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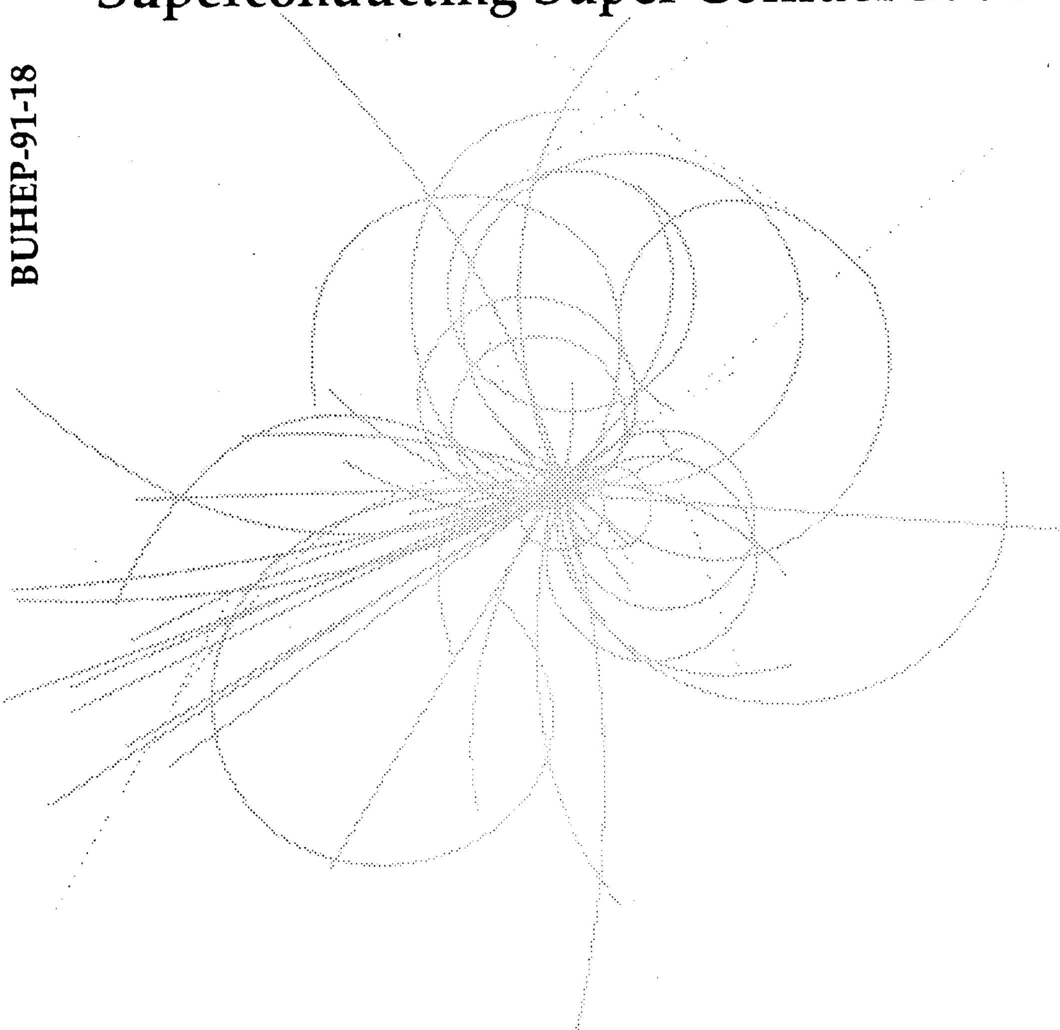

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On the Hilbert Space of the Heavy Quark Effective Theory

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We give a careful description of the Hilbert Space of the Heavy Quark Effective Theory. States must carry both velocity and momentum labels. We show that this formulation is necessary to resolve a paradox which arises in the effective theory when considering the matrix element of a light quark current between heavy meson states.

