THE QUANTUM MECHANICS OF CLOSED SYSTEMS*

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ABSTRACT

A pedagogical introduction is given to the quantum mechanics of closed systems, most generally the universe as a whole. Quantum mechanics aims at predicting the probabilities of alternative coarse-grained time histories of a closed system. Not every set of alternative coarse-grained histories that can be described may be consistently assigned probabilities because of quantum mechanical interference between individual histories of the set. In "Copenhagen" quantum mechanics, probabilities can be assigned to histories of a subsystem that have been "measured". In the quantum mechanics of closed systems, containing both observer and observed, probabilities are assigned to those sets of alternative histories for which there is negligible interference between individual histories as a consequence of the system's initial condition and dynamics. Such sets of histories are said to decohere. We define decoherence for closed systems in the simplified case when quantum gravity can be neglected and the initial state is pure. Typical mechanisms of decoherence that are widespread in our universe are illustrated.

Copenhagen quantum mechanics is an approximation to the more general quantum framework of closed subsystems. It is appropriate when there is an approximately isolated subsystem that is a participant in a measurement situation in which (among other things) the decoherence of alternative registrations of the apparatus can be idealized as exact.

Since the quantum mechanics of closed systems does not posit the existence of the quasiclassical domain of everyday experience, the domain of the approximate aplicability of classical physics must be explained. We describe how a quasiclassical domain described by averages of densities of approximately conserved quantities could be an emergent feature of an initial condition of the universe that implies the approximate classical behavior of spacetime on accessible scales.

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0. Preface

Charlie Misner was one of the pioneers of quantum cosmology — the effort to understand the universe as a whole as a quantum mechanical system. The minisuperspace models that he introduced and analysed with such characteristic elegance and clarity remain standard arenas in which to test ideas of the subject [1,2]. As he was well aware, in the applications of quantum mechanics to cosmology one must confront the characteristic features of quantum theory in a striking and unavoidable manner. A central problem is simply obtaining a coherent formulation of quantum mechanics for closed systems such as the universe. This essay on the occasion of his 60th birthday is a pedagogical summary of the efforts of Murray Gell-Mann and myself to provide such a formulation [3,4].

I. Introduction

It is an inescapable inference from the physics of the last sixty years that we live in a quantum mechanical universe — a world in which the basic laws of physics conform to that framework for prediction we call quantum mechanics. We perhaps have little evidence of peculiarly quantum mechanical phenomena on large and even familiar scales, but there is no evidence that the phenomena that we do see cannot be described in quantum mechanical terms and explained by quantum mechanical laws. If this inference is correct, then there must be a description of the universe as a whole and everything in it in quantum mechanical terms. The nature of this description and its observable consequences are the subject of quantum cosmology.

Our observations of the present universe on the largest scales are crude and a classical description of them is entirely adequate. Providing a quantum mechanical description of these observations alone might be an interesting intellectual challenge, but it would be unlikely to yield testable predictions differing from those of classical physics. Today, however, we have a more ambitious aim. We aim, in quantum cosmology, to provide a theory of the initial condition of the universe which will predict testable correlations among observations today. There are no realistic predictions of any kind that do not depend on this initial condition, if only very weakly. Predictions of certain observations may be testably sensitive to its details. These include the large scale homogeneity and isotropy of the universe, its approximate spatial flatness, the spectrum of density fluctuations that produced the galaxies, the homogeneity of the thermodynamic arrow of time, and the existence of classical spacetime. Recently, there has been speculation that even the coupling constants of the effective interactions of the elementary particles at accessible energy scales may be probabilistically distributed with a distribution which may depend, in part, on the initial condition of the universe [5,6,7]. It is for such reasons that the search for a theory of the initial condition of the universe is just as necessary and just as fundamental as the search for a theory of the dynamics of the elementary particles. They may even be the same searches.

The physics of the very early universe is likely to be quantum mechanical in an essential way. The singularity theorems of classical general relativity suggest that an early era preceded ours in which even the geometry of spacetime exhibited significant quantum fluctuations. It is for a theory of the initial condition that describes this era, and all later ones, that we need to spell out how to apply quantum mechanics to cosmology. Recent years have seen much promising progress in the search for a theory of the quantum initial condition. However, it is not my purpose to review these developments here.* Rather, I shall argue that this somewhat obscure branch of astrophysics may have implications for the formulation and interpretation of quantum mechanics on day-to-day scales. My thesis will be that by looking at the universe as a whole one is led to an understanding of quantum mechanics which clarifies many of the long standing interpretative difficulties of the subject.

* For a recent review of quantum cosmology see [8].

The Copenhagen frameworks for quantum mechanics, as they were formulated in the '30s and '40s and as they exist in most textbooks today, are inadequate for quantum cosmology. Characteristically these formulations assumed, as *external* to the framework of wave function and Schrödinger equation, the quasiclassical domain we see all about us. Bohr [9] spoke of phenomena which could be alternatively described in classical language. In their classic text, Landau and Lifschitz [10] formulated quantum mechanics in terms of a separate classical physics. Heisenberg and others stressed the central role of an external, essentially classical, observer.* Characteristically, these formulations assumed a possible division of the world into "obsever" and "observed", assumed that "measurements" are the primary focus of scientific statements and, in effect, posited the existence of an external "quasiclassical domain". However, in a theory of the whole thing there can be no fundamental division into observer and observed. Measurements and observers cannot be fundamental notions in a theory that seeks to describe the early universe when neither existed. In a basic formulation of quantum mechanics there is no reason in general for there to be any variables that exhibit classical behavior in all circumstances. Copenhagen quantum mechanics thus needs to be generalized to provide a quantum framework for cosmology.

In a generalization of quantum mechanics which does not *posit* the existence of a quasiclassical domain, the domain of applicability of classical physics must be *explained.* For a quantum mechanical system to exhibit classical behavior there must be some restriction on its state and some coarseness in how it is described. This is clearly illustrated in the quantum mechanics of a single particle. Ehrenfest's theorem shows that generally

$$M\frac{d^2\langle x\rangle}{dt^2} = \left\langle -\frac{\partial V}{\partial x}\right\rangle . \tag{1.1}$$

However, only for special states, typically narrow wave packets, will

* For a clear statement of this point of view, see [11].

this become an equation of motion for $\langle x \rangle$ of the form

$$M\frac{d^2\langle x\rangle}{dt^2} = -\frac{\partial V(\langle x\rangle)}{\partial x}. \tag{1.2}$$

For such special states, successive observations of position in time will exhibit the classical correlations predicted by the equation of motion (1.2) *provided* that these observations are coarse enough so that the properties of the state which allow (1.2) to replace the general relation (1.1) are not affected by these observations. An *exact* determination of position, for example, would yield a completely delocalized wave packet an instant later and (1.2) would no longer be a good approximation to (1.1). Thus, even for large systems, and in particular for the universe as a whole, we can expect classical behavior only for certain initial states and then only when a sufficiently coarse grained description is used.

