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# PREDICTIONS IN SU(5) SUPERGRAVITY GRAND UNIFICATION WITH PROTON STABILITY AND RELIC DENSITY CONSTRAINTS

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## Abstract

It is shown that in the physically interesting domain of the parameter space of SU(5) supergravity GUT, the Higgs and the Z poles dominate the LSP annihilation. Here the naive analyses on thermal averaging breaks down and formulae are derived which give a rigorous treatment over the poles. These results are then used to show that there exist significant domains in the parameter space where the constraints of proton stability and cosmology are simultaneously satisfied. New upper limits on light particle masses are obtained.

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**I. INTRODUCTION :** Recently there have been extensive investigations of SU(5) supergravity models[1−7] with electro–weak symmetry broken via radiative corrections[8,9]. Analyses of Refs (6)–(7) were carried out in the framework of No–Scale models[9] while the analysis of Refs (1–5) are for the standard SU(5) supergravity case[10]. In this letter we shall discuss only the standard SU(5) Model[10,8]. Here, after fixing the Z–boson mass the model depends on four arbitrary parameters, aside from the top quark mass $m_t$, which may be chosen to be $m_o$ (the universal scalar mass), $m_{1/2}$ (the universal gaugino mass), $A_o$ (the cubic soft SUSY breaking parameter) and tan $\beta =< H_2 > / < H_1 >$ where $< H_1 >$ gives mass to the down quarks and the leptons and $< H_2 >$ gives mass to the up quarks. The analyses of Refs (1–4) investigated the full parameter space of the theory, Ref (5) investigated the space under one more constraint ($B_0 = A_0 - m_0$ where $B_0$ is the quadratic soft SUSY breaking parameter) while Refs (1–3) also included in the analysis the constraint of proton stability[11]. The inclusion of proton stability constraints were seen to lead to a number of simple mass relations among the neutralino, chargino and gluino mass spectra[1−3]. One finds for most of the parameter space $2m_{\tilde{z}_1} \cong m_{\tilde{z}_2} \cong m_{\tilde{W}_1}$, and $m_{\tilde{W}_1} \simeq (1/4)m_{\tilde{g}}$ for $\mu > 0$ and $m_{\tilde{W}_1} \simeq (1/3)m_{\tilde{g}}$ for $\mu < 0$. (Here $\tilde{Z}_{1,2}$ are the two lightest neutralinos, $\tilde{W}_1$ is the lightest chargino and $\tilde{g}$ is the gluino.) Thus the gluino mass (approximately) determines the light neutralino and chargino spectrum.

Remarkably the standard SU(5) supergravity model under the constraint of proton stability also leads to the prediction that the lightest neutralino is the lightest supersymmetric particle (LSP)[1−3]. We investigate here the implications on the parameter space of the constraint that the relic density of the lightest neutralino not overclose the universe i.e. $\Omega_{\tilde{z}_1} h^2 \leq 1$, where $\Omega_{\tilde{z}_1} = \rho_{\tilde{z}_1}/\rho_c$, with $\rho_{\tilde{z}_1}$ the matter density of the lightest neutralino $\tilde{Z}_1$ and $\rho_c = 3H_0^2/(8\pi G_N)$ the critical density. Here $H_0 = h \times 100km/(s.Mpc)$ and h is the Hubble parameter with $\frac{1}{2} \leq h \leq 1$. Recently[12], it was pointed out that the dominant annihilation of neutralinos occurs near the lightest neutral Higgs (h) pole in the s–channel for the domain of the parameter space satisfying $CDF, LEP$ and proton stability constraints and the finetuning requirement that $m_{\tilde{q},\tilde{g}} \leq 1TeV$. It is known that the expansion in powers of $v$ of the thermally averaged quantity $< \sigma v >$ (where $\sigma$ is the spin averaged annihilation cross–section of two neutralinos, $v$ is the relative velocity defined by $\sqrt{s} \cong 2m_{\tilde{z}_1} + \frac{1}{4}m_{\tilde{z}_1}v^2$, and $s$ is square of the center–of–mass energy), breaks down when $\sqrt{s}$ is in the vicinity of a pole[13]. In this case, a careful treatment of integration over the pole in the annihilation channel is needed. However, the rigorous analysis even in the non–relativistic approximation involves a double integration over the pole (which is numerically intricate) for the quantity $J = \int_0^{x_f} dx < \sigma v >$ needed to calculate the relic density. ($x_f = kT_f/m_{\tilde{z}_1}$, where $T_f$ is the freeze out temperature.) Here we derive rigorous formulas where the integrations over one of the variables is analytically carried out and the remaining integration is smooth over the pole. The analysis here is complete and includes the direct channel Higgs and Z–poles as well as t–channel fermion exchange diagrams. Using the rigorous analysis for $J$ outlined above we explore the full five dimensional parameter space of the theory characterized by $m_0, m_{1/2}, A_0$, tan $\beta$ and $m_t$ under the combined constraints of $CDF$ and $LEP$ data, proton stability and relic density. We show that while the parameter space is strongly constrained, significant domains in the parameter space remain where all the constraints mentioned above are satisfied. Also new limits on the light Higgs, the light chargino and on the $LSP$ result.