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1. Introduction

Over the past decade, there has been a great deal of study of the charm and bottom hadrons. From their decays fundamental parameters of the standard model, such as quark masses and mixing angles, can be extracted. Rare decays will test our understanding of fundamental interactions and a measurement of CP violation in B–meson decays will for the first time test the Kobayashi–Maskawa theory of CP violation with experiments not involving strange quark decay.

In making theoretical predictions that confront experimental results, one faces the difficulty of computing amplitudes that necessarily involve strong interactions. A method of calculation is afforded by the use of the newly discovered symmetries of QCD. These symmetries apply only to Green functions involving heavy quarks that are almost on shell. They result from the observation that, in a restricted kinematic region, the Green functions are finite in the limit of an infinitely massive heavy quark. The resulting mass independent Green functions therefore display a flavor symmetry. Moreover, since spin–flip interactions vanish in the infinite mass limit, there is, in addition, a symmetry between the different spin degrees of freedom of the heavy quarks.

A heavy quark effective theory (HQET) can be used to compute these mass independent Green functions[1]. There are several advantages to using a HQET. The Feynman rules are simpler than in the finite mass theory. More importantly, the renormalizable divergences in the HQET dictate the violations to the flavor symmetry[2]–[3] of the large mass limit that arise because of violations to scaling[4].

As it is usually formulated, the states of the HQET depend on the velocity v of the heavy quark. Since the effective theory is an expansion around infinite mass, the velocity of the heavy quark cannot be changed except by insertion of external operators. This is called the "velocity superselection rule"[5].

In this paper we attempt to understand the meaning of the superselection rule. That there is an issue can be seen in a variety of ways. Consider the relation between the field of velocity v_μ and the heavy–light meson that contains this quark. Is the meson's momentum Mv_μ or does it differ by some amount k_μ of order of the hadronic scale Λ? Is M the mass of the meson or of the quark?

We consider an example which nicely illustrates these concerns — the form factors of the light quark current in a heavy meson state. We will see that in order to properly interpret results of the HQET, it is important to formulate the theory carefully. We will comment on a related issue brought up in a recent paper on strong decays of heavy mesons[6].

2. A Paradox

One major result of the HQET is that the form factors for decays of $B \to D$ mesons are all (to lowest order in $1/M$) determined by a single function of the velocities involved [2]:

$$\begin{aligned}
\langle \bar{D}(v)|V_\mu|B(v')\rangle &= \sqrt{M_c M_b}\,\xi(v'\cdot v)(v_\mu + v'_\mu) \\
\langle \bar{D}^*(v),\varepsilon|A_\mu|B(v')\rangle &= \sqrt{M_c M_b}\,\xi(v'\cdot v)(\varepsilon^*_\mu(1+v'\cdot v) - v_\mu v'\cdot\varepsilon^*) \\
\langle \bar{D}^*(v),\varepsilon|V_\mu|B(v')\rangle &= \sqrt{M_c M_b}\,\xi(v'\cdot v) i\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}v'^\alpha v^\beta
\end{aligned} \tag{2.1}$$

Here $V_\mu(A_\mu)$ stands for the vector (axial) current $\bar{b}\gamma_\mu(\gamma_5)c$. The so–called Isgur–Wise function $\xi(v'\cdot v)$ is in fact the b–number current form factor of the B–meson

$$\langle B(v)|\bar{b}\gamma_\mu b|B(v')\rangle = M_b\xi(v'\cdot v)(v_\mu + v'_\mu). \tag{2.2}$$

Since b–number is conserved, one has

$$\xi(1) = 1. \tag{2.3}$$

In contrast, consider the electromagnetic form factor of the B meson. The contribution from the heavy quark current is given by (2.2), up to a factor of the charge Q_b of the heavy quark. Focus, then, on the contribution from the light quark current, $\bar{q}\gamma_\mu q$. Consider

$$\langle B(v')|\bar{q}\gamma^\mu q|B(v)\rangle = M_b\chi(v'\cdot v)(v_\mu + v'_\mu). \tag{2.4}$$

