INTERFEROMETRIC STUDIES OF THE SUN

AT MICROWAVE AND MILLIMETER WAVELENGTHS

by

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Interferometric Studies of the Sun at Microwave and Millimeter Wavelengths

Thesis directed by Professor Frances Bagenal

The work presented in this thesis relied mainly upon two radio interferometers, the Owens Valley Solar Array and the Owens Valley Millimeter Array. I used the Owens Valley Millimeter Array during the eclipse of July 11, 1991 to observe the brightness temperature profile of the solar limb. These observations showed that photospheric-temperature gas exists at an altitude of about 5500 km above the visible photosphere, far beyond the expected location of the transition region where the temperature increases to about one million degrees. I proposed that this result can be explained in terms of a two-component model of the upper chromosphere. Spicules would compose a relatively cool and dense component embedded in a hot, tenuous medium like that described by the so-called VAL model (Vernazza, Avrett, and Loeser 1981).

Later I used data from the Solar Array to make a detailed investigation of the gyrosynchrotron spectrum of the flare of July 16 1992. The results of this investigation led me to propose a solution to a long-standing problem in solar microwave bursts, that of the constant peak frequency of the bursts as they evolve in brightness temperature, and the steep slope on the low-frequency side of the spectrum. I proposed that the Razin effect is at work, and I developed the theory of Razin suppression for solar microwave burst conditions. The Razin effect is the suppression of radiation from an
electron in a medium in which the index of refraction is less than unity. I demonstrated that in a medium with density $2 \times 10^{11} \text{cm}^{-3}$ and magnetic field 300 Gauss, conditions not uncommon for solar microwave bursts, the gyrosynchrotron spectrum can be suppressed for frequencies up to at least 10 GHz. Finally, I made use of the X-ray data from the Yohkoh spacecraft to learn more about this particular flare, and to check some of the results from the investigation of the radio spectrum. I found that the ambient density and electron spectral index inferred from the X-ray data matched those deduced from the microwave data.
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CONTENTS

Chapter                                                                 Page

I. INTRODUCTION ........................................................................... 1

1.1 Why Study Radio Emission from the Sun? ......................... 1
1.2 Objective of the Thesis and Important Results .......... 3
1.3 Overview of the Solar Atmosphere ................................. 4
1.4 The Nature of Solar Flares .................................................. 8
1.5 Overview of Flare Microwave Emission ......................... 9
  1.5.1 Introduction .............................................................. 9
  1.5.2 Gyrosynchrotron Emission ........................................ 11
  1.5.3 The Razin Effect: Essential Features ....................... 17
  1.5.4 Bremsstrahlung ......................................................... 17
1.6 Radio Interferometry Basics ............................................. 20
1.7 Overview of the Owens Valley Solar Array ................... 29
1.8 Origin of Data Used in this Thesis ................................. 36

II. FLARE OF JULY 16 1992: OBSERVATIONS .......................... 

WITH THE OWENS VALLEY SOLAR ARRAY ............................. 42

2.1 Introduction ......................................................................... 42
2.2 Observations and Analysis ............................................... 45
  2.2.1 Overview of the Flare ............................................... 45
  2.2.2 Distinction Between Low and High Frequency Components 49
  2.2.3 Images of the Radio Burst ......................................... 53
2.2.4 Brightness Temperature Spectra and Source Size Spectra of the Radio Burst 61

2.3 Gyrosynchrotron Emission and Application of the Simplified Theory to Solar Bursts 67

2.4 Difficulty of Explaining the Spectra Using Simplified Theory 75

2.5 Numerical Modeling 79

2.5.1 The Gyrosynchrotron Code 79

2.5.2 The Razin Effect 84

2.5.3 Modeling the Results 97

2.6 Discussion 120

III. FLARE OF JULY 16 1992: MULTISPECTRAL OBSERVATIONS 123

3.1 Introduction 123

3.2 Flare Location with Respect to Nearby Active Regions 124

3.3 Magnetograph Observations 126

3.4 Yohkoh Soft X-ray Images 133

3.5 Comparison of Microwave and Soft X-ray Images 137

3.6 Temperature and Emission Measure Determinations From Yohkoh 140

3.7 Implications of Temperature and Emission Measure Results 143

3.8 Hard X-ray Images 148

3.9 Hard X-ray Spectrum 153
### TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 Characteristics of the Yohkoh</td>
<td></td>
</tr>
<tr>
<td>Soft X-ray Telescope</td>
<td>39</td>
</tr>
<tr>
<td>1.2 Characteristics of the Yohkoh</td>
<td></td>
</tr>
<tr>
<td>Hard X-ray Telescope</td>
<td>40</td>
</tr>
<tr>
<td>Characteristics of Microwave Spectra</td>
<td>68</td>
</tr>
<tr>
<td>2.2 Range of Index of Refraction For Which Razin Effect Occurs</td>
<td></td>
</tr>
<tr>
<td>Razin Effect Occurs</td>
<td>92</td>
</tr>
<tr>
<td>2.3 Results of Fits to Gyrosynchrotron Spectra</td>
<td>113</td>
</tr>
<tr>
<td>3.1 Region of Interest Characteristics</td>
<td>144</td>
</tr>
<tr>
<td>3.2 Photon Spectral Index</td>
<td>157</td>
</tr>
</tbody>
</table>
## FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Temperature versus Height</td>
<td>7</td>
</tr>
<tr>
<td>1.2</td>
<td>Power Spectrum: Particle in Cyclotron Motion</td>
<td>14</td>
</tr>
<tr>
<td>1.3</td>
<td>The Two-Element Interferometer</td>
<td>21</td>
</tr>
<tr>
<td>1.4</td>
<td>Power Reception Pattern of a Two-Element Interferometer</td>
<td>22</td>
</tr>
<tr>
<td>1.5</td>
<td>Position Vectors</td>
<td>25</td>
</tr>
<tr>
<td>1.6</td>
<td>The Visibility Function</td>
<td>26</td>
</tr>
<tr>
<td>1.7</td>
<td>Components of an Interferometer Array</td>
<td>32</td>
</tr>
<tr>
<td>2.1</td>
<td>Time Plot of the Flare Total Power</td>
<td>46</td>
</tr>
<tr>
<td>2.2</td>
<td>Total Power at the Peak of the Flare</td>
<td>48</td>
</tr>
<tr>
<td>2.3</td>
<td>Time Evolution of the Total Power Spectrum</td>
<td>50</td>
</tr>
<tr>
<td>2.4</td>
<td>Comparison of Total Power and Correlated Amplitude</td>
<td>54</td>
</tr>
<tr>
<td>2.5</td>
<td>Cleaning of Flare Images Using Frequency Synthesis</td>
<td>58</td>
</tr>
<tr>
<td>2.6</td>
<td>Comparison of LCP and RCP Maps</td>
<td>60</td>
</tr>
<tr>
<td>2.7</td>
<td>Deconvolution of the Source Size</td>
<td>64</td>
</tr>
<tr>
<td>2.8</td>
<td>Source Size Spectrum</td>
<td>65</td>
</tr>
<tr>
<td>2.9</td>
<td>Brightness Temperature Spectra</td>
<td>69</td>
</tr>
<tr>
<td>2.10</td>
<td>Numerical Calculations of Gyromagnetic Emission</td>
<td>72</td>
</tr>
</tbody>
</table>
2.11 Universal Curves of Bremsstrahlung and Gyrosynchrotron Emission .......................................................... 74
2.12 Temporal Evolution of a Microwave Burst Spectrum .................................................................................. 78
2.13 Effect of Low-Frequency Cutoff ..................................................................................................................... 80
2.14 Geometry of a Particle Emitting Synchrotron Radiation ............................................................................. 85
2.15 Square of the Index of Refraction For Various Values of the Razin Parameter ........................................... 93
2.16 Power Spectrum from Non-Relativistic Particle ......................................................................................... 96
2.17 Effect of Varying the Density of Accelerated Particles ............................................................................. 104
2.18 Effect of Varying Exponent $\delta$ .................................................................................................................. 105
2.19 Effect of Varying the Field Strength .......................................................................................................... 106
2.20 Effect of Varying the High-Energy Cutoff ................................................................................................. 107
2.21 Effect of Varying the Low-Energy Cutoff ................................................................................................. 108
2.22 Effect of Varying the Path Length to the Source ....................................................................................... 109
2.23 Effect of Varying Viewing Angle $\theta$ ......................................................................................................... 110
2.24 Summary Figure ......................................................................................................................................... 111
2.25 Overlay of Data and Model Fits ................................................................................................................ 114
2.26 Temporal Evolution of Model Parameters ................................................................................................. 115
2.27 Fits Without Razin Suppression .............................................................................................................. 119
3.1 H$_{\alpha}$ Image from MSFC .......................................................................................................................... 125
3.2 White-Light Image from Yohkoh and 11.2 GHz Radio Source .................................................................... 127
3.3 White-Light Image from Sacramento Peak .................................................................................................. 128
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.4 MSFC Longitudinal Field Plot</td>
<td>130</td>
</tr>
<tr>
<td>3.5 MSFC Magnetogram and Hα Images</td>
<td>132</td>
</tr>
<tr>
<td>3.6 Response Function of SXT</td>
<td>138</td>
</tr>
<tr>
<td>3.7 SXT Source Morphology</td>
<td>136</td>
</tr>
<tr>
<td>3.8 Evolution of the SXT Source</td>
<td>138</td>
</tr>
<tr>
<td>3.9 Radio Source Overlaid on SXT Image</td>
<td>139</td>
</tr>
<tr>
<td>3.10 Temperature Sensitivity in SXT Filter Ratios</td>
<td>141</td>
</tr>
<tr>
<td>3.11 Region of Interest Locations</td>
<td>145</td>
</tr>
<tr>
<td>3.12 Density and Temperature in ROI 3</td>
<td>146</td>
</tr>
<tr>
<td>3.13 HXT L and H band Images</td>
<td>149</td>
</tr>
<tr>
<td>3.14 Radio and HXT Lightcurves</td>
<td>158</td>
</tr>
<tr>
<td>4.1 Path of the Moon With Respect to the Sun</td>
<td>166</td>
</tr>
<tr>
<td>4.2 Amplitude and Phase of Visibility Function</td>
<td></td>
</tr>
<tr>
<td>Around the Time of First Contact</td>
<td>170</td>
</tr>
<tr>
<td>4.3 Model of Occultation in Amplitude and Phase</td>
<td>172</td>
</tr>
<tr>
<td>4.4 Comparison of Data with Real and Computed Phases</td>
<td>175</td>
</tr>
<tr>
<td>4.5 Limb Profile Derived Using Ephemeris Phases</td>
<td>178</td>
</tr>
<tr>
<td>4.6 Off-band Hα Photograph of</td>
<td></td>
</tr>
<tr>
<td>Region Near the Limb</td>
<td>182</td>
</tr>
</tbody>
</table>
1.1 Why Study Radio Emission from the Sun?

The study of the Sun is unique in that it is a branch of astronomy that has practical applications, as well as being interesting in its own right. Solar radiation affects the earth’s climate, human activity in space, and communications. The Sun is also a laboratory for the physics of hot magnetized gases, and it presents us with the opportunity to examine stellar activity and evolution close-up. For these reasons solar physics is broader than any other subject in astronomy.

The study of solar activity is particularly relevant to human activity. One manifestation of solar activity is flares, which consist of intense bursts of electromagnetic radiation and the brightening and energizing of magnetic loops over active regions. Eruptive events such as coronal mass ejections (CMEs), which are sometimes associated with flares, are another manifestation of solar activity. Coronal mass ejections produce energetic particles which pose the danger of radiation sickness or death to astronauts. Astronauts in space during a week of high solar activity might receive a lifetime dose of radiation, even if shielded by aluminum plates 1/4 inch thick. Solar activity can also trigger geomagnetic storms which disrupt radio communications. A well-known example of the sun-earth connection is the temporary loss of electric power throughout much of Quebec as a result of solar activity one day in March 1989.
Because of the importance of solar-terrestrial effects, laboratories such as the Space Environment Laboratory of the U.S. Department of Commerce collect data on solar activity and provide forecasts of conditions in space. Forecasting flares or eruptive events is a difficult problem, however, analogous to the forecasting of earthquakes or avalanches. The problem is not like that of predicting, for example, the chance of a river overflowing its banks. This type of forecasting can be done quite accurately and with hours or days of lead time with adequate monitoring, in this case monitoring of water levels in upstream tributaries. The conditions for solar flares, and probably CMEs, do not build in such a linear fashion. While the energy stored in solar magnetic fields may accumulate through a gradual reorientation of the field, the explosive release of energy in a flare is triggered by an instability which occurs on a scale too small to be observed. Large flares probably occur when instabilities propagate to neighboring sites, so that, as with avalanches, the size of the event or flare is not related to the magnitude of the instability that triggered the original release of energy.

Research into the nature of solar activity leads to valuable new tools for forecasting flares, but most solar research, such as is presented in this thesis, is undertaken simply with the goal of improving our understanding of flares or of the solar atmosphere. Many pieces of the puzzle remain to be found. We have only a sketchy understanding of how particles are accelerated to high energies during flares, for example, or of how emission at different wavelengths and from different parts of magnetic loops is related.

A large part of this thesis is devoted to the analysis of microwave emission from flares. Microwaves are useful to trace both the energetic particles and the magnetic fields in flaring regions. Microwave emission can
be observed from the ground, in contrast to shorter-wavelength radiation which is absorbed by the earth’s atmosphere. Although the portion of flare energy flux in microwaves is small compared to the energy in X-rays, the microwaves are associated with the higher energy particles, often having similar time profiles and in some cases having the same source.

The ideal case, of course, is to study flare emission in different parts of the spectrum collectively. We have endeavored to do this in comparing the microwave and X-ray emission from a solar flare that was observed by several instruments simultaneously. Such studies of individual flares help form the big picture of solar flares: where flares occur with respect to the magnetic field of an active region, how particles are energized, and what the ambient conditions are.

1.2 Objective of the Thesis and Important Results

The objective of this thesis is to study solar microwave bursts with the high spatial, spectral, and temporal resolution of the Owens Valley Solar Array and to take advantage of the high spatial resolution afforded by the solar eclipse of 1991 to study the brightness temperature profile of the solar limb with the Owens Valley Millimeter Array. The techniques of radio interferometry are applied in both cases to achieve high spatial resolution.

The flare of July 16, 1992 provided a particularly interesting case study of the spectrum of a microwave burst, and detailed numerical modeling of the gyrosynchrotron emission was carried out in Chapter 2 to determine certain flare parameters such as the density of the ambient medium and of the accelerated particles. Observations of this flare were also available from other instruments and permitted a multi-spectral analysis of the flare conditions (Chapter 3). The eclipse observations (Chapter 4) pertain to a lower portion
of the solar atmosphere than the flare observations. The eclipse data gave a direct measurement of the temperature above the visible limb of the sun and constrain models of the chromosphere and transition region.

Important results include:

1) The discovery that microwave flare spectra may be suppressed, particularly at frequencies below 10 GHz, through the mechanism known as the Razin effect;

2) The development of the theory, first derived by Razin, of suppression of electromagnetic radiation in a medium with index of refraction less than unity;

3) The placing of the flare radio emission in the context of the active region magnetic field structure and relative to the emission at X-ray wavelengths;

4) The conclusion that spicules, or jets of material seen in Hα at the limb of the sun, may be associated with the unexpectedly low temperature (6500 K) plasma seen at 3 mm wavelength at relatively high altitude (5500 km) above the photosphere.

1.3 Overview of the Solar Atmosphere

We briefly review the solar atmosphere from the photosphere, or visible surface, to the corona, which extends a distance of several solar radii from the sun. This sets the stage for discussion of coronal plasma conditions in Chapters 2 and 3, and references to chromospheric and low coronal models in Chapter 4.

The photosphere, chromosphere, transition region and corona may be differentiated by their associated temperatures and densities and by their
magnetic structures. At the photosphere, the level at which sunspots appear, the temperature is 6000 K (decreasing to 4000 K at the photosphere-chromosphere transition) and the density of hydrogen atoms is $\sim 10^{17}$ cm$^{-3}$. Magnetic fields at the level of the photosphere are measured by the Zeeman splitting of magnetically sensitive absorption lines. Measured magnetic field values range from $\sim 500$ G in the so-called intranetwork regions outside sunspots (Lin, 1995) to 2000-3000 Gauss within spots. At the low end of the range the cutoff may determined by the sensitivity of the field-measuring technique and the low filling factor of the fields.

At the boundary between the photosphere and the chromosphere the temperature declines to a minimum of $\sim 4000$ K at an altitude about 400 km above the visible surface. (We measure altitude in the solar atmosphere with reference to the visible surface, the level at which the optical depth at 5000 Å is $\tau_{5000} = 2/3$.) At this boundary between the photosphere and chromosphere the density is $\sim 10^{12}$ cm$^{-3}$. Above this level H$_\alpha$ emission lines are produced which give the chromosphere its red color and its name. In the chromosphere the degree of ionization increases rapidly toward unity along with a rise in temperature. Despite the high degree of ionization the H$_\alpha$ emission line arising from neutral hydrogen is a useful wavelength in which to image chromospheric structures known as spicules. Spicules are jets of material seen in emission at the limb of the sun, forming a "forest" which extends (in H$_\alpha$) to an altitude of about $10^4$ km. Another characteristic of the chromosphere is the presence of metal lines with a temperature of $4 \times 10^3$ K,
and hydrogen emission with a temperature of 4000 K, suggesting that both "hot" and "cold" components form this layer.

Above an altitude of about $2 \times 10^4$ km a transition is made extremely quickly to the temperatures and densities characteristic of the corona. The transition region must be less than 1000 km in vertical extent, but very little is known about its structure. Figure 1.1, adapted from Stix (1989), shows a model of the temperature as a function of height. The temperature of formation of various atomic species is indicated on the plot, to show which elements are characteristic of the chromosphere and corona.

Above the chromosphere and transition region we know from observations of forbidden transitions in elements such as highly ionized iron and calcium that the kinetic temperature reaches one or two million degrees Kelvin, with densities near $10^8$ cm$^{-3}$ in the low corona. Over active regions the temperature may reach 50 million degrees. The density decreases slowly with distance from the sun, approaching interplanetary values of $\sim 10^4$ cm$^{-3}$ at about 5 solar radii.

Chapter 4 of this thesis discusses models of the chromosphere and transition region, focusing in particular on the temperature structure and inhomogeneity of these regions. In the rest of the thesis we are mostly concerned with the corona, where radio and X-ray emission from flares arises. Our understanding of the corona has blossomed in the past 30 or 40 years with the discovery that magnetic fields control the structure and dynamics of the corona, and with the availability of space-based X-ray and ultraviolet instruments and plasma wave and particle detectors. The instruments which have contributed to our understanding of the X-ray corona include the telescopes on board Skylab (launched 1973), the Solar Maximum Mission
Figure 1.1 Temperature as a function of height in a model solar atmosphere. The temperature of formation of various atomic species is indicated with closed and open circles. Figure adapted from Stix (1989).
(launched 1980), and the Japanese Hinotori mission (launched 1981). More recently an international collaboration led to the design and construction of low and high energy ("soft" and "hard") X-ray telescopes and spectrometers flown on the Japanese satellite Yohkoh launched by the Japanese space agency in 1991.

At the turn of the century the corona was known from observations made in visible light during solar eclipses. Electrons in the corona scatter white light from the photosphere (this is known as the K-corona) and also scatter light from the interplanetary medium (the F-corona). When viewed in white light the corona shows regions of enhanced intensity shaped like rays or like loops with cusped ends extending two to three solar radii above the photosphere. These are called coronal rays and helmet streamers. Our view of the corona is now dominated by magnetic loops and arcades of loops about $10^5$ km in height, glowing in X-ray emission, and the contrasting dark areas called coronal holes. Acton et al. (1992) review the wide array of coronal X-ray structures observed by Yohkoh.

1.4 The Nature of Solar Flares

Solar physicists agree that the energy released in a typical solar flare, of the order $10^{32}$ erg, is stored in twisted magnetic loops connecting regions of opposite magnetic polarity, much as potential energy is stored in a twisted rubber band. Twisted or sheared fields have been shown theoretically to provide the necessary potential energy, and no other source of flare energy seems plausible. According to this hypothesis the magnetic field should assume a configuration of lower potential energy after the release of energy. Measurement of the sheared state of magnetic fields before and after flares is currently an area of particular interest (see, e.g., Hagyard et al., 1995).
Still largely unknown is the mechanism by which particles are accelerated as a result of the energy release. Chupp (1990) reviews theories of particle acceleration, including magnetic reconnection in which oppositely-directed field lines come together and interact within a small volume, and acceleration by electric fields parallel to the magnetic field. A few of the questions to be answered with respect to particle acceleration are the following: are particles accelerated continuously or recurrently during the flare, or does the acceleration take place only at the onset? What determines the distribution of particle energies, and its upper limit? What is the volume over which the acceleration takes place?

Flares may give rise to emission in the entire spectrum of electromagnetic radiation from gamma rays to kilometer-wavelength radio waves. In this thesis we are concerned primarily with flare emission in the microwave regime, specifically in the range from 1-20 GHz (1.5 to 30 cm) and emission in hard (15-100 keV) and soft (0.24-4 keV) X-rays.

1.5 Overview of Flare Microwave Emission

1.5.1 Introduction

Solar burst emission in the microwave regime can be produced by a number of mechanisms, although gyrosynchrotron emission is the most prevalent. The electrons which produce the gyrosynchrotron emission at radio frequencies are thought to come from the same population of accelerated particles producing flare X-ray emission: the microwave emission is created by the spiraling of electrons around magnetic field lines, and the hard X-ray emission comes from bremsstrahlung radiation through the precipitation of these particles in the denser layers of the atmosphere.
Bremsstrahlung is another likely emission mechanism associated with flares in the gigahertz range, although this mechanism is not impulsive in nature. The classic flat spectrum of optically thin bremsstrahlung is sometimes seen persisting for tens of minutes after a large flare, along with enhanced soft X-ray emission from post-flare loops (e.g., Gary et al. 1995).

Plasma radiation and electron cyclotron maser emission are other mechanisms which may operate in the microwave regime. “Spike” bursts, so-called because their duration is on the order of 25 milliseconds or less, have brightness temperatures near $10^{13}$ K (Gary et al., 1991) and polarization near 100 percent which strongly suggests that a coherent mechanism such as the electron cyclotron maser is at work. Other forms of narrow-band bursts, generally of duration 2-4 seconds and in the 1-3 GHz range, may be produced via plasma emission. In this scenario longitudinal, non-travelling waves are converted to electromagnetic waves at the local plasma frequency or its low harmonics. See Dulk (1985) for a review of these emission mechanisms. In the following sections we discuss in more detail the gyrosynchrotron and bremsstrahlung mechanisms which are more relevant to the type of microwave burst analyzed in this thesis, broadband bursts with brightness temperatures in the range $10^7 - 10^9$ K. We include in section 1.5 a brief introduction to Razin suppression, which is discussed more fully in Chapter 2.
1.5.2 Gyrosynchrotron Emission

We categorize the radiation from particles spiraling around magnetic field lines as gyroresonance, gyrosynchrotron, or synchrotron emission, depending on the particle energy, with the middle category of gyrosynchrotron emission being occupied by particles of Lorentz factor up to 2 or 3. In astrophysical situations this gyro emission (a term we will use to generalize about all three energy classes) is often in competition with free-free emission; free-free emission or bremsstrahlung is proportional to $\frac{n^2}{T^{3/2}}$ and gyro emission to $nT^{\alpha}B^\beta$, with $\alpha, \beta > 1$, so that gyro emission dominates when the density is low or the field strength high. Astrophysical examples of gyro emission include the emission from the Crab Nebula in the constellation Taurus, extending from radio frequencies to the extreme ultra-violet, and the decimetric emission from Jupiter's radiation belts.

Most of the non-thermal radio spectra observed in astrophysics are believed to be due to synchrotron sources, but non-thermal solar radio spectra may be attributed to lower energy particles and even thermal distributions. In the following discussion we consider thermal and non-thermal distributions, among other important distinctions, in a general description of gyro radiation. We provide more detail about gyrosynchrotron emission than about bremsstrahlung because gyrosynchrotron radiation is central to the ideas presented in this thesis. All equations in this section, unless otherwise noted, are from Rybicki and Lightman (1979).
Single-Particle Emission

Consider first the case of a single electron gyrating around a magnetic field line of strength $B$. The particle orbits in a circular path with gyrofrequency $\nu_g = 2.8 \times 10^6 \, B$ (Gauss). The particle gradually loses energy through radiation, although this process is slow: for an electron of energy about 0.1 MeV, in a plasma with a magnetic field of a few hundred Gauss, the e-folding time of the energy loss is on the order of an hour. In any case the small radiation loss does not change the gyrofrequency because as the orbital radius decreases the velocity decreases also. In the low energy case radiation is emitted in all directions and may be polarized in the right circular sense, left circular, linear, or elliptical. When an electron orbit is viewed edge-on one sees the linear component. When the line of sight is along the magnetic field one sees the right circular component if $B$ points toward the observer and the left circular component otherwise. At intermediate angles one sees elliptical polarization.

As the energy or velocity increases the emitted radiation becomes beamed. The beam is aligned with the instantaneous velocity vector of the particle in circular orbit and has a width given by $\theta$ (degrees) $\sim \frac{56}{E}$ where $E$ is in MeV. The polarization seen is generally elliptical (the line of sight would have to lie along the orbit normal for the observer to see purely circular polarization).

Another effect of increasing the particle energy is that radiation is emitted at harmonics of the gyrofrequency as well as the fundamental frequency. Radiation may be emitted at hundreds of harmonics, or more, and the frequency of the harmonic which dominates the radiated power increases with increasing energy or increasing $B$. Figure 1.2 shows the power spectrum
for a low-energy particle emitting only two harmonics and that for a higher-energy particle, where the frequency of the dominant harmonic or critical frequency is given by

\[ \nu_g = \frac{3\gamma^2 q B \sin \alpha}{4\pi mc} \]  

(1.1)

(Symbols are defined in the table at the back of this thesis.)

