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DYNAMICAL MONOPOLES AND CONFINEMENT*

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ABSTRACT

It is known that an $SU(N)$ lattice gauge theory may be exactly mapped into a $Z(N)$ gauge theory coupled to dynamical monopoles of $Z(N)$ flux, with couplings and monopole current distributions determined by the $SU(N)/Z(N)$ dynamics. Using this representation for $N = 2$, several rigorous inequalities are derived giving bounds on long-distance order parameters. These reduce the problem of confinement for **large** β to estimates on expectations of monopole currents. It is shown how this can lead to permanent confinement for space-time dim ≤ 4, and a string-tension bound exhibiting the expected non-perturbative β-dependence.

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It has been known for some time that an $SU(2)$ lattice gauge theory (LGT) rewritten in terms of $Z(2)$ and $SU(2)/Z(2)$ variables assumes the form of a $Z(2)$ gauge theory with fluctuating couplings, and coupled to dynamical monopole currents[1,2]. The couplings and monopoles are determined by the $SU(2)/Z(2)$ dynamics. This generalizes to general $SU(N)$, but the simplest case $N = 2$ already exhibits all pertinent features. These $SU(2)/Z(2) \sim SO(3)$ monopoles possess one unit of $Z(2)$ flux, and may be viewed as endpoints of vortices – thus they are monopoles with strings attached to them. They are dynamically generated topological objects whose presence and interactions are revealed by an exact, gauge-invariant, non-perturbative mapping of the measure into new variables. They should **not** be confused with occasional, more or less ad-hoc attempts in the literature to isolate Abelian $U(1)$ monopoles in $SU(N)$ theories by a process of gauge fixing:[3] this "Abelian projection" and the resulting "monopole" configurations depend on the choice of gauge.

The effect of the dynamical monopoles, and associated vortices, on the phase diagram (mostly determined from bulk properties e.g. internal energy) in the standard $SU(2)$ model, and in generalizations involving chemical potentials for monopoles, and/or actions including other gauge group representations, have been discussed fairly extensively in the literature.[1,2,4–7] Effects on long-distance order parameters include the result[1] that if monopoles are rigorously excluded from the theory (MP model), the magnetic-disorder parameter ('Hooft loop) exhibits *area* law at large β. The interpretation of this[2] is that the 't Hooft operator is an external source for such monopoles, which in the absence of dynamical monopoles cannot be screened. Restoring dynamical monopoles in the theory results in screening and the expected length-law behavior. Since the MP model is expected to be confining[8], these results demonstrated that the 't Hooft loop does not provide by itself a sufficient criterion for confinement.

In what follows we will examine the role of dynamical monopoles in the study of the standard confinement criteria, the Wilson loop and the electric-flux free energy. Our results will show how the problem of confinement at arbitrarily large

β can be reduced to estimates on the distribution of monopole currents.

We work on a hypercubic lattice Λ in d dimensions with bonds b, plaquettes p, cubes c, etc. denoting its elementary cells. The $SU(2)$ LGT is defined in terms of the bond variables $U_b \in SU(2)$ with partition function:

$$Z_\Lambda = \int \prod_{b \in \Lambda} dU_b \exp \sum_{p \in \Lambda} \beta \,\mathrm{tr}\, U_p \tag{1}$$

The action is the standard (Wilson) action with $U_p \equiv \prod_{b \in \partial p} U_b$ the product of U_b's around the boundary of the plaquette p and $\beta = 1/g^2$ is the inverse bare gauge coupling. To isolate the presence of dynamical monopoles, one rewrites the theory in terms of $Z(2)$ and $SU(2)/Z(2) \sim SO(3)$ variables. Specifically, define a $Z(2)$ variable $\sigma_p = \{\pm 1\}$ residing on plaquettes, and let $\sigma_c \equiv \prod_{p \in \partial c} \sigma_p$ denote the product of σ_p's around the boundary of a cube c. Also introduce the notation

$$\eta_p \equiv \mathrm{sign}\,\mathrm{tr}\, U_p \quad , \quad \eta_c \equiv \prod_{p \in \partial c} \eta_p \tag{2}$$

