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Abstract

The decay constants for the D and D_S mesons, denoted f_D and f_{D_S} respectively, are equal in the $SU(3)_V$ limit, as are the amplitudes for $B_S - \overline{B}_S$ and $B^0 - \overline{B}^0$ mixing. The leading contribution to $(f_{D_s}/f_D) - 1$ and to the ratio of hadronic matrix elements relevant for $B_S - \overline{B}_S$ and $B^0 - \overline{B}^0$ mixing amplitudes are calculated in chiral perturbation theory. We discuss the formalism needed to include both meson and anti-meson fields in the heavy quark effective theory.

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The decay constants for the D and D_S mesons are defined by

$$\langle 0| \bar{d}\gamma_\mu\gamma_5 c |D^+(v)\rangle = if_D p_\mu,\tag{1}$$

and

$$\langle 0| \bar{s}\gamma_\mu\gamma_5 c |D_S(v)\rangle = if_{D_S} p_\mu.\tag{2}$$

These decay constants are likely to be measured in the future using the leptonic decays $D^+ \to \mu^+\nu_\mu$ and $D_S \to \mu^+\nu_\mu$. In the chiral limit, where the up, down and strange quark masses go to zero, flavor $SU(3)_V$ is an exact symmetry and so $f_{D_S}/f_D = 1$. However in nature, where $m_s \neq 0$, this ratio will deviate from unity. Neglecting the up and down quark masses, in comparison with the strange quark mass, this deviation has the form

$$f_{D_S}/f_D = 1 + \kappa\frac{M_K^2}{16\pi^2 f^2} \ln\left(M_K^2/\mu^2\right) + \lambda(\mu)M_K^2 + ...\tag{3}$$

where the ellipsis denote terms with more powers of the strange quark mass (recall $M_K^2 \sim m_s$). The dependence of λ on the subtraction point μ cancels that of the logarithm [1]. If μ is of order the chiral symmetry breaking scale then λ has no large logarithms and for very small m_s the term proportional to κ dominates the deviation of f_{D_S}/f_D from unity. Here we compute this logarithmic correction. Of course, in nature, the strange quark mass is not small enough to justify the neglect of the term proportional to λ. However the logarithmic correction is interesting for two reasons. Firstly, as we have already mentioned, in chiral perturbation theory it is formally the leading contribution to the deviation of f_{D_S}/f_D from unity. Secondly, for the pion and kaon decay constants [2], the analogous logarithmic term gives the correct sign for $(f_K/f_\pi - 1)$. The magnitude, however, is too small by about a factor of two.

$B^0 - \overline{B}^0$ mixing and $B_S - \overline{B}_S$ mixing give valuable information on elements of the Cabibo-Kobayashi-Maskawa matrix. One approach is to measure both these mixings, and then extract $|V_{td}/V_{ts}|^2$ from their ratio. This method has the advantage that in the $SU(3)_V$ symmetry limit all dependence on non-perturbative hadronic matrix elements cancels out. (However, because $B_S - \overline{B}_S$ mass mixing is large, it will be very difficult to measure.) The hadronic matrix elements needed for the analysis of $B - \overline{B}$ mixing are

$$\frac{8}{3}f_B^2 B_B = \langle \overline{B}(v)| \bar{b}\gamma^\mu(1-\gamma_5)d\,\bar{b}\gamma^\mu(1-\gamma_5)d |B(v)\rangle,\tag{4}$$

$$\frac{8}{3}f_{B_s}^2 B_{B_s} = \langle \overline{B}_S(v)| \bar{b}\gamma^\mu(1-\gamma_5)s\,\bar{b}\gamma^\mu(1-\gamma_5)s |B_S(v)\rangle,\tag{5}$$

where the f constants for the B meson system are defined by equations analogous to Eqs. (1)(2) for the D meson system. The parameters B_{B_s} and B_B defined by Eqs. (4)(5) are equal in the $SU(3)_V$ symmetry limit. For non-zero strange quark mass, the ratio is no longer unity, and can be written in the form Eq. (3). The logarithmic term will be computed in this paper using chiral perturbation theory. It is convenient to perform the computation for the ratio of B_B's, rather than for the combination $B_B f_B^2$ that occurs in Eqs. (4)(5). Most of the diagrams that occur for $B - \overline{B}$ mixing are the same as those that occur in the computation of the decay constants f_B, and can therefore be dropped in computing the renormalization of B_B.