**II. BASIC FORMULAE :** We follow standard procedure[14] and write the equation governing the number density n at time t of the lightest neutralino $\tilde{Z}_1$ in a Friedman–Robertson–Walker universe with isotropic mass density in the form

$$\frac{df}{dx} = \frac{m_{\tilde{z}_1}}{k^3}\left(\frac{8\pi^3 N_F G_N}{45}\right)^{-\frac{1}{2}} < \sigma v > (f^2 - f_0^2) \tag{1}$$

where $f = n/T^3, x = kT/m_{\tilde{z}_1}$ ($k$ is the Boltzman constant), $N_F$ is the number of degrees of freedom at temperature T, $G_N$ is the Newtonian constant and $f_0 = n_0/T^3$ where $n_0$ is the number density at thermal equilibrium. The relic density of the $LSP$ is then given by the following (approximate) formula[14]:

$$\rho_{\tilde{z}_1} = 4.75 \times 10^{-40}\left(\frac{T_{\tilde{z}_1}}{T_\gamma}\right)^3\left(\frac{T_\gamma}{2.75^\circ K}\right)^3 N_F^{\ 1/2}\left(\frac{GeV^{-2}}{J(x_f)}\right)\frac{g}{cm^3} \tag{2}$$

where $(T_{\tilde{z}_1}/T_\gamma)^3$ is a reheating factor, $T_\gamma$ is the current temperature and $J(x_f)$ is given by $J(x_f) = \int_0^{x_f} < \sigma v > dx$ and:

$$< \sigma v > = \int_0^\infty dv v^2 (\sigma v) e^{-v^2/4x} \Big/ \int_0^\infty dv v^2 e^{-v^2/4x} \tag{3}$$

The freezeout temperature $T_f$ is determined by the relation[14]

$$x_f^{-1} = \ell n \left[ x_f^{\frac{1}{2}} < \sigma v > \sqrt{45} m_{\tilde{z}_1} \Big/ (4\pi^3 N_F^{\frac{1}{2}} G_N^{\frac{1}{2}}) \right] \tag{4}$$

In Eq. (4) $< \sigma v >$ is the thermally average of $\sigma v$ evaluated at $x_f$.

**III. INTEGRATION OVER HIGGS AND Z–POLES:** $J(x_f)$ appearing in Eq.(2) can be decomposed as $J = J_h + J_Z + J_{sf}$ where $J_h$, $J_Z$ are the contributions of the s–channel Higgs and Z poles, and $J_{sf}$ is the t–channel contribution from the exchange of squarks and sleptons. In the domain of physical interest with finetuning constraints $m_{\tilde{q},\tilde{g}} \le 1TeV$, onl y the lightest neutral Higgs $h$ makes a significant contribution to the cross–section. For the Higgs pole, using the non–relativistic approximation, we write $\sigma v$ in the form

$$(\sigma v)_h = \frac{A_h}{m_{\tilde{z}_1}^2} \frac{v^2}{((v^2 - \epsilon_h)^2 + \gamma_h^2)} \tag{5}$$

