stress tensor $T(m)_{ab}$ is,

$$\frac{1}{2}\partial^{c}\partial_{c}\bar{\gamma}_{ab} = -K\,T(m)_{ab}, \qquad (18)$$

where

$$\bar{\gamma}_{ab} = \gamma_{ab} - \frac{1}{2}\eta_{ab}(\eta^{cd}\gamma_{cd}) \qquad \text{and} \qquad \gamma_{ab} = g_{ab} - \eta_{ab}.$$

Since (18) is derived independently of Einstein equation [22,23], it is therefore is different from the so-called "linearization".

Equation (18) is the basis of a very accurate Einstein-Infeld-Hoffman equation for tracking planets spacecraft [8]. Moreover, highly accurate experiments on binary pulsars also support 8) the Maxwell-Newton Approximation [22,23,31,35]. Accordingly, it has been predicted [31] that the Gravity Probe-B gyroscopes [36] on the precessions would further confirm this approximation and Einstein's equivalence principle. Therefore, it is also expected also that the experiments on horizontal local light speeds [26] will directly support the Maxwell-Newton Approximation, while rejecting the Schwarzschild solution. Thus, this is a new test since the first order isotropic of local light speeds are not shared by metrics that are incompatible with this approximation.

6. Discussions and Conclusions

It has been shown that misinterpretations of, or objections to, Einstein's equivalence principle are due to inadequate understanding of Einstein's theory and physics in general. In particular, Hong's arguments are based on a number of beliefs that are unverified or simply invalid in physics. Hong's basic belief that "a homogeneous field is characterized by the fact that any part of it is representative of the whole", implicitly assumes that phenomena depend on gravitation field but not on the gravitational potential. This belief is based on naïve intuition rather than full consideration of the facts of the entire situation in gravity, and a proper accounting the constraints imposed by observation.

A crucial difference between Einstein and Hong is that Einstein realistically treats uniform gravity only as a local idealization, whereas Hong unrealistically considered uniform gravity existing in a region of infinite extent with hypothetical characteristics. Experimentally, it is known that the gravitational potential, but not the gravitational field, determines gravitational redshifts [1,2,14]. Moreover, this belief is equally untrue in electrodynamics since experiments [37] show that the electromagnetic potential actually has physical influence just as Aharonov & Bohm predicted [38].

Einstein's equivalence principle is proposed for a *physical space*, where all physical requirements are sufficiently satisfied. Nevertheless, Zel'dovich & Novikov [15] believed the equivalence principle means only that a particle follows the geodesic. This is incorrect even in the simple case of special relativity. Following Landau & Lifshitz [29], they also believed incorrectly that everybody had the same acceleration in an accelerated frame. A subtle way of criticizing Einstein's equivalence principle is to simply ignore it [26]. For instance, Wald [10] ignored Einstein's equivalence principle, but regarded the equivalence principle just as the equivalence of inertial mass and the passive gravitational mass as Ohanian & Ruffini [14] did. Wald probably realized that. Einstein's equivalence principle is incompatible with the belief that diffeomorphic manifolds are identical in physics. Peng & Xu [39] incorrectly believed that such an issue should be decided by future experiments, although they correctly pointed out that general relativity is separated from the rest of physics because the physical meaning of space-time coordinates is not clear. For similar reasons, some theorists regarded Einstein's principles as merely heuristic arguments [25].

Currently, it is difficult to find a book on general relativity that presents Einstein's equivalence principle correctly, except those by Einstein [1,2] and Eddington [3]. Moreover, according to Fock [6] and Whitehead [40], even Einstein himself seemed to be unable to explain his own principle *precisely* since Einstein's notion of coordinates is ambiguous. Their view is obviously supported by the fact that few of Einstein's disciples were able to interpret the physics of Einstein's equivalence principle adequately. A possible exception seemed to be Zhou Pei-Yuan [41,42], who designed an experiment on local light speeds 9).

Attempts to disprove or misinterpret Einstein's equivalence principle will be continued and encouraged so long as the physical meaning of coordinates is not well understood. The crucial role of Einstein's equivalence principle in general relativity becomes even more obvious after the physical meaning of space-time coordinates are clarified 10) since a valid realistic gauge is unique for a given frame of reference [26]. In addition, many theorists probably were also unaware that the physical meaning of symmetry in the metric solutions is actually based on the meaning of coordinates6).

