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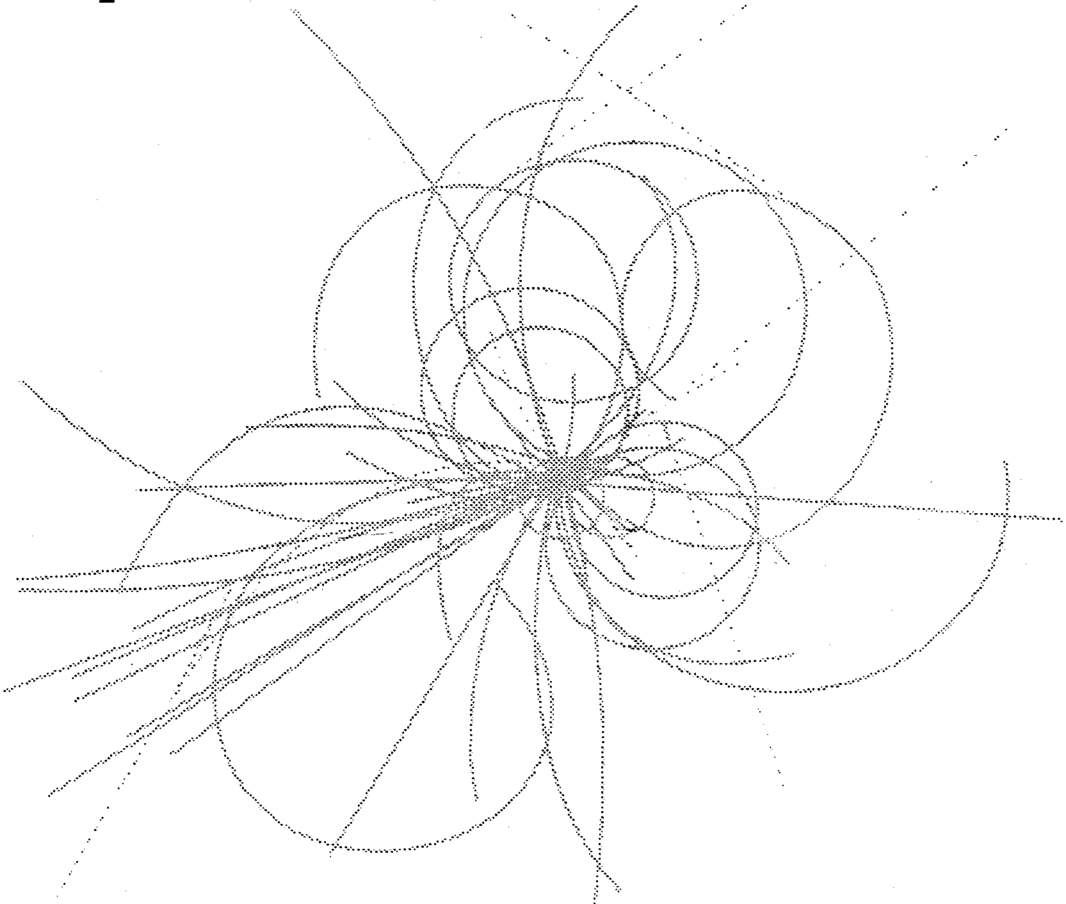

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RF Noise Tolerances at the SSC

H.-J. Shih, J.A. Ellison, B.S. Newberger, D. Coleman, and J. Ferrell

Abstract

We apply the diffusion in action theory developed by Dôme, Krinsky, and Wang to the determination of SSC rf noise tolerances using the emittance-doubling time as a criterion. We present results for white amplitude and white phase noise, noise measured from a HP synthesizer and from a SLAC PEP klystron. We also derive a scaling law for white amplitude and phase noise that allows one to understand the dependence of the diffusion process on the rf frequency. Lastly we qualitatively discuss the implementation of feedback loops to reduce rf noise.

CONTENTS

FIGURES

1.0 INTRODUCTION

In this report we use the Dôme, Krinsky, Wang (DKW) diffusion in action theory[1–3] to discuss acceptable noise levels in the Superconducting Super Collider (SSC) rf cavity. Our criteria will be stated in terms of longitudinal emittance-doubling times.

The longitudinal equations of motion with amplitude and phase noise are obtained from the Hamiltonian:

$$H = \frac{1}{2}P^2 + \Omega^2\big(1 + a(t)\big)U(\phi) + P\dot{\varphi}(t), \tag{1.1}$$

where $a(t)$ and $\varphi(t)$ are amplitude and phase noise, respectively, $U(\phi) = 1 - \cos\phi$, $P = (2\pi h\eta/T_0)(\Delta p/p_s) = \omega_{\mathrm{rf}}\eta\,(\Delta p/p_s)$, ϕ is the rf phase, and $\Omega^2 = \omega_{\mathrm{rf}}\eta(eV/p_s v_s)/T_0$ is the square of the small amplitude synchrotron frequency. At the SSC, $f_{\mathrm{rf}} = \omega_{\mathrm{rf}}/2\pi = 360\,\mathrm{MHz}$, $eV = 20\,\mathrm{MeV}$, $p_s = 20\,\mathrm{TeV/c}$, and $v_s \simeq c$; thus, $\Omega = 26.6\,\mathrm{rad\,sec^{-1}}$, corresponding to 4.23 Hz, which is small compared with the beam frequency $1/T_0 = 3.44\,\mathrm{kHz}$. Because of this and because of the "mixing" due to the nonlinearity of the potential, the DKW theory predicts that the action $J = \frac{1}{2\pi}\oint P d\phi$ evolves approximately according to

$$\frac{\partial\rho}{\partial t} = \frac{\partial}{\partial J} D(J)\,\frac{\partial\rho}{\partial J},$$
$$\rho(J,0) = \rho_0(J), \qquad \rho(J_s,t) = 0, \tag{1.2}$$

where $\rho(J,t)$ is the action density related in the obvious way to the phase space density in (ϕ, P), $\rho_0(J)$ can be computed from the initial (ϕ, P) density, and we take an absorbing boundary condition at the separatrix $J = J_s$. This assumes that once a particle crosses the separatrix, it is forever lost from the bunch. A code has been developed to solve Eq. (1.2) numerically, using the method of lines; results have been compared with simulation results for Eq. (1.1). This is discussed in detail in Reference 3. There is good agreement, and this gives us confidence in using Eq. (1.2) rather than Eq. (1.1) for SSC design criteria.

The diffusion coefficient is given by

$$D(J) := D_a(J) + D_\varphi(J), \tag{1.3}$$

where D_a and D_φ are the amplitude and phase noise diffusion coefficients, respectively, and are given by

$$D_a(J) = 4\sum_{m=2,4,\ldots}^{\infty} \frac{\big(m\omega_s(J)\big)^4}{\sinh^2 mv(J)}\, S_a\big(m\omega_s(J)\big), \tag{1.4a}$$

$$D_\varphi(J) = 4 \sum_{m=1,3,\ldots} \frac{\left(m\omega_s(J)\right)^4}{\cosh^2 mv(J)} S_\varphi\left(m\omega_s(J)\right), \tag{1.4b}$$

where $S_a(\omega)$ and $S_\varphi(\omega)$ are the amplitude and phase noise spectral densities,

$$J = 2\Omega\, k^2 \frac{4}{\pi} B(k), \tag{1.5a}$$

$$\omega_s(J) = \Omega \frac{\pi}{2K(k)}, \tag{1.5b}$$

$$v(J) = \frac{\pi}{2} \frac{K'(k)}{K(k)}. \tag{1.5c}$$

Here $0 \leq k \leq 1$, $k = 1$ corresponds to the separatrix $J_s = 8\Omega/\pi$, ω_s is the action-dependent synchrotron frequency, and $B(k)$ and $K(k)$ are elliptic integrals defined in Jahnke and Emde (see Reference 1, pp. 382–383).