If classical behavior is *in general* a consequence only of a certain class of states in quantum mechanics, then, as a particular case, we can expect to have classical spacetime only for certain states in quantum gravity. The classical spacetime geometry we see all about us in the late universe is not property of every state in a theory where geometry fluctuates quantum mechanically. Rather, it is traceable fundamentally to restrictions on the initial condition. Such restrictions are likely to be generous in that, as in the single particle case, many different states will exhibit classical features. The existence of classical spacetime and the applicability of classical physics are thus not likely to be very restrictive conditions on constructing a theory of the initial condition.

It was Everett who, in 1957, first suggested how to generalize the Copenhagen frameworks so as to apply quantum mechanics to cosmology.* Everett's idea was to take quantum mechanics seriously and apply it to the universe as a whole. He showed how an observer could be considered part of this system and how its activities — measuring, recording, calculating probabilities, etc. — could be described within quantum mechanics. Yet the Everett analysis was not complete. It did not adequately describe within quantum mechanics the origin of the "quasiclassical domain" of familiar experience nor, in an observer independent way, the meaning of the "branching" that replaced the notion of measurement. It did not distinguish from among the vast number of choices of quantum mechanical observables that are in principle available to an observer, the particular choices that, in fact, describe the quasiclassical domain.

* The original reference is [12]. For a useful collection of reprints see [13].

In this essay, I will describe joint work with Murray Gell-Mann [3,4] which aims at a coherent formulation of quantum mechanics for the universe as a whole that is a framework to explain rather than posit the quasiclassical domain of everyday experience. It is an attempt at an extension, clarification, and completion of the Everett interpretation. It builds on many aspects of the, so called post-Everett development, especially the work of Zeh [14], Zurek [15,16], and Joos and Zeh [17]. At important points it coincides with the, independent, earlier work of Bob Griffiths [18] and Roland Omnès (*e.g.*, as reviewed in [19]).

Our work is not complete, but I hope to sketch how it might become so. It is by now a very long story but I will try to describe the important parts in simplified terms.

II. Probabilities in General and Probabilities in Quantum Mechanics

Even apart from quantum mechanics, there is no certainty in this world and therefore physics deals in probabilities. It deals most generally with the probabilities for alternative time histories of the universe. From these, conditional probabilities can be constructed that are appropriate when some features about our specific history are known and further ones are to be predicted.

To understand what probabilities mean for a single closed system, it is best to understand how they are used. We deal, first of all, with probabilities for *single* events of the *single* system. When these probabilities become sufficiently close to zero or one there is a definite prediction on which we may act. How sufficiently close to 0 or 1 the probabilities must be depends on the circumstances in which they are applied. There is no certainty that the sun will come up tomorrow at the time printed in our daily newspapers. The sun may be destroyed by a neutron star now racing across the galaxy at near light speed. The earth's rotation rate could undergo a quantum fluctuation. An error could have been made in the computer that extrapolates the motion of the earth. The printer could have made a mistake in setting the type. Our eyes may deceive us in reading the time. Yet, we watch the sunrise at the appointed time because we compute, however imperfectly, that the probability of these things happening is sufficiently low.

Various strategies can be employed to identify situations where probabilities are near zero or one. Acquiring information and considering the conditional probabilities based on it is one such strategy. Current theories of the initial condition of the universe predict almost no probabilities near zero or one without further conditions. The "no boundary" wave function of the universe, for example, does not predict the present position of the sun on the sky. However, it will predict that the conditional probability for the sun to be at the position predicted by classical celestial mechanics given a few previous positions is a number very near unity.

Another strategy to isolate probabilities near 0 or 1 is to consider ensembles of repeated observations of identical subsystems in the closed system. There are no genuinely infinite ensembles in the world so we are necessarily concerned with the probabilities for deviations of the behavior of a finite ensemble from the expected behavior of an infinite one. These are probabilities for a single feature (the deviation) of a single system (the whole ensemble).

The existence of large ensembles of repeated observations in identical circumstances and their ubiquity in laboratory science should not, therefore, obscure the fact that in the last analysis physics must predict probabilities for the single system that is the ensemble as a whole. Whether it is the probability of a successful marriage, the probability of the present galaxy-galaxy correlation function, or the probability of the fluctuations in an ensemble of repeated observations, we must deal with the probabilities of single events in single systems. In geology, astronomy, history, and cosmology, most predictions of interest have this character. The goal of physical theory is, therefore, most generally to predict the probabilities of histories of single events of a single system.

Probabilities need be assigned to histories by physical theory only up to the accuracy they are used. Two theories that predict probabilities for the sun not rising tomorrow at its classically calculated time that are both well beneath the standard on which we act are equivalent for all practical purposes as far as this prediction is concerned. It is often convenient, therefore, to deal with approximate probabilities which satisfy the rules of probability theory up to the standard they are used.

The characteristic feature of a quantum mechanical theory is that not every set of alternative histories that may be described can be assigned probabilities. Nowhere is this more clearly illustrated than in the two slit experiment illustrated in Figure 1. In the usual "Copenhagen" discussion if we have not measured which of the two slits the electron passed through on its way to being detected at the screen, then we are not permitted to assign probabilities to these alternative histories. It would be inconsistent to do so since the correct probability sum rule would not be satisfied. Because of interference, the probability to arrive at y is not the sum of the probabilities to arrive at y going through the upper or lower slit:

$$p(y) \neq p_U(y) + p_L(y) \tag{2.1}$$

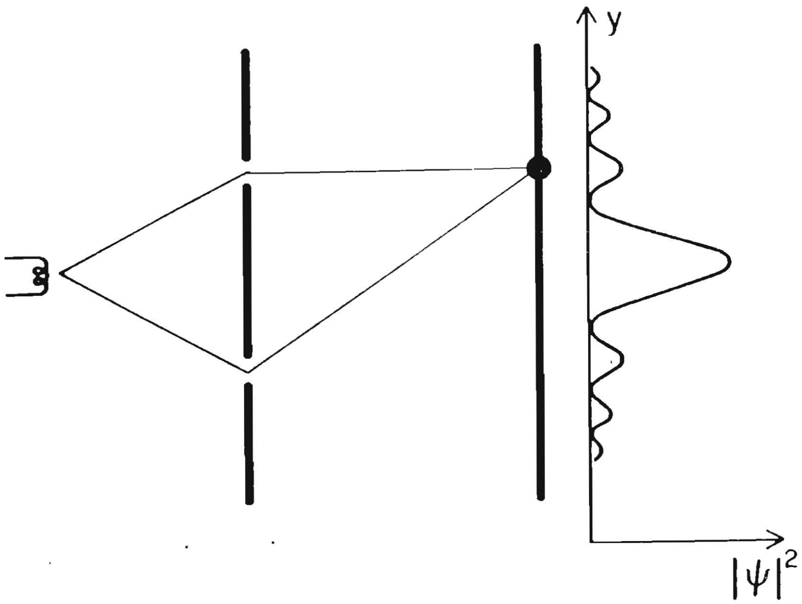

Fig. 1: *The two-slit experiment. An electron gun at right emits an electron traveling towards a screen with two slits, its progress in space recapitulating its evolution in time. When precise detections are made of an ensemble of such electrons at the screen it is not possible, because of interference, to assign a probability to the alternatives of whether an individual electron went through the upper slit or the lower slit. However, if the electron interacts with apparatus that measures which slit it passed through, then these alternatives decohere and probabilities can be assigned.*

because

$$|\psi_L(y) + \psi_U(y)|^2 \neq |\psi_L(y)|^2 + |\psi_U(y)|^2 \quad . \tag{2.2}$$

If we *have* measured which slit the electron went through, then the interference is destroyed, the sum rule obeyed, and we *can* meaningfully assign probabilities to these alternative histories.

A rule is thus needed in quantum theory to determine which sets of alternative histories may be assigned probabilities and which may not. In Copenhagen quantum mechanics, the rule is that probabilities are

assigned to histories of alternatives of a subsystem that are *measured* and not in general otherwise.

III. Probabilities for a Time Sequence of Measurements

To establish some notation, let us review in more detail the usual rules for the probabilities of time sequences of ideal measurements of subsystem using the two-slit experiment of Figure 1 as an example.

Alternatives for the electron are represented by projection operators in its Hilbert space. Thus, in the two slit experiment, the alternative that the electron passed through the lower slit is represented by the projection operator

$$P_U = \Sigma_s \int_U d^3x \, |\vec{x}, s\rangle\langle\vec{x}, s| \tag{3.1}$$

where $|\vec{x}, s\rangle$ is a localized state of the electron with spin component s, and the integral is over a volume around the upper slit. There is a similar projection operator P_L for the alternative that the electron goes through the lower slit. These are exclusive alternatives and they are exhaustive. These properties, as well as the requirements of being projections, are represented by the relations

$$P_L P_U = 0 \ , \quad P_U + P_L = 1, \quad P_L^2 = P_L, \quad P_U^2 = P_U \ . \tag{3.2}$$

There is a similarly defined set of projection operators $\{P_y\}$ representing the alternative positions of arrival at the screen.

We can now state the rule for the joint probability that the electron initially in a state $|\psi(t_0)\rangle$ at $t = t_0$ is determined by an ideal measurement at time t_1 to have passed through the upper slit and measured at time t_2 to arrive at point y on the screen. If one likes, one can imagine the case in which the electron is in a narrow wave packet in the horizontal direction with a velocity defined as sharply as possible consistent with the uncertainty principle. The joint probability is negligible unless t_1 and t_2 correspond to the times of flight to the slits and to the screen respectively.

The first step in calculating the joint probability is to evolve the state of the electron to the time t_1 of the first measurement

$$|\psi(t_1)\rangle = e^{-iH(t_1-t_0)/\hbar}|\psi(t_0)\rangle \ . \tag{3.3}$$

The probability that the outcome of the measurement at time t_1 is that the electron passed through the upper slit is:

$$(\text{Probability of } U) = \left\| P_U |\psi(t_1)\rangle \right\|^2 \tag{3.4}$$

where $\| \cdot \|$ denotes the norm of a vector in the electron's Hilbert space. If the outcome was the upper slit, and the measurement was an "ideal" one, that disturbed the electron as little as possible in making its determination, then after the measurement the state vector is reduced to

$$\frac{P_U|\psi(t_1)\rangle}{\|P_U|\psi(t_1)\rangle\|} \ . \tag{3.5}$$

This is evolved to the time of the next measurement

$$|\psi(t_2)\rangle = e^{-iH(t_2-t_1)/\hbar}\frac{P_U|\psi(t_1)\rangle}{\|P_U|\psi(t_1)\rangle\|} \ . \tag{3.6}$$

The probability of being detected at point y on the screen at time t_2 *given* that the electron passed through the upper slit is

$$(\text{Probability of } y \textit{ given } U) = \left\| P_y|\psi(t_2)\rangle \right\|^2 \ . \tag{3.7}$$

The *joint* probability that the electron is measured to have gone through the upper slit *and* is detected at y is the product of the conditional probability (3.7) with the probability (3.4) that the electron passed through U. The latter factor cancels the denominator in (3.6) so that combining all of the above equations in this section, we have

$$(\text{Probability of } y \textit{ and } U) = \left\| P_y e^{-iH(t_2-t_1)/\hbar} P_U e^{-iH(t_1-t_0)/\hbar}|\psi(t_0)\rangle \right\|^2 \ . \tag{3.8}$$

With Heisenberg picture projections this takes the even simpler form

$$(\text{Probability of } y \text{ and } U) = \left\| P_y(t_2)P_U(t_1)\ |\psi(t_0)\rangle \right\|^2 \ . \tag{3.9}$$

where, for example,

$$P_U(t) = e^{iHt/\hbar}P_U e^{-iHt/\hbar} \ . \tag{3.10}$$

The formula (3.9) is a compact and unified expression of the two laws of evolution that characterize the quantum mechanics of measured subsystems — unitary evolution in between measurements and reduction of the wave packet at a measurement.* The important thing to remember about the expression (3.9) is that everything in it — projections, state vectors, Hamiltonian — refer to the Hilbert space of a subsystem, in this example the Hilbert space of the electron that is measured.

