Misinterpretation of $E = Mc^2$

and

Einstein's Theory of General Relativity

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Abstract

Currently, the famous formula $E = mc^2$ is often misinterpreted as the unconditional equivalence between inertial mass and each type of energy (i.e., $m = E/c^2$). It is shown that, according to Einstein's general relativity, such a claim is incorrect. The root of this problem is due to an inadequate understanding of special relativity that produced the famous equation $E = m c^2$ that, as Einstein clarified, must be understood in terms of energy conservation. For example, the Reissner-Nordstrom metric illustrated that electromagnetic energy and mass are different in terms of gravity. Concurrently, it is pointed out that this error is a problem in Will's book, "Theory and experiment in gravitational physics."

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Key Words: Einstein's Equivalence Principle, Einstein-Minkowski Condition, Euclidean-like Structure, $E = mc^2$.

1. Introduction.

General relativity is established on two principles [1, 2], namely: 1) Einstein's equivalence principle, which requires the Einstein-Minkowski condition that a free falling point-like particle in a gravitational field is along a geodesic and results in a co-moving local Minkowski space; and 2) the principle of general relativity, that is "The law of physics must be of such a nature that they apply to systems of reference in any kind of motion." However, in current theory, there are other additional implicit assumptions such as the universal coupling [3].

In Newtonian theory of gravity, the universal coupling is limited to massive matter. Einstein extended the universal coupling to include the electromagnetic energy-stress tensor. On the other hand, as pointed out by Pauli [4], the theoretical framework of general relativity actually allows the existence of the (antigravity) coupling of different sign. However, some theorists rejected such a possibility because they interpreted $E = mc^2$ as and unconditional equivalence between mass and energy. Although for the universal coupling to include electromagnetism such an interpretation is not needed [3], there is no other justification for the unconditional universal coupling.

Moreover, the unconditional universal coupling is a crucial assumption for the singularity theorems of Hawking and Penrose [5]. Thus, theorists who believe in the notions of black holes and big bang, would accept the interpretation that $E = mc^2$ as and unconditional equivalence between mass and energy.

Naturally, such an interpretation was not questioned until the universal coupling was proven to be incorrect [6, 7] by the Hulse-Taylor experiment. By then, such a misinterpretation of $E = mc^2$ has become a prevailingly accepted assumption. For instance, this assumption is implicitly used in Will's interpretation of the Kreuzer Experiment [8]. An examination of Will's interpretation led to a paper [9] that criticized his interpretation of the Kreuzer Experiment and $E = mc^2$ as invalid. After more than four years of deliberation, the paper [9] that criticizes misinterpretation of special relativity was published in the Astrophysical Journal and Will was an open referee.

However, although Will was defeated because he cannot defend his interpretation of $m = E/c^2$, he did not admit his mistake. This is evident since Will did not make any change in his book [10], ***Theory and experiment in gravitational physics*** that used his misinterpretations $m = E/c^2$ to the Kreuzer Experiment and other experiments. In fact, after more than eight years, not only he did not add a note on his mistake to help his readers, but also keep the comments of Nature and Science in the cover that have the effect of misleading the readers. Such a behavior seems very strange for a scientist.[1)] Recently, I discovered an explanation. I found a new misinterpretations of the Reissner-Nordstrom metric [11, 12] that Will failed to reconcile with $m = E/c^2$. These new interpretations seems to be able to reconcile the metric and $m = E/c^2$. Unfortunately, these new interpretations are also inconsistent with general relativity.

2. Invalid Interpretation **of the Reissner-Nordstrom metric**

An implicit assumption, i.e., any type of energy is equivalent to mass

$$m = E/c^2, \tag{1}$$

is used in Will's book and papers. Unfortunately, according to general relativity, this is simply not true. This is due to that the source of an Einstein equation is an energy-stress tensor [11], and thus the equivalence of energy-mass is restricted. For example, the electromagnetic energy and mass are not equivalent, since an electromagnetic stress tensor is traceless. This has been explicitly manifested by the Reissner-Nordstrom metric [11, 12],

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right)dt^2 - \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right)^{-1} dr^2 - r^2 d\Omega^2, \tag{2}$$

where q and M are the charge and mass of a particle and r is the radial distance (in terms of the Euclidean-like structure [2]) [13, 14] that Einstein [1] called as "in the sense of Euclidean geometry") from the particle center. In this metric (2), the gravitational components generated by mass and electricity have different signs and furthermore, very different radial coordinate dependence.