If the matrix element is written as a function of v, the superselection rule immediately gives

$$\chi(v'\cdot v) = \begin{cases} 1 & \text{if } v'\cdot v = 1; \\ 0 & \text{if } v'\cdot v \neq 1. \end{cases} \tag{2.5}$$

For $v'\cdot v \neq 1$ there is no way for the light quark current to change the velocity of the heavy quark, since all of the heavy quark interactions preserve velocity. On the other hand, interactions are allowed for $v' = v$, and while this is a nonperturbative calculation, q–number conservation gives the desired normalization at $v'\cdot v = 1$.

If, like $\xi(v'\cdot v)$, we interpret $\chi(v'\cdot v)$ as a physical form factor, then this behaviour seems incorrect. In general, a form factor is an analytic function of the momentum transfer, and χ is not. Even including $1/M$ corrections will not improve the situation. In the next two sections we will explain what went wrong.

3. The Hilbert Space of the Heavy Quark Effective Theory

In this section we will briefly review the formulation of the heavy quark effective theory and then we will describe the Hilbert space of the theory in some detail. This will allow us, in the next section, to resolve the paradox of the previous section.

We begin by considering the case of a free field theory of a single massive fermion ψ. To obtain the lagrangian of the HQET proceed as follows[5]. First, change variables according to

$$Q_v = \exp(iMv\cdot x)\psi \ , \tag{3.1}$$

where v is an arbitrary velocity vector with $v^2 = 1$. Then project out the particle field

$$h_v = \frac{1+\not v}{2} Q_v \tag{3.2}$$

and integrate out the antiparticle field

$$h_v^- = \frac{1-\not v}{2} Q_v \ . \tag{3.3}$$

Finally, sum over velocities v. To leading order in $1/M$, the HQET lagrangian is[1]

$$\mathcal{L} = \sum_v \mathcal{L}_{(v)} = \sum_v \bar{h}_v iv\cdot\partial h_v \ . \tag{3.4}$$

The spinor h_v satisfies the constraint

$$\not v h_v = h_v \ . \tag{3.5}$$

In this simple theory, the equation of motion

$$v\cdot\partial h_v = 0 \ , \tag{3.6}$$

can be solved exactly. The propagator is

$$\langle \mathrm{T}(h_v(x)\bar{h}_{v'}(0))\rangle = \delta_{vv'}\left(\frac{1+\not v}{2}\right)\theta(x^0)\frac{1}{v^0}\delta^{(3)}\left(\vec{x} - \frac{\vec{v}x^0}{v^0}\right) \ . \tag{3.7}$$

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[1] In ref. [5] the sum was represented as an integral. Georgi emphasized in ref. [7] that it must be represented as a sum. The presence of the Kronecker rather than the Dirac delta function in equation (3.15) below makes it clear that a sum is required. This may be thought of as a sum over a dense but countable set of velocities distributed uniformly on the mass shell.

What is the Hilbert space upon which the theory (3.4) is built? To answer this question we canonically quantize. The momentum conjugate to the field h_v is

$$\pi_v = \frac{\delta \mathcal{L}}{\delta(\partial_0 h_v)} = i\bar{h}_v v^0 \ . \tag{3.8}$$

The canonical equal-time anti-commutation relation is

$$\{\pi_{v'}(x), h_v(y)\}|_{x^0=y^0} = i\delta_{v'v}\left(\frac{1+\not{v}}{2}\right)\delta^{(3)}(\vec{x}-\vec{y}) \ . \tag{3.9}$$