Moreover, as illustrated by the local distance formula 5) of Landau & Lifshitz [29], Pauli's version incorrectly leads to the acceptance of any Lorentz manifolds as valid in physics [30]. Fortunately, Einstein's equivalence principle, which enables us to have a clear physical meaning of coordinates, would reject existing unphysical Lorentz manifolds. Note that both Whitehead [40] and Fock [10] rejected general relativity because of their philosophies on "uniformity" of space-time. However, their required "uniformity" is, in a way, implied by Einstein's equivalence principle [26,43].

Since Einstein's equivalence principle is a crucial physical requirement for choosing a solution, the theory of black holes can no longer be based on the presumed physical validity of the Schwarzschild solution [14,44]. Understandably, theorists such as Wald [10], Lue & Weinberg [44], and Landau & Lifshitz [29] actually rejected Einstein's equivalence principle. They probably were not aware that unrestricted covariance would mean that the event horizon of a black hole is related to an arbitrary integral constant [45]. Moreover, Zhou's experiment of local light speeds would appear to be irrelevant if one believed in Liu's "definition" of light speeds [13], which are always isotropic for any othorgonal coordinate system (see Appendix).

Since the Maxwell-Newton Approximation is supported by observation, it is no longer justifiable to question, as the board of Classical and Quantum Gravity did 11), the validity of Einstein's equivalence principle (i.e., eq. (106) in [2]). Unlike Einstein [1,2], they failed to see that the physical realization of a local coordinate system in terms of a free floating is a local Minkowski space. (Instead of as a physically must, Pauli considered a local Minkowski space to be obtained by just a mathemat ical transformation.) It should be noted also that Einstein's equivalence principle is compatible with the principle of causality, whereas Pauli's version is not. Moreover, since the Schwarzschild solution has been demonstrated with the help of experiments to be invalid in physics [22,23,31], general relativity is not a product of just pure thought [29]. Nature is the ultimate authority of science.

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Appendix: The Questionable Local Distance Formula of Landau & Lifshitz

Although most physicists can use special relativity to do certain calculations, many of them, including Nobel Laurels Pauli and Landau, actually do not understand the some aspects of special relativity. This is evident since these physicists accepted Pauli's version of the equivalence principle. This will be illustrated with the local distance formula of Landau & Lifshitz [29].

We have already learned from special relativity that a coordinate system may not be physically realizable [1]. Nevertheless, Landau & Lifshitz [29] believed that one could start from an arbitrary triplet of coordinates x^1, x^2, x^3, and the time coordinate x^0, and then determine the "actual distances intervals" as follows:

$$dl^2 = -[g_{\alpha\beta} - g_{0\alpha}\, g_{0\beta}/g_{00}]\, dx^\alpha\, dx^\beta \qquad \text{where } \alpha, \beta = 1, 2, 3 \qquad \text{(A1)}$$

for

$$ds^2 = g_{00}(dx^0)^2 + 2\, g_{0\alpha} dx^0 dx^\alpha + g_{\alpha\beta}\, dx^\alpha\, dx^\beta, \qquad \text{(A2)}$$

if the metric satisfies the so-called "physical condition", $g_{00} > 0$,

$$\begin{vmatrix} g_{00} & g_{01} \\ g_{10} & g_{11} \end{vmatrix}(-1) > 0, \qquad \begin{vmatrix} g_{00} & g_{01} & g_{02} \\ g_{10} & g_{11} & g_{12} \\ g_{20} & g_{21} & g_{22} \end{vmatrix} > 0, \qquad \text{and} \qquad \begin{vmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{vmatrix}(-1) > 0. \qquad \text{(A3)}$$

This condition (A3) is based on the requirement of a proper metric signature such that a local Minkowski metric must exist [8].

The derivation is logically as follows: For an arbitrary metric $g_{\mu\nu}$, one has

$$ds^2 = g_{00}[dx^0 + g_{0\alpha}dx^\alpha/g_{00}]^2 + [g_{\alpha\beta} - g_{0\alpha}g_{0\beta}/g_{00}]\, dx^\alpha\, dx^\beta . \qquad \text{(A4)}$$

Then

$$ds^2 = g_{00}(dx')^2 + g'_{\alpha\beta}\, dx^\alpha\, dx^\beta, \qquad \text{(A5)}$$

where

$$dx'^0 = dx^0 + g_{0\alpha}dx^\alpha/g_{00} \qquad \text{and} \qquad g'_{\alpha\beta} = g_{\alpha\beta} - g_{0\alpha}g_{0\beta}/g_{00}$$

are respectively a new time coordinate and new metric elements. Thus, one could start with an arbitrary coordinate system and derive an orthogonal coordinate system. Unfortunately, the local frame (dx^1, dx^2, dx^3) is not always physically realizable. Einstein [1] requires that one *"be obliged to define time in such a way that the rate of a clock depends upon where the clock may be."* Therefore, the underlying physics of this derivation should be examined.