In "Copenhagen" quantum mechanics, it is measurement that determines which histories of a subsystem can be assigned probabilities and formulae like (3.9) that determine what these probabilities are. We cannot have such rules in the quantum mechanics of closed systems. There is no fundamental division of a closed system into measured subsystem and measuring apparatus. There is no fundamental reason for the closed system to contain classically behaving measuring apparatus in all circumstances. In particular, in the early universe none of these concepts seem relevant. We need a more observer-independent, measurement-independent, quasiclassical domain-independent rule for which histories of a closed system can be assigned probabilities and what these probabilities are. The next section describes this rule.

IV. Post-Everett Quantum Mechanics

To describe the rules of post-Everett quantum mechanics, I shall make a simplifying assumption. I shall neglect gross quantum fluctuations in the geometry of spacetime, and assume a fixed background spacetime geometry which supplies a definite meaning to the notion

* As has been noted by many authors, *e.g.*, [20] and [21] among the earliest.

of time. This is an excellent approximation on accessible scales for times later than 10^{-43} sec after the big bang. The familiar apparatus of Hilbert space, states, Hamiltonian, and other operators may then be applied to process of prediction. Indeed, in this context the quantum mechanics of cosmology is in no way distinguished from the quantum mechanics of a large isolated box, perhaps expanding, but containing both the observed and its observers (if any).

A set of alternative histories for a closed system is specified by giving exhaustive sets of exclusive alternatives at a sequence of times. Consider a model closed system initially in a pure state that can be described as an observer and two slit experiment, with appropriate apparatus for producing the electrons, detecting which slit they passed through, and measuring their position of arrival on the screen (Figure 2). Some alternatives for the whole system are:

1. Whether or not the observer decided to measure which slit the electron went through.
2. Whether the electron went through the upper or lower slit.
3. The alternative positions, $y_1, \cdots, y_N$, that the electron could have arrived at the screen.

This set of alternatives at a sequence of times defines a set of histories whose characteristic branching structure is shown in Figure 3. An individual history in the set is specified by some particular sequence of alternatives, *e.g.*, measured, upper, y_9.

Many other sets of alternative histories are possible for the closed system. For example, we could have included alternatives describing the readouts of the apparatus that detects the position that the electron arrived on the screen. If the initial condition corresponded to a good experiment there should be a high correlation between these alternatives and the position that the electron arrives at the screen. In a more refined model we could discuss alternatives corresponding to thoughts in the observer's brain, or to the individual positions of the atoms in the apparatus, or to the possibilities that these atoms

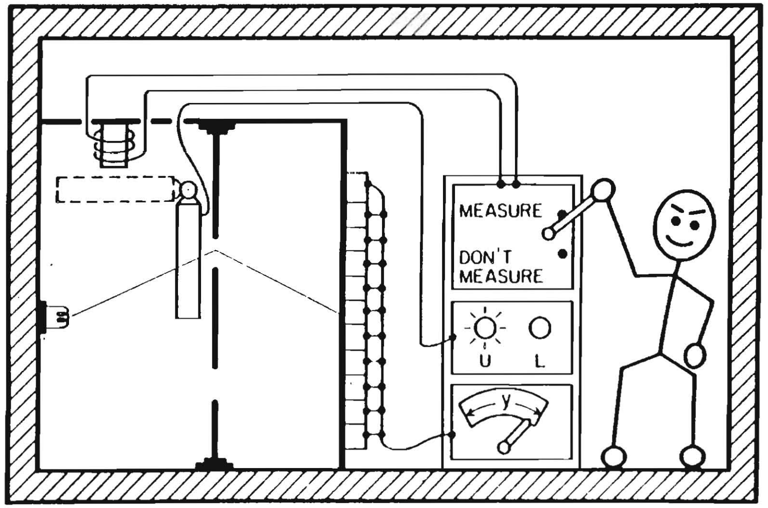

Fig. 2: *A model closed quantum system containing an observer together with the necessary apparatus for carrying out a two-slit experiment. Alternatives for the system include whether the observer measured which slit the electron passed through or did not, whether the electron passed through the upper or lower slit, the alternative positions of arrival of the electron at the screen, the alternative arrival positions registered by the apparatus, the registration of these in the brain of the observer, etc., etc., etc. Each exhaustive set of exclusive alternatives is represented by an exhaustive set of orthogonal projection operators on the Hilbert space of the closed system. Time sequences of such sets of alternatives describe sets of alternative coarse-grained histories of the closed system. Quantum theory assigns probabilities to the individual alternative histories in such a set when there is negligible quantum mechanical interference between them, that is, when the set of histories decoheres.*

A more refined model might consider a quantity of matter in a closed box. One could then consider alternatives such as whether the box contains a two-slit experiment or does not as well as alternative positions of atoms.

reassemble in some completely different configuration. There are a vast number of possibilities.

Characteristically the alternatives that are of use to us as observers are very coarse grained, distinguishing only very few of the degrees of freedom of a large closed system. This is especially true if we recall that our box with observer and two-slit experiment is only an idealized model. The most general closed system is the universe itself, and, as I hope to show, the only realistic closed systems are of cosmological dimensions. Certainly, we utilize only very, very coarse-grained descriptions of the universe as a whole.

I would now like to state the rules that determine which coarse-grained sets of histories may be assigned probabilities and what those probabilities are. The essence of the rules I shall describe can be found in the work of Bob Griffiths [18]. The general framework was extended by Roland Omnès [19] and was independently, but later, arrived at by Murray Gell-Mann and myself [3]. The idea is simple: The failure of probability sum rules due to quantum interference is the obstacle to assigning probabilities. Probabilities can be assigned to just those sets of alternative histories of a closed system for which there is negligible interference between the individual histories in the set as a consequence of the *particular* initial state the closed system has, and for which, therefore, all probability sum rules *are* satisfied. Let us now give this idea a precise expression.

Sets of alternatives at one moment of time are represented by sets of orthogonal projection operators. Employing the Heisenberg picture these can be denoted $\{P^k_{\alpha_k}(t_k)\}$. The superscript k denotes the set of alternatives being considered at time t_k (for example, the set of alternative position intervals $\{y_1, \cdots, y_N\}$ at which the electron might arrive at the screen at time t_3), α_k denotes the particular alternative in the set (for example y_9) and t_k is the time. The set of P's satisfy

$$\sum_{\alpha_k} P^k_{\alpha_k}(t_k) = 1 \, , \quad P^k_{\alpha_k}(t_k) P^k_{\alpha'_k}(t_k) = \delta_{\alpha_k \alpha'_k} P^k_{\alpha_k}(t_k) \qquad (4.1)$$

showing that they represent an exhaustive set of exclusive alternatives.