If the motion of the particle is not wholly perpendicular to \( B \), but also along \( B \) so that the velocity vector makes an angle \( \alpha \) to the magnetic field, the radiation is beamed into a cone of opening angle \( 2\alpha \). The emission appears at harmonics of \( \frac{\nu_g^2}{\sin^2 \alpha} \) instead of \( \nu_g \); this is one of the major factors in creating a continuum of emission when an ensemble of particles is considered, because there will generally be a distribution of pitch angles so that not all particles are emitting in the same bandpass. The critical frequency defining the approximate upper limit of the emitted spectrum is given by

\[ \nu_c = \frac{3}{4\pi} \gamma^3 \omega_B \sin \alpha \]  

(1.2)

and the total power emitted by

\[ P = \frac{2q^4 B^2 \gamma^2 \beta^2 \sin^2 \alpha}{3m^2 c^3}. \]  

(1.3)
Figure 1.2 Power spectrum for a charged particle in cyclotron motion. Top panel shows two harmonics emitted by a low-energy particle; bottom panel shows the spectrum emitted by a higher-energy particle. Figure adapted from Rybicki and Lightman (1979).
Ensemble Effects

As we mentioned in the previous section, the “spike” structure of the power spectrum of single-particle gyro emission is generally smoothed to a continuum when a population of particles is present. Variations in energy, pitch angle, magnetic field strength and the direction of the field contribute to this effect.

Similarly the elliptical polarization of the radiation tends to be cancelled, leaving only some residual linear polarization. In some cases, such as the decimetric emission from Jupiter, a small residual circularly polarized component can also be observed, due to the resultant of elliptical polarization in one sense being slightly higher than the resultant in the other (Carr et al. 1983). (This happens when one of the mirror points of the electrons in the magnetosphere is viewed more nearly face-on than the other.) Dulk (1985) gives expressions for the degree of circularly polarized emission as a function of frequency and temperature or energy distribution index for the cases of thermal or power-law particle distributions.

When the radiation is partially linearly polarized so that it can be specified by the power per frequency parallel \( P_{\perp}(\omega) \) and perpendicular \( P_{\parallel}(\omega) \) to the projection of the magnetic field on the plane of the sky, we define the degree of linear polarization to be:

\[
\Pi(\omega) = \frac{P_{\perp}(\omega) - P_{\parallel}(\omega)}{P_{\perp}(\omega) + P_{\parallel}(\omega)}. \tag{1.4}
\]

For particles with a power-law distribution of energy, i.e. with \( N(\gamma)d\gamma = C \gamma^{-p} d\gamma \), the degree of polarization is

\[
\Pi(\omega) = \frac{p + 1}{p + \frac{7}{3}}. \tag{1.5}
\]
In the case of a power-law distribution of electrons, which is commonly assumed, the total power per unit volume and per unit frequency is

\[ P_{\text{tot}}(\omega) \propto \omega^{-(p-1)/2}. \] (1.6)

The spectral index \( s \) of the emission is related to the particle distribution index \( p \):

\[ s = \frac{p - 1}{2}. \] (1.7)

For optically thick or self-absorbed synchrotron radiation the absorption coefficient may be derived for a thermal or non-thermal distribution of particles. Rybicki and Lightman (1979) show that for a power-law distribution of particles the absorption coefficient

\[ \alpha_\nu \propto \nu^{-(p+4)/2}. \] (1.8)

The source function of the radiation is a combination of equations 1.6 and 1.8 therefore the intensity of radiation in the optically thick part of the spectrum is

\[ S_\nu \propto \nu^{-(p-1)/2+(p+4)/2} \text{ or } \nu^{5/2}. \] (1.9)
1.5.3 The Razin Effect: Essential Features

The Razin effect is the suppression of radiation which occurs when the beaming and intensity of synchrotron or gyrosynchrotron emission are modified by the ambient medium. The effect occurs when the index of refraction is less than unity, as is always the case in a plasma, and is more pronounced for larger deviations from unity. The effect becomes apparent when the equations for radiation in vacuum are modified to apply to radiation in a medium; then the speed of light $c$ must be replaced by the phase speed $\frac{c}{n}$, leading to an effective Lorentz factor which is smaller than the Lorentz factor in vacuum. Razin suppression is not a propagation effect, but a suppression of emission at the source: the reduction of the Lorentz factor of a particle in the medium compared to the factor in vacuum means that the power radiated is also reduced. We discuss the Razin effect in more detail in Chapter 2.

1.5.4 Bremsstrahlung

Bremsstrahlung, from the German for "braking radiation," is emitted by particles as they are deflected or accelerated in the Coulomb field of other charges. These deflections are called collisions even though the most common interactions, those contributing the most to emission at radio wavelengths, are distant encounters producing small-angle deflections in the path of the radiating particle.

Not all two-particle systems produce bremsstrahlung. Encounters of like particles, such as an electron-electron pair, cannot produce dipole radiation because the accelerations of the two electrons are equal and opposite, hence the electric dipole moment is zero. Quadrupole radiation from such systems becomes significant only for energies greater than $\sim 0.5$ MeV
(Bekefi, 1966). In general, bremsstrahlung is due mainly to radiation from electron-ion encounters.

The simplest case of bremsstrahlung, which we will use to summarize the derivation of the basic equations, is the encounter of an electron with an ion consisting of at least one proton. We consider the ion to be relatively massive and at rest with respect to the electron, and producing a fixed Coulomb field from its positive charge. We must assume that the photons emitted as a result of the encounter do not have energy comparable to the energy of the radiating electron, otherwise a quantum-mechanical treatment is necessary. (The high-energy photons produced in such electron-ion interactions are an important source of the hard X-ray emission from solar flares.) We also assume that many-body collective effects are not important; an electron may radiate almost continuously because of interactions with all the ions in a Debye sphere, but the particles are uncorrelated in their motions (Melrose and McPhedran, 1991). This assumption would break down if we considered emission near the plasma frequency.

A detailed derivation of bremsstrahlung equations may be found in Rybicki and Lightman (1979). The emission from a single particle undergoing small-angle scattering is found by assuming that an electron moves in a straight line at velocity \( v \) past an ion, passing at a distance \( b \), called the impact parameter, at the closest point. The energy radiated per unit frequency interval, per unit volume and per unit time is:

\[
\frac{dW}{d\omega \, dV \, dt} = \frac{16\pi e^6}{3\sqrt{3}c^3m^2_v} n_e n_i Z^2 g_{ff}(v, \omega)
\]  (1.10)

where \( g_{ff}(v, \omega) \), known as the free-free Gaunt factor, is a function of the energy of the electron and the frequency of the emission. (All other variables
are as defined in the table at the back of this thesis.) In the case of small-angle deflections the Gaunt factor depends on the range of impact parameters for which the collision can be treated classically.

In a thermal plasma we find the total emission by integrating equation 1.10 over the thermal speed distribution. We find:

$$
\epsilon_{ff} = \frac{dW}{dV dt d\nu} = 6.8 \times 10^{-38} Z^2 n_e n_i T^{-1/2} e^{-h\nu/kT} \bar{g}_{ff}
$$

(1.11)

(in cgs units) where $\bar{g}_{ff}$ is a velocity-averaged Gaunt factor. The emission is a flat function of frequency up to values of $h\nu \sim kT$.

Sometimes we are interested in the opposite of bremsstrahlung emission, namely free-free absorption, in which electron-ion collisions remove energy from electrons which are in resonance with a wave. In the case of a thermal plasma the absorption coefficient is derived from the emission coefficient by applying Kirchhoff’s Law. For $h\nu \ll kT$ the absorption coefficient is:

$$
\alpha_{ff} = 0.018 T^{-3/2} Z^2 n_e n_i \nu^{-2} \bar{g}_{ff}
$$

(1.12)

in c.g.s. units. Dulk (1985) gives the following Gaunt factors:

$$
\bar{g}_{ff} = 18.2 + \ln T^{3/2} - \ln \nu \text{ for } T < 2 \times 10^5 \text{K},
$$

(1.13)

$$
\bar{g}_{ff} = 24.5 + \ln T - \ln \nu \text{ for } T > 2 \times 10^5 \text{K}.
$$

(1.14)

For a fully ionized hydrogen plasma, $n_e \approx n_i$, this means that the opacity of bremsstrahlung radiation is proportional to the square of the electron density, and explains the dominance of bremsstrahlung over gyrosynchrotron radiation in cool, dense plasmas.
1.6 Radio Interferometry Basics

The basic component of interferometer arrays such as the Owens Valley Solar and Millimeter Arrays is the two-element interferometer, or two antennas separated by a baseline $b$. Observations are made at high spatial resolution by taking advantage of the interference between the waveforms received at each antenna from a distant source. The principle of interference is illustrated in Figure 1.3, adapted from Thompson, Moran, and Swenson (1986). The diagram shows parallel rays from a distant source. The difference in the path length of each ray to the antenna is $d = D \sin \theta$. When the path length difference is an integer multiple of the observing wavelength, the signals at each antenna combine constructively, while if the path length difference is a multiple of 1/2 the wavelength, the signals combine destructively. As the source moves through the sky and $\theta$ varies, the combination of signals cycles through the constructive and destructive states. An alternative way to view the situation is that the two-element interferometer has a power reception pattern that has peaks and nulls across the sky. Figure 1.4, also from Thompson, Moran, and Swenson, illustrates this concept. The separation of the antennas and the observing frequency determine the rate at which the source interference pattern evolves, or alternatively, the width of the lobes of the power reception pattern.

The amplitude variation of the received signal as the source moves in and out of the lobes of the power reception pattern is known as the Fringe Visibility. If the source is a point source, or very small compared to the angular width of the lobes or “beamwidth” $\theta$ of the interferometer system, then the amplitude variation as the source moves in and out of the peaks in the power reception pattern is a maximum. (In this case the source is said
Figure 1.3 The two-element interferometer. The difference in path length travelled by the signal to each antenna is $D \sin \theta$. Figure adapted from Thompson, Moran, and Swenson (1986).
Figure 1.4 Power reception pattern of a two-element interferometer. Figure adapted from Thompson, Moran, and Swenson (1986).
to be “unresolved.”) If the source is large compared to the beamwidth, then power from the source spills into adjacent lobes, and the amplitude variation as the source or power pattern moves is less. If the source is very large, there will be little or no amplitude variation as the source moves, and the source is said to be “overresolved.” The size of the source can be determined by noting the amplitude variation or fringe visibility $V$ as a function of the beamwidth. The beamwidth may be varied by moving the antennas, or, with less effort, by varying the frequency of observation. If the frequency of observation is varied, one must consider the possibility that the source size itself is not constant with frequency.

The components of the interferometer baseline along the projected East-West and North-South axes in the sky are called the $(u,v)$ components. The third component, $w$, is along the third direction, into the sky. The components are measured in wavelengths at the center frequency of the observing bandwidth. As long as the baseline vectors of a 2-dimensional array remain coplanar in $(u,v,w)$ space the $w$ component may be ignored. For a synthesized beam width of 1 arcsecond, the size of the field of view that may be imaged without considering the $w$ component, without significant resulting phase errors, is about 2.5 arcminutes. The $w$ component can safely be ignored when imaging solar flares.

How is the observed visibility function related to the source brightness distribution and the interferometer power reception pattern? Let us denote by $s_0$ the unit vector in the direction of the center of the field of view, or the phase center of the array, relative to which we measure the phase of the visibility. Let $\sigma$ be the vector in the plane of the sky from the phase center to
a point of interest. These vectors are shown in Figure 1.5. The source brightness distribution $B(\sigma)$ and the normalized antenna beam pattern $A_N(\sigma)$ are related to the Visibility as follows:

$$V(D_\lambda) = \int_{4\pi} A_N(\sigma)B(\sigma)e^{-i2\pi D_\lambda \cdot \sigma} d\Omega$$

(1.15)

where $D_\lambda$ is the baseline vector of two antennas. This equation has the form of a Fourier Transform. The source brightness distribution $B$ can thus be obtained by performing an inverse Fourier Transform on the visibility data; this is the basis of synthesis imaging.

To give a better understanding of the imaging problem we show in Figure 1.6 three one-dimensional brightness distribution functions and the corresponding visibilities. The parameter $\eta$ is the angular variable on the plane of the sky, $a$ is the characteristic width of the model distribution, and $u$ is the projected baseline in units of the wavelength. The visibilities have different functional forms depending on the brightness distribution. A measurement on a given baseline, at a given frequency, samples only one point along the $u$ axis of the visibility plot. The width of the brightness distribution function could be estimated from this single measurement only by fitting this one point to a model such as a Gaussian. To better characterize the visibility function other points may be sampled, using projected baselines for different positions of the source in the sky, for example, or by changing the frequency of observation so that the antenna separation is effectively altered. Combining data from a range of frequencies to fill in the $(u,v)$ plane is known as frequency synthesis.
Figure 1.5 Position vectors specifying the source and phase center of the interferometer. Figure adapted from Thompson, Moran, and Swenson (1986).
Figure 1.6 Three models of a one-dimensional brightness distribution and the corresponding Visibility Functions. \( \eta \) is the angular variable on the sky, \( a \) is the angular width of the model, and \( u \) is the spacing of the interferometer elements in wavelengths. Solid lines in the Visibility curves indicate the modulus of the Fourier Transform of the brightness distribution, and the broken lines indicate negative values of the transform. Figure adapted from Thompson, Moran, and Swenson (1986).
The Clean Procedure

The basic idea of the "Clean" procedure is that real sources can be represented by a superposition of point sources, or delta functions. A raw image consists of the convolution of these point sources with the instrument response function, and the sidelobes in the instrument response function lead to spurious features in the image. In the Clean procedure, which has been in use in radio astronomy since it was devised by Högbom in 1974, the point sources are found and an image is reconstructed from them together with an idealized instrument response function without sidelobes.

The details of the Clean algorithm are given, e.g., in Cornwell and Braun (1989). We summarize the steps here. First, the irregularly-sampled complex visibilities are gridded and resampled on a regular grid. This allows a Fast Fourier Transform (FFT) (actually an inverse transform) to be performed, leading to the so-called dirty map. The dirty beam is the inverse transform of the set of measured \((u,v)\) points with amplitude set to unity and phase set to 0.

Step 1: Find the position and strength of the peak in the dirty map. Usually one constrains the number of degrees of freedom of the Clean procedure by specifying that only one region in the dirty map, or a limited number of regions, will be examined. This selection is called the Clean box.

Step 2: At the location of the brightest point or points in the dirty map, subtract the instrument response function, including the sidelobe pattern, scaled to the amplitude of the source and then additionally multiplied by a damping factor or gain factor. In other words, do not remove the entire source at once. Usually the gain factor is on the order of 0.25 or less.
Step 3: While keeping track of the sources (delta functions) already found, search for sources that remain in the dirty map. This process continues until a satisfactory end is reached. The end may be defined by a limiting flux level in the remaining dirty map sources, or by noting when the rms flux level of the dirty map stops decreasing.

Step 4: Restore the delta functions to the map, convolved not with the dirty beam but with a Clean beam. The Clean beam is often chosen to be a Gaussian function with half-amplitude width equal to that of the dirty beam, or simply a truncation of the dirty beam outside the first zero-crossing. Illustrations of dirty and clean maps are given in Chapter 2.

The Clean procedure has been found to work well even for extended sources. Some problems with the procedure include the underestimation of the short-spacing flux of extended sources and ripples or modulations in the cleaned images at spatial frequencies corresponding to unsampled parts of the $(u,v)$ plane. A disadvantage of the Clean deconvolution, from one point of view, is that it is a very non-linear procedure, difficult to express mathematically. An alternative method of image deconvolution is the Maximum Entropy Method (MEM). In this method the final product is the smoothest image which is consistent with the data. The image is compared with a default image or prior distribution which incorporates some information about the source; for example, a low-resolution image of the source may be used as a starting point for the deconvolution. This method is discussed in more detail in Bastian (1987) and Cornwell and Braun (1989).
1.7 Overview of the Owens Valley Solar Array

The Owens Valley Radio Observatory, of which the Solar Array is a part, is located about 250 miles north of Los Angeles, California, near the town of Big Pine. The Owens Valley Solar Array (OVRO Solar Array, sometimes OVSA for short) consists of 5 antennas. The two largest antennas, which formed part of the original installation in the 1958, are 27 m in diameter. Three smaller antennas, 2 m in diameter, were added beginning in 1989 to improve \((u,v)\) coverage—the range of spatial scales on the source to which the interferometer is sensitive. The antennas may be used individually for total power observations. The field of view of a 2-m dish at a typical observing frequency of 5 GHz is 2 degrees, larger than the disk of the sun which subtends an angle of about half a degree. The field of view of a 27-m antenna is about 0.2 degrees.

The 27-m dishes are equipped with two feeds or receiving horns, one measuring Stokes I and the other measuring either left or right circular polarization (LCP, RCP). The smaller dishes are equipped with I feeds only. The receivers operate between 1.0 and 18.0 GHz. The observer typically selects 45 frequency channels to observe within this range, with frequencies spaced at least 200 MHz apart.

The antennas are generally arranged in a T formation, although they are on rails and may be moved to suit the purposes of the observation. The longest baseline available is 2200 ft (670 m) and the shortest commonly used is 200 ft (60 m). The spatial resolution depends on the frequency of observation and on the location of the sun in the sky. At 5 GHz the narrowest dimension of the beam is about 5 arcsec.
The Owens Valley Solar Array is one of the premier instruments in the world for microwave observations of the sun. It is most similar for this purpose to the Very Large Array (VLA) in New Mexico operated by the National Radio Astronomy Observatories. The VLA has 27 antennas so its \((u,v)\) coverage is better than that of Owens Valley, and the VLA can image lower-intensity features such as solar active regions more quickly. However, the VLA does not have the dense frequency coverage that is necessary to resolve spectral features in the microwave emission, and is not dedicated to solar observations as is the Owens Valley Solar Array. The Nobeyama radio telescope array in Japan is dedicated to solar observing, but presently operates only at 17 GHz.

**Calibration of the Owens Valley Solar Array**

Calibration is the standardization of the output of a measuring system. The process of calibration usually involves an absolute calibration in which measured values are compared to accepted or defined values. For example, the flux recorded by an interferometer when observing a calibrator source is referred to the accepted flux of the source as listed in a catalog; the scale or graduations of an instrument are determined in this way. Another sense of the word is the removal of known instrumental factors from the measurements. An example of this kind is the correction for the increase in system temperature which occurs as the antenna is pointed to larger zenith angles in the sky.

The major components of the system being calibrated are shown in Figure 1.7, adapted from Thompson (1986). The components include the feed horns at the focus of the antennas, the mixers converting the signal from the radio frequency (RF) to an intermediate frequency (IF) for processing,
delay components to compensate for the delay between the signals received at different antennas, and the correlator where the fringe amplitude and phase are detected.

The output of any pair of antennas in an interferometer is the complex visibility, which we write as $V_{i,j}$. The subscripts $i$ and $j$ identifying the antennas range from 1 to $N$, where $N$ is the number of elements in the array. We will first describe the procedure of obtaining the true visibility from the measured one in general terms, following the discussion of Bignell and Perley (1986). The measured visibility is a product of the true visibility and the complex gain of the system (the primed quantity is the measured one):

$$I V_{i,j}(t, \nu) = G_{i,j}(t, \nu) V_{i,j}(t, \nu).$$

(1.16)

In the case of continuum, not spectral-line observations, we may drop the dependence on frequency $\nu$. Henceforth we will also drop the explicit time dependence, and keep in mind that all quantities are time-dependent.

We may separate the gain term into a product of two antenna-based gains and a term representing a correlator-based gain: $G_{i,j} = G_i G_j^* G_{i,j}$. Generally there are no instrumental effects which cannot be identified with one or other element of the interferometer pair, so the correlator-based term is taken to be 1. Now the equation may be written:

$$V'_{i,j} = G_i G_j^* V_{i,j}.$$  

(1.17)

The individual gain terms and the true complex visibilities for all $i, j$ combinations are obtained by solving the set of equations. For $N$ antennas there are $\frac{N(N-1)}{2}$ equations and $2N - 1$ unknowns, which in practice are determined by a least squares fit to the solutions.
Figure 1.7 Components of an interferometer array. Figure adapted from Thompson (1986).
The solutions to the equation are usually obtained in terms of amplitude and phase, rather than the equivalent form of real and imaginary parts, because the various sources of error corrupt the amplitude and phase in easily distinguishable ways. As the signal from the source propagates through the atmosphere, for example, the amplitude is generally decreased and the phase rotated. Another reason to consider the amplitude and phase calibration separately is that the phase calibration needs to be done on a shorter timescale. Drifts in the phases occur on the timescale of hours, while the amplitude is stable to 10 percent or less over the course of a day. A noise diode is used to monitor the IF gain on a daily basis, but the frequency dependence of the amplitude variation can be assumed to be fixed between amplitude calibrations.

**Phase Calibration**

Many instrumental and environmental factors affect the interferometer phase. One of the most significant for the Owens Valley Solar Array is that temperature variations cause stretching and contracting of the cables carrying the signal from the antennas to the correlator, and changes in the signal path length of even a fraction of a millimeter correspond to measurable delays at the correlator. Other important effects are due to the propagation of the signal through the atmosphere and variations in the system temperature.

The phase calibration is done on two levels. The reference calibration is a set of phase and amplitude measurements made about once a month, and the solution to the complex visibility equation derived from these measurements takes care of changes to the system that occur on long timescales, such as the repositioning of antennas in the array. Phase-calibration sources
are bright, distant sources at fixed and well-established positions, such as the quasar 3C84. It is preferable to use several phase-calibration sources distributed over the sky.

The second level is the daily calibration. Observations of calibrator sources, lasting about 15 minutes, are interleaved with observations of the sun at intervals of about two hours. These secondary observations help remove the effect of cable-length variations, atmospheric variations, pointing errors, and random errors caused by instabilities in the electronic components of the receiver.

In practice the phase calibration of solar data is made as follows. First the phase spectrum of the reference calibrator source is established. Then the data from the daily calibrator observations are reduced: the measurements from an entire 15-minute scan are averaged, giving a single amplitude and phase for each frequency. The difference between the daily and reference calibrator phases is expected to be linear with frequency, so a linear slope is fit to the phase difference spectrum. It is sometimes difficult to establish a good fit because the calibrator sources are relatively weak and the phase data are noisy; in some cases the data are examined by eye and bad points affecting the fit are deleted. When a good fit has been obtained, the phase calibration is known for that day and can be applied to the daily calibrator and solar data.

A good fit to the phase difference spectrum preserves phase closure. Phase closure is a geometric relationship between the phases measured on baselines involving three antennas (A,B,C) of the array (see, for example, Thompson, Moran, and Swenson 1986):

\[ \Phi_{CA} + \Phi_{BC} - \Phi_{AB} = 0 \]  \hspace{1cm} (1.18)
after allowances are made for phases induced by the backend instrumentation.

**Total Power and Fringe Amplitude Calibration**

The total power and fringe amplitude are measured separately. The total power is obtained from individual feeds on the antennas (the two 27-meter dishes each have one Stokes I polarization feed, and one of either RCP or LCP; the 2-meter dishes have one I feed). The fringe amplitude, on the other hand, is the correlated amplitude between the signal from one feed at one antenna and another feed at the other, e.g. the correlation of the I feed on antenna 1 with the RCP feed on antenna 2, yielding the correlated amplitude in the RCP polarization. The total power and fringe amplitude measurements are recorded separately during solar observations, and require separate calibration.

Total power calibration involves the observation of an absolute flux calibrator source over the available spectrum. The flux calibrator source (Cas A) is assumed to be completely unpolarized, so that the power (per polarization) is the same in RCP or LCP, and $I = R + L$.

The amplitude calibration is part of the reference calibration mentioned earlier. The calibrator source must either be unresolved (much smaller than the fringe width at all of the frequencies of observation) or else we must have a model for the brightness distribution, so that we know what fringe amplitude or flux to expect for a given baseline length and observing frequency. The flux calibrator used is 3C84. The flux calibration is good to a few percent in the middle range of the frequencies of observation, and is poorer at the higher end of the microwave spectrum because of the loss of sensitivity of
the solar array at high frequencies. Above 14 GHz the calibration the error may reach 50 percent.

One of the main instrumental factors affecting the amplitude is the gain of the receiver. This is monitored once or twice a day using a noise diode. A noise source simulating the emission of a blackbody is injected into the RF signal path. The resulting IF level change calibrates the gain.

Observations in the microwave regime are affected much less by atmospheric emission and absorption than are observations in the millimeter or optical wavelengths, which explains the fact (remarkable to the layman) that observations may continue even in the case of rain or heavy cloud cover. The optical depth at 15 GHz can be written as:

\[ \tau = 0.013 + 0.0002 P_v \]  

where \( P_v \) is the water-vapor partial pressure in millibars (Fomalont and Perley 1988). The opacity at this frequency rarely attains 0.2.

We have not discussed polarization calibration as this is not explicitly done for Owens Valley solar data. As mentioned previously, the I, RCP, and LCP data are calibrated separately on sources which have very low polarization, taken to be zero. At the time of the observations reported in this thesis the measured polarization is not accurate above 14 GHz because of uncertainties in the calibration.

1.8 Origin of Data used in this Thesis

Besides the Owens Valley Solar Array, the following instruments provided data which were used or cited in this thesis.
GOES

At any given time two or more Geostationary Operational Environmental Satellite (GOES) spacecraft monitor the space environment. At the time of writing the satellites are GOES-7 and GOES-8, although GOES-6 is still functioning also. Soft X-ray detectors have been a part of the space monitoring instrument package since 1974. The soft X-ray detectors measure the integrated flux from the sun in two channels, the $0.5 - 4\,\text{Å}$ and $1 - 8\,\text{Å}$ bands.

Yohkoh

The Yohkoh spacecraft, designed as a successor to the Skylab, Solar Maximum Mission and Hinotori instruments, was launched in August 1991 from the Kagoshima Space Center in Japan. It carries four instruments: the Soft X-ray Telescope (SXT), the Hard X-ray Telescope (HXT), the Bragg Crystal Spectrometer (BCS), and the Wide Band Spectrometer (WBS). The spacecraft orbits at an altitude near 600 km. It has an orbit period of 97 minutes, about half an hour of which is spacecraft night.

The SXT instrument was jointly developed by the Lockheed Palo Alto Research Laboratory and the National Astronomical Observatory of Japan. It is a grazing-incidence telescope with a focal length of 1.54 m, forming X-ray images in the range from 0.25 to 4 keV on a $1024 \times 1024$ pixel charge-coupled device (CCD). Observations are made through two sets of filters in a sequence of combinations. The filters select the energy band and provide extra attenuation to increase the dynamic range of the instrument, while a shutter device controls the time exposure. A co-aligned aspect telescope, which functioned until late in 1992, provides an optical counterpart to the
soft X-ray images. Table 1.1 lists the important parameters of the SXT instrument.

The Hard X-ray telescope (HXT) is an instrument of novel design. It consists of a collimator, detector and an electronics unit. The collimator is a tube about 1.4 m long with X-ray grid plates at both ends. Photons passing through the collimator are detected by a set of 64 detector modules consisting of a scintillation crystal and photomultiplier tube (see Kosugi 1991 for more details). The principle behind the reconstruction of images is similar to that of Fourier synthesis, and the subcollimators are arranged in a symmetrical polar configuration which was chosen to optimize the \((u,v)\) coverage for solar X-ray sources. In the electronics unit the signal is converted to a digital signal representing counts in four energy bands. The properties of HXT are summarized in Table 1.2.