2.0 EMITTANCE-DOUBLING TIME AND A SIMPLE APPROXIMATION

If we let f denote the fraction of particles under consideration, then we define the f-emittance, $J_f(t)$, by

$$\int_0^{J_f(t)} \rho(J,t)dJ = f, \qquad J_f(t) < J_s \tag{2.1}$$

and the emittance-doubling time, t_d, by

$$J_f(t_d) = 2J_f(0). \tag{2.2}$$

Here we assume that $J_f(t)$ is monotonically increasing with t. The emittance-doubling time can be found directly from Eqs. (2.1) and (2.2); however, an alternative, which is easily integrated into the method of lines code, is to solve the differential equation for $J_f(t)$:

$$\frac{dJ_f}{dt} = -D(J_f) \frac{\partial}{\partial J_f} \ln \rho(J_f, t), \tag{2.3a}$$

$$J_f(0) = J_0. \tag{2.3b}$$

Eq. (2.3a) is obtained by differentiating Eq. (2.1), and J_0 is determined from the initial density via Eq. (2.1). Eqs. (2.3) will not be considered further in this report.

In the next two sections we will discuss our calculations of $J_f(t)$ and t_d. Here we discuss a simple approximation to $J_f(t)$ that was used in the 1986 Conceptual Design Report (Reference 4, p. 162) to determine noise levels corresponding to a 50-h emittance-doubling time.

It is perhaps not unreasonable to presume that $J_f(t)$ evolves roughly like the mean of $J(t)$ conditioned on $J(t) < J_s$; that is,

$$\bar{J}(t) := E[J(t)|J(t) < J_s] = \int_0^{J_s} J\rho_c(J,t)dJ, \tag{2.4}$$

where $\rho_c(J,t) = \rho(J,t)/\int_0^{J_s}\rho(J,t)dJ$. Differentiating Eq. (2.4) and using Eq. (1.2) yields

$$\frac{d\bar{J}(t)}{dt} = \int_0^{J_s} D'(J)\rho_c(J,t)dJ + (J_s - \bar{J}(t))\, D(J_s)\,\frac{\partial \rho_c}{\partial J}(J_s,t). \tag{2.5}$$

In the white noise, small oscillation case ($U(\phi) = \frac{1}{2}\phi^2$),

$$D(J) = \frac{1}{2}\Omega^2\left(\Omega S_\varphi J + S_a \frac{1}{2} J^2\right), \tag{2.6}$$

which gives

$$\frac{d\bar{J}}{dt} = \frac{1}{2}\Omega^2(\Omega S_\varphi + S_a \bar{J}) + H(t, J_s), \tag{2.7}$$

where $H(t, J_s)$ is the second term on the right hand side of Eq. (2.5). Equation (2.6) can be derived in the small k asymptotics for Eqs. (1.4) and (1.5), or directly from Eq. (3.1), to be discussed shortly. In the small oscillation approximation, the action density, for a Gaussian beam in P and ϕ with the rms longitudinal bunch spread σ_l, is $\rho(J) = \frac{1}{\mu}e^{-J/\mu}$ and $\mu = \bar{J} = \Omega(2\pi\sigma_l/\lambda_{\rm rf})^2$. If we ignore H and define $X = \bar{J}/\Omega$, then Eq. (2.7) is exactly Eq. (4.4-21) of Reference 4:

$$\frac{dX}{dt} = \frac{1}{2}\Omega^2(S_\varphi + S_a X). \tag{2.8}$$

The doubling time of the mean is now easily calculated. For amplitude noise,

$$t_d = 2\ \ln 2/\Omega^2 S_a; \tag{2.9}$$

and for phase noise,

$$t_d = 2\bar{J}_0/\Omega^3 S_\varphi. \tag{2.10}$$

Using the values in Reference 4, p. 163 ($\lambda_{\rm rf} = 0.8\,\text{m}$, $\sigma_l = 0.07\,\text{m}$ and $\Omega = 43.98\,\text{rad}\,\text{sec}^{-1}$), $X_0 = 0.3023$ and $\bar{J}_0 = 13.30\,\text{rad}\,\text{sec}^{-1}$, and we find, for a 50-h doubling time, $S_\varphi = 1.74 \times 10^{-9}\,\text{rad}^2\text{Hz}^{-1}$ and $S_a = 3.98 \times 10^{-9}\,\text{Hz}^{-1}$. Note that the quoted CDR value of $S_a = 6 \times 10^{-9}$ is an error. For the present SSC parameters ($\lambda_{\rm rf} = 0.83\,\text{m}$, $\sigma_l = 0.051\,\text{m}$ and $\Omega = 26.6\,\text{rad}\,\text{sec}^{-1}$), $X_0 = 0.1479$ and $\bar{J}_0 = 3.937\,\text{rad}\,\text{sec}^{-1}$, and we obtain $S_\varphi = 2.32 \times 10^{-9}\,\text{rad}^2\text{Hz}^{-1}$ and $S_a = 10.9 \times 10^{-9}\,\text{Hz}^{-1}$, which are larger (and therefore better) than the previous values. Solving Eq. (2.7) with $H = 0$ and using the new values of S_φ and S_a, we obtain

$$t_d = \frac{2}{\Omega^2 S_a} \ln\left(\frac{\Omega S_\varphi + 2\bar{J}_0 S_a}{\Omega S_\varphi + \bar{J}_0 S_a}\right) \simeq 24.76\text{ h}. \tag{2.11}$$

This is less than 50 h, as it should be; however, because phase and amplitude noise enter Eq. (2.7) differently, it is surprising and probably coincidental that $t_d \simeq 50/2\,\text{h}$.

3.0 WHITE AMPLITUDE AND WHITE PHASE NOISE CASES

In the case of white amplitude and white phase noise the diffusion coefficients can be written:

$$D_a(J) = \frac{1}{2} S_a \frac{4\sqrt{2}\Omega^5}{2\pi\omega_s(J)} \int_0^{a(J)} U'(\phi)^2 \sqrt{U(a(J)) - U(\phi)}\, d\phi, \tag{3.1a}$$

$$D_\varphi(J) = \frac{1}{2} S_\varphi \frac{4\sqrt{2}\Omega^5}{2\pi\omega_s(J)} \int_0^{a(J)} \left(U''(\phi)\right)^2 \sqrt{U(a(J)) - U(\phi)}\, d\phi, \tag{3.1b}$$

where $a(J)$ is the amplitude of the synchrotron oscillation. Actually, these are valid as long as the correlation time is short relative to the synchrotron period.[5] In the small oscillation approximation, $\left(U(\phi) = \frac{1}{2}\phi^2\right)$, Eq. (3.1) reduces to

$$D_a(J) = \frac{1}{4} S_a \Omega^2 J^2 \tag{3.2a}$$

$$D_\varphi(J) = \frac{1}{2} S_\varphi \Omega^3 J, \tag{3.2b}$$

which is consistent with Eq. (2.6). The diffusion coefficients of Eqs. (3.1) and (3.2) are shown in Figures 1 and 2 for the values of S_a and S_φ of Section 2.0. Notice that the small oscillation approximation gives good agreement out to $J \simeq 0.3 J_s$.

The amplitude noise calculations for the evolution of ρ and emittance growth are shown in Figures 3 and 4. In Figure 3 notice that $\rho(0,t)$ is fixed, which causes the narrowing of