In Ref. [3] the formalism for applying chiral perturbation theory to mesons containing a heavy quark was developed. It is important that the effective Lagrangian that describes the low momentum interactions of the D and B mesons with the pseudo-Goldstone bosons π, K and η be invariant not only under chiral $SU(3)_L \times SU(3)_R$ symmetry but also under heavy quark spin symmetry. For example, even if one is interested in processes involving only a real D meson, the D^* meson will occur as a virtual particle in Feynman diagrams. The heavy quark symmetry causes the D^* to be almost degenerate with the D so its effects cannot be neglected. It is also important to write the chiral Lagrangian for matter fields such as the D's in terms of velocity dependent fields, to restore the validity of the chiral expansion. The situation here is similar to the case of baryon chiral perturbation theory [4].

The effective Lagrangian that describes the strong interactions of the pseudo-Goldstone bosons with the ground state mesons containing a heavy quark Q is

$$
\begin{aligned}
\mathcal{L} = & \frac{f^2}{8} \operatorname{Tr}\left(\partial^\mu \Sigma \partial_\mu \Sigma^\dagger\right) + \lambda_0 \operatorname{Tr}\left[m_q \Sigma + m_q \Sigma^\dagger\right] - i \operatorname{Tr} \overline{H}^{(Q)a} v_\mu \partial^\mu H_a^{(Q)} \\
& + \frac{i}{2} \operatorname{Tr} \overline{H}^{(Q)a} H_b^{(Q)} v^\mu \left[\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger\right]^b{}_a + \frac{ig}{2} \operatorname{Tr} \overline{H}^{(Q)a} H_b^{(Q)} \gamma_\nu \gamma_5 \left[\xi^\dagger \partial^\nu \xi - \xi \partial^\nu \xi^\dagger\right]^b{}_a \\
& + \lambda_1 \operatorname{Tr} \overline{H}^{(Q)a} H_b^{(Q)} \left[\xi m_q \xi + \xi^\dagger m_q \xi^\dagger\right]^b{}_a + \lambda_1' \operatorname{Tr} \overline{H}^{(Q)a} H_a^{(Q)} \operatorname{Tr}\left[m_q \Sigma + m_q \Sigma^\dagger\right] \\
& + \frac{\lambda_2}{m_Q} \operatorname{Tr} \overline{H}^{(Q)a} \sigma^{\mu\nu} H_a^{(Q)} \sigma_{\mu\nu} + ...
\end{aligned}
$$

(6)

where the ellipsis denote terms with more derivatives, more factors of the light quark mass matrix

$$
m_q = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}
$$

(7)

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or more factors of $1/m_Q$ associated with the violation of heavy quark spin symmetry. The pseudoscalar and vector meson fields $P_a^{(Q)}$ and $P_{a\mu}^{*(Q)}$ form the matrix [5]

$$H_a^{(Q)} = \frac{(1 + \not{v})}{2} \left[P_{a\mu}^{*(Q)} \gamma^\mu - P_a^{(Q)} \gamma_5 \right] .$$

For $Q = c$, $(P_1^{(c)}, P_2^{(c)}, P_3^{(c)}) = (D^0, D^+, D_S^+)$, and similarly for $P_a^{*(c)}$. The field $H_a^{(Q)}$ is a doublet under the heavy quark spin symmetry, and a $\bar{3}$ under flavor $SU(3)_V$,

$$H_a^{(Q)} \to S \left(H^{(Q)} U^\dagger \right)_a .$$

The field $H^{(c)}$ describes D and D^* mesons with definite velocity v. The subscript v on H, P and P_μ^* has been omitted, to avoid complicating the notation. The hermitian conjugate field is defined by

$$\overline{H}^{a(Q)} = \gamma^0 H_a^{(Q)\dagger} \gamma^0 . \tag{8}$$

The pseudo-Goldstone bosons appear in the Lagrangian through $\xi = e^{iM/f}$ ($\Sigma = \xi^2$) where

$$M = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \overline{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix} , \tag{9}$$

and the pion decay constant $f \simeq 135$ MeV. The Lagrangian Eq. (6) is the most general Lagrangian invariant under both the heavy quark and chiral symmetries to first order in m_q and $1/m_Q$.