**The Owens Valley Millimeter Array**

At the time of the solar eclipse observations in July 1991 the Owens Valley Millimeter Array consisted of three 10.4-m telescopes operating in the 3mm and 1.3mm bands. (Six telescopes are now operational.) It is primarily used for spectral-line and continuum aperture synthesis of objects such as star-forming regions and galaxies. Solar observations require special precautions such as attenuation in the RF signal path to avoid saturating the receivers. In 1991 the system temperature was approximately 300 K at 100 GHz.
### Table 1.1

<table>
<thead>
<tr>
<th>SXT Characteristics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Instrument</td>
<td>Modified Wolter type I grazing incidence mirror + CCD with coaligned optical telescope</td>
</tr>
<tr>
<td>Spectral range</td>
<td>3-45 Å at 1% of peak response</td>
</tr>
<tr>
<td>Field of View</td>
<td>Full solar disk</td>
</tr>
<tr>
<td>CCD array size</td>
<td>$1024 \times 1024$</td>
</tr>
<tr>
<td>Angular resolution</td>
<td>$2.4528 \pm 0.0005$ arcsec</td>
</tr>
<tr>
<td>Time resolution</td>
<td>0.5 sec (special mode) 2.0 sec (normal mode)</td>
</tr>
<tr>
<td>Dynamic range</td>
<td>$&gt; 5 \times 10^9$</td>
</tr>
</tbody>
</table>
Table 1.2

<table>
<thead>
<tr>
<th>HXT Characteristics</th>
<th>Fourier Synthesis collimator (64 sub-collimators)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instrument</td>
<td></td>
</tr>
<tr>
<td>Energy bands</td>
<td>L: 14-23 keV</td>
</tr>
<tr>
<td></td>
<td>M1: 23-33 keV</td>
</tr>
<tr>
<td></td>
<td>M2: 33-53 keV</td>
</tr>
<tr>
<td></td>
<td>H: 53-93 keV</td>
</tr>
<tr>
<td>Field of View</td>
<td>Full solar disk</td>
</tr>
<tr>
<td>Angular resolution</td>
<td>( \sim 5 ) arcsec</td>
</tr>
<tr>
<td>Time resolution</td>
<td>0.5 sec (special mode)</td>
</tr>
</tbody>
</table>
Marshall Space Flight Center Vector Magnetograph

The presence of a magnetic field in a plasma means that some spectral lines (either in emission or absorption) are split, i.e. components of the line form shifted to either side of the original line center frequency. This is known as Zeeman splitting. In the case of a longitudinal field, that is, B along the line of sight, one observes two components of opposite circular polarization called the $\sigma$ components. In the case of a field transverse to the line of sight three components are seen, one at the original (unshifted) line center, known as the $\pi$ component, and the other two to either side. If the line is an absorption line the $\pi$ component is linearly polarized perpendicular to B and the $\sigma$ components are linearly polarized parallel to B. Stix (1989) gives a complete description of the Zeeman effect.

Measurement of magnetic field is based on the polarization of the observed components. Some vector magnetograms are made by measuring the line profiles in the Stokes I, Q, U, V polarizations and inverting them to obtain the field strength. With the Marshall Space Flight center instrument measurements are made instead at a number of discrete frequencies within the Fe I 5250.22 absorption line to determine the state of polarization. The polarized intensities are related via a model of the solar atmosphere and sunspots to the strength and inclination of the magnetic field. The azimuth of the transverse field is determined from the plane of polarization of the linearly polarized intensity (see Hagyard and Kineke 1995 for more details).
CHAPTER II

FLARE OF JULY 16 1992: OBSERVATIONS WITH
THE OWENS VALLEY SOLAR ARRAY

2.1 Introduction

We selected the flare of July 16 1992 for analysis because X-ray data for this event were available from the instruments on board the Yohkoh spacecraft (see section 1.7), complementing the microwave imaging and spectral data from the Owens Valley Solar Array. A major facet of solar flare research today is "multispectral" analysis in which radiation or particles from different energy regimes or different wavelengths are examined together in an effort to uncover the flare mechanism. A good example is the study of Wang et al. (1994) of a solar limb flare observed in microwaves, Hα, and hard X-rays. By combining these data the authors were able to show the time development of the flare, as material descended along magnetic loops to the site of the flare, the magnetic structure of the loop (with field strength \( \sim 160 \) G at the loop top, and \( \sim 300 \) G at the footpoints), and the change in the energy distribution of the dominant electrons from the loop-top to the footpoints. The flare we selected gives us an opportunity to further study the relationship between the microwave and hard and soft X-ray emission in a flare.

While the flare was selected because of the availability of complementary observations, we found that the microwave data alone raised some interesting questions. The radio data from the Owens Valley Solar Array can be used without reference to other observations to determine the source location, the source size, and the likely emission mechanism deduced from
the microwave spectrum. Furthermore, all of these measurements are made as a function of time. Therefore, before embarking on a comparison of flare characteristics at different wavelengths in Chapter 3, we may use the radio data alone to determine attributes of the flare plasma. In particular, we have for the first time been able to follow the temporal evolution of the brightness temperature spectrum of a flare during the impulsive phase. Thus chapter 2 will be limited to the analysis of the radio data.

We find that the brightness temperature spectra for most of the duration of the flare are best explained as due to gyrosynchrotron emission. Our spectra are in many ways typical of the those observed in the microwave regime. Bursts attributed to gyrosynchrotron emission have peak brightness temperatures between $10^7$ K and $10^9$ K and usually have spectral peaks between 3 and 10 GHz. However the spectra exhibit features, namely a narrow bandwidth and a constant frequency of maximum emission (or constant "peak frequency"), which demand an explanation. As we will show in section 2.4, the basic theory (incorporating only emission and absorption effects) predicts broader bandwidths or less steep low-frequency slopes and an evolution of the peak frequency with peak brightness temperature. The features we mention have been seen before in type IV meter- to centimeter-wavelength emission (Ramaty and Lingenfelter, 1967, and references therein) and their rate of occurrence noted in a survey of microwave bursts (Stähli et. al, 1989). However, in previous observations measurements were made of flux or total power spectra, not the brightness temperature spectra of spatially resolved sources, so that that variation in source size could not be ruled out as a cause of the special features in the spectra. Our observations confirm the
appearance of features not explained in the basic theory of gyrosynchrotron emission.

We used a numerical code developed by Ramaty (Ramaty et al., 1994) to model the flare brightness temperature spectra. The code takes into account the suppression of radiation due to the ambient medium, known as the Razin effect or medium suppression (see section 2.5.2). We find that when Razin suppression is a significant factor, the modeling of the spectra is simpler: the maintenance of a nearly constant peak frequency while the brightness temperature evolves can be achieved simply with a variation in the number of accelerated particles. In the absence of Razin suppression, constancy of peak frequency requires significant variations in the system parameters such as the magnetic field and the index in the electron energy distribution. Our findings have implications for our understanding of particle acceleration during flares, and tell us something about the conditions of flaring plasmas.

Section 2.2 describes our acquisition of the flare data and the formation of images giving the flare spectrum. Section 2.3 gives a summary of the theory of gyrosynchrotron emission as applied to solar bursts. Section 2.4 explains the difficulty of interpreting the spectra without Razin suppression. Section 2.5 describes the results we obtained from modeling the spectra with a gyrosynchrotron code developed for this purpose. In section 2.6 we discuss our results, and we summarize and conclude in Section 2.7.
2.2 Observations and Analysis

2.2.1 Overview of the Flare

The flare occurred in Active Region 7220, 63 degrees West of disk center and 10 degrees South, and peaked near 16:55 UT. It had a soft X-ray classification of M 6.8 on a logarithmic scale of X-ray flux in which A, B, and C range from $10^{-8}$ to $10^{-5}$ Watts per square meter, M is from $10^{-5}$ to $10^{-4}$, and the highest level, X, refers to levels above $10^{-4}$ Watts per square meter. The flare was observed by Yohkoh and the GOES spacecraft as well as by the Owens Valley Solar Array. A number of instruments listed in Section 1.4 were used to observe the active region before or after the flare; these other observations are discussed in Chapter 3.

Figure 2.1 shows a time plot of the total power (Stokes I parameter) at 11.2, 4.4, and 1.2 GHz. Antenas 1 and 2 measure the I total power independently, and we have plotted the geometric mean of these two signals. The time intervals which were chosen to form images and spectra are shown by lines labelled A, B, C, D, E, F. The times are as follows. A: 16:54:53-16:55:04; B: 16:55:05-16:55:16; C: 16:55:29-16:55:40; D: 16:56:41-16:56:52; E: 16:57:05-16:57:16; F: 17:00:05-17:00:52. It was not possible to image the flare closer to the time of maximum emission due to source complexity. The issue of complexity is discussed in section 2.2.3.

The total power plots show that the flare reached a peak flux of about 800 Solar Flux Units (SFU) at 11.2 GHz, with a rise time less than one minute. (One SFU is $10^4$ jansky (Jy); 1 Jy = $10^{-26}$ W m$^{-2}$ Hz$^{-1}$.) The initial decay occurred in about a minute also, but low-level flux enhancement persisted for several minutes. A background level was subtracted from this
Figure 2.1 Time plot of the flare total power (Stokes I parameter) in Solar Flux Units. From top to bottom, 11.2, 4.4, and 1.2 GHz. The signal plotted is the geometric mean of the total power from antennas 1 and 2. The solid lines labelled A, B, C, D, E, F show the times at which the flare was imaged. For the F image, data was averaged over nearly one minute.
and all interferometric data. The background subtraction is discussed in section 2.2.3 on the imaging of the burst.

Figure 2.2 shows the agreement between the Owens Valley data and the flux measured at Radio Solar Telescope Network (RSTN) and other sites at the peak of the flare (Solar Geophysical Data, Comprehensive Reports for July 1992). There are five stations that monitor the solar flux at one or more frequencies within the range measured by Owens Valley: Cuba, Palheua (RSTN, Hawaii), Penticton (Canada), Sagamore Hill (RSTN, Massachusetts), and San Vito (RSTN, Italy). A comparison of the fluxes measured at Owens Valley and at these sites provides a useful rough check on the absolute calibration of the total power. (Section 1.6 discusses the calibration of the Owens Valley Solar Array data.)

The Owens Valley spectrum at 16:56:05 UT is shown using squares to represent the geometric mean of antennas 1 and 2, and dotted lines above and below to show the antenna 1 and 2 values separately. We also show the antenna 1 and 2 data for an earlier time, 16:55:05 UT. The RSTN data are overlaid on the OVRO plot using five distinct symbols. The fluxes noted for the RSTN sites are the peak fluxes at the measured frequency. Some of the RSTN sites recorded the peak flux at 16:55 UT and some at 16:56 UT; the time is not given with high precision. Except for the data above 15 GHz, for which absolute calibration is difficult at all sites, the RSTN data are in good agreement with the range of values measured by antennas 1 and 2 at OVRO. The RSTN data are particularly close to the fluxes measured by antenna 1, suggesting that the offset between the curves measured by antennas 1 and 2 is due to a problem with the absolute calibration of antenna 2, amounting to an excess gain of about 43 percent.
Figure 2.2  Total power at the peak of the flare, as measured at OVRO and other sites. The Owens Valley (OVRO) data from antennas 1 and 2 is shown with a dotted line, for 16:55:05 UT (middle of the figure) and for 16:56:05 UT (near the bottom axis). The geometric mean of antennas 1 and 2 data for 16:55:05 UT is shown with square symbols. Data from the Radio Solar Telescope Network (RSTN) and other sites are overlaid using various symbols as shown in the insert. The other fluxes, though not measured at precisely the same time as the OVRO data (see text) provide a rough check of the calibration of the OVRO total power.
2.2.2 Distinction Between Low and High Frequency Components

Figure 2.1 from the previous section shows that the flux peaked at different times depending on the radio frequency. In the time plots for 11.2 and 4.4 GHz the peak occurs around 16:55:53 UT. In the plot for 1.2 GHz some data near the peak are missing, but the peak must have occurred after 16:56:20 UT. Indeed, the peak at 1.2 GHz occurs after the flux at the higher frequencies has decayed by at least 30 percent. The delay is not abrupt; intermediate frequencies are not shown, but the relative delay in the flare peak occurs smoothly with frequency. The delay is the same in the RC and LC polarizations.

Figure 2.3 is another view of the evolution of the flare. We show the total power spectrum of the flare from antenna 1 at several representative times, using log-log axes. The first three plots (those along the top row) show the rise phase of the flare. The second row of plots shows the spectra during the times of maximum emission, when the spectrum was relatively flat. The last three plots are samples from the decay phase of the flare. The total time spanned in the figure is from 16:54:59 UT to 16:58:29 UT.

We can distinguish at least two features or components of the spectrum based on the behavior in time. There is a high-frequency component with a local intensity peak near 11.2 GHz and a middle-range component with a broad maximum near 4.5 GHz. There is a distinct relative minimum between these two components at about 7.5 GHz. Late in the flare there may be a low-frequency component with a maximum near 1 GHz, corresponding to bumps in the time plot for the low frequencies seen in the left circular polarization.
Figure 2.3  Spectrum of microwave total power from antenna 1 at several representative times. The total power is measured in Solar Flux Units and the frequency in GHz, both on logarithmic axes. The UT corresponding to each sample is given above each plot. The total time spanned in the figure is from 16:54:59 to 16:58:29 UT. The spectra show a spectral component with peak near 11 GHz.
The flux enhancement during the flare occurs at the high frequencies first and then at low frequencies. Initially (starting around 16:54:50 UT) the enhancement is strongest near 11 GHz and tapers off steeply above that; on the low frequency side, the enhancement is seen primarily above 2 or 3 GHz. Beginning at 16:55:40 the middle and high frequency components have nearly equal flux, so the spectrum as a whole looks quite flat. After about 16:56 UT both the middle and high-frequency components decay, but the flux at 11.2 GHz decays faster than the flux at lower frequencies. The temporal behavior we see is reminiscent of that noted by Lee et al. (1994) for several large flares, but occurs on a much shorter timescale.

The fact that the flare does not peak simultaneously across the measured bandwidth suggests that the 11.2 and 4.5 GHz sources are distinct in some way. The delay in the peak between 11.2 GHz and 1.2 GHz is about 30 seconds. Even in a large region encompassing a range of magnetic field strengths there is no reason to expect that radiation produced at harmonics of a lower gyrofrequency will be delayed with respect to that produced in the regions of higher field strength, and electron time-of-flight effects for a typical 50 keV electron could produce delays of only tenths of a second even in a loop $10^5$ km in length. There are many free parameters that could be invoked to explain the delay: multiple sites of the flare, evolution of the energy distribution of the accelerated particles over the 30-second time period, or expansion and evolution of the source, for example.

The expansion of the source is a likely hypothesis which we may consider here without detailed modeling of gyrosynchrotron emission. We suppose that the peak at 11.2 GHz represents emission at the third harmonic of the gyrofrequency in the original source, and that the peak at 1.2 GHz
represents emission at the third harmonic at a later time when the loop has expanded and the magnetic field strength decreased. Then the magnetic field strength would have changed from about 1300 G to about 150 G in 30 seconds. These field values are possible, but it is not clear whether we can expect the expansion to occur so quickly, without invoking also the movement of the source from the loop footpoints to the loop top. Another possibility is that emission near 1 GHz is dominated by bremsstrahlung radiation from outside the flux tube region, as in the core-halo model of Böhme et al. (1977), and that the delay in the peak between the low and high frequencies represents the time for the energetic particles outside the flux tubes to reach their maximum emission.

A probe of source size provides another clue to the distinction between the flare source at the lower and higher frequencies. We may obtain a measure of the source size as a function of frequency by comparing the total power from the flare with the correlated amplitude measured on the shortest baseline—that which measures power on the largest spatial scale. Figure 2.4 illustrates the comparison. We compare the total power in the left circular polarization (LCP) from antenna 1 with the LCP correlated amplitude from baseline 15. Early in the development of the flare, at 16:55:05 UT, all of the flux is registered on the shortest baseline (top panel). At 16:55:53, on the other hand, near the peak of the flare, the ratio of the correlated amplitude and total power drops sharply toward lower frequencies. Above 9 GHz the correlated amplitude and total power agree, within the limits of the noise in each signal; between 6 and 9 GHz, the ratio of correlated amplitude to total power is about 75 percent; below 6 GHz the ratio is about 50 percent, improving slightly again to about 70 percent near 2 GHz. The same analysis
comparing the I polarization total power from antenna 1 with the sum of RCP and LCP from baseline 15 gives essentially the same result.

Thus, below 6 GHz the source is much larger than the source at 10 GHz, and is so large that a significant fraction of the flux is lost to the interferometer. The source is not necessarily larger than the field of view, but all the flux is not measured by the interferometer because the sensitivity of any antenna pair is limited to emission on the spatial scale defined by the observing frequency and the separation of the antennas. This problem is known as overresolution and is discussed in section 1.5. The fact that the source is overresolved below 6 GHz means that we can obtain only an upper limit to the brightness temperature at frequencies lower than 6 GHz. The upper limit would be obtained by assuming that the source is the size of the fringe spacing of the shortest baseline of the interferometer. Because the source is, in fact, larger, the flux is spread over a larger area and the brightness temperature is lower than estimated.

This comparison of source sizes at low and high frequencies has shown that the narrow, high-frequency component in the spectrum of total power with a peak near 11.2 GHz corresponds to a small kernel of emission, and the broader component below 7 GHz to a larger source.

2.2.3 Images of the Radio Burst

The data are originally in the form of correlated phase and amplitude between seven antenna pairs, and total power measured independently by two of the dishes. The observing mode we used gave us visibilities measured in both left circular polarization (LCP) and right circular polarization (RCP) at 45 frequencies between 1.0 and 18.0 GHz, although in practice data above 16.4 GHz rarely survived the automatic flagging of bad data (for bad tracking
Figure 2.4 Comparison of the total power and of the correlated amplitude measured on the shortest baseline, to assess the degree of overresolution. Left circularly polarized (LCP) data from antenna 1 are compared with LCP data from baseline 1-5. Before the start of the flare the source is overresolved. After flare onset the source is overresolved below 6 GHz.
by the dishes, or lack of phase lock in the receivers, for example). The data
are calibrated for daily and monthly phase variations by reference to the
calibrator sources. After calibration, a Fourier Transform was performed on
the correlated data to obtain the spatial components of the source or sources
for display in map form.

Before making images from the data we selected a time period just
prior to the flare and vector-subtracted the spectrum corresponding to this
time period from all subsequent data. This removes the emission spectrum
from the underlying active region. The subtracted spectrum has very low
amplitude compared to that at the peak of the flare, and the emission it
corresponds to is too complex to map in the snapshot data; to image the
active region requires a long time integration, usually 4 hours or more. How-
ever, the pre-flare subtraction is useful to isolate the flare spectrum from the
active region spectrum during the times of relatively low-level emission. The
selection of the time interval for pre-flare subtraction is made by examining
the time plots at many different frequencies and finding the latest time before
the flare which shows no systematic change in the amplitude.

The next step is to create images of the source. The identification of
flare sources in the raw or dirty maps can be difficult. Weak sources and the
presence of strong sidelobes in the synthesized beam give rise to “wallpaper”
type images in which it is impossible to identify the real source among all
the repetitions. To identify the real source location we increase the number
of (u,v) points (components of the antenna baseline vectors on the plane
of the sky) by performing frequency synthesis. When the location of the
source has been determined we revert to single-frequency maps and use this
information to select clean boxes. Section 1.5 explains the "clean" procedure in more detail.

Frequency synthesis has been described in section 1.5, and we summarize it briefly here. The source location is better defined if we increase the coverage in the (u,v) plane, and one way to do this is to combine visibility data from different frequencies. Because we are only concerned with identifying the source location, we may effectively integrate the data over a range of frequencies, ignoring for the moment any variations in the source configuration as a function of frequency. Experience tells us that source configuration is relatively stable over factors up to 2 in frequency, and in this case we have used a factor of about 1.5.

Figure 2.5 illustrates the process described above. Part a) shows the dirty source map and beam for a representative map, that of 10.0 GHz at the D time sample shown in Figure 2.1. There is a source near the center of the map where we expect to find one, but it appears indistinguishable from its neighbors. Part b) shows a frequency synthesis map and corresponding beam made at the same time, incorporating data at 9.4, 10.0, 11.2, and 11.8 GHz. This map narrows the field of candidate sources to two in the East-West direction. In comparing the map and beam we notice that the sidelobe pattern to the left (East) of the strong sources looks like the beam pattern to the left of the central part of the beam, but that the sidelobe patterns on the right sides of the map and beam don't match. This suggests that either there is some emission extending to the right side of the main source, adding some complexity to the sidelobe pattern on that side, or there is a
frequency-dependent shift in the source position, which can also distort the frequency-synthesis map.

In the first panel of Figure 2.5 part c) we show the dirty single-frequency map and the clean box chosen on the basis of the frequency-synthesis map. (The contours plotted are the same as in a) and b), ranging from 10 percent of peak to the 99 percent level.) The second panel shows the resulting clean map. There is an indication of a secondary source or of some extended emission to the West (right) of the main source, as we expected. In this extended region the flux we can map is only ten percent of the peak flux in the main source. In the third panel we show, using the same contour levels, the clean map which results if we choose a clean box to the right of center. This choice clearly leads to an inferior map, as do other clean box choices not shown, and confirms the validity of the central source.

For the six time samples A through F we produced single-frequency maps and cleaned them using the frequency-synthesis maps for comparison. Unfortunately the LCP data were largely unusable for further analysis because of a combination of technical problems and the fact that the source appears to have been more complex in LCP than in RCP. On several baselines and at intermittent times the correlated amplitude values were anomalously low. In some cases the data from one or more baselines were flagged by the software as problematic and are actually missing. This problem does not prevent us from forming images, but leads to inadequately sampled flux and hence, misleading brightness temperature values. In Figure 2.6 we illustrate this problem by comparing results in LCP (top line) and RCP for a representative data set at 7.0, 10.0, and 11.2 GHz. All of the maps have been cleaned; the LCP maps, however, show high residuals. In the case of the 7.0
Figure 2.5 a, b, c  Cleaning the flare images using frequency synthesis to locate the source. Contours are at 10 percent intervals from -10% to +99% of the peak flux. Part a) shows the dirty source and beam for the map at 10.0 GHz only, D time sample. Part b) shows a frequency synthesis map incorporating data from the same time sample at 9.4, 10.0, 11.2, and 11.8 GHz. The field of possible sources is reduced. The first panel of part c) shows the dirty single-frequency map and the clean box chosen on the basis of the frequency-synthesis map. The second panel shows the resulting clean map, and the third panel shows the clean map which results is we select an alternative clean box.
GHz LCP map the residuals may be due to inadequate u-v coverage of the source. In the 10.0 and 11.2 GHz maps we see from the shape and orientation of the beam, shown in the box in the top left of the panel, that data are missing. The RCP maps show how full-coverage beams should appear.

We attempted to clean the maps below 6 GHz but found that the source is complex and substantially over-resolved at the lower frequencies. This problem was noted already in section 2.2.2 when we compared the total power with the correlated amplitude measured on the shortest baseline. Overresolution is a problem which manifests itself only in the comparison of the total power with the correlated amplitude. If a source is too complex to be mapped with existing baselines the cleaned maps will show significant residuals, which may vary from one clean iteration to the next. Overresolution itself, on the other hand, may not cause any obvious problems in making the images.

To recapitulate, we have selected for further analysis only the RCP data above 6.2 GHz. The source at frequencies below 6.2 GHz was over-resolved, even on the shortest baseline, which means that the brightness temperature cannot be reliably determined. The LCP data at all frequencies could not be adequately imaged, due to source size and complexity and problems with missing data. Unfortunately this means that we do not have reliable, quantitative information on the polarization of the source emission.

In the next section we discuss the brightness temperature of the source. The cleaned maps provide a measure of the brightness temperature, but these values can be refined with a more detailed evaluation of the source size.
Figure 2.6 Comparison of left circularly polarized (LCP) and right circularly polarized (RCP) clean maps. The residuals in the LCP maps illustrate the difficulty of cleaning the LCP maps due to source complexity and missing data.
2.2.4 Brightness Temperature Spectra and Source Size Spectra of the Radio Burst

We have made images not only to study the source size and morphology, but also to establish the brightness temperature spectrum of the source. The flux spectrum, by contrast, is not an unambiguous guide to the emission mechanism because of the likelihood that the source size varies with frequency. The brightness temperature spectrum is more informative, and requires that we determine the source size. In this section we derive the source size spectra and brightness temperature spectra for the six representative time samples. In later sections we consider in more detail what we can learn from the brightness temperature spectra.

The observed source in the OVRO maps is a convolution of the true source and the interferometer beam. Therefore to recover the true source dimensions and orientation we perform a deconvolution, following the method of Wild (1970) which is summarized below. The deconvolution reduces to a simple geometrical problem if we assume that the source and beam can be represented by functions with elliptical isophotes and Gaussian profiles. The first step, then, is to measure the size and orientation of the source and beam ellipses in the OVRO maps. We did this using the data-analysis program IDL. The built-in function ‘surface_fit’ was used to fit smooth surfaces to the two-dimensional brightness temperature images in the clean maps. The surface-fitting yielded a consistent (reproducible) estimate of the peak brightness temperature in each single-frequency map, and allowed us to measure the major and minor axes and orientations of the observed source and beam in each map.
In the deconvolution procedure the observed source, the deconvolved or true source, and the beam are represented by functions of the form

\[ F_j(x, y) \propto \exp\left(-\frac{(x \cos \theta_j + y \sin \theta_j)^2}{a_j^2} - \frac{(x \sin \theta_j - y \cos \theta_j)^2}{b_j^2}\right) \]  

(2.1)

where \( a \) and \( b \) denote the major and minor semi-axes of the ellipse and \( \theta \) is the inclination of the major axis to the horizontal. The subscripts 0, 1, and 2 in place of \( j \) are used respectively for the observed source, deconvolved source, and the beam. \( F_0 \) is the convolution \( F_1 \ast F_2 \), therefore the Fourier Transform of the function can be written as the product of the transforms:

\[ \tilde{F}_0 = \tilde{F}_1 \tilde{F}_2. \]

Taking the Fourier Transform of the ellipse function, substituting into the convolution relation, and collecting like terms gives us three equations defining \( a_1, b_1, \) and \( \theta_1 \) in terms of the known variables. It is convenient to combine the \( a, b, \) and \( \theta \) variables into a single vector, \( D_j = (a_j - b_j) \exp(2i\theta_j) \). Then the requirement that \( D_0 = \sum_{j=1}^{2} D_j \) leads to the following results for \( a_1, b_1, \) and \( \theta_1 \):

\[ a_1^2 = \frac{1}{2}(a_0^2 + b_0^2 - a_2^2 - b_2^2 + \sqrt{\text{factor 1}}), \]

\[ b_1^2 = \frac{1}{2}(a_0^2 + b_0^2 - a_2^2 - b_2^2 - \sqrt{\text{factor 1}}), \]  

(2.2)

\[ \tan 2\theta_1 = \frac{(a_0^2 - b_0^2) \sin 2\theta_0 - (a_2^2 - b_2^2) \sin \theta_2}{(a_0^2 - b_0^2) \cos 2\theta_0 - (a_2^2 - b_2^2) \cos \theta_2} \]

where \( \text{factor 1} = (a_0^2 - b_0^2)^2 + (a_2^2 - b_2^2)^2 - 2(a_0^2 - b_0^2)(a_2^2 - b_2^2) \cos (\theta_0 - \theta_2) \). These equations allow deconvolution of the source parameters from the observed source and beam.