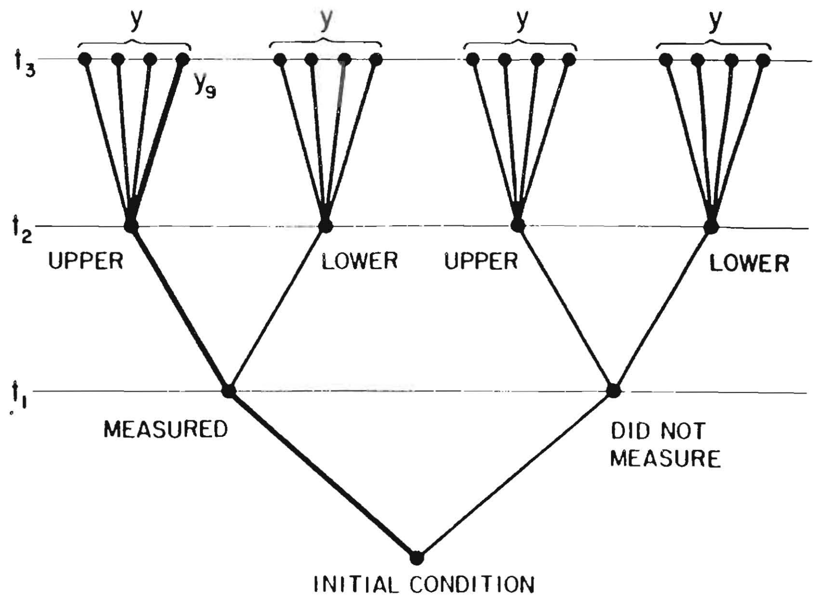

Fig. 3: *Branching structure of a set of alternative histories. This figure illustrates the set of alternative histories defined by the alternatives of whether the observer decided to measure or did not decide to measure which slit the electron went through at time t_1, whether the electron went through the upper slit or through the lower slit at time t_2, and the alternative positions of arrival at the screen at time t_3. A single branch corresponding to the alternatives that the measurement was carried out, the electron went through the upper slit, and arrived at point y_9 on the screen is illustrated by the heavy line.*

The illustrated set of histories does not decohere because there is significant quantum mechanical interference between the branch where no measurement was carried out and the electron went through the upper slit and the similar branch where it went through the lower slit. A related set of histories that does decohere can be obtained by replacing the alternatives at time t_2 by the following set of three alternatives: (a record of the decision shows a measurement was initiated and the electron went through the upper slit); (a record of the decision shows a measurement was initiated and the electron went through the lower slit); (a record of the decision shows that the measurement was not initiated). The vanishing of the interference between the alternative values of the record and the alternative configurations of apparatus ensures the decoherence of this set of alternative histories.

Sets of alternative histories are defined by giving sequences of sets of alternatives at definite moments of time, *e.g.*, $\{P^1_{\alpha_1}(t_1)\}$, $\{P^2_{\alpha_2}(t_2)\}$, $\cdots, \{P^n_{\alpha_n}(t_n)\}$. Different choices for $\{P^1_{\alpha_1}(t_1)\}$, $\{P^2_{\alpha_2}(t_2)\}$, etc. describe different sets of alternative histories of the closed system. An individual history in a given set corresponds to a particular sequence $(\alpha_1, \cdots, \alpha_n) \equiv \alpha$ and, for each history, there is a corresponding chain of projection operators

$$C_\alpha \equiv P^n_{\alpha_n}(t_n) \cdots P^1_{\alpha_1}(t_1) \ . \tag{4.2}$$

For example, in the two slit experiment in a box illustrated in Figure 2, the history in which the observer decided at time t_1 to measure which slit the electron goes through, in which the electron goes through the upper slit at time t_2, and arrives at the screen in position interval y_9 at time t_3, would be represented by the chain

$$P^3_{y_9}(t_3) P^2_U(t_2) P^1_{\mathrm{meas}}(t_1) \tag{4.3}$$

in an obvious notation. The only difference between this situation and that of the "Copenhagen" quantum mechanics of measured subsystems is the following: The sets of operators $\{P^k_{\alpha_k}(t_k)\}$ defining alternatives for the closed system act on the Hilbert space of the closed system that includes the variables describing any apparatus, observers, and anything else. The operators defining alternatives in Copenhagen quantum mechanics act only on the Hilbert space of the measured subsystem.

When the initial state is pure, it can be resolved into *branches* corresponding to the individual members of any set of alternative histories. The generalization to an impure initial density matrix is not difficult [3], but for simplicity we shall assume a pure initial state throughout this article. Denote the initial state by $|\Psi\rangle$ in the Heisenberg picture. Then

$$|\Psi\rangle = \sum\nolimits_\alpha C_\alpha|\Psi\rangle = \sum_{\alpha_1,\cdots,\alpha_n} P^n_{\alpha_n}(t_n) \cdots P^1_{\alpha_1}(t_1)|\Psi\rangle \ . \tag{4.4}$$

This identity follows by applying the first of (4.1) to all the sums over α_k in turn. The vector

$$C_\alpha|\Psi\rangle \tag{4.5}$$

is the branch corresponding to the individual history α and (4.4) is the resolution of the initial state into branches.

When the branches corresponding to a set of alternative histories are sufficiently orthogonal the set of histories is said to *decohere*. More precisely a set of histories decoheres when

$$\langle\Psi|C^\dagger_{\alpha'} C_\alpha|\Psi\rangle \approx 0 \ , \quad \text{for any} \quad \alpha'_k \neq \alpha_k \ . \tag{4.6}$$

We shall return to the standard with which decoherence should be enforced, but first let us examine its meaning and consequences.