The left handed current $L_a^\nu = \bar{q}_a \gamma^\nu (1 - \gamma_5) Q$ in QCD can be written in the low energy chiral theory as [3]

$$L_a^\nu = \left(\frac{i\alpha}{2} \right) \operatorname{Tr} \gamma^\nu (1 - \gamma_5) H_b \xi^{\dagger b}{}_a + ..., \tag{10}$$

where the ellipsis denote higher dimension operators in the chiral and heavy quark expansions. The parameter f_P is obtained by taking the matrix element of the current in the pseudoscalar meson state. At lowest order, this fixes $\alpha = f_D \sqrt{m_D}$. The graphs which contribute at one loop are shown in fig. 1.

The computation of the one loop graphs is straightforward. Graph (c) vanishes because of the identity $v^\mu (g_{\mu\nu} - v_\mu v_\nu) = 0$. The renormalization of f_{D_s}/f_D is independent of the overall magnitude α of the current, and is

$$f_{D_s}/f_D = 1 - \frac{5}{6} \left(1 + 3g^2 \right) \frac{M_K^2}{16\pi^2 f^2} \ln \left(M_K^2/\mu^2 \right) . \tag{11}$$

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(The same formula also holds for f_{B_s}/f_B.) The contribution of pion loops is proportional to $M_\pi^2 \ln M_\pi^2$, and has been neglected. The η loops have been written in terms of M_K using the Gell-Mann–Okubo formula $M_\eta^2 = 4M_K^2/3$. The one loop graphs with intermediate P^* states depend on the mass difference $\Delta = m_{P^*} - m_P$ only at order Δ^2. Δ has terms of order $1/m_Q$ as well as terms of order m_s, and is numerically of order M_π. The Δ^2 terms are comparable to the M_π^2 terms, and can be neglected since they are numerically small, and formally of higher order. This simplifies the computation somewhat, since Δ can be set equal to zero before evaluating the Feynman diagrams.

Numerically, the result is that

$$f_{D_s}/f_D = 1 + 0.064\,(1 + 3g^2), \tag{12}$$

using $\mu = 1$ GeV [6]. The experimental limit on $\Gamma(D^* \to D\pi)$ constrains g^2 to be less than 3. The quark model estimate [7] for g is that $g^2 \approx 0.7$ so that we expect $f_{D_s}/f_D \approx 1.2$. The formula (11) can be written in terms of either f_π or f_K. Formally, this ambiguity is of higher order, but it can make a sizeable difference in estimating the correction, since $f_K = 1.25 f_\pi$. The most important terms in Eq. (11) come from virtual K mesons, so we have chosen to use f_K to estimate the correction.

The Lagrangian for $\overline{B}$ mesons is identical in form to Eq. (6), except that the field $H^{(Q)}$ now has $Q = b$ and $P^{(b)}$ and $P_\mu^{*(b)}$ destroy $\overline{B}$ and $\overline{B}^*$ mesons respectively. In the heavy quark effective theory, the field $P_v^{(Q)}$ destroys a meson of velocity v containing a heavy quark Q, but it does not create an anti-meson containing the heavy anti-quark $\overline{Q}$. To describe mesons containing heavy anti-quarks, we have to introduce two new fields, $P_\mu^{*(\overline{Q})}$ and $P^{(\overline{Q})}$ which destroy mesons containing a heavy anti-quark $\overline{Q}$. The phases of the fields $P_\mu^{*(\overline{Q})}$ and $P^{(\overline{Q})}$ are fixed relative to $P_\mu^{*(Q)}$ and $P^{(Q)}$ by charge conjugation. We use the usual phase convention

$$\mathcal{C}\xi\mathcal{C}^{-1} = \xi^T, \qquad P_\mu^{*(\overline{Q})a} = -\mathcal{C}P_{a\mu}^{*(Q)}\mathcal{C}^{-1}, \qquad P^{(\overline{Q})a} = \mathcal{C}P_a^{(Q)}\mathcal{C}^{-1}. \tag{13}$$

The field $H^{(\overline{Q})a}$ is defined by

$$H^{(\overline{Q})a} = c\left(\mathcal{C}H_a^{(Q)}\mathcal{C}^{-1}\right)^T c^{-1} = \left[P_\mu^{*(\overline{Q})a}\gamma^\mu - P^{(\overline{Q})a}\gamma_5\right]\frac{(1-\slashed{v})}{2}. \tag{14}$$

The matrix c is the charge conjugation matrix for Dirac spinors, $c = i\gamma^2\gamma^0$, and the transpose is on the spinor matrix indices. The transpose and c matrices are necessary to ensure