In Figure 2.7 we show the observed source, beam, and deconvolved source for nine frequencies in the range from 6.2 to 16.4 GHz, at the D time
sample. The beam is shown with cross-hatching, and the deconvolved source shown shaded in black. The deconvolved source is elongated in the E-W direction (left to right in the figure). In most cases the source is resolved by the beam along the major axis of the source, and not resolved along the minor axis of the source.

In Figure 2.8 we show the results of the deconvolution, plotting source area against frequency for each time sample. We have also plotted for each time a size spectrum in which source area varies linearly with frequency, as has been commonly found in interferometric studies (Wang et al. (1994), Lim et al., 1994). The linear spectrum is forced to agree with the observed area at 11.2 GHz where the source was strongest. The linear spectrum is for comparison only. We did not use it to amend the source size spectrum.

The source sizes sometimes vary by factors of two or three at neighboring frequencies, as is the case for example in the A time sample for 10.0, 10.6 and 11.2 GHz. The reason for the high variability can be traced to the fact that the source is small or unresolved in one dimension. Numerical deconvolution experiments on observed sources and beams of typical sizes show that 10 percent errors in the determination of the observed source size along the minor radius of the source can lead to errors in the area of factors of 1.6 or more, depending on the aspect of the source. Uncertainty in the observed source dimensions can be inferred from the variations which arise from the choice of isophote level used to define the source and beam: the source as defined by the contour level which is 50 percent of the maximum intensity may be more or less elongated than the source as defined by the
Figure 2.7 Deconvolution of source size and orientation from the observed source and beam. The observed source, beam, and deconvolved source are shown for 9 frequencies at the D time sample. The beam is shown with horizontal cross-hatching, and the deconvolved source is shaded in black. In most cases the source is resolved by the beam along the major axis of the source, and not resolved along the minor axis of the source.
Figure 2.8  Source size spectrum, with error bars, for each time sample. The data are shown with the x symbol. A spectrum in which the source size varies linearly with frequency, as is commonly observed, is plotted using square symbols. The linear spectrum is required to fit the observed spectrum at 11.2 GHz, where the images are of the highest quality.
10 percent contour level. We have plotted error bars which represent the extremes of source area calculated with 10 percent errors in the determination of the observed source size.

Judging from the behavior at 11.2 GHz, where the source was easiest to map, we determined that the source grew by about a factor of 1.6 in area through the D, E, and F time samples, from 16:56:41 to 16:57:16 UT. The estimated size at 11.2 GHz grows from 59 to 94 square arcseconds during this decay period. A comparison of the beam and deconvolved source sizes indicates that, owing to this growth, the source became resolved or nearly resolved for all frequencies after UT 16:55:40, the peak of the flare. Before the peak the source was resolved only for frequencies above 10.6 GHz.

The deconvolved sizes are used to estimate the true brightness temperature of the source. This is necessary because the peak brightness temperature in the single-frequency maps is underestimated if the source is unresolved, or much smaller than the beam. In this case the brightness temperature, which is a measure of the flux divided by the area, has been determined with an assumed source area that is too large. We compare the true source and beam areas and correct the brightness temperature for the filling factor of the source in the beam. At the peak of the flare the correction amounts to an increase in inferred brightness temperature by a factor of 1.6. The fact that the correction is highest at the peak of the flare suggests that the brightness distribution of the source at the time of maximum emission was more sharply peaked than at other times. We assumed at all times that the brightness distribution had a Gaussian profile and defined the source by the 50 percent contour level, while at the time of maximum emission the 50
percent contour level may have enclosed a smaller fraction of the area of the source than at other times.

Figure 2.9 is a plot of the brightness temperature spectra for each of the six time samples. The uncorrected brightness temperatures are denoted by crosses and the corrected ones by squares. The correction we made for the filling factor of the source in the beam raised the overall brightness temperature at the peak of each spectrum, but did not substantially affect the shape of the spectra. Table 2.1 gives the peak brightness temperatures, peak frequencies, bandwidths, and high and low-frequency slopes measured from Figure 2.9 for all the time samples. The important thing to note in Figure 2.9 is that the frequency of maximum emission, or peak frequency, remains nearly constant as the spectrum evolves in brightness temperature. This is the first time that data have been obtained with high enough time resolution to obtain the brightness temperature spectrum during the rise and decay of the impulsive phase. We will return in section 2.4 to the problem of explaining the constant peak frequency.

2.3 Gyrosynchrotron Emission and Application of the Simplified Theory to Solar Bursts

The steep low-frequency slopes in the brightness temperature spectra we obtain for this flare, shown in Figure 2.9, indicate that thermal bremsstrahlung or thermal gyrosynchrotron radiation are not the preferred emission mechanisms to explain the observed spectra. As noted in section 1.4, thermal sources have a constant brightness temperature in the optically thick part of the spectrum below the turnover frequency. The high temperature at the peak of the flare, $6 \times 10^8$ K, is also not characteristic of thermal emission. We thus turn our attention to the gyrosynchrotron process and
<table>
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<th>$\nu_{\text{peak}}$</th>
<th>Bandwidth</th>
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<td></td>
<td></td>
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Figure 2.9 Brightness temperature spectra for the A, B, C, D, E, F time samples. The data were corrected for the filling factor of the source in the beam (see text). The uncorrected points are shown using x symbols and the corrected ones with squares.
point out the features of the spectra which must be addressed by the gyrosynchrotron model. In section 2.5.3 we will give quantitative examples of the theoretical predictions of the numerical code we used; in this section we summarize the simplified theory and its application to solar bursts with reference to the papers of Dulk and Marsh (1982) and Gary and Hurford (1988).

Dulk and Marsh (1982) explored the gyrosynchrotron emission from mildly relativistic electrons with energy in the range $\sim 10$ keV to $\sim 1$ MeV. Electrons with energy in this range are important in radio and hard X-ray emission from solar flares. The authors sought approximations to the theoretical expressions for gyrosynchrotron emission from these electrons. The approximations would allow simple comparisons to be made between models with thermal and nonthermal particle distributions.

The variables influencing gyrosynchrotron emission are: the field strength $B$, the total electron number $N$ above the energy $E_0$, the index in the particle energy distribution function $\delta$, and $\theta$, the viewing angle to the magnetic field, as we saw in section 1.4.2 on gyro emission. The general theoretical expressions relating these parameters in the emission and absorption coefficients $\eta_\nu$, and $\kappa_\nu$ are unwieldy, and therefore for their numerical calculations of emission from non-thermal particles the authors used formulae developed for the widely applicable case in which the index of refraction is near unity. As noted by the authors, the requirement that deviations of the index of refraction from unity be ignored implies that Razin suppression is unimportant in the cases examined.

The authors calculate the following parameters as functions of the harmonic number $\frac{\nu}{\nu_b}$: the normalized emission and absorption coefficients
the effective temperature \( T_{\text{eff}} \) inferred from the emission and absorption coefficients, and the polarization coefficient \( r_c \) which gives the degree of circular polarization. We are particularly interested in the effective temperature, which is integrated along the path to the source to give the brightness temperature, so that \( T_b = T_{\text{eff}} \) for those frequencies for which the source is optically thick, and \( T_b = T_{\text{eff}} \tau_\nu \) if the source is optically thin. Figure 2.10 from Dulk and Marsh (1982) shows the results of the numerical calculations, with three curves in each plot for three values of the electron power-law index \( \delta \).

In the regime of high harmonic numbers, \( \frac{\nu}{\nu_B} \geq 10 \), Dulk and Marsh found that the slopes of the curves were nearly constant, and were able to derive simplified expressions for the parameters as power-law functions of the harmonic number. The simplified expressions are good approximations to the full expressions within the following ranges: \( \frac{\nu}{\nu_B} \geq 10 \), \( 2 \leq \delta \leq 7 \), \( \theta \geq 20 \). The simplified expressions, including one for the frequency of peak emission \( \nu_{\text{peak}} \), are:

\[
\frac{\eta \nu}{BN} \approx 3.3 \times 10^{-24} \ 10^{-0.52\delta} \sin(\theta)^{-0.43+0.65\delta} \left( \frac{\nu}{\nu_B} \right)^{1.22-0.90\delta} \tag{2.3}
\]

\[
\frac{\kappa \nu B}{N} \approx 1.4 \times 10^{-9} \ 10^{-0.22\delta} \sin(\theta)^{-0.09+0.72\delta} \left( \frac{\nu}{\nu_B} \right)^{-1.30-0.98\delta} \tag{2.4}
\]

\[
T_{\text{eff}} \approx 2.2 \times 10^{9} \ 10^{-0.31\delta} \sin(\theta)^{-0.36-0.06\delta} \left( \frac{\nu}{\nu_B} \right)^{0.50+0.085\delta} \tag{2.5}
\]
Figure 2.10 Figure 3 of Dulk and Marsh (1982). Their figure caption reads: Characteristics of gyromagnetic emission for the x-mode calculated numerically from the formulae of Takakura and Scalise (1970). Curves are given for three values of the electron power-law index $\delta$ and viewing angles $40^\circ$ (solid lines) and $80^\circ$ (dashed lines). The low-energy cutoff in the electron distribution function is 10 keV. Shown are: (a) emission coefficient, (b) absorption coefficient, (c) effective temperature, and (d) degree of polarization.
\[ r_c \approx 1.26 \times 10^{0.035\delta} 10^{-0.071 \cos \theta} \left( \frac{\nu}{\nu_B} \right)^{-0.782 + 0.545 \cos(\theta)} \quad \text{(if } \tau_\nu \ll 1) \quad (2.6) \]

\[ \nu_{\text{peak}} \approx 2.72 \times 10^3 10^{0.275} \sin(\theta)^{0.41 + 0.035} (NL)^{0.32 - 0.035} B^{0.68 + 0.035}. \quad (2.7) \]

Using the expressions for gyrosynchrotron emission derived by Dulk and Marsh, Gary and Hurford (1988) constructed ‘universal’ spectra based on simple theoretical sources. The spectra show the shapes of the curves that would be observed for homogeneous sources, that is, sources which have uniform magnetic field strength, temperature, and density, so that the source size does not change with frequency. Figure 2.11 shows the universal spectra for two homogeneous sources, one with a thermal particle distribution function and the other with a power-law distribution function. The bottom panels show the normalized flux density spectra and the top panels show the corresponding brightness temperature spectra that would be observed with an instrument capable of resolving the source.

The curves for the thermal and non-thermal models are distinctive. In the case of flux density, with logarithmic axes, the slope of the low-frequency, optically thick part of the spectrum is exactly 2 for the thermal model, and generally greater than 2, depending on the index of the energy distribution function, for the non-thermal model. The high-frequency slope in the flux density curves is shallower for the non-thermal case than for the thermal case. The brightness temperature spectra for the two cases are even more easily distinguished because the thermal spectrum is flat up to the turnover
Figure 2.11 Figure 7 of Gary and Hurford (1988). Their caption reads: Universal curves, from theory, for emission by various mechanisms from a homogeneous source. The arrows represent the magnitudes and directions of shift for an increase of parameters by a factor of 2. The top panels show normalized brightness temperature spectra (requiring measurements with spatial resolution) and the bottom show the corresponding flux density spectra. The arrow labeled $B$ in the thermal gyrosynchrotron curves actually represents the quantity $B(nL)^{0.11}(\sin \theta)^{2/3}$. 
frequency and rolls off steeply with a slope of -10 above the turnover frequency. The slopes in the non-thermal gyrosynchrotron case depend on the index in the energy distribution function, but for typical values of $\delta$ between 2 and 7, the low-frequency slope is between 0.67 and 1.09.

Vectors attached to the curves show what the effect is of varying by a factor of 2 the key parameters such as field strength $B$, viewing angle to the source $\theta$, and the parameter $NL$, where $N$ is the number of electrons having energy above a given limit, and $L$ the path length to the source. In the case of the flux density curves a vector is also attached to indicate the way the curve changes for an increase $\Delta \Omega$ in the angular size of the source. The parameter $NL$ in the non-thermal gyrosynchrotron case has the same effect on the spectrum as the temperature $T$ in the thermal case in the sense that when the temperature of the electrons is increased, or when the number of electrons above a given limit in the power-law distribution is increased, the curves move to the right and up: the turnover frequency increases and the amplitude of the spectrum increases.

2.4 Difficulty of Explaining the Spectra Using Simplified Theory

We have seen that the simplified theory of non-thermal gyrosynchrotron emission predicts brightness temperature spectra with certain characteristic features, depending mainly on the index $\delta$ in the energy distribution function, and specific changes of shape, depending on the changes in energy or in the average magnetic field of the source, for example. The spectra we obtained for the flare in section 2.2.4 are not easily explained with reference to these theoretical curves. The slopes of the optically thick or low-frequency parts of the observed spectra are very steep, apparently requiring unusual indices $\delta$, and the evolution in time of the observed spectra is in amplitude
only, with no discernible change in the turnover or "peak" frequency. In this section we therefore consider the question of how well actual observations are fit by the universal curves. We illustrate the difficulty of explaining our observed spectra by reviewing the findings of Stähli, Gary and Hurford (1989).

In 1981, while the imaging capability of the Owens Valley Solar Array was still under development, a number of solar bursts were observed and flux density spectra acquired with high spectral resolution in the 1-18 GHz band. Stähli, Gary, and Hurford (1989) made a statistical analysis of 49 bursts observed with this system. The observations were made with 10 s resolution, which allowed them to analyze the rise, maximum, and decay phases of the bursts separately.

Noting that most events had multiple spectral components that would have led to misrepresentations of the spectrum in systems with less dense spectral coverage, the authors fit the dominant component in each spectrum to obtain a statistical sample of four key parameters: the low-frequency slope on a double-logarithmic plot (they sometimes called this the spectral index), the high-frequency slope, the peak frequency (i.e., the frequency of the spectral peak) and the FWHM bandwidth. Their Table 2.2 shows the results. The average low-frequency slope was close to 3, whether measured in the rise, maximum, or decay phase. The average high-frequency slope was -3.7, the average peak frequency was about 7.4 GHz, and the average bandwidth about 85 percent. The low-frequency slopes were noted to be particularly steep: slopes as high as 10 were observed, and during the rising part of the flare, when the slopes were generally steepest, 48 percent of the events had slopes greater than 3. The slopes are thus steeper than expected from the...
simplified theory of gyrosynchrotron emission. Furthermore, inhomogeneity in the source cannot account for the discrepancy, since inhomogeneity is expected to make the low-frequency slope more shallow.

The authors considered Razin suppression as a process that might steepen the low-frequency side of their spectra, but judged the implied ambient density of $2 \times 10^{10} \text{ cm}^{-3}$ to be too high to occur commonly. They suggested that the exact theory of gyrosynchrotron emission should be applied, and noted that in the case of low harmonics ($\frac{\nu}{\nu_B} < 10$) the optically thick part of the spectrum is steeper than in the region approximated by the simplified expressions.

Another result which has a bearing on our observations is that the authors found that the peak frequency stayed remarkably constant throughout almost all of the bursts they observed. The smallest increase in the peak frequency is that expected for the thermal case, as opposed to the non-thermal case. They note that during the rise phase, a flux density increase by a factor of $e$ should lead to a minimum of 32 percent increase in the peak frequency. Figure 2.12 reproduced from their paper shows the expected shift in peak frequency compared with a typical burst during the rise phase and during the decay phase. Although a small shift does occur, the discrepancy with theory is large. Most interestingly, only 2 events, or 4 percent of the sample, showed a shift as large as that expected from theory.

We can use the results of Dulk and Marsh to estimate the shift in peak frequency we would expect for the nearly two-order-of-magnitude increase in brightness temperature that occurs in our flare. Extrapolating the $T_{eff}$ curve in their figure 3c), we find that a peak $T_\nu$ of $2 \times 10^7 \text{ K}$ would occur at a frequency $\nu \sim 2 \nu_B$, while a peak $T_\nu$ of $6 \times 10^8 \text{ K}$ would occur near $8 \nu_B$. For
Figure 2.12 Figure 3 from Stähli, Gary, and Hurford (1989). Their caption reads, in part: Example of the temporal evolution of a microwave burst spectrum: The observed flux density is plotted versus the frequency for several times during the rising phase of the burst (a) and during the decay of the event (b). The dashed arrows indicate the minimum peak frequency shifts expected for gyrosynchrotron emission of a homogeneous source.
higher peak brightness temperatures where the simplifying expressions are valid, a similar increase in $T_b$ of a factor of 30 would lead to an even larger shift in the peak frequency, as can be seen from the low slope of the $T_{eff}$ curve. The problem with the simplified theory of gyrosynchrotron emission is immediately apparent, because the observed shift in peak frequency for the burst studied here is less than 2 GHz.

Some low-frequency absorption or cutoff is apparently needed to explain the observed spectra. Figure 2.13 shows how a low-frequency cutoff can produce a constant peak frequency and steep low-frequency slope. In the next section we investigate the possibility that the cutoff is provided by Razin suppression. Free-free absorption of gyrosynchrotron emission plays a role, as well; the high ambient density required for Razin suppression means that some free-free absorption must accompany it. However, our concern here is to determine to what degree the observed spectra may be explained by Razin suppression. In Chapter 3 we show that indeed, the temperature of the flare plasma is too high to allow free-free absorption to account for the low-frequency cutoff in this particular flare.

2.5 Numerical Modeling

2.5.1 The Gyrosynchrotron Code

We used a gyrosynchrotron code described in Ramaty et al. (1994) and provided to us by the lead author. The code is based on the equations for the gyrosynchrotron emission and absorption coefficients presented by Ramaty (1969) (except for the omission of a spurious term; see Ramaty et al., 1994). The code treats the case of a distribution of electrons radiating in a cold, collisionless, magnetoactive plasma. The index of refraction in the
Figure 2.13 Sketch to illustrate how a low-frequency cutoff, shown by a dashed line, can produce an apparently constant peak frequency and steep low-frequency slope.
plasma depends on the gyrofrequency $\nu_B$ as well as the plasma frequency $\nu_p$ and frequency of observation $\nu$, i.e. it is not assumed that the gyrofrequency is low and that the simpler form of the index of refraction ($n^2 = 1 - \frac{\nu^2}{\nu_B^2}$) pertaining to an isotropic plasma may be applied. The exact index of refraction used in the code has two forms corresponding to wave propagation in the ordinary (+) and extraordinary (-) modes, and is written as follows, where $\theta$ is the angle between the magnetic field and the direction of wave propagation:

$$n_{\pm}^2(\theta) = 1 + \frac{2\nu_p^2(\nu_p^2 - \nu^2)}{\pm[\nu^4\nu_B^4 \sin^4 \theta + 4\nu^2\nu_B^2(\nu_p^2 - \nu^2)^2 \cos^2 \theta]^{1/2} - 2\nu^2(\nu_p^2 - \nu^2) - \nu^2\nu_B^2 \sin^2 \theta}.$$  

(2.8)

The importance of the use of the exact index of refraction is that when $n$ differs from unity the medium affects the generation of radiation as well as its propagation and absorption, and this is taken into account by the code. The index of refraction may differ significantly from unity in one or both modes of propagation. The suppression of gyrosynchrotron radiation in a plasma when the index of refraction is less than unity is called the Razin effect and is discussed further in section 2.5.2.

We used the code to evaluate the emission and absorption coefficients, and hence the brightness temperature, resulting from a power-law distribution of electrons with an isotropic pitch angle distribution. We now briefly describe the equations used and the input and output of the code. In Chapter 1 we showed that the volume emissivity corresponding to a single electron is obtained by summing the radiation over some range of harmonics $s$ of the gyrofrequency, where the range of harmonics that is relevant is determined
by the Lorentz factor $\gamma$ and pitch angle $\phi$ of the electron as well as the index of refraction in the ambient medium. The volume emissivity of a single electron is integrated over the particle distribution function to obtain the total emissivity $\eta$. The reabsorption of photons by electrons in the magnetic field is written in terms of the emissivity by way of the Einstein coefficients, to give the absorption coefficient $\kappa$. We have:

$$\eta_{\pm}(\nu, \theta) = \frac{BN}{V} e^{3} G_{\pm}(\frac{\nu}{\nu_B}, \theta); \quad \kappa_{\pm}(\nu, \theta) = \frac{N}{BV} (2\pi)^{2} e H_{\pm} \left( \frac{\nu}{\nu_B}, \theta \right),$$

(2.9)

where $B$ is the magnetic field strength, $N$ the total number of electrons with energy above a specified minimum energy $E_0$, and $V$ the total volume of the source. The functions $G_{\pm}$ and $H_{\pm}$ are as follows ($J_s$ are Bessel functions of the first kind of order $s$, $u(\gamma)$ is the energy distribution function, $g(\phi)$ is the pitch angle distribution function, and $\beta = \frac{\nu}{c}$):

$$G_{\pm} = \frac{2\pi}{\cos \theta} \int_{1}^{\infty} d\gamma \frac{u(\gamma)}{\beta \gamma} \sum_{s=s_1}^{s_2} \frac{s}{1 - n_{\pm} \beta \cos \theta \cos \phi_s} g(\phi_s)
\times [-\beta \sin \phi_s J_s'(x_s) + a_{\theta \pm} \left( \frac{\cot \theta}{n_{\pm}} - \beta \frac{\cos \phi_s}{\sin \theta} \right) J_s(x_s)]^2;$$

(2.10)

$$H_{\pm} = \frac{2\pi}{\cos \theta} \int_{1}^{\infty} d\gamma \frac{u(\gamma)}{\beta \gamma} \sum_{s=s_1}^{s_2} \frac{s}{1 - n_{\pm} \beta \cos \theta \cos \phi_s} g(\phi_s)
\times [-\beta \sin \phi_s J_s'(x_s) + a_{\theta \pm} \left( \frac{\cot \theta}{n_{\pm}} - \beta \frac{\cos \phi_s}{\sin \theta} \right) J_s(x_s)]^2
\times \frac{1}{n_{\pm}^2} \left( \frac{\nu_B}{\nu} \right)^2 \left[ -\frac{\beta \gamma^2}{u(\gamma)} \frac{d}{d\gamma} \frac{u(\gamma)}{\beta \gamma^2} + \frac{n\beta \cos \theta - \cos \phi}{\gamma \beta^2 \sin \phi} \frac{1}{g(\phi)} \frac{dg}{d\phi} \right],$$

(2.11)

where
The inputs to the code are as follows. To define the ambient medium the user specifies a uniform magnetic field strength, the cosine of the angle $\theta$ between the field and the line of sight, and the so-called Razin parameter $\alpha = 1.5 \nu_B / \nu_p$ (Ramaty and Lingenfelter, 1967). The $\alpha$ parameter serves as an index for the influence of the medium: Ramaty (1968) demonstrated that the emission is suppressed when $\alpha \gamma < 1$. The $\alpha$ parameter is further discussed in section 2.5.2. The density of the electrons in the ambient medium, which determines the plasma frequency, is set by $\alpha$, since the field strength and hence the gyrofrequency is already specified.

To define the electron energy spectrum the user specifies the exponent $\delta$ and normalization factor $A$ of the distribution function $N(E) = A E^{-\delta} \text{ (MeV}^{-1})$, as well as the lower and upper bounds of electron energy $E_0$ and $E_{\text{max}}$ used in the integration over the distribution function, which in the analytic expression is carried out from $\gamma = 1$ to $\infty$. The user also specifies the angular size of the source, which is relevant when the total flux is of interest. Finally, the user specifies the range of observation frequencies examined by the code, and the electron energy value at which the computation shifts from the full gyrosynchrotron formulation to the faster ultrarelativistic formulae of Ginzburg and Syrovatskii (see Ramaty 1994).
The outputs include the x and o-mode emissivities and absorption coefficients, the x and o-mode flux densities at Earth, and the degrees of polarization. For our purposes we combined the emissivities and absorption coefficients and assumed a fixed path length to the source L to calculate the brightness temperatures in each of the two modes according to the formula:

\[ T_b = \frac{n_\nu}{\kappa_\nu} \frac{c^2}{\nu^2 k_B} (1 - e^{-\kappa_\nu L}). \]

2.5.2 The Razin Effect

In this section we give a general description of the Razin effect and derive the conditions under which the effect takes place, principally following the arguments of Ginzburg and Syrovatskii (1965) and Ramaty (1968). It is generally assumed in discussions of the Razin effect that the plasma frequency is large compared to the gyrofrequency, and that the frequency of observation is large compared to either of those. We consider the conditions that pertain in the case of the flare of 1992 July 16 and discuss of the consequences of a relatively high gyrofrequency.

The Razin effect is the suppression of radiation which occurs when the beaming and intensity of synchrotron or gyrosynchrotron emission are modified by the ambient medium. The effect occurs when the index of refraction is less than unity. Razin suppression is not a propagation effect; it is a suppression of emission at the source, and as such, as pointed out by Ramaty (1969), it leads to a corresponding reduction in the absorption process. This means that medium suppression and gyrosynchrotron re-absorption must be treated consistently in modeling the emission.

A complete derivation of the Razin effect is beyond the scope of this work. However, we compare the expressions for emission in plasmas with and without an index of refraction differing from unity, and summarize the
arguments leading to the Razin condition in terms of beaming considerations. We also develop the theory of Razin suppression for mildly relativistic particles.