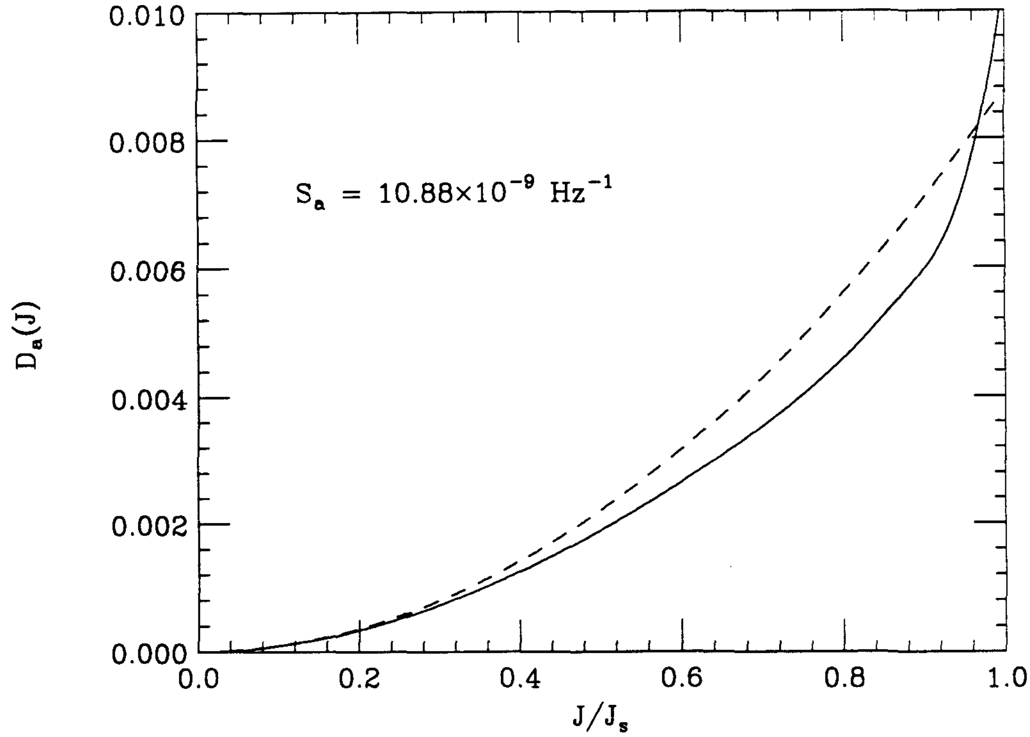

Figure 1. Diffusion Coefficient for White Amplitude Noise with $S_a = 10.88 \times 10^{-9}\,\mathrm{Hz}^{-1}$. Solid: DKW diffusion theory; dashed: small oscillation approximation.

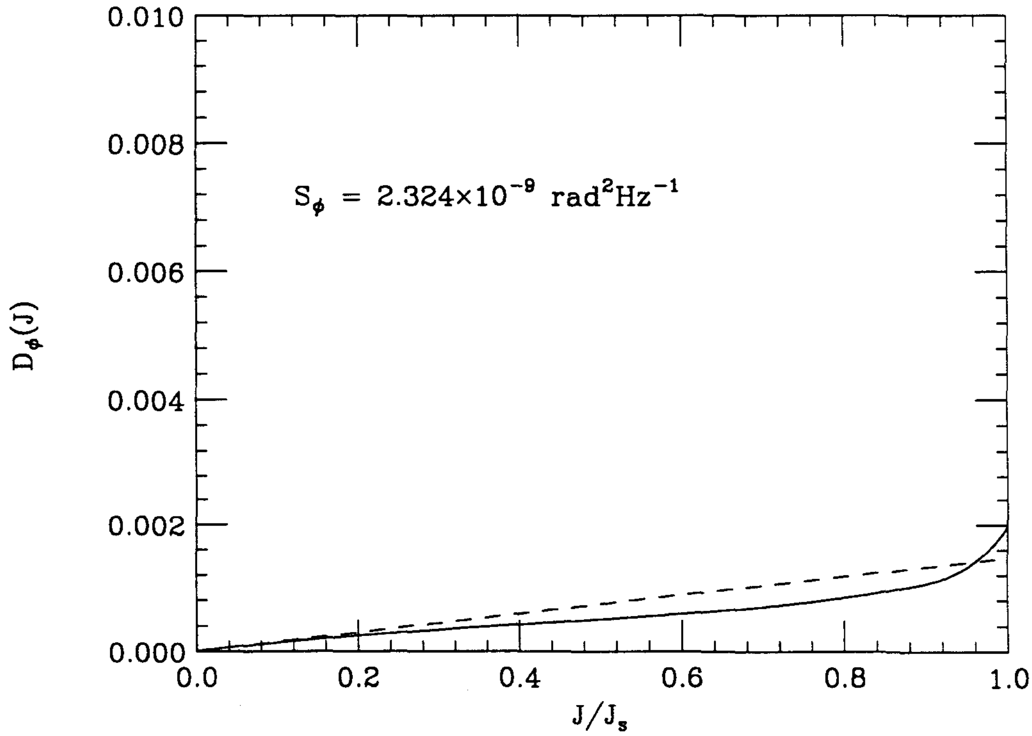

Figure 2. Diffusion Coefficient for White Phase Noise with $S_\varphi = 2.324 \times 10^{-9}\,\mathrm{rad}^2\mathrm{Hz}^{-1}$. Solid: DKW diffusion theory; dashed: small oscillation approximation.

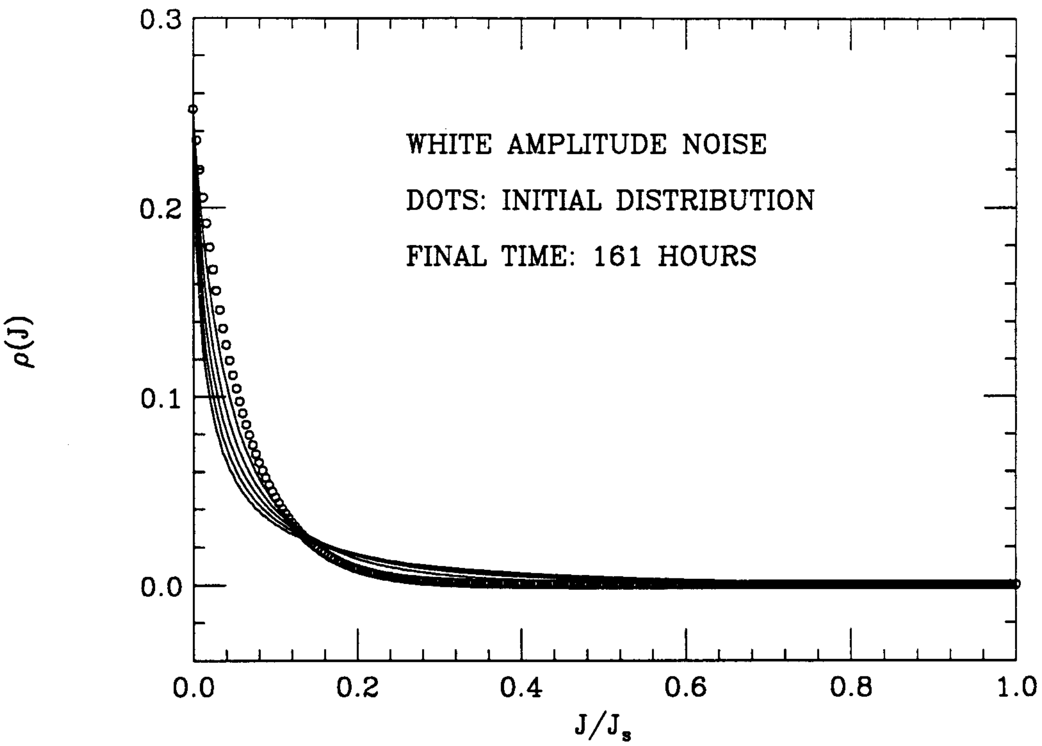

Figure 3. Evolution of Action Density for Case of Figure 1 in Increments of 161/5 h. Initial density is given by circles; final density is at 161 h.

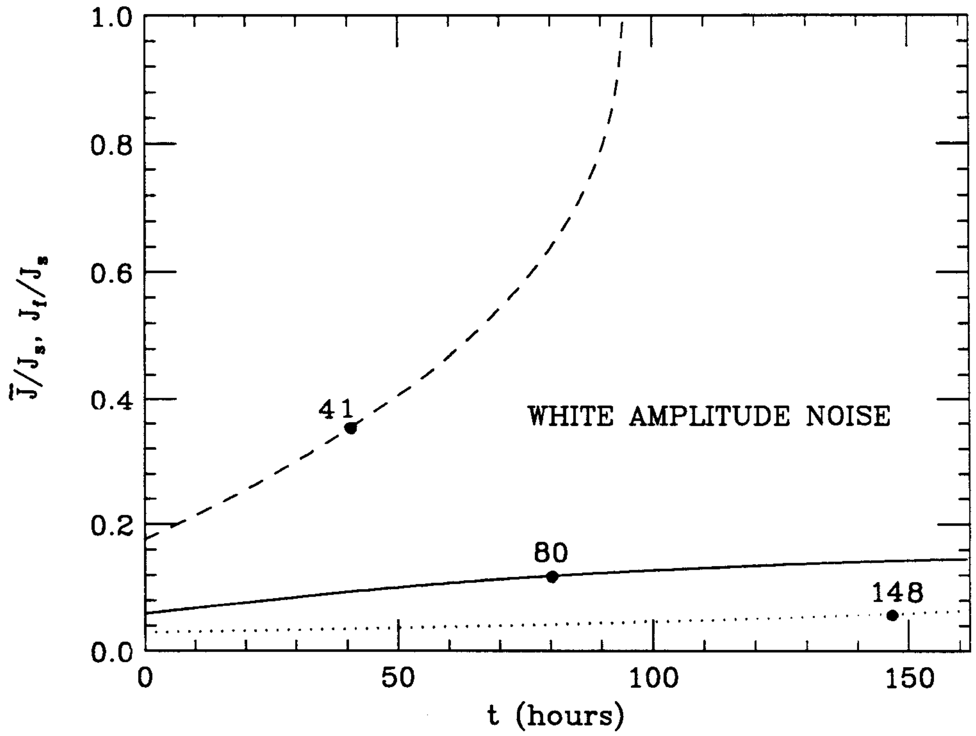

Figure 4. Emittance Growth for Case of Figure 1. Solid: mean emittance; dashed: 95% emittance; dotted: 39% emittance. The doubling times are indicated by full circles.