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that $H^{(\overline{Q})}$ transforms as a bispinor under the Lorentz group in the same respresentation as $H^{(Q)}$. $H^{(\overline{Q})}$ transforms as a $(\overline{2},3)$ under the heavy spin $\otimes$ $SU(3)_V$ flavor symmetry,

$$H^{(\overline{Q})a} \rightarrow \left(UH^{(Q)}\right)^a S^\dagger.$$

The hermitian conjuate field is defined by

$$\overline{H_a^{(\overline{Q})}} = \gamma^0 H^{(\overline{Q})a\dagger}\gamma^0.$$

The Lagrangian for B mesons in terms of $H^{(\overline{b})}$ fields is obtained from Eq. (6) by setting $Q = b$ and applying charge conjugation.

The $\Delta b = 2$ operator which produces $B - \overline{B}$ mixing in the standard model is

$$\mathcal{O}^{aa} = \overline{b}\gamma_\mu(1 - \gamma_5)q^a \ \overline{b}\gamma^\mu(1 - \gamma_5)q^a,$$

where $a = 2, 3$ for B^0 and B_S mixing respectively. (Note that the repeated index a is not summed.) The operator $\mathcal{O}^{aa}$ transforms as the 22 (or 33) component of the six dimensional representation of flavor $SU(3)_L$. In the effective theory, the operator $\mathcal{O}^{aa}$ can be written as

$$\mathcal{O}^{aa} = \beta \ \text{Tr} \ (\xi\overline{H}^{(b)})^a\gamma_\mu(1 - \gamma_5) \ \text{Tr} \ (\xi H^{(\overline{b})})^a\gamma^\mu(1 - \gamma_5) + \dots. \tag{15}$$

Evaluating the traces gives

$$\mathcal{O}^{aa} = 4\beta \left[(\xi P_\mu^{*(b)\dagger})^a(\xi P^{*(\overline{b})\mu})^a + (\xi P^{(b)\dagger})^a(\xi P^{(b)})^a\right] + \dots, \tag{16}$$

so that the amplitude for $B - \overline{B}$ mixing is the negative (since the polarization of a physical B^* is spacelike) of that for $B^* - \overline{B}^*$ mixing. This relation between the two amplitudes can be proved directly by an application of the heavy quark spin-symmetry. The operator $\mathcal{O}^{aa}$ can potentially match onto many different operators in the effective theory, such as $\text{Tr} \ (\xi\overline{H}^{(b)})^a\gamma_\mu(1 - \gamma_5)(\xi H^{(\overline{b})})^a\gamma^\mu(1 - \gamma_5)$. However, all the operators are proportional to Eq. (15), because the spin symmetry requires that the B and B^* mixing amplitudes be the negative of each other. The $SU(3)_L \otimes SU(3)_R$ transformation property of $\mathcal{O}^{aa}$ then uniquely fixes the chiral structure of the operator. The chiral corrections to B_B are given by the graphs in fig. 2. Only η graphs contribute to the correction to B_{B_S}/B_B. K mesons cannot contribute to the graphs in Fig. 2a because of flavor conservation. The $BB^*\pi$ coupling constant is the negative of the $\overline{B}\,\overline{B}^*\pi$ coupling constant, because of the phase convention for charge conjugation chosen in Eq. (13). The two meson vertex in Fig. 2b

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is obtained by expanding Eq. (16) to second order in the meson fields. The terms where either ξ is expanded to second order in the meson fields are identical to those that occur in the renormalization of f_B, and can be omitted. The term where each ξ is expanded to first order in the meson field is new. It has terms of the form ξ_a^a, and so only η mesons contribute to Fig. 2b. The chiral correction to B_{B_s}/B_B is

$$\frac{B_{B_s}}{B_B} = 1 - \frac{2}{3}\left(1 - 3g^2\right)\frac{M_K^2}{16\pi^2 f^2}\ln\left(M_K^2/\mu^2\right), \tag{17}$$

where we have again used $M_\eta^2 = 4M_K^2/3$. Numerically, the correction is $B_{B_s}/B_B \approx 0.9$, using $\mu = 1$ GeV, $f = f_K$, and $g^2 = 0.7$ as before. The renormalization of B_{B_s}/B_B is a violation of factorization in the hadronic matrix elements for $B - \overline{B}$ mixing. (The perturbative QCD corrections to $B - \overline{B}$ mixing that contain large logarithms of m_b/Λ_{QCD} do factorize [8].) The overall ratio of the hadronic matrix elements for $B - \overline{B}$ mixing is obtained by combining Eqs. (11) and (17),

$$\frac{B_{B_s}f_{B_s}^2}{B_B f_B^2} = 1 - \left(\frac{7}{3} + 3g^2\right)\frac{M_K^2}{16\pi^2 f^2}\ln\left(M_K^2/\mu^2\right), \tag{18}$$

which is numerically about 1.3 for $\mu = 1$ GeV, $f = f_K$, and $g^2 \approx 0.7$.