We begin by comparing the beaming of synchrotron radiation in vacuum and in a medium with index of refraction not equal to unity. (See section 1.4.2 for background information on synchrotron and gyrosynchrotron radiation.) In the vacuum case the frequency of radiation emitted by a relativistic particle may be obtained by a transformation from a frame in which the electron is non-relativistic and the emitted frequency is

$$\omega = \omega_i = \frac{eB}{mc}$$  \hspace{1cm} (2.15)

to a system in which the particle has the relativistic velocity $v$ (Ginzburg and Syrovatskii, loc. cit.) In this system the observed frequency is Doppler shifted by the motion of the particle, so

$$\omega = \omega_i \frac{\sqrt{1 - (\frac{v^2}{c^2})}}{1 - \frac{v}{c} \cos \Phi},$$  \hspace{1cm} (2.16)

where $\Phi$ is the angle between the velocity vector and the wave vector of the observed radiation. Figure 2.14 illustrates this geometry.

*Figure 2.14 Sketch of the geometry of a particle emitting synchrotron radiation. The velocity of the particle is shown by the vector $v$. The wave vector is labeled $k$. $\Phi$ is the angle between them.*
We can show that both the frequency of emission and the intensity of radiation are maximized when the angle $\Phi$ is approximately equal to the quantity $\frac{1}{\gamma}$. If $\Phi \approx \frac{1}{\gamma}$ then

$$\omega \approx \frac{\omega_i}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}} = \gamma \omega_i$$

(2.17)

so that the emitted frequency is large compared to the reference frequency $\omega_i$, and if $\Phi > \frac{1}{\gamma}$, the emitted frequency is sharply reduced. The expression for the intensity of radiation $dI$ per unit solid angle $d\Omega$ also contains the expression $\left(1 - \frac{v}{c} \cos \Phi\right)$ in the denominator (see Landau and Lifschitz, 1951) so that the intensity of the radiation is similarly peaked in a cone or beam of opening angle $\frac{1}{\gamma}$.

In the case of radiation in a plasma it can be shown that the equations for radiation in a vacuum must be modified by formally replacing the velocity of light $c$ by the phase velocity $c_p = \frac{c}{n}$ of the radiated wave in the medium (Ginzburg and Syrovatskii, 1965, Melrose and McPhedran, 1991). Then the factor $\left(1 - \frac{v}{c} \cos \Phi\right)$ in the denominator of the expressions for the frequency and angular distribution of intensity becomes $\left(1 - \frac{v}{c} \cos \Phi\right)$.

What is the effect on the beaming of radiation, and on the intensity? In vacuum the beam of emission from a single particle has an opening angle

$$\theta_b \approx \frac{1}{\gamma} = \sqrt{1 - \frac{v^2}{c^2}},$$

(2.18)

so in the presence of a medium of refractive index $n$, the opening angle will be given by

$$\theta_b \approx \frac{1}{\gamma} = \sqrt{1 - n^2 \frac{v^2}{c^2}}.$$

(2.19)
For \( n \) sufficiently less than unity, and for relativistic particles for which \( v \approx c \), we may write
\[
\theta_b \approx \sqrt{1 - n^2}.
\] (2.20)

If in addition the gyrofrequency is low compared to the plasma frequency, as assumed by Ginzburg and Syrovatskii and Ramaty, then the index of refraction takes the simplified form \( n^2 = 1 - \frac{\nu^2}{\nu_p^2} \) and the opening angle of the beam can be expressed in terms of the ratio of plasma frequency to the frequency of observation:
\[
\theta_b \approx \sqrt{1 - n^2} = \frac{\omega_p}{\omega}. 
\] (2.21)

From this we derive the condition in Ginzburg and Syrovatskii (loc cit.) for the medium to have an effect: if
\[
\frac{\omega_p}{\omega} > \frac{1}{\gamma} 
\] (2.22)
i.e., the opening angle of the beam is large compared to that in the vacuum case, the medium determines the beaming.

**Frequency dependence of the Razin effect when \( \nu_B \) is low**

To obtain an expression for the frequency interval over which the effect of the medium is *not* appreciable, Ginzburg and Syrovatskii write \( \omega \) in terms of the critical frequency \( \nu_c = \frac{3 e B_{\perp}}{4\pi m c} \gamma^2 \) (see section 1.4.2). Substituting into the condition \( \frac{\omega}{\omega^2} << \frac{1}{\gamma} \) for the medium *not* to be important, we obtain
\[
\nu^2 >> \frac{4}{3} e \frac{N e}{B_{\perp}} \nu_c, 
\] (2.23)
or, since the maximum intensity is emitted close to $\nu_c$,

$$\nu >> \nu_R \simeq 20 \frac{N_e}{B_\perp}. \quad (24)$$

Melrose and McPhedran (1991) write this condition equivalently in terms of the plasma frequency, to derive the so-called Razin-Tsytovich frequency $\omega_{RT}$ below which the Razin effect steepens the spectrum of synchrotron radiation:

$$\omega_{RT} = \frac{2\omega_p^2}{3\Omega_\perp}, \quad \text{where} \quad \Omega_\perp = \frac{eB_\perp}{mc}. \quad (2.25)$$

Microwave-burst emitting regions of the solar corona are typically found to have magnetic fields in the range from 50 to somewhat less than 500 Gauss, and densities $10^8$ to a few times $10^{10}$ cm$^{-3}$. Applying equation 25 we see that Razin suppression is not important above 0.6 GHz for a typical situation of $n_e \sim 5 \times 10^8$ cm$^{-3}$ and $B = 100$ Gauss, although a plasma density of only $2.0 \times 10^9$ cm$^{-3}$ boosts the Razin-Tsytovich frequency into the range of microwave observations.

The Razin condition when the general expression for $n$ applies

We have shown that the Razin effect is strongest at low frequencies, and have derived the condition for the Razin effect in the case in which the gyrofrequency is low compared to the plasma frequency. The frequency dependence of the Razin effect in Ginzburg and Syrovatskii's derivation has led to suggestions by a number of authors that Razin suppression plays a role in solar spectra at both metric and microwave wavelengths (Ramaty and Lingenfelter (1967) and Gopalswamy and Kundu (1990) in the metric case; Klein (1987) in the microwave case). To show rigorously why the Razin effect may be important in the microwave regime, we must examine the effect of
the medium when the gyrofrequency is of the same order of magnitude as the plasma frequency and we must also consider the case of mildly relativistic, or non-relativistic electrons.

The derivation we laid out was based on a comparison of the beaming of radiation in vacuum and in a dispersive medium. The derivation thus applies to relativistic electrons for which the beaming is a strong effect. A different treatment is required for mildly relativistic electrons, which in fact contribute the most to microwave gyrosynchrotron emission. In the remainder of this section we give an indication of the frequency range affected by Razin suppression in the relativistic case, then consider in more detail the case of mildly relativistic electrons.

In the relativistic case ($\gamma$ greater than 2 or 3) the effect of the medium can be recognized from the comparison of the vacuum and dispersive-medium expressions for the angular distribution of intensity. In vacuum the spectral distribution of power radiated by a single electron is

$$p(\nu) = \frac{\sqrt{3} \ e^3 \ B_\perp}{mc^2} \ \frac{\nu}{\nu_c} \ \frac{\nu}{\nu_c} \ \int_{\nu/c}^{\infty} K_{\frac{3}{2}} (\eta) d\eta, \quad (2.26)$$

(Ginzburg and Syrovatskii, loc. cit.) where $K$ is a Bessel function of the second kind used to describe the electric field of the radiating particle in a Fourier expansion. In a medium with $n < 1$ this expression becomes:

$$p(\nu) = \frac{\sqrt{3} \ e^3 \ B_\perp}{mc^2} \left[ 1 + (1 - n^2) \left( \frac{E}{mc^2} \right)^2 \right]^{-1/2} \ \frac{\nu}{\nu^1_c} \ \frac{\nu}{\nu^1_c} \ \int_{\nu^1_c/c}^{\infty} K_{\frac{3}{2}} (\eta) d\eta, \quad (2.27)$$

where

$$\nu^1_c = \nu_c \left[ 1 + (1 - n^2) \left( \frac{E}{mc^2} \right)^2 \right]^{-3/2}. \quad (2.28)$$
Clearly, the condition for the effect of the medium to be negligible, or for the two expressions to be equivalent, is

\[(1 - n^2) \ll \left( \frac{mc^2}{E} \right)^2. \tag{2.29} \]

Unlike the expression \( \frac{\omega_p}{\omega} \ll \frac{1}{\gamma} \), this statement of the condition for the absence of Razin suppression does not depend on the assumption that the gyrofrequency contributes negligibly to the expression for the index of refraction \( n \).

Combining the general condition for Razin suppression \((1 - n^2) \geq \frac{1}{\gamma} \) with the general expression for the index of refraction (equation 2.8) does not lead to a simple formulation of the condition required for Razin suppression, because we are interested in the case in which the plasma frequency and gyrofrequency are of the same magnitude. Instead we approach the problem graphically, comparing \((1 - n^2)\) with \( \frac{1}{\gamma} \) for a range of situations. We find that Ramaty's parameter \( \alpha = 1.5 \frac{\nu_B}{\nu_p} \) is a useful index for the strength of Razin suppression, even in the case of high gyrofrequency.

Table 2.2 shows, for representative values of the particle Lorentz factor \( \gamma \), the range of \( n^2 \) for which Razin suppression occurs. For example the radiation from a particle with Lorentz factor \( \gamma = 3 \), corresponding to a kinetic energy of 1.02 MeV, will be affected by the medium in cases in which the square of the index of refraction is less than or approximately equal to 0.890. As the energy of the particle is increased, the effect of the medium is felt for smaller deviations from unity in the index of refraction. However,
one should keep in mind that particles of higher energy radiate mostly at higher frequencies, where the index of refraction is generally closer to 1.

We plotted $n^2$ against frequency for a range of values of magnetic field $B$, viewing angle $\theta$, and $\alpha$. (Fixing $B$ means that $\alpha$ stands for the reciprocal of the plasma density.) Figure 2.15 shows an example of the results. The $\alpha$ parameter ranges from 1.5 down to 0.25 in the four panels read from left to right. The square symbols illustrate the behavior of $n$ for propagation of the ordinary mode and the diamond symbols are used for the extraordinary mode. The plasma frequency defines one edge of a stop band for the o mode, and the upper-hybrid frequency $\omega_{\text{UH}}^2 = \omega_B^2 + \omega_p^2$ is a resonance frequency for the x mode.

The curves were not particularly sensitive to viewing angle except for values less than 40°, but showed a strong dependence on $\alpha$. For the example given with $\gamma = 2$, Figure 2.14 shows that $n^2 \leq 0.75$, or that Razin suppression is important below about 3 GHz when $\alpha = 1$ and that the effect of Razin suppression extends to 5 GHz when $\alpha = 0.5$.

Razin suppression extends to even higher frequencies as $\alpha$ decreases further, corresponding to an increasing plasma frequency, but a practical limit is reached in real astrophysical conditions. On the low-frequency side, the extent of Razin suppression is limited in frequency by the resonances which lead to $n^2 \to \infty$.

We now address the case of non-relativistic or mildly-relativistic electrons. The general procedure is to modify the equation for the cyclotron (non-relativistic) power to take into account a dispersive medium, then evaluate using approximations which are valid in this energy range.
<table>
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<td>$\leq 0.960$</td>
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</table>
Figure 2.15 Index of refraction squared, plotted versus frequency, for values of the Razin parameter $\alpha$ ranging from 1.5 (very little or no medium suppression) to 0.25 (strong Razin effect). Square symbols are for propagation of the ordinary mode and diamonds are for the extraordinary mode. The gyrofrequency is 0.84 GHz and the plasma frequency ranges from 0.84 GHz in the case where $\alpha = 1.5$ to 5.04 GHz in the case where $\alpha = 0.25$. 
Bekefi (1966) gives the total radiation in a given harmonic \( m \) as follows:

\[
\eta_m = \frac{e^2 \omega_0^2}{2 \pi \varepsilon_0 c} \frac{1 - \beta_0^2}{\beta_0} \left[ m \beta_0^2 J'_{2m} (2m \beta_0) - m^2 (1 - \beta_0^2) \int_0^{\beta_0} J_{2m} (2mt) dt \right] \tag{2.30}
\]

where

\[
\beta_0 \equiv \frac{\beta_\perp}{\sqrt{1 - \beta_\parallel^2}}, \tag{2.31}
\]

and the subscripts refer to velocity components perpendicular and parallel to the magnetic field.

Radiation at harmonic \( m \) is \( 2^m \) electric multipole emission. In a dispersive medium the power radiated is proportional to the index of refraction to the power \( 2m - 1 \), where \( m \) is the multipole index written as a power of 2 (Melrose and McPhedran, 1991). We therefore multiply equation 2.30 by \( n^{2m-1} \). The next step is to evaluate the Bessel functions in the general expression for the radiated power. When these Bessel functions are evaluated in the context of synchrotron emission, for example, the Airy integral forms are used. In the non-relativistic situation the leading terms in the power series are retained, and the Carlini approximation is appropriate to evaluate these (Melrose and McPhedran 1991). The useful equations are:

\[
J_{2s} \approx \frac{(sx)^{2s}}{(2s)!}, \tag{2.31}
\]

\[
J'_x \approx \frac{J_{2s}(2sx)}{x}, \tag{2.33}
\]

\[
\int_0^x dy J_{2s}(2sy) \approx \frac{x J_{2s}(2sx)}{2s}, \tag{2.34}
\]
$J_{2s}(2sx) \approx \frac{[Z(x)]^{2s}}{(4\pi s)^{1/2}(1-x^2)^{1/2}}, \quad (2.35)$

$Z(x) = \frac{xe^{(1-x^2)^{1/2}}}{1+(1-x^2)^{1/2}}, \quad (2.36)$

leading to

$J'_{2s}(2sx) \approx \frac{(1-x^2)^{1/2}J_{2s}(2sx)}{x} \quad (2.37)$

$\int_0^x dy J_{2s}(2sy) \approx \frac{xJ_{2s}(2sx)}{2s(1-x^2)^{1/2}}. \quad (2.38)$

We illustrate the suppression effect as a function of frequency for typical values of $B$, $n_e$, and $\gamma$. For the sake of simplicity we assume that the particle pitch angle is equal to the viewing angle; radiation is maximized in this direction. In future work we will investigate the effect of viewing angle and particle pitch angle. Figure 2.16 shows the normalized power radiated as a function of harmonic $m$, relative to the power radiated in vacuum. The plot shows that radiation is strongly suppressed for the low harmonics. The parameters used for this plot were $B = 300$ Gauss, $n_e = 2 \times 10^{11}$ cm$^{-3}$, $\gamma = 1.5$, and viewing angle and pitch angle both 60°. (These values are chosen based on the results of modeling the spectra; see next section.) At harmonic $m = 12$ corresponding to $\nu \sim 10$ GHz, the emission in both x and o modes is only about 10 percent that in vacuum; the suppression due to the influence of the medium thus extends far above the plasma frequency, which is 4 GHz or $m \approx 5$ in this case.
Figure 2.16  Power radiated as a function of harmonic number $m$, $m = \frac{\nu}{\nu_{\text{gyro}}}$. The gyrosynchrotron emission in vacuum is shown with asterisk symbols, the x-mode radiation in a medium with squares, and o-mode radiation in a medium with diamonds.
2.5.3 Modeling the Results

In this section we give the results of modeling the data with Ramaty's gyrosynchrotron code. Note that all of the results apply to the x mode. O-mode results are generally similar, but with lower intensity. Unfortunately, as remarked in section 2.3, we were not able to make polarization measurements due to source complexity and other problems in the left circular sense of polarization (LCP). However, we were able to identify the mode corresponding to RCP from the magnetograms, which are discussed in Chapter 3.

In this section we discuss the influence on the brightness temperature spectra of each of the input parameters, present the results for the fitting to the spectra, and note the effect of Razin suppression. We also remark on the validity of the simplified expressions of Dulk and Marsh for the peak brightness temperature and frequency of maximum emission for our particular flare.

We begin by addressing the question of the uniqueness of the results of our fits to the data. We did not formally apply inverse theory to uncover the best fit to the data, but made a "first-order" determination of the gyrosynchrotron model parameters in a space that clearly constituted a global minimum in the model fit to the data.

The gyrosynchrotron model equations depend non-linearly on a set of eight parameters. For the determination of the brightness temperature spectra these are: δ, the exponent in the energy distribution function; B, the magnetic field strength; θ, the angle between the field and the line of sight; $E_0$, the low-energy cutoff in the electron energy spectrum; $E_{\text{max}}$, the high-energy cutoff; $\alpha$, which defines the ambient density; $n_e (\text{accel})$, the accelerated particle density, and $L$, the path to the source. The data to be fit consist of 7, 8, or 9 points in the spectrum, depending on the time sample.
The B and C spectra consist of only 7 data points, the A and E spectra of 8, and the D and F spectra of 9. Formally, then, the problem of fitting the model to the B and C spectra is underdetermined, although a priori information can be added to the problem, such as requiring the magnetic field to lie in the range from 10 to 1000 Gauss, to aid in finding a solution.

We did not attempt an eight-dimensional grid search for the best fit to the data because our goal is to understand the shapes and evolution of the spectra, not to determine precise values of the parameters. We conducted a sensitivity analysis to understand the effects of the variables in the gyrosynchrotron model, and used the results of the sensitivity analysis to locate the global minimum to the fit.

The sensitivity analysis is an investigation of the hierarchy of model parameters in terms of their influence, and of the interplay of parameters. A given spectrum may be fit, for example, with a magnetic field that is high or low, depending on the energy distribution function and densities of energetic and ambient electrons. However, our sensitivity analysis showed that some combinations of parameters that would be required to fit a feature of the spectrum, such as the peak frequency, corresponded to very unusual or artificial conditions. Ruling out those solutions is equivalent to adding a priori covariance in a formal model-fitting. One advantage of our heuristic approach is that we did not have to think up all the possible a priori information ahead of time.

The evolution of the spectrum in time is a particularly strong constraint, as we shall see. Many of the alternative fits that we rejected would require two or more parameters to offset each other in just the right way
to keep the peak frequency approximately constant. Such changes can be considered unlikely on the grounds of inordinate complexity.

Although the fits we derived look very good, as will be shown later in this section, we caution that the main import of our modeling is to explain the shape and evolution of the spectra and to derive an accurate first-order set of parameters, and not to determine precise values of the model inputs. In keeping with this philosophy some limitations were imposed on the range of input parameters to speed the model fitting. For example, we kept the path length to the source constant at $10^9$ cm, and have stayed with a fixed lower bound to the integration in energy. We took the angle between the magnetic field and the line of sight to be $60^\circ$ because the active region was located about $60^\circ$ from disk center and it is fair to assume that the dominant component of the magnetic field is radial. These choices affect the details of the emitted spectrum. We have given some indication of the errors in our fits in quoting the results.

**Cartoon Results**

To help illustrate the influence of the various parameters on the shape of the spectrum we give a number of pictorial demonstrations. In Figures 2.17 to 2.23 we show the results of varying the eight free parameters in the gyrosynchrotron model. In each case the left panel illustrates the behavior of the free parameter in the presence of strong Razin suppression ($\alpha = 0.320$) and the panel on the right shows the behavior when $\alpha = 1$, when there is negligible suppression.

Figure 2.17 shows the effect of varying the accelerated particle density, with and without the effect of Razin suppression. All other parameters, such
as magnetic field strength or the exponent in the energy distribution function, were kept constant. The accelerated particle density is one of the four dominant variables affecting the gyrosynchrotron spectrum, along with the Razin parameter $\alpha$, the exponent $\delta$ in the energy distribution function, and the magnetic field $B$. The figure shows, in the case without Razin suppression, that the peak brightness temperature ranges from about $10^8$ to about $2 \times 10^9$ K when the accelerated particle density is increased from $1 \, \text{cm}^{-3}$ to $10^7 \, \text{cm}^{-3}$. If Razin suppression occurs, with $\alpha = 0.320$ corresponding in this case to an ambient density of $2.6 \times 10^{11} \, \text{cm}^{-3}$, the change in peak brightness temperature is from $5 \times 10^5$ K to $2 \times 10^9$ K. The comparison of the two panels shows clearly the effect of Razin suppression: the low-frequency emission is suppressed, so that resonance structures are not apparent, and due to the cutoff on the low-frequency side of the spectrum, the frequency of maximum emission remains constant.

Figure 2.18 shows the effect of varying $\delta$, the index in the energy distribution function. In the absence of Razin suppression the peak of the curve moves toward higher frequencies and lower peak brightness temperature as the index is increased from 3 to 7. With Razin suppression the curves corresponding to lower indices are more affected, so that as $\delta$ increases, the curve initially moves up. Again, resonant structures are suppressed along with the low-frequency emission.

In Figure 2.19 $B$ is varied from 50 to 450 Gauss. Increasing $B$ shifts the peak of the curve to the right, because the harmonics of the gyrofrequency with high optical depth are shifted to higher frequencies, but the peak in the spectrum also shifts down to lower brightness temperature, because the efficiency of the emitting process is reduced. In the curves with
Razin suppression the peak frequencies are slightly lower than in the curves without, because the low-frequency side is cut off.

Figure 2.20 shows the effect of varying the high-energy cutoff in the energetic particle distribution from 0.32 MeV to 3.2 MeV. As the maximum energy level is increased the peak brightness temperature increases slightly, and the bandwidth of the curve is broadened from the optically thin side.

Figure 2.21 shows the minimal effect of varying the low-energy cutoff, which in fact was not varied in our modeling. Electrons with energy below the cutoff contribute to the thermal background population. In Figure 2.2 we show that the minimum energy may range from 0.01 MeV to 0.1 MeV with no appreciable effect on the gyrosynchrotron spectrum in the case of strong Razin suppression. In the case of $\alpha = 1$, eliminating the lower energy electrons caused the resonant structures at low frequency to disappear. This result suggests that the gyrosynchrotron emission between 6 and 16 GHz is not strongly dependent on the electrons with energy in the range 0.01 MeV to 0.1 MeV.

Figure 2.22 shows the effect of varying the path length to the source. This is a large effect, but in practice it is related to the size of the source and the accelerated particle density. Although we expect the optical depth to the source to increase as the source grows during the flare, experience suggests that growth in the source size is accompanied by an increase in the source density, i.e. the expansion is homologous and the increased optical depth cannot be disentangled from the increase in particle density without an independent measurement of the source size along the line of sight. We also note that for an increase in path length from $10^9$ to $10^{10}$ cm$^{-3}$ (and no variation in the accelerated particle density) the peak frequency shifts to higher
frequencies, unlike the behavior we see for an increase in the accelerated particle density in the case with Razin suppression, so that a simple expansion of the source without an increase in the density of energetic particles could not account for the observed behavior of the brightness temperature spectrum. For simplicity in the modeling, then, we have attached all of the increase in source size to the increase in accelerated particle density. We must keep in mind, however, that the choice of a different path length to the source would alter the results of our modeling. A path length smaller than $10^9$ cm, although unlikely based on typical observations, would mean that milder Razin suppression would be needed to fit the data.

Figure 2.23 shows the effect of varying $\theta$, the angle between the line of sight and the magnetic field. The effect is substantial if the angle is less than 60 degrees. However, we reiterate that we did not vary this angle in the modeling, and consider the active region longitude as a good representation of this parameter.

In Figure 2.25 we summarize the effects in one figure, with arrows showing the direction of evolution of the spectrum corresponding to an increase in each of the eight parameters. The arrows indicate direction only, not relative magnitude of the effect. As mentioned earlier, we found that, in addition to the very pronounced effect of Razin suppression, the most important parameters are the accelerated particle density and (tied for third place in the hierarchy of influence) the exponent $\delta$ and the field $B$. The figure shows that $\alpha$, $\theta$, and $N$ are the parameters capable of shifting the peak of the curve vertically, when Razin suppression is in effect. (Note that there are two arrows for $N$, as a reminder that the effect of $N$ varies with the magnitude of this parameter.) In the case of $\alpha = 1$, with less or no
Razin suppression, no single parameter shifts the curve purely in the vertical direction.

Results of the modeling

Table 2.3 lists the best fit parameters to the data, along with errors which are further discussed below. Figure 2.25 shows the results graphically. The data for the A, B, C, D, E and F time samples are overlaid with plots of the best model fits to each curve. Figure 2.26 shows the evolution of the model parameters at the six times during the flare.

The errors quoted were determined by varying each parameter in turn, in a range centered on its best value for that fit. For example, starting with the C time sample best-fit parameters, we varied $B$ and found that values from 320 to 390 Gauss led to acceptable fits. This procedure was repeated for each free parameter. The errors show that the 16.4 GHz data points did not constrain the bandwidth of the curve very well. This is expected because of the higher noise and difficulty of imaging the source at this frequency.

In modeling the data we concentrated on fitting the curves for the A and C time samples, because they represented extremes in the physical conditions required to reproduce the observations. For example, the spectrum with peak brightness temperature near $10^9$ K requires an accelerated particle density of at least $10^4$ cm$^{-3}$; there does not seem to be any other way to achieve such high brightness temperature. This is in line with what other researchers have found, e.g Kucera (1992) found $\sim 3 \times 10^3$ cm$^{-3}$ and Willson et al. (1990) found an accelerated particle density $\sim 4 \times 10^6$ cm$^{-3}$. Increasing the accelerated particle density beyond a certain point, however, does not increase the peak brightness temperature; a “saturation” effect is
Figure 2.17 Effect on the brightness temperature spectrum of varying the density of accelerated particles from $10$ to $10^7$ cm$^{-3}$. The left panel shows the effect with Razin suppression, and the right panel shows the effect without Razin suppression.
Figure 2.18 Effect on the brightness temperature spectrum of varying the exponent $\delta$ in the energy distribution function. The range of $\delta$ shown, from 3 to 7, is typical for microwave bursts. The left panel shows the effect with Razin suppression, and the right panel without.
Figure 2.19 Effect on the brightness temperature spectrum of varying the magnetic field strength $B$ from 50 to 450 Gauss. The left panel shows the effect with Razin suppression, and the right panel shows the effect without.
Figure 2.20 Effect on the brightness temperature spectrum of varying the high-energy cutoff in the electron energy distribution. The cutoff ranges from 0.32 MeV to 3.2 MeV. The left panel shows the effect with Razin suppression, and the right panel shows the effect without.
Figure 2.21 Effect on the brightness temperature spectrum of varying the low-energy cutoff in the electron energy distribution. The cutoff was varied from 0.01 MeV to 0.1 MeV with no appreciable effect on the spectrum. The left panel shows the effect with Razin suppression, and the right panel shows the effect without.
Figure 2.22 Effect on the brightness temperature spectrum of varying the path length to the source $L$. The effect is significant, but not distinguishable from varying the particle density or source size. The path length was kept fixed at $10^9$ cm in the modeling. The left panel shows the effect with Razin suppression, and the right panel shows the effect without.
Figure 2.23 Effect on the brightness temperature spectrum of varying the viewing angle to the magnetic field $\theta$. The effect is pronounced in the presence of Razin suppression. The viewing angle was not varied in the modeling as we consider it to be fixed, to first order, by the longitude of the source. The left panel shows the effect with Razin suppression, and the right panel shows the effect without.
Figure 2.24 Figure summarizing the effect of the physical parameters in the gyrosynchrotron model. The arrows indicate the direction the curve evolves when the variable is increased. The arrows indicate direction only, not relative magnitude of the effect. There are two arrows for N in the left panel because increasing N can shift the curve both ways, depending on the value of N.
seen. Having fixed the accelerated particle density, a turnover frequency of about 10 GHz requires a magnetic field strength of about 350 Gauss. The main problem with fitting the "C" time sample is to get high enough brightness temperature and narrow enough bandwidth. If $\alpha$ is too high, resonant structures appear, in which case the bandwidth is very large. In the case of the A time sample, the accelerated particle density must be reduced to fit the low brightness temperature spectrum, but the turnover frequency is then naturally low. The effect of Razin suppression is well demonstrated here, because while the higher-$T_b$ curves might be fit with low Razin suppression if we allow a large bandwidth, for the lower-$T_b$ spectrum, low Razin suppression leads to a low turnover frequency, which in turn would require a big boost in the magnetic field strength.