the density near $J = 0$. That this must be so follows from the PDE (1.2) and the fact that $D \propto J^2$ for small J. Figure 4 gives a doubling time of 80 h for the mean emittance $\bar{J}$, compared to the 50 h given by the approximate model of Section 2.0. The fact that $\bar{J}(t)$, defined by Eq. (2.7) with $S_\varphi = 0$, is not growing exponentially is due to the fact that H cannot be ignored in Eq. (2.7) because $\frac{\partial \rho}{\partial J}(J_s, t)$ is becoming significant. Notice also that $J_f(t)$ for $f = 0.39$ is growing linearly, contrary to the exponential behavior predicted by Eq. (2.8) with $S_\varphi = 0$. The blow-up for $f = 0.95$ at $\sim$ 95 h is due to the loss of 5% of the particles.

The phase noise calculations for the evolution of ρ and emittance growth are shown in Figures 5 and 6. In Figure 6 we see that J_f and $\bar{J}$ are roughly linear and are in reasonable agreement with the approximate model of Section 2.0, although the mean emittance-doubling time of 59 h is somewhat larger.

Figures 3–6 are universal in that the times scale directly with the corresponding spectral densities.

4.0 EMITTANCE-DOUBLING TIME FOR CURRENT SSC DESIGN

4.1 Synthesizer Noise

The phase noise for a synthesizer of a type being considered for use in the SSC rf system is shown in Figure 7. Notice that the carrier frequency in Figure 7 is 420 MHz; however, we expect the spectrum to remain essentially the same at 360 MHz. Nevertheless, a measurement at the operating frequency is desirable. A reasonable fit to the spectrum is given by

$$S_\varphi(\omega) = \begin{cases} 1.3 \times 10^{-5}/\omega^{2.65} & \omega < 628.3 \\ 0.5 \times 10^{-12} & \omega \geq 628.3. \end{cases} \tag{4.1}$$

The diffusion coefficient is shown in Figure 8; surprisingly, it is nearly linear, as the straight dashed line indicates. Figure 9 shows $\rho(J,t)$ vs. J for various t, and Figure 10 shows $\bar{J}(t)$ and $J_f(t)$ for two values of f. The doubling times of the mean, 39% and 95% emittances, are $\sim$ 55 h, which is on the order of the CDR design criterion. This may be satisfactory; however, depending on the level of design conservatism, a feedback loop might be desirable. In Section 6.0, feedback is considered.

The amplitude spectral density has a shape similar to Eq. (4.1) but is of lower magnitude. For a worst-case estimate we take $S_a = S_\varphi$. Figure 11 shows the diffusion coefficient, Figure 12 the evolution of ρ, and Figure 13 the emittance growth curves. The mean emittance-doubling time is greater than 3230 h, so amplitude noise should be negligible.

The effect of using a superconducting rf cavity is under consideration.

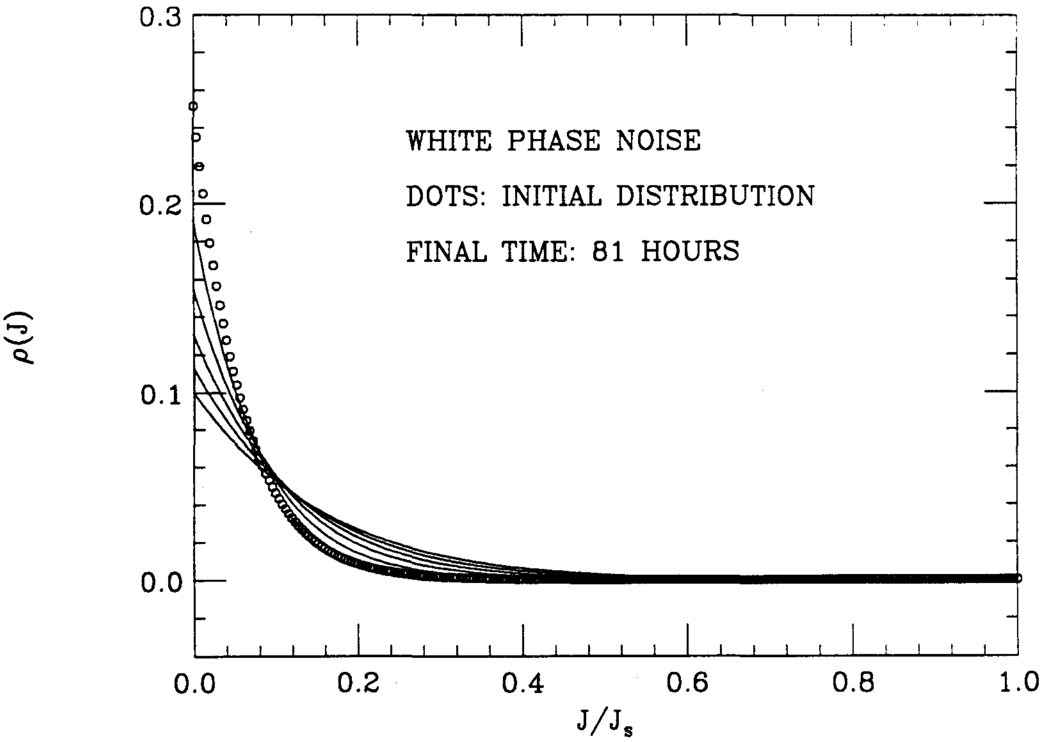

Figure 5. Evolution of Action Density for Case of Figure 2 in Increments of 81/5 h. Initial density is given by circles; final density is at 81 h.

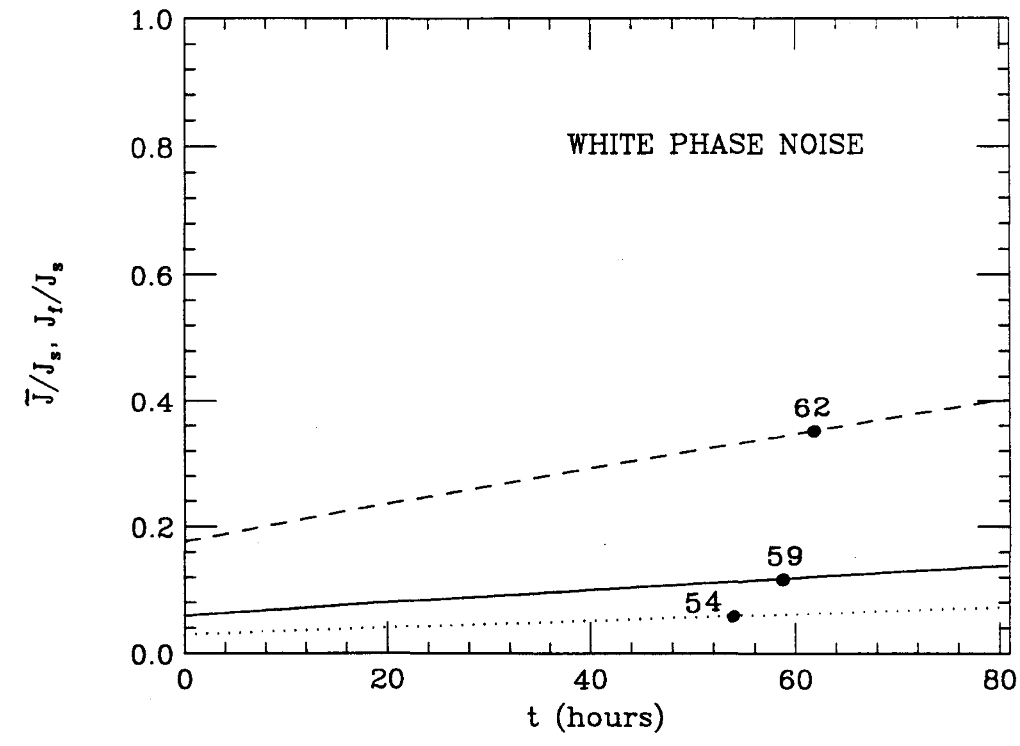

Figure 6. Emittance Growth for Case of Figure 2. Solid: mean emittance; dashed: 95% emittance; dotted: 39% emittance. The doubling times are indicated by full circles.