If the frame of reference (x^1, x^2, x^3.) is physical realizable, the passage of time at a given point in space would be that $dx^1 = dx^2 = dx^3 = 0$ [29]. Then, ds will be the time separation between two nearby events; i.e., $ds = c\, d\tau$. Consequently,

$$d\tau = [(g_{00})^{1/2}/c]\, dx^0 , \qquad \text{and} \qquad \tau = \frac{1}{c}\int \sqrt{g_{00}}\, dx^0. \qquad \text{(A6)}$$

Then, it follows from (A6) that (A5) is obtained. The problem is that (A6) may not be the local time since, as shown by the Michelson-Morley experiment [1] and special relativity, a mathematical frame of reference may not be realizable. Moreover, since some Lorentz manifolds cannot be diffeomorphic to a physical space [22,31], their formula is also misleading in physics.

Nevertheless, Zel'dovich & Novikov [15] applied (A1) successfully to Einstein's [1,2] rotating disk since this frame is realizable. Here, a counterexample will be given. Now consider the flat metric (1) and the Galilean transformation,

$$x = x', \qquad y = y', \qquad z = z' - vt', \qquad \text{and} \qquad t = t'. \tag{A7}$$

Then

$$ds^2 = [dz' + (c - v)dt'][-dz' + (c + v)dt'] - dx'^2 - dy'^2. \tag{A8}$$

where the units are also "cm" for length and "sec" for time, because the subspace (x', y', z') has a Euclidean-like structure. Since a Galilean transformation is proven physically not realizable (for instance, by the Michelson-Morley experiment [1]), the system (x' y', z', t') is only a mathematical coordinate system, but not an inertial system. In other words, metric (A8) cannot be used for tasks involved physical interpretations. Thus, we may expect that the formula of Landau and Lifshitz would fail. Based on formula (A1), from metric (A8) one obtains,

$$dl^2 = dx'^2 + dy'^2 + (1 - v^2/c^2)^{-1}dz'^2. \tag{A9a}$$

and

$$cdt'' = cdt + (v/c)dz' \, [1 - (v/c)^2]^{-1} \tag{A9b}$$

Formula (A9a) is clearly incorrect for a local distance in the case of no gravity, and t" is not the local time. (A9) is related to

$$ds^2 = (1 - v^2/c^2)\, c^2dt''^2 + dx'^2 + dy'^2 + (1 - v^2/c^2)^{-1}dz'^2. \tag{A10}$$

which is a constant metric, but not the flat metric of the Minkowski space for special relativity. Moreover, the dl in (A9) is obviously not the "actual distance interval", since the meaning of space coordinates has been clarified (see Sections 3 & 5). Thus, (A9) does not satisfy Einstein's equivalence principle [1] although Pauli's version is satisfied. Therefore, metric (A10) is not realizable, and this contradicts the original assumption of Landau & Lifshitz [29].

Nevertheless, Liu [13, p.38] argued that metric (A10) could be used to recover a flat metric, by using the transformation,

$$dT = (1 - v^2/c^2)^{1/2}\, dt'', \qquad \text{and} \qquad dZ = (1 - v^2/c^2)^{-1/2}dz' \tag{A11}$$

and

$$ds^2 = c^2dT^2 + dx'^2 + dy'^2 + dZ^2 \tag{A12}$$

is obtained. However, since there is no physical cause for (A11), it is only a rescaling. In other words, (A11) cannot be regarded as consequences of Einstein's equivalence principle. Thus, in metric (A12) the unit of the time T would be in $(1 - v^2/c^2)^{-1/2}$ (sec) and the unit of the Z-axis is in $(1 - v^2/c^2)^{1/2}$ (cm). And the light speed in the x- and the z-direction would respectively be

$$(1 - v^2/c^2)^{1/2}c \qquad \text{and} \qquad (1 - v^2/c^2)c. \tag{A13}$$

Thus, transformation (A11) cannot justify metric (A10). Besides, the issue here is whether (A1) presents the "actual distance interval", but not whether metric (A10) can be transformed into a physically valid space-time metric. It should be noted that if one of two diffeomorphic manifolds is a physical space, but the other is not, the diffeomorphism is not a physical transformation.