**Evolution of the Spectrum in the Absence of Razin Suppression**

In the absence of Razin suppression it is difficult both to fit the spectra and to find fits in which the dominant parameters ($B$, $N$, and $\delta$) vary in a credible way from one time sample to the next. In Figure 2.27 we show an example of fits to the A and C spectra for the case in which the medium has no effect ($\alpha = 2$). For the C spectrum the parameters were: $B = 400$ Gauss, $n_e(accel) = 1.2 \times 10^3$ cm$^{-3}$, $\delta = 4$, and $E_{max} = 3.16$ MeV. For the A spectrum, which was much more difficult to fit, the parameters were: $B = 700$ Gauss, $n_e(accel) = 1.0 \times 10^{-4}$ cm$^{-3}$, $\delta = 7$, and $E_{max} = 1$ MeV. The C model has too wide a bandwidth on the low-frequency side, but otherwise is not altogether unsatisfactory; on the other hand the A model does not have the right shape at all. To get the total emission down to the right level the accelerated particle density had to be dropped to extremely low
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<tr>
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<td>$+0.3/ -0.2 \times 10^{29}$</td>
<td>$+0.3/ -0.9 \times 10^{30}$</td>
<td>$+0.1/ -0.2 \times 10^{30}$</td>
<td>$+0.2/ -0.5 \times 10^{29}$</td>
<td>$+0.4/ -0.5 \times 10^{28}$</td>
</tr>
<tr>
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<td>$3.0 \times 10^4$</td>
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<tr>
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<td>4.0</td>
<td>4.0</td>
<td>4.0</td>
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<td>$+0.3/-0.5$</td>
<td>$+0.1/-0.2$</td>
<td>$+0.1/-0.1$</td>
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<td>$+40/-10$</td>
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</tr>
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</tr>
<tr>
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<td>$+0.369/-0.130$</td>
<td>$+0.995/-0.369$</td>
<td>$+0.000/-0.103$</td>
<td>$+0.585/-0.206$</td>
<td>$+99./-0.206$</td>
</tr>
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<td>0.319</td>
<td>0.312</td>
<td>0.310</td>
</tr>
<tr>
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<td>$+0.008/-0.015$</td>
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</table>
Figure 2.25 Overlay of the data for the A, B, C, D, E, F times with the model fits.
Figure 2.26 Evolution in time of the parameters of the model. Continued on the next page.
Figure 2.26 Evolution in time of the parameters of the model. Continued on the next page.
Figure 2.26 Evolution in time of the parameters of the model. Continued from the previous page.
levels \((1.0 \times 10^{-4}\text{cm}^{-3})\); then the magnetic field and the exponent in the energy distribution function were manipulated to put the optically-thin fall-off at approximately the right frequency. The difficulty of fitting the spectra is strong evidence that a low-frequency cutoff is needed, and that Razin suppression works very well to explain the shape and evolution of the spectra.

**Validity of the Simplifying Expressions of Dulk and Marsh**

We have shown that the effect of the medium alters the shape of the gyrosynchrotron spectra, not only on the low frequency side but also in reducing the brightness temperature overall. In this section we check the applicability of the expressions of Dulk and Marsh (loc cit.) when Razin suppression is at work. The equations to check (from section 2.3) are:

\[
T_{\text{eff}} \sim 2.2 \times 10^9 10^{-0.31\delta} (\sin \theta)^{-0.36-0.06\delta} \left(\frac{\nu}{\nu_B}\right)^{0.5+0.085\delta}
\]

and

\[
\nu_{\text{peak}} \sim 2.72 \times 10^3 10^{0.27\delta} (\sin \theta)^{0.41+0.03\delta} NL^{0.32-0.03\delta} B^{0.68+0.03\delta}.
\]

For \(\theta = 60^\circ\), \(\delta = 4\), \(B = 325\ \text{G}\), \(n_e\ (\text{accel}) = 1.4 \times 10^3\ \text{cm}^{-3}\), \(L = 10^9\ \text{cm}\), the expressions of Dulk and Marsh yield a peak frequency of 0.8 GHz, while in fact, with Razin suppression, the gyrosynchrotron model produces a peak frequency of about 7 GHz. Calculating the effective temperature at the peak frequency, the expressions of Dulk and Marsh give 7.7 \times 10^8\ K, while the same model gives a temperature of \(\sim 2 \times 10^7\ K\). The comparison shows that the effect of the medium is to produce an artificially high turnover frequency in the spectrum because the lower frequencies are suppressed.
Figure 2.27 Example of fits to the A and C spectra without Razin suppression. See text.
2.6 Discussion

We found that Razin suppression can solve the mystery of constant peak frequency in microwave flare spectra. The most important argument is that Razin suppression can explain not only an individual spectrum, but also the evolution in time of the spectra. Without Razin suppression we would have to vary several parameters (such as the magnetic field strength, the ambient density, and the index in the electron energy distribution function) in just the right way to keep the peak frequency constant (or nearly constant) while the brightness temperature varies over nearly two orders of magnitude. If Razin suppression is present, only the accelerated particle density is required to vary in order to change the peak brightness temperature. We do not rule out variations other than in the accelerated particle density, but we have shown that it is possible to model the evolution of the spectra with a simple variation in the accelerated particle density, once the remaining parameters (field strength, ambient density, electron energy distribution, etc.) have been chosen appropriately.

The ambient density required in our model ($\sim 2 \times 10^{11}$ cm$^{-3}$) is higher than commonly assumed in models of the microwave emission. The ambient density is generally thought to lie in the range from $10^8$ to about $3 \times 10^{10}$ cm$^{-3}$, although recently some authors have reported higher densities. For example Doschek (1994) used SXT and BCS data from Yohkoh (see section 1.8 and Chapter 3) to infer the temperature and density at the top of a flare loop and found $T \sim 10 - 20 \times 10^6$ K and $n_e \sim 3.2 \times 10^{11}$ to $1.2 \times 10^{12}$ cm$^{-3}$.

Benka and Holman (1992) also address the steep low-frequency slope and constant peak frequency in microwave burst spectra. They attribute the
steep low-frequency slope to thermal absorption and interpret the constancy of the peak frequency in terms of a model of electron heating and acceleration in current sheets. In this model the evolution of the microwave spectra with constant peak frequency can be accounted for by a changing electric field strength, but this result is so far only qualitative.

The Razin effect and the role of free-free and self-absorption in microwave sources has been investigated by Klein (1987) and his colleagues. Many of our results support his conclusions. He finds, as we do, that the Razin effect creates a steep low-frequency slope in the microwave burst spectrum when the ambient density is on the order of $10^{11}$ cm$^{-3}$. Our conclusions differ from his in that he requires a source size $l \sim 10^7$ cm and a density of accelerated particles near $10^9$ cm$^{-3}$, or about $10^{-2} \times n_e$(ambient). These differences may arise from Klein's interpretation of the millimeter flux data from which he inferred the column density of non-thermal electrons.

One limitation of our modeling is that we did not investigate the emission with a non-isotropic pitch-angle ($\phi$) distribution. Some investigations reported in the literature show that power is preferentially radiated in the direction $\phi = \theta$ ($\theta$ the angle between the magnetic field line and the line of sight), and that for large ($\phi - \theta$) the peak frequency and slopes of the spectra are affected (Tarnstrom, 1976).

In future work it would be useful to have polarization data and to make use of this diagnostic in modeling with Razin suppression.

In summary, the flare scenario implied by our results is that the microwave emission takes place in a plasma of high ambient density. The accelerated particle density increases and decreases along with the peak brightness
temperature during the flare, while \( \delta \) and other parameters stay roughly constant. The *accelerated* particle density we infer is in line with that found by several other researchers as noted in section 2.5.2. We do not require the accelerated particle density to be a significant fraction, more than 1 percent, of the ambient density, as required in the models of Kaufmann et al. (1986) or McClements and Brown (1986). The high turnover frequency in the spectrum is determined by the suppression of low frequencies known as the Razin effect.
CHAPTER III

FLARE OF JULY 16 1992: MULTISPECTRAL OBSERVATIONS

3.1 Introduction

In Chapter 2 we analyzed the microwave emission from the flare of July 16 1992 in detail. Modeling the gyrosynchrotron emission with a numerical code allowed us to determine flare parameters such as the ambient plasma density and the magnetic field strength in the flaring loop. In this chapter we examine data at other wavelengths to arrive at a comprehensive view of the flare. Where possible we also check the flare parameters derived from these observations with the results of modeling the radio spectra.

Our primary source of supporting data is the soft X-ray imaging telescope (SXT) on the Yohkoh spacecraft. (See section 1.8 for background information on the instruments referred to in this chapter.) We also make use of the Yohkoh hard X-ray images and lightcurves. Magnetograph observations from the Marshall Space Flight Center provide the context for this flare. Some low-resolution Hα images were available for reference from a small telescope located at Caltech and from the Marshall Space Flight Center; data from the Big Bear Observatory were not available because of damage to the site during the Landers-Big Bear earthquake of June 1992.

We find that the SXT and HXT data, in particular, shed light on the interpretation of the radio data. The plasma density which we infer from the intensity of soft X-rays emitted at the radio source location is in the range $2.0 \times 10^{10} \text{ cm}^{-3}$ to $3.3 \times 10^{11} \text{ cm}^{-3}$, which brackets the density inferred from the radio data in Chapter 2, $2 \times 10^{11} \text{ cm}^{-3}$. We also find that the flare at
16:56 UT was followed by at least 15 minutes of activity in soft X-rays, none of which had any counterpart in the microwave data.

### 3.2 Flare Location with Respect to Nearby Active Regions

The flare originated from a point between two large well-developed active regions, AR 7222 at 10° South, 47° West, and AR 7220 even closer to the West limb at 12° South, 67° West. Preliminary reports in the Solar Geophysical Data bulletins attribute the flare to one or the other active region, depending on the site. The MSFC magnetograms and the Yohkoh SXT images show that the flare region was closer to AR 7222. Figure 3.1 from MSFC shows sunspots of the two regions and the flare in Hα at a late stage in the flare, 16:58:49 UT, when the Hα emission was most complex. (See Figure 2.1 for a guide to the time history of the flare.)

We determine the position of the radio emission relative to the active regions by superimposing the contours of the microwave source on the white-light image from Yohkoh. The center of the Owens Valley radio images corresponds to the phase center of the array, and this location is known from the observing log. The vertical axis of the image is aligned with celestial (Earth) North and the right side of the image is to the West. The Yohkoh images, on the other hand, are created with solar North at the top (after removing the effects of spacecraft roll) and West at the right. To compare the images one takes into account the inclination of the solar rotation axis with respect to the plane of the ecliptic. On this date the solar P angle was 4.5 degrees, which significantly affects the offset between the two coordinate systems for sources near the solar limb.

Figure 3.2 shows a white-light image from Yohkoh, in both greyscale and contours, taken at 16:56:16 UT. The image covers an area 157 X 157
Figure 3.1  \( H_\alpha \) image from MSFC showing the \( H_\alpha \) emission in relation to the sunspots of AR 7222 (left) and AR 7220 (right).
arcsec (64 pixels on a side, at 2.45 arcseconds per pixel). A plus symbol is overlaid on this figure at the position of the center of the radio emission. The center of the radio source is about 25 arcseconds to the northwest of the leading spot of AR 7222. At 6.2 GHz the radio emission extends roughly 10 arcseconds in each direction.

No sunspots are visible at the location of the radio source in this low-resolution white-light image. Having determined the location of the radio source with respect to the active regions, however, we may look for spots in higher resolution images. Figure 3.3 is a white-light image from Sacramento Peak Observatory taken on July 16 before the flare. AR 7222 and AR 7220 are labelled A and B, respectively. Three small spots are visible to the north of AR 7222, and a penumbral region labelled C lies further to the north. By comparing the separations of the spots and the radio source in the white light images from Sacramento Peak and from Yohkoh, we found that the radio emission lies just to the south of the penumbral spot seen in the Sacramento Peak photograph.

3.3 Magnetograph Observations

Vector magnetogram observations were available from both the Marshall Space Flight Center (MSFC) in Huntsville, Alabama and the Mees Solar Observatory in Hawaii. The data consist of line-of-sight and transverse field measurements. Low-resolution $H_\alpha$ images during the flare were also provided by MSFC and were helpful in determining the flare location in the magnetograms.

The active region which produced the flare is close to the solar limb. This creates ambiguity in the field orientation. A field line which is nearly radial with respect to sun center has a small line-of-sight component when
Figure 3.2 Location of 11.2 GHz radio source (plus symbol) overlaid on white-light image from Yohkoh.
Figure 3.3 White-light image from Sacramento Peak Observatory, showing the small spots and penumbral fragments attending AR 7222.
viewed near the solar limb, and the direction of the field may be viewed as toward or away from the observer with only a small change in viewing angle near the limb. Thus while the large-scale pattern of magnetic field may be determined for both active regions, the orientation and strength of fields for AR 7220 are not reliably indicated.

Figure 3.4 is a magnetogram from MSFC taken about one hour after the flare. For clarity we show only the longitudinal, or line-of-sight data. The plot covers an area of approximately 200 by 200 arcseconds (about 27 percent larger than the area in the Yohkoh white-light image), centered between AR 7222 on the left (East) and AR 7220 on the right. The large-scale fields for both active regions follow the classical pattern for northern-hemisphere spots in solar cycle 22 of negative polarity in the leader spot and positive polarity in the trailing spot or spots. Positive polarity is shown by solid lines, and negative polarity by dashed lines. The positive-polarity regions of AR 7222 and AR 7220 reach maximum values of 2000 Gauss. The negative polarity region of AR 7222 has a maximum about 100 Gauss and that of AR 7220, about 500 Gauss. To the north of the positive-polarity region of AR 7222 is a small patch of negative polarity field of 10 Gauss. The error in the longitudinal component is on the order of one or two Gauss. The small region of negative polarity is approximately 65 arcseconds north of the leading spot of AR 7222, and therefore approximately 40 arcseconds north of the center of radio emission.

A comparison of the MSFC magnetograms on July 13 and 16 suggests that AR 7222 advanced toward AR 7220 more than can be explained by projection effects, and that the flare appears in the area of interaction of the two active regions (M. Hagyard, private communication).
Figure 3.4 Longitudinal Field plot from MSFC.
To understand the detailed structure of the flare with respect to the magnetic field environment we compare the longitudinal field and the $H\alpha$ emission from the MSFC instruments in Figure 3.5. The color of the background in 3.5 b indicates the polarity of the field, black for negative and white for positive. Note that the colors of the contours were chosen for visibility (black contours over white areas, and vice versa) and do not indicate polarity.

These images from 16:56:52 UT show that the core of the $H\alpha$ emission, corresponding to the location of the radio source and, as we shall see later, to the compact source seen in soft X-rays, overlays a region of positive longitudinal field. The northward "arm" of $H\alpha$ emission, which is also seen in soft X-rays, extends toward a negative-polarity spot intruding into the general area of positive polarity. The other arm of emission in $H\alpha$ and in soft X-rays which extends to the northwest appears to connect with a negative polarity area which is part of AR 7220. The picture which emerges from a comparison of the white-light photographs, radio images and magnetograms is that a loop formed between the penumbral region of positive polarity, with a field strength about 500 Gauss, and a small, possibly emerging region of negative polarity directly to the north. The radio emission came from the southern end of this asymmetric loop, where the field strength is higher. It would be interesting to know if the negative-polarity intruding spot in AR 7222 is an emerging flux region, but the necessary high-resolution data from Mees Solar Observatory in Hawaii is not available (data not taken) for comparison on previous days.
Figure 3.5  Panel on the left shows longitudinal field overlaid on Hα image from MSFC. Panel on the right shows the longitudinal field contours overlaid on a magnetogram showing positive polarity areas in white and negative in black.
3.4 Yohkoh Soft X-ray Images

The soft X-ray data for this event consist of a cyclical sequence of images taken through 3 filters which have different transmission functions over the range of solar temperatures observed. The filters used in this observing sequence were Al1, Al12, and Be119 (see section 1.8 for a more detailed description of the SXT instrument). The temperature response function of the SXT instrument is obtained by convolving the instrument response function (the effective area of the detector as a function of wavelength) with theoretical X-ray continuum and line spectra (Tsuneta et al., 1991). As illustrated in Figure 3.6 the response function of the telescope with the Al1 filter in place rises sharply above \( \log(T) \sim 6.2 \). With the Be119 or Al12 filter the signal is lower and peaks at higher temperature. In the case of the Al12 filter the sensitivity is greatest for \( \log(T) \sim 7.1 \) while with the Be119 filter in place, the sensitivity rises toward \( \log(T) \sim 8.0 \). A neutral-density filter which attenuates the signal uniformly throughout the band may be used along with the entrance filter to avoid saturating the detector when the intensity of the solar emission is high.

Before we examined the soft X-ray images we calibrated them using a standard program in the Yohkoh software package (the "sxLprep" procedure). Those images with a minimum level of data quality (images with maximum data number counts over 100, and with less than 10 percent of the data missing,) were co-registered and corrected for instrumental effects. The dark current in the detector and stray visible light from entrance filter pinholes were subtracted. The calibration procedure also keeps track of pixels that are saturated, and calculates the errors resulting from the fact that the
Figure 3.6 Response function of the Soft X-ray Telescope on Yohkoh with different filters in place.
output of the CCD camera is converted from 12 bits to 8 bits for telemetry, and restored to 12 bits with some small loss of information.

The images taken in any one of the filters around the time of the microwave burst show a compact region with areas of lower temperature emission extending particularly in two "arms" to the north and west of the compact source. Figure 3.7 shows the source in the Be119, Al.1, and Al12 filters. (For greater clarity only the inner 28 x 28 pixels of the original 64 x 64 image are displayed.) The images illustrate the morphology of the source as seen through different filters and at different times in the period from 16:55 to 16:58 UT when the radio emission was detectable. The source is almost twice as large in the first image, taken with the Al.1 filter at 16:55:28 UT, as in the third image, taken with the Al12 filter at 16:56:22 UT. This is primarily due to the difference in the response functions (as shown in Figure 3.6) but may also be due to time variability. The compact source seen in the Be119 filter data is about 10 arcsec (7500 km) in the north-south direction and 3 arcsec (2200 km) wide.

The sequence of (high-quality) SXT data for this flare begins at 16:55:18 UT, soon after the spacecraft emerged from its orbital night. This is less than one minute prior to the onset of the microwave burst, but the compact soft X-ray source is already very bright.

The temporal evolution of the SXT source may be summarized as follows. From the beginning of the SXT sequence to the time of the radio burst the source becomes more compact. This may indicate some heating of the region prior to the start of the SXT sequence, and a concentration of soft X-ray emission at the loop top just before the onset of radio emission. After the radio burst the soft X-ray emission extends to the north, forming a
Figure 3.7 Source morphology in soft X-rays at the time of the peak radio emission, in different filters: From left to right, top to bottom: Al.1 at 16:55:28 UT, Be119 at 16:55:38 UT, Al12 at 16:56:22 UT, and Al.1 again at 16:57:00 UT. In all SXT images presented, pixel sizes are 2.45 by 2.45 arcseconds.
loop approximately 40 arcsec in the North-South direction. Twenty minutes after the radio burst another loop is seen in the Al12 filter data (images were no longer recorded with the thick Be119 filter). This new loop appears both longer (50 arcseconds) and thinner than the previous loop, and may have formed higher in the atmosphere. It is not clear whether the new configuration consists of two North-South loops in series (similar to those shown in Strong et al., 1994) or whether the original compact source loop shares its southern footpoint with the larger loop, which terminates further to the North (a schematic drawing of this type of loop is given in Inda-Koida et al. 1994). About twenty minutes after the radio burst another loop is seen in the Al12 filter (images were no longer recorded with the thick Be119 filter). This new loop appears both longer and thinner than the previous loop, and may have formed higher in the atmosphere. This evolution is shown in Figure 3.8.

3.5 Comparison of Microwave and Soft X-ray Images

In Figure 3.9 we show the full-width half-maximum ellipse representing the 11.2 GHz radio source overlaid on the soft X-ray image from 16:56:12 UT. The figure shows that the radio source is located along the southern end or footpoint of the compact source, with its center displaced about 10 arcsec from the brightest point of the compact source. The radio source location remains constant during the period (less than one minute) when the radio flux was high enough to allow imaging.

It is not unusual for the position of the radio source to be displaced with respect to the center of the soft X-ray emission. Lim et al. (1994) and Wang et al. (1994) have shown that radio emission dominates the soft X-ray emission near the loop footpoints where the magnetic field is relatively high.
Figure 3.8 Evolution of the source in the Al.12 filter from the beginning of the SXT data sequence to the end of the flare episode. From top to bottom the times are: 16:55:40, 16:56:14, 17:02:02, and 17:16:44 UT.
Figure 3.9 Radio source (contour) overlaid on SXT source in the Be119 filter, at 16:56:46 UT.
The soft X-ray emission is linked to areas of higher density and arises near the loop top or, in the case of an asymmetric loop, from the footpoint with the lower field strength.

### 3.6 Temperature and Emission Measure Determinations from Yohkoh

The response functions plotted in Figure 3.6 vary slowly with temperature above $\log(T) \sim 6.5$, but the curves are sufficiently different in slope and amplitude that ratios of pair of curves provide sensitive temperature diagnostics in the range 1 to $50 \times 10^6$ Kelvin. Figure 3.10 shows the signal in the ratio of three pairs of filters. The ratio of intensities through the Be119 and Al12 filters is useful at temperatures above $\log(T) \sim 7.5$, while the ratio of the AlMg and Al1 intensities is more appropriate for lower temperatures. The temperatures derived from the filter ratios are accurate only for the case of an isothermal plasma. For isothermal sources the uncertainty in the temperature is about 0.1 in the logarithm.

The plasma density cannot be determined directly from the image intensity. Instead, the quantity obtained is the emission measure (EM), defined as

$$ EM = \int n_e^2 \, dV $$

where $n_e$ is the electron density and $V$ is the volume of the X-ray emitting plasma along the line of sight. The temperature response function of Figure 3.6 is the basis for determining the flux and hence the emission measure empirically. The flux from an isothermal plasma is proportional to the emission measure multiplied by a function $f$ which depends on the temperature and
Figure 3.10 Temperature sensitivity of three filter ratios for the SXT instrument.
wavelength of observation (Thomas et al., 1985). The function $f(T, \lambda)$ contains factors related to the processes forming the relevant emission lines and the ionization balance in the plasma; it is essentially the theoretical spectrum of an isothermal plasma normalized to unit emission measure.

We estimate the density by assuming that the volume $V$ of the source is given by the source area $A$ in pixels times the depth $L$. We have no direct measurement of $L$, but for sources near the limb, such as ours, the width of the source is a reasonable estimate. We thus take $L = 10^9$ cm, corresponding to the width of the soft X-ray source where the intensity is greatest. One pixel in an SXT image corresponds to an area $3.24 \times 10^{16}$ cm; the volume is then

$$V = N_{pix} \times 3.24 \times 10^{25} \text{cm}^3.$$  \hspace{1cm} (3.2)

We used the combination of Be119 and Al12 filters to determine the temperature and emission measure and to infer the density during the times of enhanced radio emission, i.e. 16:55 to 16:58 UT. The temperature and emission measure may be found for the integrated source or for subregions, known as regions of interest or ROI, at different locations in the SXT image. In the image taken at 16:55:22 UT we selected five regions of interest, one being the source as a whole, encompassing 64 pixels or $2 \times 10^{18}$ cm$^2$ on the sun, and the four smaller, 4-pixel areas at the loop top and along the narrower extensions of the source to the south. These ROI were applied to the sequence of data in the Be119 and Al12 filters to determine the temperature, emission measure, and inferred density as a function of time.

The ROI are shown in Figure 3.11. The results of the temperature and emission measure determinations, as well as the lower left and upper
right coordinates of the ROI, are shown in Table 3.1. (The coordinates are those of the nearest pixel edge; the box extends half a pixel on each side of the given coordinate.) An illustration is given in Figure 3.12 of the time sequence of temperature and density in the ROI called "Box 3" in the table. For the integrated source the average temperature ranges from $8.4 \times 10^6$ K to $2.1 \times 10^7$ K and the average density ranges from $3.4 \times 10^{10}$ to $1.8 \times 10^{11}$ cm$^{-3}$. For the smaller ROI along the southern leg of the loop, nearer to the center of radio emission, the density ranges from a low of $2 \times 10^{10}$ cm$^{-3}$ to a high of $3.3 \times 10^{11}$ cm$^{-3}$. The high values occurred in each case near 16:57 UT, about one minute after the time of the peak radio emission.

We may have underestimated the electron density in the smaller ROI because our model loop is of uniform depth or thickness, whereas in reality the loop may become thinner at the footpoints. If the thickness of the loop were a factor of 2 less in the smaller ROI, the derived density would be 40 percent higher, since $n_e \propto L^{-1/2}$.

3.7 Implications of Temperature and Emission Measure Results

The range of ambient density derived from the soft X-ray data is in good agreement with the ambient density inferred in Chapter 2 of $2 \times 10^{11}$ cm$^{-3}$.