One can then show[1,2] that (1) may be written in the form

$$Z_\Lambda = \int \prod_b dU_b \prod_p d\sigma_p \prod_c \delta[\eta_c \sigma_c] \exp \sum_p K_p(U) \sigma_p \tag{3}$$

where

$$K_p(U) \equiv \beta |\,\mathrm{tr}\, U_p| \tag{4}$$

and $\delta(\tau) = \begin{cases} 1 & \tau = 1 \\ 0 & \tau = -1 \end{cases}$ is the δ-function on $Z(2)$. Now note that η_c is invariant under $U_p \to U_b \gamma_b$, for any $\gamma_b \in Z(2)$, and, trivially, so is also $K_p(U)$. This implies that the integrand is (3) depends only on the coset variables $\hat{U}_b \in SU(2)/Z(2) \sim SO(3)$, i.e. each U_b may be any representative of the coset $\hat{U}_b$, and the U_b-integration is replaced by integration over the cosets.

The physical interpretation of the form (3) is easily obtained by going over to additive $Z(2)$ notation. Let us write

$$\sigma_p = \exp i\pi F_{\mu\nu}(x) \tag{5}$$

$$\eta_c = \exp i\pi m_{\mu\nu\lambda}(x) \tag{6}$$

introducing explicit tensor notations $F_p = F_{\mu\nu}(x)$, $m_c = m_{\mu\nu\lambda}(x)$ corresponding to plaquettes $p = (x, \mu\nu)$, and cubes $c = (x, \mu\nu\lambda)$ extending from site x in the positive μ, ν, λ directions. The δ-function constraint $\sigma_c = \eta_c$ then becomes

$$m_{\mu\nu\lambda} = \nabla_{[\mu} \ F_{\nu\lambda]} \tag{7}$$

Furthermore, definition (2) implies that η_c satisfies the constraint $(d \geq 4)$

$$\prod_{c\epsilon\partial h} \eta_c = 1 \tag{8a}$$

where the product is over all cubes forming the boundary of the hypercube h. In additive language (8a) becomes

$$\epsilon_{\mu\nu\kappa_\lambda} \nabla_\mu \, m_{\nu\kappa_\lambda} = 0 \tag{8b}$$

(7) is recognized as Dirac's modification of Maxwell's equations in the presence of a magnetic monopole current m_c, whereas (8) is the equation of magnetic current conservation.

Expression (3) for the partition function is thus seen to cast the original theory (1) in the form of a $Z(2)$ LGT of the variables σ_p with fluctuating (positive) coupling constants K_p, and coupled to dynamical monopole current η_c. As noted above, $K_p = K_p(\hat{U})$, $\eta_c = \eta_c[\hat{U}]$, $\hat{U} \ \epsilon \ SU(2)/Z(2)$. Thus the monopoles arise solely due to the non-Abelian part of the dynamics which allows configurations with $\eta_c = -1$. No such sources of $Z(2)$ flux can arise in an Abelian theory where $\eta_c = 1$, or, more generally, $\prod_{p\epsilon S} \eta_p = 1$ for any closed surface S, is always, trivially, satisfied.

Geometrically, the condition (8) means that the cubes on which $\eta_c = -1$ form co-closed sets of cubes (i.e. closed sets on the dual lattice Λ^*). In $d = 4$, where a cube is dual to a bond, a co-closed set of cubes is a closed loop of bonds on Λ^* representing a magnetic current loop. In $d = 3$ cubes are trivially co-closed – a set of two cubes is a pair of sites on Λ^* representing a monopole-(anti)monopole pair (monopoles are "instantons" in $d = 3$). Now a co-closed set of cubes $\mathbf{C}$ forms the co-boundary of a set of plaquettes D. If $\eta_c = -1$ on every $c \in \mathbf{C}$, $\eta_c = +1$ otherwise, one must have $\eta_p = -1$ on every $p \in D$. On Λ^* in $d = 3$, D is a set of bonds forming a path connecting a pair of monopole sites; in $d = 4$, D is a set of plaquettes spanning the closed magnetic current loop of bonds. Thus D is the Dirac string (sheet) attached to a monopole (monopole world-line). It is easily seen that the location of D is arbitrary – it can be moved around by a change of variables in (3).