The fourier transform of h_v is

$$h_v(x) = \sum_s \int \frac{d^3k}{(2\pi)^3 v^0} e^{-ik\cdot x} b^{(s)}_{v,k} u^{(s)}(v) \ , \tag{3.10}$$

where in order to satisfy eq. (3.5), the spinor u obeys $\not{v}u = u$, and, using the equation of motion (3.6), k^0 is defined through $v \cdot k = 0$. One obtains the following anti-commutation relations for the creation and annihilation operators:

$$\{b^{(s)}_{v,k}, b^{(s')\dagger}_{v',k'}\} = (2\pi)^3 v^0 \delta_{ss'}\delta_{vv'}\delta^{(3)}(\vec{k}-\vec{k}') \ . \tag{3.11}$$

It is now straightforward to construct the Hilbert space of this HQET. The single particle states $|v, k, s\rangle$ are obtained by letting the creation operator $b^{(s)\dagger}_{v,k}$ act on the vacuum. The full Hilbert space is the Fock space built out of these single particle states. However, in this paper we will focus on states containing at most one heavy particle.

What is the relation between this Hilbert space and that of the original theory? To investigate this question we consider the relation between the momentum operator of the full and effective theories. The momentum operator for a fermion is

$$P^\mu = \int d^3x\, \bar{\psi}\gamma^0 i\partial^\mu \psi \ . \tag{3.12}$$

We wish to consider the operator $P^\mu - Mv^\mu N$, where N is the number operator $\int d^3x\, \bar{\psi}\gamma^0\psi$. If we perform the field redefinitions leading to the HQET, we find

$$P^\mu - Mv^\mu N = \int d^3x\, \bar{h}_v v^0 \partial^\mu h_v + \dots \ , \tag{3.13}$$

where the elipsis represents terms higher order in $1/M$. It is easy to see that this agrees with what one obtains by deriving the momentum operator directly from $\mathcal{L}_{(v)}$. We call this lowest order operator $K^\mu_{(v)}$. In terms of the creation and annihilation operators

$$K^\mu_{(v)} = \sum_s \int \frac{d^3k}{(2\pi)^3 v^0} b^{(s)\dagger}_{v,k} b^{(s)}_{v,k} k^\mu \tag{3.14}$$

where, in the integral, k^0 is defined by $v \cdot k = 0$. The state $|v, k, s\rangle$ is an eigenvector of $K_{(v)}$ with eigenvalue k.

The operators $b^{(s)}_{v,k}$ above create states normalized according to:

$$\langle v', k', s'|v, k, s\rangle = (2\pi)^3 v^0 \delta_{s,s'} \delta^3(k - k') \delta_{v,v'} \ , \tag{3.15}$$

where $\delta_{v,v'}$ is a Kronecker delta function: 1 if $v = v'$, 0 otherwise. It is there to enforce the velocity superselection rule. Note that these states are dimension $-3/2$; they differ by a factor of $1/\sqrt{M}$ from a one particle state in the more conventional normalization. If the states in the full theory are normalized so as to refrain from referring to the mass M of the heavy quark in the normalization of the states:

$$\langle p'|p\rangle = (2\pi)^3 \frac{E}{M} \delta^3(p - p') \ , \tag{3.16}$$

then the normalization choice (3.15) would simply be the limit $M \to \infty$ of this normalization of states in the full theory.

The HQET overcounts (infinitely many times) the degrees of freedom of the original theory. To leading order, any one-particle state with a given value of $p = Mv + k$ can be used to represent the state $|p\rangle$ of the full theory —though the approximation degrades as $|\vec{k}|$ approaches M. The different sectors do not talk to each other. Therefore to any given order in $1/M$ the HQET has a symmetry: $v^\mu \to v^\mu + \delta v^\mu$, for any δv^μ which preserves $v^2 = 1$. This symmetry group is rather trivial – there is no dynamics associated to the label v. To any order in $1/M$, the Lagrangian will never couple states built on two different velocities. This is the origin of the velocity superselection rule.