In Eq.(5) $\epsilon_h = (m_h^2 - 4m_{\tilde{z}_1}^2)/m_{\tilde{z}_1}^2$ and $\gamma_h = m_h \Gamma_h / m_{\tilde{z}_1}^2$ where $m_h$ is the Higgs mass an d $\Gamma_h$ is the Higgs decay width and $A_h$ is[15]

$$A_h = \frac{1}{8\pi} \left( \frac{g_2}{2M_W} \frac{\sin\alpha}{\cos\beta} \right)^2 \frac{g_2^2}{\cos^2\theta_W} (n_{11}\cos\theta_W - n_{12}\sin\theta_W)^2$$
$$(n_{13}\sin\alpha + n_{14}\cos\alpha)^2 \sum_i C_i m_{fi}^2 \left( 1 - \frac{m_{fi}^2}{m_{\tilde{z}_1}^2} \right)^{\frac{3}{2}} . \tag{6}$$

Here $\sin 2\alpha = -(m_A^2 + m_Z^2)(m_H^2 - m_h^2)^{-1} \sin 2\beta$ and where $m_A$ is the mass of the CP–odd Higgs and $m_H$ is the mass of the CP–even heavy neutral Higgs. $C_i$ i s a color factor which is (3,1) for (quarks, leptons) and $n_{1i}$ are components of $\tilde{Z}_1$ eigen–vector in the basis defined in Ellis et al in Ref (14). Using Eq.(5) one can carry out the $x$–integration in $J_h(x_f)$ and get

$$J_h(x_f) = \frac{A_h}{2\sqrt{2} m_{\tilde{z}_1}^2} \left[ I_{1h} + \frac{\epsilon_h}{\gamma_h} I_{2h} \right] \tag{7}$$

where

$$I_{1h} = \frac{1}{2} \int_0^\infty d\xi \xi^{-\frac{1}{2}} e^{-\xi} \ell n \left[ \frac{(4\xi x_f - \epsilon_h)^2 + \gamma_h^2}{\epsilon_h^2 + \gamma_h^2} \right] \tag{8}$$

$$I_{2h} = \int_0^\infty d\xi \xi^{-\frac{1}{2}} e^{-\xi} \left[ \tan^{-1}\left(\frac{4\xi x_f - \epsilon_h}{\gamma_h}\right) + \tan^{-1}\left(\frac{\epsilon_h}{\gamma_h}\right) \right] \tag{9}$$

A similar analysis can be carried out for the Z–Pole and here one finds

$$J_Z = \frac{1}{2\sqrt{\pi} m_{\tilde{z}_1}^4} \left[ A_Z \frac{I_{1Z}}{\epsilon_Z} + \frac{\epsilon_Z}{\gamma_Z} B_Z (I_{1Z} + I_{2Z}) \right] \tag{10}$$

where $I_{1Z}$ and $I_{2Z}$ are defined anologously to $I_{1h}$ and $I_{2h}$ with $m_h, \Gamma_h$ replaced by $M_Z, \Gamma_Z$. In Eq.(10) $A_Z$ is given by

$$A_Z = \frac{\pi}{8} \frac{\alpha_2^2}{\cos^4\theta_W} (n_{13}^2 - n_{14}^2)^2 \left(1 - \frac{4m_{\tilde{z}_1}^2}{M_z^2}\right)^2$$
$$\times \left[ 3m_b^2 \left(1 - \frac{m_b^2}{m_{\tilde{z}_1}^2}\right)^{\frac{1}{2}} + m_\tau^2 \left(1 - \frac{m_\tau^2}{m_{\tilde{z}_1}^2}\right)^{\frac{1}{2}} + 3m_c^2 \left(1 - \frac{m_c^2}{m_{\tilde{z}_1}^2}\right)^{\frac{1}{2}} \right] \tag{11}$$

where we have retained only the dominant $b$, $c$ and $\tau$–contributions, while $B_Z$ (in the zero–fermion mass approximation) is

$$B_Z = \frac{\pi}{6} \frac{\alpha_2^2}{\cos^4\theta_W} m_{\tilde{z}_1}^2 (n_{13}^2 - n_{14}^2)^2 \left[ \frac{21}{2} + \frac{80}{3} \sin^4\theta_W - 20 \sin^2\theta_W \right] . \tag{12}$$