Liu's argument, like many others, showed only that he does not understand the physical meaning of space-time coordinates. Further evidence of this is supplied by the fact that Liu [13, p. 39], in disagreement with Einstein, defined a coordinate light speed as follows:

$$\frac{d\ell}{dt} = \omega_{cor}(n^i) = \frac{c\sqrt{g_{00}}}{\gamma_i n^i + 1} \qquad \text{where} \qquad \gamma_i \equiv -g_{0i}/\sqrt{g_{00}}\,, \tag{A14}$$

and n^i is the directional vector. For an orthogonal metric, one has simply $\omega_{cor} = c\sqrt{(g_{00})}$, which is essentially the invalid 1911 formula of Einstein [1]. Also, from the metric (A10), one would obtain the incorrect light speed $(1 - v^2/c^2)^{1/2}c$.

ENDNOTES

1) According to Einstein [1,2], a space-time metric g_{ik} of a (pseudo-) Riemannian space M is determined by the distribution of matter. Moreover, since Einstein's Riemannian space-time models reality, all the physical requirements must be sufficiently satisfied by the space-time metric g_{ik}. However, physical requirements are often understood through long processes. For instance, the notion of energy was started with only mechanical energy.

Moreover, his equivalence principle remains to be clarified since his space-time coordinates were ambiguous. Einstein has indicated the difficulty of presenting general relativity "precisely" as mathematics. Einstein wrote in 1916 [1] the following:

> "It is not my purpose in this discussion to represent the general theory of relativity as a system that is as simple and logical as possible, and with the minimum number of axioms; but my main object is to develop this theory in such a way that the reader will feel that the path we have entered upon is psychologically the natural one, and that the underlying assumptions will seem to have the highest possible degree of security."

Another problem is that he has not been able to describe precisely the physical process (due to a free fall), which transforms a metric near a point to a local Minkowski space although he infers the correct result.

2) Einstein also praised Eddington's book of 1923 [3] to be the finest presentation of the subject ever written [46].

3) While Einstein's equation was guessed, the Maxwell-Newton Approximation is derived independent of Einstein's equation. The energy conservation law $\nabla^i T_{ik} = 0$ was used, but the requirement is actually only that $\nabla^i T_{ik}$ is of the first order [22].

4) This is commonly, but mistakenly, known as "Einstein's elevator" (due to Bergmann) [9]. However, to avoid the usual association of an elevator with the gravity of the earth, Einstein actually used the word "chest" [47].

5) Landau & Lifshitz [29] made the same erroneous claim, "A body of arbitrary mass, freely moving in such a system of reference, clearly has relative to this system a constant acceleration, equal and opposite to the acceleration of the system itself." It is known that in special relativity the "sum" of two velocities u and v is $(u + v)/(1 + uv/c^2)$. If a particle has a constant velocity v in the direction of the acceleration $a= du/dt$, the acceleration with respect to this particle is a, with a factor $[(1 - v^2/c^2)^{1/2}/(1 + uv/c^2)]^3$. In addition, they are confused on the physics of space-time coordinates (see Appendix). Note that Pauli [3], Tolman [12], and Fock [6], also made errors in special relativity [48,49].

6) The time-tested assumption that phenomena can be explained in terms of identifiable causes is called the *principle of causality*. This principle is the basis of relevance for all scientific investigations. This principle implies that any parameter in a physical solution must be related to some physical causes. Thus, if an unphysical parameter exists in the metric, such a manifold would not be diffeomorphic to a physical space [22,30], and symmetry is preserved unless some causes break it [1-16]. In general relativity, Einstein and subsequent theorists have used this principle implicitly in symmetry considerations [1-16]. However, many failed to recognize this requirement on symmetry is also a restriction for a valid source [50]. For example, if a source was a bounded (in amplitude) function f(ct - z), then the Maxwell equation is incompatible with such a requirement [51]. Moreover, this requirement on symmetry is crucial to show that there is no dynamic solution for the Einstein equation [31]. Theorists, including Feymann [52] and Einstein [53], had incorrectly assumed the existence of such dynamic solutions.

7) Einstein [1] declared, "The general laws of nature are to be expressed by equations which hold good for all systems of coordinates, that is, are covariant with respect to any substitutions whatever (generally covariant)." However, this is different from the principle of general relativity [1], in at least two points: - First, a tensor equation is physically meaningful only if the coordinate systems are realizable. Second, not all the laws are tensor equations. For instance, Einstein's equivalence principle and the principle of causality are not just tensor equations, and thus can be incompatible with covariance [26]. Also, Zhou's theory [41], which is based on Einstein's equivalence principle, is incompatible with general covariance. Nevertheless, this crucial point does not seem to be recognized by Peng & Xu [39].