How do we match the currents in the full and effective field theories? To address this question, we include in both a light quark q, still without turning on QCD. First, we consider the heavy quark current $\bar{\psi}\Gamma\psi$. The steps leading to the HQET would give

$$\bar{\psi}\Gamma\psi \to \sum_{v,v'} e^{iM(v-v')\cdot x} \bar{h}_v \Gamma h_{v'} \ . \tag{3.17}$$

This operator satisfies the condition that we can use any state with $p = Mv+k$ to represent the full theory state $|p\rangle$. The term above with $v = v'$ would be useful when both the initial and final state quarks have velocities near v, but not in the general case. If the current transfers a momentum large compared to M, then at least one of the initial or final states will have very large k, and the HQET will be outside of its domain of validity. When this happens we should use states with different v's, so that the initial state has momentum near Mv and the final state near Mv'. Both the intial and final state k's will be small.

The matching of the current $\bar{q}\Gamma\psi$ is similar to the case above, but what about $\bar{q}\Gamma q$? The obvious matching is to $\bar{q}\Gamma q$, since the HQET is exactly the same as the full theory as far as the light quarks are concerned. Since we are still talking about free field theory, the heavy and light quarks are completely decoupled.

Nonetheless, in the one heavy quark sector, there are other terms in the representation of $\bar{q}\Gamma q$! Consider the operator $\mathbf{1}$ in the full theory. Its matrix elements in the one heavy particle sector are given by equation (3.16). Because any state with $p = Mv+k$ can be used to represent the full theory state $|p\rangle$, the operator $\mathbf{1}$ in the full theory is *not* represented by $\mathbf{1}$ in the HQET. Instead it is

$$\mathbf{1} \rightarrow \mathbf{1}^{full} = \sum_{v,v'} \mathbf{1}_{vv'} \tag{3.18}$$

where

$$\mathbf{1}_{vv'} = \sum_{s} \int \frac{d^3k}{(2\pi)^3\sqrt{v^0 v'^0}} b^{(s)\dagger}_{v',k'} b^{(s)}_{v,k} \ , \tag{3.19}$$

where $k' = M(v-v') + k$. The operator $\mathbf{1}^{full}$ is an operator in the HQET. It has matrix elements

$$\langle v', k', s' | \mathbf{1}^{full} | v, k, s\rangle = \delta_{s,s'} v^0 (2\pi)^3 \delta^3(M(\vec{v}-\vec{v}') + \vec{k} - \vec{k}') + O(1/M) \tag{3.20}$$

which is the correct HQET analogue of (3.16). It is important to emphasize that the derivation of (3.19) assumes that v is near v', *ie.* that $v - v'$ is of order $1/M$. While it is in principle possible to extend the operator $\mathbf{1}^{full}$ to the case where v is not near v', for the free field theory it is not particularly useful. In that case, at least one of the residual momenta k or k' in (3.20) would have to be large, out of the domain of validity of the HQET.

The operator $\mathbf{1}_{vv'}$ satisfies $\mathbf{1}^{\dagger}_{vv'} = \mathbf{1}_{v'v}$. We may write it in terms of the fields h:

$$\mathbf{1}_{vv'} = \sqrt{v^0 v'^0} \int d^3\vec{x}\, \bar{h}_{v'}(x'^0, \vec{x} - \vec{\delta}_-) h(x^0, \vec{x} + \vec{\delta}_-) e^{iM(\vec{v}-\vec{v}')\cdot(\vec{x}-\vec{\delta}_+)} \ , \tag{3.21}$$

where

$$\delta_{\pm} = \frac{1}{2}\left(\frac{x^0}{v^0}\vec{v} \pm \frac{x'^0}{v'^0}\vec{v}'\right) \ . \tag{3.22}$$

This expression appears to depend on x^0 and x'^0, the time components of the positions at which the two fields are evaluated. Using the equations of motion for the fields h it is possible to show that this dependence is fictitous.

Note that $\mathbf{1}_{vv'}$ is not a local operator. This is not too surprising, since $\mathbf{1}$ wasn't local in the full theory.