In the vicinity of the Higgs (or Z–Pole), $J_{sf}$ is typically much smaller than $J_h$ (or $J_Z$) and thus we shall use the conventional approximation[14–15] of $a x_f + \frac{1}{2} b x_f^2$ in computing $J_{sf}$.

**IV. ANALYSIS AND RESULTS :** We begin by exhibiting the result that the computation of $J_{approx}$ using power expansion in $v^2$ on $< \sigma v >$ is a poor approximation to the full analysis of J where rigorous thermal averaging on the Higgs and Z–Poles is carried out. The ratio of $\Omega_{approx}/\Omega = J/J_{approx}$ is exhibited in Fig. 1. The results of Fig. 1 show that $\Omega_{approx}$ can be inaccurate by up to 3 orders of magnitude and show a total breakdown of the approximate result near the Higgs pole or $Z$ pole.

To proceed further we must include proton stability constraints. In supergravity SU(5), the dominant proton decay proceeds via dimension five operators and involves the Higgs color triplet exchange. The most dominant decay mode is $p \to \bar{\nu} K^+$, and proton stability may be conveniently characterized by the value of the dressing loop function B that enters in $p \to \bar{\nu} K^+$ decay and is defined in Ref 1. The current Kamiokande bound of[16] $\tau(p \rightarrow \overline{\nu}K^+) > 1 \times 10^{32} yr$ translates to a bound on B of[17]

$$B < 105 \left(\frac{M_{H_3}}{M_G}\right) GeV^{-1} \tag{13}$$

where $M_{H_3}$ is the Higgs triplet mass and $M_G$ is the GUT mass. We also note that the simplest GUT sector in SU(5)[18] leads to the relation $M_{H_3}/M_V = (\alpha_\lambda/\alpha_G)^{\frac{1}{2}}$ between the Higgs triplet mass $M_{H_3}$ and the massive vector boson mass $M_V$. [Here $\alpha_\lambda = \lambda_2^2/4\pi$ and $\lambda_2$ enters the GUT superpotential via the term $\lambda_2 H_1(\Sigma + 3M)\bar{H}_2$ where $H_1, \bar{H}_2$ are the 5, $\bar{5}$ and $\Sigma$ is the 24–plet representation of SU(5).] An upper limit on the Higgs triplet mass emerges if one assumes that the Yukawa couplings be perturbative at the GUT scale. Estimates on $M_{H_3}$ that lead to perturbative $\lambda_2$ lie in the range $M_{H_3} < 3M_G$[1,2] to $M_{H_3} < 10M_G$[19]. Here as a guideline we shall use a benchmark limit of $M_{H_3} < 6M_G$.

We discuss now the result of the analysis. We start at the GUT scale with SU(5) supergravity boundary conditions and use the renormalization group equations to evolve masses and coupling constants to low energy where a radiative breaking of the electro–weak symmetry is achieved. Solutions are subjected to constraints of the $CDF$ and $LEP$ data which give lower limits on the SUSY mass spectra, the proton decay constraint of Eq.(13) with $M_{H_3} < 6M_G$, and the relic density constraints discussed in secs I–III. We shall also impose the fine tuning condition $m_{\tilde{q},\tilde{g}} < 1TeV$. The analysis shows that there exists significant domains in the parameter space for both $\mu > 0$ and $\mu < 0$ ($\mu$ is the Higgs mixing parameter which enters the superpotential via the term $\mu H_1 H_2$) where all the desired constraints are satisfied. The allowed parameter space is found to be larger for the case $\mu > 0$.