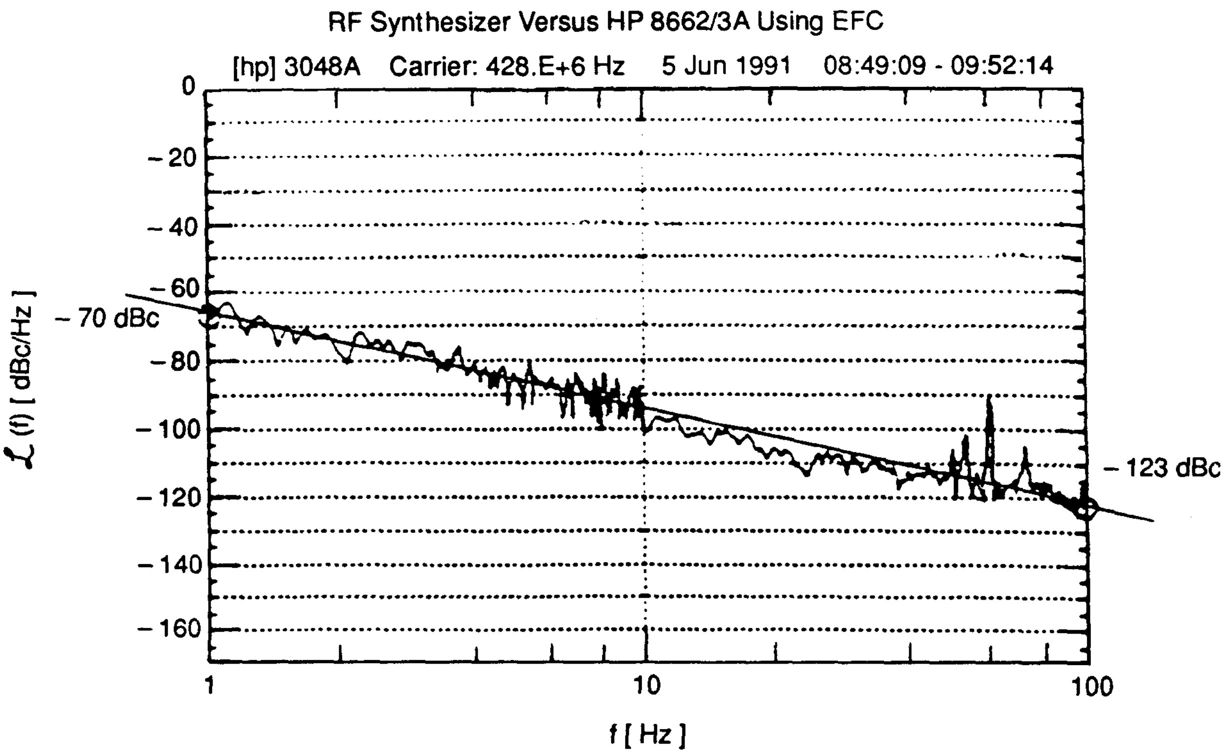

Figure 7. Noise Spectral Density Measured on HP8662 Synthesizer. The straight line is the fit (Eq. (4.1)).

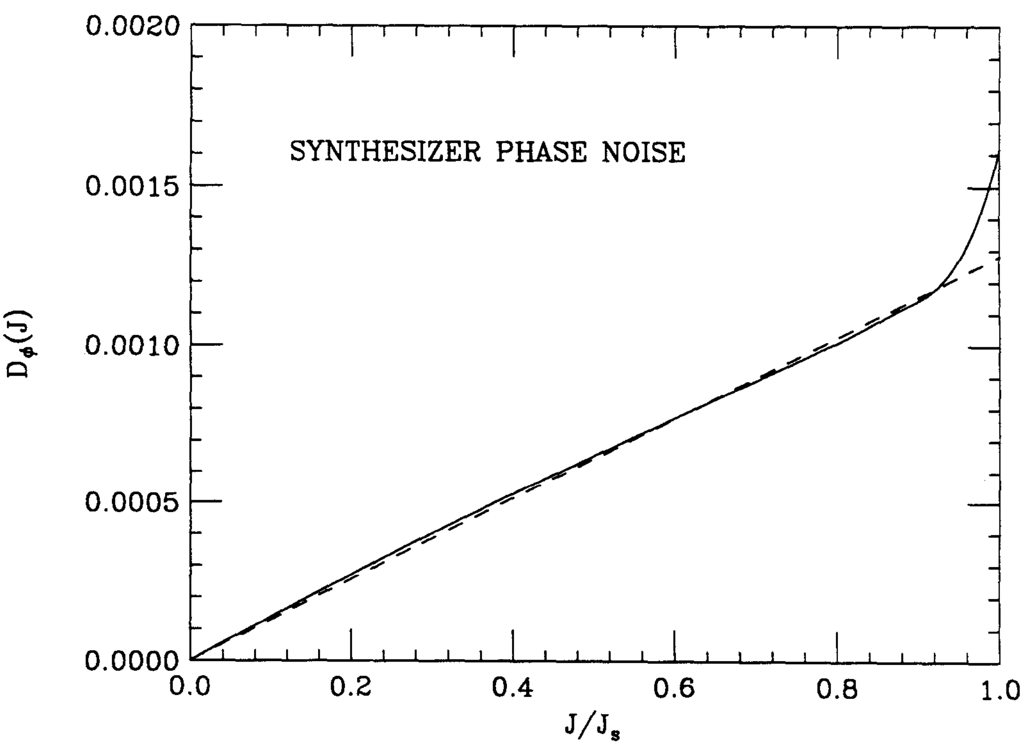

Figure 8. Diffusion Coefficient for Synthesizer Phase Noise with the Spectrum (Eq. (4.1)). Solid: DKW diffusion theory; dashed: linear approximation.