Turning back to the current $\bar{q}\Gamma q$ of the full theory, we see that in the one-heavy-quark sector, the HQET operator for it to match onto is $\bar{q}\Gamma q \mathbf{1}^{full}$. This operator is not local either! Only if we confine ourselves to the part which is diagonal in v and v' do we get to use the simple $\bar{q}\Gamma q$.

We now switch on QCD. The spectrum no longer contains free quarks, only mesons and baryons. A meson state in the full theory $|B(p)\rangle$ may be approximated by any state in the effective theory $|B, v, k\rangle$, with $p = Mv + k$. Of course, not all such states are created equal: we note that inside a heavy–light meson, the heavy quark always has a velocity near that of the meson. Thus we choose v to be near the velocity of the meson. When the interactions are turned on, the matching of the currents is more complicated than in the free theory case, and in general will require the evaluation of diagrams with loops of gluons[8]. However, the basic results of this section will continue to hold. The current representing the operator $\bar{q}\Gamma q$ is a sum of terms, each of which connects different velocity superselection sectors, and only the terms in which $v = v'$ are local. The heavy quark bilinear will look like (3.17)– it is renormalized, but it remains local.

4. The Resolution of the Paradox

We can now use the results of the previous section to resolve the paradox of section two.

Consider first the derivation of the relationships (2.1). The operator representing the the heavy quark bilinear is inserted between states representing the initial and final state mesons. Since we can chose any state with $p = Mv + k$ to represent the meson, we pick[2] the one in which $\vec{k}$ is parallel to $\vec{v}$, and therefore both are parallel to $\vec{p}$. This is implicitly what authors have been doing when they have written the states in the HQET as depending only on velocity, $|B(v)\rangle$. Using the symmetries of the lagrangian (3.4), which allow the rotation

[2] We can't chose $k = 0$, because the mass of the meson is not equal to the mass M of the quark, which we have used to construct the HQET.

of a heavy quark of one spin into a heavy quark of any spin and flavor, the relationships (2.1) hold to lowest order in $1/M$.

How is the light quark current different? What we want is the matrix element

$$\langle B, v', k' | (\bar{q}\Gamma q) \mathbf{1}^{full} | B, v, k \rangle \; . \tag{4.1}$$

In principle, we are free to choose $\vec{k}, \vec{k}'$ parallel to $\vec{v}, \vec{v}'$ as before. However, if we do so, we will have to confront calculations involving the non–local operator $\mathbf{1}_{vv'}$. Moreover $\mathbf{1}_{vv'}$ is a rapidly varying function of $v - v'$. (Recall that we had to assume that $v - v'$ was of order $1/M$ in order to construct $\mathbf{1}_{vv'}$.) Instead, it is much simpler to chose both the initial and final states to be built over the *same* velocity vector[3] v, and therefore to have at least one of k and k' not parallel to v.

Consider for definiteness the light quark vector current. In the full theory the matrix element looks like

$$\langle B(p') | (\bar{q}\gamma^\mu q) | B(p) \rangle = g((p - p')^2)(p^\mu + p'^\mu) \tag{4.2}$$

where we have used the conservation of the light quark current to eliminate the other possible form factor. In the HQET we will have

$$\langle B, v, k | (\bar{q}\gamma^\mu q) | B, v, k' \rangle = 2M g((k - k')^2) v^\mu \tag{4.3}$$

Note that just because the initial and final state velocity state labels are equal does not mean that the momentum transfer is zero. We have chosen instead to write the matrix element as a function of $q = p - p' = k - k'$. One may choose v to be parallel to either p or p' but not both!

Using the spin and flavor symmetries of the lowest order HQET lagrangian, we conclude that the form factor above is independent of the spin and flavor of the heavy meson. The function $g(q^2)$ is normalized to 1 at $q^2 = 0$, and varies on the hadronic scale Λ^2. This is simply the statement that the momentum of the light quark inside the meson does not grow with M.