Decoherence means the absence of quantum mechanical interference between the individual histories of a coarse-grained set.* Probabilities can be assigned to the individual histories in a decoherent set

* The term "decoherence" is used in several different ways in the literature. Therefore, for those familiar with other work, a comment is in order to specify how we are employing the term in this simplified presentation. We have followed our previous work [3], [4] in using the term "decoherence" to refer to a property of a set of alternative time *histories* of a closed system. A decoherent set of histories is one for which the quantum mechanical interference between individual histories is small enough to guarantee an appropriate set of probability sum rules. Different notions of decoherence can be defined by utilizing different measures of interference. The weakest notion is just the consistency of the probability sum rules that was called "consistency" by Griffiths [18] and Omnès [19] and that term is used by some to refer to all measures of interference. Vanishing of the real part of (4.6) is a sufficient condition for the consistency of the probability sum rules called the "weak decoherence condition". We are using the stronger condition (4.6) because it characterizes widespread and typical mechanisms of decoherence. Eq (4.6) has been called the "medium decoherence condition". "Decoherence" in the context of this paper, thus, means the medium decoherence of sets of histories.
In the literature the term "decoherence" has also been used to refer to the decay in time of the off-diagonal elements of a reduced density matrix defined by tracing the full density matrix over a given set of variables [22]. The two notions of "decoherence of reduced density matrices" and "decoherence of histories" are not generally equivalent

of alternative histories because decoherence implies the probability sum rules necessary for a consistent assignment. The probability of an individual history α is

$$p(\alpha) = \|C_\alpha|\Psi\rangle\|^2 \ . \tag{4.7}$$

To see how decoherence implies the probability sum rules, let us consider an example in which there are just three sets of alternatives at times t_1, t_2, and t_3. A typical sum rule might be

$$\sum_{\alpha_2} p(\alpha_3, \alpha_2, \alpha_1) = p(\alpha_3, \alpha_1) \ . \tag{4.8}$$

We show (4.6) and (4.7) imply (4.8). To do that write out the left hand side of (4.8) using (4.7) and suppress the time labels for compactness.

$$\sum_{\alpha_2} p(\alpha_3, \alpha_2, \alpha_1) = \sum_{\alpha_2} \left\langle \Psi | P^1_{\alpha_1} P^2_{\alpha_2} P^3_{\alpha_3} P^3_{\alpha_3} P^2_{\alpha_2} P^1_{\alpha_1} | \Psi \right\rangle \ . \tag{4.9}$$

Decoherence means that the sum on the right hand side of (4.9) can be written with negligible error as

$$\sum_{\alpha_2} p(\alpha_3, \alpha_2, \alpha_1) \approx \sum_{\alpha_2' \alpha_2} \left\langle \Psi | P^1_{\alpha_1} P^2_{\alpha_2'} P^3_{\alpha_3} P^3_{\alpha_3} P^2_{\alpha_2} P^1_{\alpha_1} | \Psi \right\rangle \ . \tag{4.10}$$

the extra terms in the sum being vanishingly small. But now, applying the first of (4.1) we see

$$\sum_{\alpha_2} p(\alpha_3, \alpha_2, \alpha_1) \approx \left\langle \Psi | P^1_{\alpha_1} P^3_{\alpha_3} P^3_{\alpha_3} P^1_{\alpha_1} | \Psi \right\rangle = p(\alpha_3, \alpha_1) \tag{4.11}$$

so that the sum rule (4.8) is satisfied.

Given an initial state $|\Psi\rangle$ and a Hamiltonian H, one could, in principle, identify all possible sets of decohering histories. Among these will be the exactly decohering sets where the orthogonality of the branches is exact. Indeed, trivial examples can be supplied by resolving $|\Psi\rangle$ into a sum of orthogonal vectors at time t_1, resolving those vectors into sums of further vectors such that the whole set is orthogonal at time t_2, and so on. However, such sets of exactly decohering histories will not, in general, have a simple description in terms of fundamental fields nor any connection, for example, with the quasiclassical domain of familiar experience. For this reason sets of histories that approximately decohere are of interest. As we will argue in the next two Sections, realistic mechanisms lead to the decoherence of histories constituting a quasiclassical domain to an excellent approximation. When the decoherence condition (4.6) is approximately enforced, the probability sum rules such as (4.8) will only be approximately obeyed. However, as discussed earlier, these probabilities for single systems are meaningful up to the standard they are used. Approximate probabilities for which the sum rules are satisfied to a comparable standard may therefore also be employed in the process of prediction. When we speak of approximate decoherence and approximate probabilities we mean decoherence achieved and probability sum rules satisfied beyond any standard that might be conceivably contemplated for the accuracy of prediction and the comparison of theory with experiment.

Decoherent sets of histories of the universe are what we may utilize in the process of prediction in quantum mechanics, for they may be assigned probabilities. Decoherence thus generalizes and replaces the notion of "measurement", which served this role in the Copenhagen interpretations. Decoherence is a more precise, more objective, more observer-independent idea and gives a definite meaning to Everett's branches. For example, if their associated histories decohere, we may assign probabilities to various values of reasonable scale density fluctuations in the early universe whether or not anything like a "measurement" was carried out on them and certainly whether or not there was an "observer" to do it.

but also not unconnected in the sense that in particular models certain physical processes can ensure both. (See, e.g. the remarks in Section II.6.4 of [23]).

V. The Origins of Decoherence in Our Universe

What are the features of coarse-grained sets of histories that decohere in our universe? In seeking to answer this question it is important to keep in mind the basic aspects of the theoretical framework on which decoherence depends. Decoherence of a set of alternative histories is not a property of their operators *alone*. It depends on the relations of those operators to the initial state $|\Psi\rangle$, the Hamiltonian H, and the fundamental fields. Given these, we could, in principle, *compute* which sets of alternative histories decohere.

We are not likely to carry out a computation of all decohering sets of alternative histories for the universe, described in terms of the fundamental fields, anytime in the near future, if ever. It is therefore important to investigate specific mechanisms by which decoherence occurs. Let us begin with a very simple model due, in its essential features to Joos and Zeh [17]. We consider the two-slit example again, but this time suppose that in the neighborhood of the slits there is a gas of photons or other light particles colliding with the electrons (Figure 4). Physically it is easy to see what happens, the random uncorrelated collisions can carry away delicate phase correlations between the beams even if the trajectories of the electrons are not affected much. The interference pattern will then be destroyed and it will be possible to assign probabilities to whether the electron went through the upper slit or the lower slit.