Co-closed sets of plaquettes D (strings forming closed loops in $d = 3$, sheets forming closed 2-dim surfaces in $d = 4$) are vortices. Such configurations may also occur in Abelian LGT. Monopoles may be viewed as arising from "cut" vortices (endpoints in $d = 3$, edges of sheets in $d = 4$), and can only exist as non-Abelian $SO(3)$ configurations. They are then "stringy" monopoles – they are topologically characterized by $\pi_1(SO(3)) = Z_2$ and correspond to its non-trivial element. Stringless "monopoles" correspond to the trivial element of $\pi_1(SO(3))$, and may be thought of as the cube (point) where two vortices come together and annihilate. Thus if one imagines $SO(3)$ broken down to $U(1)$ (e.g. by an adjoint Higgs field), the stringless monopoles would be the of Hooft-Polyakov monopoles given by integer multiples in the Dirac quantization condition; whereas the stringy monopoles by half-integer multiples. Upon removing the breaking restoring the full $SO(3)$, the stringless monopoles become topologically trivial (i.e. regularized Wu-Yang monopole configurations).

As expected on physical grounds, monopoles become rare at large β. In fact it can be proven that the probability of exciting monopoles on a set G containing

$|G|$ cubes obeys the bound:[1]

$$\langle\prod_{c\epsilon G}\theta[-\eta_c]\rangle_\Lambda \quad \leq \quad (\text{const.})\ e^{-c\beta|G|} \quad , \quad \beta\to\infty. \tag{9}$$

Numerically, approximate equality results for $c\sim 2$. Thus monopoles become rare as $\beta\to\infty$. This in itself does not mean, however, that they become unimportant since, e.g. , they can strongly influence any observable whose variation is also exponential. In fact, it was observed in Ref. [7] that the density of monopoles $N(\beta)/a(\beta)^3$ ($N(\beta)$ = number of monopoles is given by the above estimate, and a = the lattice spacing) diverges in the continuum limit if $a(\beta)$ is given by the standard RG 2-loop expression. We refer to the literature[5-7] for discussions of the influence of monopoles in the determination of various observables. Here we concentrate on long distance effects, in particular the role of monopoles in confinement at large β.

We next perform a duality transformation on the $Z(2)$ variables (i.e. a Fourier transform on $Z(2)$) in (3). The transformation trades σ_p for a $Z(2)$ variable ω_c defined on cubes, and gives:

$$Z_\Lambda = \int\prod_b dU_b\prod_c d\omega_c\prod_c\chi_{\eta_c}[\omega_c]\exp\sum_p\Big[\hat{M}_p(\hat{U})+\hat{K}_p(\hat{U})\omega[\hat{\partial}_p]\Big] \tag{10a}$$

with

$$\begin{aligned}\hat{M}(\hat{U}) &\equiv \tfrac{1}{2}ln\tfrac{1}{2}\sinh 2K_p\\ \hat{K}_p(\hat{U}) &\equiv \tfrac{1}{2}ln\coth K_p\end{aligned} \tag{10b}$$

and the notation $\omega[\hat{\partial}_p]\equiv\prod_{c\epsilon p}\omega_c$, i.e. the product of all ω_c's on cubes whose boundary contains a given plaquette p. The quantities $\chi_{\eta_c}[\omega_c]$ are the characters of the $Z(2)$ group defined by

$$\chi_\tau[\gamma]=\begin{cases}1 & \text{if } \tau=1\\ \gamma & \text{if } \tau=-1\end{cases} \tag{11}$$

for any two elements $\tau,\gamma\ \epsilon\ Z(2)$.

Consider now the standard confinement order parameters, the Wilson loop or, more conveniently, the electric-flux free energy[9]. Recall that the electric flux free energy $e^{-F_{el}}$ ("sourceless Wilson loop") may be viewed as obtained from the Wilson loop $W[C]$ by removing the (physically inessential) external source current on the loop C, while trapping the color-electric flux it creates in a topologically non-trivial configuration in a lattice with periodic boundary conditions (torus). Performing the steps that led from (1) to (3) to (10) in the case of the electric-flux free energy one obtains[10]

$$\langle e^{F_{el}} \rangle = \frac{1}{Z_\Lambda} \int \prod_b dU_b \prod_c d\omega_c \prod_p e^{\hat{M}_p} \eta_S \prod_c \chi_{\eta_c}[\omega_c] \exp \sum_p \hat{K}_p \omega[\partial p](-1)^{E_S[p]} \tag{12}$$