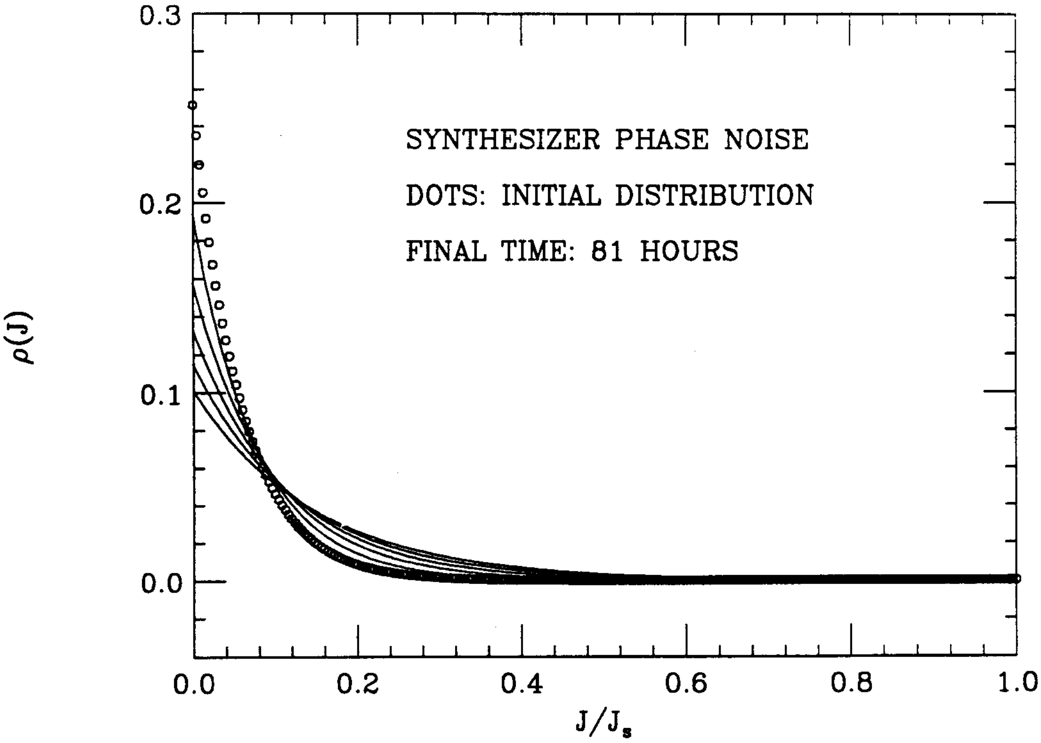

Figure 9. Evolution of Action Density for Case of Figure 8 in Increments of 81/5 h. Initial density is given by circles; final density is at 81 h.

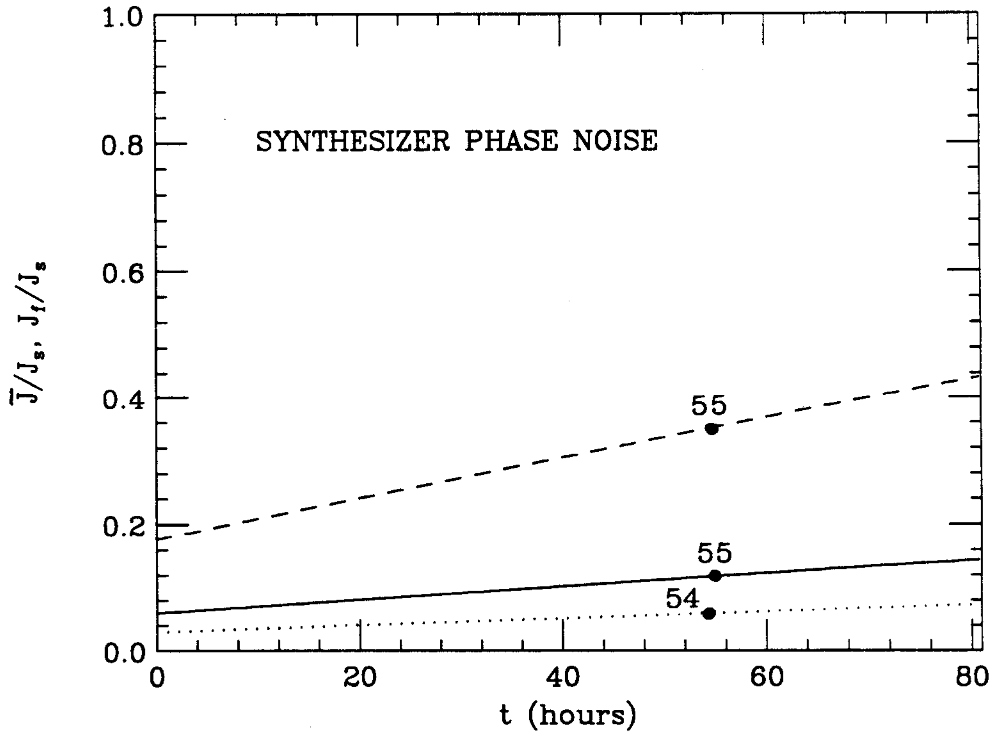

Figure 10. Emittance Growth for Case of Figure 8. Solid: mean emittance; dashed: 95% emittance; dotted: 39% emittance. The doubling times are indicated by full circles.

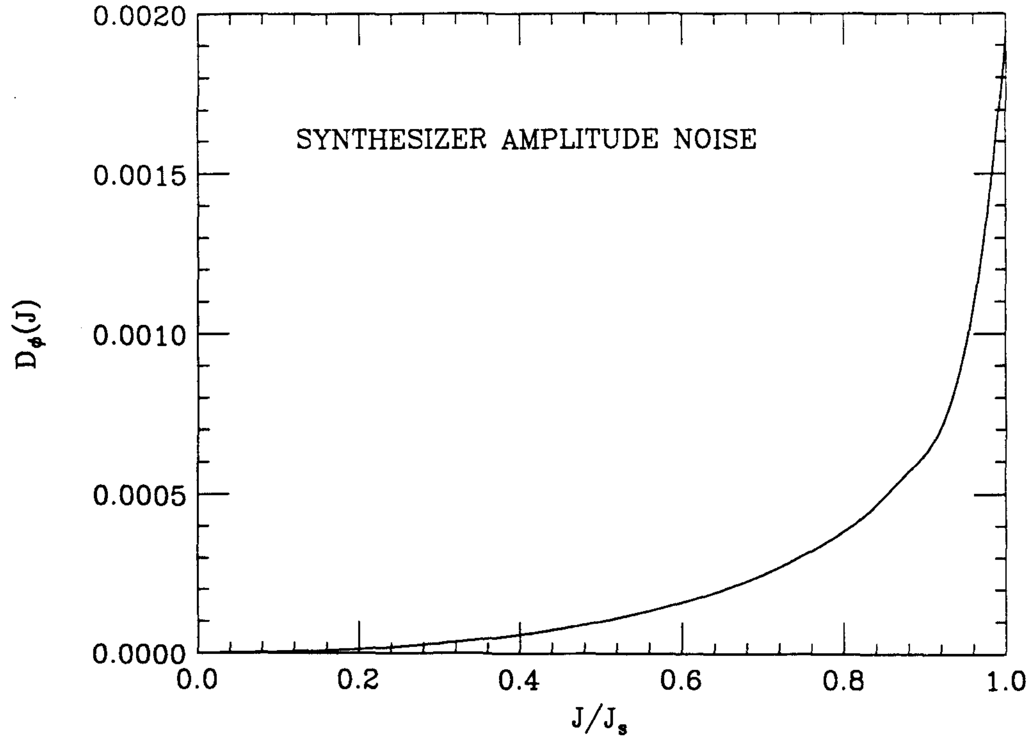

Figure 11. Diffusion Coefficient for Synthesizer Amplitude Noise with the Spectrum (Eq. (4.1)).

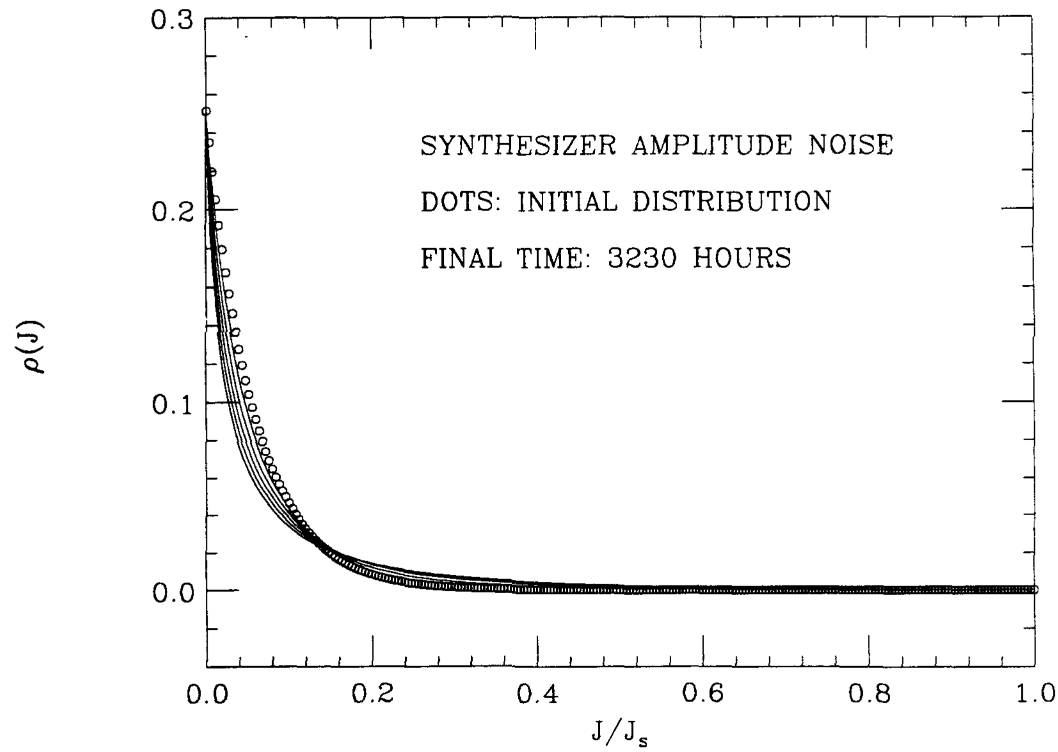

Figure 12. Evolution of Action Density for Case of Figure 11 in Increments of 3230/5 h. Initial density is given by circles; final density is at 3230 h.