Let us see how this picture in words is given precise meaning in mathematics. Initially, suppose the state of the entire system is a state of the electron $|\psi>$ and N distinguishable "photons" in states $|\varphi_1\rangle$, $|\varphi_2\rangle$, etc., *viz.*

$$|\Psi\rangle = |\psi\rangle|\varphi_1\rangle|\varphi_2> \cdots |\varphi_N\rangle \ . \tag{5.1}$$

Suppose further that $|\psi\rangle$ is a coherent superposition of a state in which the electron passes through the upper slit $|U\rangle$ and the lower slit $|L\rangle$. Explicitly:

$$|\psi\rangle = \alpha|U\rangle + \beta|L\rangle \ . \tag{5.2}$$

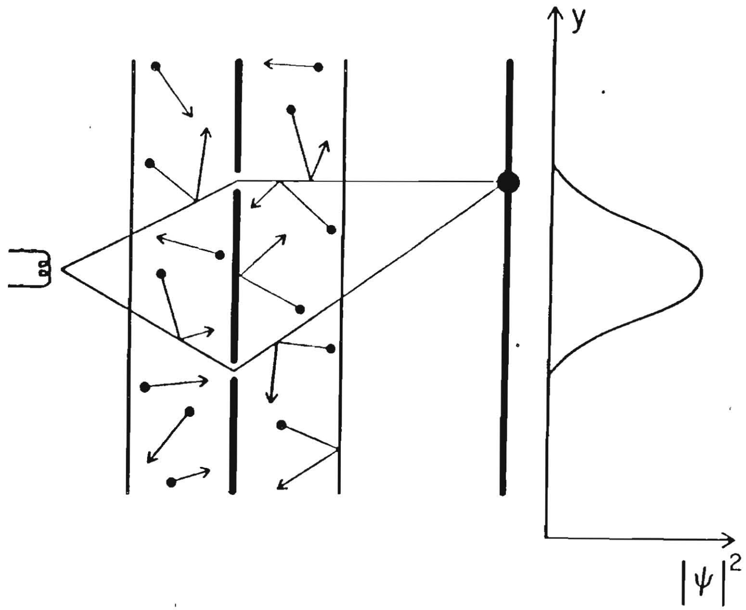

Fig. 4: *The two slit experiment with an interacting gas. Near the slits light particles of a gas collide with the electrons. Even if the collisions do not affect the trajectories of the electrons very much they can still carry away the phase correlations between the histories in which the electron arrived at point y on the screen by passing through the upper slit and that in which it arrived at the same point by passing through the lower slit. A coarse graining that described only of these two alternative histories of the electron would approximately decohere as a consequence of the interactions with the gas given adequate density, cross-section, etc. Interference is destroyed and probabilities can be assigned to these alternative histories of the electron in a way that they could not be if the gas were not present (cf. Fig. 1). The lost phase information is still available in correlations between states of the gas and states of the electron. The alternative histories of the electron would not decohere in a coarse graining that included both the histories of the electron and operators that were sensitive to the correlations between the electrons and the gas.*

This model illustrates a widely occuring mechanism by which certain types of coarse-grained sets of alternative histories decohere in the universe.

Both states are wave packets in x, so that position in x recapitulates history in time. We now ask whether the history where the electron passes through the upper slit and arrives at a detector at point y on the screen, decoheres from that in which it passes through the lower slit and arrives at point y as a consequence of the initial condition of this "universe". That is, as in Section 4, we ask whether the two branches

$$P_y(t_2)P_U(t_1)|\Psi\rangle \quad , \quad P_y(t_2)P_L(t_1)|\Psi\rangle \tag{5.3}$$

are nearly orthogonal, the times of the projections being those for the nearly classical motion in x. We work this out in the Schrödinger picture where the initial state evolves, and the projections on the electron's position are applied to it at the appropriate times.

Collisions occur, but the states $|U\rangle$ and $|L\rangle$ are left more or less undisturbed. The states of the "photons", of course, are significantly affected. If the photons are dilute enough to be scattered once by the electron in its time to traverse the gas the two branches (5.3) will be approximately

$$\alpha P_y|U\rangle S_U|\varphi_1\rangle S_U|\varphi_2\rangle \cdots S_U|\varphi_N\rangle \ , \tag{5.4a}$$

and

$$\beta \ P_y|L\rangle S_L|\varphi_1\rangle S_L|\varphi_2\rangle \cdots S_L|\varphi_N\rangle \ . \tag{5.4b}$$

Here, S_U and S_L are the scattering matrices from an electron in the vicinity of the upper slit and the lower slit respectively. The two branches in (5.4) decohere because the states of the "photons" are nearly orthogonal. The overlap of the branches is proportional to

$$\langle\varphi_1|S_U^\dagger S_L|\varphi_1\rangle\langle\varphi_2|S_U^\dagger S_L|\varphi_2\rangle \cdots \langle\varphi_N|S_U^\dagger S_L\ |\varphi_N\rangle \ . \tag{5.5}$$

Now, the S-matrices for scattering off the upper position or the lower position can be connected to that of an electron at the orgin by a translation

$$S_U = \exp(-i\mathbf{k}\cdot\mathbf{x}_U)S\ \exp(+i\mathbf{k}\cdot\mathbf{x}_U) \ , \tag{5.6a}$$

$$S_L = \exp(-i\mathbf{k}\cdot\mathbf{x}_L)S\ \exp(+i\mathbf{k}\cdot\mathbf{x}_L) \ . \tag{5.6b}$$

Here, $\hbar\mathbf{k}$ is the momentum of a photon, $\mathbf{x}_U$ and $\mathbf{x}_L$ are the positions of the slits and S is the scattering matrix from an electron at the origin.

$$\langle\mathbf{k}'|S|\mathbf{k}\rangle = \delta^{(3)}(\mathbf{k}-\mathbf{k}') + \frac{i}{2\pi\omega_{\mathbf{k}}} f(\mathbf{k},\mathbf{k}')\delta(\omega_k - \omega_k') \ , \tag{5.7}$$

where f is the scattering amplitude and $\omega_k = |\vec{k}|$.

Consider the case where initially all the photons are in plane wave states in an interaction volume V, all having the same energy $\hbar\omega$, but with random orientations for their momenta. Suppose further that the energy is low so that the electron is not much disturbed by a scattering and low enough so the wavelength is much longer than the separation between the slits, $k|\mathbf{x}_U - \mathbf{x}_L| << 1$. It is then possible to work out the overlap. The answer according to Joos and Zeh [17] is

$$\left(1 - \frac{(k|\mathbf{x}_U - \mathbf{x}_L|)^2}{8\pi^2 V^{2/3}}\sigma\right)^N \tag{5.8}$$

where σ is the effective scattering cross section and the individual terms have been averaged over incoming directions. Even if σ is small, as N becomes large this tends to zero. In this way decoherence becomes a quantitative phenomenon.