Here S is any 2-dim surface completely winding through the lattice in two fixed directions, say $[\mu\nu] = [12]$, i.e. it is a topologically non-trivial closed surface. $E_S[p] = 1$ if $p \epsilon S$, 0 otherwise, is its characteristic function. η_S denotes the product $\prod_{p \epsilon S} \eta_p$. (12) has a rather transparent structure. With periodic b.c., η_S depends only on cosets $\hat{U}_b$, and represents the part of $e^{-F_{el}}$ directly coupled to the $SU(2)/Z(2)$ dynamics. The part coupling to the $Z(2)$ degrees of freedom appears, after the $Z(2)$ duality transformation, as a magnetic-flux free energy ("sourceless" 't Hooft loop), as expected. The two parts interact through the coupling $\chi_{\eta_c}[\omega_c]$ of the monopoles to the ω's. It is, of course, important that the expectation (12) is independent of the choice of the surface S. It is indeed easy to verify that S in the exponent in (12) can be moved to a different surface S' by a shift of the ω integration variables; and that the resulting shift in the $\chi_{\eta_c}[\omega_c]$ factors is precisely what is needed to change η_S to $\eta_{S'}$.

Consider first the case where monopole excitation is forbidden (MP model). Now for large β, $\hat{K}_p(U) \sim e^{-4\beta} \ll 1$ for almost all U's. Thus the ω variables give a strongly-coupled pure $Z(2)$ gauge theory for which the 't Hooft operator in its action can only have "length-law" behavior, i.e. S-independent contribution to (12). This can be proven rigorously – the exceptional U configurations being of "small measure" can be shown not to alter this result[11]. It follows that in the absence of monopoles confining behavior can only come from the $SU(2)/Z(2)$ part of the operator, i.e. η_S. η_S probes vortices winding around the lattice in the $\mu = 3, 4, \cdots$ directions. The way in which lateral flux spreading of such vortices can, if rapid enough, disorder η_S has been discussed in the literature[12,4,2,13]. Unfortunately, demonstrating such behavior of spread-out vortices requires a non-perturbative treatment such as block-spinning, and thus very difficult to demonstrate rigorously.

When monopole excitation is allowed on a lattice with periodic b.c., there are two possibilities for the associated Dirac sheet. It can span a magnetic current loop as a surface either (a) topologically equivalent, or (b) topologically inequivalent to the minimal area surface.

In case (a) a loop in $d = 4$ encircling a $x_\nu =$ const. ($\nu = 1$ or 2) slice through S (equivalently, a monopole pair whose two members are on oppose sides of S in $d = 3$, or in a 3-dim slice $x_\nu =$ const., $\nu = 3$ or 4, of $d = 4$) couples to both operators in (12), i.e. it contributes a minus sign to η_S and excites ω_c's that couple to $\omega[\hat{\partial}p]$, $p\epsilon S$. The net effect cancels, however: a change of ω_c-variables in (12) can move S to another S' not intersected by the Dirac sheet.

In case (b) the loop couples either to the observable in the $Z(2)$ action, or, if S is moved to S' away from the loop location, to η_S, but not both or neither. The physical situation is now quite close to that of a vortex. A magnetic loop with the Dirac winding around the lattice is effectively a vortex but one with a "puncture" or "gap". Placing S through the gap allows these configurations to couple directly to F_{el} through its $Z(2)$ part.

Let us now exclude all vortices winding around the lattice in directions perpendicular to S ($\mu = 3, 4, \cdots, d$). This can be done by inserting a factor $\theta[\eta_S] \equiv \Theta_S$ in the measure in (12) thus restricting η_S to unity. Freezing η_S to unity means that F_{el} is now given solely by the operator in the $Z(2)$ action in (12). We saw that without monopoles this operator cannot give confining behavior. Note also that the constraint $\eta_S = 1$ implies that a monopole loop "encircling" (a slice through) S must necessarily have the associated Dirac sheet winding around the lattice.