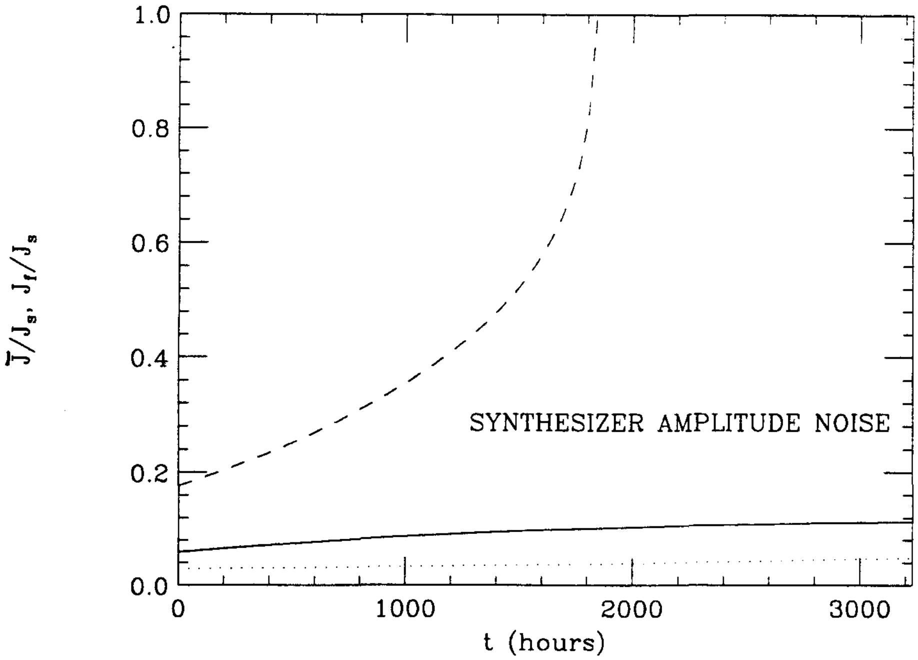

Figure 13. Emittance Growth for Case of Figure 11. Solid: mean emittance; dashed: 95% emittance; dotted: 39% emittance.

4.2 Klystron Noise

The klystron amplifiers can be another source of noise in the collider rf system. We can use the results of recent measurements made on the PEP klystrons at SLAC and the model described earlier to estimate the emittance growth due to both phase and amplitude noise from those klystrons in that system as applied to the SSC. A typical spectrum is shown in Figure 14. Because the critical components of the noise spectral density are those that occur at the first few harmonics of the synchrotron frequency (odd for phase noise, even for amplitude noise), a reasonable approximation is to take a flat spectrum for $0 < f < 25\,\text{Hz}$. In this region, we use a value of $-75\,\text{dBc}$ for phase noise and $-85\,\text{dBc}$ for amplitude noise in amplifiers operating unsaturated. These correspond to spectral densities of

$$S_\varphi = 3.2 \times 10^{-8}\,\text{rad}^2\text{Hz}^{-1},$$
$$S_a = 3.2 \times 10^{-9}\,\text{Hz}^{-1}.$$

The emittance-doubling times can be estimated by using the scaling arguments established earlier. From these, t_d = 4 h for phase noise and t_d = 272 h for amplitude noi. would be inferred. The first is quite severe. However, some care must be exercised in interpreting these estimates. The most important point to recognize is that the measurements also ider.:ified the source of the klystron noise as fluctuations in the cathode supply voltage. Several strong lines appear in the spectrum of the cathode voltage in the critical frequency band. These lines must be dependent on the operating environment, so a too-literal reading of the emittance-doubling times is not justified. A feedback loop around the amplifier should suffice to reduce the klystron noise to acceptable levels. It is prudent to contemplate such a loop in the design until actual measurements on an SSC prototype system are available.

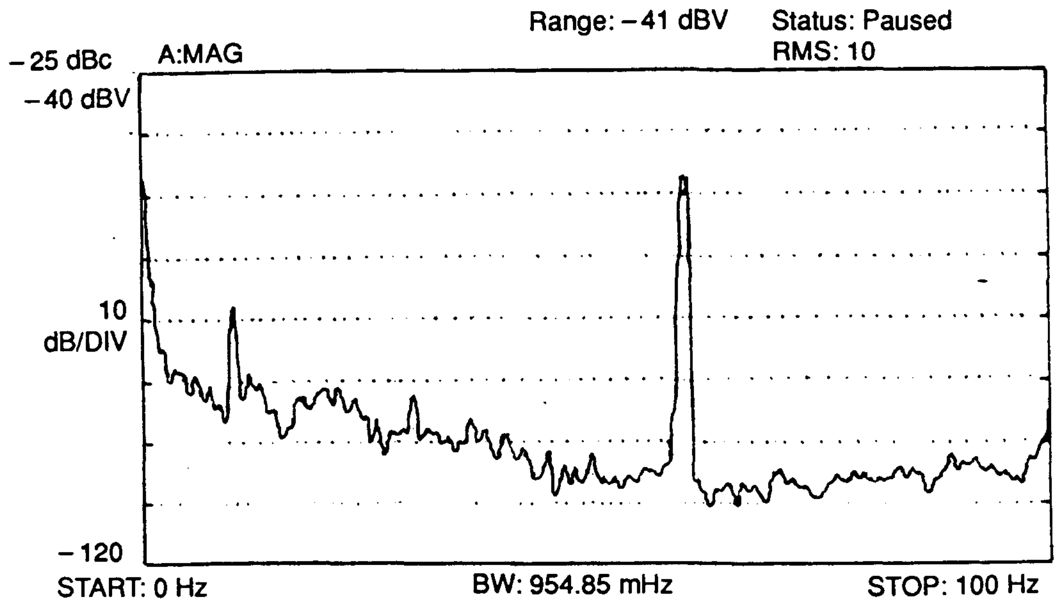

Figure 14. Typical Noise Spectrum Measured on a SLAC PEP Klystron.

5.0 A SCALING LAW FOR WHITE AMPLITUDE AND PHASE NOISE

In this section we discuss a scaling law that will allow us to qualitatively understand the dependence on $\omega_{\rm rf}$. At the time of this report the SSC rf frequency is not fixed. If we let $P = \Omega\tilde{P}$, then the $(\tilde{P}, \phi)$ phase space is independent of Ω (and $\omega_{\rm rf}$), and $2\pi\, J = \oint P d\phi = \Omega \oint \tilde{P} d\phi =: \Omega\, 2\pi \tilde{J}$, where the last equality defines $\tilde{J}$. The $\tilde{J}$ density $p(\tilde{J}, t) = \Omega\, \rho(J, t)$, and Eq. (1.2) becomes

$$\frac{\partial p}{\partial t} = \frac{\partial}{\partial \tilde{J}} \frac{D(\Omega\tilde{J}, \Omega)}{\Omega^2} \frac{\partial p}{\partial \tilde{J}}, \tag{5.1}$$

where $D(J) = D(J, \Omega)$. From Eq. (1.5a), the relation between $\tilde{J}$ and k is independent of Ω, so v depends only on $\tilde{J}$, and for white amplitude and phase noise,

$$D(\Omega\tilde{J}, \Omega) = \Omega^4 \mathcal{D}(\tilde{J}),$$

where $\mathcal{D}(\tilde{J})$ is independent of Ω. Letting $\tau = \Omega^2 t$, we obtain the initial-boundary value problem for $p(\tilde{J}, t)$ as

$$\frac{\partial p}{\partial \tau} = \frac{\partial}{\partial \tilde{J}} \mathcal{D}(\tilde{J}) \frac{\partial p}{\partial \tilde{J}},$$

$$p(\tilde{J}_s, \tau) = 0, \qquad p(\tilde{J}, 0) = \Omega \rho_0(\Omega \tilde{J}). \tag{5.2}$$