What such models convincingly show is that decoherence is frequent and widespread in the universe for histories of certain kinds of variables. Joos and Zeh calculate that a superposition of two positions of a grain of dust, 1mm apart, is decohered simply by the scattering of the cosmic background radiation on the timescale of a nanosecond. The existence of such mechanisms means that the only realistic isolated systems are of cosmological dimensions. So widespread is this kind of phenomena with the initial condition and dynamics of our universe, that we may meaningfully speak of habitually decohering variables such as the center of mass positions of massive bodies.

VI. The Copenhagen Approximation

What is the relation of the familiar Copenhagen quantum mechanics described in Section III to the more general "post-Everett" quantum mechanics of closed systems described in Sections IV and V? Copenhagen quantum mechanics predicts the probabilities of the histories of measured subsystems. Measurement situations may be described in a closed system that contains both measured subsystem and measuring apparatus. In a typical measurement situation the values of a variable not normally decohering become correlated with alternatives of the apparatus that decohere because of *its* interactions with the rest of the closed system. The correlation means that the measured alternatives decohere because the alternatives of the apparatus with which they are correlated decohere.

The recovery of the Copenhagen rule for when probabilities may be assigned is immediate. Measured quantities are correlated with decohering histories. Decohering histories can be assigned probabilities. Thus in the two-slit experiment (Figure 1), when the electron interacts with an apparatus that determines which slit it passed through, it is the decoherence of the alternative configurations of the apparatus that enables probabilities to be assigned for the electron.

There is nothing incorrect about Copenhagen quantum mechanics. Neither is it, in any sense, opposed to the post-Everett formulation of the quantum mechanics of closed systems. It is an *approximation* to the more general framework appropriate in the special cases of measurement situations and when the decoherence of alternative configurations of the apparatus may be idealized as exact and instantaneous. However, while measurement situations imply decoherence, they are only special cases of decohering histories. Probabilities may be assigned to alternative positions of the moon and to alternative values of density fluctuations near the big bang in a universe in which these alternatives decohere, whether or not they were participants in a measurement situation and certainly whether or not there was an observer registering their values.

VII. Quasiclassical Domains

As observers of the universe, we deal with coarse-grained histories that reflect our own limited sensory perceptions, extended by instruments, communication and records but in the end characterized by a large amount of ignorance. Yet, we have the impression that the universe exhibits a much finer-grained set of histories, independent of us, defining an always decohering "quasiclassical domain", to which our senses are adapted, but deal with only a small part of it. If we are preparing for a journey into a yet unseen part of the universe, we do not believe that we need to equip ourselves with spacesuits having detectors sensitive, say, to coherent superpositions of position or other unfamiliar quantum variables. We expect that the familiar quasiclassical variables will decohere and be approximately correlated in time by classical deterministic laws in any new part of the universe we may visit just as they are here and now.

Since the post-Everett quantum mechanics of closed systems does not posit a quasiclassical domain, it must provide an explanation of this manifest fact of everyday experience. No such explanation can be provided from the dynamics of quantum theory alone. Rather, like decoherence, the existence of a quasiclassical domain in the universe must be a consequence of both initial condition of the universe and the Hamiltonain describing evolution.

Roughly speaking, a quasiclassical domain should be a set of alternative histories that decoheres according to a realistic principle of decoherence, that is maximally refined consistent with that notion of decoherence, and whose individual histories are described largely by alternative values of a limited set of quasiclassical variables at different moments of time that exhibit as much as possible patterns of classical correlation in time. To make the question of the existence of one or more quasiclassical domains into a *calculable* question in quantum cosmology we need measures of how close a set of histories comes to constituting a "quasiclassical domain". A quasiclassical domain cannot be a *completely* fine-grained description for then it would not decohere. It cannot consist *entirely* of a few "quasiclassical variables" repeated over and over because sometimes we may measure something highly quantum mechanical. Quasiclassical variables cannot be *always* correlated in time by classical laws because sometimes quantum mechanical phenomena cause deviations from classical physics. We need measures for maximality and classicality [3].

It is possible to give crude arguments for the type of habitually decohering operators we expect to occur over and over again in a set of histories defining a quasiclassical domain [3]. Such habitually decohering operators are called "quasiclassical operators". In the earliest instants of the universe the operators defining spacetime on scales well above the Planck scale emerge from the quantum fog as quasiclassical. Any theory of the initial condition that does not imply this is simply inconsistent with observation in a manifest way. A background spacetime is thus defined and conservation laws arising from its symmetries have meaning. Then, where there are suitable conditions of low temperature, density, etc., various sorts of hydrodynamic variables may emerge as quasiclassical operators. These are integrals over suitably small volumes of densities of conserved or nearly conserved quantities. Examples are densities of energy, momentum, baryon number, and, in later epochs, nuclei, and even chemical species. The sizes of the volumes are limited above by maximality and are limited below by classicality because they require sufficient "inertia" resulting from their approximate conservation to enable them to resist deviations from predictability caused by their interactions with one another, by quantum spreading, and by the quantum and statistical fluctuations resulting from interactions with the rest of the universe that accomplish decoherence [24]. Suitable integrals of densities of approximately conserved quantities are thus candidates for habitually decohering quasiclassical operators. These "hydrodynamic variables" *are* among the principal variables of classical physics.

It would be in such ways that the classical domain of familiar experience could be an emergent property of the fundamental de-

scription of the universe, not generally in quantum mechanics, but as a consequence of our specific initial condition and the Hamiltonian describing evolution. Whether the universe exhibits a quasiclassical domain, and, indeed, whether it exhibits more than one essentially inequivalent domain, thus become calculable questions in the quantum mechanics of closed systems.

VIII. Conclusion

Quantum mechanics is best and most fundamentally understood in the context of quantum mechanics of closed systems, most generally the universe as a whole. The founders of quantum mechanics were right in pointing out that something external to the framework of wave function and the Schrödinger equation *is* needed to interpret the theory. But it is not a postulated classical domain to which quantum mechanics does not apply. Rather it is the initial condition of the universe that, together with the action function of the elementary particles and the throws of the quantum dice since the beginning, is the likely origin of quasiclassical domain(s) within quantum theory itself.

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References

In the spirit of providing a simplified introduction to the quantum mechanics of closed systems rather than a comprehensive review, no attempt has been made to provide anything more than the references that are directly relevant to the points raised in the text. These are not always the earliest nor are they the latest. More extensive, but still incomplete, lists can be found in [3] and [23].

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