Some might argue that the total electric energy outside a sphere of radius r is $q^2/2r$, and thus the effective mass is

$$M - \frac{q^2}{2r}. \tag{3}$$

Thus (3) could be interpreted as supporting $m = E/c^2$ at least for electromagnetic energy. Such a view aims at a narrow point but misses the whole picture. There are several difficulties raised from such a view:

1) If any energy has a mass equivalence, an increase of energy should lead to an increment of gravitational strength. However, although energy increases by the presence of a charge, the strength of a gravitational force, as shown by metric (3), decreases everywhere.
2) If the electric energy is assigned a mass, should it be considered as part of the gravitational mass of the particle or not. If it is then gravitational mass and inertial mass are different. If it is not that means any electromagnetic energy should assign a mass.
3) If any electromagnetic energy should assign a mass equivalence, then this means a rejection of the notion that a photon is massless and that special relativity is valid.

In the above, problems were created because one ignores that the electromagnetic energy-stress tensor is traceless, but the massive energy-stress tensor is not. However, to interpret that electromagnetic energy has a mass equivalence, the coincidence (3) is far from adequate.

In fact, it is invalid in physics that a particle with additional energy would result in less mass since (3) was interpreted as the mass inside a sphere of radius r. Thus, such a frivolous conclusion of equivalence is a manifestation of inadequate understanding of general relativity.

3. Another Invalid Interpretation of the Reissner-Nordstrom metric

If one interpreted M in (6) as a "total mass" that includes the electric energy,[3)] problems 2) and 3) remain unsolved although this new interpretation of M would alleviate problem 1), in addition to double counting of the electric energy. Thus, the general validity of $m = E/c^2$ is incorrect.

It is shown that their efforts achieved only exposing further their inadequate understanding in the theory of relativity. According to Einstein, the field equation for the metric is [12],

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi T_{\mu\nu}, \tag{4}$$

or

$$R_{\mu\nu} = -8\pi[T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T], \qquad \text{where} \qquad T = g^{\alpha\beta} T_{\alpha\beta}.$$

In this equation, the energy stress tensor $T_{\mu\nu}$ is the sum of any type of energy-stress tensor. For the Reissner-Nordstrom metric, it includes at least the massive energy-stress tensor and the electromagnetic energy-stress tensor. They differ by that the electromagnetic energy-stress tensor is traceless whereas the massive energy-stress tensor is not.

If one assumes that the metric has the following form,

$$ds^2 = f dt^2 - h dr^2 - r^2 (d\theta^2 + \sin\theta^2 d\phi^2), \tag{5}$$

then, as shown by Wald [12], at the region out side the particle ($r > r_0$) we have

$$-R_{00} = \frac{1}{2} (fh)^{-1/2} \frac{d}{dr} \left[(fh)^{-1/2} f' \right] + (fhr)^{-1} f', \tag{6a}$$

$$-R_{11} = -\frac{1}{2} (fh)^{-1/2} \frac{d}{dr} \left[(fh)^{-1/2} f' \right] + (h^2 r)^{-1} h', \tag{6b}$$

$$-R_{22} = -\frac{1}{2} (rfh)^{-1} f' + \frac{1}{2} (h^2 r)^{-1} h' + r^{-2} (1 - h^{-1}) \tag{6c}$$

Moreover, outside the particle we have

$$T(m)_{\mu\nu} = 0 \qquad \text{for} \quad r > r_0 . \tag{7a}$$

But

$$T(m)_{00} = \rho(r), \quad T(m)_{11} = T(m)_{22} = T(m)_{33} = P(r), \ \text{when } r < r_0 \tag{7b}$$

where P(r) is the pressure of the perfect fluid model.

Because of the electric energy-stress tensor $T(E)_{\mu\nu}$ is traceless, we also have, for $r > r_0$,

$$R_{00} = -R_{11} = R_{22} = -E^2, \qquad \text{where} \qquad \vec{E} = \frac{q}{r^3}\vec{r}\,, \tag{8}$$

according to Misner et al. [15; p. 841]. If h = 1/f as in metric (2), then (6) is reduced to

$$-R_{00} = R_{11} = \frac{1}{2}f'' + r^{-1}f' = E^2 \tag{9a}$$

and

$$-R_{22} = -r^{-1}f' + r^{-2}(1-f) = E^2 \tag{9b}$$

Moreover, if $f = \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right)$ as in metric (2), then we have, in consistent with (8),

$$\frac{q^2}{r^2} = r^2E^2 \tag{10}$$

Thus, it seems there is no restriction on M of metric (2). However, from (7), it is clear that M in metric (2) cannot include the electric energy (out side the particle) since it has been represented in (8). In other words, to interpret $(M - q^2/2r)$ as representing the mass inside a sphere of radius r is incorrect.