We discuss the $\mu > 0$ case now in greater detail. Fig. 2 exhibits the allowed domain consistent with proton stability and relic density for the case $m_0 = 700 GeV$ as a function of $A_t$ (where $A_t$ is the t-quark Polonyi constant at the electro–weak scale) when tan $\beta$ and $m_{1/2}$ are varied over the allowed range. Fig. (3) exhibits the allowed domain as a function of $\alpha_H (\tan\alpha_H \equiv \cot\beta)$ at $A_t = 0$ when $m_0$ and $m_{1/2}$ are varied over the allowed range of values. In each of the two cases one finds that the domain of the parameter space consistent with $CDF, LEP$ data, proton stability and relic density is quite substantial even at the lower bound of $B < 300$ GeV$^{-1}$ ($M_{H_3} = 3M_G$).

New upper limits on the Higgs mass and on the chargino mass also emerge. One finds that $m_h \leq 105$ GeV and $m_{\tilde{W}_1} < 100$ GeV for $B < 600$ GeV$^{-1}$. Thus the chargino should be seen at $LEP2$ while for much of the allowed parameter space, the light CP even Higgs should also be seen. The lightest neutralino mass has an upper limit of $\lesssim$50 GeV and the maximum t-quark mass is $\simeq$ 165 GeV. These bounds are lower than the ones given in Ref. 2 where no relic density constraint was imposed. The $h$, $\tilde{W}_1$ and $\tilde{Z}_1$ mass bounds also decrease if one lowers the bound on $B$.

**IV. CONCLUSION :** It is shown that in the physically interesting domain of the parameter space of the standard SU(5) supergravity, annihilation of the relic neutralinos is dominated by the light Higgs and the Z poles. Analysis is given which treats the thermal averaging over the poles rigorously. Previous approximate analyses are found to be inaccurate by several orders of magnitude near the Higgs pole and also significantly inacurate near the Z pole. It is found that for both $\mu > 0$ and $\mu < 0$ significant domains of the parameter space exists where all the desired constraints are satisfied.

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## Figure Captions

Fig. 1 : $\Omega_{approx}/\Omega$ as a function of $m_{gluino}$ for top masses of 110 GeV (dashed curve), 125 GeV (solid curve) and 140 GeV (dotted curve), showing massive breakdown of the approximation near the Higgs and Z poles. The poles occur close to where $\Omega_{approx}/\Omega$ decreases sharply. Note that $\Omega_{approx}$ is least accurate in the region prior to the poles, which is also where $\Omega h^2 < 1$.

Fig. 2 : Allowed domains in the $B - A_t$ plane for top mass of 110 GeV (dashed curves),

125 GeV (solid curves) and 140 GeV (dotted curves) when $m_0 = 700$ GeV. The domain allowed by relic density constraints is the region between the upper and lower curves. The domain allowed by proton stability lies below the solid horizontal line when $M_{H_3} < 6M_G$. The gap in the central region for $m_t = 110$ GeV is due to the requirement that $m_h > 60$ GeV.

Fig. 3 : Allowed domains in the $B - \alpha_H$ ($\tan\alpha_H \equiv ctn\ \beta$) plane for top quark masses of 110 GeV (dashed curves), 125 GeV (solid curves), 140 GeV (dotted curves) and 160 GeV (dot–dash curves), when $A_t = 0$. The domain allowed by relic density constraints and proton stability is as in Fig. 2.