The high temperatures found in the area of the microwave source support another of our conclusions from Chapter 2, i.e. that free-free absorption cannot account for the low-frequency cut-off in the microwave spectrum. The opacity of free-free emission for temperatures above $2 \times 10^5$ K is as follows (Dulk 1985):

$$\kappa_{ff} = 9.78 \times 10^{-3} \frac{n_e}{\nu^2 T^{3/2}} \sum_i Z_i^2 n_i (24.5 + \ln T - \ln \nu)$$

(3.3)
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<th>No. pixels</th>
<th>$T_e$ range (Kelvin)</th>
<th>$n_e$ range (cm$^{-3}$)</th>
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<td>64</td>
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<td>$7.7 \cdot 10^6$-$3.8 \cdot 10^7$</td>
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<tr>
<td>Box 3</td>
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<td>$7.8 \cdot 10^6$-$2.8 \cdot 10^7$</td>
<td>$4.0 \cdot 10^{10}$-$1.3 \cdot 10^{11}$</td>
</tr>
<tr>
<td>Box 4</td>
<td>(28,33) (29,34)</td>
<td>4</td>
<td>$8.0 \cdot 10^6$-$4.3 \cdot 10^7$</td>
<td>$2.0 \cdot 10^{10}$-$8.4 \cdot 10^{10}$</td>
</tr>
</tbody>
</table>
Figure 3.11 Region of Interest (ROI) locations for the determination of temperature and emission measure from the SXT images. Box 1 is closest to the center of the source. Axes are in units of arbitrary pixel number.
Figure 3.12 Time sequence of density and temperature in the region of interest called "Box 3" in Table 3.1.
where the sum is taken over elements $i$ to give the total density of electrons from hydrogen, helium, and higher-Z elements. The optical depth $\tau_{ff} = \kappa_{ff} L$ where $L$ is the depth of the source. Taking $n_e \sum_i Z_i^2 n_i = n_e^2$ and substituting the SXT-derived temperatures and densities for the radio source, we find that the optical depth at 10 GHz ranges from 0.17 to 0.30. At lower frequencies we might therefore expect some free-free absorption, but not enough to account for the very steep low-frequency slopes in the observed spectra. We saw in Chapter 2 that the disparity between the observed 10 GHz brightness temperature and that derived theoretically without Razin suppression is very large.

The thermal content of the electrons in the plasma is given by

$$E_e = \frac{3 EM k_B T}{n_e}.$$ \hspace{1cm} (3.4)

The thermal content of the plasma, derived from the range of temperature and emission measure for the integrated source and assuming equal electron and positive ion temperatures, is $2.4 \times 10^{29}$ to $3.2 \times 10^{30}$ erg.

The presence of widespread high-temperature, high-emission-measure areas before the onset of the radio emission, and before the start of the Yohkoh sequence, suggests that there may have been flaring in soft X-rays prior to 16:55 UT. This activity could have raised the temperature and density in the loops before the time of the microwave burst. This also provides a likely explanation for the high density medium and the suppression of the microwave source inferred in Chapter 2.
3.8 Hard X-ray Images

The Yohkoh Hard X-ray Telescope (HXT) forms images in four energy bands. They are the Low or L channel, 14-23 keV; the Medium 1 or M1 channel, 23-33 keV; the Medium 2 or M2 channel, 33-53 keV, and the High channel, 53-93 keV. We created HXT images in these bands with the same scale and with the same center as the SXT images by converting the SXT pointing coordinates to the HXT reference. (The conversion must be done for each time frame separately, because the offset between the centers of the HXT and SXT images is time-variable.) We re-calibrated the SXT images without removing the spacecraft roll angle, because that is the same for both telescopes.

Overlays of the SXT and HXT images are shown in Figures 3.13 (a)-(e). For clarity, the SXT images are greyscale images that have been inverted so that the darkest pixels correspond to the highest counts. The HXT data are plotted as contours. In Figure 3.13 (a) and (b) we show, respectively, the low and high channel HXT data from 16:55:36 and 16:55:34 UT, before the peak of the microwave fluxes, overlaid on the SXT image from 16:55:22 UT. Figures 3.13 (c) and (d) show the overlays for the HXT L and H-band data from 16:55:56 UT and the SXT data from 16:56:12 UT. This time is near the peak of the radio flux. The third image shows the low-channel HXT data from 16:58:06 UT and the SXT image from 16:58:06 UT. In all cases the HXT contours are plotted at 20, 40, 60, 80, and 99 percent of the source maximum.

In the first pair of images we see that the hard X-rays from the lowest energy channel lie directly over the maximum soft X-ray emission, although the L-band X-rays are more compact and do not show any loop structure.
Figure 3.13 (a) (top panel) and (b) (bottom panel). Hard X-ray source before the peak of the radio emission. Top panel: L-band hard X-ray image from 16:55:36 UT overlaid on an SXT image from 16:55:22 UT. Bottom panel: H-band image from 16:55:34 UT overlaid on the same SXT image.
Figure 3.13 (c) (top panel) and (d) (bottom panel). Hard X-ray source near the time of peak radio emission. Top panel: L-band hard X-ray image from 16:55:56 UT overlaid on an SXT image from 16:56:12 UT. Bottom panel: H-band image from 16:55:56 UT overlaid on the same SXT image.
Figure 3.13 (e) Hard X-ray source after the peak of the radio emission. L-band hard X-ray image from 16:58:06 UT overlaid on an SXT image from 16:58:06 UT. (No H-band image.)
The hard X-rays from the high channel form a compact source which is displaced to the left (east) of the SXT loop by about 1 pixel, or 2.45 arcsec, and is slightly elongated in the northwest-southeast direction. The apparent displacement to the east can be interpreted as a difference in height of the HXT and SXT emission, because of the high viewing angle to the source. The offset implies that the HXT emission came from an altitude about 2000 km below the main source of soft X-rays.

The second pair of images is similar to the first. The SXT source has become more compact since the time of the first pair of images.

In the third image the L-channel X-ray source fills the SXT loop region, and in addition there is a source to the northeast (upper left) of the compact source region. The high-energy source was too weak to image at this time.

The sources we see here are typical in many ways of the flare loops that have been studied in the Skylab and Yohkoh databases. In impulsive flares the L-band hard X-ray emission is similar to the loop-like SXT source, possibly an indication of contamination, at this low energy, from thermal emission of plasma at about $2 \times 10^7$ Kelvin (Kosugi 1994). At higher energies, corresponding to emission from nonthermal particles, the sources break up into smaller patches. In the highest channel, compact sources are seen at one or both footpoints of the loop (e.g. Kosugi 1994, and references therein).

Recently, hard X-ray sources have been found above the tops of soft X-ray loops (e.g., Masuda et al., 1994). The source found by Masuda was located about 10 arcseconds above the apex of a soft X-ray loop and varied rapidly on a timescale less than a few tens of seconds. The variations resembled those seen in the footpoints. The significance of these so-called
“loop-top impulsive sources” is that they may be visible manifestations of shock heating of the plasma resulting from the reconnection of coronal current sheets above the loop. We do not see a loop-top impulsive source, but the conditions are not right to detect such a source in our data. The loop-top impulsive sources occur for a short time during the rise phase in the flare, when we cannot create reliable images because we lack data prior to the flare for background subtraction.

3.9 The Hard X-ray Spectrum

We used data from the four energy channels of HXT to investigate the energy spectrum of the radiating electrons, in order to compare the spectral index derived from the hard X-rays with that derived from the radio emission. We worked with the integrated hard X-ray flux, not spatially resolved kernels of emission. This approach is justified by the proximity of the microwave and hard X-ray sources, and by the fact that the hard X-ray source is a simple, single source until well after the peak of the radio emission.

We found that the hard X-ray spectrum from the four channels of data could be fit with a single power-law distribution of energetic electrons from 16:55:28 to 16:55:56 UT, which is the time period of interest. After 16:55:56 UT the signal in the highest energy channel is low and introduces a greater uncertainty in the fit. A single power-law does not fit the data very well after 16:55:56 UT, but more data points would be required to fit to a thermal or thermal plus power-law spectrum.

The energy spectrum is calculated using the Yohkoh software procedure “hxt_fsp” which removes the instrumental response function from the distribution of counts in each of the four energy channels. The background signal level, to be subtracted from the data, had to be taken after the flare,
because the signal in the highest energy channel was already rising at the beginning of the data sequence. The background was taken from 17:04:30 to 17:04:34 UT.

The derivation of the electron energy spectrum from the observed photon spectrum can be made on the basis of models of the electron acceleration and deceleration in the medium. The simplest model, and that most often invoked to explain the hard X-ray emission, is the so-called thick target model (e.g., Brown 1971, Lin and Hudson 1976). In this model the energetic electrons rapidly lose all their energy in one or more collisions when they reach the lower corona or chromosphere. This is in contrast to the thin-target model in which the electrons lose a negligible part of their energy through collisions, the lower-energy electrons losing their energy first because the collisional loss time increases with particle energy.

Brown (1971) derived the deconvolution of the electron energy spectrum from the photon spectrum. The photon energy distribution is written:

$$\frac{dN_e}{dE} = A \left(\frac{E}{E_m}\right)^{-\gamma} \text{photons cm}^{-2} \text{s}^{-1} \text{keV}$$

(3.5)

where \(A\) and \(\gamma\) are constants determined from the fit to the observed photon spectrum and \(E_m\) is a reference energy level. The electron spectrum, on the other hand, is written:

$$I(E) = BE^{-\delta} \text{electrons keV}^{-1} \text{s}^{-1}$$

(3.6)

where

$$B = AE_m^\gamma \times 10^{33} \gamma(\gamma - 1)^2 b(\gamma - 1/2, 1/2)$$

(3.7)

and

$$\delta = \gamma + 1.$$

(3.8)
The function $b$ is the beta function,

$$b(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m + n)}.$$  \hspace{1cm} (3.9)

The flux of electrons with energy greater than the assumed lower cutoff $E_0$ in the distribution is

$$\frac{B}{\gamma} E_0^{-\gamma} \text{ electrons s}^{-1} \hspace{1cm} (3.10)$$

while the power flux is

$$\frac{B}{(\gamma - 1)} E_0^{-\gamma + 1} \text{ keV s}^{-1}.$$

The power flux is multiplied by the time interval to get the energy flux.

As shown in the second column of Table 3.2, the index in the photon spectrum varied from 2.6 to 3.7. Table 3.2 lists, for 2-second time intervals during the peak of the radio emission, the photon spectral index and statistical error from the fit, the electron flux, and the energy flux calculated with low-energy cutoffs of 20 and 30 keV, respectively. The sum of the energy released during the tabulated period is $4 \times 10^{29}$ to $10^{30}$ erg, depending on whether the low-energy cutoff is taken at 20 or 30 keV. As expected, this is roughly in agreement with the thermal content of the plasma inferred from the temperature and emission measure in section 3.7, $2.4 \times 10^{29}$ to $3.2 \times 10^{30}$ erg. The energy released in the hard X-ray emission is also of the order of the energy that would be available from the reconnection of 300 Gauss magnetic fields in a volume of about $2 \times 10^{27} \text{ cm}^3$, which is the volume of the integrated source in Figure 3.11.

The total number of energetic electrons, from summing the numbers in the third column of Table 3.2, is $3 \times 10^{37}$. In Chapter 2 we determined
that the total number of accelerated electrons during the time of peak radio emission was $3 \times 10^{30}$. Thus the fraction of electrons which are trapped and produce gyrosynchrotron emission is on the order of one electron in $10^7$.

Equation 3.5 shows that the electron spectral index is one greater, (i.e., the spectrum is softer) than the photon spectral index in the thick-target model. Thus between 16:55:28 and 16:56:00 UT the electron spectral index derived from the hard X-ray spectrum ranges from 3.6 to 4.7. These values agree very well with the electron spectral index of $4 (+0.3, -0.5)$ inferred from the microwave spectrum.

### 3.10 Hard X-ray and Radio Time Profiles

In Figure 3.14 we show the total power lightcurve at 11.2 GHz plotted on the same axes as the hard X-ray lightcurve from each of the four energy channels. In each case we have scaled the microwave peak to the value of the X-ray lightcurve at the time of that peak, to make the comparison easier.

The microwave emission is nearly as impulsive as the emission in the 53-93 keV band, but the peak in the microwaves is delayed by about 30 seconds with respect to the hard X-ray peak, and the decay of the microwave emission is slower. The microwave lightcurve drops to the background level from the peak in about one minute, whereas the hard X-ray lightcurve in the highest energy channel decays in about 30 seconds.

In chapter 2 we found that the microwave emission increased and decreased along with the density of accelerated electrons. We also found that the gyrosynchrotron spectrum was insensitive to electrons with energy much less than about 100 keV. We might then expect the microwave lightcurve to more closely resemble the lightcurve of the X-rays from the 53-93 keV channel (and resemble less the lightcurves of lower energy X-rays). In fact,
Table 3.2

<table>
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<th>Time Interval UT</th>
<th>spectral index electrons s^{-1} electron flux 20-500 keV</th>
<th>E (erg s^{-1}) 20-500 keV</th>
<th>E (erg s^{-1}) 30-500 keV</th>
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<td>3.24</td>
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<td></td>
<td>$\pm 0.03$</td>
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<tr>
<td>16:55:30-32</td>
<td>3.26</td>
<td>$3.8 \times 10^{35}$</td>
<td>$1.54 \times 10^{28}$</td>
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<tr>
<td></td>
<td>$\pm 0.03$</td>
<td></td>
<td></td>
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<tr>
<td>16:55:32-34</td>
<td>3.01</td>
<td>$4.5 \times 10^{35}$</td>
<td>$1.70 \times 10^{28}$</td>
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<td>$\pm 0.02$</td>
<td></td>
<td></td>
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<tr>
<td>16:55:34-36</td>
<td>2.78</td>
<td>$4.9 \times 10^{35}$</td>
<td>$1.77 \times 10^{28}$</td>
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<td></td>
<td>$\pm 0.01$</td>
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<td>2.58</td>
<td>$4.6 \times 10^{35}$</td>
<td>$1.61 \times 10^{28}$</td>
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<td>$1.82 \times 10^{28}$</td>
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<td>$3.65 \times 10^{28}$</td>
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<td>$\pm 0.02$</td>
<td></td>
<td></td>
</tr>
<tr>
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<td>$1.0 \times 10^{36}$</td>
<td>$4.30 \times 10^{28}$</td>
</tr>
<tr>
<td></td>
<td>$\pm 0.02$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16:55:50-52</td>
<td>3.08</td>
<td>$1.0 \times 10^{36}$</td>
<td>$3.86 \times 10^{28}$</td>
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<td>$\pm 0.01$</td>
<td></td>
<td></td>
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<tr>
<td>16:55:52-54</td>
<td>3.03</td>
<td>$1.0 \times 10^{36}$</td>
<td>$3.87 \times 10^{28}$</td>
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<td>$\pm 0.01$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16:55:54-56</td>
<td>3.39</td>
<td>$1.2 \times 10^{36}$</td>
<td>$5.10 \times 10^{28}$</td>
</tr>
<tr>
<td></td>
<td>$\pm 0.02$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16:55:56-58</td>
<td>3.47</td>
<td>$1.3 \times 10^{36}$</td>
<td>$5.47 \times 10^{28}$</td>
</tr>
<tr>
<td></td>
<td>$\pm 0.02$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16:55:58-60</td>
<td>3.68</td>
<td>$1.5 \times 10^{36}$</td>
<td>$6.77 \times 10^{28}$</td>
</tr>
<tr>
<td></td>
<td>$\pm 0.02$</td>
<td></td>
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</tbody>
</table>
Figure 3.14 Radio total power lightcurve at 11.2 GHz (open diamonds) scaled to the four hard X-ray counting rate lightcurves of HXT. From top to bottom, these hard X-ray lightcurves represent HXT channels: L (14-23 keV), M1 (22-33 keV), M2 (33-54 keV), and H (53-94 keV).
the microwave lightcurve does not closely resemble the behavior in any of the hard X-ray channels. However, the population of electrons radiating via gyrosynchrotron emission and via collisional bremsstrahlung may not be the same. The radio emission may peak at a later time and decay more slowly than the hard X-ray emission if those electrons producing radio emission have larger pitch angles and are trapped in the loop. The collisional energy loss time (Aschwanden and Schwartz, 1995)

\[ t^{\text{Coll}}(n_e) = \frac{e[eV]^{3/2}}{2.91 \cdot 10^{-6}\ln \Lambda n_e}, \]

where $e$ is the average electron energy and $\ln \Lambda$ is the Coulomb logarithm, implies that electrons with energy between 100 and 300 keV can be trapped on the order of tens of seconds to minutes if the density in the trapping area is on the order of $10^{10}$ to $10^{11}$ cm$^{-3}$ and the temperature is $10^7$ to $10^8$ K.

The comparison of the microwave lightcurve and the HXT L-band lightcurve is interesting because it shows a second peak in the L-band, reaching a maximum at about 16:58 UT, which has no counterpart in the microwave radiation and only minor counterparts in the other energy channels. The lack of microwave counterpart is not surprising considering the low energy of the particles showing this second peak. Furthermore, we believe that the microwave spectrum was highly suppressed as a result of the dense medium, so it is likely that any further increase in the ambient density, as implied by the rise in L-band emission and the continued high emission measure in soft X-rays, would quench any associated gyrosynchrotron emission entirely. The second peak in L-band, which was seen filling the soft X-ray loop area in Figure 3.13 c), is probably due to heating of the plasma in response to the flare two minutes earlier. In this scenario thermal conduction
results in evaporation of chromospheric material into the loop (Antonucci et al., 1984).

3.11 Summary and Conclusions

The flare came from a compact region to the northwest of AR 7222. The main source of X-rays was a loop approximately 40 arcseconds long, oriented in the north-south direction, although magnetic loops connecting to other elements of AR 7222 to the north and to elements of AR 7220 to the west were excited as well. These secondary loops were seen primarily in H$_\alpha$ and give the H$_\alpha$ flare a U-shape extending 30 to 40 arcseconds in each arm.

The microwave source is close to the main north-south loop seen in soft X-rays, but is displaced a little to the south of the most intense soft X-ray emission, near the footpoint with the highest field strength (about 500 Gauss at the photospheric level). This is in agreement with recent work comparing OVRO and SXT source locations, which show the radio emission concentrated near the footpoint with the strongest photospheric field, and soft X-ray emission dominating at the loop top and in regions of lower magnetic field.

The pattern seen in hard and soft X-rays is typical of many impulsive flares. The highest energy X-rays form compact sources at the loop footpoints, while lower-energy hard X-rays slowly fill the loop structure delineated by the soft X-ray emission.

The main result of this chapter is that we have found agreement between the values of the ambient density $n_e$ and electron spectral index $\delta$ determined from analyses of the radio emission and hard and soft X-ray emission. In the previous chapter we found that a high ambient density leads to Razin suppression and provides a viable and satisfactory explanation for the
shape and evolution of the microwave spectrum. Without a direct measurement of the path length to the source or the viewing angle to the field in the region of microwave emission we could not place tight constraints on the ambient density; however we showed that an ambient density of $2 \times 10^{11}$ cm$^{-3}$, coupled with reasonable assumptions for the path length and viewing angle, led to excellent fits to the data. The density inferred from the soft X-ray emission is an independent measure and brackets the density found from the analysis of the microwave emission: $2 \times 10^{10}$ cm$^{-3} < n_e < 3 \times 10^{11}$ cm$^{-3}$. The electron spectral index derived from the hard X-ray data, assuming a thick-target model, is in the range 3.6 to 4.7, while the spectral index derived from the analysis of the radio data is 3.5 to 4.3.

A comparison of the total number of energetic electrons producing the microwave emission and those producing the hard X-ray emission indicates that a small fraction, or 1 in $10^7$ electrons, are trapped in magnetic loops producing the microwave emission, while the rest precipitate directly from the acceleration region to the chromosphere at the base of the loop footpoints.
Chapter IV

INTERFEROMETRY OF THE SOLAR LIMB AT 3 MILLIMETERS


We observed the solar limb at the point of first contact during the eclipse of 1991 July 11, with a spatial resolution $\sim 1''.6$. The observations were carried out at the Owens Valley Radio Observatory millimeter interferometer, operating at 3 mm (99.46 GHz, with a bandwidth of 350 MHz). (Section 1.8 contains background information on the Owens Valley Millimeter Array.) The visibility amplitude and phase (see section 1.6 on radio interferometry) were modeled to yield the height of the 3 mm limb above the visible photosphere, and the data were differentiated to yield the brightness profile of the limb in strips $\sim 1''.6$ wide. We found that the 3 mm limb extends $7''.5 \pm 0.'8$ or 5500 km above the visible photosphere, with no evidence of a limb spike. This result, and the overall shape of the limb profile, are similar to the interferometric results of Wannier, Hurford, and Seielstad (1983) obtained with the same instrument but without the benefit of an eclipse. The 3 mm limb, at a temperature of $\sim 6500$ K, extends to altitudes far beyond the expected location of the transition region in the model of Vernazza, Avrett, and Loeser (1981). (See section 1.3 for an overview of the solar atmosphere.) A comparison of the 3 mm profile and an off-band $H_\alpha$ photograph of the
limb reveals a close correspondence between the 3 mm limb and the height of Hα spicules.

4.1 Introduction

On 1991 July 11, during a partial solar eclipse at Owens Valley, we determined the brightness temperature profile of the solar limb at a wavelength of 3 mm (99.46 GHz, $\Delta \nu = 350$ MHz), with $\sim 1''$ resolution. High-resolution observations of the solar limb in the millimeter wavelengths are of interest in constructing models of the lower atmosphere. In particular, we seek to elucidate the role of inhomogeneities or features such as spicules, which may dominate the temperature profile at the limb while playing a small role near disk center.

The 3 mm brightness is due to free-free emission from the chromosphere above the temperature minimum region. This belief is based on the density and temperature of the so-called upper chromosphere in semiempirical models such as the VAL (Vernazza, Avrett, and Loeser 1981); even if the vertical scale of the models is uncertain, free-free emission is the most likely because hydrogen ionization sets in at a height of about 1200 km above the visible photosphere and ensures that free-free interactions are the dominant source of opacity for this and higher altitudes, and through the millimeter band. The optical depth of the corona is negligible.

While the emission mechanism pertaining to our observations is well understood, the interplay between atmospheric structures and viewing geometry is not. If the chromosphere were spherically symmetric, uniform, and horizontally stratified, we would see limb brightening for wavelengths above 200 $\mu$m because of the temperature rise with altitude. At 3 mm, smooth-chromosphere models predict not just radial brightening toward the limb but
also a limb spike, for as we move off the limb optical depth is provided by material at transition region temperatures. Many previous millimeter and sub-millimeter studies, however, fail to show the predicted limb brightening or limb spike, and many authors have argued that large-scale or small-scale features introduce such important variations in the temperature and density structure that limb profiles cannot be predicted with models that preserve spherical symmetry and hydrostatic equilibrium (see, for example, Simon and Zirin 1969; Lindsey and Hudson 1976). Here we attempt to identify the features that evidently dominate the brightness temperature profile at the extreme limb.

Section 4.2 describes the eclipse geometry and our observing procedure. In section 4.3 we discuss the analysis and results: First we explain what we can learn from the simple record of the occultation as a function of time, and then we derive the brightness profile of the limb averaged over narrow strips in the sky. In section 4.4 we place our results in the context of other findings in the millimeter and submillimeter bands, and in section 5 we summarize.

4.2 Viewing Geometry and Observing Procedures

Figure 4.1 shows the path of the Moon with respect to the Sun during the eclipse as seen from the Owens Valley Radio Observatory (north is at the top of the figure and west at the right.) The relative size and approximate location of the primary beam of the Owens Valley (OVRO) 10 m dishes is also shown. The interferometer tracked a point 10" above the photospheric limb during the first contact. Although the eclipse as viewed from Owens Valley was a partial one, so that the motion of the Moon had a nonradial component with respect to Sun center, the motion of the Moon within the
beam was predominantly radial and successive intervals of time corresponded to a linear progression of the lunar limb in radial distance. The geometry of the observing conditions was also simplified by the fact that the solar limb has negligible curvature within the 89″ FWHM size of the main beam. The distance between the limb and the corresponding chord across the field of view was, at worst, 1″, which is about the scale of roughness of the lunar limb due to mountains.

We prepared for the eclipse observations by inserting a 6 dB “Bakelite” absorber in the RF signal path. The calibrator sources 3C 273, Mercury, and Mars were observed through the absorber. We also configured the three antennas of the array to create suitable fringe position angles for observing the first and fourth contacts. The desired fringe position angle is such that the fringes are parallel to the common solar/lunar tangent, as this maximizes the spatial resolution in the radial direction. Furthermore, with this orientation of the fringes the solar source looks like a step function, and the effect of tracking jitter is much less than in the case of single-dish observations; a single dish sees a source with a steep spatial brightness gradient, so that small shifts in pointing have a significant effect on the flux, while the interferometer, insensitive to the large-scale flux in the step function, responds to tracking jitter in a way that is similar to the response to a point source at the center of the beam. For the observation of first contact the angle between the fringes and the solar tangent was only 1°.6, although the main beam took in 5° of arc in position angle along the perimeter of the Sun, so that near the north and south edges of our field of view the angle was 0.9 and 4.1, respectively. The two 10.4 m antennas that formed optimum fringes
Figure 4.1 Path of the Moon with respect to the Sun during the eclipse (north is at the top of the figure and West to the right). The relative size of the main beam is shown by a circle. The interferometer tracked a point 10" above the photospheric limb.
for first contact were located at 20 m north of the array reference position and 50 m west. At the time of the eclipse the fringe spacing was 13''.5.

We supplemented the OVRO observations with Hα observations made at Big Bear Solar Observatory (BBSO) and used data from the Normal Incidence X-ray Telescope (NIXT), provided by Leon Golub, to judge the coronal activity level around the point of first contact. The NIXT rocket flight occurred only a few minutes before our observations, and from its images we find that the point of first contact is exceptionally quiet, with a low coronal brightness off the limb.

At OVRO, we recorded the visibility amplitude and phase from all three baselines (including the two for which the fringes were not parallel to the solar/lunar tangent) with an integration time of 3.24 s. Some uncertainty was introduced by the fact that transient system errors, such as a loss of phase-lock in one of the receivers, suspended the collection of data. This means that more than 3.24 s of elapsed time was required to obtain 3.24 s of integration time. We have no way to identify the period or periods during the sample interval when the 3.24 s of integration occurred. To understand the effect of the noncontinuous integration, we worked with two versions of the data (later folded together), one in which the assignment of a time to a sample was based on the assumption that all the integration occurred at the beginning of the sample interval, and the other, the end of the sample interval. Since the flux generally decreased with time as the occultation progressed, the assumption that all the integration occurred at the beginning of the sample interval tends to underestimate the flux at the assigned time, while the assumption that all the integration occurred at the end tends to
overestimate the flux. The two versions of the data, which we call “early-sampled” and “late-sampled,” thus represent the two extreme cases, which determine the amplitude errors indicated on our plots.