The Ω dependence now enters only in two ways: (1) through the initial density and (2) through the time scaling. For a Gaussian beam in P and ϕ, $\rho_0(J) = \frac{1}{\mu(\Omega)} e^{-J/\mu(\Omega)}$ at small J, where $\mu(\Omega) = \bar{J}_0(\Omega) = \Omega^4 \nu$, and ν is an Ω-independent constant if longitudinal emittance $\epsilon_L = \sigma_E \sigma_t$ is fixed. Thus, recalling that $\Omega^2 \propto \omega_{\rm rf}$, we see that increasing $\omega_{\rm rf}$ broadens the initial distribution. If we assume that a broader beam leads to a faster deterioration of the beam, then

$$\tau_{c2} < \tau_{c1}, \tag{5.3}$$

where τ_c is a critical time for loss of beam quality. Therefore, $\Omega_2^2 t_{c2} < \Omega_1^2 t_{c1}$ and

$$t_{c2} < \frac{\omega_{\rm rf1}}{\omega_{\rm rf2}} t_{c1}. \tag{5.4}$$

If $f_{\rm rf1} = 360\,\text{MHz}$ and $f_{\rm rf2} = 480\,\text{MHz}$, then we expect $t_{c2} < \frac{3}{4} t_{c1}$ because we have a broader initial beam in case 2. However, for phase noise of $S_\varphi = 2.32 \times 10^{-9}\,\text{rad}^2\text{Hz}^{-1}$ and an initial emittance of $\epsilon_L = 0.233\,\text{eV-sec}$, our calculations show that the mean emittance-doubling time in case 1 is 86 h and that it increases to 142 h in case 2, in contrast to the expectation in Eq. (5.4). To understand this we note that the narrower beam has steeper gradients; thus the diffusion process works faster, giving a shorter doubling time even though the resulting beam is still relatively narrow and could be narrower than the initial beam for the larger $\omega_{\rm rf}$. This points out that emittance-doubling times may not be an appropriate design criterion. A more appropriate criterion may be the time it takes for the beam to reach a certain critical size relative to the bucket area. We are presently studying this.

6.0 FEEDBACK LOOPS

Single-bunch phase feedback loops (sometimes called longitudinal dampers) have been employed at the Tevatron and at SpS for the control of coherent oscillation of the bunch centroid (dipole oscillation), of incoherent emittance blow-up, or of both. A similar scheme was anticipated for the SSC collider rings, and an estimate of its effect on the emittance increase due to noise in the rf system was made.[4] At the SSC, the loop cannot act bunch-by-bunch. It seems feasible[6] to update the phase correction approximately every 100 bunches. But since it is noise at the synchrotron frequency that is important, the noise is not changing on this scale, and the loop operation should be like a single bunch system insofar as its effect on emittance is concerned. This begs the question of high-frequency noise at harmonics of the revolution frequency. (These do not seem to be significant for the synthesizer noise, but there will undoubtedly be other sources of noise as yet unconsidered, including those in the loop components themselves.) Only those harmonics that lie inside the cavity bandwidth are important. For a superconducting cavity, there are probably none. A normal cavity may have up to 14 harmonics inside its bandwidth. If the phase correction can indeed be updated as expected, then the loop should behave as in the single bunch case for these as well.

It is easy to write down a steady-state description of a generic loop and show that its effect in terms of the theory of the diffusion due to noise is to modify the spectral density of the noise in the cavity relative to the source. In particular, the spectral density in the neighborhood of the synchrotron frequency can easily be seen to be reduced significantly because of the peak in the bunched beam dispersion function. Modifying the numerical diffusion calculations to include the effect of a single-bunch phase feedback is straightforward, although some prescription for computing the beam dispersion function must be implemented. Once this is done, we will attempt to benchmark the computations against the old ISR data,[7] which seems to be one of the more complete sets of data available. As other sources of noise beyond the synthesizer—e.g., magnet supply noise and loop noise—are defined, their inclusion in the theory should also be straightforward.

7.0 SUMMARY AND DISCUSSION

In this report, we have examined the effect of phase and amplitude noise in the rf system on the growth of longitudinal emittance in the collider ring. The description we have adopted—due to Dôme and, independently, Krinsky and Wang—has become conventional. Noise in the rf system was identified as a concern in the SSC Conceptual Design Report,[4] particularly because of the low synchrotron frequency at the SSC. A simple *ad hoc* model equation, which we discussed in Section 2.0, was used there to obtain an estimate of noise spectral density level that would be required to attain a 50-h doubling time of the phase area occupied by a "typical" bounding trajectory.

The results reported here are aimed at going beyond these estimates using numerical solutions of the DKW diffusion equation. However, we have shown how an equation of the form of the CDR model can be obtained. While we believe the numerical results are trustworthy, it is prudent to observe several caveats. Not the least is to note that the derivation of the equations involves one or more heuristic arguments that relate to the noise correlation time and the action dependent synchrotron period. (Some of our work in progress seeks to systematize these by applying the asymptotic theory of stochastic differential equations.[5]) While the qualitative features of the model have been found to agree with the observed behavior on several accelerators, there appears to be a paucity of quantitative data with which to make a comparison. One attempt was made using very old Bevatron data,[8] but we were not particularly successful; the span of time makes it unlikely that the reason for the discrepancy can be determined. It may be possible to obtain the data necessary to benchmark the theory in the context of a proposed test at the Tevatron (P-853) of the concept of beam halo extraction using a bent crystal. In this test, rf noise will be used to control the beam halo formation rate, and the crystal essentially serves as an instrumented collimator.

Because the low-level signal generators inevitably have a band of noise about the carrier, attention has focused on them. Although the noise spectrum diminishes rapidly away from the carrier, the low synchrotron frequency at the SSC has made it a source of concern. We have computed emittance-doubling times for synthesizer noise using a fit to the noise spectrum of an HP8662 synthesizer that is being considered for the SSC rf system. For the phase noise, these times are on the order of 50 h (the CDR design criterion), and a feedback loop might be desirable. Based on a plausible estimate of the spectral density, our results indicate that synthesizer amplitude noise is negligible. We have also discussed the implications for noise in the klystrons, taking the SLAC PEP testbed as typical. The phase noise seems quite large, but it appears to result less from processes intrinsic to the device than from the power supplies driving it. However, the noise can be controlled by a feedback loop around the klystron. Other "environmental" sources of noise may also cause concern, again largely due to the low synchrotron frequency in the collider rings. The issue of ground motion and other mechanical vibrations coupling into the rf system due to microphonics in the cavity, klystrons, or other mechanical components in the rf system has not been addressed here. Furthermore, anything that might introduce noise in a neighborhood of a harmonic of the revolution frequency would be as detrimental as introducing noise at Ω. Electronic circuits that might interact with the beam should be quiet at these frequencies. The totality of noise arising from each of the different physical sources is an incoherent superposition, and the net diffusion coefficient is the sum of the diffusion coefficient for each of the processes.

We have also considered the effect of boosting the collider rf frequency to 480 MHz. There are two effects worth noting. For a fixed injection emittance, a bunch fills a larger fraction of the bucket at the higher frequency; this might be expected to increase the rate of beam deterioration, although it is probably a weak effect. More important, it is a straightforward consequence of the DKW model that in the case of white noise the diffusion coefficient scales linearly with the rf frequency, again giving an increased rate of beam deterioration. (For more complicated spectral shapes, no rigorous scaling like this obtains. For the synthesizer we have been considering, the rapid decrease of the noise spectral density dominates and the rate of beam deterioration diminishes slightly at the higher rf frequency.) However, contrary to the above, we find that for the white noise case, the emittance-doubling time increases. This raises the question of whether the doubling time is really the appropriate figure of merit, as discussed at the end of Section 5.0. Further study of this is needed.

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