Fermilab

TS-SSC 90-092

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Subj: Notes on Tangential Probes: Basic Equations II

In this note I "publish" a few more of my notes on tangential probes. This extends the discussion in my previous note[1] to allow for multiple turns on the bucking coils and to allow for the case that there is only one bucking coil. The latter is relevant for Peter Mazur's "DDL" probes in which a "belly band" winding which is parallel to the tangential winding does most of the bucking. In these notes I only work out the values of the bucking factor(s) but do not give the probe sensitivity factors F_n and γ_n.

I reproduce page 1 from the previous set of notes[1] to define the probe geometry. On page 2 I calculate the bucking factors for two tangential windings in the "general" case that each winding segment has its own radius and phase angle and that each of the three windings (tangential and two bucking) have their own number of turns. I use the notation, introduced but not explicitly defined on pp. 5.2 and 5.3 of my previous note,

$$c_{12} = \cos\phi_1 - \cos\phi_2,$$
$$s_{12} = \sin\phi_1 - \sin\phi_2, \text{ etc.}$$

On page 3 I compute S_o and C_o for the case of only one bucking coil. This yields two equations in one unknown which cannot be solved. The solution is to minimize the sensitivity of the bucked signal to the dipole component. This is done on page 4, yielding the "general" bucking factor. In the case of an ideal probe with all windings at the same radius, of the same length, and at ideal angular positions the bucking factor takes the simple form on page 5. With judicious choices of number of windings and the tangential winding opening angle the bucking factor can be set to -1. The tangential and bucking windings can be wired in series and out of phase to yield a probe with zero sensitivity to the dipole field. Note that in this case S_o on page 3 is zero so, to the extent that the probe geometry is perfect, the bucking is also perfect.

[1] J. Strait, Notes on Tangential Probes: Basic Equations, TS-SSC 90-082, 11/9/90.

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$$B_r = B_0 \sum_{n=0}^{\infty} \left(\frac{r}{\rho}\right)^n \left(a_n \cos\bar{n}\,\theta + b_n \sin\bar{n}\,\theta\right) \qquad \bar{n} \equiv n+1$$

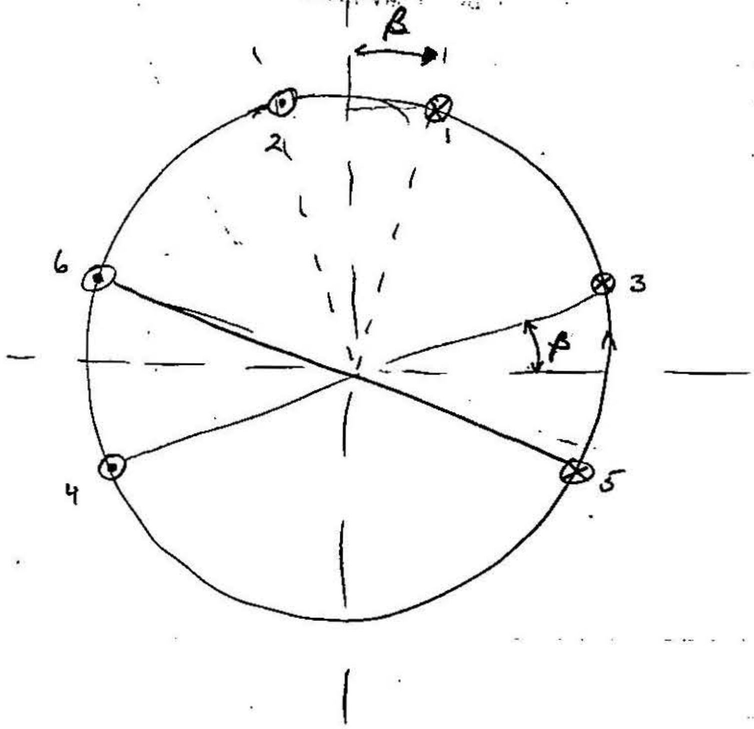
$$\theta = (\omega t + \phi)$$

$$V^{(t)}_{1\,WIRE} = \omega\, l\, B_0 \sum_{n=0}^{\infty} \frac{r^{\bar{n}}}{\rho^n}\left[a_n \cos(\bar{n}\omega t + \bar{n}\phi) + b_n \sin(\bar{n}\omega t + \bar{n}\phi)\right]$$

for the i-th winding

wire sense x bucking factor

$$v_i = B_0\, \omega\, f_i\, N_i\, l_i \sum_{n=0}^{\infty} \frac{r_i^{\bar{n}}}{\rho^n}\left[a_n \cos(\bar{n}\omega t + \bar{n}\phi_i) + b_n \sin(\bar{n}\omega t + \bar{n}\phi_i)\right]$$