We conclude that *in the absence of monopoles (MP model) confining behavior can only come from vortices; whereas in the absence of vortices confinement, if it persists, can only be due to monopoles with Dirac sheets completely winding around the lattice. Finally, eliminating both vortices and monopoles leads to non-confining ("length-law") behavior of the electric-flux free energy.*

Since vortices tend to disorder the system, their exclusion should result into increasing the expectation (12). The proof, though not trivial, is a straightforward consequence of the reflection positivity properties of the measure (3), and one can indeed show

$$\langle e^{-F_{el}}\rangle_\Lambda \;\leq\; \langle 2\Theta_S e^{-F_{el}}\rangle_\Lambda \tag{13}$$

with planar S, i.e. S any [12]-plane : x_μ =const., $\mu = 3, \cdots, d$. As already discussed, the r.h.s. cannot produce confinement at large β in the absence of monopoles. Since it gives a rigorous *upper-bound* on the exact expectation, this means that *demonstration of confinement at large β is tantamount to estimating the effect of the monopole loops with topologically non-trivial Dirac sheets.*

To do this consider the difference between the r.h.s. in (13) and the corresponding expectation where the monopoles have also been excluded. In particular, let $\langle e^{-F_{el}}\rangle_{\Lambda,K^o,1}$ denote the same expectation as (12) but with $K_p(U)$ replaced its maximum $K^o \equiv 2\beta$, and η_S and η_c (all c) set equal to unity. One can then prove

$$\langle e^{-F_{el}}\rangle_{\Lambda,K^o,1} - \langle \Theta_S \, e^{-F_{el}}\rangle_\Lambda \;\geq\; 0 \tag{14}$$

Inequality (14), which is expected by physical reasoning, is proven by an argument resembling Ginibre's method of proving Griffiths and related "comparison" inequalities for Abelian systems[14]. (The $Z(2)$ gauge system forming part of the integrand allows an extension of such a method to go through). Now the probability of exciting a monopole current on a single co-closed set of cubes ℓ (monopole instanton pair in $d = 3$, monopole current loop in $d = 4$, monopole closed 2-dim current surface in $d = 5$, etc.) and with topologically non-trivial Dirac sheet can be shown to satisfy:

$$\frac{1}{Z_\Lambda^{(+)}} \int d\nu_\Lambda \prod_{c\epsilon\Lambda/\ell} \theta[\eta_c] \prod_{c\epsilon\ell} \theta(-\eta_c) \quad \geq \quad \text{const.}\ e^{-k\beta|\ell| - k'\beta L^{d-4}} \tag{15a}$$

with

$$d\nu_\Lambda \equiv \prod_b dU_b \prod_p e^{\hat{M}_p(\hat{U})} \quad , \quad Z_\Lambda^{(+)} \equiv \int d\nu_\Lambda \prod_c \theta[\eta_c] \tag{15b}$$

and where k,k' constants, and L the length of the lattice $\Lambda = L^d$ in any direction (cp the upper bounds (9)). *For $d \leq 4$ this probability remains finite in the large volume limit.* Using (15) in the proof of (14) shows that (14) is actually a strict inequality.

Now consider the quantity $\langle e^{-F_{el}} \rangle_{\Lambda, K^o, \bar{\eta}}$ obtained from (12) again by replacing $K_p(U)$ by K^o and η_S by 1, but now replacing η_c (all c) by a mean value $\bar{\eta}$, $0 < \bar{\eta} \leq 1$. So $\chi_{\eta_c}[\omega_c]$ is replaced by $\frac{1}{2}[(1+\bar{\eta}) + (1-\bar{\eta})\omega_c]$. $\langle e^{-F_{el}} \rangle_{\Lambda, K^o, 1}$ in (14) may then be viewed as the limit $\bar{\eta} \to 1$. Now $\langle e^{-F_{el}} \rangle_{\Lambda, K^o, \bar{\eta}}$ is continuous in $\bar{\eta}$, whereas, as noted, (14) is actually a strict inequality. It then follows that, by continuity, there exists a neighborhood $1 \geq \bar{\eta} \geq \bar{\eta}_o$ where

$$\langle e^{-F_{el}} \rangle_{\Lambda, K^o, \bar{\eta}} \quad - \quad \langle \theta_S e^{-F_{el}} \rangle_\Lambda \quad \geq 0 \quad , \quad \bar{\eta}_o \leq \bar{\eta} \leq 1 \tag{16}$$