This fast behavior of the form factor of the light quark current is what distinguishes it from that of the heavy quark current. In the latter case the form factor is the slowly varying Isgur–Wise function, $\xi(v \cdot v')$. It is independent of the precise choice of v and v'

[3] If we had used a Dirac, rather than a Kronecker delta function in (3.15), and an integral rather than a sum in (3.4), we would not be able to use this procedure.

because, if we change $v \to v + k/M$, then $\xi(v \cdot v') \to \xi(v \cdot v') + \mathcal{O}(1/M)$. The form factor of the light current, on the other hand, has a rapid behavior, so that it cannot be written as an M independent function of $v \cdot v'$. If it did depend on $v \cdot v'$ in an M independent way, as assumed in the statement of the paradox in section 2, it would lead to great ambiguities from the choice of velocities. The resolution is to regard it as a function of the residual momenta.

5. A Related Issue

It has been suggested[6] that amplitudes for hadronic decays of excited heavy hadrons to a light hadron and another heavy hadron are related by the spin symmetry. In particular, it is claimed that the four amplitudes for the decays of the D_1 and the D_2^* mesons to a pion and either a D or D^* meson are all related. This would follow from consideration of the light current matrix element

$$\frac{1}{f_\pi}\langle (J', m'), v', k' | \bar{q}_1 \gamma^\mu \gamma_5 q_2 | (J, m), v, k\rangle \ , \tag{5.1}$$

where (J, m) labels the angular momentum state of the heavy hadron. This matrix element will be given in terms of form factors $g_i(q^2)$. One obtains the amplitude for the transition $(J, m) \to (J', m') + \pi$ by extracting the residue at the pole at $q^2 = m_\pi^2$.

The discussion of the previous section implies that the form factors g_i are rapidly varying functions of q^2. However, all the decays considered here have the same q^2, namely m_π^2. Thus, the symmetry relations derived in ref. [6] will hold to lowest order in $1/M_c$.

However, $M_{D_2^*} - M_D = 594$MeV while $M_{D_1} - M_{D^*} = 414$MeV, and the difference, 180 MeV, is a substantial fraction of the typical scale of the interactions of the light degrees of freedom, Λ. This might cause us to question whether the next–to–leading corrections are really under control. For example, in ref. [6] there were huge corrections generated by this difference when it was inserted into the "phase space factor", $|\vec{p}_h|^{2L+1}$. In $D_2^* \to D\pi$, the momentum of the pion is 504 MeV, while in $D_1 \to D^*\pi$, it is 358MeV. Since the angular momentum of the pion is two, this correction to the amplitude is a factor of $(504/358)^{5/2}$ $= 2.35$! At lowest order in $1/M_c$, we should use a common mass for the various members of a D multiplet. However, as we just saw, there are rather large mass splittings within multiplets. Formally these effects are order $1/M_c$. However, they are large numerically. The phase space factor is only one of the next–to–leading corrections.

On the other hand, the phase space factor may be the dominant $1/M_c$ correction. One situation where this occurs is in K^* decays, in which the largest SU(3) breaking effects come from the substitution of the physical masses into the phase space factors[4]. As we have seen, the phase space factors can generate factors of two in the amplitude, but one hopes that the corrections to the vertices are suppressed by Λ/M_c times a number of order 1.

It is hard to see whether the situation will be much improved for the corresponding b-quark system. There the difference between splittings is expected to be reduced from 180MeV to 60MeV.

6. Conclusions

We have seen that the proper formulation of the HQET has the states labeled both by their velocity v and their HQET momentum k. This is, at first, surprising. But, as we have seen, it is necessary if we are to understand the behavior of the form factors of currents of light quarks between heavy hadron states. For matrix elements of currents of heavy quarks between heavy hadron states we have seen why one can simplify the analysis by neglecting to include the momentum label k. Thus, happily, the existing results[9] on relations between form factors are well justified.

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[4] We thank Nathan Isgur and Mark Wise for pointing this out.

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