8) As shown by Hu, Zhang, & Ding [54], the calculated gravitational radiation depends on the approach used. However, Einstein's radiation formula based on the Maxwell-Newton Approximation is supported by observation [31].

9) Yilmaz [55] proposed a similar experiment in 1979 to test the difference between his theory and Einstein's theory. However, because of interference from mechanical stresses due to gravity, Zhou's experiment failed to reach the required accuracy [42,43].

10) One can easily imagine a curved two-dimensional space as a surface immersed in Euclidean three-dimensional space. In the same way, Dirac [56] reasoned, one can have a curved four-dimensional space immersed in a flat space of a larger number of dimensions. Such a curved space is called a Riemannian space. Dirac believed, "Einstein assumed that physical space is of this nature and thereby laid the foundation for his theory of gravitation." Therefore, Dirac continued, "For dealing with curved space one cannot introduce a rectilinear system of axes. One has to use curvilinear coordinates." However, Weinberg [7] has shown that the coordinates of a

curved space need not be curves. In fact, Einstein's curved physical space has a rectilinear system of axes for physical reasons [26,43]. However, the invariant line element ds^2 in Einstein's space-time is generally not Minkowski-like.

11) A Board member of Classical and Quantum Gravity claimed [57] that the application of Einstien's equivalence principle (that leads to the approximate eq. (106) in [2]) is a reason to question whether the basic physical and mathematical concepts involved in the interpretation of general relativity is understood. (For example, consider the isotropic solution (16b), $ds^2 = [(1 - M\kappa/2r)^2/(1 + M\kappa/2r)^2]\ c^2dt^2 - (1 + M\kappa/2r)^4(dx^2 + dy^2 + dz^2)$. Assuming its validity, Einstein's equivalence principle would imply that, for a resting observer at free fall, the metric of the local space is $ds^2 = c^2dT^2 - dX^2 - dY^2 - dZ^2$ such that $c^2dT^2 = [(1 - M\kappa/2r)^2/(1 + M\kappa/2r)^2]\ c^2dt^2$ and $dX^2 + dY^2 + dZ^2 = (1 + M\kappa/2r)^4\ (dx^2 + dy^2 + dz^2)$.) Also, some still believed incorrectly that *an ideal observer immersed in a gravitational field can choose a reference frame in which gravitation goes unnoticed*, as Pauli did. In fact, this rejection of Einstein's equivalence principle, but acceptance of Pauli's version is the dubious foundation of many articles. Moreover, to avoid incompatibility between Pauli's version and observation, it has been claimed that the local light speeds are regarded as physically meaningless and the coordinates were arbitrary. However, such a claim is in conflict with the simple fact that there are non-scalars in physics [26].

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Résumé

Le principe d'équivalence d'Einstein était initialement l'équivalence d'un cadre accéléré et la pesanteur uniforme. Bien qu'il était souvent défié, Einstein exigait sur l'importance fondamentale de son principe d'équivalence à la relativité générale. C'est montré que les critiques actuelles, commencent avec les critiques de Synge et de Fock, sont les résultats de la méprise et de la mauvaise compréhension dans la physique et/ou des considerations incohérentes. Les erreures d'interprétations de Pauli, Bergmann, Tolman, Landau et Liftshitz, Zel'dovich et Novikov, Dirac, Wheeler, Thorne, et Hawking sont comprises. Ils n'ont pas réussi à voir que le principe d'équivalence d'Einstein insinue l'originalité de la jauge pour un cadre de référence. La critique récente de Hong a la distinction de commencer, de son intuition, mais l'observation insuffisante "qu'un champ homogène est caractérisé par le détail qu'aucune partie est représentative d'entier." C'est montré que sa compréhension de la gravité uniforme n'est pas d'accord avec l'expérience sur redshift gravitationnel. Ses arguments pour l'accélération, en conflit avec la rélativité spéciale, ont répété la même erreur de Landau et Liftshitz. De plus, c'est montré que le rôle crucial du principe d'équivalence d'Einstein dans la relativité générale est bien fondé parce que l'Approximation de Maxwell-Newton, qu'elle est derivée rigoureusement du cadre théorique de la relativité générale, est approuvée sans ambiguïté par les experiences. Donc, la solution de Schwarzschild est vraiment invalide dans la physique.