The spatial resolution was degraded by the extra long sampling intervals, regardless of the fact that the total length of integration was always 3.24 s. During the course of our observation the Moon moved at a rate of 0.40 s\(^{-1}\), so the resolution, based on the best sample interval of 3.24 s, is 1".3. The spatial resolution that pertains around the time of first contact is about 1".6 and certain portions of the disk were seen with a resolution of 2".7. These variations complicated our analysis, but even in the worst case our resolution was better than has been obtained with previous observations near the same wavelength.

We will show in the analysis that the most significant source of uncertainty in our limb profiles is due not to the errant integration times but to atmospheric fluctuations and receiver noise.

4.3 Analysis And Results

4.3.1. The Occultation Record

Figure 4.2 (top and bottom panels) shows the visibility amplitude and phase of the occultation record. We emphasize for clarity that the plot shows correlated amplitude, not total power. The interferometer phase, shown in the bottom plot, generally acts as a position indicator for the dominant source in the field of view; in this case, the interferometer tracked a fixed point near the solar limb, and the initial phase represents the position of the sharp fall-off in the limb brightness. As the moon occults the solar limb, the
sharpest fall-off in brightness occurs at the lunar limb so the phase changes to reflect the changing position of the lunar limb.

The behavior of the amplitudes and phases can thus be summarized as follows. In the precontact period (to the left in the figure) the amplitudes and phases are roughly constant. When the Moon first occults a portion of the extended solar limb, near 17h 15m 40s UT, we see what may be a small dip in the amplitude followed by a rise above the precontact level. The phase appears constant around this time. Thereafter the flux level drops in proportion to the decreasing area of exposed disk in the beam, except that there are “bumps” and “dips” superposed on the curve. The phases ramp linearly as the Moon traverses the fringes nearly perpendicularly.

The “bumps and dips” in the occultation record are not to be confused with the pattern of oscillations seen in single-dish lunar occultation observations (lunar occultation as a special form of interferometry is described by Thompson, Moran, and Swenson 1986). This latter pattern is due to Fresnel or near-field diffraction around the Moon of plane waves from the Sun, and the varying amplitude of the oscillations comes from the covering-up of varying orders of Fresnel zones by the Moon. Fresnel diffraction is not relevant to our observations because at $\lambda = 3$ mm the width of the first Fresnel zone is only $0''.42$, and our resolution is at least 3 times this width. We investigated the possibility of some residual effect of Fresnel diffraction and, in our modeling, found no significant effect.

To show how the occultation record comes about we create a model of the eclipse. Figure 4.3 (top panel) shows a model of the solar limb which consists of a step function convolved with a Gaussian, 3" wide, centered 8" above the photosphere—similar to the limb found by Wannier, Hurford,
Figure 4.2 Amplitude (top panel) and phase (bottom panel) of the visibility around the time of first contact, for a baseline with fringes parallel to the limbs. The behavior of each is explained in the text. The arrows point to the time at which the photosphere was occulted.
and Seielstad (1983)—and multiplied by a Gaussian beam of width 89", centered 10" above the photosphere. The middle and bottom panels show that resulting visibility amplitude and phase as a function of time as would be measured by our interferometer, for a step function lunar limb occulting the model of the top panel from left to right. The precontact visibility amplitude reflects the spacing of the interferometer and the smoothness of the effective solar profile at the scales the interferometer is sensitive to. The rise in the visibility at contact occurs because the Moon sharply cuts off the solar limb profile and boosts the high spatial frequencies in the Fourier transform of the resulting profile. The small dip seen in the data just prior to the rise can be reproduced with limb models (not shown) that have a broad "tail" with a small step in it.

The height of the 3 mm limb above the photosphere can be found directly from the occultation record by comparing the observed time of first contact with the time at which the Moon was due to occult the photosphere. Calculations for the OVRO site obtained from the US Naval Observatory (courtesy of A. D. Fiala, Nautical Almanac Office) as well as our own calculations based on their ephemeris show that the lunar limb began to occult the photosphere at 17:16:12.9 UT. We found the observed time of first contact in our record from the behavior of the visibility phase, which is simpler to interpret than the fluctuations in the amplitude. We constructed a model of the phase as a function of time in which the slope was determined using the ephemeris to calculate the relative position of the Moon in the fringe pattern. The precontact phase for the model was adopted from the observed, immediate precontact phase in the data. We shifted the model in time with respect to the data, interpolated the model to the data sample times, and
Figure 4.3 Top panel: Model of the solar limb consisting of a step function centered 8″ above the photosphere, convolved with a Gaussian 3″ wide, and multiplied by a Gaussian beam of width 89″ centered 10″ above the photosphere. Middle panel: Amplitude of the visibility as a function of time, as would be measured by our interferometer, for a step function limb occulting from left to right in the model of the top panel. Bottom panel: Corresponding visibility phase.
searched for the minimum displacement between the two curves. The time corresponding to this minimum difference was then taken as the observed first contact time.

The observed first contact time occurred significantly before the predicted photospheric first-contact time. The “early-sampled” data (see section 2) give a first-contact time corresponding to a distance of 7″.6 above the photosphere, while the “late-sampled” data give 7″.3, well within half of a resolution element (1″.6 given by the time between samples) so we adopt the value 7″.5 ± 0″.8 for the observed height of the solar limb. We note that the fit was marginally better for the “early” version and will indicate one or two data points in the final limb profile where the earlier points appear to be more trustworthy.

The height measured by our technique is a lower limit to the 3 mm limb and turns out to be about the half-power point of the brightness profile (as will be shown in section 3.2). Any extended “tail” in the profile could not be seen directly in the phases due to noise. Some variation can be seen in the amplitudes prior to the ramp-up in phases; as will be seen in section 3.2, this corresponds to a bump in the brightness profile which occurred far above the half-power point of the limb. This feature has a low standard deviation above the mean of the precontact values and could be due to a region over the limb. The Solar Geophysical Data reports indicate that on the previous day, July 10, NOAA region 6706 (a small, rapidly decaying region) was at S 10, W 75, which puts it on the west limb on July 11. However, the NIXT image of this location showed an exceptionally quiet limb in soft X-rays, so the reality of the feature remains questionable.
4.3.2. The Strip Brightness Profile

Following Gary and Hurford (1987), we perform a vector subtraction of the visibility at time $t_n$ from the visibility sample for $t_{n+1}$, to convert the time profile in Figure 4.2 to the strip brightness profile of the solar limb. Assuming no time-varying sources are present, this gives the visibility of the strip occulted in the intervening time. In applying this procedure to our data we first linearly interpolate the "early-sampled" and "late-sampled" versions of the occultation record so that the data points occur at regular intervals of 1.8 s, which is half the mean sample interval, so that the strips on the sky have equal width. In the final result, we will remove this oversampling by using a three-point running mean.

The amplitude of the vector difference of the data in Figure 4.2 is plotted in the top panel of Figure 4.4. A high level of fluctuations is apparent in this figure, even in the precontact period when no fluctuations are expected at all. When the precontact amplitudes and phases are plotted on a polar plot, the variations are found to be mostly in the phases, probably primarily due to the atmosphere.

Because of the unusual amount of phase noise, we searched for an alternate treatment of the data that would be less sensitive to the phases. One option is to ignore the phases completely and rely on our knowledge of the geometry of the eclipse, which is very precise; we demonstrated in section 4.3.1 that the phase for the occultation of a uniform solar disk can be predicted very accurately. We expect that deviations from a uniform solar disk will appear about equally in the amplitudes and phases, so by substituting model phases, we reduce our sensitivity to these deviations, but will have less uncertainty than when using the noisy phases. Figure 4.4
Figure 4.4 Top panel: Amplitude of the vector difference of the visibilities in Figure 4.2, computed as described in the text. Bottom panel: Same as top panel, except that phases computed from the ephemeris have been substituted for the measured phases. The error bars show the amplitude range when the early and late data are used, as explained in the text.
(bottom panel) shows the dramatic reduction in the noise level when the "early" difference plot is calculated with model phases.

The substitution of model phases must be done carefully because computer modeling shows that the visibility phases that pertain to the occultation of realistic limb profiles do not start to wind or ramp up linearly at the time of first contact. They may, for complex or extended limb profiles, show a dip or a gradual change in slope, and this has a significant effect on the shape of the derived limb profile. Accordingly, we substituted artificial phases for real ones in our data, except for the five or six points that defined the onset of the phase wind.

Each point in the vector difference profile, such as shown in Figure 4.4 using the model phases, represents the total flux of a strip of the exposed Sun within the primary beam. The effect of the primary beam is to reduce the flux near the edge of the beam, as is easily seen at the left-hand side of the profile in Figure 4.4. We can convert the flux from each strip into a one-dimensional brightness temperature using the effective area of the strip at each time, under the assumption that the brightness temperature is uniform along the strip except for the effect of the primary beam. Figure 4.5 shows the data converted to brightness temperature, and smoothed with a three-point running mean to remove the oversampling we introduced when we linearly interpolated the data at half the mean sample interval. We have confidence in the correction for the primary beam out to the half-power points and display only that spatial extent in the figure.

The limb profile of Figure 4.5 displays several important features. The mean brightness temperature on the outer 20" of the disk is about 6300 K, in agreement with the 3.0 and 3.09 mm values given by Linsky (1973)
and in accord with the VAL model temperature some distance above the temperature minimum. From the photosphere to a height of ~ 5".5 (4100 km) the temperature stays within the range measured on the disk, then it falls precipitously to about 1500 K in the next 3". After this, the decrease is more gradual until at ~ 12" above the photosphere the form of the profile is lost in noise. The half-maximum point is at an altitude of about 7".4, which is essentially the same height we obtained from the behavior of the phases alone in the occultation record. The root mean square temperature of about 100 K in the precontact period is a lower limit, but reasonable approximation, to the noise level. The error bars reflect the difference between the "early-sampled" and "late-sampled" versions of the data; if we lean more toward the "early-sampled," as discussed in section 4.3.1, the only difference worth noting is that the point about 5".5 above the photosphere is likely to fall at the bottom of the error bar, making a smoother connection with the rest of the profile.

As we have already indicated, the profile we obtain does not show a uniform brightness temperature on the disk. Seven arcseconds inside the photosphere, the brightness temperature is down by almost 30%; then it rises again. Some of this nonuniformity is probably due to noise in the data, but the broad features affecting many samples are probably due to real brightness temperature variations. Full disk observations near 3 mm with spatial resolution of about 1' show amplitude variations of ~ 20% due to active regions and filaments (Hurford 1986). If the variations in Figure 4.5 are real, they could be due to plage elements or other bright or dark features on the disk, on a scale that is difficult to see on optical photographs because the region is so close to the limb and is therefore severely foreshortened.
Figure 4.5 The limb profile, derived using ephemeris phases. The attenuation due to the primary beam has been removed.
4.4. Discussion

Our detection of photospheric-temperature plasma at an altitude of 5500 km above the visible photosphere complements data from a wide spectral range in demonstrating the inadequacy of certain models—those that depict the chromosphere as smooth and that ignore spicules and other features not in hydrostatic equilibrium. A survey of the literature reveals that in the range from 30 μm to 3 mm the data display two types of phenomena that the authors surveyed cannot accommodate with hydrostatic models. The phenomena are, first, the existence of millimeter opacity at heights well above the visible photosphere, i.e., the extension of the limb beyond the photospheric radius, and, second, the absence (in almost all cases) of a limb "spike" or transient. In the range 100 μm to 1 mm, some single-dish data suggest that large-scale limb brightening is suppressed by inhomogeneity in the chromosphere, adding to the list of phenomena that cannot be accommodated, but these data suffer from the uncertainty of beam deconvolution, and we will not comment further on the limb brightening question.

We illustrate the difficulty of reconciling the data and the hydrostatic models by considering the predictions of the VAL at 3 mm. By our reckoning the VAL model predicts, at this wavelength, a limb spike of $T_b \sim 25,000$ K about 3" above the limb. The temperature is indicative of the transition region, which in this model occurs near 2200 km. The density along the line of sight is high enough to provide unit optical depth when the line of sight intersects points below 3" or $\sim 2200$ km altitude; above this, the plasma becomes optically very thin and the brightness temperature drops rapidly. There should be no significant emission above $\sim 3000$ km. The measured extensions, on the other hand, strongly suggest the presence of
cool (< 10,000 K), dense material above 5000 km. Labrum, Archer, and Smith, in an admirable 1978 paper, were perhaps the first with resolution comparable to ours to report on the 3 mm radio limb explicitly, and they found an extension of 5600±800 km. The other observations near 3 mm with resolution of a few arcseconds are those at 2.7 mm by Wannier et al. (1983), who determined a limb extension in accord with ours, and those at 3.5 mm of White and Kundu (1992), who observed the 1991 July eclipse with the Berkeley-Illinois-Maryland array and found a limb extension of about 11". All of these authors found the limb to be free of spikes. Perhaps even more difficult to reconcile with the hydrostatic models are the interferometric data at 1.3 mm of Horne et al. (1981): They found a limb extension of 8.1", similar to ours at 3 mm, whereas the difference in observing wavelength should have led, in the case of a hydrostatic model, to their finding a much smaller limb extension.

The cool, dense material we infer from the limb extensions could be present in addition to the material described by the VAL, if it were structured in such a way as to be more visible at the limb than on the disk. In this case the denser material would provide most of the opacity and would effectively obscure the hot, less dense matter. In such a two-component model, it is tempting to associate the obscuring component with spicules. Spicules are rather sparsely distributed when one views the disk of the Sun, but at the limb, spicules over a wide range of longitudes project densely against the sky to form an edge approximately 5000 kilometers above the photosphere. We are not the first to suggest that spicules provide the opacity to extend the radio limb above the photosphere; Coates (1958) did so in a remarkable early study, and the models of Thomas and Athay (1961, see chap. 8) and Lantos
and Kundu (1972) are similar in general terms. However, our high-resolution profile does permit us to compare the 3 mm and optical limbs directly. When we overlay our one-dimensional brightness profile on an off-band H$_\alpha$ image, as in Figure 4.6, we find that the sharp edge of the 3 mm profile lies very close to the edge of spicules.

The sharp edge in the radio profile probably corresponds to $\tau = 1$, in which case the relatively flat part of the limb profile is optically thick and the temperature of spicule material is only about 6000 K. A crude calculation of the electron density at the altitude corresponding to the edge in the 3 mm profile, assuming free-free opacity, gives $\sim 10^{11}$ cm$^{-3}$.

4.5 Summary

We found that at 3 mm the solar limb extends 7\".5 ± 0\".8 or 5500 km above the visible photosphere, as measured by the onset of the ramp-up in the visibility phases. The limb profile is relatively steep at this height, so the measurement of the limb extension according to the half-power point rather than the phase wind gives essentially the same result. Our data do not give information on the center-to-limb brightness profile; we have confidence in the brightness temperature only to 20\" inside the limb, as this represents the half-power point of the primary beam. We can say, however, that the disk brightness temperature at the extreme limb appears to be affected by features that vary in amplitude by about 30% over about 5\", and there is no underlying trend of limb brightening or darkening that we can detect.

The data of Wannier et al. (1983), with which ours basically agree, were taken with the same interferometer at Owens Valley but without the benefit of an eclipse and with resolution of about 6\". They determined a best-fit limb profile that is very similar to ours, with a shallow decline above
Figure 4.6 Off-band $H_\alpha$ photograph of the region near the limb, taken at BBSO on the day of the eclipse, overlaid with solar latitude and longitude lines at $10^\circ$ intervals and with a plot of the 3 mm limb profile. The vertical scale of the radio profile is arbitrary. The plus sign shows the center of the beam. The spicules can be seen as a fuzzy edge above the optical limb, nearly at the same height as the sharp cutoff in the radio profile. The contrast of the $H_\alpha$ spicules was photographically enhanced for clarity.
the limb, then a "knee" and a steeper drop in brightness temperature (see Fig. 4 in their paper). The main difference is a larger limb extension of 8" – 12". The variability of the limb extension may be a real solar effect or of instrumental origin, but in any case we believe that their results can be reconciled with ours by taking into consideration the difference in spatial resolution and the difference in the way the extension was measured.

Besides the results of Wannier et al. (1983), we find consistency only with the single-dish measurement of Labrum et al. (1978), and with the interferometric data of White and Kundu (1992), notwithstanding the diversity of published limb profiles near λ3 mm. Most of the limb information obtained to date comes from single-dish observations, and it seems likely that their inconsistency stems from poor spatial resolution and from the difficulty of deconvolving the beam profile. The interferometric data of White and Kundu (1992) at 3.5 mm are of special interest because they pertain to the same eclipse, and very nearly the same limb. Using the same definition we used in section 3.1, the onset of phase wind in the occultation record, they find a limb extension of 11", similar to, but significantly different from, our measurement of 7".5 ± 0'.8. Nor can the discrepancy be completely attributed to the 15% difference in observing frequency, since the White et al. observations at 86 and 89 GHz show no difference in onset times. The true test of the discrepancy will come when a vector-difference profile is obtained from the White et al. data, and the two limb profiles are compared over their entire heights.

In the meantime, the high spatial resolution of our limb profile has allowed us to compare the 3 mm limb with that in off-band Hα in Figure 4.6, and we find that the sharp edge of the 3 mm profile corresponds quite closely
to the apparent top edge of the spicules. This supports our hypothesis that spicules consist of relatively cool ($\sim 6000$ K) material capable of obscuring any transition-region type gas at altitudes up to at least 3500 km.
Chapter V

Concluding Remarks

The basis of this thesis is the interferometric data gathered on the solar microwave flare of July 16, 1992 and the interferometric data of the solar limb taken during the solar eclipse of July 11, 1991. In both studies the technique of radio interferometry provides high spatial resolution, on the order of a few arcseconds, which is necessary to further our understanding of the sun. In the case of the microwave flare high spatial resolution allows us to model the brightness temperature spectrum of the flare, rather than the flux spectrum, thus laying to rest any ambiguity in the interpretation of the spectrum caused by variations in the source size as a function of frequency. The high spectral and temporal resolution of the Owens Valley Solar Array is crucial also, for solar microwave flares often have rapid rise times, and the spectra often have two or more components. In the case of the limb observations with the Owens Valley Millimeter Array high spatial resolution is possible because the phase of the interferometer visibility function indicates the precise location of the moon, allowing us to infer the brightness temperature profile in narrow strips along the radial direction.

Although this was originally intended as a purely observational thesis, the difficulty of explaining the spectrum of the microwave flare led me to carry out detailed numerical modeling, and finally to develop the theory of Razin suppression in a dispersive medium with a strong magnetic field.

In this chapter I summarize the main conclusions from the research reported in Chapters 2, 3, and 4. These remarks are made with the “big
picture" in mind and end with some thoughts about future work that would build on these results.

1) In Chapter 2 we found that the gyrosynchrotron spectrum of a microwave flare can be fit if we include in the modeling the effect of Razin suppression. Including the effect of Razin suppression explains not only the individual spectra but also their evolution in time. This solves a longstanding problem in the study of solar microwave burst spectra, namely the constant peak frequency as the peak brightness temperature varies, and the steep slope on the low-frequency side of the spectrum.

2) The main problem in attempting to fit the spectra without Razin suppression is the difficulty of achieving a peak frequency (spectral turnover frequency) near 10 GHz along with a low peak brightness temperature of about $10^7$ K. The spectral fit without Razin suppression has too broad a bandwidth.

3) When Razin suppression is included in the model fitting, the following flare parameters result: Magnetic field strength $B = 300$ Gauss; electron energy distribution from 0.1 MeV at the lower end to $\sim 0.4$ to $\sim 1$ MeV at the upper; index $\delta$ in electron energy distribution 4; ambient electron density $\sim 2 \times 10^{11}$ cm$^{-3}$; accelerated particle density ranging from $5 \times 10^2$ cm$^{-3}$ to $3 \times 10^4$ cm$^{-3}$.

4) It appears that the rise and fall of the microwave spectrum can be explained by the rise and fall of the number density of accelerated particles, while the density of the ambient medium remains constant. This conclusion should be checked in further analysis of microwave gyrosynchrotron spectra, because acceptable fits to the spectra might be found with simultaneous variation of both the ambient and accelerated particle densities. If the parallel
behavior of the accelerated particles and the peak brightness temperature is correct, this raises interesting questions which are beyond the scope of this thesis: for example, does the variation of the number of accelerated particles reflect the efficiency of the acceleration mechanism, or just the time evolution of a sub-set of particles reaching the radio-emitting region?

5) In Chapter 3 we found that the temperature and emission measure results from Yohkoh support our conclusion of high ambient density in the radio-flare producing region. The density inferred from the soft X-ray data is $2 \times 10^{10} < n_e < 3 \times 10^{11} \text{cm}^{-3}$.

6) The electron spectral index derived from the hard X-ray data, assuming a thick-target model, is in the range 3.6 to 4.7, while the spectral index derived from the analysis of the radio emission is 3.5 to 4.3.

7) The overall morphology and evolution of the flare as seen in soft and hard X-rays conforms to a well-established pattern in which hard X-rays appear at the loop footpoint or footpoints, while the soft X-rays are more diffuse and are brightest near the top of the loop. After the peak of the hard X-ray emission the lower-energy hard X-rays start to fill the volume of the loop seen in soft X-rays. The second peak in the lowest energy band of hard X-rays had no associated microwave emission. This emission is probably due to heating of the plasma in response to the flare two minutes earlier.

8) In Chapter 4 we found from observations at 3 millimeters wavelength that photospheric-temperature gas exists at an altitude of about 5500 km above the visible limb of the sun. This is surprising in view of chromospheric models which predict that at this wavelength the plasma should be optically thin. The temperature remains constant at about 6500 degrees K.
up to 5500 km, then declines precipitously, suggesting that the plasma is optically thin above this level.

9) Despite the wide variety of measurements of the solar limb profile in the millimeter regime we find agreement only with interferometric measurements. Single-dish measurements appear to be suspect because of the difficulty of deconvolving the antenna response from the data.

10) One hypothesis which fits the data is that spicules are relatively cool and dense, and provide opacity at 3 mm above the visible limb. We speculate that the spicules form a cool component embedded in a relatively hot and tenuous plasma. The hot tenuous medium is that described by the chromospheric models such as that of Vernazza, Avrett and Loeser (1981).

In future work it would be of great interest to model all microwave burst spectra with a gyrosynchrotron code that includes the effect of Razin suppression, as we have done. In light of the wide range of frequency over which the Razin effect is at work, as found in Chapter 2, it appears necessary to take Razin or medium suppression into account. It would be particularly interesting to examine the spectral evolution of a number of flares to see if the accelerated particle density does indeed follow the increase and decrease of the peak brightness temperature. Such observations require a time resolution of at least 20 seconds to follow the spectrum during the impulsive rise and decay phases. The Owens Valley Solar Array must be used in order to have adequate frequency coverage—the Very Large Array has limited spectral range—but the sparsity of \((u,v)\) coverage at Owens Valley is sometimes a problem; good spectral coverage is necessary to properly image the source and obtain a reliable brightness temperature.
For further work on spicules I gathered follow-up data with the Owens Valley Millimeter Array in May 1992. These data were taken without benefit of an eclipse, but images were made of the solar limb at different positions around the limb, so that a wide range of solar latitude is sampled. Spicules near the poles are thought to extend about 20 percent higher than near the equator. These data have not been analyzed yet. In any case the temperature of spicules may be determined from investigations which others have planned with the SUMER instrument on the solar satellite SOHO, due to be launched in November 1995.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
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<tbody>
<tr>
<td>$a_{\pm}$</td>
<td>Polarization coefficient, in ordinary (+) and extraordinary (-) modes</td>
</tr>
<tr>
<td>$B$</td>
<td>Magnetic Field (Gauss)</td>
</tr>
<tr>
<td>$B_\perp$</td>
<td>Component of magnetic field which is perpendicular to line of sight</td>
</tr>
<tr>
<td>$c$</td>
<td>Speed of light</td>
</tr>
<tr>
<td>$e$</td>
<td>Charge of the electron</td>
</tr>
<tr>
<td>$eV$</td>
<td>Electron Volt, $1\ eV = 1.6 \times 10^{-12}\ erg$</td>
</tr>
<tr>
<td>$g$</td>
<td>Gaunt factor</td>
</tr>
<tr>
<td>$j_{\pm}$</td>
<td>Emissivity, in ordinary (+) and extraordinary (-) modes</td>
</tr>
<tr>
<td>$k_B$</td>
<td>Boltzmann's constant, $1.38 \times 10^{-16}\ erg\ K^{-1}$</td>
</tr>
<tr>
<td>$L$</td>
<td>Path length to the source</td>
</tr>
<tr>
<td>$m$</td>
<td>Mass of the electron, $9.11 \times 10^{-28}\ g$</td>
</tr>
<tr>
<td>$n$</td>
<td>Index of refraction</td>
</tr>
<tr>
<td>$n_e$</td>
<td>Density of ambient plasma</td>
</tr>
<tr>
<td>$N$</td>
<td>Total number of accelerated electrons</td>
</tr>
<tr>
<td>$Z$</td>
<td>Atomic number</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Razin parameter proposed by R. Ramaty; $\alpha = 1.5\nu_B/\nu_p$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$\frac{v}{c}$ of an energetic particle</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>γ</td>
<td>Lorentz factor of energetic particle; ( \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} )</td>
</tr>
<tr>
<td>γ</td>
<td>Exponent or index in photon spectrum</td>
</tr>
<tr>
<td>δ</td>
<td>Exponent or index in electron energy distribution</td>
</tr>
<tr>
<td>θ</td>
<td>Viewing angle; angle between magnetic field B and line of sight</td>
</tr>
<tr>
<td>κ±</td>
<td>Absorption coefficient, in ordinary (+) and extraordinary (-) modes</td>
</tr>
<tr>
<td>ln Λ</td>
<td>Coulomb logarithm = ( \ln(12\pi(6.9)^3\left(\frac{r}{a}\right)^{3/2}) )</td>
</tr>
<tr>
<td>ν</td>
<td>Frequency of observation</td>
</tr>
<tr>
<td>νp, ωp</td>
<td>Plasma freq. ( \omega_p = 2\pi \nu_p ); ( \omega_p^2 = \frac{4\pi n_e e^2}{m} ); ( \nu_p \sim 9000\sqrt{n} )</td>
</tr>
<tr>
<td>νB, ωB</td>
<td>Gyrofrequency ( \omega_B = \frac{eB}{mc} ); ( \omega_B \sim 2.8 \times 10^6 B ) (Gauss)</td>
</tr>
<tr>
<td>νc</td>
<td>Critical frequency; ( \nu_c = \frac{3eB_\perp}{4\pi mc} \gamma^2 )</td>
</tr>
<tr>
<td>φ</td>
<td>Pitch angle of energetic particle</td>
</tr>
<tr>
<td>τ</td>
<td>Time constant</td>
</tr>
<tr>
<td>ω</td>
<td>Angular frequency</td>
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REFERENCES


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