4. Remarks

In physics, the most famous formula is probably $E = mc^2$. Ironically, it is also this formula that many physicists do not understand properly. Einstein himself has made clear that this formula must be understood in terms of energy conservation [16]. This formula means that there is an energy related to a mass, but it does not means that, for any type of energy, there is a related mass. Moreover, general relativity also makes it explicit that the gravity generated by mass and that by the electromagnetic energy are different as shown by the Riessner-Nordstrom metric.

Since not every type of energy is equivalent to mass, Will's assumption (1) is actually invalid. Nevertheless, this nonequivalence remains compatible with Einstein's famous equation, relating the total energy E_T and mass M_T,

$$E_T = M_T c^2 \tag{11}$$

It is crucial to note that E_T is the *total* energy of the particle. (However, E_T does not include the energy of the field out side the particle.) The massive energy-stress tensor has a very specific form. This means that there should be cancellations among gravitational effects due to different types of energy [6, 7].

The misinterpretation of $E = mc^2$ as any energy has a mass correspondence, i.e., $m = E/c^2$ is a rather common one. For instance, Tolman [17; p. 49], Fock [18; p.111], and Hawking [19, p. 107] have also made this mistake explicitly. In view of that Will's book [10] gets only positive comments from two top journals [4) 5)], this error seems to have been rather universally committed. Nevertheless, there are exceptions, such as D. Bohm [20].

Although the problem was identified in 1997 [3, 9], there is little improvement in understanding this issue. A hidden agenda of the implicit assumption of universal equivalence of energy and mass is to justify the universal coupling that is a vital assumption of the singularity theorems. However, the universal coupling has already been proven to be incorrect [6, 7] by the Hulse-Taylor experiment. It is hope that this paper would call the attention to this important issue of over simplifying the relationship between mass and energy.

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ENDNOTES

1) Clifford M. Will, who is the President of the International Society on General Relativity and Gravitation (2004-2007), was an open referee of [9].

2) The existence of a Euclidean-like structure in the frame of reference is a necessary condition for a physical space [14]. For example, the Schwarzschild solution in quasi-Minkowskian coordinates [11; p.181] is the following:

$$ds^2 = (1 - 2M\kappa/\rho)c^2dt^2 - [(1 - 2M\kappa/\rho)^{-1} - 1]\,\rho^{-2}(xdx + ydy + zdz)^2 - (dx^2 + dy^2 + dz^2), \qquad \text{(E1)}$$

where

$$\rho^2 = x^2 + y^2 + z^2, \qquad x = \rho\sin\theta\cos\varphi, \qquad y = \rho\sin\theta\sin\varphi, \qquad \text{and} \qquad z = \rho\cos\theta. \qquad \text{(E2)}$$

Coordinate transformation (E2) tells that the space coordinates satisfy the Pythagorean theorem. The Euclidean-like structure represents this fact, but avoids confusion with the notion of a Euclidean subspace, determined by the metric. In this system a light speed ($ds^2 = 0$) is defined in terms of dx/dt, dy/dt, and dz/dt [1, 2].

3) This approach ignores that in general relativity gravity is decided by a tensor metric, instead of a scalar gravitational potential. For example, Einstein's equivalence principle was commonly mistaken [21], as shown in the British Encyclopedia, to be the same as the 1911 preliminary application of equivalence between acceleration and Newtonian gravity. A clear difference is that the metric of a uniform gravity must be time-dependent because of Einstein's equivalence principle [22], but in Newtonian theory such a potential can be static.

4) "Consolidates much of the literatures on experimental gravity and should be invaluable to researchers in gravitation."— Science on Will's book [10].

5) "A concise meaty book ... and a most useful reference work ... researchers and serious students of gravitation should be pleased with it." – Nature on Will's book [10].

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Professor Eric J. Weinberg, Editor
Physical Review D

Dear Professor Weinberg:

Thank you very much for your two emails that give new interpretations of the Reissner-Nordstrom. Sorry that I have to tell you that they are wrong, according to Einstein's general relativity. The physical reason why such interpretations are incorrect is presented in the attached paper, "Misinterpretation of $E = mc^2$ and Einstein's General Relativity". I believe this paper, if published in the Physical Review D, would help many physicists who were misled by theorists such as C. M. Will.

The interpretation of the Reissner-Nordstrom can be traced back to my disagreement with Dr. Will stating from 1993. He was defeated and my paper is published in the Astrophysical Journal in 1997. Apparently, he still hang on his old view of $m = E/c^2$. This only exposes further that Dr. Will actually understand very little about general relativity. It is a surprise to me that now he is the president of the International Society on General Relativity and Gravitation (2004-2007). It is rather sad that such a theorist could the leader of the field of general relativity and gravitation.

I hope that you understand that this disagreement is not personal but a matter of the correct science. Thank you for your kind attention. I am looking forward to hearing from you.

Sincerely yours,

C. Y. Lo