Our results may be useful in estimating the difference between the values of f_{D_s}/f_D and B_{B_s}/B_B in the quenched approximation to QCD and their values in nature [9]. The logarithmic corrections we have calculated necessarily involve quark loops, and so would not be seen by lattice Monte Carlo calculations that use the quenched approximation [9][10].

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Figure Captions

Fig. 1. The graphs contributing to the renormalization of f_P. The solid square denotes the axial vector current vertex. The pseudoscalar and vector mesons P and P^* can be either the D and D^*, or the B and B^*. Graph (a) is the tree level contribution. Graph (b) is the wavefunction renormalization correction and is proportional to g^2. Graph (c) vanishes identically. Graph (d) is independent of g.

Fig. 2. Graphs producing a renormalization of B_B. The dot is the $\Delta b = 2$ operator. Only virtual η particles contribute.
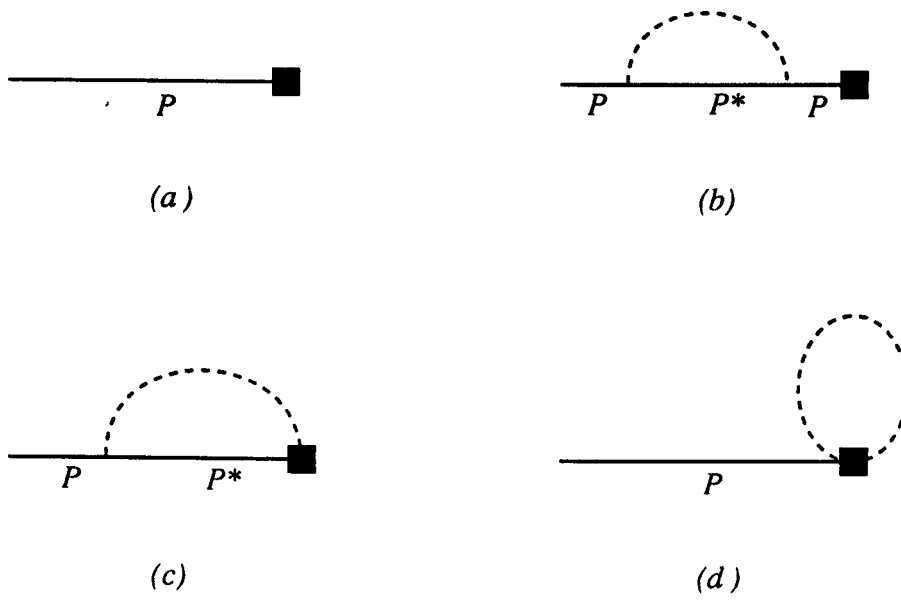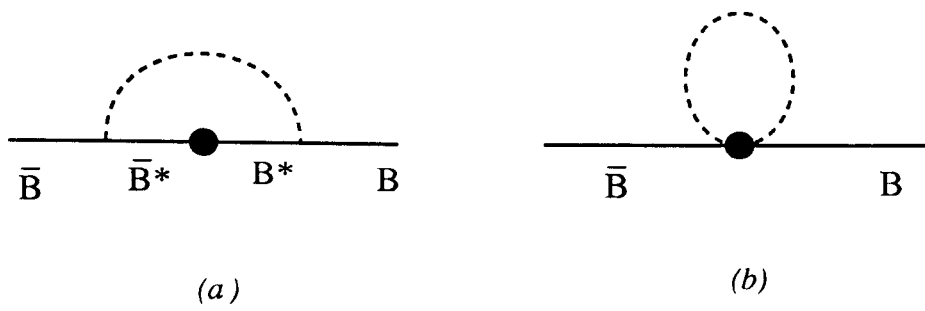
(a)

(b)

(c)

(d)

Figure 1

(a)

(b)

Figure 2