The argument leading to (16) does not determine the size of this neighborhood, in particular, any dependence of $\bar{\eta}_o$ on, say, the lattice size. Certainly, for $d > 4$ where the estimate (15) vanishes as $L \to \infty$, (14) cannot be shown to remain a strict inequality, and $\bar{\eta}_o$ shrinks to zero with increasing lattice size. For $d \leq 4$, however, the monopole excitation bound (15) remains finite regardless of lattice size. This strongly suggests, but does not by itself suffice to give a rigorous proof, that an $\bar{\eta}_o$ independent of any lattice length exists. We will not attempt to discuss here what further considerations may be needed to rigorously justify this last statement. Instead we will assume that $\bar{\eta}_o$ in (16) is a constant: indeed, it is clear from (15), (9) that $\bar{\eta}_o \sim$[1−const. exp(−const.β)].

(16), (13) provide then an explicit upper bound on the electric-flux free energy for large β which, after some manipulation, is easily shown to assume the form:

$$\begin{aligned} \langle e^{-F_{el}}\rangle_\Lambda \quad &\leq 2 \quad \langle \theta_S e^{-F_{el}}\rangle_\Lambda \\ &\leq \frac{2}{\hat{Z}_\Lambda} \int \prod_c d\omega_c \exp\Big(\sum_c R^2 \omega_c + \sum_p \hat{K}^o \omega[\hat{\partial}_p](-1)^{E_S[p]} \Big) \end{aligned} \tag{17}$$

where $\tanh R^2 \equiv \frac{1-\bar{\eta}}{1+\bar{\eta}}$, $\hat{K}^o \equiv \frac{1}{2} \ln \coth 2\beta$, and $\hat{Z}_\Lambda$ is the corresponding partition function (i.e. the numerator without the insertion of the (-1) twist on $p \in S$). This is the form of an effective $Z(2)$ gauge-Higgs system (in unitary gauge). The modulus R of the Higgs field is given by the monopole excitation probability, and, as noted above, $R^2 \sim \exp-(\text{const.})\beta$ as $\beta \to \infty$. For large β, $\hat{K}^o \ll 1$, and the r.h.s. of (17) can be evaluated in a polymer cluster expansion that can be proven to converge for sufficiently large β. As it is physically obvious, and confirmed by the explicit computation, the bound (17) gives area-law behavior with coefficient in leading approximation equal to $2(d-2)R^2 \sim$ const. $\exp-(\text{const.})\beta$, $\beta \to \infty$. This is, of course, the expected non-perturbative exponential β-dependence for the string tension.

In fact area dependence holds for any non-zero R, even though (17) holds, of course, only for $R[\bar{\eta}(\beta)]$ exponentially small corresponding to $\bar{\eta}_o < \bar{\eta} < 1$. For $R = 0$ $(\bar{\eta} = 1$, exclusion of monopoles) we have sudden crossover to length-law. This is because the Higgs coupling is necessarily in the fundamental representation, since this is the only non-trivial representation of $Z(2)$. As it is well-known no phase transition as a function of the Higgs modulus occurs in this case[15]. Thus area-law obtains even when the dynamical monopoles become very rare as $\beta \to \infty$. It should be stressed that the physical origin of this state of affairs is the "stringy" nature of the monopoles[16].

To summarize, we have seen how dynamical monopoles with topologically non-trivial Dirac sheets provide a sufficient mechanism for confinement at arbitrarily weak gauge coupling (large β) for $d \leq 4$. These monopoles and their interactions are isolated by the exact rewriting of the original measure of the $SU(2)$ LGT in terms of $SO(3)$ and $Z(2)$ variables. The mechanism is given a precise mathematical statement in the rigorous inequalities (13), (14), and (16). The remaining assumption of the lattice size independence of the quantity $\bar{\eta}_o$ in (16) is certainly physically very plausible. Removing this assumption would then result into a complete rigorous proof of confinement via dynamical monopoles for $\beta \rightarrow \infty$ on the lattice.

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