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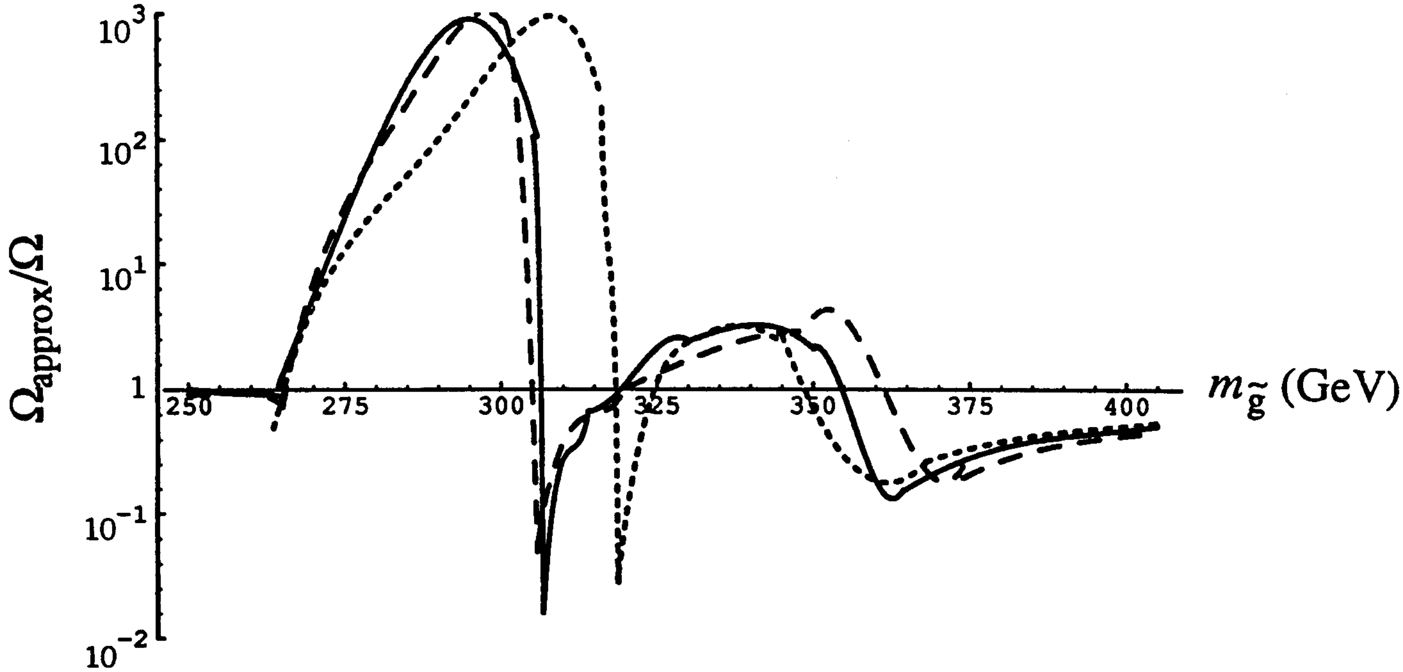

10^3
10^2
10^1
1
10^-1
10^-2
Ω_approx/Ω
250
275
300
325
350
375
400
m_g̃ (GeV)