$l\, v \times B$

i	ϕ_i	N_i	f_i	l_i	r_i
1	$90-\beta$	M	$+1$	$l_1 = 42.028$	$r_1 = \cancel{.5??}$ .596
2	$90+\beta$	M	-1	l_1	r_1
3	β	1	$-d$	$l_3 = 42.563$	$r_3 = \cancel{.595}$ .605
4	$180+\beta$	1	$+d$	l_3	r_3
5	$-\beta$	1	$-d$	$l_5 = 42.417$	r_3
6	$180-\beta$	1	$+d$	l_5	r_3

$\beta_{nominal} = 8.2$

or L_{mag} if $L_{mag} < l_i$

allow multiple turns on bucking coils

M_1 = turns on litz

M_3, M_5 = turns on bucking coils

$$
\begin{aligned}
C_0 = {} & M_1 l_1 r_1 (\cos\phi_1 - \cos\phi_2) \\
& + M_3 l_3 r_3 d_3 (\cos\phi_3 - \cos\phi_4) \\
& + M_5 l_5 r_5 d_5 (\cos\phi_5 - \cos\phi_6)
\end{aligned}
$$

$$
\begin{aligned}
S_0 = {} & M_1 l_1 r_1 (\sin\phi_1 - \sin\phi_2) \\
& + M_3 l_3 r_3 d_3 (\sin\phi_3 - \sin\phi_4) \\
& + M_5 l_5 r_5 d_5 (\sin\phi_5 - \sin\phi_6)
\end{aligned}
$$

$$\Rightarrow d_3 = \frac{M_1 l_1 r_1 (c_{12} s_{56} - s_{12} c_{56}) \cancel{M_5}}{l_3 r_3 (c_{56} s_{34} - s_{56} c_{34}) M_3 \cancel{M_5}}$$

$$d_5 = -\frac{M_1 \cancel{M_3}}{M_5 \cancel{M_3}} \frac{l_1 r_1 (c_{12} s_{34} - s_{12} c_{34})}{l_5 r_5 (c_{56} s_{34} - s_{56} c_{34})}$$

only 1 bucking coil

if M_5 (for example) $= 0$

$$C_0 = M_1 l_1 r_1 (\cos\phi_1 - \cos\phi_2) + M_3 l_3 r_3 d_3 (\cos\phi_3 - \cos\phi_4) \equiv A + B d_3$$

$$S_0 = M_1 l_1 r_1 (\sin\phi_1 - \sin\phi_2) + M_3 l_3 r_3 d_3 (\sin\phi_3 - \sin\phi_4) \equiv D + E d_3$$

$$C_0 = 0 \Rightarrow d_3 = -\frac{M_1 l_1 r_1}{M_3 l_3 r_3} \frac{\cos\phi_1 - \cos\phi_2}{\cos\phi_3 - \cos\phi_4}$$

$$S_0 = 0 \Rightarrow d_3 = -\frac{M_1 l_1 r_1}{M_3 l_3 r_3} \underbrace{\frac{\sin\phi_1 - \sin\phi_2}{\sin\phi_3 - \sin\phi_4}}_{0/0}$$

$0/0$ for lity perfectly placed about 90°
& bucking coil at 0 & 180°

$$F^2 = C_0^2 + S_0^2$$

if only 1 bucking coil, minimize this

$$F^2 = A^2 + 2ABd + B^2 d^2 + D^2 + 2DEd + E^2 d^2$$

$$\frac{d(F^2)}{d(d)} = \cancel{2}AB + \cancel{2}B^2 d + \cancel{2}DE + \cancel{2}E^2 d = 0$$

$$d = -\left(\frac{AB + DE}{B^2 + E^2}\right)$$

$$= -\frac{(M_1 l_1 r_1 c_{12})(M_3 l_3 r_3 c_{34}) + (M_1 l_1 r_1 s_{12})(M_3 r_3 l_3 s_{34})}{(M_3 l_3 r_3 c_{34})^2 + (M_3 l_3 r_3 s_{34})^2}$$

$$\boxed{d_3 = -\frac{M_1 l_1 r_1}{M_3 l_3 r_3}\,\frac{c_{12} c_{34} + s_{12} s_{34}}{c_{34}^2 + s_{34}^2}}$$

"Perfect" bucking coil has $\phi_3 = 0$, $\phi_4 = 180°$

$$c_{34} = \cos\phi_3 - \cos\phi_4 = 2$$

$$s_{34} = \sin\phi_3 - \sin\phi_4 = 0$$

"Perfect" tangential winding has $\phi_1 = 90-\beta$

$\phi_2 = 90+\beta$

$$c_{12} = \cos\phi_1 - \cos\phi_2$$
$$= 2\sin\beta$$

$$\Rightarrow \quad d_3 = -\frac{M_1\, l_1\, r_1}{M_3\, l_3\, r_3}\sin\beta$$

for $d_3 = -1$

$$\frac{M_3\, l_3\, r_3}{M_1\, l_1\, r_1} = \sin\beta$$

for "ideal" probe with $l_1 = l_3$

$r_1 = r_3$

$$\sin\beta = \frac{M_3}{M_1}$$

for $M_3 = 1$:

M_1	β
5	11.54°
10	5.74°
15	3.82°