Fig. 1

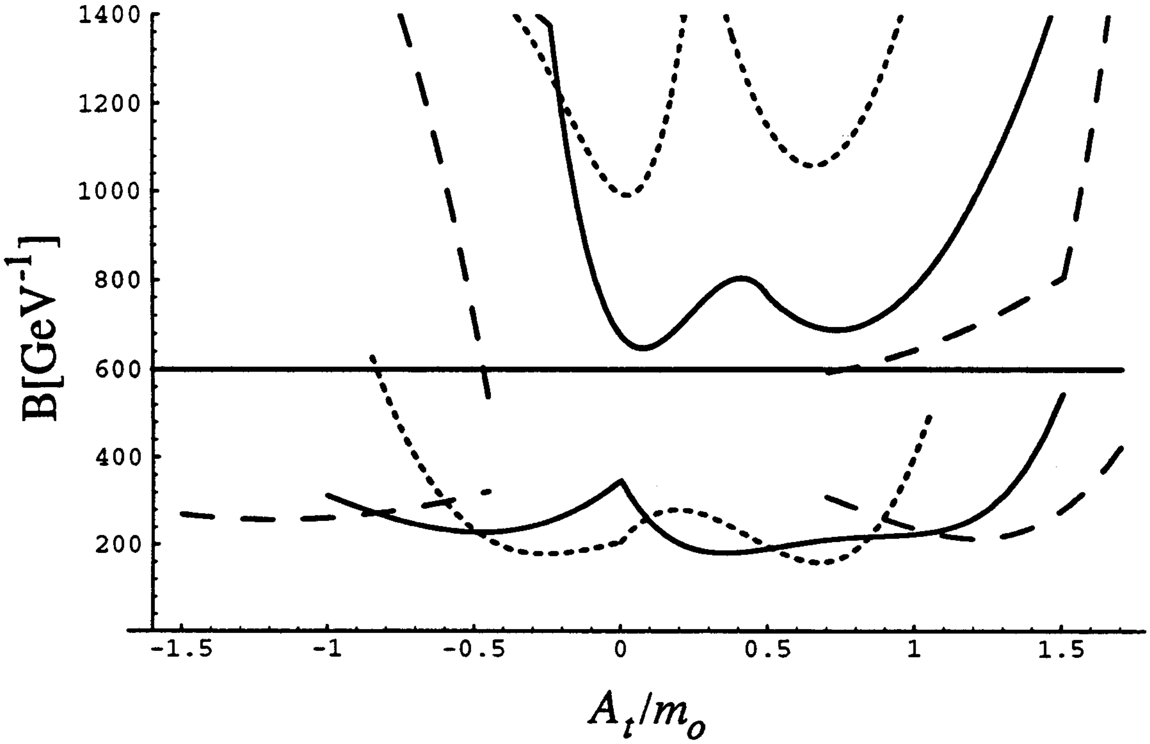


Fig. 2

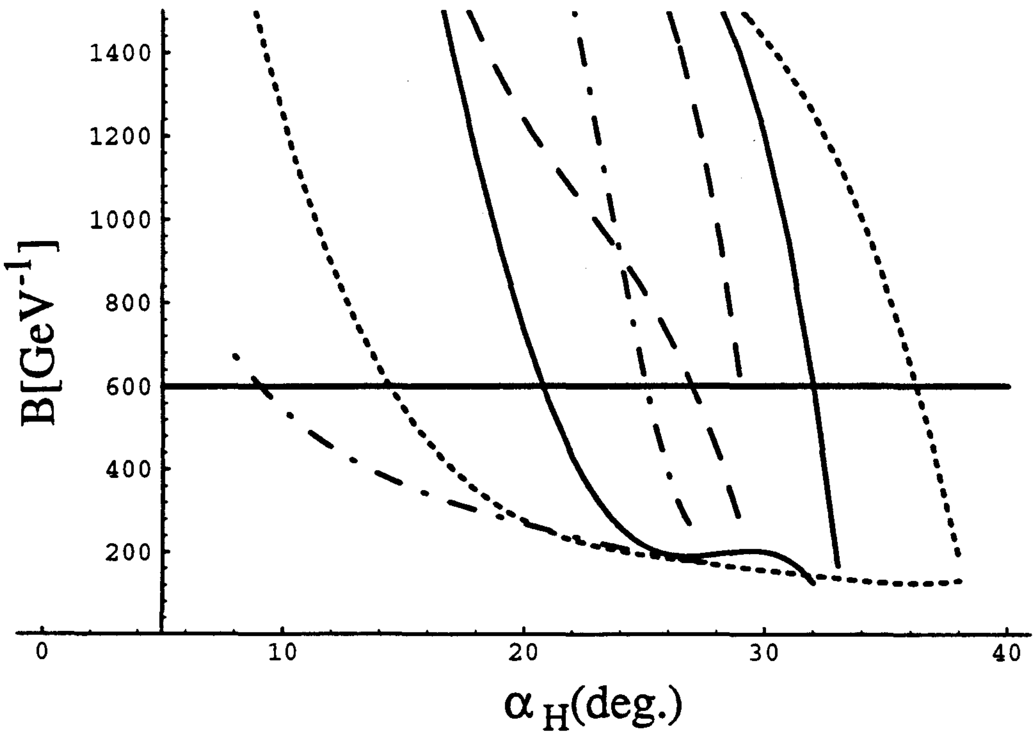


Fig. 3